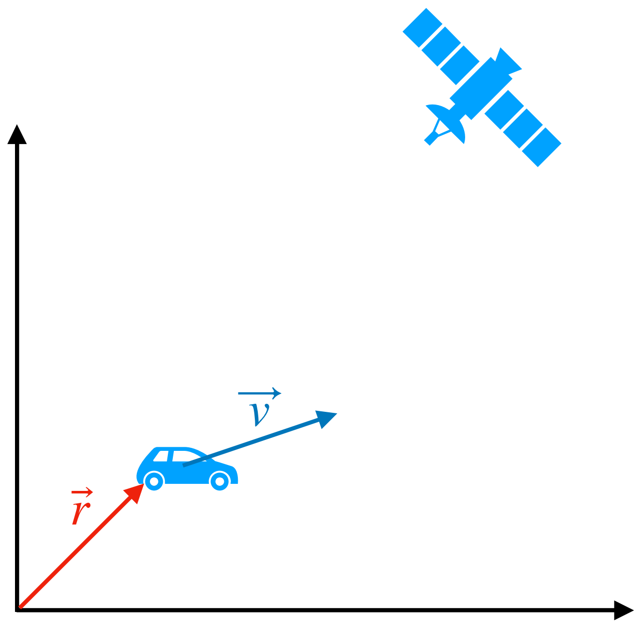
Lecture 20: State-space models - Kalman filters

Professor Ilias Bilionis

Kalman filter - The complete algorithm

PREDICTIVE
SCIENCE LABORATORY

Reminder: Object tracking equations in 2D



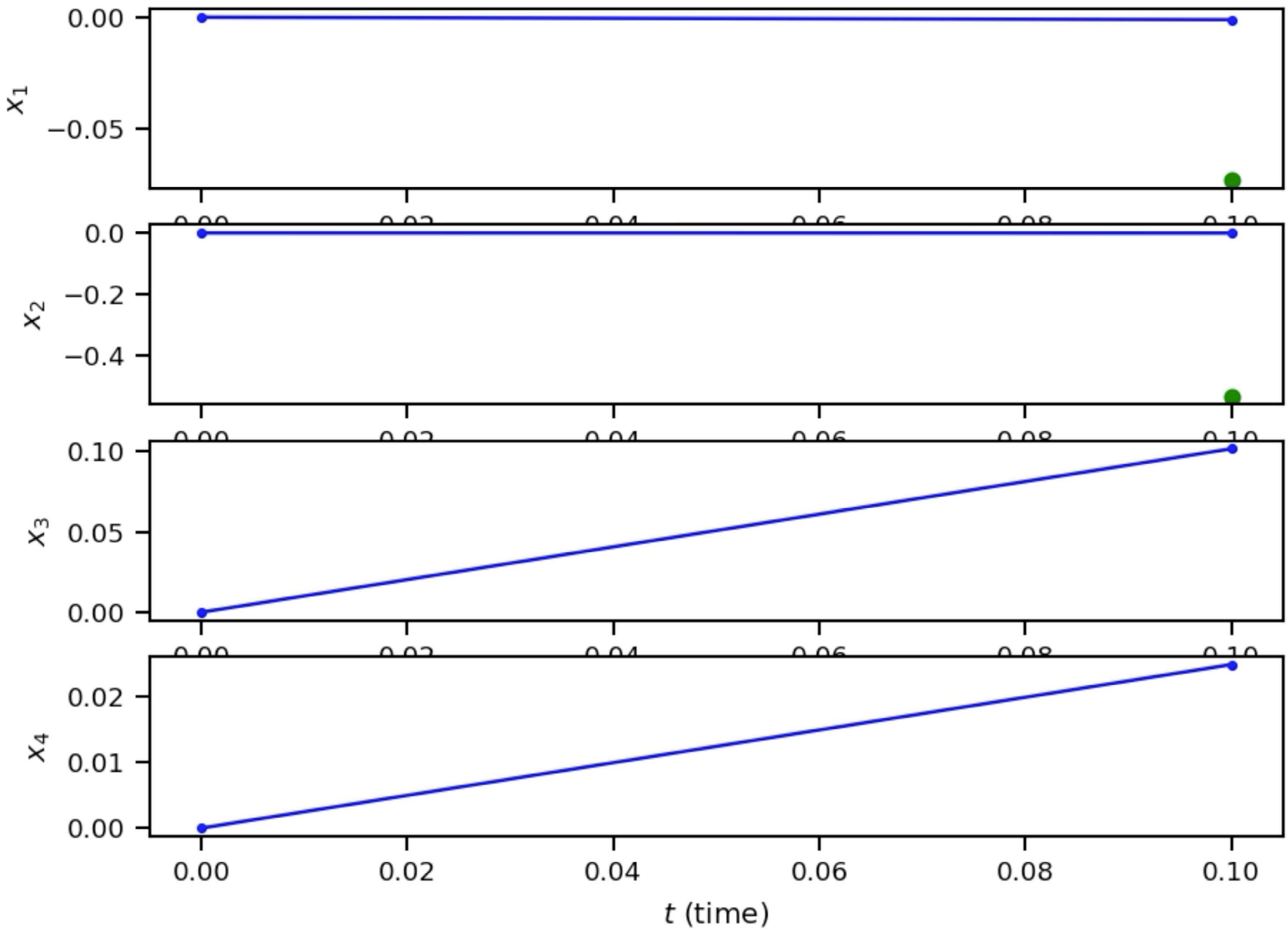
$$m\ddot{\vec{r}} = u_1\hat{i} + u_2\hat{j}$$

$$\mathbf{x} = \begin{bmatrix} r_1 \\ r_2 \\ v_1 \\ v_2 \end{bmatrix}$$

Discretize in time:

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n + \mathbf{B}\mathbf{u}_n + \mathbf{z}_n, \mathbf{z}_n \sim N(0,\mathbf{Q})$$

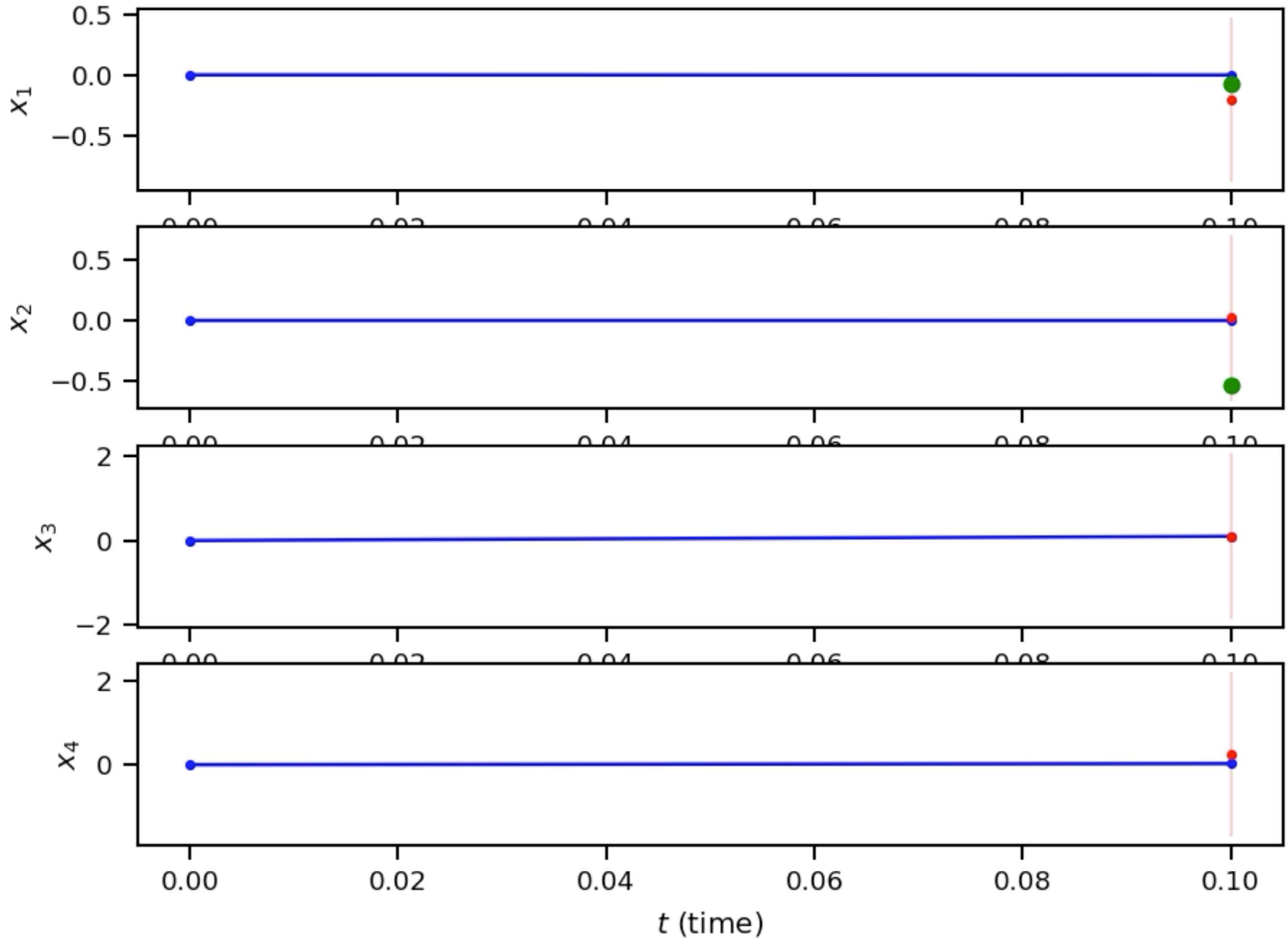
$$\mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \mathbf{w}_n, \mathbf{w}_n \sim N(0,\mathbf{R})$$



0.00
−0.05
x_1
0.0
−0.2
−0.4
x_2
0.10
0.05
0.00
x_3
0.02
0.01
0.00
x_4
0.00
0.02
0.04
0.06
0.08
0.10
t (time)

Kalman filter algorithm

- Initialize state.
- **Predict** new using transition probabilities.
- Make observation.
- **Update** belief using observation.
- Go to predict.



0.5
0.0
−0.5
x_1
0.00
0.02
0.04
0.06
0.08
0.10
0.5
0.0
−0.5
x_2
2
0
−2
x_3
2
0
x_4
t (time)