Lecture 9: Monte Carlo estimates of various statistics

Professor Ilias Bilionis

Application - Propagating uncertainties through an ordinary differential equation

PREDICTIVE
SCIENCE LABORATORY

Example ODE: Exponential decay

exponential decay rate const.

- Consider the ODE:

$$\dot{y} = \frac{dy}{dt} = -a y$$

uncertainty

- With initial conditions:

$$y(0) = y_0$$

- The solution is:

$$y = y_0 e^{-at}$$

?

PREDICTIVE
SCIENCE LABORATORY

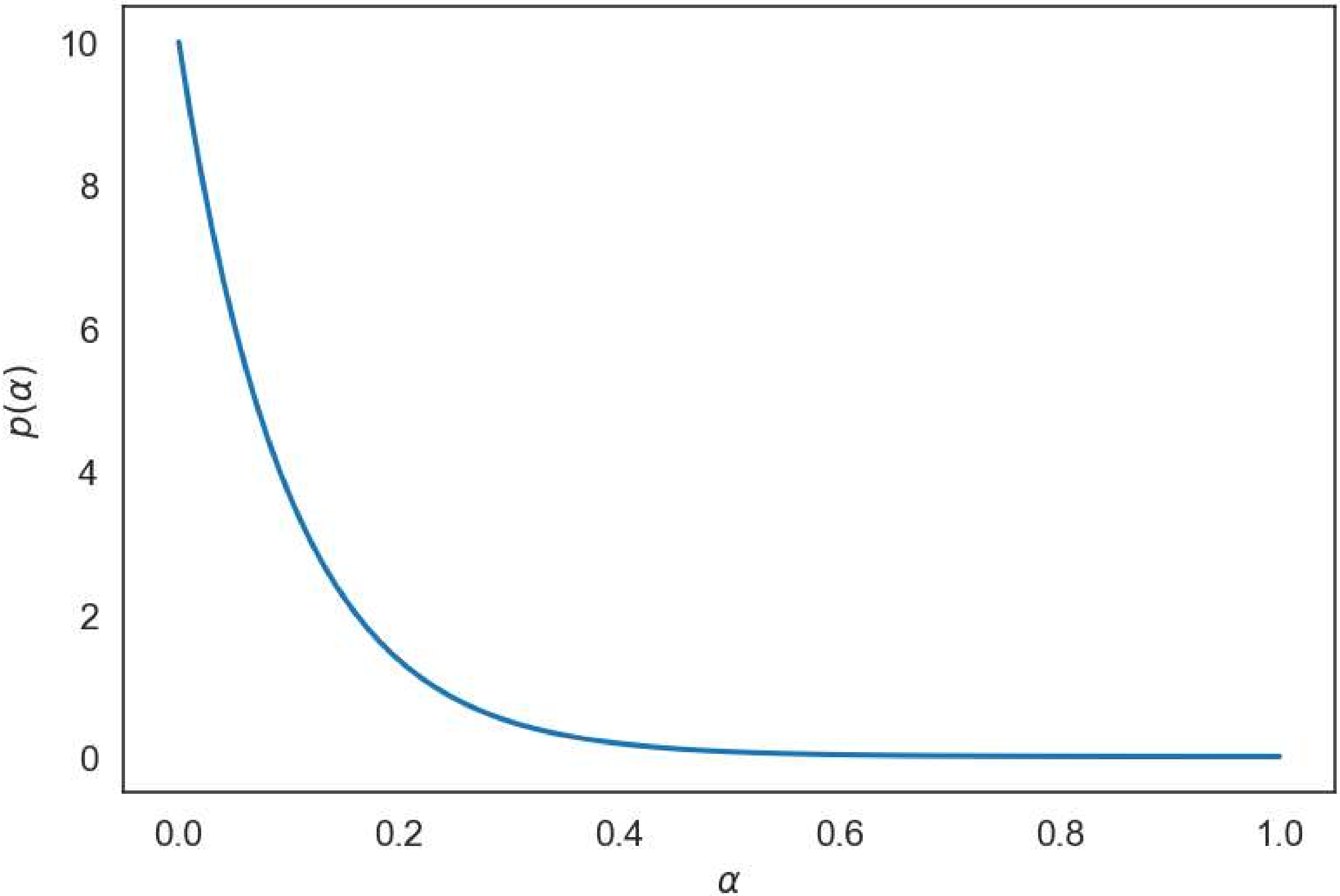
Example ODE: Assigning random variables (a)

- Start with the decay rate coefficient a.
- We know that it is positive.
- Assume that we know that $\mathbb{E}[a] = 0.1$.
- What random variable should we assign to it?

$a \sim \text{Exponential}(\lambda)$

$\mathbb{E}[a] = \lambda^{-1} \Rightarrow \lambda = 10.$

Example ODE: Assigning random variables (α)



Example ODE: Assigning random variables (y_0)

- Take the initial condition y_0.
- We know that it is positive.
- Assume that we know that $\mathbb{E}[y_0] = 10$ and $\mathbb{V}[y_0] = 1$.
- What random variable should we assign to it?

