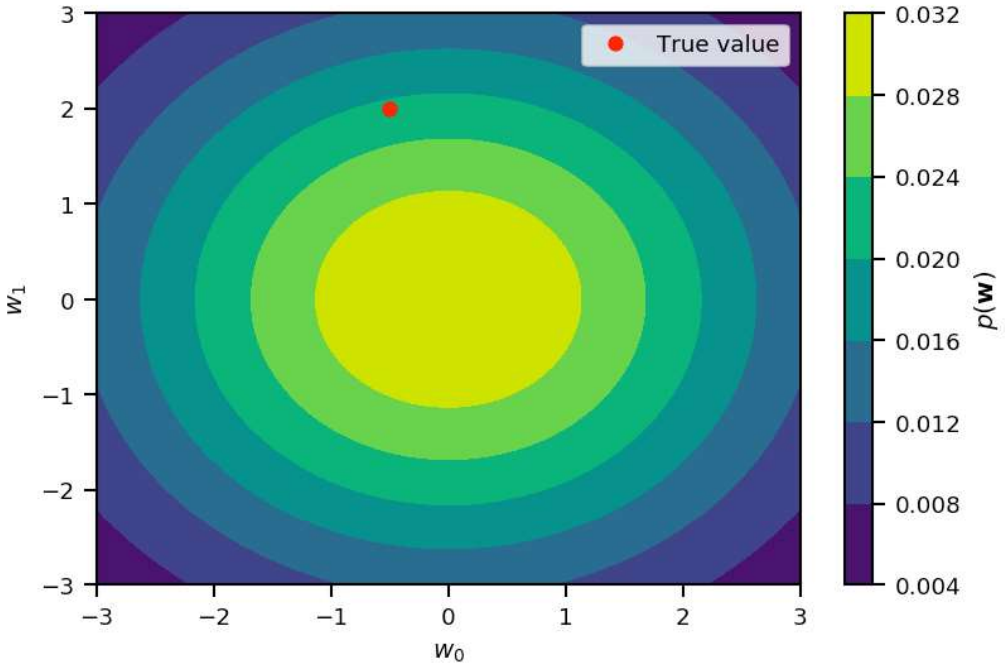
Open questions

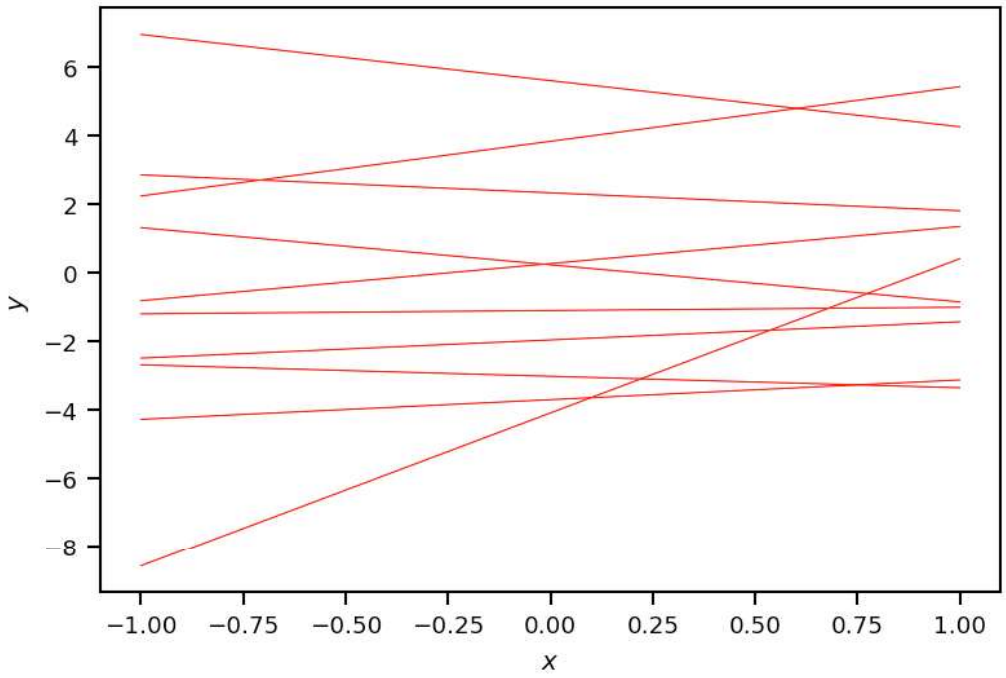
- How do I quantify the measurement noise?
- How do we avoid overfitting?
- How do I quantify epistemic uncertainty induced by limited data?
- How do I choose any remaining parameters?
- How do I choose which basis functions to keep?

