Aufgabe 1

(1) a) $1^\circ = \frac{1^\circ}{360^\circ} * 2\pi\,\mathrm{rad} \approx 0.0174\,\mathrm{rad}$

b) $1' = \frac{1'}{21\,600'} * 2\pi\,\mathrm{rad} \approx 0.2909\,\mathrm{mrad}$

c) $1'' = \frac{1''}{1\,296\,000''} * 2\pi\,\mathrm{rad} \approx 4.848\,\mu\mathrm{rad}$

(2) $\frac{\Delta t}{1\,\mathrm{a}} = 10^{-9} \iff \Delta t \approx 3.154 \times 10^{7}\,\mathrm{s} * 10^{-9} = 0.031\,54\,\mathrm{s}$

Aufgabe 2

(1) Für $c = 299\,792\,458\,\mathrm{m/s}$:

$$1\,\mathrm{ly} = c * 1\,\mathrm{a} \approx c * 3.154 \times 10^{7}\,\mathrm{s} \approx 9.455 \times 10^{15}\,\mathrm{m} = 9.455\,\mathrm{Pm}$$

(2)

$$\tan 0.5\alpha = \frac{0.5\,\mathrm{AE}}{1\,\mathrm{pc}} \iff 1\,\mathrm{pc} = \frac{0.5\,\mathrm{AE}}{\tan 0.5\alpha} \approx \frac{0.5 * 1.496 \times 10^{11}\,\mathrm{m}}{\tan 0.5''} \approx 3.086 \times 10^{16}\,\mathrm{m}$$

$$3.086 \times 10^{16}\,\mathrm{m} * \frac{1\,\mathrm{ly}}{9.455\,\mathrm{Pm}} \approx 3.264\,\mathrm{ly}$$

(3) a) $4.3 \times 10^{16}\,\mathrm{m} * \frac{1\,\mathrm{ly}}{9.455\,\mathrm{Pm}} \approx 4.548\,\mathrm{ly}$

b) $4.3 \times 10^{16}\,\mathrm{m} * \frac{1\,\mathrm{pc}}{3.086 \times 10^{16}\,\mathrm{m}} \approx 1.393\,\mathrm{pc}$

Aufgabe 3

(1) $\overline{V} = \frac{1}{14} \sum_{i=1}^{14} V_i = \frac{13\,980\,\mathrm{ml}}{14} \approx 998.57\,\mathrm{ml}$

(2) $\sigma = \sqrt{\frac{1}{13} \sum_{i=1}^{14} (V_i - \overline{V})^2} \approx 6.086\,\mathrm{ml}$

(3) Nur Flasche 11, denn $998.57\,\mathrm{ml} - 985\,\mathrm{ml} > 2 * 6.086\,\mathrm{ml}$

Aufgabe 4

(1) $\exp 0 = \sum_{n=0}^{\infty} \frac{0^n}{n!} = 1 + \sum_{n=1}^{\infty} \frac{0^n}{n!} = 1$

(2)

$$\begin{aligned}\exp(x+y) &= \sum_{n=0}^{\infty} \frac{(x+y)^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{n!} \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \right) \\ &= \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} \frac{x^k y^{n-k}}{k!(n-k)!} \right) \\ &=1+ \\ &\quad y+x+ \\ &\quad \frac{y^2}{2} + xy + \frac{x^2}{2} + \dots \\ &=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\dots \\ &+y+xy+\frac{x^2y}{2}+\frac{x^2y}{6}+\dots \\ &+\frac{y^2}{2}+\frac{xy^2}{2}+\frac{x^2y^2}{4}+\frac{x^3y^2}{12}+\dots \\ &= \sum_{n=0}^{\infty} \frac{x^n}{n!} * \sum_{n=0}^{\infty} \frac{y^n}{n!}\end{aligned}$$

(3) $\frac{\mathrm{d}}{\mathrm{d}x} \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{\mathrm{d}}{\mathrm{d}x} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\mathrm{d}}{\mathrm{d}x} x^n = \sum_{n=1}^{\infty} \frac{1}{n!} * n x^{n-1} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

(4) $\frac{\mathrm{d}}{\mathrm{d}x} \sin x = \frac{\mathrm{d}}{\mathrm{d}x} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} \frac{\mathrm{d}}{\mathrm{d}x} x^{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{(2n+1)x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \cos x$

(5) $\frac{\mathrm{d}}{\mathrm{d}x} \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} \frac{\mathrm{d}}{\mathrm{d}x} x^{2n} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n-1}}{(2n-1)!} = \sum_{n=-1}^{\infty} (-1)^{n+1} \frac{x^{2n+1}}{(2n+1)!} = -1 * \sum_{n=-1}^{\infty} (-1)^n$

(6) $\exp(0) = 1 \iff \exp(x-x) = 1 \iff \exp(x)\exp(-x) = 1 \iff \exp(-x) = \frac{1}{\exp(x)}$

(7) Für $x \geq 0$ gilt offensichtlich $\exp(x) > 0$. Für $x < 0$ gilt $\exp(x) = \frac{1}{\exp(-x)}$, was ebenfalls positiv ist.

