GaussCAD - Gaussian beam field simulator.

Schultze, Alexander

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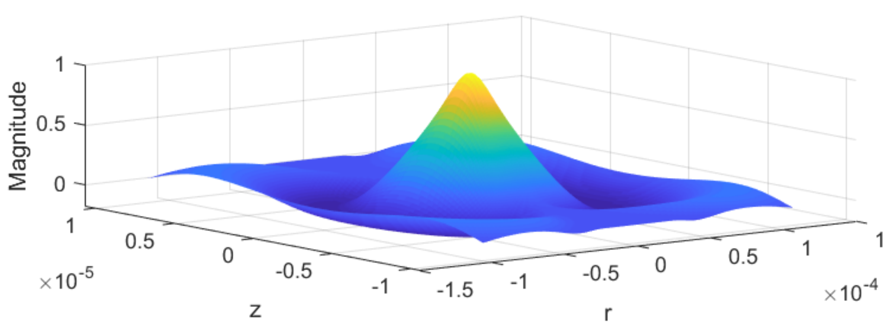
GaussCAD is a matlab based simulator for evaluating Gaussian optics and simulate projection onto a photodiode. Visual representation of intensity/phase fields as described by Gaussian optics. A simple paraxial optics (ABCD matrix) tracer functionality is included to propagate gaussian beams. It makes uses of paraxial optics to propagate Gaussian Rays through optical elements.

The Repository is available at: https://github.com/alexschultze/GaussCAD.



1 Gaussian Optics Beam (Class @gaussian.gauss-beam)

Gaussian Beam

The Gaussian beam is described as a transversal electrical mode (TEM) is description of monochromatic electromagnetic radiation. Tthe fundamental (TEM_{00} can be described according to the following definition.

$$\mathbf{E}(r,z) = E_0\,\hat{\mathbf{x}}\,\frac{w_0}{w(z)}\exp\left(\frac{-r^2}{w(z)^2}\right)\exp\left(-i\left(kz + k\frac{r^2}{2R(z)} - \psi(z)\right)\right) \tag{1}$$

r .. is the radial distance from the center axis of the beam,
z .. is the axial distance from the beam's focus (or "waist"),
i .. is the imaginary unit,
$k = 2\pi n/\lambda$ .. is the wave number (in radians per meter) for a free-space wavelength λ, and n is the index of refraction of the medium in which the beam propagates,
E_0 = E(0, 0) .. the electric field amplitude (and phase) at the origin at time 0,
w(z) .. is the radius at which the field amplitudes fall to 1/e of their axial values (i.e., where the intensity values fall to 1/e2 of their axial values), at the plane z along the beam,
w0 = w(0) .. is the waist radius,
R(z) .. is the radius of curvature of the beam's wavefronts at z, and
(z) .. is the Gouy phase at z, an extra phase term beyond that attributable to the phase velocity of light.

GaussCAD The software can be used to describe beams in the beam reference system $E(r,z)$. The beam can be transformed in cartesian coordinates. In cartesian definition the vector p describes the location of the beam waist and the vector n describes the axial direction vector along along which z distance propagates.

2 Gaussian Screen (Class @gaussian.field-beam)

Field screen represents a two dimensional pixel sensor array. It accepts two input beams and evaluates their field at each pixel. The screen is described by its parameters of position r, normal vector n, its physical dimension (2D) and pixel count (2D).

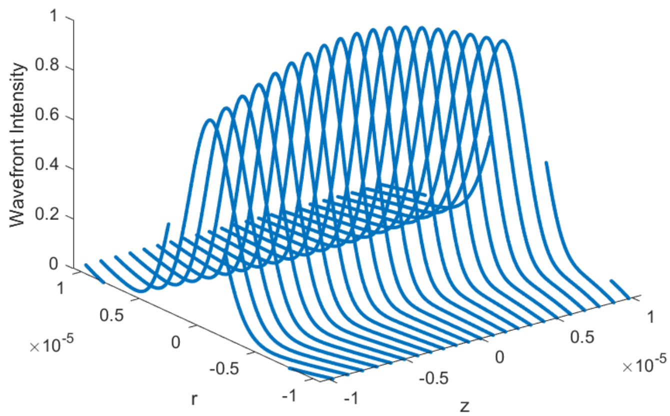


Figure 1: Wavefront progression with intensity for a Gaussian beam in beam coordinates r and z.

