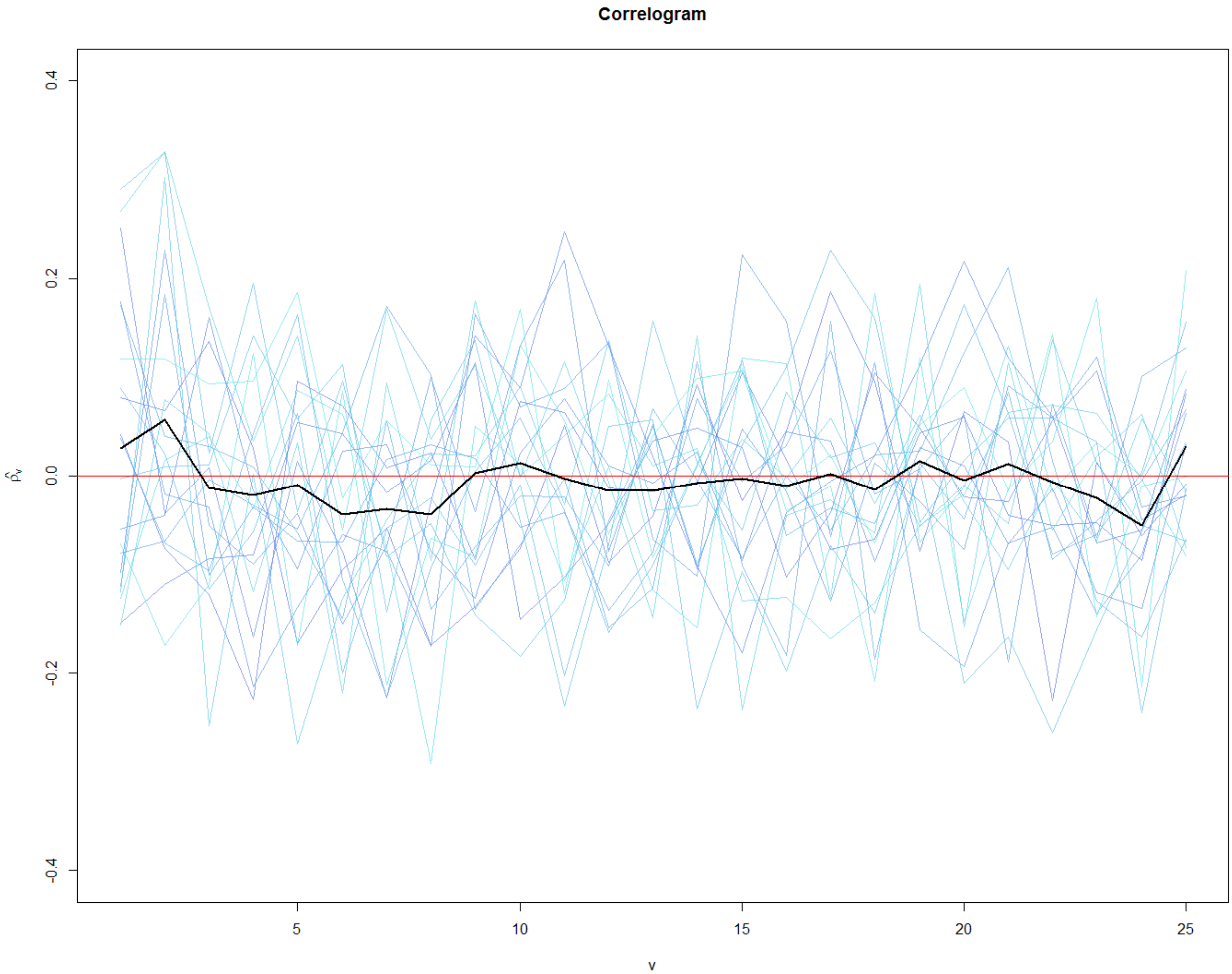
C4.2



Red line is the reference line at 0; black line – autocorrelation of the mean values of the realizations.

```
gen_wn_corr <- function(n, nreps=20, m=25, sig2=1){
      data = vector()
      for (i in 1:nreps){ # Creating the Ensemble
            data = rbind(data, corr(rnorm(n,0,sig2), m)$corr)}
      max = max(data); max = 0.4
      min = min(data); min = -0.4
      R = 0.3; G = 0.4; B = 0.9;
      color = rgb(R, G, B, 0.6)
      plot(data[1,1:m], ylim=c(min, max), type="l", xlab = "v", col=color,
            ylab = expression(hat(rho[v])), main = "Correlogram")
      for (i in 2:nreps){
            greens = seq(0, 0.5, length.out=nreps)
            color = rgb(R, G + greens[i], B, 0.6)
            lines(data[i,1:m], type="l", col=color) }
      abline(h=0.0, col = 'red', lwd=1.5)
      means = colMeans(data)
      lines(means, type="l", lwd=2)
      return(means)
}
cors.mean = gen_wn_corr(75, nreps=20, m=25, sig2=1)
mean(cors.mean) # 0.001
```

a) I expect to observe random behavior for the collection of sample correlograms of different realizations of Gaussian white noise process. I expect correlograms that do not show any particular pattern – look random, like the white noise itself. I also expect the correlograms of the realizations to be centered around zero because we showed that autocovariance function for a white noise process is: $R_v = \sigma_v^2 \delta_v$ and autocorrelation is $\rho_v = \frac{R_v}{R_0} = 0$

