DS 223: Marketing Analytics

Homework 3 - Survival Analysis

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To build AFT models, we first need to load the data. We will be working with the Telco Customer Churn dataset, which has the following columns:

- ID: subscriber's ID
- region: region code
- tenure: lifetime (in months)
- age: subscriber's age
- marital: subscriber's marital status
- address: number of years living in the same address
- income: subscriber's annual income (K)
- ed: subscriber's education level
- retire: retired (Yes/No)
- gender: subscriber's gender (Male/Female)
- voice: voice service (Yes/No)
- internet: internet service (Yes/No)
- forward: call forwarding (Yes/No)
- custcat: customer category
- churn: whether the customer churned (Yes/No)

```r
# Read the CSV file
telco <- read.csv("telco.csv")
telco$churn = ifelse(telco$churn=='Yes', 1, 0)

head(telco)
```

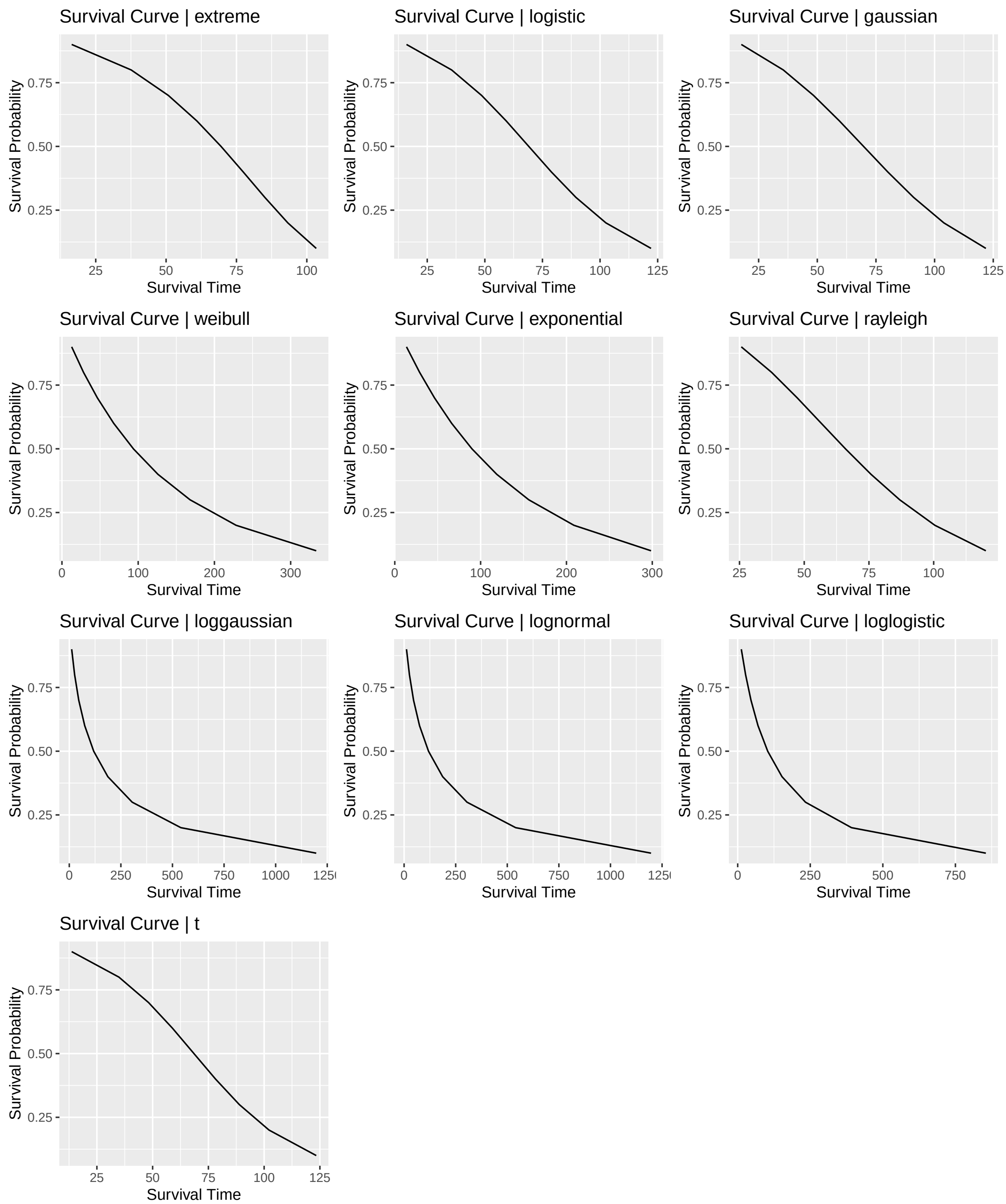
```
##   ID region tenure age   marital address income                           ed
## 1  1 Zone 2     13  44   Married       9     64              College degree
## 2  2 Zone 3     11  33   Married       7    136    Post-undergraduate degree
## 3  3 Zone 3     68  52   Married      24    116 Did not complete high school
## 4  4 Zone 2     33  33 Unmarried      12     33          High school degree
## 5  5 Zone 2     23  30   Married       9     30 Did not complete high school
## 6  6 Zone 2     41  39 Unmarried      17     78          High school degree
##   retire gender voice internet forward       custcat churn
## 1     No   Male    No       No     Yes Basic service     1
## 2     No   Male   Yes       No     Yes Total service     1
## 3     No Female    No       No      No  Plus service     0
## 4     No Female    No       No      No Basic service     1
## 5     No   Male    No       No     Yes  Plus service     0
## 6     No Female    No       No      No  Plus service     0
```

