

Lecture 3: Discrete Random Variables

Professor Ilias Bilonis

Variance of a discrete random variable

Expectation of a random variable

- The variance of a random variable is:

$$\mathbb{V}[X] := \mathbb{E} \left[(X - \mathbb{E}[X])^2 \right]$$

$$= \sum_x (x - \mathbb{E}[X])^2 p(x)$$

- You can think of the variance as the spread of the random variable around its expectation.
- However, do not take this too literally for discrete random variables.

Properties of the variance

- Take any constant c :

$$\mathbb{V}[X + c] = \mathbb{V}[X]$$

Proof:

$$\begin{aligned}\mathbb{V}[X + c] &= \mathbb{E}[(X + c - \mathbb{E}[X + c])^2] \\ &= \mathbb{E}[(X - \mathbb{E}[X])^2] \quad \square\end{aligned}$$

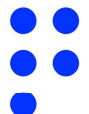
Properties of the variance

- Take any constant λ :

$$V[\lambda X] = \lambda^2 V[X]$$

Proof:

$$\begin{aligned} V[\lambda X] &= E[(\lambda X - E[\lambda X])^2] \\ &= E[(\lambda X - \lambda E[X])^2] \\ &= E[\lambda^2 (X - E[X])^2] \\ &= \lambda^2 E[(X - E[X])^2] \end{aligned}$$



Properties of the variance

- It holds that:

$$V[X] = E[X^2] - (E[X])^2$$

Proof: $V[X] = E[(X - E[X])^2]$

$$= E[X^2 - 2X E[X] + (E[X])^2]$$
$$= E[X^2 - 2X E[X]] + (E[X])^2$$
$$= E[X^2] - 2(E[X])^2 + (E[X])^2$$

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