

# Lecture 5: Collections of Random Variables

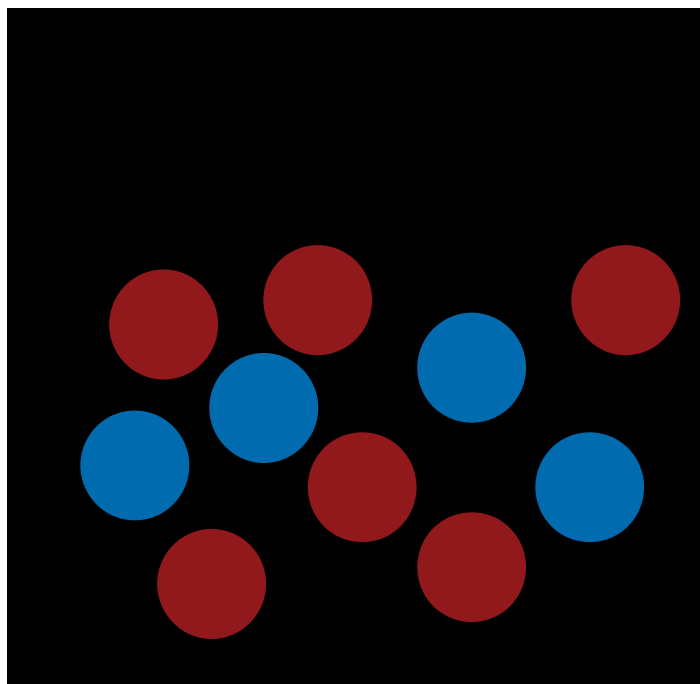
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**Example: Drawing two balls from a box without replacement**

# Example: Drawing balls from a box without replacement

Consider the following example of prior information I:

*We are given a box with 10 balls 6 of which are red and 4 of which are blue. The box is sufficiently mixed so that when we get a ball from it, we don't know which one we pick. When we take a ball out of the box, we do not put it back.*



- Assume that we represent red balls with a 0 and blue balls with 1.
- $X =$  be the random variable corresponding to the outcome of the first draw and  $Y$  of the second.
- Let's find the joint pdf  $p(x, y)$ .

# Example: Drawing balls from a box without replacement

Using results from previous lectures.

$$p(X = 0, Y = 0) = p(X = 0)p(Y = 0 | X = 0) = \frac{3}{5} \cdot \frac{5}{9} \approx 0.33$$

$$p(X = 0, Y = 1) = p(X = 0)p(Y = 1 | X = 0) = \frac{3}{5} \cdot \frac{4}{9} \approx 0.27$$

$$p(X = 1, Y = 0) = p(X = 1)p(Y = 0 | X = 1) = \frac{2}{5} \cdot \frac{2}{3} \approx 0.27$$

$$p(X = 1, Y = 1) = p(X = 1)p(Y = 1 | X = 1) = \frac{2}{5} \cdot \frac{1}{3} \approx 0.13. \quad +$$

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# Example: Drawing balls from a box without replacement

- Here is how we marginalize:

$$p(Y = 0) = \sum_x p(X = x, Y = 0) = p(X = 0, Y = 0) + p(X = 1, Y = 0) \approx 0.33 + 0.27 = 0.6$$

$$p(Y = 1) = \sum_x p(X = x, Y = 1) = p(X = 0, Y = 1) + p(X = 1, Y = 1) \approx 0.27 + 0.13 = 0.4.$$

# Example: Drawing balls from a box without replacement

- Let's find the covariance of the two random variables.

$$\mathbb{E}[X] = 0 \cdot p(X = 0) + 1 \cdot p(X = 1) = 0.4,$$

$$\mathbb{E}[Y] = 0 \cdot p(Y = 0) + 1 \cdot p(Y = 1) = 0.4.$$

$$\begin{aligned}\mathbb{C}[X, Y] &= \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] \\ &= \sum_{x,y} p(X = x, Y = y)(x - 0.4)(y - 0.4) \\ &\approx 0.33 \cdot (0.16) + 0.27 \cdot (-0.24) + 0.27 \cdot (-0.24) + 0.13 \cdot (0.36) \\ &= -0.03.\end{aligned}$$