Now, we will build basic models (intercept-only) with all the different distributions available in `survreg` package.

```r
surv_obj = Surv(time=telco$tenure, event=telco$churn)
reg_models <- list()

for(distribution in names(survreg.distributions)){
    # get the regression model
    reg_m = survreg(formula=surv_obj~1, dist=distribution)

    # print the summary
    # summary(reg_m)

    # add reg_m to reg_models
    reg_models[[distribution]] <- reg_m
}
```

As we have the models now, let's visualize the probability of churn during customer lifetime using the models in different plots and have an initial look at them.

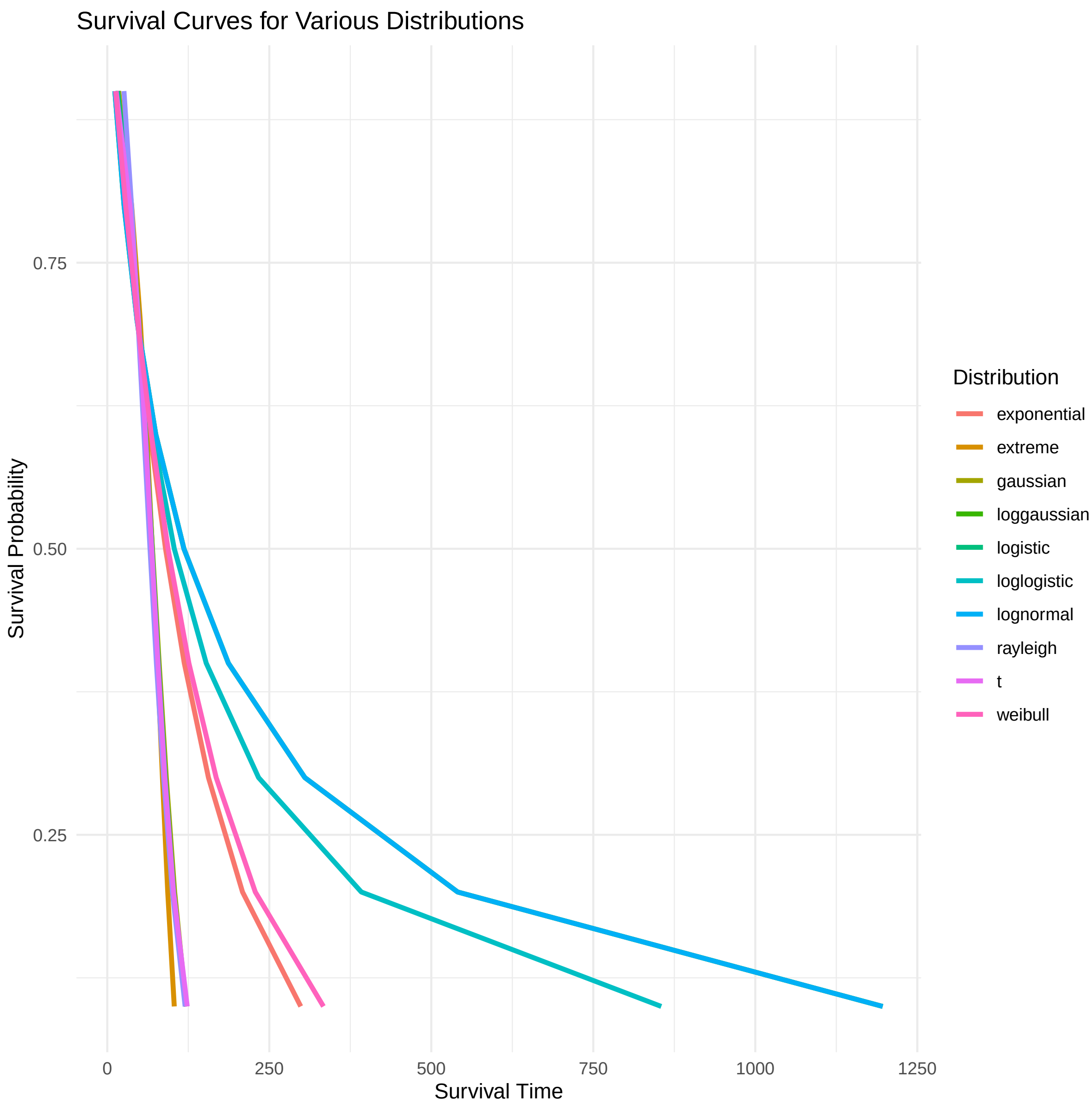
```r
# Initialize an empty list for storing plots
plot_list <- list()

for (distribution in names(survreg.distributions)) {
    reg_m <- reg_models[[distribution]]

    probs <- seq(.1, .9, length=9)
    pred <- predict(reg_m, type="quantile", p=1-probs, newdata=data.frame(1))

    df <- data.frame(Time=pred, Probabilities=probs)

    # Generate the plot for current distribution
    p <- ggplot(df, aes(x = Time, y = Probabilities)) +
        geom_line() +
        labs(title = paste("Survival Curve |", distribution),
             x = "Survival Time",
             y = "Survival Probability")

    # Store the plot in the list
    plot_list[[distribution]] <- p
}

# Combine the plots into a grid (4x3) and leave the last two positions blank
plot_grid <- wrap_plots(plot_list, nrow = 4, ncol = 3) +
             plot_spacer() + plot_spacer()

# Print the combined plot grid
print(plot_grid)
```



As we can see, there are indeed some differences between the models. We can plot all the model curves in one graph to be able to compare the models.

```
# Initialize an empty data frame for storing combined data
combined_df <- data.frame()

for (distribution in names(survreg.distributions)) {
    reg_m <- reg_models[[distribution]]

    probs <- seq(.1, .9, length=9)
    pred <- predict(reg_m, type="quantile", p=1-probs, newdata=data.frame(1))
```

```
    df <- data.frame(Time = pred, Probabilities = probs, Distribution = distribution)

    # Combine data
    combined_df <- rbind(combined_df, df)
}

# Generate a single plot with curves for all distributions
p <- ggplot(combined_df, aes(x = Time, y = Probabilities, color = Distribution)) +
    geom_line(linewidth=1.2) +
    labs(title = "Survival Curves for Various Distributions",
         x = "Survival Time",
         y = "Survival Probability") +
    theme_minimal()

# Print the plot
print(p)
```



