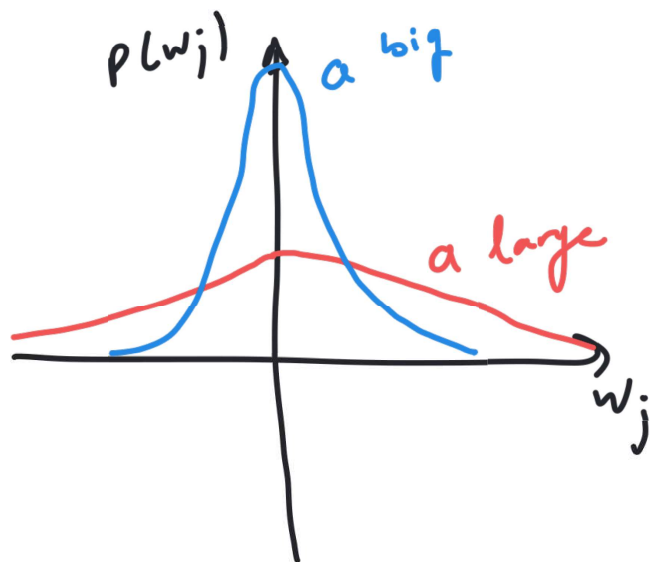# Open questions

- How do I quantify the measurement noise?

- How do we avoid overfitting?

- How do I quantify epistemic uncertainty induced by limited data?

- How do I choose any remaining parameters?

- How do I choose which basis functions to keep?

Gaussian prior on weights

$$w_j \sim N\left(0, \underset{\text{precision} = \frac{1}{\text{variance}}}{a^{-1}}\right)$$

$$p(\underline{w}) = \prod_{j=1}^{m} p(w_j)$$

$$\propto \exp\left\{ -\frac{a}{2} \sum_{j=1}^{m} w_j^2 \right\}$$

Maximum a posteriori estimate

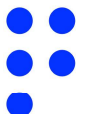posterior $\propto$ likelihood $\times$ prior

$$p(\underline{w} \mid x_{1:N}, y_{1:N}, \sigma) \propto p(y_{1:N} \mid x_{1:N}, \sigma^2) \, p(\underline{w})$$

$$\max_{\underline{w}} \log \text{post} = \log \text{like} + \log p(w)$$

$$= -\frac{1}{2\sigma^2} \sum_{i=1}^{N} \left(y_i - \varphi^T(x_i)\underline{w}\right)^2 - \frac{a}{2} \sum_{j=1}^{m} w_j^2 + \text{const.}$$

$$\min_{\underline{w}} \underbrace{\sum_{i=1}^{N} \left(y_i - \varphi^T(x_i)\underline{w}\right)^2}_{SE} + \underbrace{\frac{a}{2} \sum_{j=1}^{m} w_j^2}_{\text{Regularization term}}$$

