9.6 Rao [220, Sec. 5g] presented an example on genetic linkage of 197 animals in four categories (also discussed in [67, 106, 171, 266]). The group sizes are

$(125, 18, 20, 34)$. Assume that the probabilities of the corresponding multinomial distribution are

$$\left(\frac{1}{2} + \frac{\theta}{4}, \frac{1-\theta}{4}, \frac{1-\theta}{4}, \frac{\theta}{4}\right).$$

Estimate the posterior distribution of θ given the observed sample, using one of the methods in this chapter.

9.7 Implement a Gibbs sampler to generate a bivariate normal chain (X_t, Y_t) with zero means, unit standard deviations, and correlation 0.9. Plot the generated sample after discarding a suitable burn-in sample. Fit a simple linear regression model $Y = \beta_0 + \beta_1 X$ to the sample and check the residuals of the model for normality and constant variance.

9.8 This example appears in [40]. Consider the bivariate density

$$f(x, y) \propto \binom{n}{x} y^{x+a-1}(1-y)^{n-x+b-1}, \quad x = 0, 1, \ldots, n, \ 0 \le y \le 1.$$

It can be shown (see e.g. [23]) that for fixed a, b, n, the conditional distributions are Binomial(n, y) and Beta$(x + a, n - x + b)$. Use the Gibbs sampler to generate a chain with target joint density $f(x, y)$.