Lecture 20: State-space models - Kalman filters

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Beyond linear models and Gaussian noise

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Non-linear systems with non-linear observations with Gaussian noise

$$\left.\begin{aligned} x_{n+1} &= f(x_n, u_n, z_n), \quad z_n \sim p(z_n) \\ y_n &= h(x_n, w_n), \quad w_n \sim p(w_n) \end{aligned}\right\} \text{The most general case.}$$

$$\Downarrow$$

$$\left.\begin{aligned} x_{n+1} &= f(x_n, u_n) + z_n, \quad z_n \sim N(0, Q) \\ y_n &= h(x_n) + w_n, \quad w_n \sim N(0, R) \end{aligned}\right\| \text{Extended Kalman Filter (Unscented KF)}$$

$$\Downarrow$$

Linearize Everything.

$\underline{n}$: $p(x_{n-1} | y_{1:n-1}, u_{0:n-2}) \approx N(x_{n-1} | \mu_{n-1}, \Sigma_{n-1})$

Predict: $\mu_n^p = f(\mu_{n-1}, u_{n-1})$

~~Matrix~~ $d \times d$

$A_n(u)$

$$f(x, u) \underset{\text{Exp.}}{\overset{\text{Taylor}}{\approx}} f(\mu_n^p, u) + \nabla_x f(\mu_n^p, u) \cdot (x - \mu_n^p) + \dots$$

$$h(x) \approx h(\mu_n^p) + \nabla_x h(\mu_n^p) \cdot (x - \mu_n^p)$$

matrix $m \times d$

C_n

$$\Sigma_n^p = \nabla_x f(\mu_n^p, u_{n-1})^T Q \nabla_x f(\mu_n^p, u_{n-1})$$

Update: Same but $A_n(u) = \nabla_x f(\mu_n^p, u_{n-1})$ instead of A

and $C_n = \nabla_x h(\mu_n^p)$ instead of C.

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