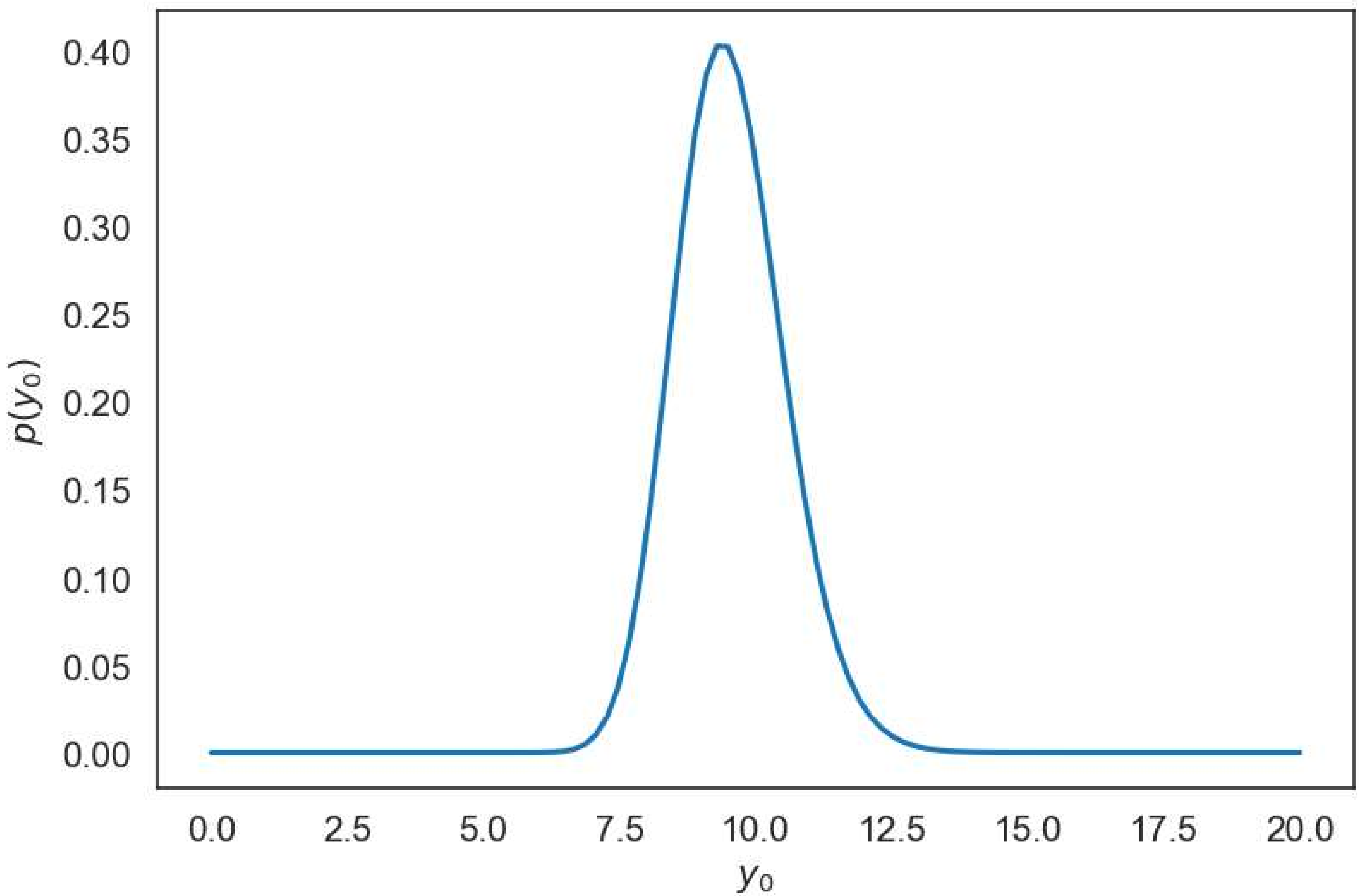
$y_0 \sim \text{LogNormal}(\mu, \sigma^2)$

$\mathbb{E}[y_0] = \exp\{\mu + \frac{1}{2}\sigma\} = 10 \quad (1)$

$\mathbb{V}[y_0] = [e^{\sigma^2} - 1] \cdot \exp\{2\mu + \sigma^2\} = 1 \quad (2)$

