Lecture 15: Advanced topics in Bayesian linear regression

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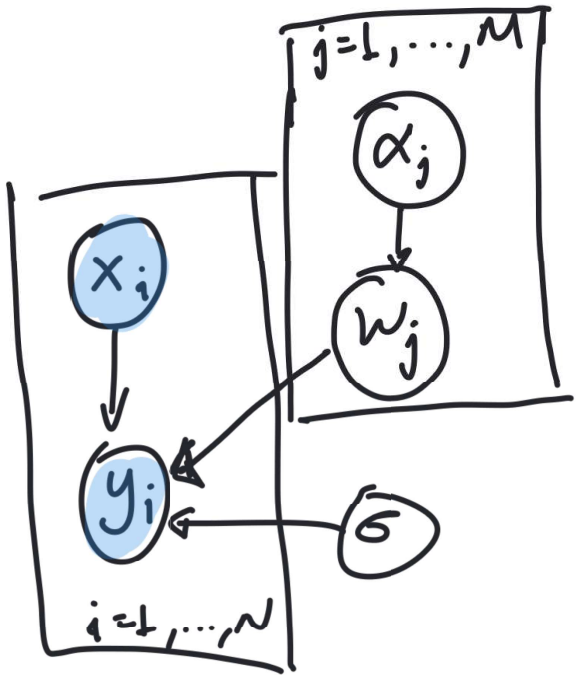
Automatic relevance determination

Open questions

- How do I quantify the measurement noise?
- How do we avoid overfitting?
- How do I quantify epistemic uncertainty induced by limited data?
- How do I choose any remaining parameters?
- How do I choose which basis functions to keep?

Idea: Different hyper-prior per weight



Prior:

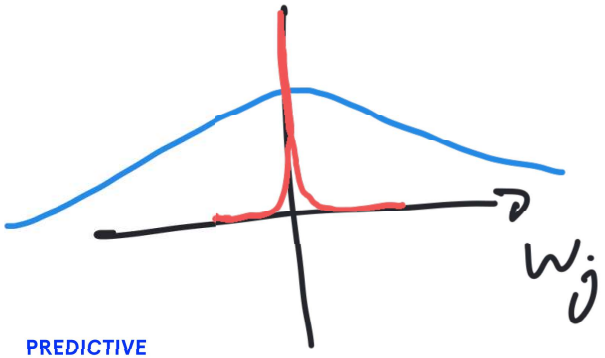
$$\alpha_j \sim p(\alpha_j)$$

$$w_j | \alpha_j \sim p(w_j | \alpha_j) = N(w_j | 0, \alpha_j^{-1})$$

$$\sigma \sim p(\sigma)$$

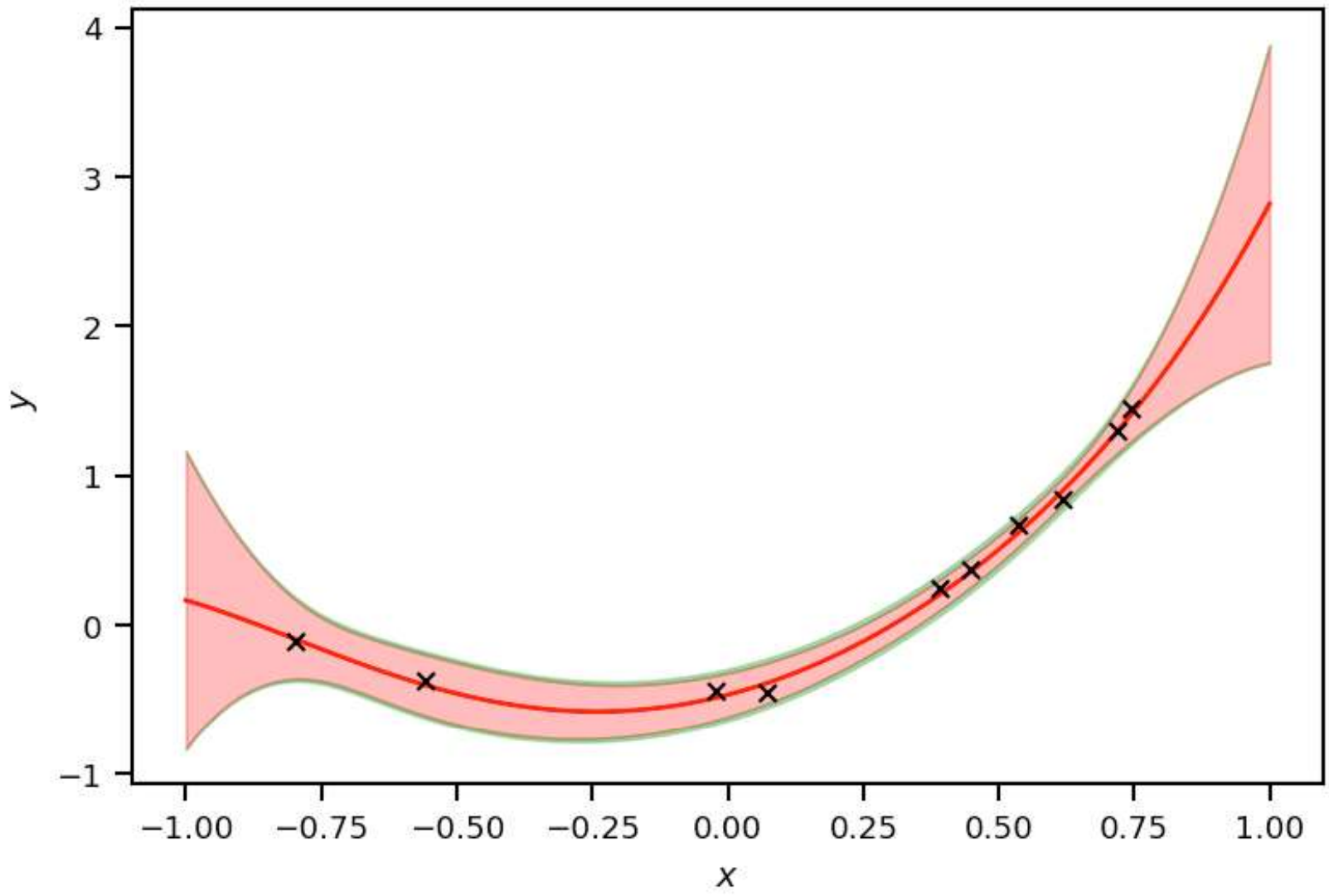
Likelihood:

$$\underline{\alpha}^*, \sigma^* = \arg\max_{\underline{\alpha}, \sigma} p(\underline{\alpha}, \sigma | x_{1:N}, y_{1:N})$$

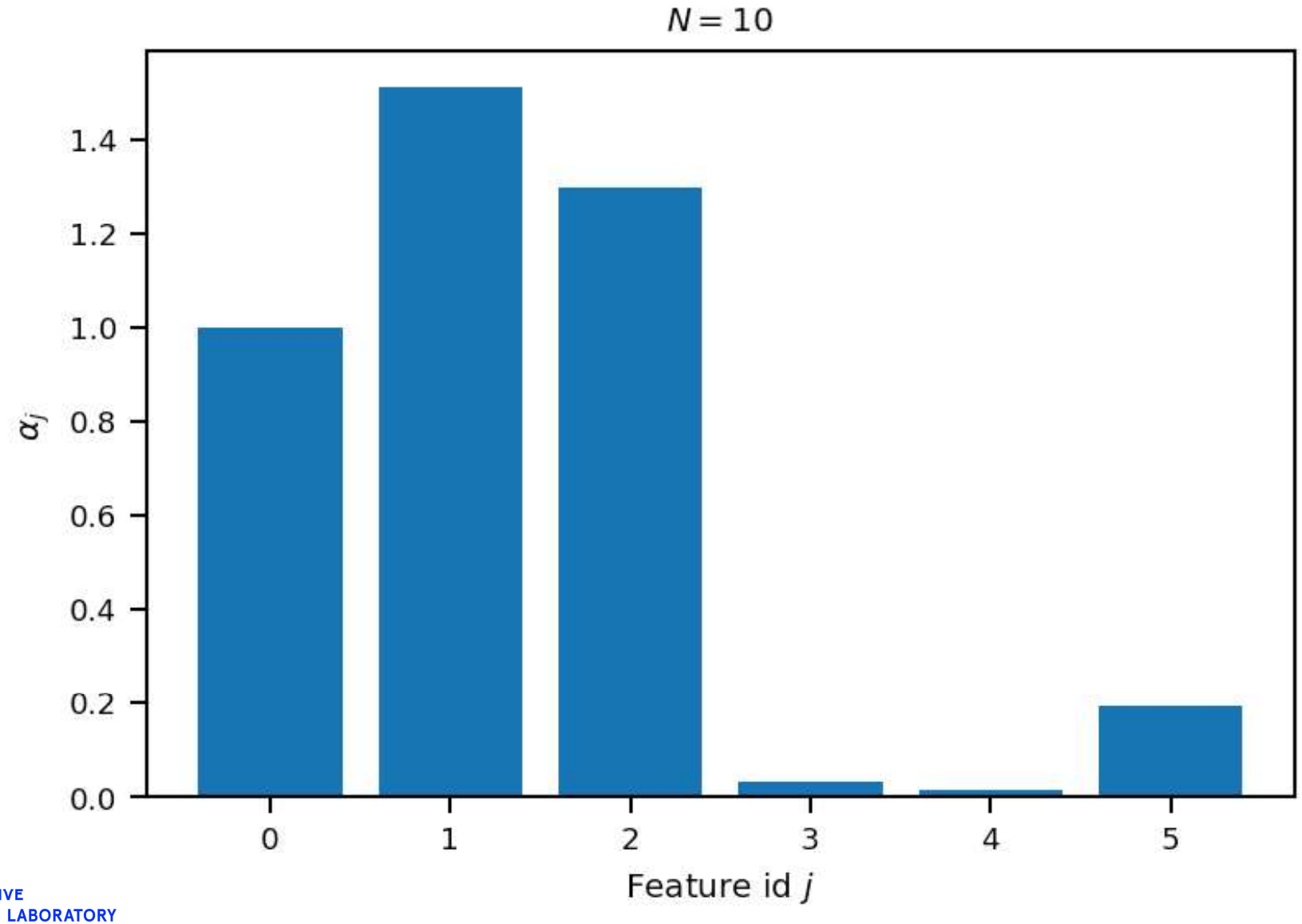


$\alpha_j >> 1$

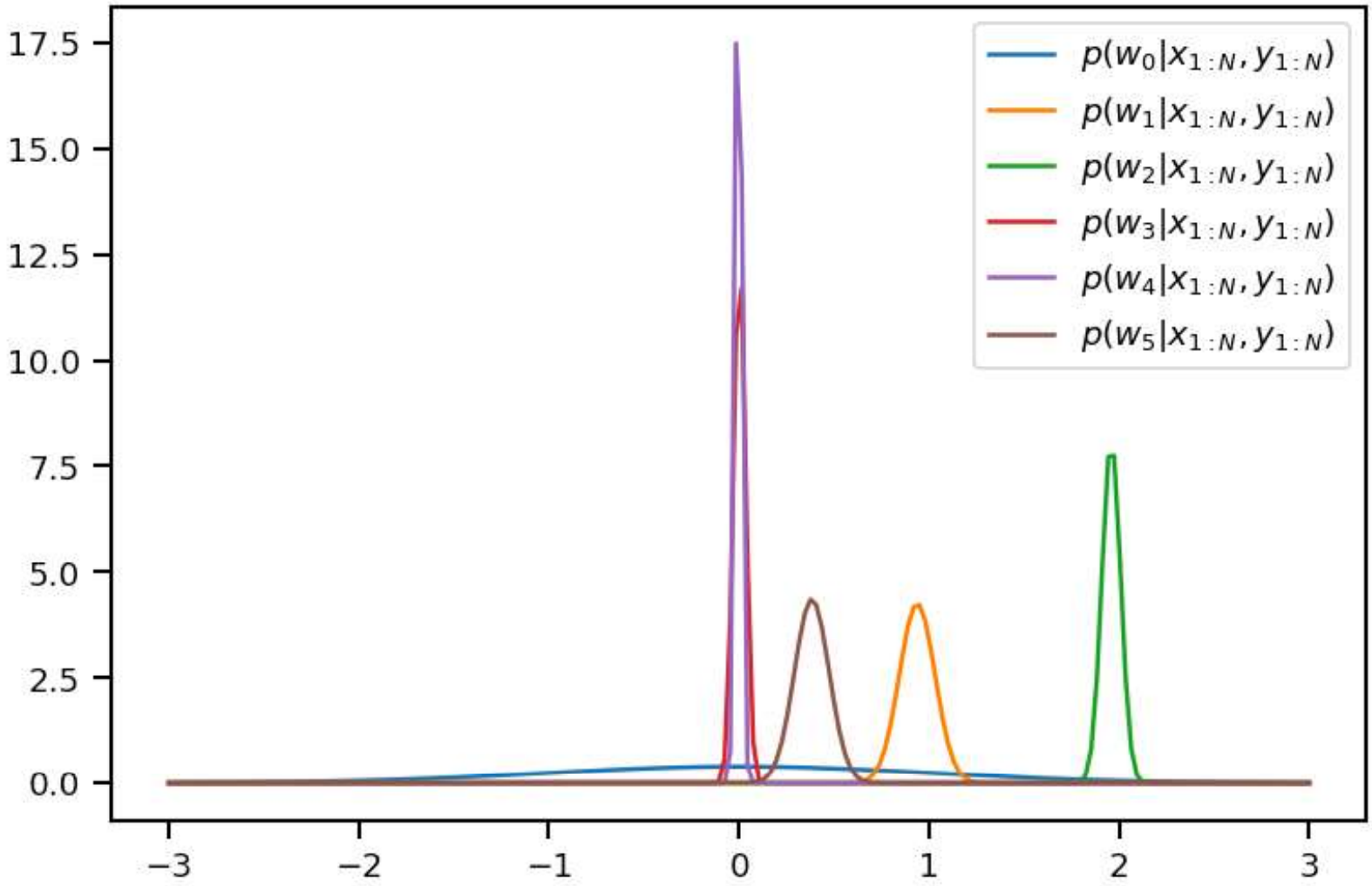
Example



Optimized values for the α_j's



Marginal posteriors for the weights



Open questions

- Cannot be used to compare generalized models with other models (e.g., of completely different functional form). For this, we will need Bayesian model selection.

- How can we model the fact that our noise is input-dependent (heteroscedastic)?