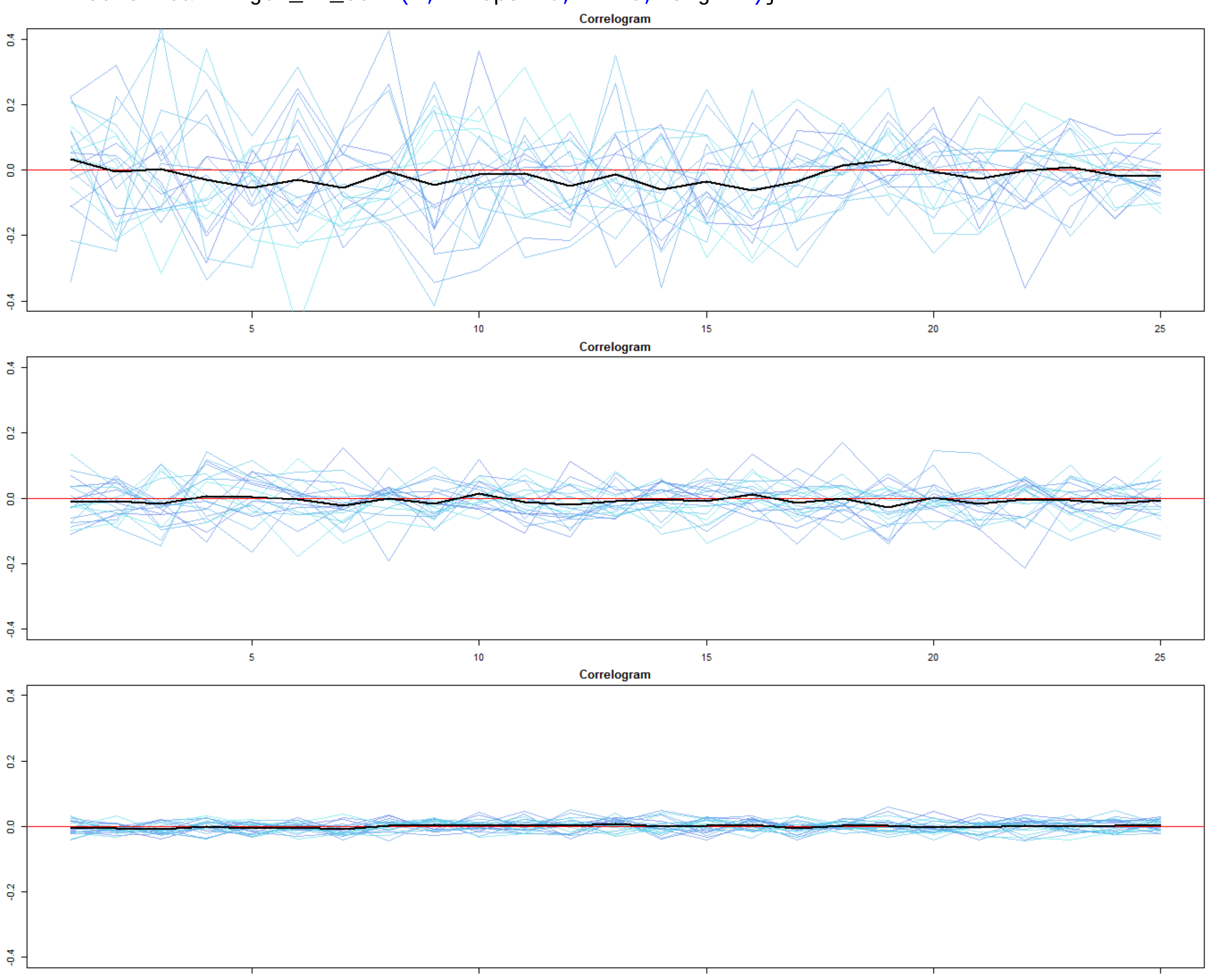
b) In the graph of all sample correlograms I observe randomness. Each of the correlograms look like white noise itself. They all seem to be centered around zero indeed. The mean of all realization is also seem to be centered around zero. My observations agree with my expectations. (Mean of correlogram means is equal zero, too).

c) By keeping the number of lags **m** and the number of realizations **nreps** constant (25 and 20, respectively) while **increasing** the length of each of the realizations **n**, we observe $\hat{\rho}_v$ to come **closer** to the true $\rho_v = 0$; the variability between $\hat{\rho}_v$ decreases (see plot below), seemingly approaching zero.

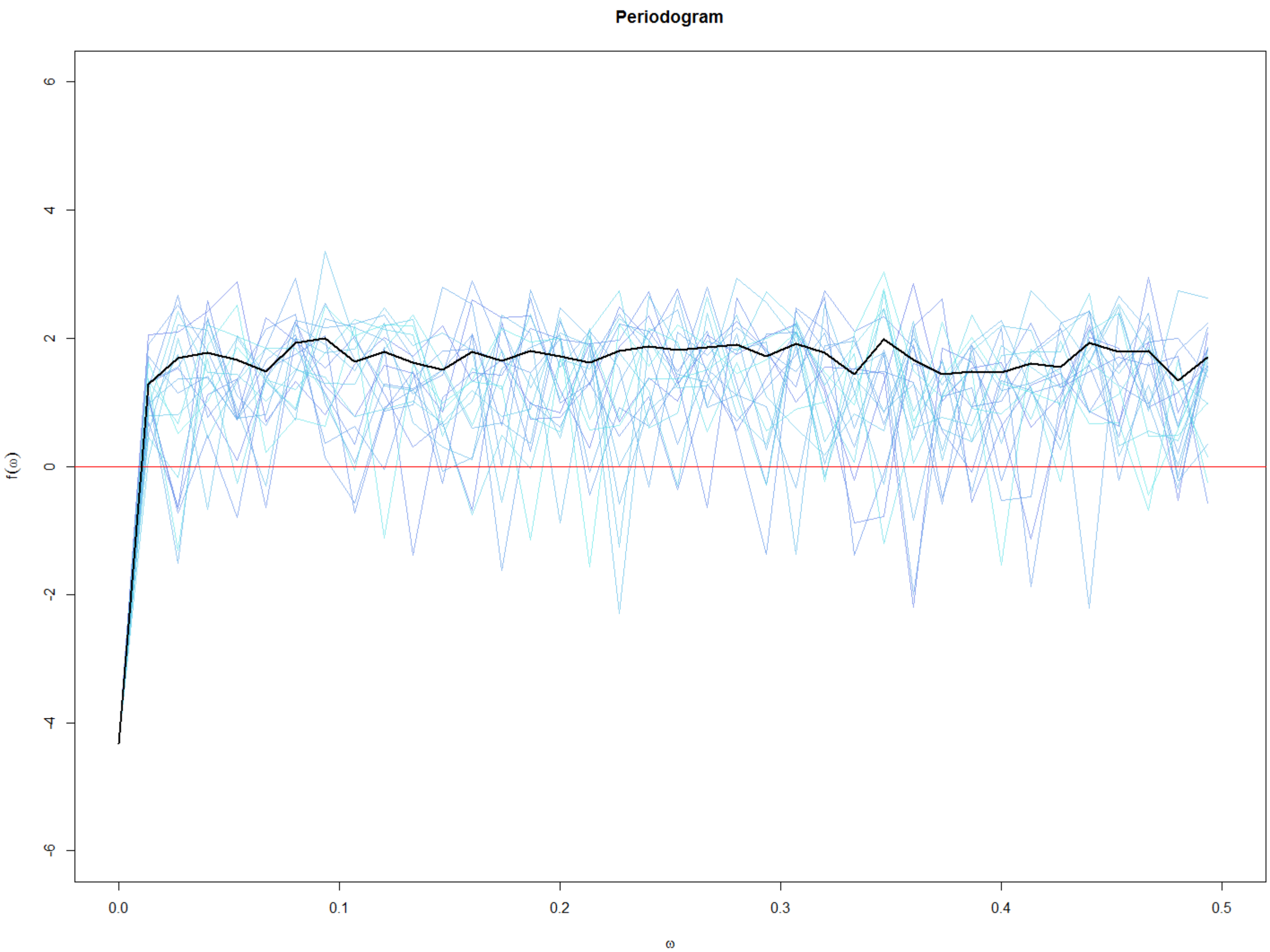
d) The estimator is consistent estimator if with the increase (to positive infinity) of data points used for estimation, the sequence of estimates converges to the parameter. $\hat{\rho}_v$ appears to be a consistent estimator of ρ_v as we observe it come closer to ρ_v at any value of v.