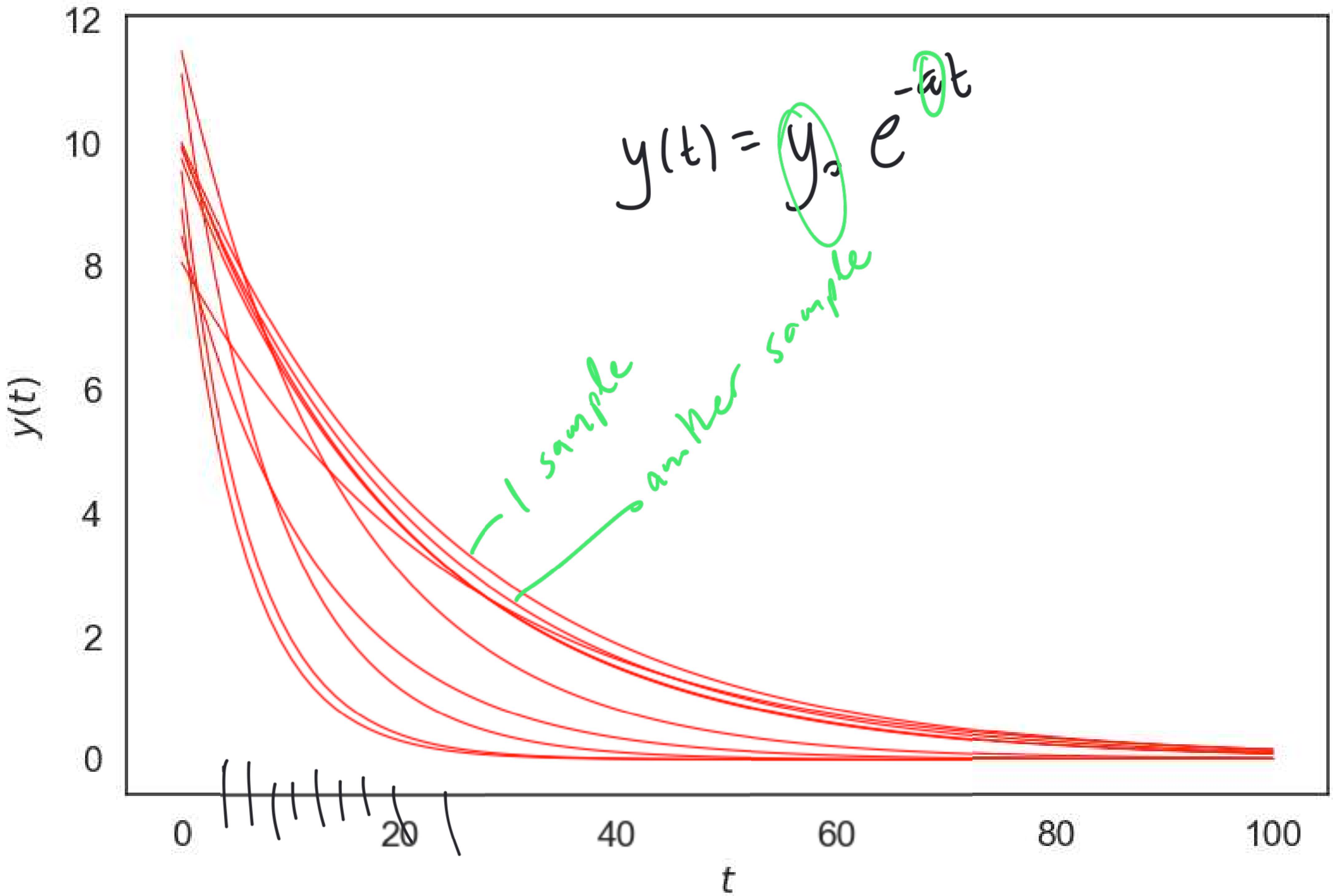
PREDICTIVE
SCIENCE LABORATORY

Example ODE: Assigning random variables (y_0)



