Lecture 10: Quantifying uncertainties in Monte Carlo estimates

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The central limit theorem

The Central Limit Theorem

- Take $X_1, X_2, \ldots$ to be iid random variables with mean μ and variance σ^2.
- Consider their average:

$$S_N = \frac{X_1 + \ldots + X_N}{N}$$

- The Central Limit Theorem States that:

$$S_N \sim N\left(\mu, \frac{\sigma^2}{N}\right) \text{ for large } N.$$

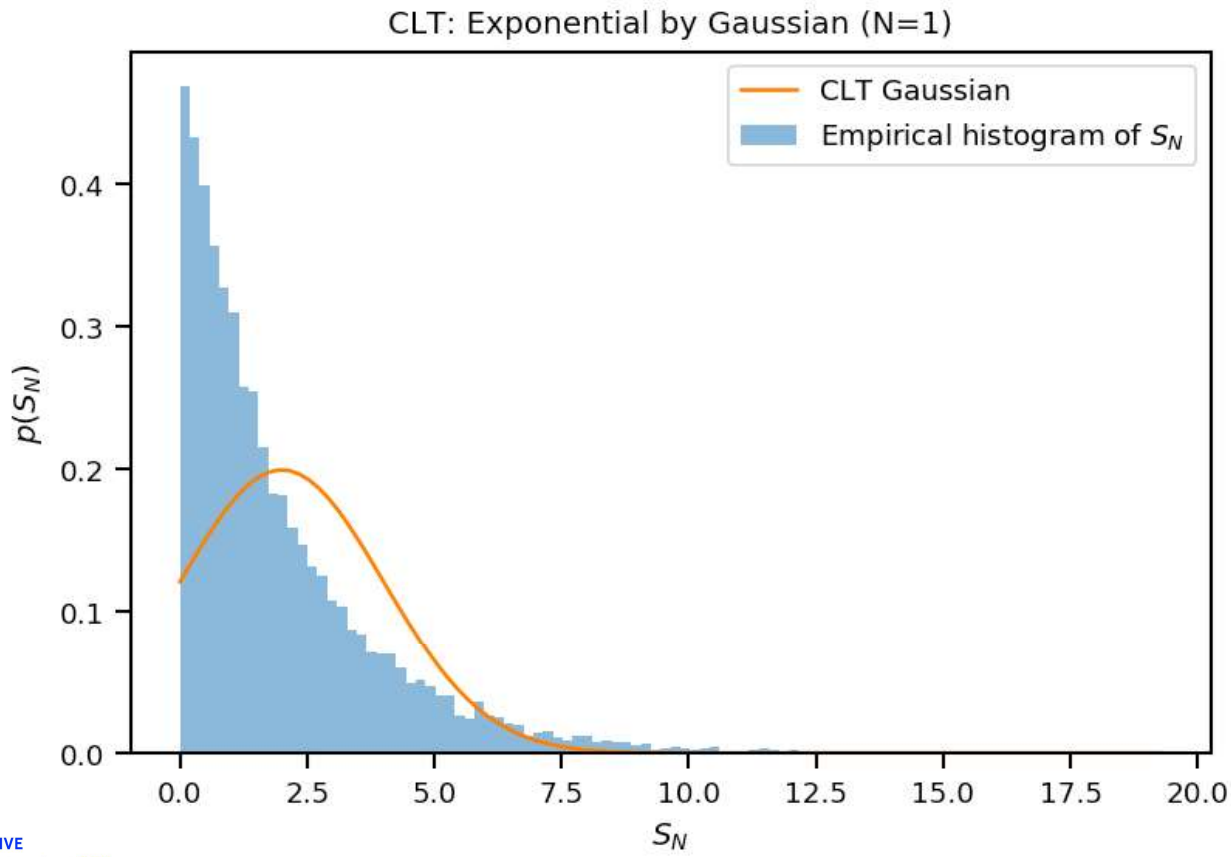
Example of the Central Limit Theorem

- Take $X_i \sim \mathrm{Exp}(r)$ with r fixed.
- Define the average of N such variables:

$$S_N = \frac{X_1 + \ldots + X_N}{N}$$

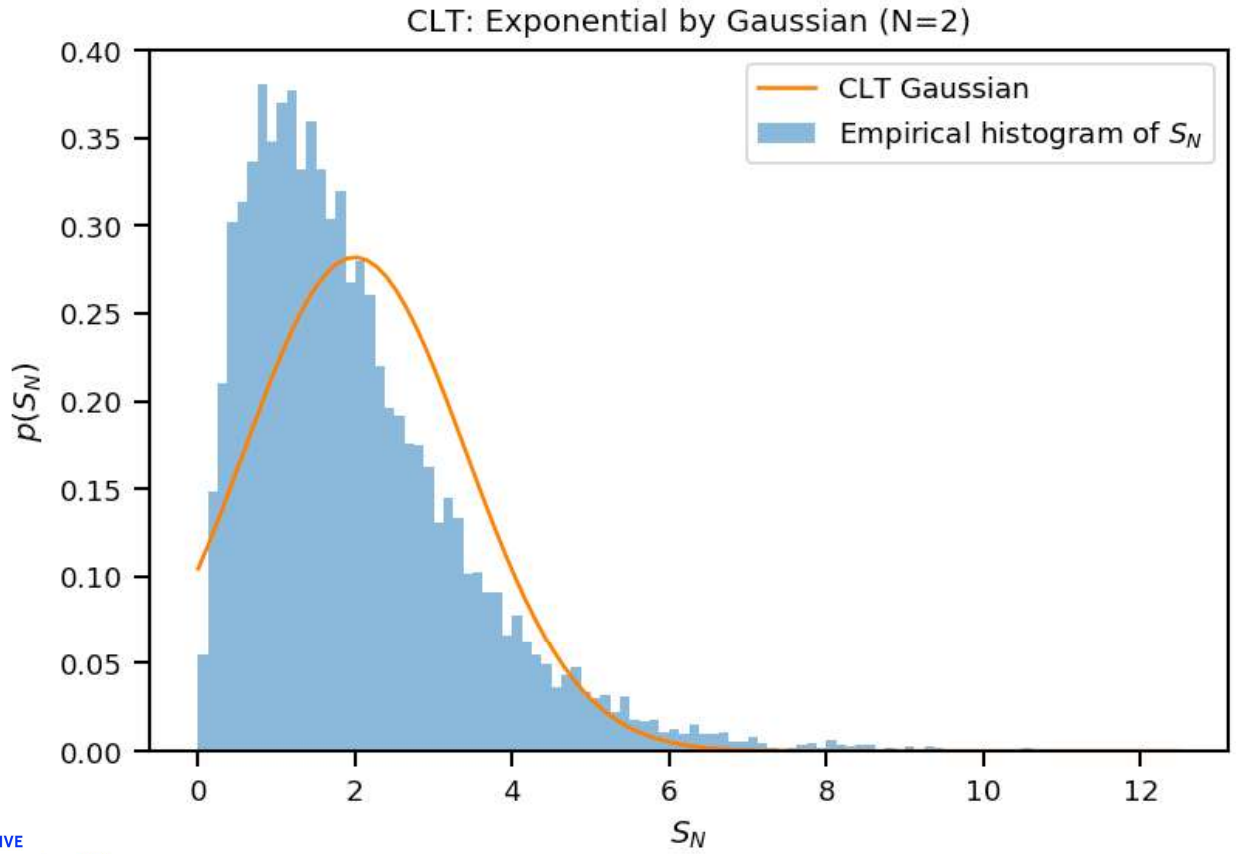
- Characterize probability density function of the average via samples.
- Compare to the CLT prediction.