When rendered, the current field value for each beam is calculated and saved for each pixel.
The class provides additional functions for masking and segmentation to represent the shape of actual quadrant photodiode with gap.
The pixels irradiance (intensity) is equal to the magnitude of sum of the two phasors squared. The AC, DC is determined by the maximum and minimum values of the phasors magnitude and their difference. The phase information (interferogram) for each location pixel is the difference between the two beam phasor angles.

$$AC(x, y, z) = min(mag(E1(x, y, z)), mag(E2(x, y, z))) \tag{2}$$

$$DC(x, y, z) = max(mag(E1(x, y, z)), mag(E2(x, y, z))) - AC; \tag{3}$$

$$Phase(x, y, z)\phi = angle(E1(x, y, z)) - angle(E2(x, y, z)) \tag{4}$$

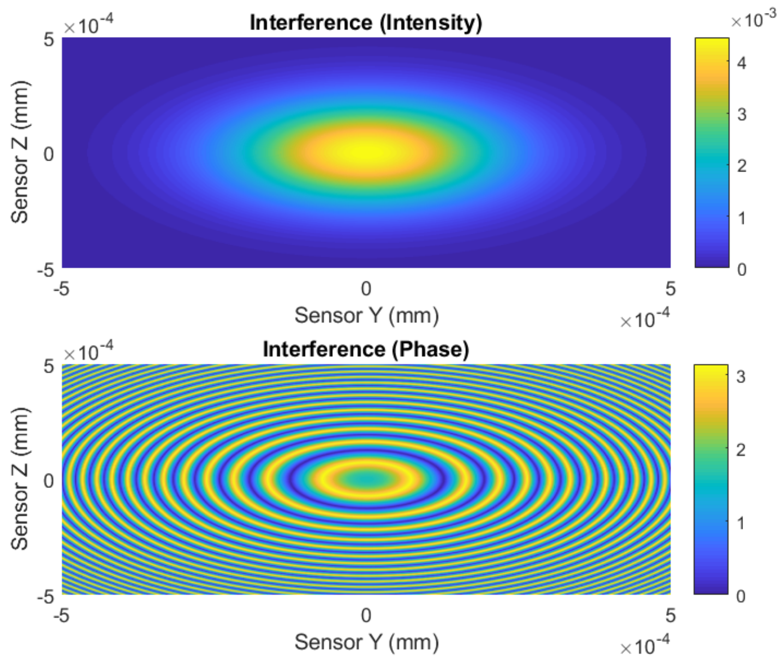
$$Int(x, y, z) = \sum_{n=1}^{2} E_n(x, y, z) \tag{5}$$

The interferogram may contain phase jumps which is in the next setup removed by 2D unwrapping based on total-variation (TV) denoising. [1]
Because each pixel has a different impact on the overall phase information recovered, the phase is weighed by the AC amplitude.

$$\phi_w(x, y, z) = AC(x, y, z) \cdot e \exp -i \cdot \phi \tag{6}$$

The actual phase when combining several pixel arrays (full screen, masked screen, quadrants) is calculated as the resulting phase of the sum of all pixel elements.

$$\Phi = angle(\sum \phi_w(x, y, z)) \tag{7}$$



*0.8*0.8

Figure 2: Exemple interferogram of two beams on a Screen class object.

Photodiode Masks / Quadrant Photodiodes The field screen class supports masking to represent round photodetector surfaces. A gap between the segments can be added.

Differential Wave-front Sensing DWS The class implements Differential Wave-front Sensing evaluation by masking quadrants either orthogonal or diagonally. The definition for calculating the phase difference between quadrants is is given in the following. Q1..Q4 denote the 4 quadrants.

$$DWS_y = \frac{1}{2} \cdot \frac{\Phi_{Q2} + \Phi_{Q3}}{\Phi_{Q1} + \Phi_{Q4}}; DWS_z = \frac{1}{2} \cdot \frac{\Phi_{Q3} + \Phi_{Q4}}{\Phi_{Q1} + \Phi_{Q2}} \tag{8}$$

Figure 3: Two gaussian beam interference showing interfering (AC) and non-interfering (DC) components on a sensor.

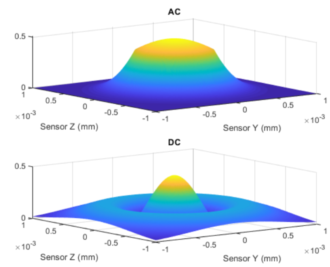


Figure 4: Showing the (phase dependent) current irradiance for a given phase and the resulting interferogram.