To demonstrate the above answers, consider fixing **xmax** and **xmin** to **0.4** and **0.4** respectively and vary **n to be 30, 250 and 2500**.

```
par(mfrow=c(3,1),mar = c(2,2,2,2))
for (n in c(30, 250, 2500)) {
      cors.mean = gen_wn_corr(n, nreps=20, m=25, sig2=1)}
```



C4.3



Red line is the logarithm of normalized variance $log\left(\frac{\sigma^2}{\sigma^2}\right) = log(1) = 0$; black line – periodogram of the mean values of the realizations.

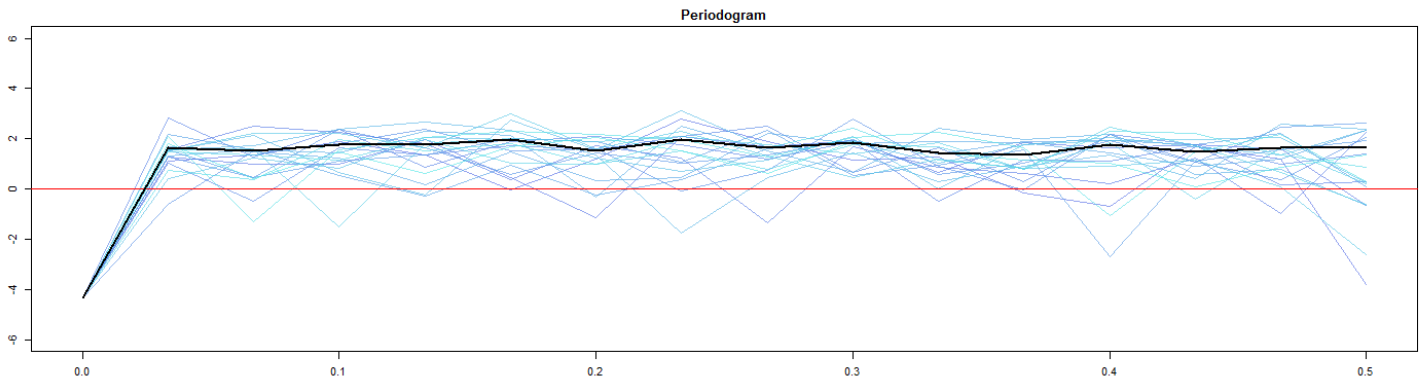
```
gen_wn_perdgm <- function(n, nreps=20, sig2=1) {
	lognorm = function(x){log(stdf(x, sig2, exp(-6), exp(6)))}
	data = vector()
	for (i in 1:nreps){ # Creating the Ensemble
		data = rbind(data, perdgm(rnorm(n,0,sig2))) }
	R = 0.3; G = 0.4; B = 0.9;
	color = rgb(R, G, B, 0.6)
	m = (n/2)+1
	plot(freqs(n), lognorm(data[1,1:m]), type="l", col=color,
		xlim = c(0, 0.5), ylim = c(-6, 6), main = "Periodogram",
		xlab = expression(omega), ylab = expression(hat(f)*(omega)))
	for (i in 2:nreps){
		greens = seq(0, 0.5, length.out=nreps)
		color = rgb(R, G + greens[i], B, 0.6)
		lines(freqs(n), lognorm(data[i,1:m]), type="l", col=color) }
	abline(h=lognorm(sig2), col = 'red', lwd=1.5)
	means = colMeans(data)
	lines(freqs(n), lognorm(means), type="l", lwd=2)
	return(means)
}
perd.mean = gen_wn_perdgm(n=75, nreps=20, sig2=1)
mean(perd.mean) # 1.85
```

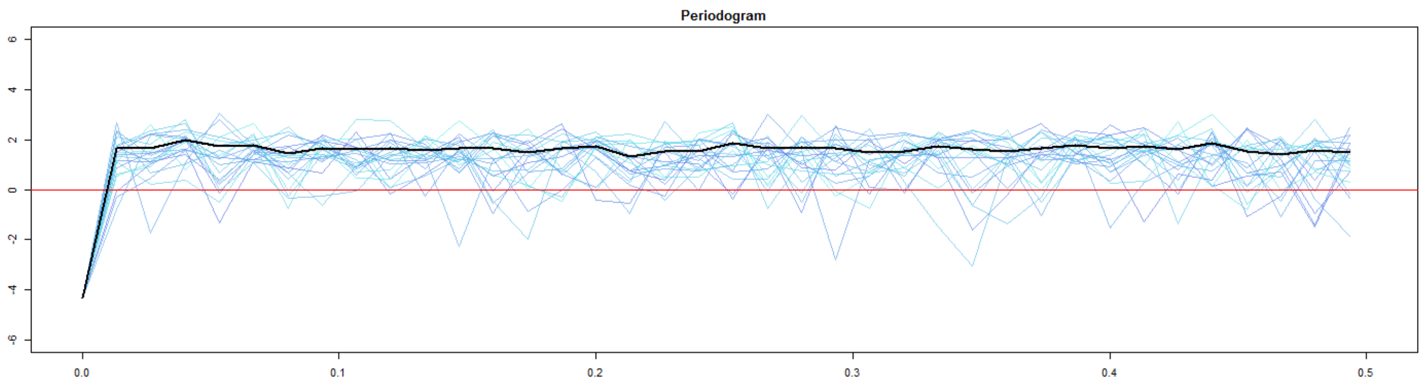
a) I expect to observe random behavior for the collection of periodograms of different realizations of Gaussian white noise process. I expect periodograms that do not show any particular pattern – no prevalence of any frequency. I expect sample periodograms not to be centered around zero since we have learned in chapter 2 that periodogram is biased estimator.

