Lecture 26: Physics-informed deep neural networks

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Physics-informed regularization: Solving ODEs

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From ODE to a loss function

$$\begin{cases} \frac{du}{dt} = f(t,u) & (1) \\ u(0) = u_0 & (2) \end{cases} \quad \text{initial value problem}$$

<u>IDEA</u>: Represent the solution w/ a DNN.

$$\rightarrow u(t) = u_0 + t \cdot \underbrace{N}_{\text{DNN}}(t; \underbrace{\theta}_{\text{param.}}) \quad ; \quad u(0) = u_0$$

$$(1) \longrightarrow L_T(\theta) = ?$$

$$L(\theta) = \int_0^T \Big\{ \underbrace{\frac{du}{dt} - f(t,u)}_{\text{residual}} \Big\}^2 dt$$

$$= \int_0^T \Big\{ N(t;\theta) + t\frac{\partial N(t;\theta)}{\partial t} - f(t, u_0 + tN(t;\theta)) \Big\}^2 dt$$

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Solving the problem with stochastic gradient descent

$$L(\theta) = \int_0^T \ell(t,\theta)\, dt$$

M times, integer

$T_j \sim U([0,T])$, indipendent

$$L(\theta) = \mathbb{E}\left[\frac{T}{M}\sum_{j=1}^{M} \ell(T_j,\theta)\right].$$

- θ_0
- Sample times t_{jk} of T_j
- $\theta_{k+1} = \theta_k - \alpha_k \frac{T}{M}\sum_{j=1}^{M} \nabla_\theta \ell(t_{jk},\theta_k)$

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