Documentation Software: GaussCAD v.03

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GaussCAD is a matlab based simulator for evaluating Gaussian optics and simulate projection onto a photodiode. Visual representation of intensity/phase fields as described by Gaussian optics. A simple paraxial optics (ABCD matrix) tracer functionality is included to propagate gaussian beams. It makes uses of paraxial optics to propagate Gaussian Rays through optical elements.
The Code is available at https://github.com/alexschultze/GaussCAD .

1 Gaussian Optics Beam (Class @gaussian.gaussbeam)

$$\mathbf{E}(r,z) = E_0\,\hat{\mathbf{x}}\,\frac{w_0}{w(z)}\exp\left(\frac{-r^2}{w(z)^2}\right)\exp\left(-i\left(kz + k\frac{r^2}{2R(z)} - \psi(z)\right)\right) \tag{1}$$

r is the radial distance from the center axis of the beam,
z is the axial distance from the beam's focus (or "waist"),
i is the imaginary unit,
k = 2n/ is the wave number (in radians per meter) for a free-space wavelength
, and n is the index of refraction of the medium in which the beam propagates,
E0 = E(0, 0), the electric field amplitude (and phase) at the origin at time 0,
w(z) is the radius at which the field amplitudes fall to 1/e of their axial values (i.e., where the intensity values fall to 1/e2 of their axial values), at the plane z along the beam,
w0 = w(0) is the waist radius,
R(z) is the radius of curvature of the beam's wavefronts at z, and
(z) is the Gouy phase at z, an extra phase term beyond that attributable to the phase velocity of light.

The software can be used to describe beams in the beam reference system E(r,z). Alternatively the beam can be evaluated in cartesian coordinates. In cartesian definition the vector p describes the location of the beam waist and the vector n describes the axial direction vector along along which z distance propagates.

2 Gaussian Screen (Class @gaussian.field-beam)

Field screen represents a two dimensional pixel sensor array. It accepts two input beams and evaluates their field at each pixel. The screen is described by its parameters of position r, normal vector n, its physical dimension (2D) and pixel count (2D).
When rendered, the current field value for each beam is calculated and saved for each pixel.
The class provides additional functions for masking and segmentation to represent the shape of actual quadrant photodiode with gap.
The pixels irradiance (intensity) is equal to the magnitude of sum of the two phasors squared. The AC, DC is determined by the maximum and minimum values of the phasors magnitude and their difference. The phase information (interferogram) for each location pixel is the difference between the two beam phasor angles.

$$AC(x,y,z) = min(mag(E1(x,y,z)), mag(E2(x,y,z))) \tag{2}$$

$$DC(x,y,z) = max(mag(E1(x,y,z)), mag(E2(x,y,z))) - AC; \tag{3}$$

$$Phase(x,y,z)\phi = angle(E1(x,y,z)) - angle(E2(x,y,z)) \tag{4}$$

$$Int(x,y,z) = \sum_{n=1}^{2} E_n(x,y,z) \tag{5}$$

The interferogram may contain phase jumps which is in the next setup removed by 2D unwrapping based on total-variation (TV) denoising. [1]
Because each pixel has a different impact on the overall phase information recovered, the phase is weighed by the AC amplitude.

$$\phi_w(x,y,z) = AC(x,y,z) \cdot e \exp -i \cdot \phi \tag{6}$$

The actual phase when combining several pixel arrays (full screen, masked screen, quadrants) is calculated as the resulting phase of the sum of all pixel elements.

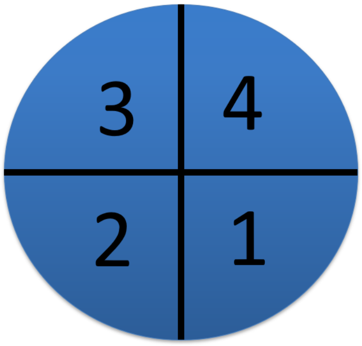
$$\Phi = angle(\sum \phi_w(x,y,z)) \tag{7}$$

The class implements Differential Wavefront Sensing evluation by masking quadrants either orthogonal or diagonally. The definition for calculating the phase difference between quadrants is is given in the following. Q1..Q4 denote the 4 quadrants.

$$DWS_y = \frac{1}{2} \cdot \frac{\Phi_{Q2} + \Phi_{Q3}}{\Phi_{Q1} + \Phi_{Q4}}; DWS_z = \frac{1}{2} \cdot \frac{\Phi_{Q3} + \Phi_{Q4}}{\Phi_{Q1} + \Phi_{Q2}} \tag{8}$$

$$DWS_d1 = \frac{\Phi_{Q1}}{\Phi_{Q3}}; DWS_d2 = \frac{\Phi_{Q2}}{\Phi_{Q4}} \tag{9}$$

Figure 1: Definition of Segments



3 Paraxial Optics Bench (Class @paraxial.bench)

The paraxial bench can be used to visualise and calculate the interaction of optical elements for geometrical rays. Change of gaussian beam parameters $q(z) = z + iz_R$, where z is the distance from the beam waist and $z_r = z_0$ the Rayleigh length, is calculated according to the following formula. The multiplication is performed on each optical element individually, for gaussian propagation the matrices of optical elements can not be combined by multiplication.

$$q_1(z) = \frac{Aq_0 + B}{Cq_0 + D} \tag{10}$$

References

[1] Howard Y. H. Huang, L. Tian, Z. Zhang, Y. Liu, Z. Chen, and G. Barbastathis. Path-independent phase unwrapping using phase gradient and total-variation (tv) denoising. *Opt. Express*, 20(13):14075–14089, Jun 2012.