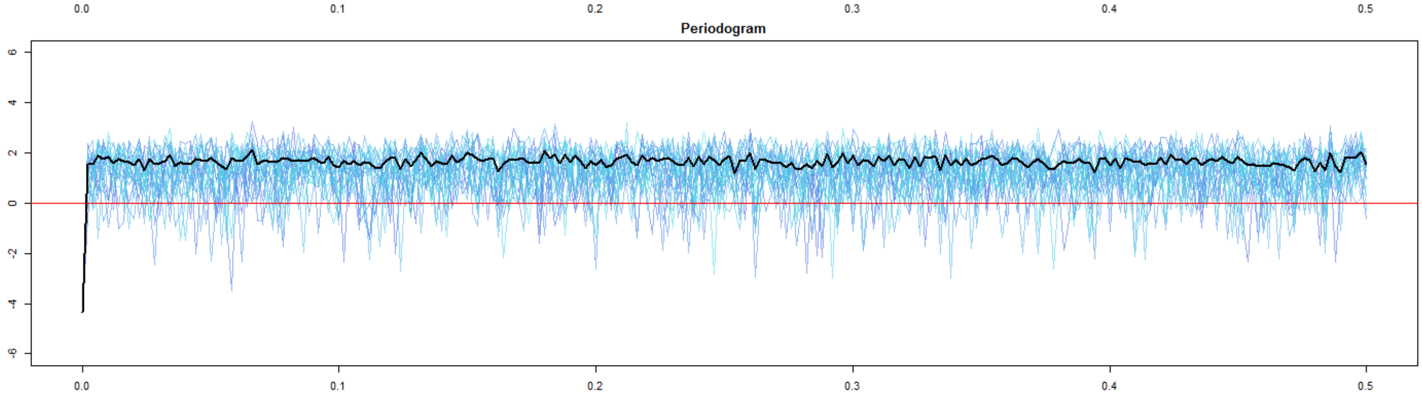
b) I observe no particular pattern. I do not observe the prevalence of any frequency. I observe seemingly constant variability in periodograms of realizations. They seem to be centered around $2(\sigma^2)^2$. They behave as I expected.

c) As the length of the series **n** increases (other variables kept constant), values of $\hat{f}(\omega_j)$ seem not to come closer to the true value of $f(\omega_j) = \sigma^2$ (see plot below, keep in mind both estimate and true spectral density are normalized; σ^2 is plotted as red line at zero); variability between the $\hat{f}(\omega_j)$ seems to stay same.

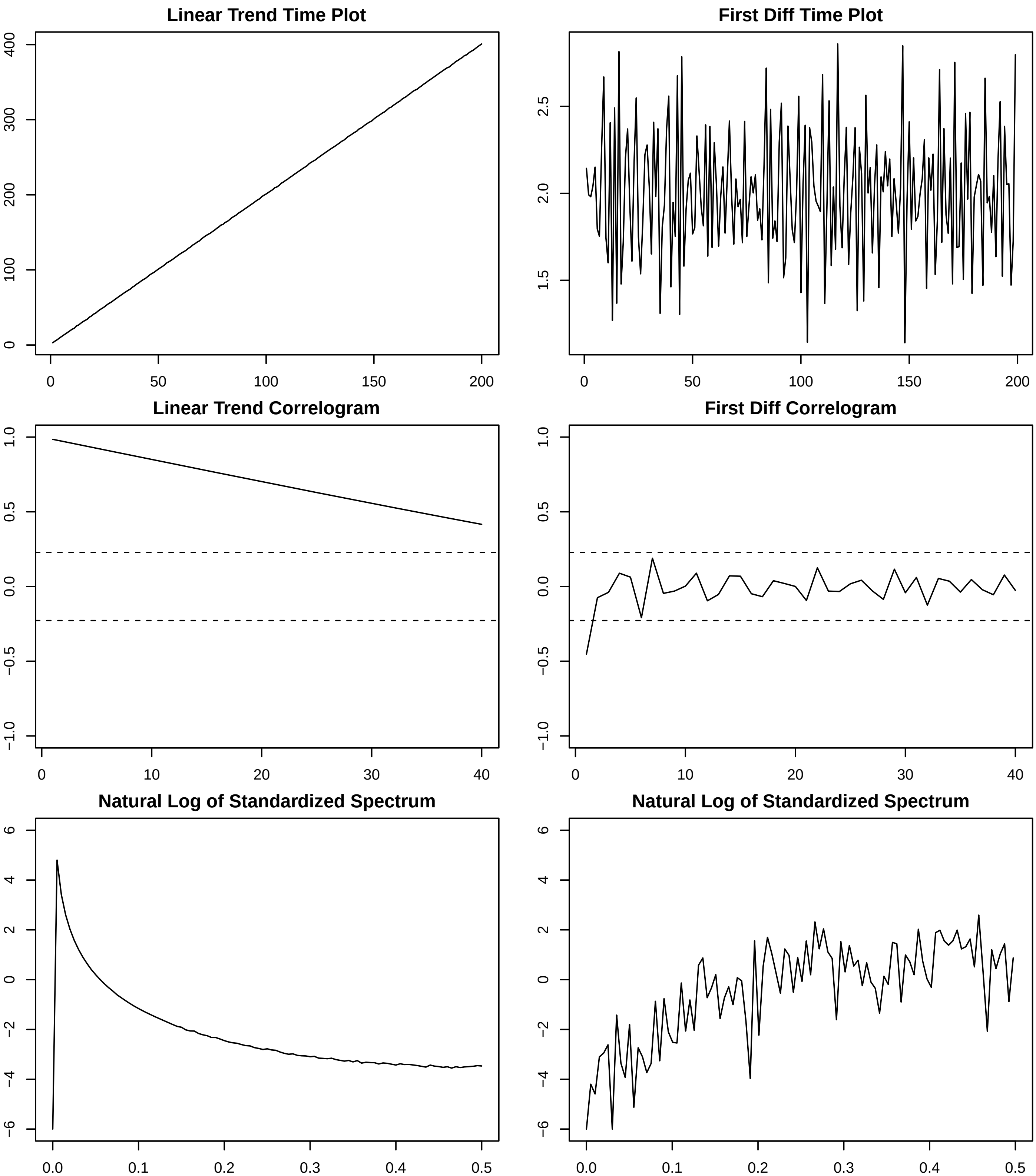
d) Using the definition of the consistent estimator from 4.2, for each value ω_j, $\hat{f}(\omega_j)$ does not appear to be a consistent estimator of $f(\omega_j)$, because with the increase of the number of observation, our estimated values do not appear to converge to the true parametr values.