(8) $\sinh(-x) = \frac{1}{2}(e^{-x} - e^{x}) = -\frac{1}{2}(e^{x} - e^{-x}) = -\sinh(x)$

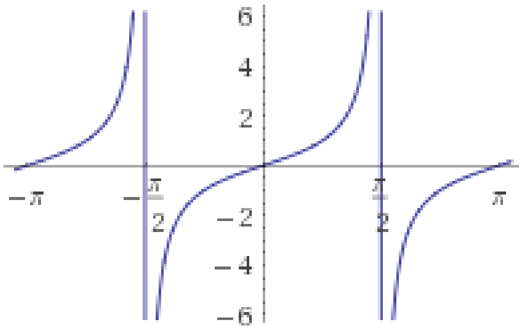
(9) $\cosh(-x) = \frac{1}{2}(e^{-x} + e^{x}) = \cosh(x)$

(10) $\cosh(x+y) = \frac{1}{2}(e^{x+y} + e^{-x-y}) = \frac{1}{4}(2e^{x+y} + 2e^{-x-y}) = \frac{1}{4}(e^{x+y} + e^{x-y} + e^{y-x} + e^{-x-y}) + \frac{1}{4}(e^{x+y} - e^{x-y} - e^{y-x} + e^{-x-y}) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

(11) $\cosh^2(x) - \sinh^2(x) = \frac{1}{4}(e^{2x} + 2 + e^{-2x}) - \frac{1}{4}(e^{2x} - 2 + e^{-2x}) = \frac{4}{4} = 1$

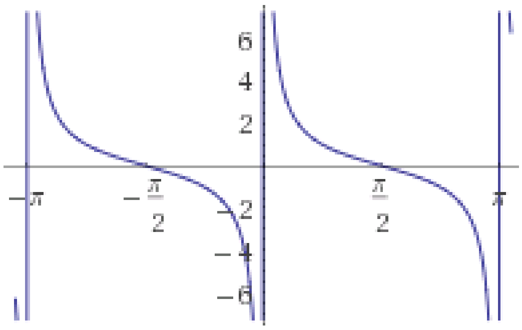
(12) $y = \frac{1}{2}(e^{x} - e^{-x}) \iff 2y = e^{x} - e^{-x} \iff 0 = (e^{x})^2 - 2ye^{x} - 1 \implies e^{x} = y + \sqrt{y^2+1} \iff x = \ln(y + \sqrt{y^2+1})$

Aufgabe 5

(1) 

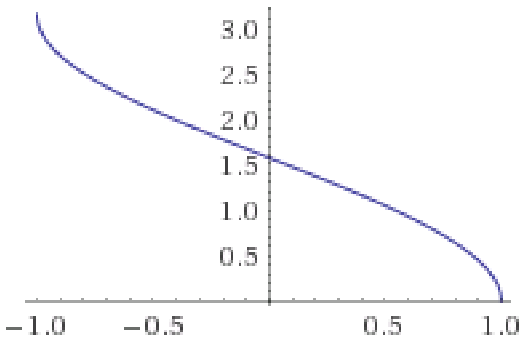
(x from −3 to 3)

- $\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \frac{\mathrm{d}}{\mathrm{d}x}\frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \tan^2 x$
- $\frac{\mathrm{d}}{\mathrm{d}x}(1 + \tan^2 x) = 2(1 + \tan^2 x)\tan x$

(2) 

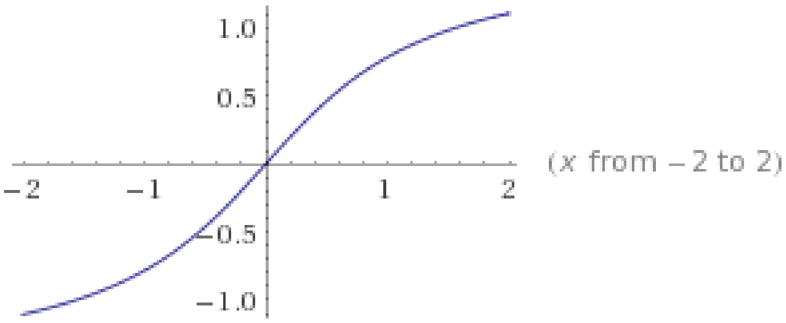
(x from −3 to 3)

