# Lecture 18: Dimensionality Reduction

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# Principal component analysis: Basics

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# Principal component analysis as linear dimensionality reduction

$\underline{x}_1, \dots, \underline{x}_n \in \mathbb{R}^D \longrightarrow \underline{z}_1, \underline{z}_2, \dots, \underline{z}_n \in \mathbb{R}^d, \quad d \ll D.$

Projection

$\underline{x} \longrightarrow \underline{z} = f(\underline{x}) = \underline{\underline{W}}^T \underline{x}, \quad \underline{\underline{W}} \in \mathbb{R}^{D\times d}$ ?

principal components of $\underline{x}$

$\underline{\underline{V}} \in \mathbb{R}^{D\times d}$

(affine map)

Reconstruction

$\underline{\underline{V}}\,\underline{z} + \underline{x}_0 = g(\underline{z}) = \tilde{\underline{x}} \longleftarrow \underline{z}$

? ?

$\underline{x}_0 \in \mathbb{R}^D$

Parameters: $\underline{\underline{W}}, \underline{\underline{V}}, \underline{x}_0$ ?

# Minimum-error formulation of principal component analysis

loss = reconstruction error $x_{1:n}$

= $\sum$ error in reconstruction of the projection of each observation

$$= \sum_{i=1}^{n} \| \underbrace{g}_{\text{reconstruction}}(\underbrace{f}_{\text{projection}}(\underline{x}_i)) - \underline{x}_i \|_2^2$$

$$= \sum_{i=1}^{n} \| \underline{\underline{V}} ( \underline{\underline{W}}^T \underline{x}_i ) + \underline{x}_0 - \underline{x}_i \|^2$$

min over $\underline{\underline{V}}, \underline{\underline{W}}, \underline{x}_0$

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