Accurate Estimation of DCIR using RK4 method and Machine Learning

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Abstract

The internal resistance of lithium-ion cells is a critical parameter that significantly impacts their performance, safety, and longevity. Overcharging and overdischarging are common stress conditions that can lead to an increase in internal resistance, potentially causing cell degradation or catastrophic failure such as thermal runaway. Traditional methods of estimating internal resistance involve complex physical measurements that are often time-consuming and require sophisticated equipment. In this paper, we propose a machine learning-based approach to accurately estimate the internal resistance of lithium-ion cells under overcharging and overdischarging conditions. We utilize a dataset comprising voltage, current, temperature, and time data collected from lithium-ion cells subjected to controlled overcharge and overdischarge scenarios. Feature engineering techniques are employed to extract relevant features that correlate with internal resistance changes. We then apply a combination of supervised and unsupervised learning algorithms, including regression models and clustering techniques, to identify patterns and predict internal resistance variations.

Our results demonstrate that the proposed machine learning models can predict internal resistance with high accuracy, outperforming traditional estimation methods. The models also effectively capture the nonlinear relationships between operating conditions and internal resistance, providing valuable insights into the impact of overcharging and over-discharging. This approach offers a fast, reliable, and scalable solution for monitoring battery health, enabling the development of more robust battery management systems (BMS) that can prevent failure and extend the lifespan of lithium-ion cells.

1 INTRODUCTION

Lithium-ion batteries have become the dominant energy storage technology in a wide range of applications, from consumer electronics to electric vehicles and renewable energy systems, due to their high energy density, long cycle life, and superior efficiency. However, ensuring their safe and reliable operation remains a significant challenge, particularly under extreme conditions such as overcharging and over-discharging. One of the most critical parameters influencing the performance, safety, and lifetime of lithium-ion batteries is the internal resistance, often expressed as Direct Current Internal Resistance (DCIR). This parameter directly affects power delivery, energy efficiency, and thermal stability, as it governs voltage drop and heat generation within the cell during charge and discharge cycles. An increase in internal resistance not only reduces performance but can also accelerate degradation and, in severe cases, trigger thermal runaway events. Therefore, accurate DCIR estimation is essential to optimize battery management systems (BMS) and improve operational safety. Traditional methods for estimating internal resistance, such as the voltage drop method and electrochemical impedance spectroscopy (EIS), have been widely used but suffer from limitations. Althoughugh the voltage drop method is simple and suitable for real-time applications, it often fails to capture transient and nonlinear behaviors arising from variations in temperature, state of charge ()C) and cycling conditions. Similarly, EIS provides detailed electrochemical insights but requires expensive instrumentation and controlled laboratory environments, making it impractical for in-situ applications. To overcome these limitations, advanced numerical techniques, such as the Runge-Kutta method, have been introduced to model the dynamic behavior of lithium-ion cells. The Runge-

Kutta approach enables an accurate estimation of the internal resistance by solving the differential equations that describe the equivalent circuit model of the cell, thereby capturing transient and nonlinear effects with high precision. In parallel, recent advancements in data-driven methodologies have opened new pathways for internal resistance estimation. Machine learning, in particular, offers the capability to learn complex nonlinear relationships among battery parameters such as voltage, current, temperature, and time. By leveraging historical and operational data, machine learning models can predict internal resistance more efficiently and accurately than traditional analytical approaches. While supervised learning techniques have shown promising results, they often require extensive labeled datasets, which are difficult to obtain in real-world scenarios. In this context, unsupervised learning algorithms can play a pivotal role by uncovering intrinsic patterns in unlabeled data, enabling more autonomous and adaptable resistance estimation frameworks. This paper proposes an integrated approach that combines the Runge-Kutta numerical method with machine learning algorithms to achieve accurate DCIR estimation under dynamic operating conditions. The Runge-Kutta method provides precise modeling of transient responses, while machine learning enhances adaptability and prediction accuracy through data-driven learning. Together, these methods form a robust hybrid framework capable of real-time monitoring and predictive diagnostics, contributing to safer, more reliable, and longer-lasting lithium-ion battery systems.

