Lecture 6: Random Vectors

Professor Ilias Bilionis

The multivariate normal - marginalization

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Marginalization

- Assume that you have a random vector $\mathbf{X}$ made out of two sub-random vectors $\mathbf{X}_1$ and $\mathbf{X}_2$, i.e.:

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- What is the probability density of $\mathbf{X}_1$?

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Marginalization

- Assume that:

$$\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \; ; \quad \underline{X} = \begin{pmatrix} \underline{X}_1 \\ \underline{X}_2 \end{pmatrix}$$

- Decompose mean and covariance in blocks:

$$\boldsymbol{\mu} = \begin{pmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{pmatrix} \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \underline{\underline{\Sigma}}_{11} & \underline{\underline{\Sigma}}_{12} \\ \underline{\underline{\Sigma}}_{12}^T & \underline{\underline{\Sigma}}_{22} \end{pmatrix}$$

- To find the probability density of $\mathbf{X}_1$, we marginalize:

$$p(\mathbf{x}_1) = \int p(\mathbf{x}_1, \mathbf{x}_2) dx_2 = \int N\left(\begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} \middle| \begin{pmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{pmatrix}, \begin{pmatrix} \underline{\underline{\Sigma}}_{11} & \underline{\underline{\Sigma}}_{12} \\ \underline{\underline{\Sigma}}_{12}^T & \underline{\underline{\Sigma}}_{22} \end{pmatrix} \right) d\underline{x}_2$$

$$= \ldots = N\left(\underline{x}_1 \,\middle|\, \underline{\mu}_1, \underline{\underline{\Sigma}}_{11} \right)$$

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