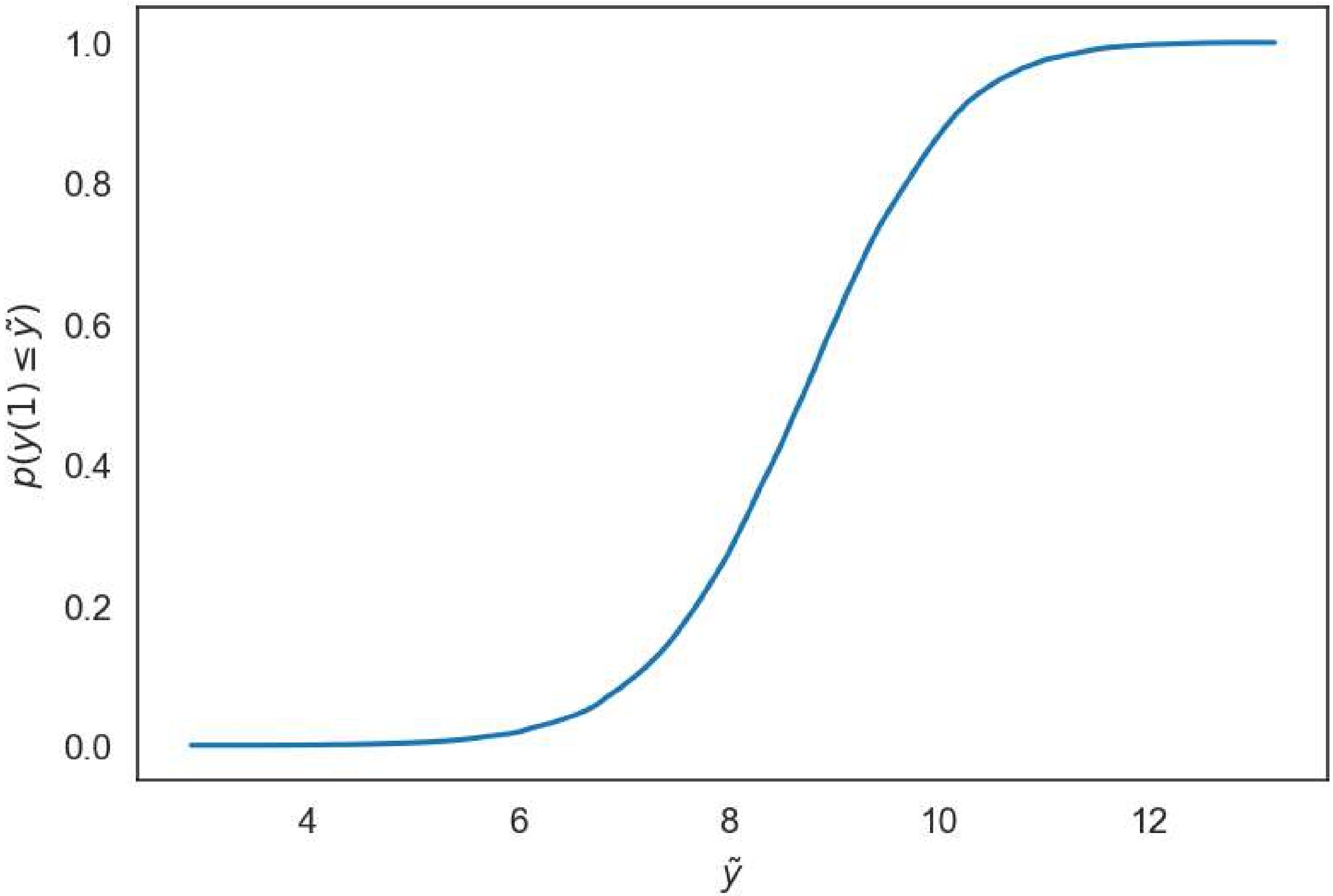
PREDIC
SCIENCE LABORATORY

Example ODE: Sampling possible random paths



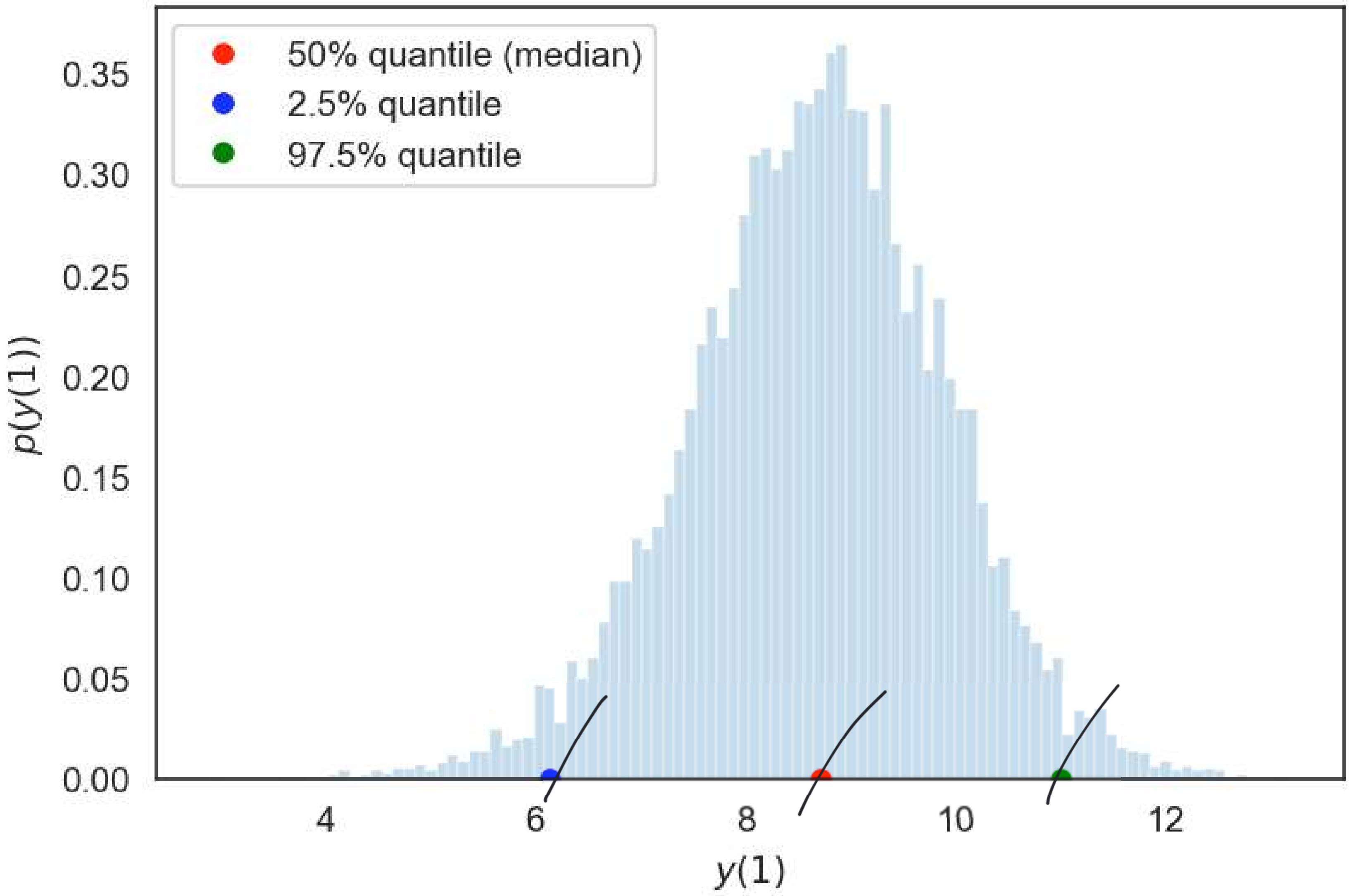
PREDICTI
SCIENCE LABORATORY

Example ODE: Estimating the CDF at $y(t=1)$