- $\frac{\mathrm{d}}{\mathrm{d}x}\cot x = \frac{\mathrm{d}}{\mathrm{d}x}\frac{\cos x}{\sin x} = \frac{-\sin^2(x) - \cos^2(x)}{sin^2 x} = -1 - \cot^2 x$
- $\frac{\mathrm{d}}{\mathrm{d}x}(-1 - \cot^2 x) = 2(1 + \cot^2 x)\cot x$

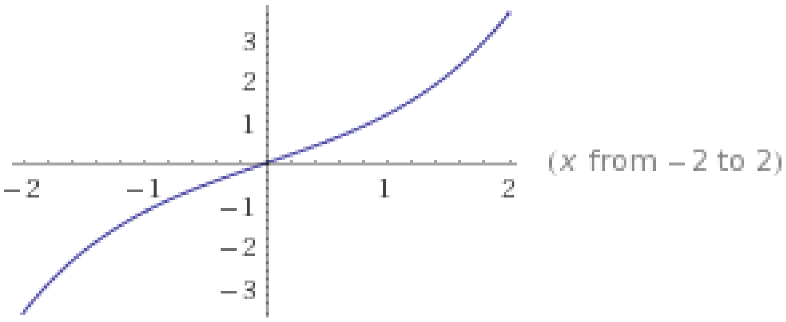
(3) 

(x from −1 to 1)

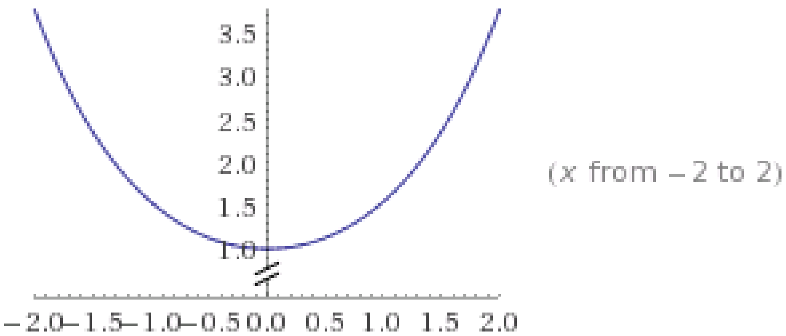
- $y = \arccos(x) \iff \cos y = x \implies \frac{\mathrm{d}}{\mathrm{d}x}\cos y = \frac{\mathrm{d}}{\mathrm{d}x}x \iff -\frac{\mathrm{d}y}{\mathrm{d}x}\sin y = 1 \iff -\frac{\mathrm{d}y}{\mathrm{d}x}\sqrt{1 - \cos^2 y} = 1 \iff \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{\mathrm{d}}{\mathrm{d}x} - \frac{1}{\sqrt{1-x^2}} = -2x * \frac{-1}{2}(1 - x^2)^{-3/2} = \frac{x}{\sqrt{(1-x^2)^3}}$

(4) 

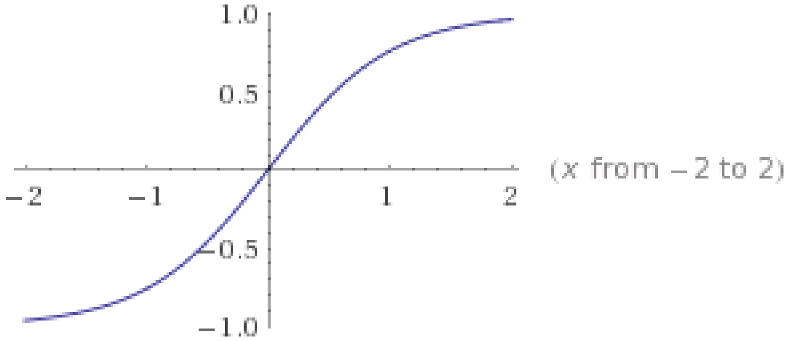
- $y = \arctan x \iff \tan y = x \implies \frac{\mathrm{d}}{\mathrm{d}x} \tan y = 1 \iff \frac{\mathrm{d}y}{\mathrm{d}x}(1 + \tan^2 y) = 1 \iff \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x^2}$
- $\frac{\mathrm{d}}{\mathrm{d}x} \frac{1}{1+x^2} = -\frac{2x}{(1+x^2)^2}$

(5) 