Lognormal seems to be the better one, however, let's also compare the AIC and BIC.

```r
combined_scores <- data.frame(Distribution = character(), AIC = numeric(), BIC = numeric())


for(distribution in names(survreg.distributions)){
    reg_m <- reg_models[[distribution]]

    extracted_scores <- extract(
      reg_m,
      include.aic = TRUE,
      include.bic = TRUE
    )

    # Extract AIC and BIC names
    score_names <- extracted_scores@gof.names

    # Find indices of AIC and BIC in the names
    aic_index <- which(score_names == "AIC")
    bic_index <- which(score_names == "BIC")

    # Extract AIC and BIC scores
    aic <- extracted_scores@gof[aic_index]
    bic <- extracted_scores@gof[bic_index]

    combined_scores <- rbind(combined_scores,
                             tibble(Distribution = distribution,
                                    AIC = aic,
                                    BIC = bic))
}
# Order the scores by AIC and BIC
combined_scores_ordered <- combined_scores[order(combined_scores$AIC, combined_scores$BIC), ]

# Print the combined scores data frame
print(combined_scores_ordered)
```

```
## # A tibble: 10 x 3
##    Distribution   AIC   BIC
##    <chr>        <dbl> <dbl>
##  1 loggaussian  3209. 3219.
##  2 lognormal    3209. 3219.
##  3 loglogistic  3214. 3224.
##  4 exponential  3216. 3221.
##  5 weibull      3217. 3227.
##  6 gaussian     3433. 3443.
##  7 logistic     3472. 3482.
##  8 rayleigh     3481. 3486.
##  9 extreme      3498. 3508.
## 10 t            3500. 3510.
```

As we can see the models with `loggaussian` and `lognormal` distributions have lower AIC and BIC. Let's pick the `lognormal` one go on with it. We'll train a new model adding some of the variables to it. But first, we'll define the order to some of the factor variables.

```r
# Define the education order
ed_order <- c("Did not complete high school", "High school degree", "Some college", "College degree"

# Apply the education order to the respective variable
telco$ed <- factor(telco$ed, levels = ed_order)
```

Let's add gender all the columns to the model and then remove those that are not statistically significant to the model (we assume that the p-values of the models with only one covariate and the model with said covariate and some others are not very different).

```
reg_f= survreg(surv_obj ~ region + age + marital + address + income + ed + retire + gender + voice +
               data=telco, dist="lognormal")
summary(reg_f)
```

```
##
## Call:
## survreg(formula = surv_obj ~ region + age + marital + address +
##     income + ed + retire + gender + voice + internet + forward +
##     custcat, data = telco, dist = "lognormal")
##                                Value Std. Error     z       p
## (Intercept)                  2.73588    0.31345  8.73 < 2e-16
## regionZone 2                -0.09704    0.14277 -0.68   0.497
## regionZone 3                 0.04822    0.14154  0.34   0.733
## age                          0.03267    0.00725  4.50 6.7e-06
## maritalUnmarried            -0.45515    0.11543 -3.94 8.0e-05
## address                      0.04254    0.00890  4.78 1.8e-06
## income                       0.00140    0.00092  1.52   0.129
## edHigh school degree        -0.05768    0.18724 -0.31   0.758
## edSome college              -0.10129    0.20103 -0.50   0.614
## edCollege degree            -0.37361    0.20159 -1.85   0.064
## edPost-undergraduate degree -0.40797    0.26920 -1.52   0.130
## retireYes                    0.02248    0.44407  0.05   0.960
## genderMale                   0.05188    0.11429  0.45   0.650
## voiceYes                    -0.43379    0.16895 -2.57   0.010
## internetYes                 -0.77150    0.14348 -5.38 7.6e-08
## forwardYes                  -0.19813    0.18004 -1.10   0.271
## custcatE-service             1.06642    0.17053  6.25 4.0e-10
## custcatPlus service          0.92495    0.21575  4.29 1.8e-05
## custcatTotal service         1.19860    0.25045  4.79 1.7e-06
## Log(scale)                   0.27577    0.04600  6.00 2.0e-09
##
## Scale= 1.32
##
## Log Normal distribution
## Loglik(model)= -1457   Loglik(intercept only)= -1602.5
##  Chisq= 291.01 on 18 degrees of freedom, p= 3.4e-51
## Number of Newton-Raphson Iterations: 5
## n= 1000
```