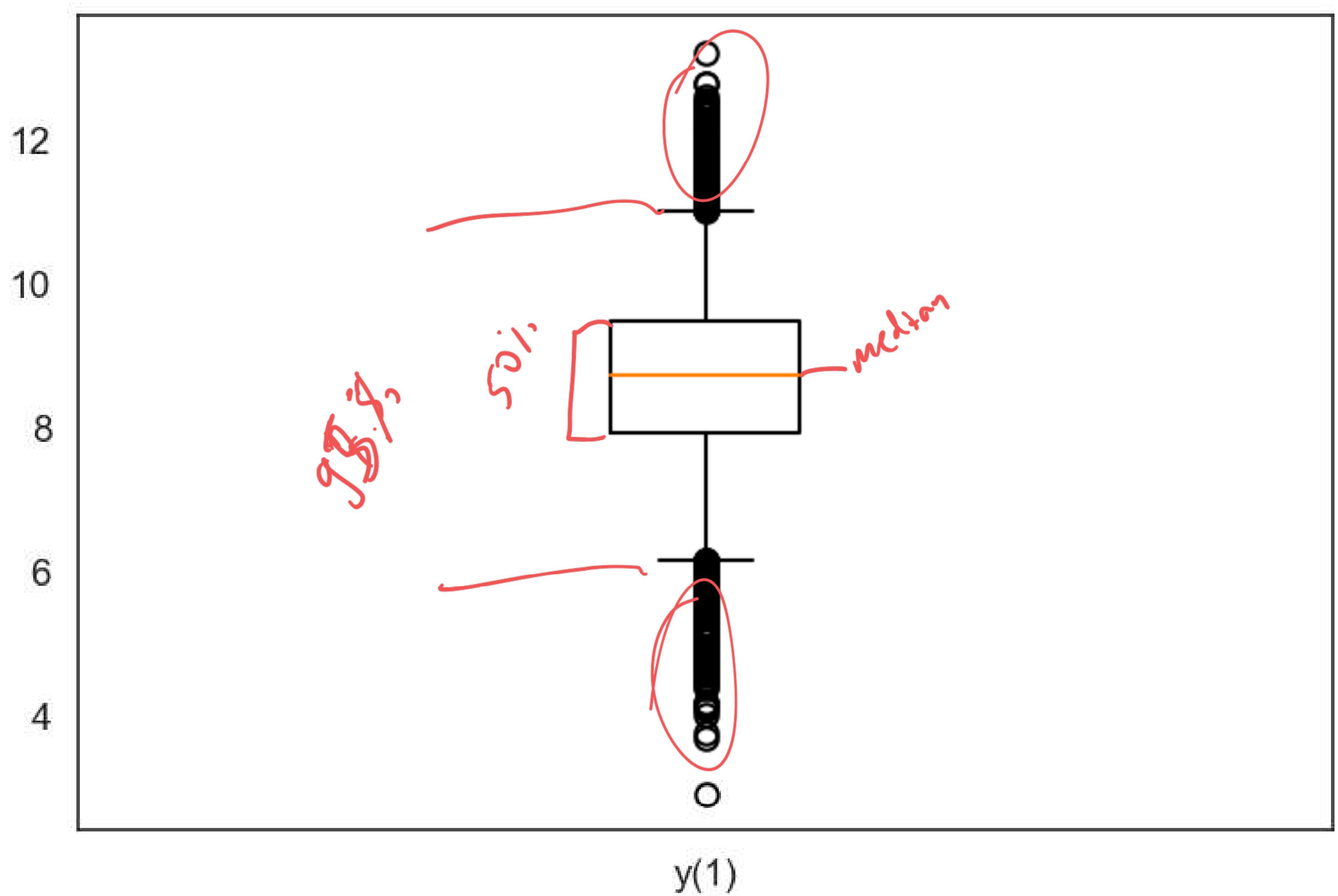
PREDICTIVE
SCIENCE LABORATORY

Example ODE: Estimating the PDF and quantiles at $y(t=1)$

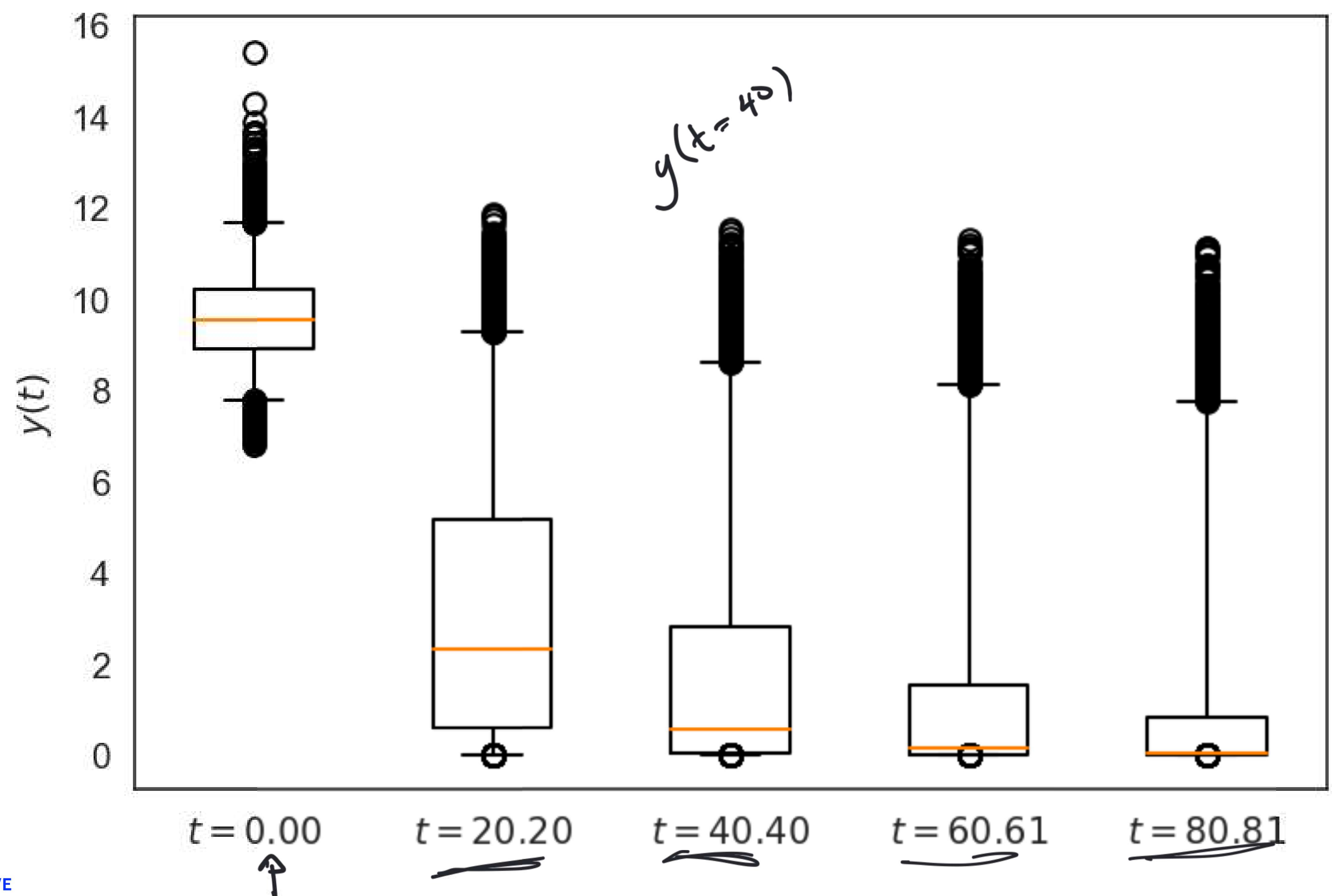


PREDIC
SCIENCE LABORATORY

Example ODE: Estimating the PDF and quantiles at $y(t = 1)$

