Lecture 2: Basics of Probability Theory

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Dynamics of a coin toss

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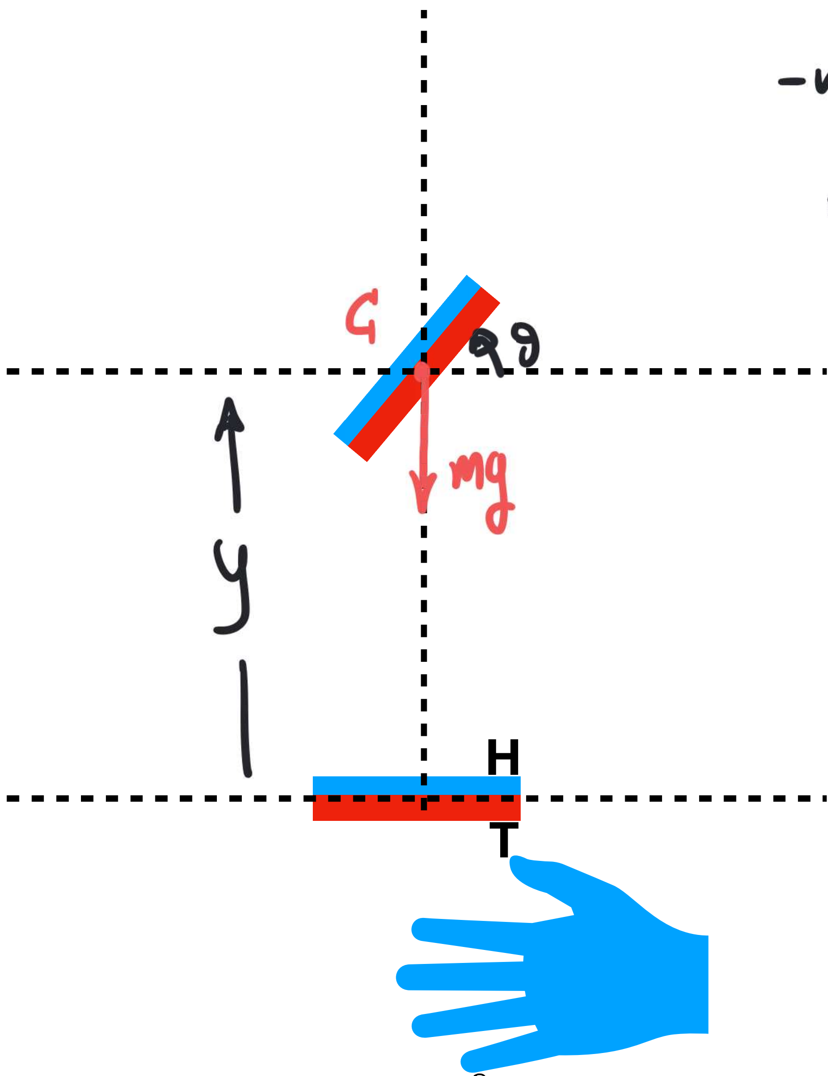
Dynamics of a coin toss



$$\sum F_y = m a_y$$

$$-mg = m\ddot{y}$$

$$\Rightarrow \ddot{y} = -g \quad (1)$$

$$\sum M_c = I\alpha$$

$$0 = I\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = 0 \quad (2)$$

Dynamics of a coin toss

$$
\begin{aligned}
\ddot{y} &= -g, \\
\ddot{\theta} &= 0, \\
\underline{y(0)} &= 0, \\
\underline{\theta(0)} &= 0, \\
\underline{\dot{y}(0)} &= v_0, \quad \text{init. vel.} \\
\underline{\dot{\theta}(0)} &= \omega_0 \,. \quad \text{int. ang. vel.}
\end{aligned}
$$

Solve the initial value problem:

$$
\boxed{y(t) = -\frac{1}{2} g t^2 + v_0 t,}
$$
$$
\boxed{\theta(t) = \omega_0 t \,.}
$$

