Lecture 7: Basic Sampling

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Sampling the categorical

Example: Sampling from the Bernoulli distribution

$$X \sim \text{Bernoulli}(\theta) \; ; \; X = \begin{cases} 1, & w/pr. \ \theta \\ 0, & \text{otherwise} \end{cases}$$

To sample from it, we do the following steps:

- Sample a uniform number u. $\sim U([0,1])$

- If $u \leq \theta$, then set $x = 1$.

- Otherwise, set $x = 0$.

$$U \sim U([0,1])$$

$$X = \begin{cases} 1, & \text{if } U \leq \theta \\ 0, & \text{otherwise} \end{cases}$$

$$p(X=1) = P(U \leq \theta) = F_U(\theta) = \theta$$

$$p(X=0) = 1 - \theta$$

Sampling discrete distributions
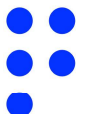
K : possibilities

$0, 1, 2, \ldots, K-1$

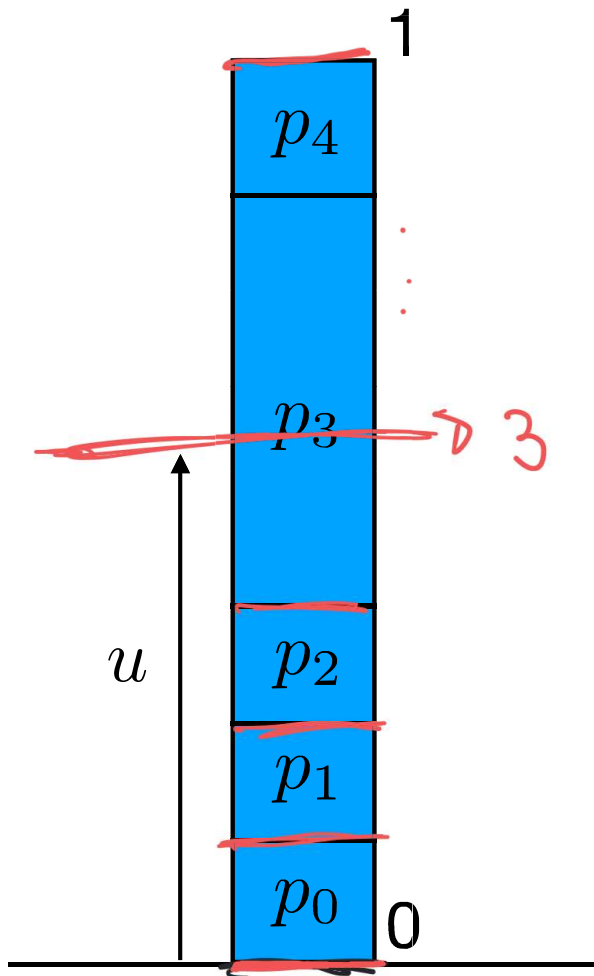$p_0, p_1, p_2, \ldots, p_{K-1}$

- Consider a generic discrete random variable taking different values, with probability:

$$p(X = k) = p_k.$$

$$X \sim \text{Categorical}(p_0, p_1, \ldots, p_{K-1}) \; ; \quad X = \begin{cases} 0, & \text{w/ pr. } p_0 \\ 1, & \text{w/ pr. } p_1 \\ \vdots \\ K-1, & \text{w/ pr. } p_{K-1} \end{cases}$$

Sampling Discrete Distributions

- Draw a uniform number u. $\sim U([0,1])$

- Find j such that:

$$\sum_{k=0}^{j-1} p_k \leq u < \sum_{k=0}^{j} p_k$$

- j is your sample

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Sampling Discrete Distributions

$$U \sim U([0,1])$$

$$X = \begin{cases} 0, & \text{if } U \leq p_0 \\ 1, & \text{if } p_0 \leq U \leq p_0 + p_1 \\ 2, & \text{if } p_0 + p_1 \leq U \leq p_0 + p_1 + p_2 \\ \vdots \\ K-1, & \text{otherwise} \end{cases}$$

$$P(X=0) = P(U \leq p_0) = p_0$$

$$P(X=1) = P(p_0 \leq U \leq p_0 + p_1)$$

$$= F_U(p_0 + p_1) - F_U(p_0)$$

$$= p_0 + p_1 - p_0 = p_1$$

$$P(X=2) = \ldots$$

p_4

p_3

$u \longrightarrow 2$

p_2

p_1

p_0

1

0

Sample 1

$p_4 \longrightarrow 4$

p_3

p_2

u

p_1

p_0

1

0

Sample 2