As we can see, only the coefficients of `age`, `marital`, `voice`, `internet`, and `custcat` are statistically significant. Let's rebuild the model using only those. Let's also add `ed`, because, I suppose, it might also have some impact on the model (the p-value of some education levels are almost statistically significant).

```
reg_f= survreg(surv_obj ~ age + marital +  voice + internet + custcat,
               data=telco, dist="lognormal")
summary(reg_f)
```

```
##
## Call:
## survreg(formula = surv_obj ~ age + marital + voice + internet +
##     custcat, data = telco, dist = "lognormal")
##                         Value Std. Error     z       p
## (Intercept)            2.1416     0.2346  9.13 < 2e-16
```

```
## age                    0.0576     0.0053 10.86 < 2e-16
## maritalUnmarried      -0.4227     0.1167 -3.62 0.00029
## voiceYes              -0.5279     0.1707 -3.09 0.00198
## internetYes           -0.8980     0.1412 -6.36 2.0e-10
## custcatE-service       1.0905     0.1719  6.34 2.2e-10
## custcatPlus service    0.8823     0.1729  5.10 3.4e-07
## custcatTotal service   1.1313     0.2141  5.28 1.3e-07
## Log(scale)             0.3090     0.0462  6.69 2.3e-11
##
## Scale= 1.36
##
## Log Normal distribution
## Loglik(model)= -1474.1   Loglik(intercept only)= -1602.5
##  Chisq= 256.9 on 7 degrees of freedom, p= 9.4e-52
## Number of Newton-Raphson Iterations: 5
## n= 1000
```

Now, as we have the model, let's have a look at AIC and BIC.

```
extract(
    reg_f,
    include.aic = TRUE,
    include.bic = TRUE
)
```

```
##
##
##                             coef.       s.e.            p
## (Intercept)            2.14158992 0.23455078 6.811916e-20
## age                    0.05760635 0.00530462 1.794220e-27
## maritalUnmarried      -0.42269964 0.11670057 2.922356e-04
## voiceYes              -0.52785517 0.17067406 1.982995e-03
## internetYes           -0.89799989 0.14122107 2.033043e-10
## custcatE-service       1.09048902 0.17190558 2.245524e-10
## custcatPlus service    0.88228952 0.17293336 3.362522e-07
## custcatTotal service   1.13131652 0.21410967 1.265190e-07
## Log(scale)             0.30901747 0.04621185 2.278320e-11
##
##                       GOF dec. places
## AIC              2966.131        TRUE
## BIC              3010.301        TRUE
## Log Likelihood -1474.066        TRUE
## Num. obs.        1000.000       FALSE
```

As we can see the AIC and BIC are better than of the intercept-only model. Now, let's interpret the coefficients. Since we're using a `lognormal` distribution for the model, we'll need to exponentiate the coefficients returned by the model to understand the real effect of the covariates.

```
exp(coef(reg_f))
```

```
##          (Intercept)                  age     maritalUnmarried
##            8.5129618            1.0592979            0.6552754
##             voiceYes          internetYes     custcatE-service
##            0.5898688            0.4073837            2.9757289
##  custcatPlus service custcatTotal service
##            2.4164258            3.0997347
```

Report

As we can see the coefficients for `maritalUnmarried`, `voiceYes`, and `internetYes` are less than one. From this we can conclude that unot married individuals, as well as individuals using voice and/or internet services have less life time, that is, they are more prone to churn earlier compared to married, individuals not using voice and internet services, respectively.

On the contrary, `age` has a coefficient, which is greater than 1. This means as people get older, they are less prone to churn (the lifetime is longer). Same goes for the different customer categories; those using Plus service tend to have longer lifetime (2.4 times more), those using E-service and Total service are even less prone to churn (about 3 times longer lifetime), compared to individuals with Basic service.