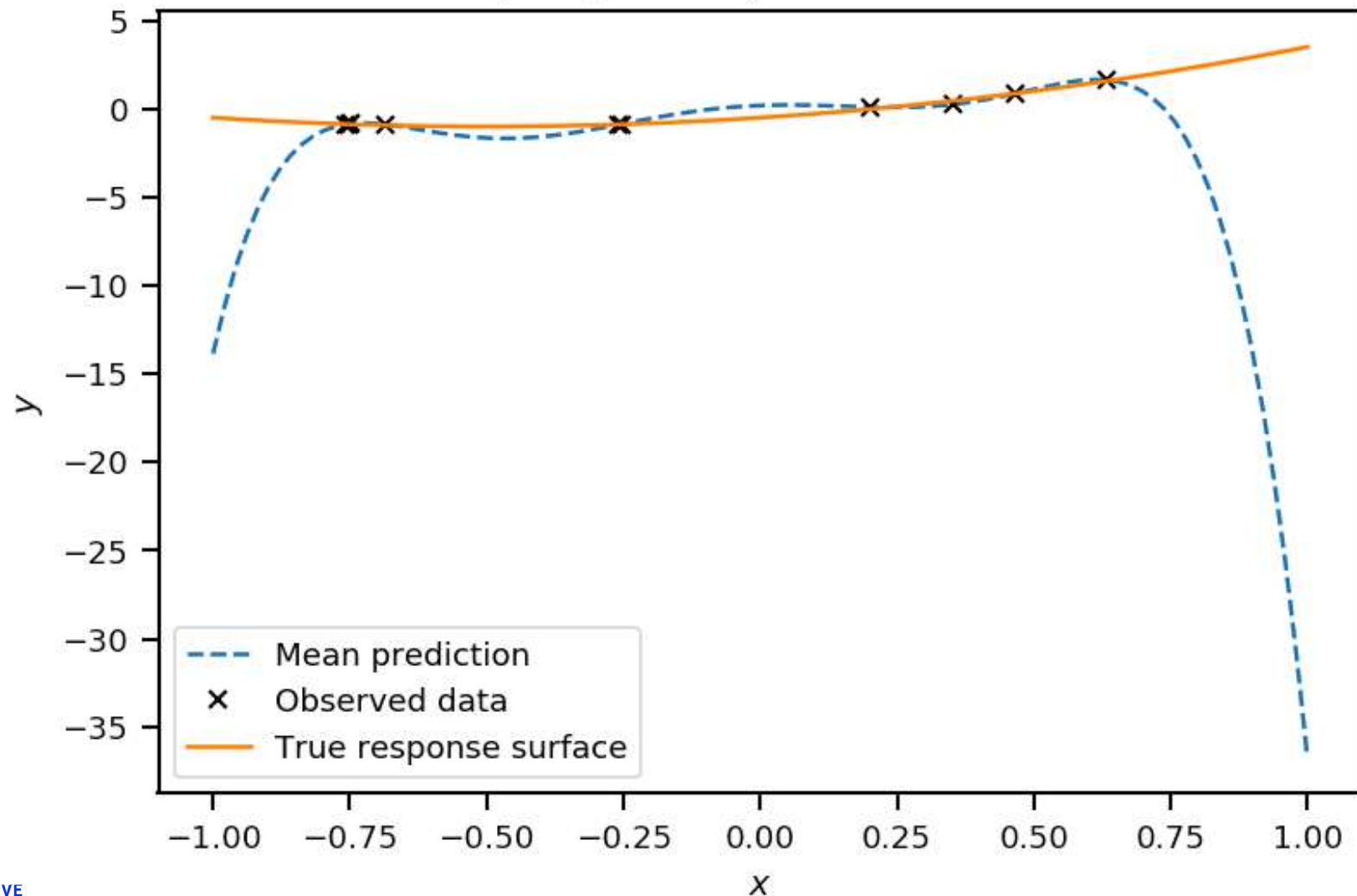# Example: Degree 6 polynomial $(\alpha = 0)$