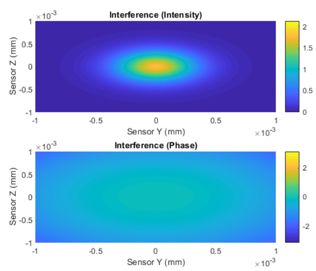


Figure 5: Study of impact of beam displacement into path length with one beam being offset on the sensor.

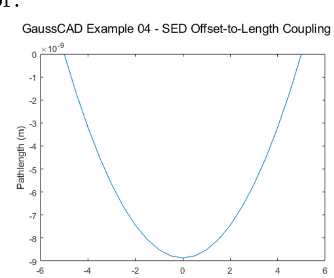


Figure 6: Definition of Quadrant Segments

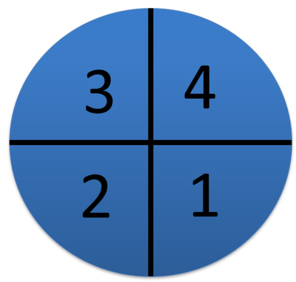
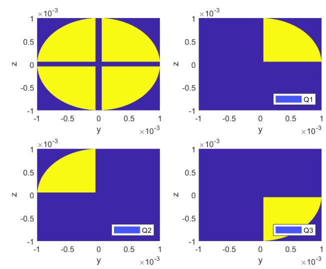


Figure 7: Definition of Quadrant Segments



$$DWS_d1 = \frac{\Phi_{Q1}}{\Phi_{Q3}}; DWS_d2 = \frac{\Phi_{Q2}}{\Phi_{Q4}} \tag{9}$$

3 Paraxial Optics Bench (Class @paraxial.bench)

The paraxial bench can be used to perform operations of ray transfer matrix analysis, e.g. ABCD matrix analysis. It can be used to visualise and calculate the interaction of optical elements for geometrical rays, as well as propagation of Gaussian beam parameters through optical elements.

Change of Gaussian beam parameters $q(z) = z + iz_R$, where z is the distance from the beam waist and $z_r = z_0$ the Rayleigh length, is calculated according to the following formula. The multiplication is performed on each optical element individually, for Gaussian propagation the matrices of optical elements can not be combined by multiplication.

$$q_1(z) = \frac{Aq_0 + B}{Cq_0 + D} \tag{10}$$

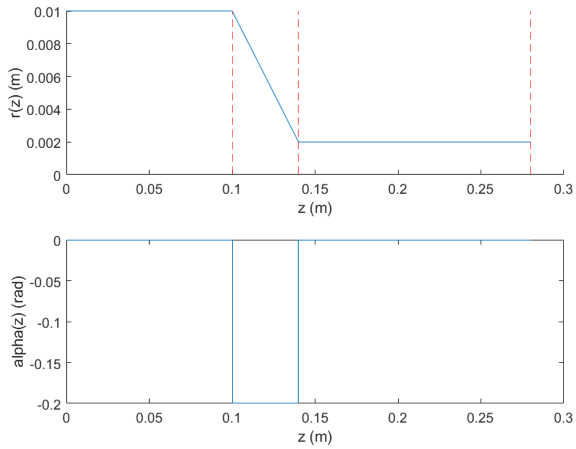


Figure 8: Example of paraxial matrix optics bench beam propagation for a simple example of a galileo telescope with two lenses and magnification of 5:1.

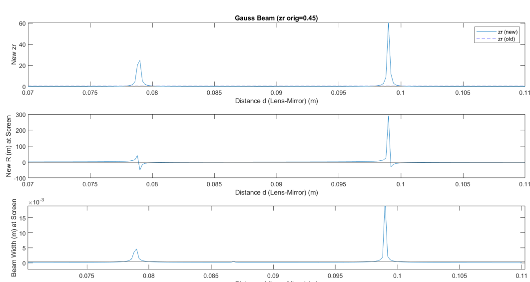


Figure 9: Example of paraxial matrix optics used for Gaussian beam propagation.

References

[1] Howard Y. H. Huang, L. Tian, Z. Zhang, Y. Liu, Z. Chen, and G. Barbastathis. Path-independent phase unwrapping using phase gradient and total-variation (tv) denoising. *Opt. Express*, 20(13):14075–14089, Jun 2012.

[2] Dieter Meschede. *Optik, Licht und Laser*. Teubner Studienbücher Physik. Vieweg+Teubner Verlag, Wiesbaden and s.l., 2., überarbeitete und erweiterte auflage edition, 2005.