STA4273_HW4

Alex Jones

2023-04-23

Question 9.6

Using the Metropolis-Hastings random walk sampler with the proposal distribution being uniform:

```
groups <- c(125,18,20,34)
w <- 0.1
n <- 20000
x <- numeric(n)
burn <- 5000

prop <- function(a) {
  if (a > 0 && a < 1) {
    (2+a)^groups[1]*(1-a)^(groups[2]+groups[3])*a^groups[4]
  }
  else 0
}

x[1] <- runif(1,0,1)
u <- runif(n)
v <- runif(n, -w,w)

for (i in 2:n) {
  y <- x[i - 1] + v[i]
  if (u[i] <= prop(y)/prop(x[i-1])) {
    x[i] <- y
  }
  else {
    x[i] <- x[i-1]
  }
}

x_burned <- x[(burn+1):n]
theta <- mean(x_burned)
theta
```

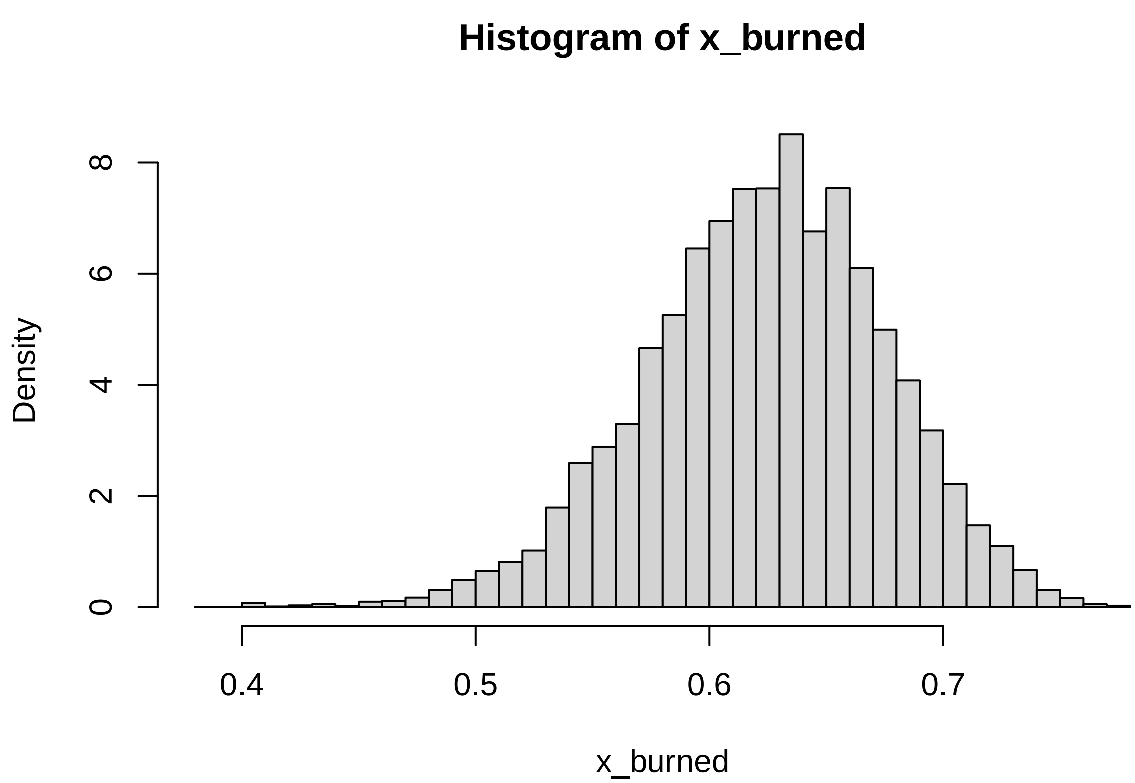
```
## [1] 0.6237187
```

```
p <- c(0.5+theta/4, (1-theta)/4, (1-theta)/4, theta/4)

p * sum(groups)
```

```
## [1] 129.21815  18.53185  18.53185  30.71815
```

```
hist(x_burned, prob = TRUE, breaks = "Scott")
```



It seems our generated distribution aligns with both the sample and the target distribution.