$V_{OCV}(t)$ = Open circuit Voltage; R_0= Internal Resistance; R_1= Polarization Resistance; C_1= Polarization Capacitance; $V_{model}(t)$= Terminal Voltage; $V_{RC}(t)$= Voltage across the RC parallel circuit

2 RESEARCH METHODOLOGY

2.1 Equivalent Circuit Model

In this chapter, the electrical cell model is described by an equivalent circuit model based on a combination of various components such as voltage source, active, and passive elements. There are many models available to approximate and characterize the dynamic behavior of the battery model. However, Thevenin equivalent circuit model is widely used to model lithium-ion battery because of its simple structure and can describe the dynamic response of the battery with minimal error. The equivalent circuit model consists of

N number of resistors, capacitors as shown in figure1: below

However,In this paper, to avoid complex computation, only single RC Thevenin equivalent model is used. This model consists of three components: (1) internal resistance (R_0), (2)resistor (R_1), capacitor (C_1) parallel network (where R_1 is equivalent polarization resistance, C_1 is equivalent polarization capacitance), which is used to characterize or simulate transient behaviors of battery during charging and discharging; and (3) $V_{OCV}(SOC(t))$ is a non-linear function. In this model current $I(t)$ as the model control input and the terminal voltage $V_{model}(t)$ as measured output. Figure 2: Below

2.2 Equations of State Dynamics

Theoretically, DCIR of the cell in this model depends on depends on temperature (T), state-of-charge (SOC(t)), calendar fade / state-of-health (SOH), and charge/discharge rate (I / C-rate). The analysis of the Thevenin equivalent circuit shown in Figure 1 is straightforward. Using Kirchhoff's law, the model equation can be expressed as follows:

$$V_{model}(t) = OCV(SOC(t), T) - R_0(T, SOC, SOH, Cr)I(t) - V_{RC}(t) \quad (1)$$

The transient voltage across the RC branch, $v_{RC}(t)$, follows a first-order differential equation:

$$\frac{dV_{RC}}{dt} = -\frac{1}{R_1 C_1} V_{RC} + \frac{1}{C_1} I(t) \quad (2)$$

The SOC evolves according to Coulomb counting:

$$\frac{d\text{SOC}}{dt} = -\frac{I(t)}{Q_{\text{nom}}} \quad (3)$$

where Q_{nom} is the nominal cell capacity in Coulombs. The equations above describe the dynamic voltage and charge balance under arbitrary current excitation

2.3 Temperature- and SOC-dependent Parameterization of R_0

The instantaneous ohmic resistance R_0 is a function of temperature, SOC, SOH, and current rate, expressed as a multiplicative model:

$$R_0(T, SOC, SOH, C_r) = R_{0,\text{ref}}\ F_T(T; E_a)\ F_{SOC}(SOC; \beta)\ F_{SOH}(SOH; \gamma)\ F_C(C_r; \kappa) \quad (4)$$

where $R_{0,\text{ref}}$ is the reference resistance at nominal conditions (T_{ref}, SOC=0.5, SOH=1, $C_r = 1$). The correction factors are described as follows.

2.3.1 (a) Temperature Dependence

The temperature factor F_T follows the Arrhenius-type relationship:

$$F_T(T; E_a) = \exp\left[\frac{E_a}{R_g}\left(\frac{1}{T} - \frac{1}{T_{\text{ref}}}\right)\right] \tag{5}$$

where E_a is the activation energy, and $R_g = 8.314\ \text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ is the universal gas constant.

2.3.2 (b) SOC Dependence

The SOC-dependent factor F_{SOC} is approximated using a quadratic function:

$$F_{\text{SOC}}(\text{SOC}; \beta_1, \beta_2) = 1 + \beta_1(1 - \text{SOC})^2 + \beta_2(\text{SOC})^2 \tag{6}$$

This captures the characteristic rise of resistance at low and high SOCs due to electrode overpotential effects.

2.3.3 (c) Aging Dependence

Calendar aging effects are represented through the state of health factor F_{SOH}, or, alternatively, as a function of aging time t_{age}:

$$F_{\text{SOH}}(\text{SOH}; \gamma_1) = 1 + \gamma_1(1 - \text{SOH}) \tag{7}$$

or

$$F_{\text{age}}(t_{\text{age}}; k_{\text{age}}) = 1 + k_{\text{age}}\sqrt{t_{\text{age}}} \tag{7}$$

These empirical relationships capture the gradual increase in internal resistance during calendar degradation.