Weight prior (linear regression)

Weight space



Model space



PREDICTIVE
SCIENCE LABORATORY

Weight posterior

posterior $\propto$ likelihood $\times$ prior

$$p(\underline{w} \mid \underline{x}_{1:N}, \underline{y}_{1:N}, \sigma) \propto p(\underline{y}_{1:N} \mid \underline{x}_{1:N}, \sigma, \underline{w}) \cdot p(\underline{w})$$

$$= \mathcal{N}\left(\underline{y}_{1:N} \mid \underline{\underline{\Phi}}\, \underline{w}, \sigma^2 \underline{\underline{I}}_N\right) \times \mathcal{N}\left(\underline{w} \mid \underline{0}, a^{-1} \underline{\underline{I}}_m\right)$$

$$\underline{\underline{\Phi}} = \begin{pmatrix} \underline{\varphi}^T(x_1)\, \underline{w} \\ \vdots \\ \underline{\varphi}^T(x_N)\, \underline{w} \end{pmatrix}$$

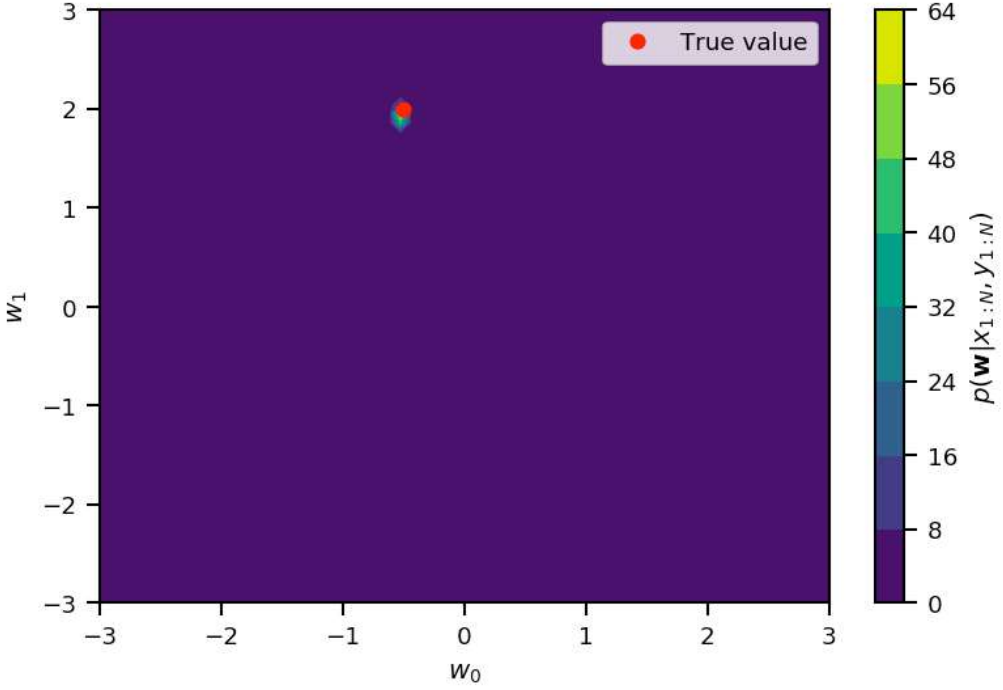
$$= \mathcal{N}\left(\underline{w} \mid \underline{m}, \underline{\underline{S}}\right)$$

$$\underline{\underline{S}} = \left(\sigma^2 \underline{\underline{\Phi}}^T \underline{\underline{\Phi}} + a \underline{\underline{I}}_m\right)^{-1}$$

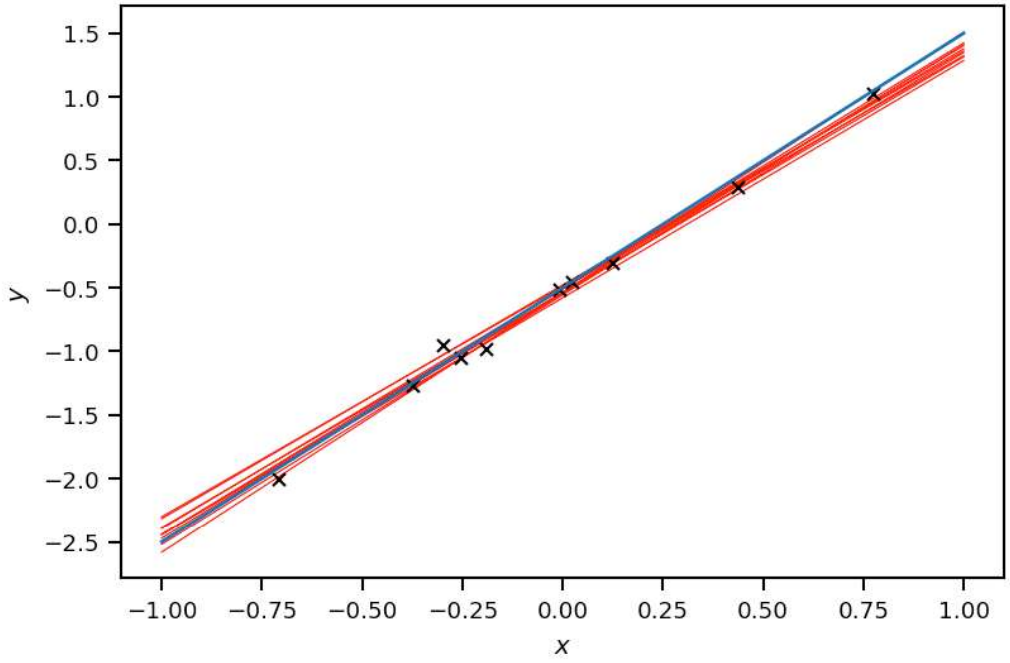
$$\underline{m} = \sigma^{-2} \underline{\underline{S}} \cdot \underline{\underline{\Phi}}^T \underline{y} .$$