Dynamics of a coin toss

$$\begin{aligned} y(t) &= -\frac{1}{2}gt^2 + v_0 t, \\ \theta(t) &= \omega_0 t\,. \end{aligned}$$

Time it takes to hit the hand:

$$y(t_1) = 0 \implies \boxed{t_1 = \frac{2v_0}{g}}\,.$$

Angle when the coin hits the hand again:

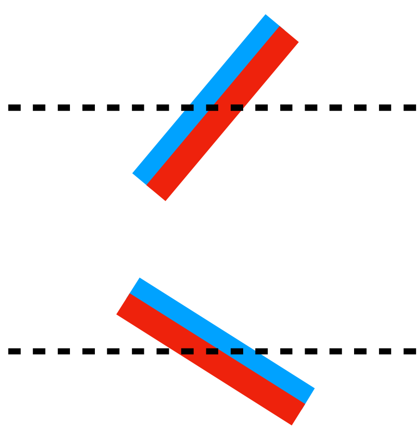
$$\boxed{\theta(t_1) = \frac{2v_0\omega_0}{g}}$$

Dynamics of a coin toss

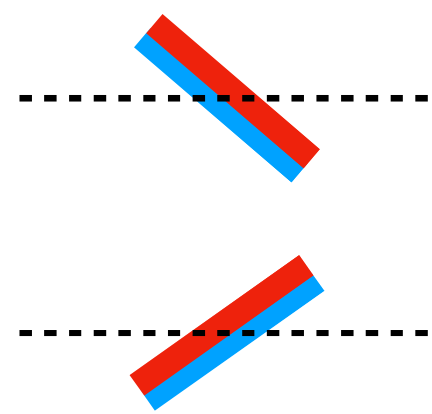
Angle when the coin hits the hand again:

$$\theta(t_1) = \frac{2v_0\omega_0}{g}$$

HEADS

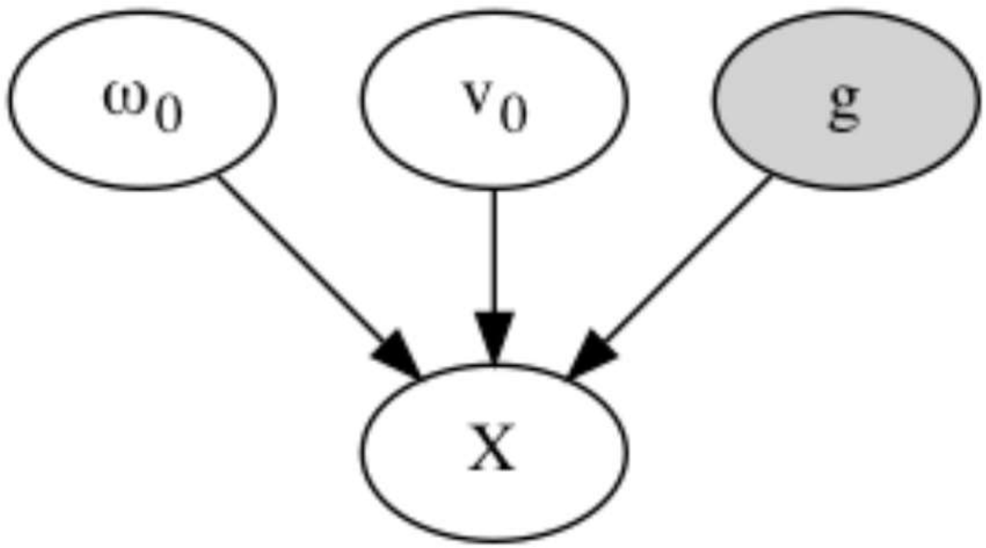


TAILS



Graphical causal model representation

$$X = \begin{cases} T, & \textbf{if } \frac{2v_0\omega_0}{g} (\textbf{mod } 2\pi) \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \\ H, & \textbf{otherwise}. \end{cases}$$



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Dynamics of a coin toss: Rise of uncertainty