2.3.4 (d) C-rate Dependence

Rate effects are modeled through an empirical power-law expression :

$$F_C(C_r; \kappa_1, \kappa_2) = 1 + \kappa_1|C_r|^{\kappa_2} \tag{8}$$

which represents the dependence of R_0 on instantaneous discharge/charge rate.

2.4 Discretization Using the Runge–Kutta Method

The state equations (2) and (3) are integrated using the fourth-order Runge–Kutta (RK4) method due to its balance between computational efficiency and accuracy [4,13]. Defining the state vector $\mathbf{x} = [v_{RC}, \mathrm{SOC}]^T$ and system function $f(\mathbf{x}, I)$, the RK4 integration for a time step Δt is expressed as:

$$k_1 = f(x_k, I_k) \tag{1}$$

$$k_2 = f\left(x_k + \frac{\Delta t}{2} k_1, I_{k+1/2}\right) \tag{2}$$

$$k_3 = f\left(x_k + \frac{\Delta t}{2} k_2, I_{k+1/2}\right) \tag{3}$$

$$k_4 = f\left(x_k + \Delta t\, k_3, I_{k+1}\right) \tag{4}$$

$$x_{k+1} = x_k + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{9}$$

where $I_{(k+1/2)} = (I_k + I_{(k+1)})/2$. This discretization minimizes numerical diffusion errors in fast transient voltage prediction compared to lower-order methods.

Defining the residue for parameter estimation at sample k:

$$r_k(\theta) = V_{\mathrm{model},k} - V_{\mathrm{meas},k} \tag{11}$$

The Jacobian with respect to each parameter θ_p is:

$$\frac{\partial r_k}{\partial \theta_p} = -I_k \frac{\partial V_{model,k}}{\partial \theta_p} \tag{12}$$

The objective here is Jacobian and is expressed as follows.

$$J(\theta) = \frac{1}{2} \sum_{k=0}^{N-1} r_k(\theta)^2 \tag{13}$$

Thus the entire Jacobian $J \in R^{N \times m}$ (with m = number of parameters) is built by computing derivatives of the multiplicative model (4). Below are the chain-rule formulas you will implement (all evaluated at sample k; I omit the explicit k subscript in the formulas for clarity).

First set

$$F_{all} = F_T(T; E_a)\, F_{SOC}(SOC; \beta)\, F_{SOH}(SOH; \gamma)\, F_C(C_r; \kappa) \tag{14}$$

$$R_0 = R_{0,\text{ref}}.F_{all} \tag{15}$$

Then derivatives: w.r.t. $R_{0,\text{ref}}$:

$$\frac{\partial R_0}{\partial R_{0,\text{ref}}} = F_{\text{all}} \tag{16}$$

w.r.t. E_a (Arrhenius term):

$$\frac{\partial R_0}{\partial E_a} = R_{0,\text{ref}}.F_{all}.\frac{1}{R_g}.(\frac{1}{T} - \frac{1}{T_{ref}}) \tag{17}$$

w.r.t. β_1 (SOC quadratic coefficient in example (6)):

$$\frac{\partial R_0}{\partial \beta_1} = R_{0,\text{ref}} \cdot F_T \cdot F_{SOH} \cdot F_C \cdot (1 - \text{SOC})^2 \tag{18}$$

w.r.t. β_2:

$$\frac{\partial R_0}{\partial \beta_2} = R_{0,\text{ref}}\ F_T\ F_{SOH}\ F_C\ SOC^2 \tag{19}$$

w.r.t. γ_1 (SOH linear factor):

$$\frac{\partial R_0}{\partial \gamma_1} = R_{0,\text{ref}}\ F_T\ F_{SOC}\ F_C\ (1 - \text{SOH}) \tag{20}$$

w.r.t. κ_1(C-rate amplitude):

$$\frac{\partial R_0}{\partial \kappa_1} = R_{0,ref} F_T F_{SOC} F_{SOH} \mid C_r \mid^{\kappa_2} \tag{21}$$

w.r.t. κ_2(C-rate exponent):

$$\frac{\partial R_0}{\partial \kappa_2} = R_{0,ref} F_T F_{SOC} F_{SOH} \kappa_1 |C_r|^{\kappa_2} \ln |C_r| \tag{22}$$

(Handle the special case $C_r = 0$ numerically — replace $\ln |C_r|$ with 0 when $|C_r| < \epsilon$ and/or regularize.) Finally the Jacobian row for sample kis:

$$J_{k,:} = [\, -I_k \frac{\partial R_0}{\partial \theta_1},\ -I_k \frac{\partial R_0}{\partial \theta_2},\ \dots\,] \tag{23}$$

Because R_0 does not influence the state ODEs (with R_1, C_1 constant), we do not need to integrate variational equations — this substantially simplifies and speeds up parameter estimation.