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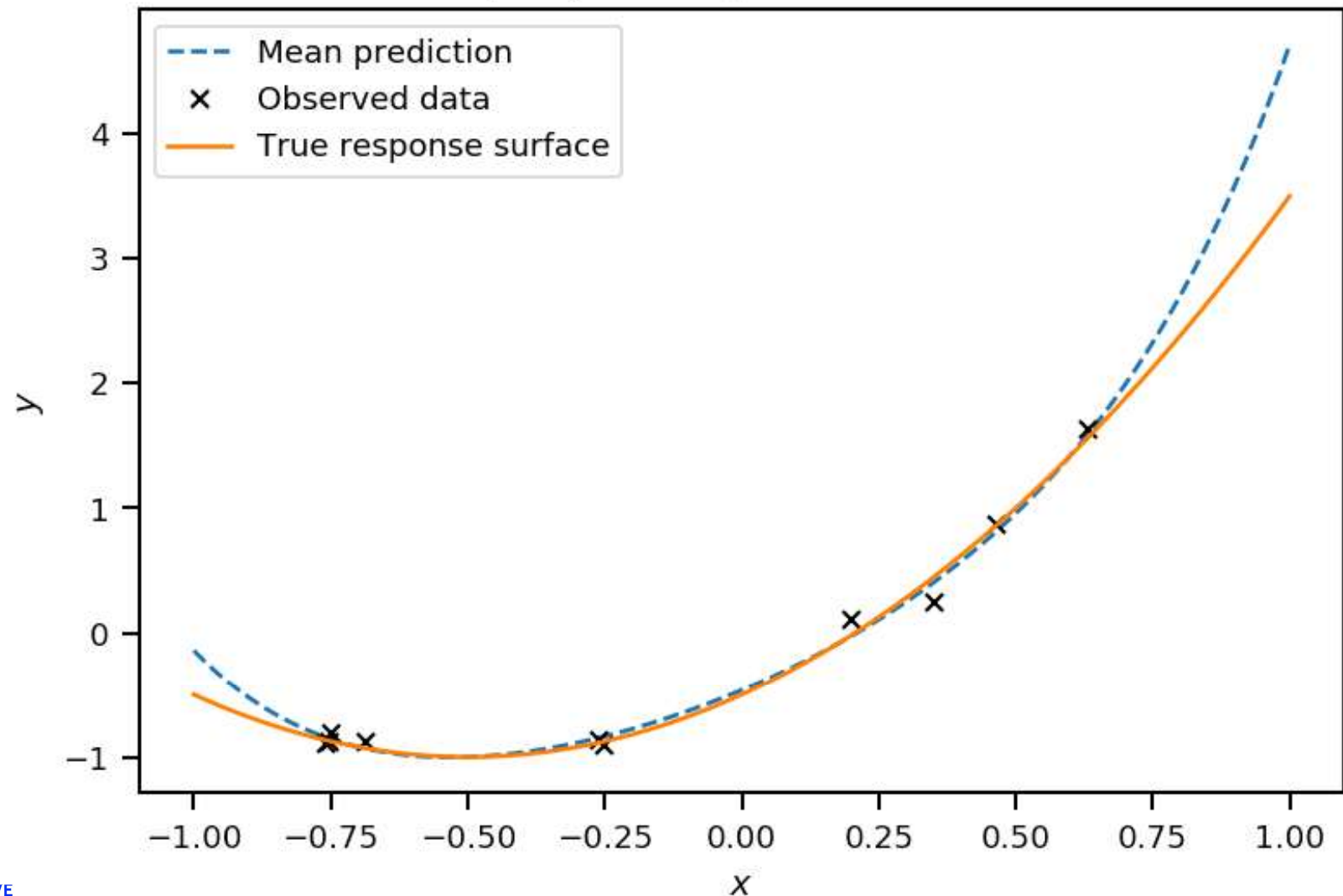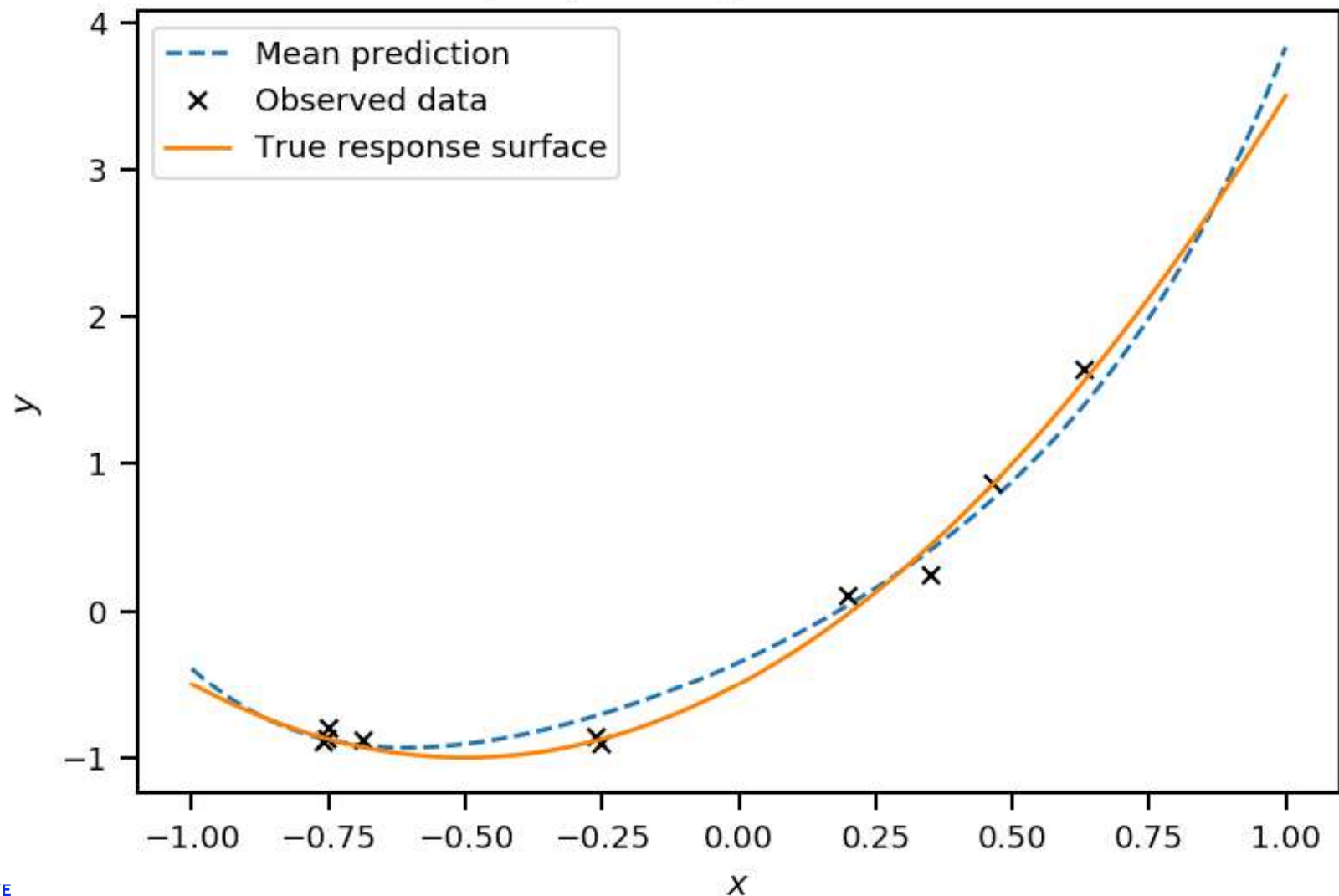# Example: Degree 6 polynomial $(\alpha = 1)$