C4.5


Linear Trend Time Plot
First Diff Time Plot
Linear Trend Correlogram
First Diff Correlogram
Natural Log of Standardized Spectrum
Natural Log of Standardized Spectrum
0
100
200
300
400
1.5
2.0
2.5
50
150
1.0
0.5
0.0
−0.5
−1.0
10
20
30
40
6
4
2
−2
−4
−6
0.0
0.1
0.2
0.3
0.4
0.5

Series: taking first difference turns a seemingly straight line (since we have a linear trend coefficient **b=2** much higher than the variance of WN $\sigma^2 = 0.25$) with the slope equal 2 into what appears to be a white noise, centered around 2, which is exactly the slope. That is expected, since by taking the first difference we are left over with:

$$a + b(t+1) + \epsilon_{t+1} - a - bt - \epsilon_t = b + \epsilon_{t+1} - \epsilon_t$$

Which is close to b itself, since b is much larger than the variance of the white noise.

Autocorrelation: taking first difference turns a seemingly downslope line shape correlogram of a line into the correlogram without any particular pattern, which is a characteristic of a correlogram of WN. The correlogram after taking the first difference does not appear to be significant.

Periodogram: of the linear trend is smooth, shows the excess of low frequency. Taking first difference turns periodogram to be wiggly.

```
# C4.5
linearTrendDiff=function(a,b,sig2,n){
	lt = b*seq(1,n) + a + rnorm(n,0,sig2)
	d = diff(lt, 1)
	#
	lt.corr=corr(lt,40)$corr; d.corr=corr(d,40)$corr
	lt.var=corr(lt,40)$var;   d.var=corr(d,40)$var
	lt.pgdm=perdgm(lt);             d.pgdm=perdgm(d)
	#
	par(mfrow=c(3,2),mar = c(2,2,2,2))
	plot(lt, type="l", main="Linear Trend Time Plot")
	plot(d, type="l", main="First Diff Time Plot")
	abline(h=b, lwd=2, lty=2)
	#
	lim <- qnorm((1+(1-0.05)^(1/40))/2)/sqrt(n)
	plot(lt.corr, ylim = c(-1,1), type="l", main="Linear Trend Correlogram")
	abline(h=lim, lty=2); abline(h=-lim, lty=2)
	plot(d.corr, ylim = c(-1,1), type="l", main="First Diff Correlogram")
	abline(h=lim, lty=2); abline(h=-lim, lty=2)
	#
	plotsp(lt.pgdm, n, lt.var)
	plotsp(d.pgdm, n-1, d.var) #n-1 for difference
}
linearTrendDiff(1,2,0.25,200)
```

C4.4

```
x = wn(n=200);                                      descplot(x, m=20)

y = filt(beta=c(0.9),               beta0=1, x=x); descplot(y, m=20)
y = filt(beta=c(-0.9),              beta0=1, x=x); descplot(y, m=20)
y = filt(beta=c(0, 0, 0, 0.8),      beta0=1, x=x); descplot(y, m=20)
y = filt(beta=c(-0.7, -0.1, 0.6),   beta0=1, x=x); descplot(y, m=20)
```

(I also fixed the xlim=c(-3, 3) in descplot plot of original TS to make it easier to compare plots).

Beta = 0.9:

1. Time plot: of filtered TS looks different, but does not appear to have any noticeable pattern . The amplitude of the filtered TS appears to be larger. That is anticipated since we combine two members of the original TS.
2. Correlogram: of the filtered TS looks very simlar to the correlogram of the original TS, except the first lag, where filtered TS' correlogram has relatively high value – only this value at the first lag appears to be significant. I expected that after completing exrecise T4.10, which showed that true autocorrelation is 1 at lag=0, ${0.9}/{0.81} = 0.49$ at lag=1 and zero at lag > 1.
3. Partial correlogram: of the filtered TS looks like evading "sawtooth" function. I did not know what to expect.
4. Standadized Residual Variances: of filtered TS seemengly decreased, especially after first lag. That is different from the original TS, where reisual variances were very high at each lag. I expected to see the decrease in residual variances, since we define it as $1 - \frac{\widehat{\sigma_v}}{\sigma_v}$, and we have significant autocorrelation at the fist lag.
5. Periodogram: periodogram of the filtered TS looks wiggly, as the one from the original TS. However, periodogram of the filtered TS appears to have downslope as w_n increases. That is expected after you calculate the values of: $1 + 1.8e^{2\pi\omega i} + 0.8e^{4\pi\omega i}$ which decrease with ω increasing.
6. Cumulative Periodogram: looks like an arc; appears to reject white noise hypothesis, not centered along the diagonal. That is expected, considering it is just a cumulative sum of the periodogram.