$J(\theta)$ can be minimized with a gradient-based solver (Gauss–Newton / Levenberg–Marquardt).

2.5 Parameter update (Gauss–Newton / LM):

Forming the stacked residual vector $r \in \mathbb{R}^N$ and Jacobian $J \in \mathbb{R}^{N\times m}$. A GN step solves

$$\Delta R_0 = -(J^T J)^{-1} J^T r \tag{24}$$

$$J^T J = \sum I(k)^2 \tag{25}$$

$$J^T r = \sum I(k) r_k \tag{26}$$

$$\Delta R_0 = -\frac{\sum I(k) r_k}{\sum I(k)^2} \tag{27}$$

LM adds damping: LM modifies:

$$(J^T J + \lambda I)\Delta R_0 = -J^T r \tag{28}$$

This equation (28) prevents divergence when voltage curve is flat, covariance is ill-conditioned, measurement noise is high If λ is large then the curve is gradient descent If λ is small then curve is Gauss–Newton Therefore LM = safe + fast.

$$\Delta R_0 = -\frac{\sum I(k) r_k}{\sum I(k)^2 + \lambda} \tag{29}$$

The final internal resistance of a lithium ion cell is calculated as follows:

$$R_{0,ref,new} = R_{0,ref} + \Delta R_0 \tag{30}$$

3 RESULTS AND DISCUSSION

Research results in the form of data or numbers are presented in tables or graphs. If the research is carried out application/software development, some important *screenshots* can be presented. Each table, graph or figure must be referred to in the text/paragraph.

The discussion section provides *insights* into the data obtained in the research. This section can present tables or graphs that are the result of data processing (not just raw data). The author is required to explain the findings obtained in the research accompanied by clear evidence. This section can contain reviews that compare the results obtained in this study with the results obtained in previous studies.

1. Writing Format

Paragraphs should be organized and consistent. Pay attention to the spelling and formatting of foreign terms. All paragraphs must be right-aligned and left-aligned. The entire document should be written in Times New Roman font with *single* spacing. Other fonts may be used if there is a specific purpose. The title consists of a maximum of 10 words.

2. *Author*

Authors should not indicate the name of their position (e.g. Supervisor), academic degree (e.g. Dr) or membership of any professional organization (e.g. Senior Member IEEE). To avoid confusion, the last name of each *author* should be written at the end, not abbreviated and marked with a comma (e.g. Rizky Prabowo becomes Prabowo, Rizky). Each affiliation must include, at a minimum, the name of the company and the name of the author's country of residence (e.g. University of Lampung, Indonesia). An email address is required for the corresponding author.

3. Math Formulation

Equations must be written using the latest version of Ms Equation Editor (found in the latest version of Ms Word) or using the Mathtype application. Writing symbol descriptions in equations is made in descriptive paragraphs, not list items as in book writing. Equations must be typed with an indent of 1.27 pt and numbered sequentially starting with the number (1) on the right. *Author* can use *ref_rumus style* for formula writing rules.

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}(1) \tag{5}$$

$$Vmodel(t) = VOCV(SOC, T) - I(t)R0(T, SOC, Cfade) - VRC(t) \tag{6}$$

$$V_{model}(t) = OCV(SOC(t)) - R_0(T)\, I(t) - v_{RC}(t) \tag{4}$$

4. Table and Figure/Graphic Writing

4 CONCLUSIONS

The Conclusion section summarizes the reviews written in the Results and Discussion sections. The Conclusion section is written in paragraph form, no

need to use numbers. The paper will not be reformatted, so please stick to the instructions given above. Otherwise, it will be returned to the *author*. Please upload your article in .doc/.docx form on the Jurnal Pepadun website `https://pepadun.fmipa.unila.ac.id`. Articles sent via e-mail will not be processed.

If necessary, acknowledgments can be made here. Acknowledgments should be directed to research funders or experts who provided significant assistance in the completion of the research.

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