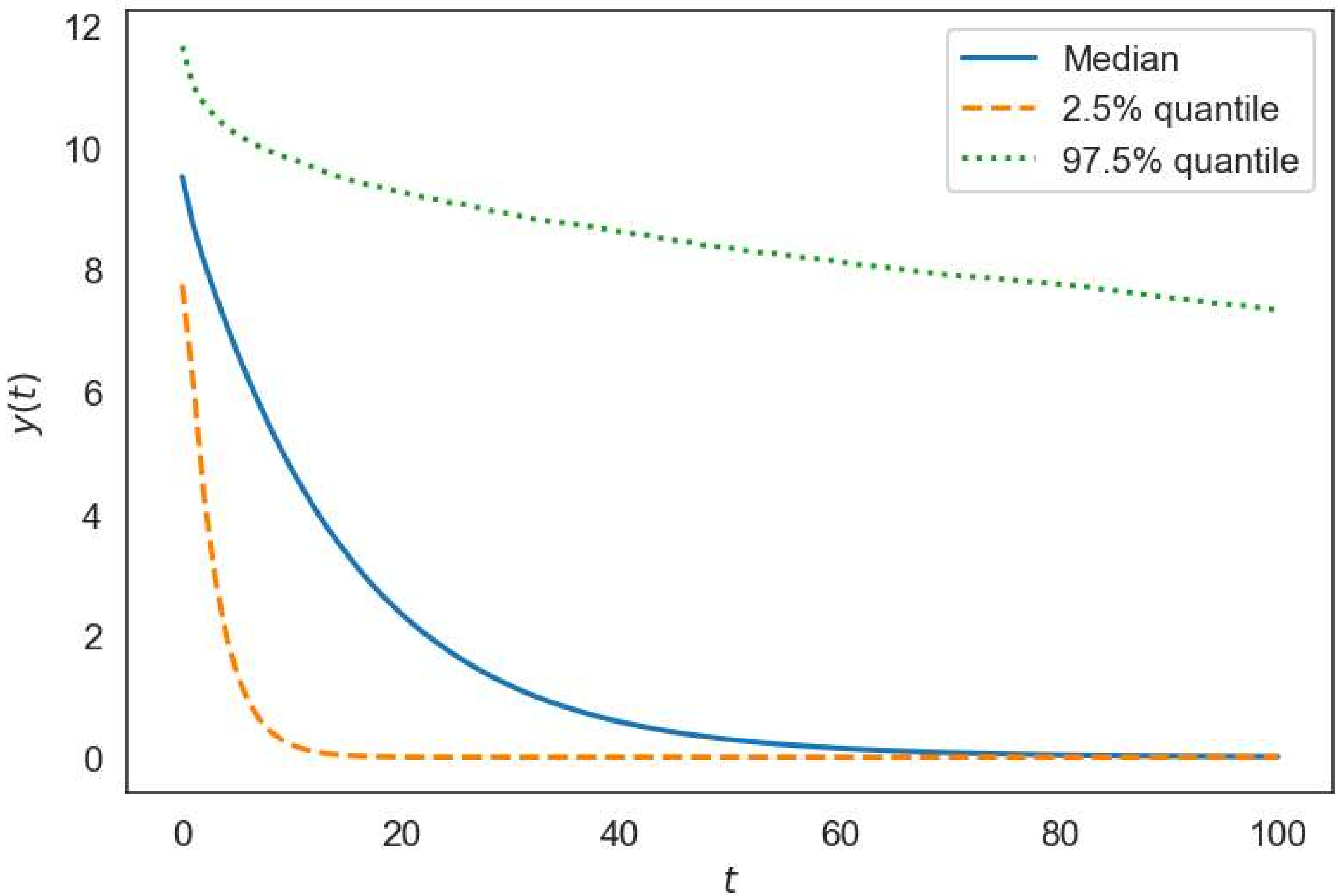


Example ODE: Visualizing the quantiles at multiple time steps



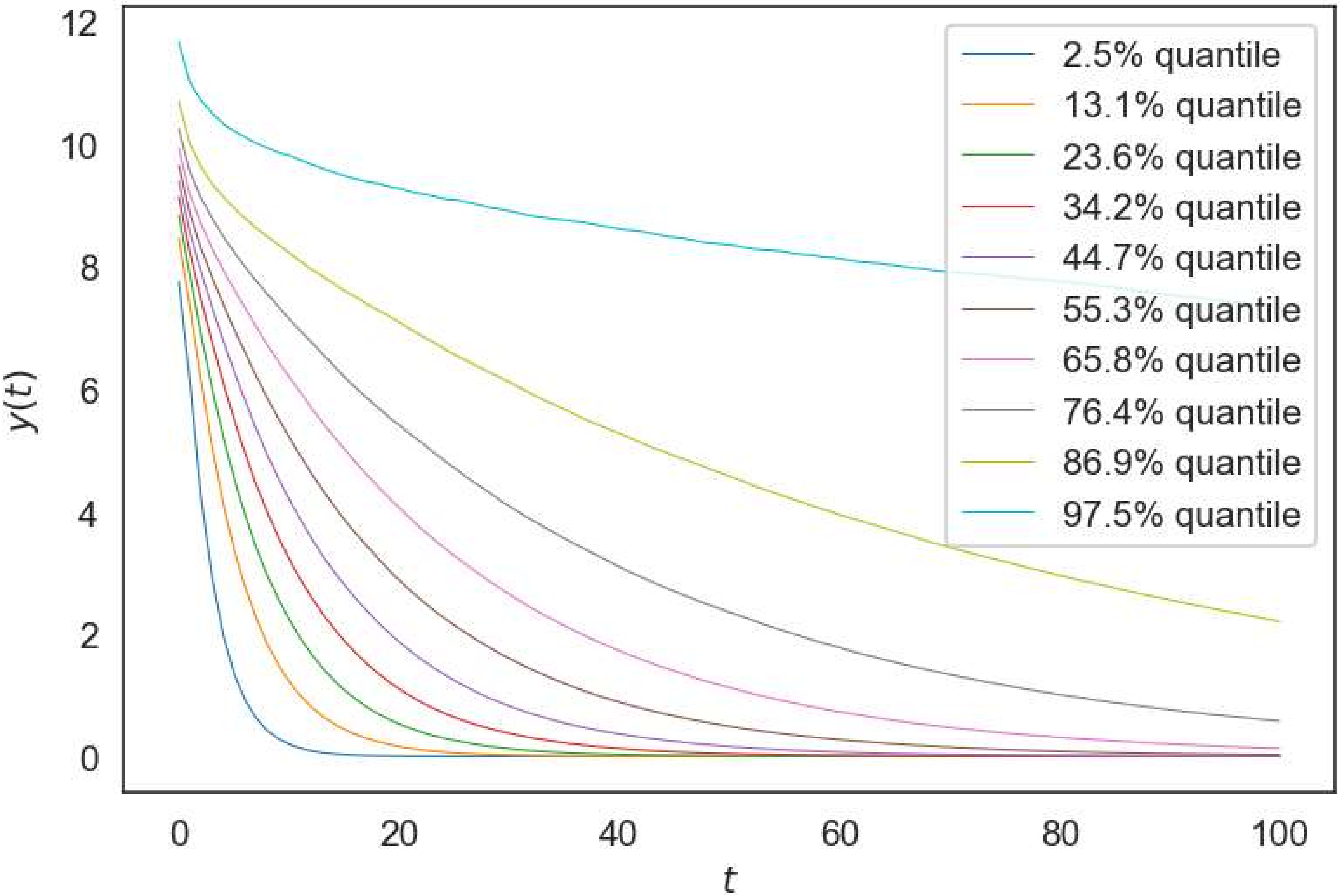
PREDICTIVE
SCIENCE LABORATORY

Example ODE: Visualizing the quantiles at all time steps