Beta = -0.9:

1. Time plot: of filtered TS looks different, but does not appear to have any noticeable pattern . The amplitude of the filtered TS appears to be larger. Also time plot of the filtered TS appears to be "wigglying" more often, than the time plot of the TS, filtered with beta=0.9. That is anticipated since we always take the inverse of the previous element.
2. Correlogram: of the filtered TS looks different from both correlogram of the original TS and TS, filtered with beta=0.9. Correlogram of the filtered TS does not appear to be significat at any lag, except 1. I expected that after completing exrecise T4.10, which showed that true autocorrelation is 1 at lag=0, ${-0.9}/{0.81} = -0.49$ at lag=1 and zero at lag > 1.
3. Partial correlogram: looks different from both previous partial correlograms. Doesn't look like a "sawtooth". Appears to be close to zero after lag=5. I did not expect anything in particular.
4. Standadized Residual Variances: looks similar to the residual variances of TS, filtered with beta=0.9. Looks like residual variances dropped values significantly after the first lag. After the lag=4, all the residual variances appear to settle around some value (0.6).
5. Periodogram: periodogram of the filtered TS looks like inverted periodogram of the TS filtered with beta=0.9. No particular pattern, has upward slope as frequency increases. That is expected, since values, obtained with $1 - 1.8e^{2\pi\omega i} + 0.8e^{4\pi\omega i}$ grow with the increase of ω.
6. Cumulative Periodogram: looks like inverted cumulative periodogram of the TS filtered with beta=0.9 – that is looks like an "canoe" (I'm sorry Dr. Harvill ☺). That is excepted by the token of how we compute it and considering periodogram itself. Appears to reject a white noise hypothesis.

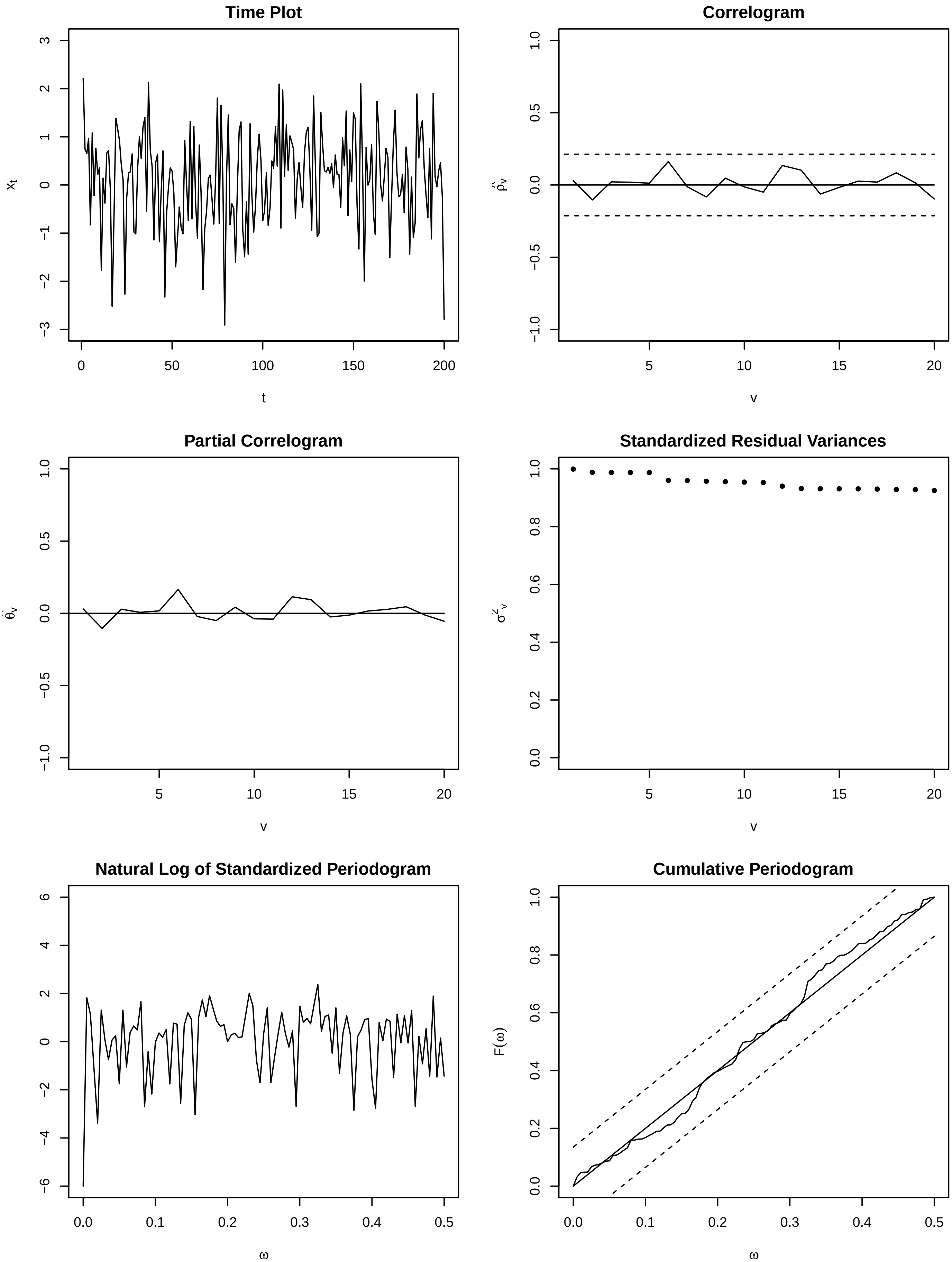
Beta = c(0, 0, 0, 0.8):

1. Time plot: of filtered TS looks random with slightly larger amplitude than in the original TS. That is what I expected.
2. Correlogram: appears to have a definite spike at some lag (seemingly v=4), and to be close to zero everywhere else. That is expected, since we showed the true autocorrelation to be ~0.5 at lag=4 and 0 otherwise.
3. Partial correlogram: looks like a set of random peaks. Different from (a) and (b).
4. Standadized Residual Variances: look similar at the first three lags, then appear to drop significantly at lag=4 and then appear to stay same thereafter.
5. Periodogram: appears to have two dips (or one bump). That is expected after we covered examples in chapter 1.
6. Cumulative Periodogram: looks like in (a) at the beginning, then has an S-shape, then looks like in (b) at the end. Appears to fit a white noise hypothesis.