Question 9.7

Using the Gibbs Sampler (something to note: usually there's different μ and σ for each variate, but since they come from identical distributions I only made one variable for both of them).

```
n <- 12000
burn <- 2000
X <- matrix(0,n,2)
rho <- 0.9
mu <- 0
sigma <- 1
s <- sqrt(1-rho^2)

for (i in 2:n) {
  x2 <- X[i-1,2]
  m1 <- mu + rho * (x2-mu)
  X[i,1] <- rnorm(1,m1,s)
  x1 <- X[i,1]
  m2 <- mu + rho * (x1-mu)
```

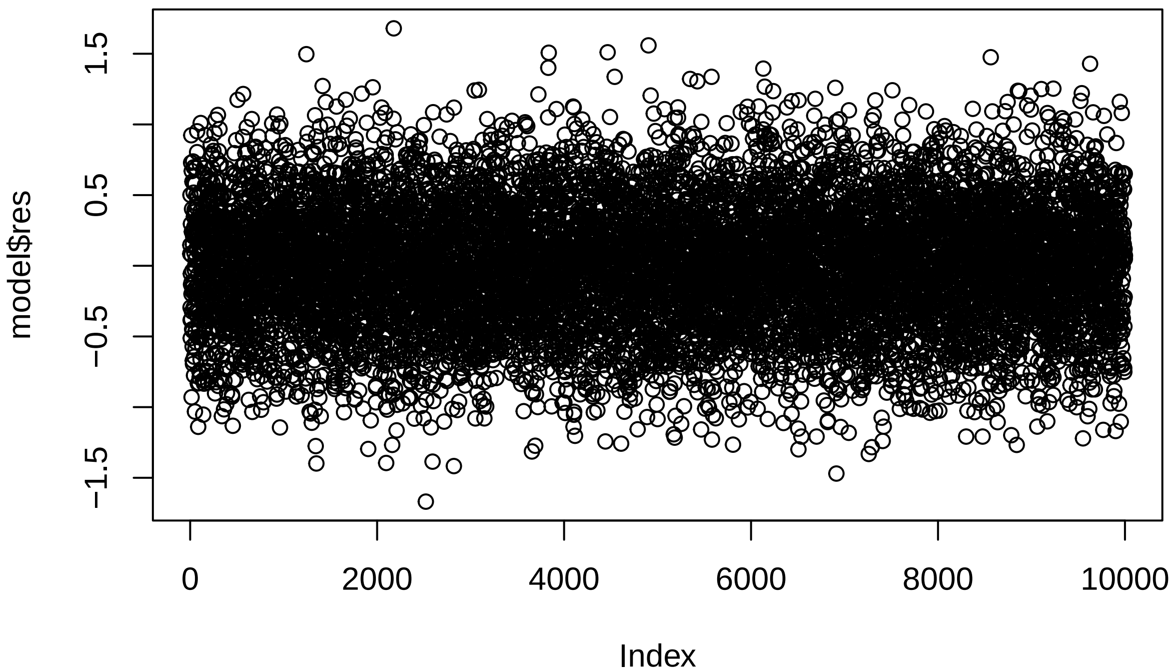
```r
  X[i,2] <- rnorm(1,m2,s)
}

x <- X[(burn+1):n, 1]
y <- X[(burn+1):n, 2]
model <- lm(y~x)
summary(model)
```

