Lecture 28: Variational Inference

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Posterior approximations

The mean-field Gaussian approximation

$$q(z;\varphi) = \mathcal{N}\Big(z \,\Big|\, \underset{\in \mathbb{R}^K}{\mu}, \underbrace{\operatorname{diag}(\sigma_1^2, \dots, \sigma_K^2)}_{\substack{\| \\ \operatorname{diag}(e^{\omega_1}, \dots, e^{\omega_K})}}\Big)$$

$$\sigma_i = \exp\{\omega_i\}$$

$$q(z;\varphi) = \mathcal{N}\left(z \,\middle|\, \mu, \operatorname{diag}\left(e^{\omega_1}, \dots, e^{\omega_K}\right)\right)$$

$$\varphi = \{\mu, \omega_1, \dots, \omega_K\}$$

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The full-rank Gaussian variational approximation

$$q(z;\varphi) = \mathcal{N}(z \mid \mu, \Sigma)$$

$$\Sigma = L \cdot L^T$$

Cholesky of Σ

$$L = \begin{pmatrix} * & & 0 \\ * & * & \\ * & * & * \\ & & * \end{pmatrix} \quad > 0$$

$$\varphi = \Big\{ \underbrace{\mu}_{K}, \underbrace{\qquad}_{K}, \underbrace{\qquad\qquad}_{\frac{K^2-K}{2}} \Big\}$$

The low-rank Gaussian variational approximation

Full rank approx. not good for high dim. x.

$$q(z;\varphi) = N\left(z \,\middle|\, \mu,\ \mathrm{diag}\left(e^{\omega_1},\dots,e^{\omega_K}\right) + U \cdot U^T\right)$$

$U \in \mathbb{R}^{K \times R}$

rank

$$\varphi = \left\{ \underbrace{\mu}_{K},\ \underbrace{\omega_1,\dots,\omega_K}_{K},\ \underbrace{U}_{KR} \right\}$$

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