Lecture 27: Physics-informed deep neural networks

Professor Ilias Bilionis

Gibbs sampling

PREDICTIVE
SCIENCE LABORATORY

Gibbs sampling

- Split variables in groups: $x = (x_1, x_2, \dots, x_G)$
- **Initialize**: $x_0 = (x_{10}, x_{20}, \dots, x_{G0})$
- For step $n = 1, 2, \dots$

$$x_{n-1} = (x_{1,n-1}, x_{2,n-1}, \dots, x_{G,n-1})$$

$$(x_{1,n}, x_{2,n-1}, \dots, x_{G,n-1})$$

$$(x_{1,n}, x_{2,n}, \dots, x_{G,n-1})$$

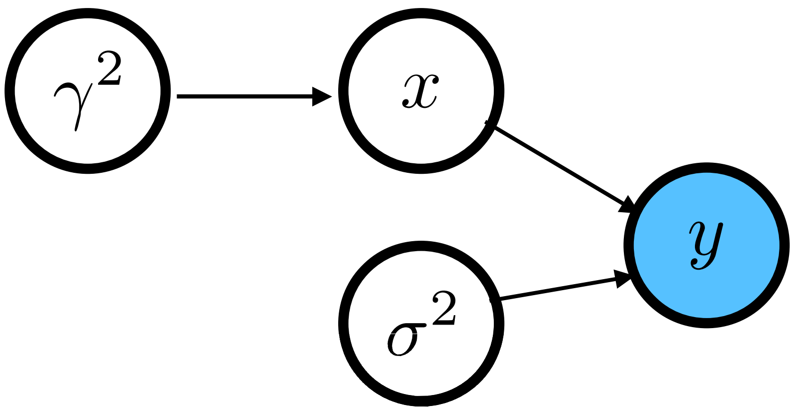
For $g = 1, \dots, G$

Sample x_{ng} from

$$p(x_g \mid x_{g'} = x_{g',n}, g' < g, x_{g''} = x_{g'',n-1}, g'' > g)$$

PREDICTIVE
SCIENCE LABORATORY

Application: Sampling from a hierarchical model



$$p(\gamma,\sigma,x|y) \propto \underbrace{p(y|x,\sigma)\,p(x|\gamma)\,p(\sigma)}_{\pi(x,\gamma,\sigma)}$$

- γ_0, x_0, σ_0
- For $n=1,2,\dots$

$$\gamma_n | x=x_{n-1}, \sigma=\sigma_{n-1}, y \sim p(\gamma | x=x_{n-1}, \sigma=\sigma_{n-1}, y) \propto \pi(x_{n-1}, \gamma, \sigma_{n-1})$$

$$x_n | \gamma=\gamma_n, \sigma=\sigma_{n-1}, y \sim p(x | \gamma=\gamma_n, \sigma=\sigma_{n-1}, y) \propto \pi(x, \gamma_n, \sigma_{n-1})$$

$$\sigma_n | \gamma=\gamma_n, x=x_n, y \sim p(\sigma | \gamma=\gamma_n, x=x_n, y) \propto \pi(x_n, \gamma_n, \sigma)$$