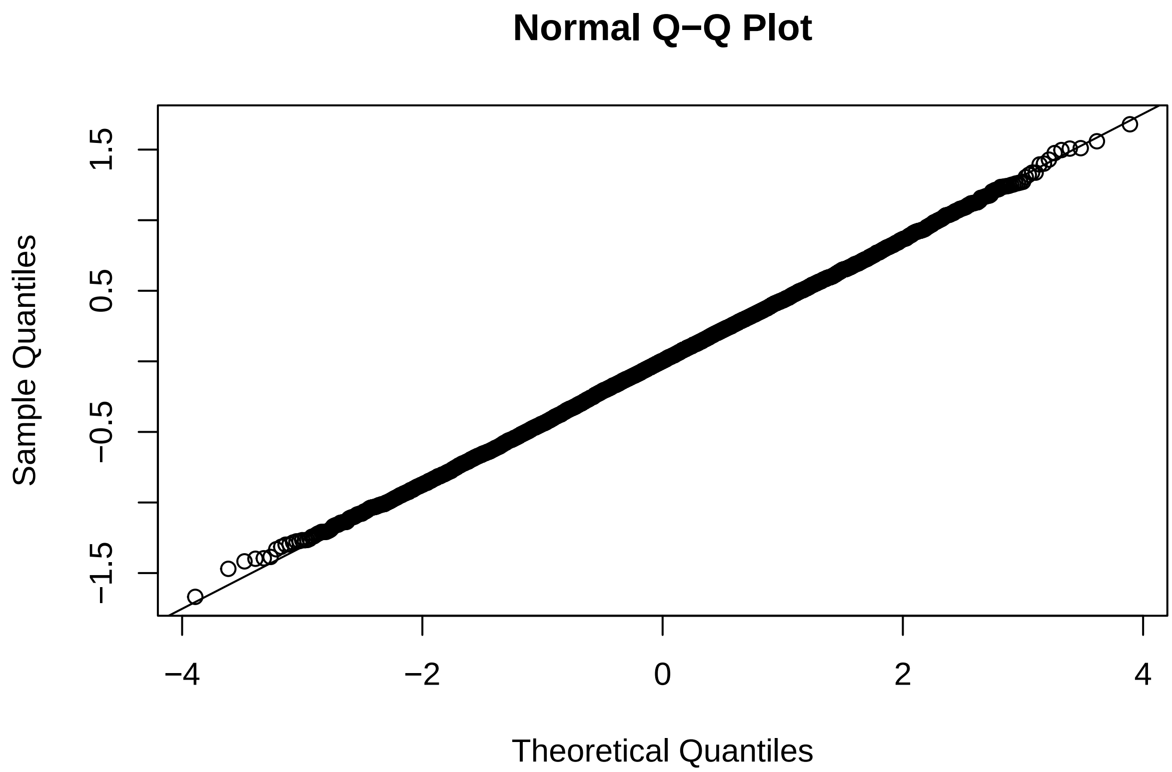
```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.66851 -0.29551  0.00286  0.29583  1.67954
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -6.584e-05  4.347e-03  -0.015    0.988
## x            9.023e-01  4.290e-03 210.340   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4347 on 9998 degrees of freedom
## Multiple R-squared:  0.8157, Adjusted R-squared:  0.8157
## F-statistic: 4.424e+04 on 1 and 9998 DF,  p-value: < 2.2e-16
```

As noted in the summary output, R^2 is approximately 0.8, which corresponds to $R \approx 0.9$, matching what we needed. Looking at the plot of the residuals and the qqline:

```r
plot(model$res)
```




```
qqnorm(model$res)
qqline(model$res, cex = 0.25)
```



The constant variance/normality condition seems to be satisfied.

Question 9.8

For the purposes of this problem, we will set $a = 4$, $b = 5$ and $n = 20$.

```
size <- 11000
burn <- 1000
a <- 4
b <- 5
n <- 20

x <- y <- rep(0,n)
x[1] <- rbinom(1, n, 0.5)
y[1] <- rbeta(1, x[1] + a, n-x[1]+b)

for (i in 2:size) {
  x[i] <- rbinom(1, n, y[i-1])
  y[i] <- rbeta(1, x[i]+a, n-x[i]+b)
}

x_burned <- x[(burn+1):size]
y_burned <- y[(burn+1):size]
table <- table(x_burned)/length(x_burned)
```

Looking at the computed marginal probability distribution $f(x|y)$:

```r
hist(x_burned, prob = TRUE)
```