Lecture 15: Advanced topics in Bayesian linear regression

Professor Ilias Bilionis

Diagnostics for posterior predictive

Standarized errors

Post. Pred. $p(y|x, data) = N(y|m(x), \sigma^2(x))$

Validation data: $x_i, y_i, \quad i=1, \dots, N^v$

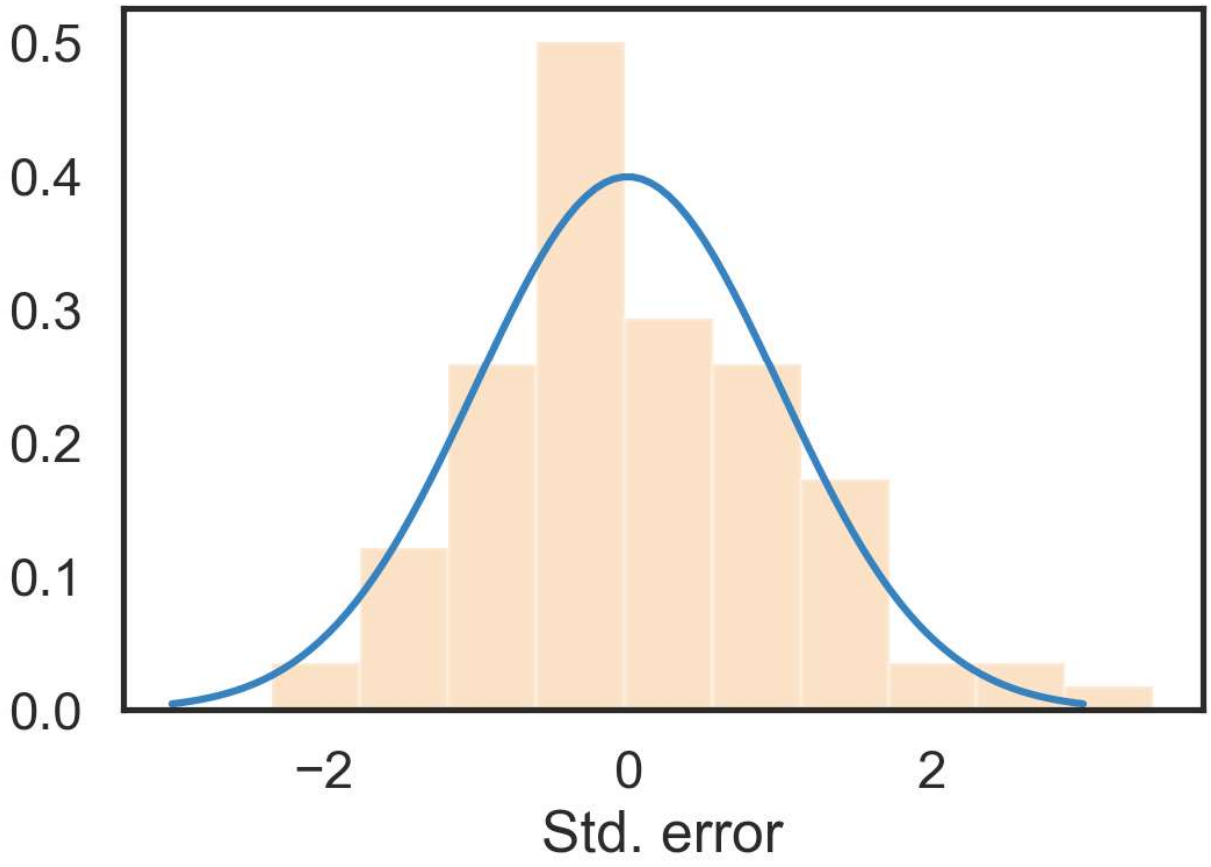
model says: $y_i | x_i \sim N(m(x_i), \sigma^2(x_i))$

Standarized Error: $z_i := \dfrac{y_i - m(x_i)}{\sigma(x_i)} \sim N(0,1)$ (If model correct)