Weight posterior (linear regression)

Weight space

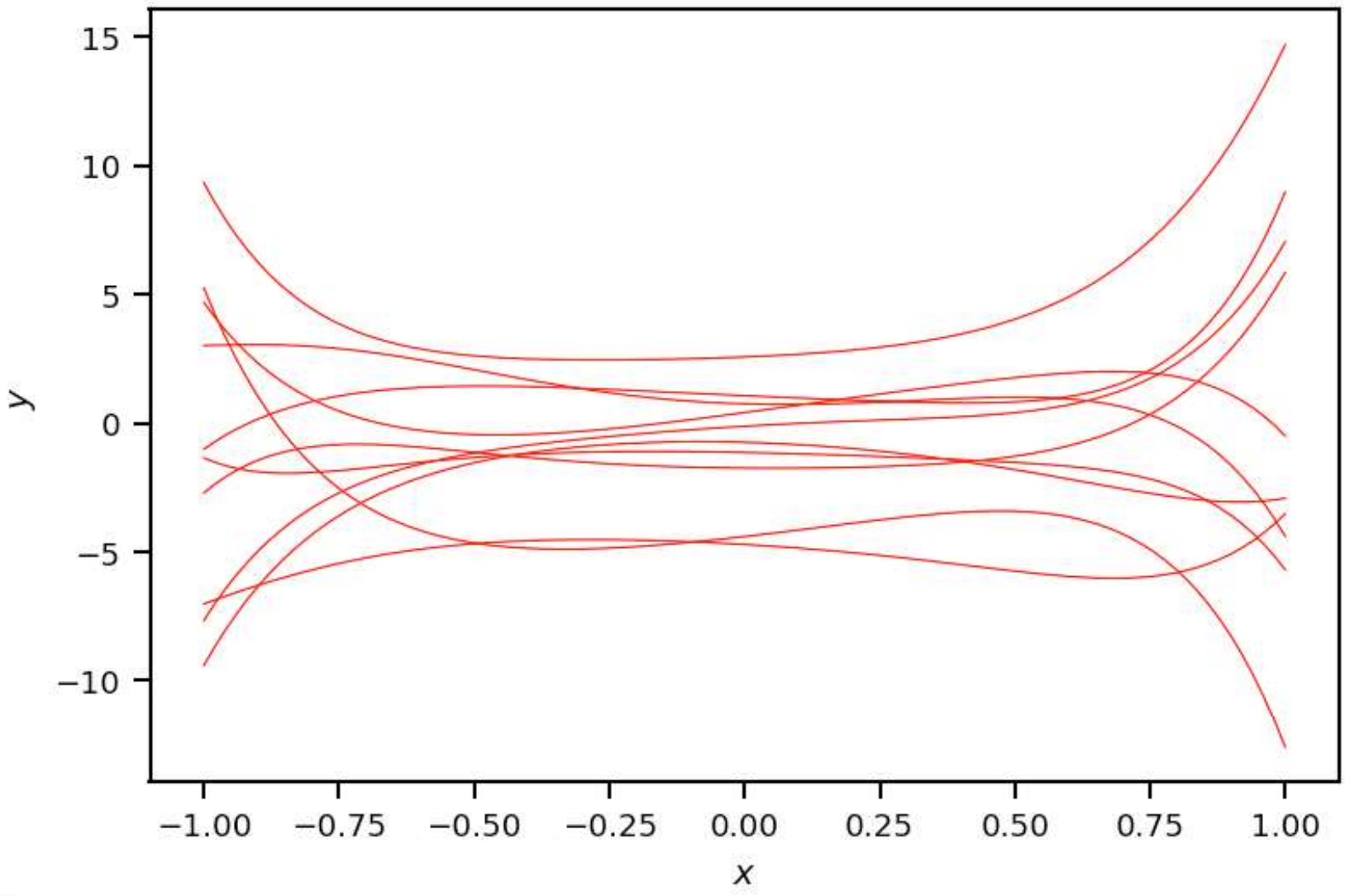


Model space



PREDICTIVE
SCIENCE LABORATORY

Example: 7th degree polynomial (prior)



Example: 7th degree polynomial (posterior)