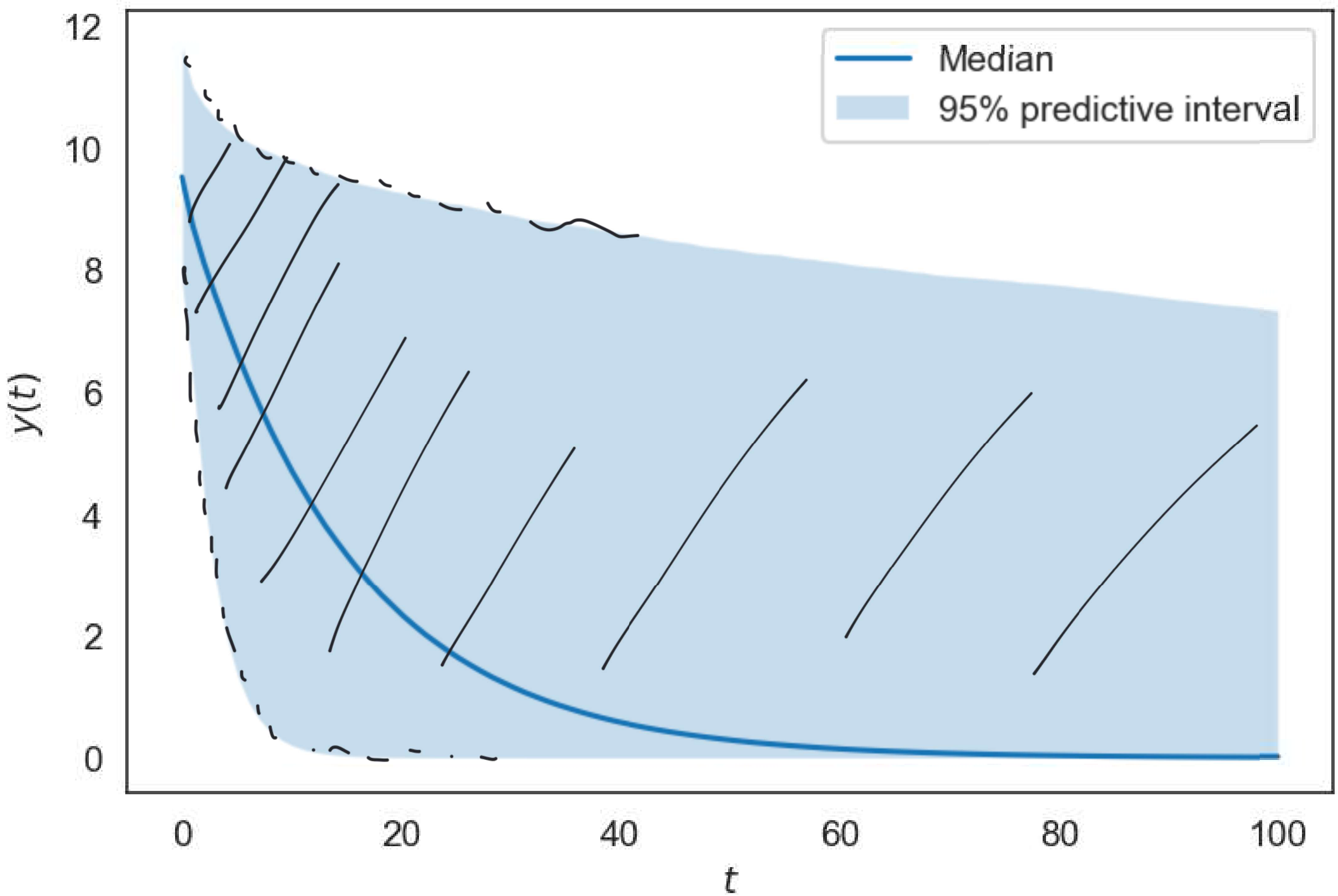
PREDICTI
SCIENCE LABORATORY

Example ODE: Visualizing the quantiles at all time steps



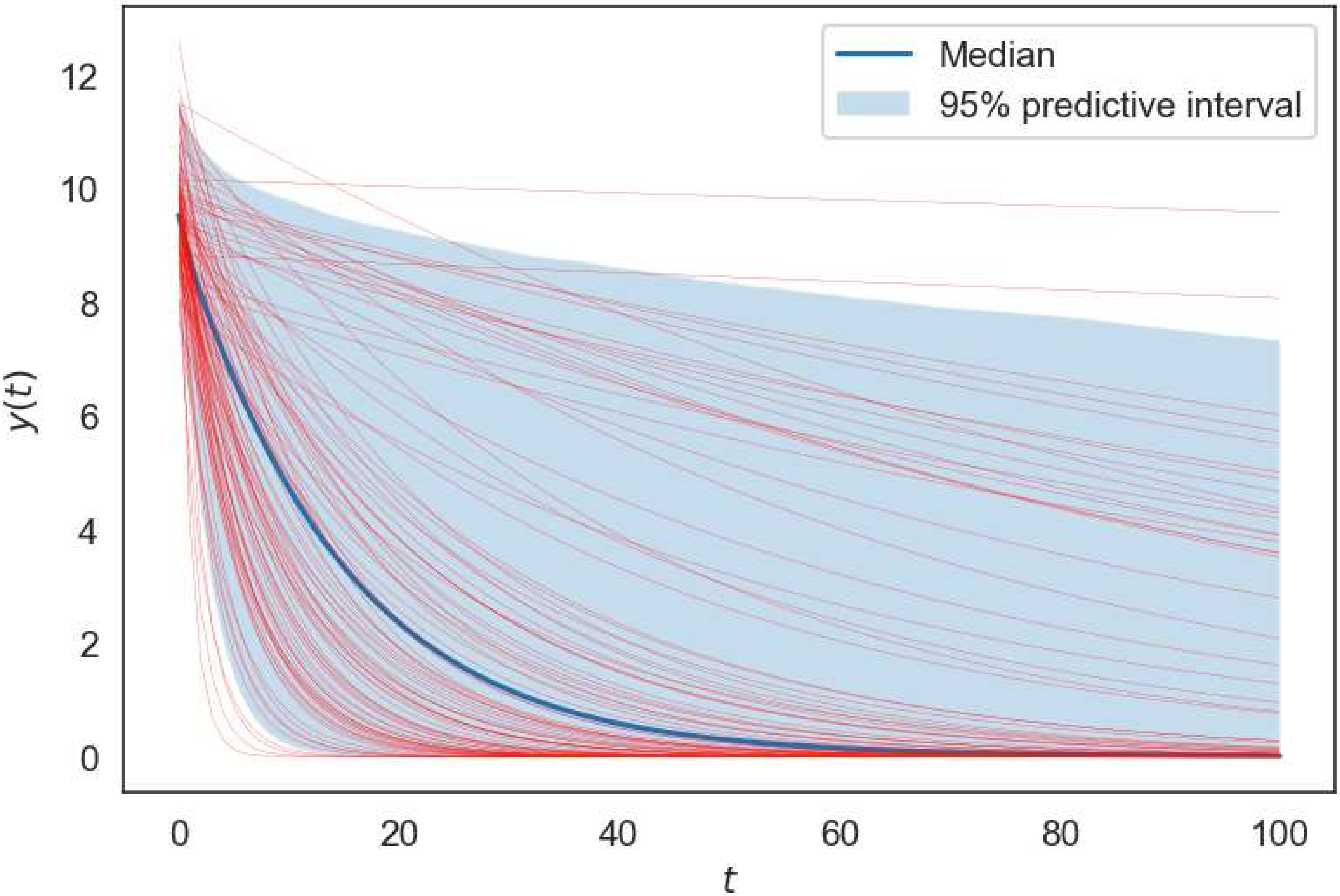
PREDICTI
SCIENCE LABORATORY

Example ODE: Visualizing the quantiles at all time steps