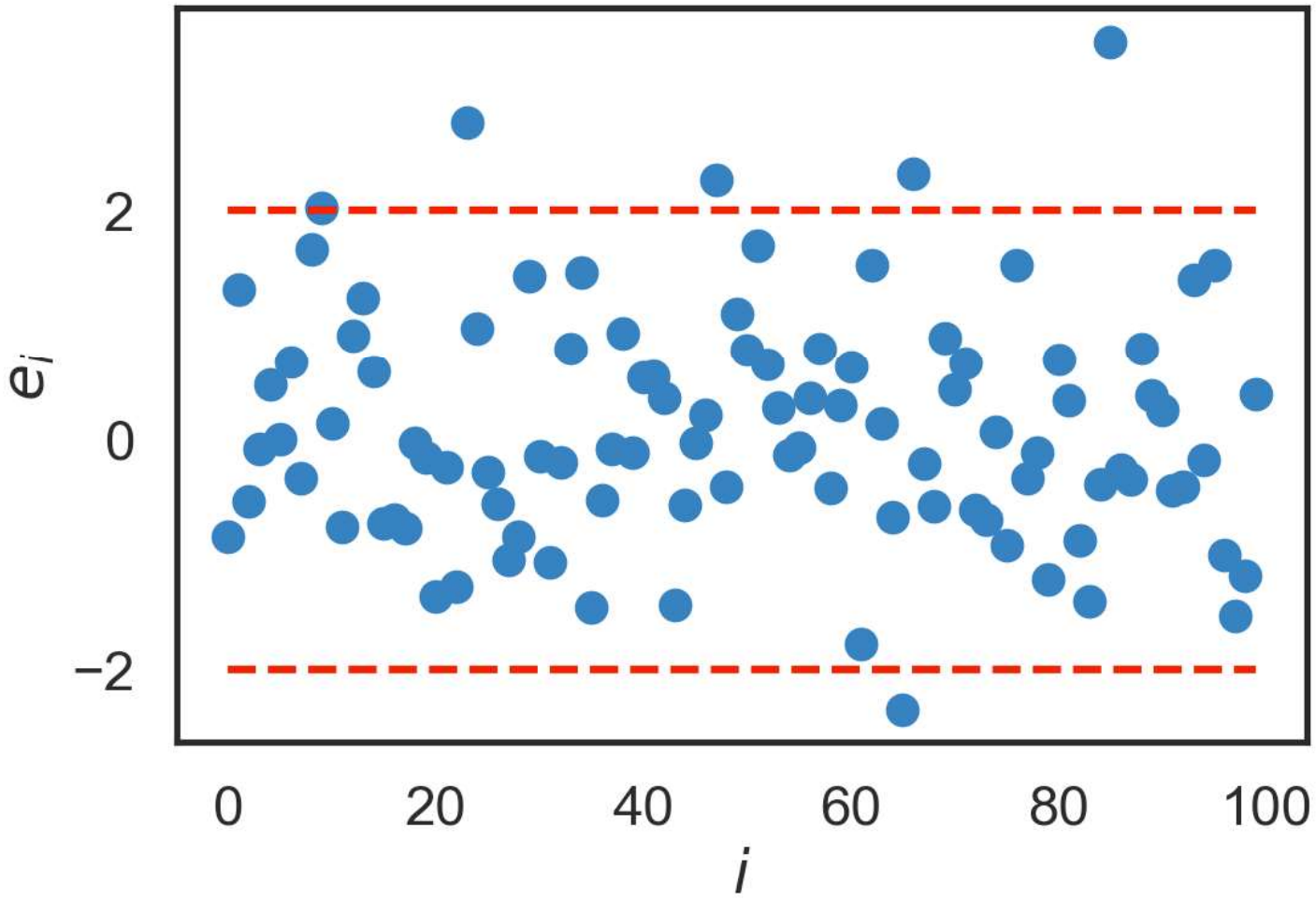
$\mathbb{E}[z_i] = \mathbb{E}\left[\dfrac{y_i - m(x_i)}{\sigma(x_i)}\right] = \dfrac{\{\mathbb{E}[y_i] - m(x_i)\}}{\sigma(x_i)} = 0$ ($\mathbb{E}[y_i] = m(x_i)$ correct)

$\mathbb{V}[z_i] = \mathbb{V}\left[\dfrac{y_i - m(x_i)}{\sigma(x_i)}\right] = \dfrac{1}{\sigma^2(x_i)}\mathbb{V}[y_i] = \dfrac{\sigma^2(x_i)}{\sigma^2(x_i)} = 1$

Standardized Errors



Standardized Errors



Standardized Errors