Lecture 11: Selecting prior information

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The principle of insufficient reason

Example: Random variable with two possible values

- You have a random variable X with two possible values, say 0 and 1.
- If that's all you know, what probability do you assign to each value?

$$p(X=0) = p(X=1) = \frac{1}{2}$$

Example: Random variable with six possible values

- You have a random variable X with six possible values, say 1, 2, 3, 4, 5, 6.

- If that's all you know, what probability do you assign to each value?

$$p(X=1) = p(X=2) = \cdots = p(X=6) = \frac{1}{6}$$

Laplace considered this obvious

"The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided about in regard to their existence, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favorable cases and whose denominator is the number of all the cases possible…"
—Pierre-Simon Laplace

https://en.wikipedia.org/wiki/Pierre-Simon_Laplace#/media/File:Laplace,_Pierre-Simon,_marquis_de.jpg

Principle of insufficient reason

- Let X be a discrete random variable taking N different values $x_1, \ldots, x_N$.
- If that all we know, then the principle of insufficient reason states that we should assign:

$$p(X = x_1) = \cdots = p(X = x_N) = \frac{1}{N}$$

Generalizations of the principle of insufficient reason

Equivalent states of knowledge should be assigned equivalent epistemic probabilities
—E. T. Jaynes

https://en.wikipedia.org/wiki/Edwin_Thompson_Jaynes#/media/File:ETJaynes1.jpg

Principle of transformation groups (advanced)

Principle of maximum entropy