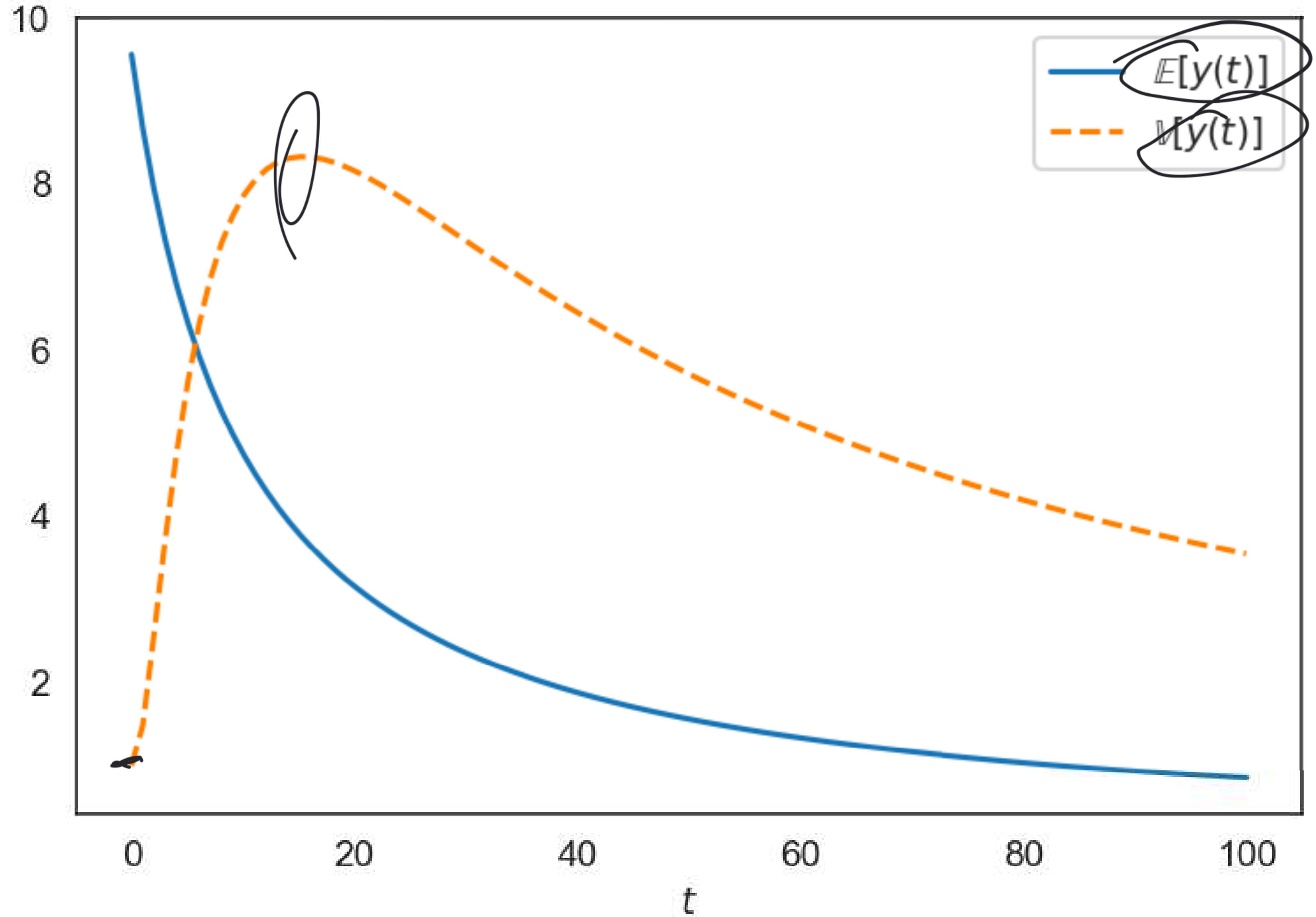
PREDICTI
SCIENCE LABORATORY

Example ODE: Visualizing the quantiles at all time steps



PREDICTI
SCIENCE LABORATORY

Example ODE: Summarizing uncertainty with the mean and the variance



PREDICTIVE
SCIENCE LABORATORY