N=1



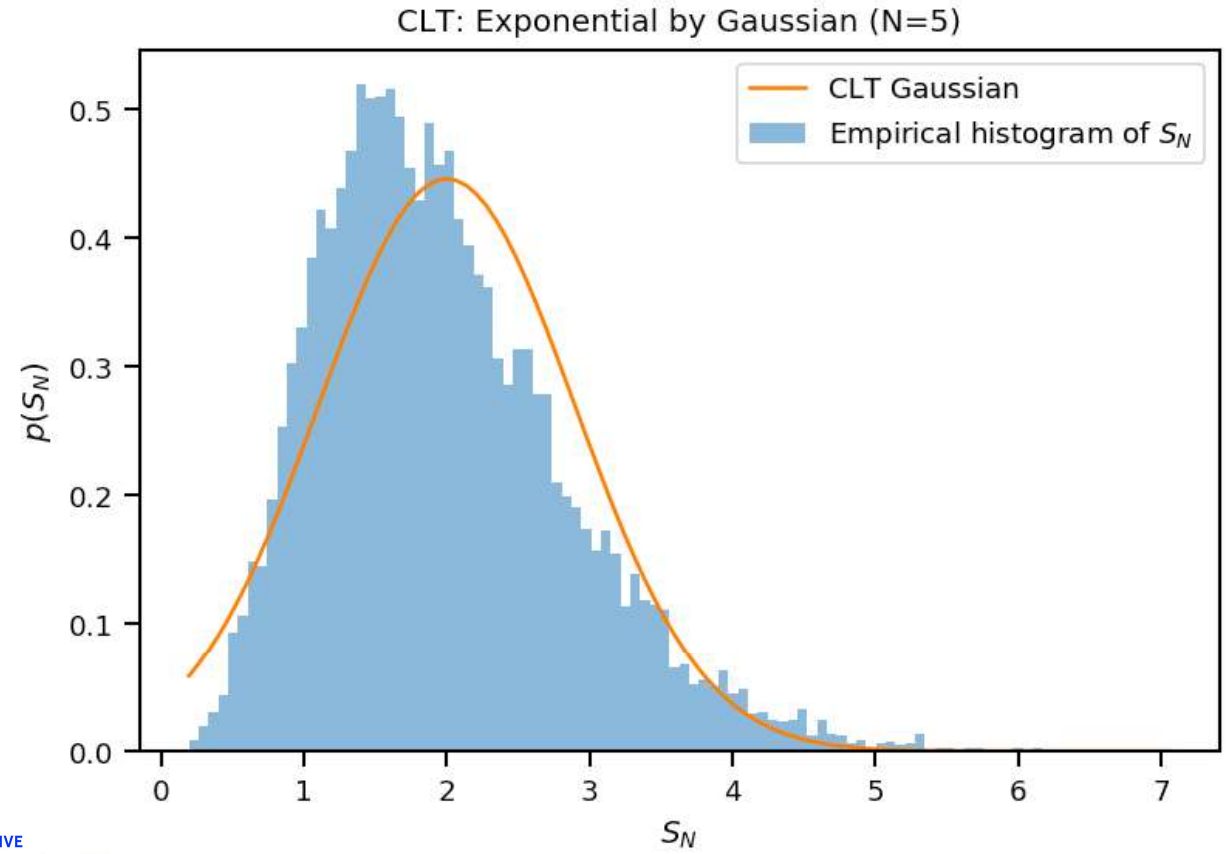
PREDICTIVE
SCIENCE LABORATORY

N=2



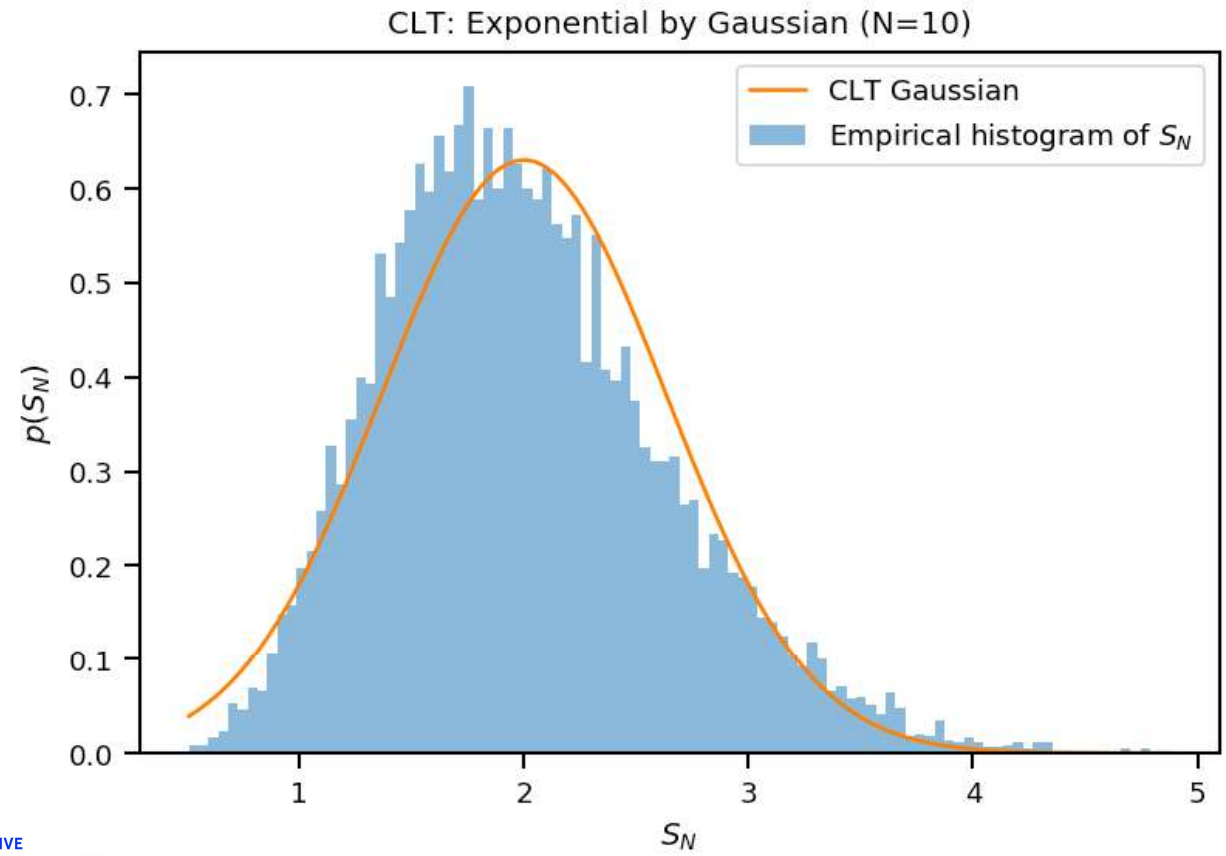
PREDICTIVE
SCIENCE LABORATORY

N=5



