Lecture 3: Discrete Random Variables

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The probability mass function

Probability mass function

Let X be a discrete random variable. The *probability mass function (pmf) of* X *is:*

$p(X = x) =$ Probability that the random variable X takes the value x

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Probability mass function

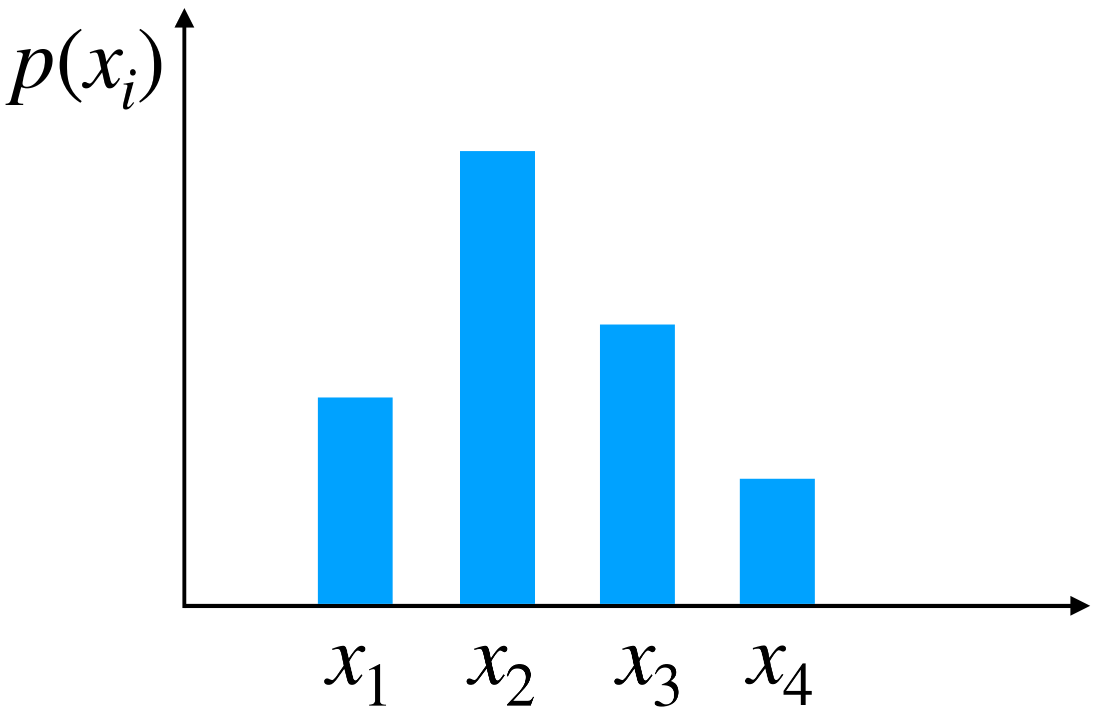
Let X be a discrete random variable. The *probability mass function (pmf) of* X *is:*

$p(X = x) =$ Probability that the random variable X takes the value x

When there is no ambiguity:

$$\boxed{p(x) \equiv p(X = x)\,.}$$

Visualization of the probability mass function



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Properties of the probability mass function

- The probability mass function is nonnegative:

$$p(x) \geq 0.$$

- The probability mass function is normalized:

$$\sum_x p(x) = 1,$$

where the summation is over all the possible values of X.

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Properties of the probability mass function

- Let X be a discrete random variable.

- The probability of X taking either the value x_1 or the value x_2 (assuming $x_1 \neq x_2$) is:

$$\underline{p(X = x_1} \text{ or } \underline{X = x_2}) \equiv p(X \in \{\underline{x_1, x_2}\}) = p(X=x_1) + p(X=x_2)$$

$$= p(x_1) + p(x_2)$$

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Properties of the probability mass function

- More generally, the probability that the random variable X takes any value in a set A is given by:

$$p(X \in A) = \sum_{x \in A} p(x)$$

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Functions of random variables

- Consider a function $g(x)$.

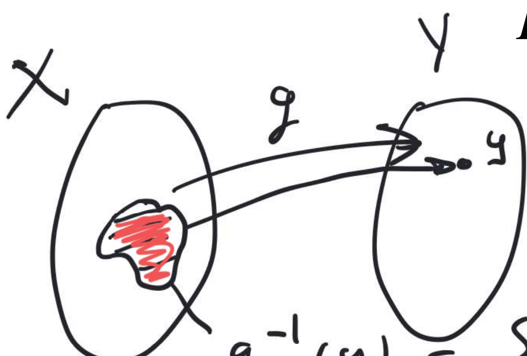
- We can now define a new random variable:

$$Y = g(X)\,.$$

- It has its own probability mass function (pmf):

$$p(y) = p(Y = y) = \sum_{x \in g^{-1}(y)} p(x)$$

where



$g^{-1}(y) = \{x : g(x) = y\}$