Lecture 3: Discrete Random Variables

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The Bernoulli distribution

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Example: The Bernoulli distribution

- Models an experiment with two outcomes.

$$X = \begin{cases} 1, & \text{with probability } \theta, \\ 0, & \text{otherwise}. \end{cases}$$

- Notation:

$$\boxed{X \sim \text{Bernoulli}(\theta)}$$

- You read: “X follows a Bernoulli with parameter θ.”

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Example: PMF of a Bernoulli

- Assume $X \sim \text{Bernoulli}(\theta)$.
- We have:

$$p(X=1)=\theta$$

- From this, because of the normalization constraint:

$$p(X=0)+p(X=1)=1$$

we get that: $p(X=0)=$

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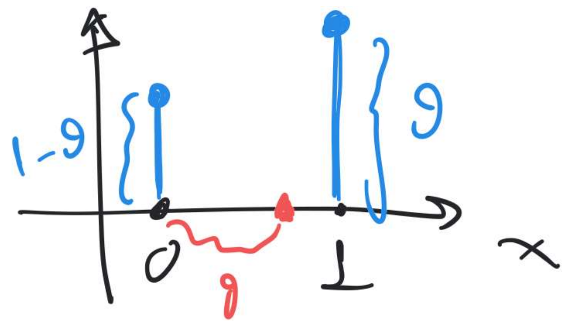
Example: Expectation and variance of a Bernoulli



- Assume $X \sim \text{Bernoulli}(\theta)$.
- The expectation is:

$$\mathbb{E}[X] = \sum_x x\, p(x) = 1 \cdot p(X=1) + 0 \cdot p(X=0)$$
$$= 1 \cdot \theta + 0 \cdot (1-\theta) = \theta$$

- The variance is: $\mathbb{E}[X^2] = \sum_x x^2 p(x) = 1^2 \cdot \theta + 0^2 \cdot (1-\theta) = \theta$

$$\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \theta - \theta^2 = \theta \cdot (1-\theta)$$