

Lecture 6: Random Vectors

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The multivariate normal - conditioning

Marginalization

- Assume that you have a random vector \mathbf{X} made out of two sub-random vectors \mathbf{X}_1 and \mathbf{X}_2 , i.e.:

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}^T & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

- What is the PDF of $\mathbf{X}_1 \mid \mathbf{X}_2 = \mathbf{x}_2$?

Marginalization

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

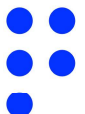
$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}^T & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

- We conditional PDF is:

$$p(\mathbf{x}_1 | \mathbf{x}_2) = \frac{p(\mathbf{x}_1, \mathbf{x}_2)}{p(\mathbf{x}_2)} \propto p(\mathbf{x}_1, \mathbf{x}_2) = N\left(\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} \middle| \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}^T & \boldsymbol{\Sigma}_2 \end{pmatrix}\right)$$

completing the square

$$= \dots = N(\mathbf{x}_1 | \mu_{1|2}, \boldsymbol{\Sigma}_{1|2})$$



Marginalization

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}^T & \boldsymbol{\Sigma}_2 \end{pmatrix}$$

- We get that:

$$\mathbf{X}_1 | \mathbf{X}_2 = \mathbf{x}_2 \sim N(\underbrace{\mu_1}_{\text{prior mean}} + \underbrace{\boldsymbol{\Sigma}_{12} \boldsymbol{\Sigma}_2^{-1} (\mathbf{x}_2 - \mu_2)}_{\text{correction}}, \underbrace{\boldsymbol{\Sigma}_1 - \boldsymbol{\Sigma}_{12} \boldsymbol{\Sigma}_2^{-1} \boldsymbol{\Sigma}_{12}^T}_{\text{correction}})$$

Handwritten annotations:
 - "prior mean" above μ_1
 - "data" above \mathbf{x}_2
 - "prior covariance" above $\boldsymbol{\Sigma}_1$
 - "correction" below the first correction term
 - "correction" below the second correction term

