Lecture 28: Variational Inference

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The variational problem in real space

The variational problem in real space

$$\max_{\varphi} \mathrm{ELBO}(\varphi)$$

$$\mathrm{ELBO}(\varphi) = \underbrace{-\mathbb{E}_{q(z;\varphi)}\left[\log q(z;\varphi)\right]}_{\substack{\mathbb{H}[q(z;\varphi)] \\ \text{analytically avail.} \\ \text{for all Gaussians.}}} + \underbrace{\mathbb{E}_{q(z;\varphi)}\left[\log p(x=T^{-1}(z),y) + \log\left|\det J_{T^{-1}}(z)\right|\right]}_{\substack{\text{not analytically} \\ \text{avail.}}}$$

The reparameterization trick - Full/low-rank Gaussian approximation

$$\text{ELBO}(\phi) = \mathbb{H}[q(z;\phi)] + \mathbb{E}_{q(z;\phi)}\left[\ln p(T^{-1}(z), y) + \left|\det J_{T^{-1}}(z)\right|\right]$$

$$\nabla_\phi \, \mathbb{E}_{q(z;\phi)}[\ \dots\ ]$$

$$n = S_\phi(z) \sim N(0, I)$$

$$n = L^{-1}(z - \mu) \sim N(0, I)$$ full-rank

$$\mathbb{E}[n] = 0, \quad \mathbb{C}ov[n] = I$$

$$\text{ELBO}(\phi) = \mathbb{H}[q(z;\phi)] + \mathbb{E}_{n \sim N(0,I)}\left[\ln p(T^{-1}(S_\phi^{-1}(n))) + \left|\det J_{T^{-1}}(S_\phi^{-1}(n))\right|\right]$$

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The reparameterization trick - mean-field Gaussian approximation