Lecture 3: Discrete Random Variables

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The Categorical distribution

Example: The Categorical distribution

- Models an experiment with K outcomes.

$$X = \begin{cases} c_1, & \text{with probability } p_1, \\ \vdots & \\ c_K, & \text{with probability } p_K, \end{cases}$$

- Notation:

$$X \sim \text{Categorical}(p_1, \ldots, p_K)$$

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Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1, 0.3, 0.6)$.

- We have $K = 3$ possible outcomes, say c_1, c_2, c_3.

- The PMF is:

$$P(X = c_1) = 0.1$$
$$P(X = c_2) = 0.3$$
$$P(X = c_3) = 0.6$$

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Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1, 0.3, 0.6)$.

- We have $K = 3$ possible outcomes, say c_1, c_2, c_3.

- The probability that X is either c_1 or c_3.

$$p(X = c_1 \text{ or } X = c_3) = p(X \in \{c_1, c_3\})$$
$$= p(X = c_1) + p(X = c_3)$$
$$= 0.1 + 0.6$$
$$= 0.7$$

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Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1, 0.3, 0.6)$.

- We have $K = 3$ possible outcomes.

- The expectation is:

$$\mathbb{E}[X] = \sum_x x \, p(x) = C_1 \cdot 0.1 + C_2 \cdot 0.3 + C_3 \cdot 0.6$$

Example: PMF of a Categorical

- Assume $X \sim$ Categorical$(0.1, 0.3, 0.6)$.

- We have $K = 3$ possible outcomes.

- The variance is: $\mathbb{V}[X] = \mathbb{E}[X^2] - \left(\mathbb{E}[X]\right)^2$

$$\mathbb{E}[X^2] = \sum_x x^2 \, p(x) = c_1^2 \cdot 0.1 + c_2^2 \cdot 0.3 + c_3^2 \cdot 0.6$$