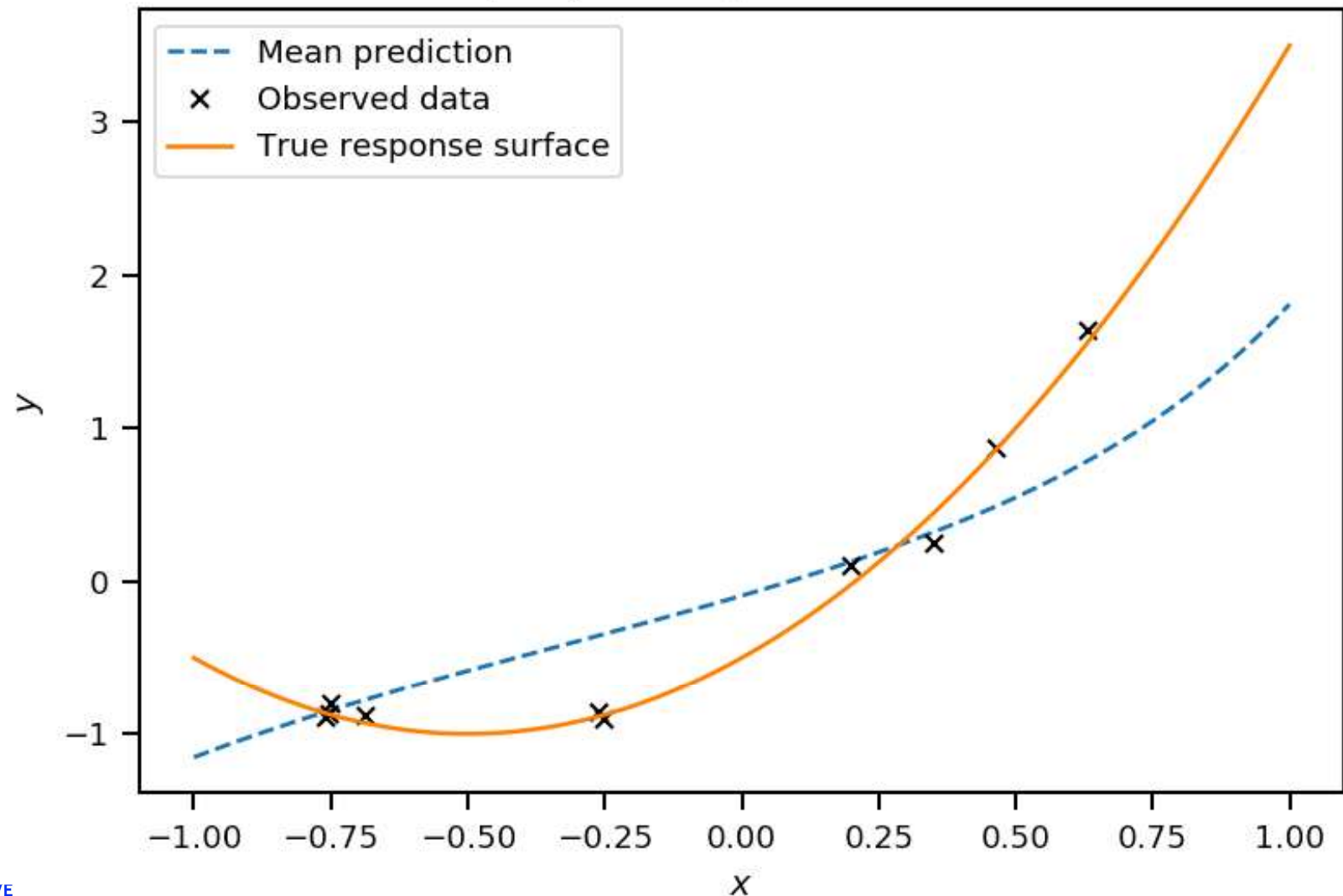$\rho = 6, \sigma = 0.10, \alpha = 1.00e + 00$

Example: Degree 6 polynomial $(\alpha = 10)$

$\rho = 6, \sigma = 0.10, \alpha = 1.00e + 01$

Example: Degree 6 polynomial ($\alpha = 100$)

$\rho = 6, \sigma = 0.10, \alpha = 1.00e + 02$

Mean square error over a validation dataset as a function of α