CLV

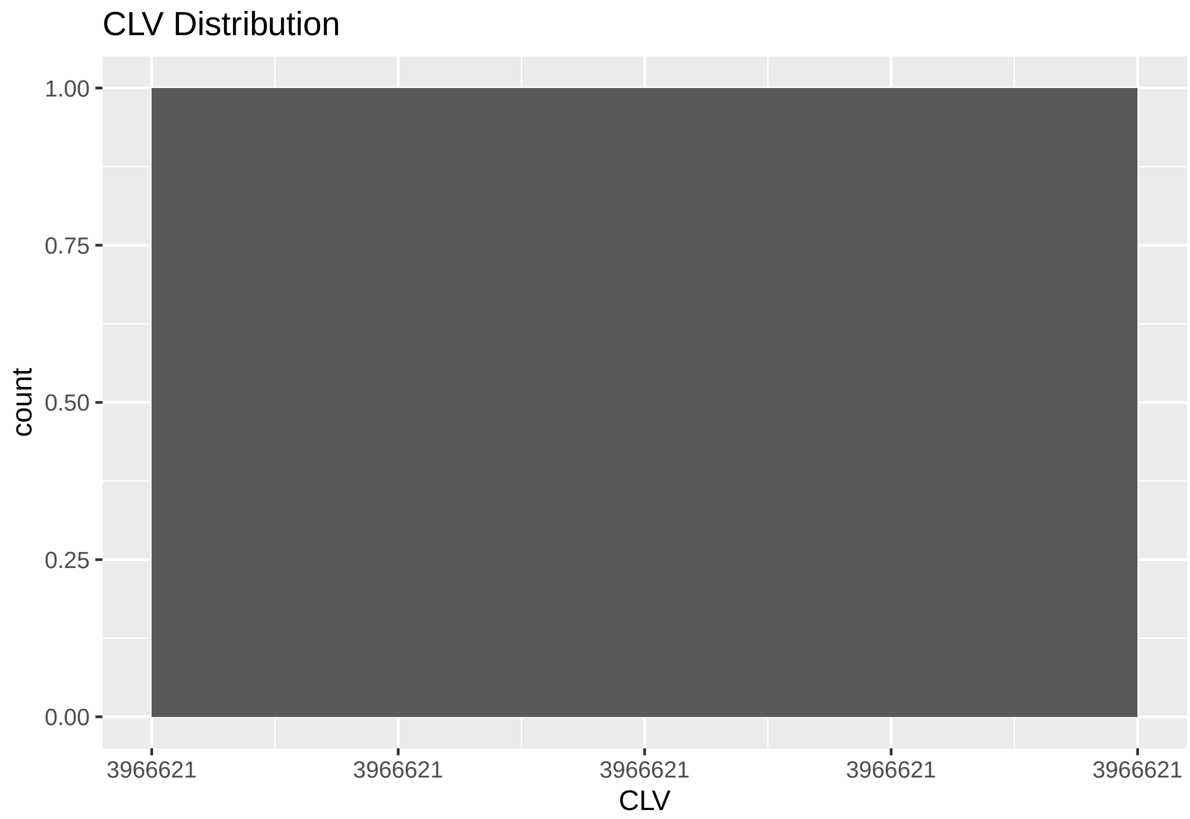
(Something is off here. I didn't have time to understand what the problem is before midnight. My next commits will solve the problem.)

```
pred=predict(reg_f, type="response")

pred_data=data.frame(t(pred))[,0:24]

sequence = seq(1,length(colnames(pred_data)),1)
MM = 1300
r = 0.1

for (num in sequence) {
    pred_data[,num]=pred_data[,num]/(1+r/12)^(sequence[num]-1)
}
```

```
pred_data$CLV=MM*rowSums(pred_data)
summary(pred_data$CLV)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 3966621 3966621 3966621 3966621 3966621 3966621
```

```
ggplot(pred_data,aes(x=CLV))+labs(title = "CLV Distribution")+
geom_histogram()
```




```
telco$CLV = pred_data$CLV
ggplot(telco,aes(x=CLV, color=gender))+
labs(title = "CLV Density By Gender")+
geom_density()
```