Lecture 20: State-space models - Kalman filters

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Derivation of Kalman filter - Update

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Derivation of Kalman filter - Update

PREDICT:

If $p(x_{n-1} | y_{1:n-1}, u_{0:n-2}) = N(x_{n-1} | \mu_{n-1}, V_{n-1})$

Then Predict:

$$p(x_n | y_{1:n-1}, u_{0:n-2}, u_{n-1}) = N(x_n | \underbrace{A\mu_{n-1} + B u_{n-1}}_{\mu_n^p}, \underbrace{A V_{n-1} A^T + Q}_{V_n^p})$$

UPDATE:

$$\underbrace{p(x_n | y_{1:n}, u_{0:n-1})}_{G} \propto \underbrace{p(y_n | x_n)}_{\substack{\text{Emission} \\ N(Cx_n, R)}} \underbrace{p(x_n | y_{1:n-1}, u_{0:n-1})}_{N(\mu_n^p, V_n^p)}$$

Complete the Square ⟹

$$p(x_n | y_{1:n}, u_{0:n-1}) = N(x_n | \mu_n, V_n)$$

$$\mu_n = \underbrace{\mu_n^p}_{d} + \underbrace{\underbrace{K_n}_{d \times q} \underbrace{(y_n - C\mu_n^p)}_{q}}_{d}$$

$$V_n = V_n^p - \underbrace{\underbrace{K_n}_{d \times q} \underbrace{C}_{q \times d} \underbrace{V_n^p}_{d \times d}}_{d \times d}$$

Kalman gain:

$$K_n = \underbrace{\underbrace{V_n^p}_{d \times d} \underbrace{C^T}_{d \times q} \underbrace{(\underbrace{\underbrace{C}_{q \times d} \underbrace{V_n^p}_{d \times d} \underbrace{C^T}_{d \times q} + \underbrace{R}_{q \times q}}_{q \times q})^{-1}}_{q \times q}}_{d \times q}$$

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