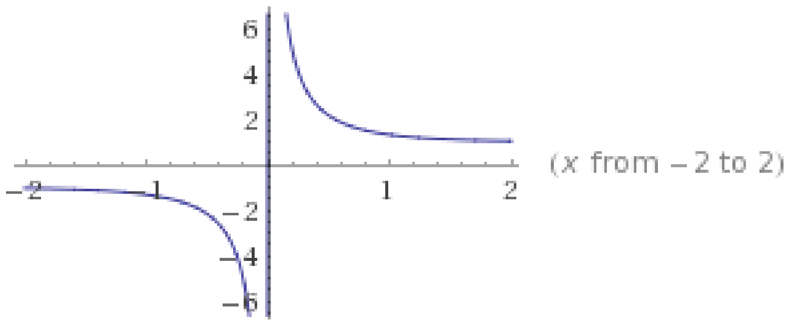
- $\frac{\mathrm{d}}{\mathrm{d}x} \sinh x = \frac{\mathrm{d}}{\mathrm{d}x} \frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}(e^x + e^{-}x) = \cosh x$
- $\frac{\mathrm{d}}{\mathrm{d}x} \cosh x = \frac{\mathrm{d}}{\mathrm{d}x} \frac{1}{2}(e^x + e^{-x}) = \frac{1}{2}(e^x - e^{-x}) = \sinh x$
- $\lim_{x\to\infty} \sinh x = \frac{1}{2} \lim_{x\to\infty} e^x = \infty$
- $\lim_{x\to-\infty} \sinh x = -\frac{1}{2} \lim_{x\to-\infty} e^{-x} = -\infty$

(6) 

- Ableitungen siehe (5)
- $\lim_{x\to\infty} \cosh x = \frac{1}{2} \lim_{x\to\infty} e^x = \infty$
- $\lim_{x\to-\infty} \cosh x = \frac{1}{2} \lim_{x\to\infty} e^x = \infty$

(7)


- $\frac{\mathrm{d}}{\mathrm{d}x}\frac{\sinh x}{\cosh x} = \frac{\cosh^2(x)-\sinh^2(x)}{\cosh^2 x} = 1 - \tanh^2 x$
- $\frac{\mathrm{d}}{\mathrm{d}x}(1-\tanh^2 x) = -2(1-\tanh^2 x)\tanh x$
- $\lim_{x\to\infty}\tanh x = \frac{\lim_{x\to\infty}\sinh x}{\lim_{x\to\infty}\cosh x} = 1$
- $\lim_{x\to-\infty}\tanh x = \frac{\lim_{x\to-\infty}\sinh x}{\lim_{x\to-\infty}\cosh x} = -1$

(8)


- $\frac{\mathrm{d}}{\mathrm{d}x}\frac{\cosh x}{\sinh x} = \frac{\sinh^2(x)-\cosh^2(x)}{\sinh^2 x} = 1 - \coth^2 x$
- $\frac{\mathrm{d}}{\mathrm{d}x}(1-\coth^2 x) = -2(1-\coth^2 x)\coth x$
- $\lim_{x\to\infty}\coth x = \frac{\lim_{x\to\infty}\cosh x}{\lim_{x\to\infty}\sinh x} = 1$
- $\lim_{x\to-\infty}\coth x = \frac{\lim_{x\to-\infty}\cosh x}{\lim_{x\to-\infty}\sinh x} = -1$

Aufgabe 7

(1) $|\vec{a}| = \sqrt{1^2+(-1)^2+2^2} = \sqrt{6}$
$\left|\vec{b}\right| = \sqrt{4^2+3^2+0^2} = 5$
$|\vec{c}| = \sqrt{(-3)^2+3^2+(-6)^2} = \sqrt{54} = 3\sqrt{6}$

(2) $\vec{a_n} = \frac{1}{\sqrt{6}}\vec{a}$
$\vec{b_n} = \frac{1}{5}\vec{b}$
$\vec{c_n} = \frac{1}{3\sqrt{6}}\vec{c}$

(3)
- $\vec{a}+\vec{b} = \begin{pmatrix}5\\2\\2\end{pmatrix}$

- $\vec{a} + \vec{c} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$
- $\vec{a} - \vec{b} = \begin{pmatrix} -3 \\ -4 \\ 2 \end{pmatrix}$
- $\vec{a} - \vec{c} = \begin{pmatrix} 4 \\ -4 \\ 8 \end{pmatrix}$
- $\vec{a} * \vec{b} = 1$
- $\vec{a} * \vec{c} = -18$
- $\vec{a} \times \vec{b} = \begin{pmatrix} -6 \\ 8 \\ 7 \end{pmatrix}$
- $\vec{a} \times \vec{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0}$

(4)
- $\vec{a}$ und $\vec{b}$: $\arccos(\frac{1}{\sqrt{6}*5}) = 85.32°$
- $\vec{a}$ und $\vec{c}$: $\arccos(\frac{-18}{\sqrt{6}*3*\sqrt{6}}) = \pi$

(5) $\frac{\vec{a}*\vec{b}}{|\vec{b}|} = \frac{1}{5}$