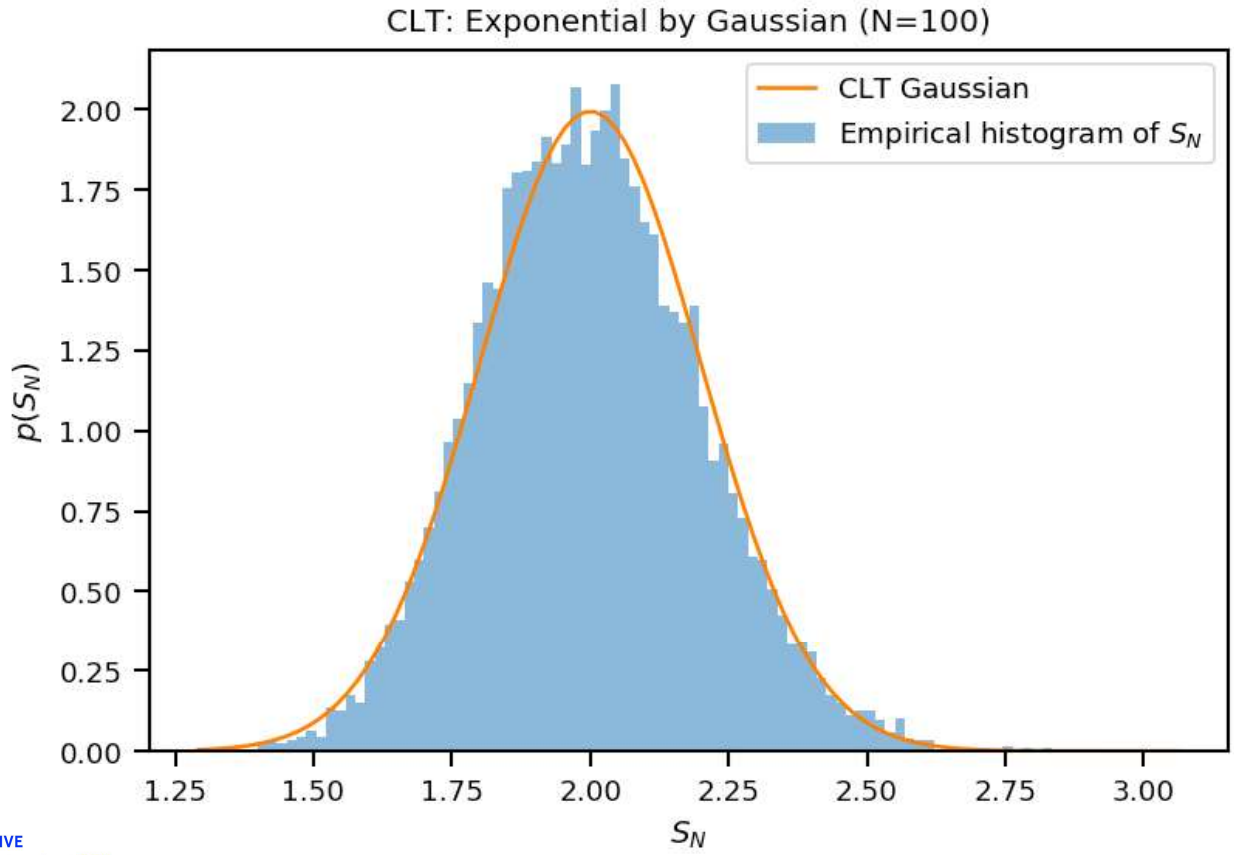
PREDICTIVE
SCIENCE LABORATORY

N=10



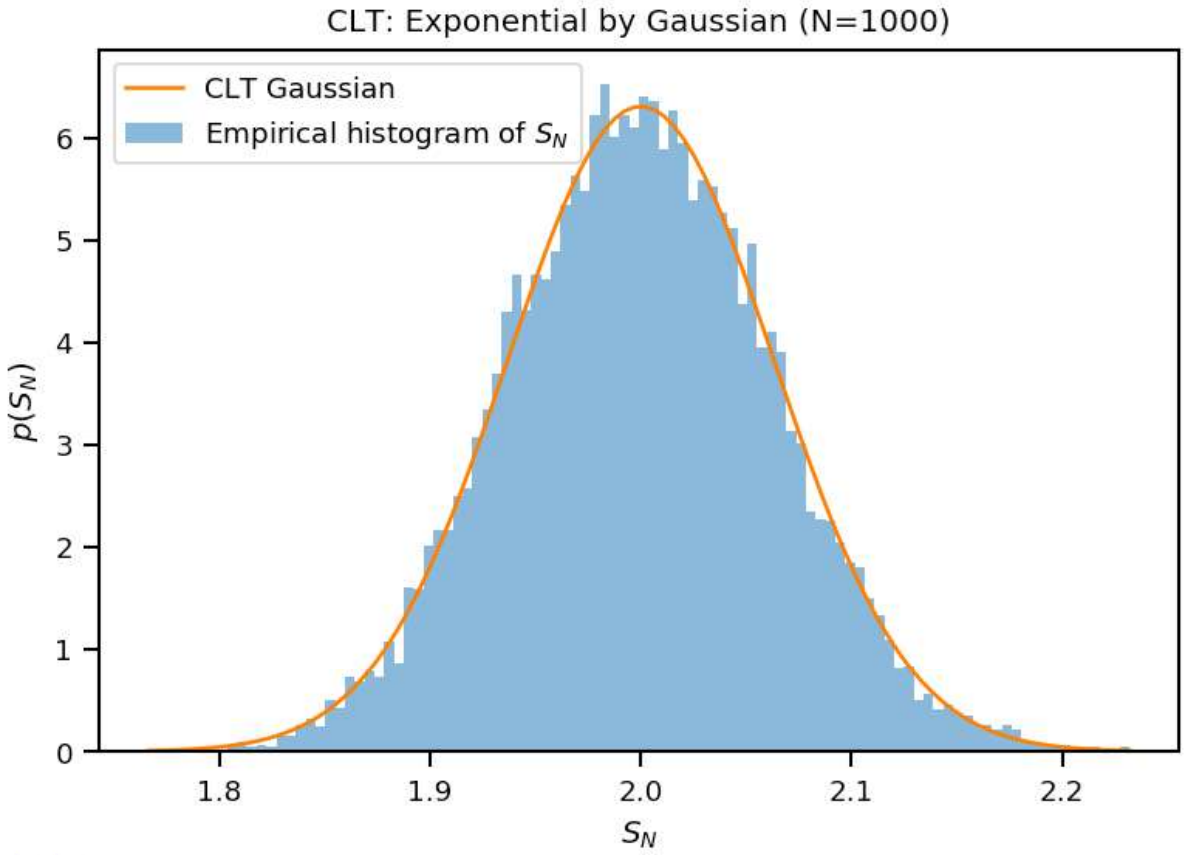
PREDICTIVE
SCIENCE LABORATORY

N=100



PREDICTIVE
SCIENCE LABORATORY

N=1000


CLT: Exponential by Gaussian (N=1000)
CLT Gaussian
Empirical histogram of S_N
$p(S_N)$
0
1
2
3
4
5
6
1.8
1.9
2.0
2.1
2.2
S_N