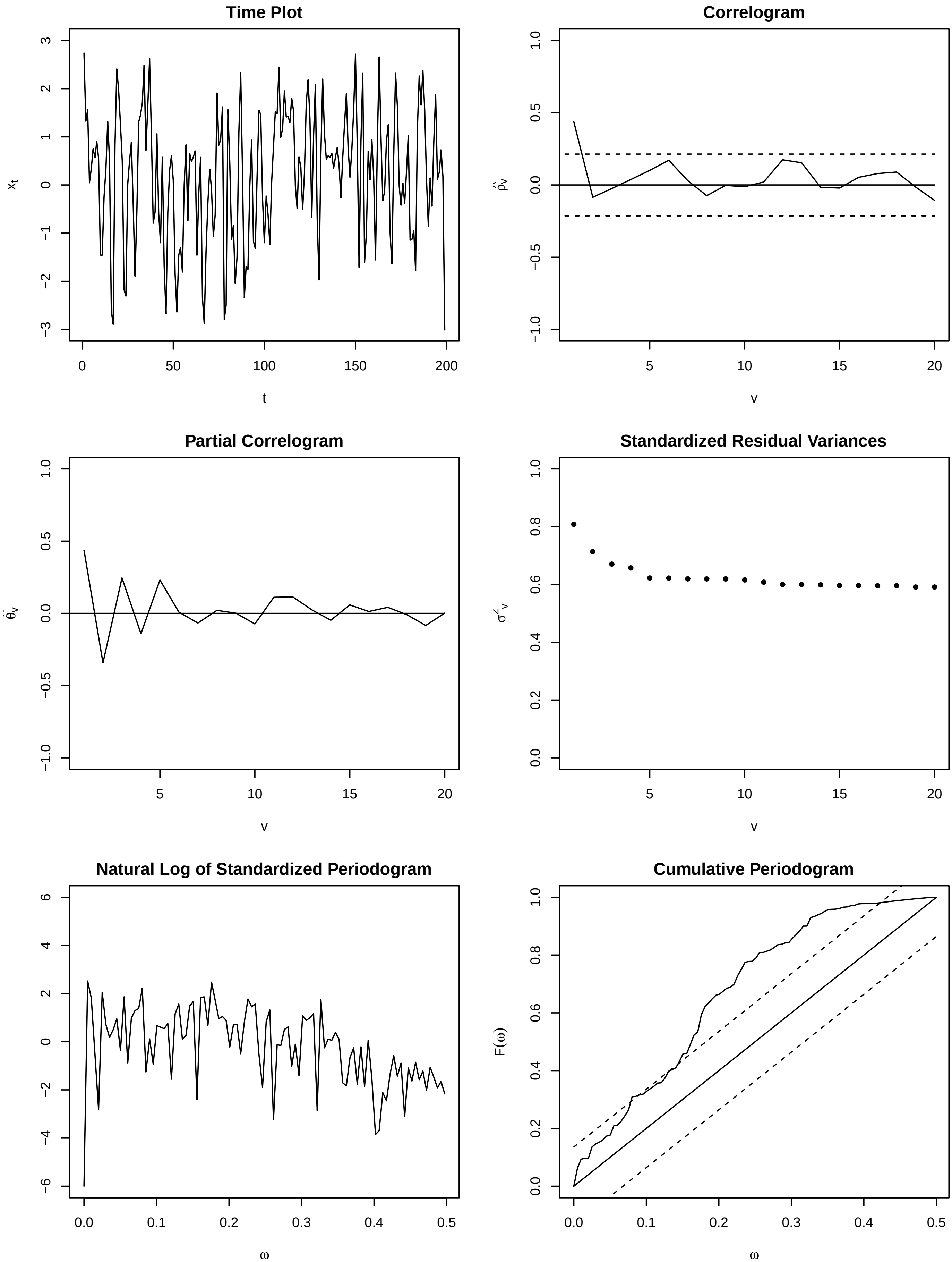
Beta = c(-0.7, -0.1, 0.6):

1. Time plot: of filtered TS appears to have more very high spikes, unlike the original TS. Other than that does not appear to have any pattern. That is expected since we use three consequent elements of the original TS.
2. Correlogram: looks like “sawtooth” pattern. Only first three lags appear to be significant. That is expected, since we showed the true autocorrelation is zero everywhere, but first three lags and at lag=0.
3. Partial correlogram: looks to have high values at the beginning, then to be close to zero.
4. Standadized Residual Variances: appears to drop after the lag=1.
5. Periodogram: looks like a “seagull”.
6. Cumulative Periodogram: appears be an S-shaped. That is what expected after the examples in chapter 1.

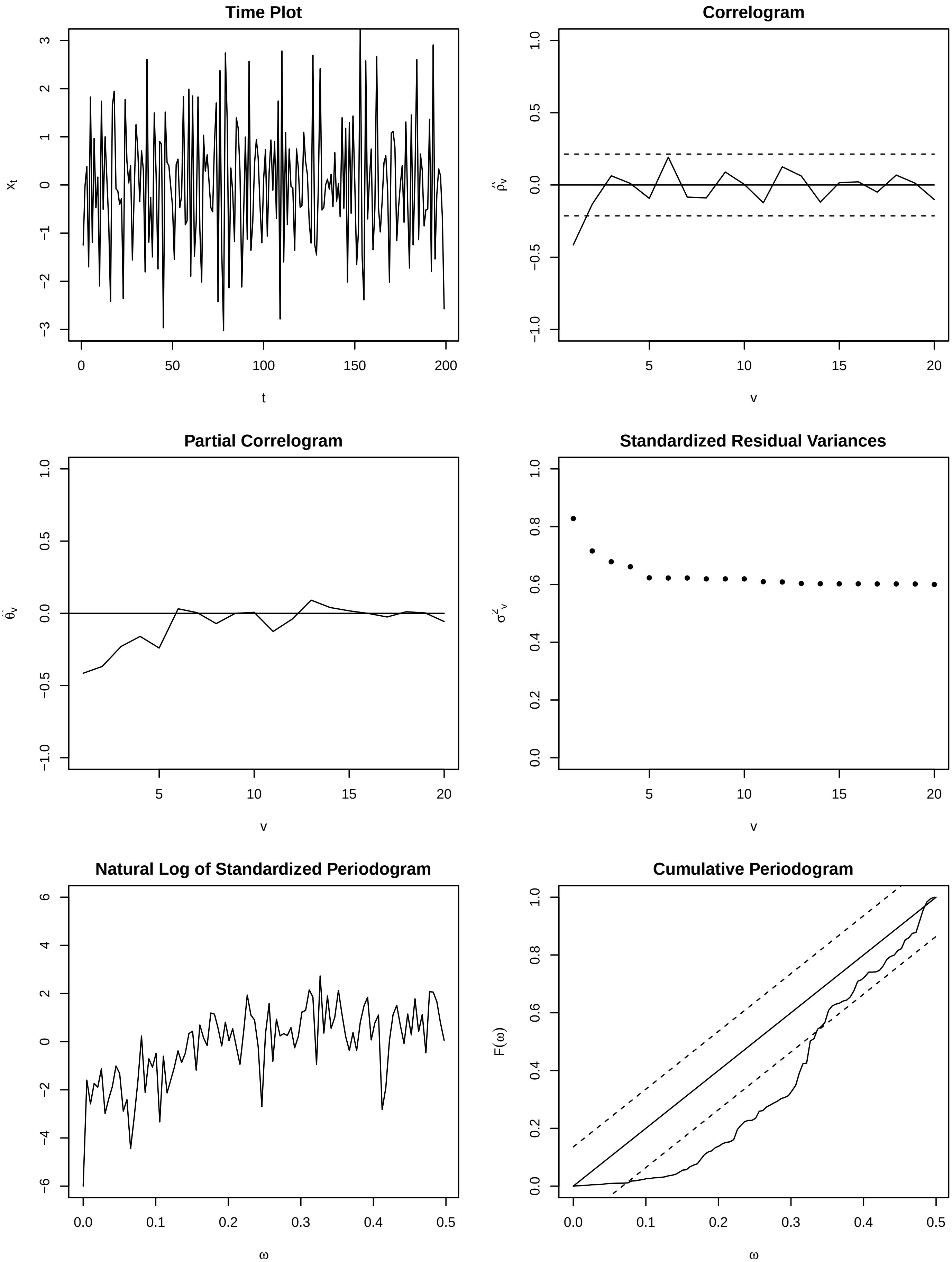
Original Series X (White Noise)



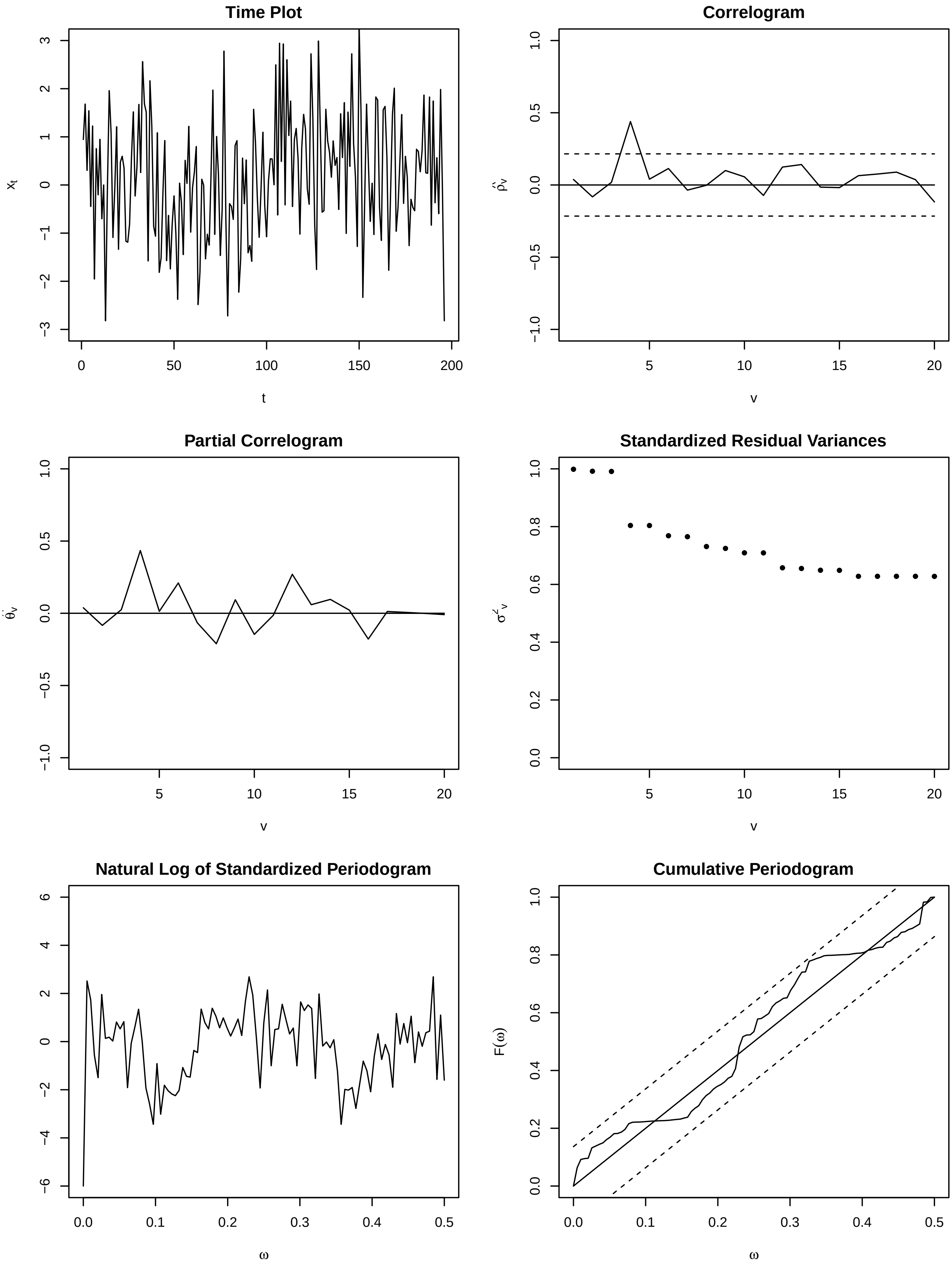
Filtered Series Y (beta = 0.9)



Filtered Series Y (beta = -0.9)



Filtered Series Y (beta = c(0, 0, 0, 0.8))



Filtered Series Y (beta = c(-0.7, -0.1, 0.6))