Lecture 1: Introduction to Predictive Modeling

Professor Ilias Bilionis

What is predictive modeling?

What can go wrong in an engineering system?

- Engineering systems can be very complicated.
- The are **known knowns.**
- There are **known unknowns.**
- There are **unknown unknowns.**

Orbiter challenger as it lifts from Pad 39A

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A shot at defining predictive modeling

Predictive modeling is the process of describing our state of knowledge about *known unknowns* in order to make informed decisions.

What about unknown unknowns?

- There is no widely accepted automated way for turning unknown unknowns into known unknowns.
- Effectively, this is what science is doing.
- Automating the process requires understanding how humans perform induction.
- It may require general artificial intelligence.

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Causal models and their graphical representation

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Why worry about causality?

- Models that explicitly encode causal relationships are the only useful models.
- Most physical and engineering models are causal models.
- When we extend them to account for known unknowns, we need to make sure that they remain causal.
- Otherwise, we are merely capturing correlations and the results cannot be trusted.
- Structural causal models give us the language we need to formalize causality.

What is a structural causal model?

- A set of variables we are interested in:
  - Y: an individual has asthma or not
  - X: the individual is treated or not
  - Z: air pollution level
- A set of functions that describes how the variables are connected:

$$Y = f(X, Z)\,.$$

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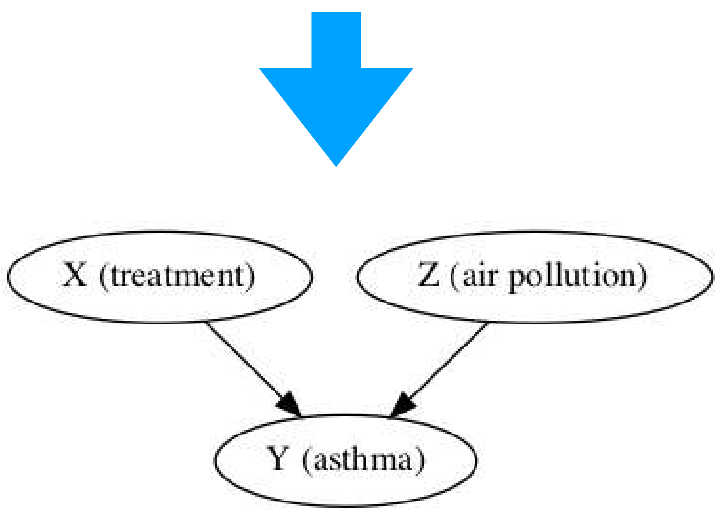
Graphical representation of causal models

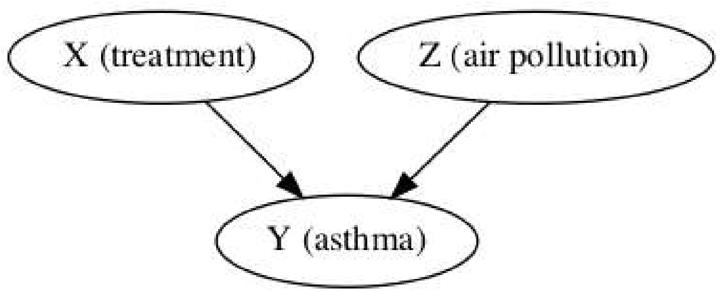
$$Y = f(X, Z)$$



- To each structural causal model there corresponds a directed acyclic graph.
- The edge represents a direct causal link between the parent and the child nodes.

We typically first build the graph and then work out the function details

$$Y = f(X, Z)$$



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Aleatory vs epistemic uncertainty

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Types of uncertainty

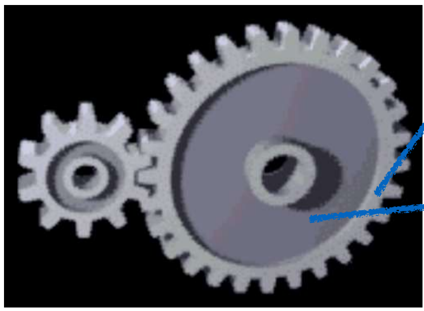
- **Aleatory**: naturally occurring randomness that we cannot (or do not know how to) reduce.

  Latin aleatorius of a gambler, from aleator gambler, from alea a dice game

- **Epistemic**: uncertainty due to lack of knowledge that we can reduce by paying a price.

  Greek επιστήμη meaning knowledge.

Unknown microstructure of a manufactured artifact



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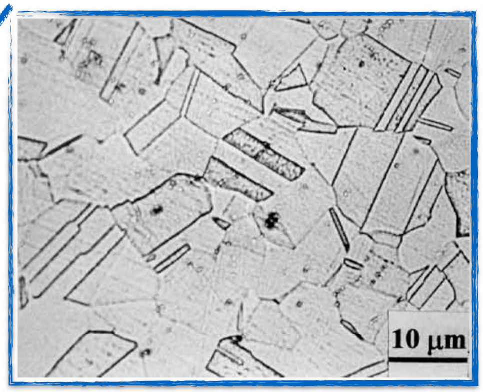

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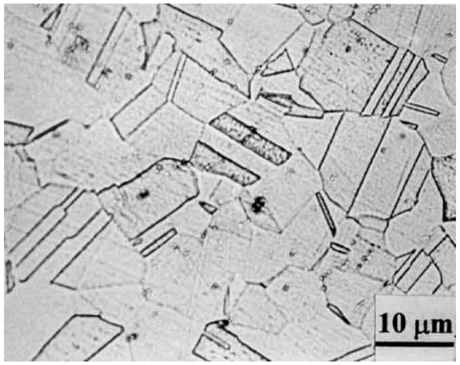
We model uncertainties using probability

$p(A \mid K)$ = "How much do we believe A is true given our current state of knowledge K"

p( |K)=?

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The uncertainty propagation problem

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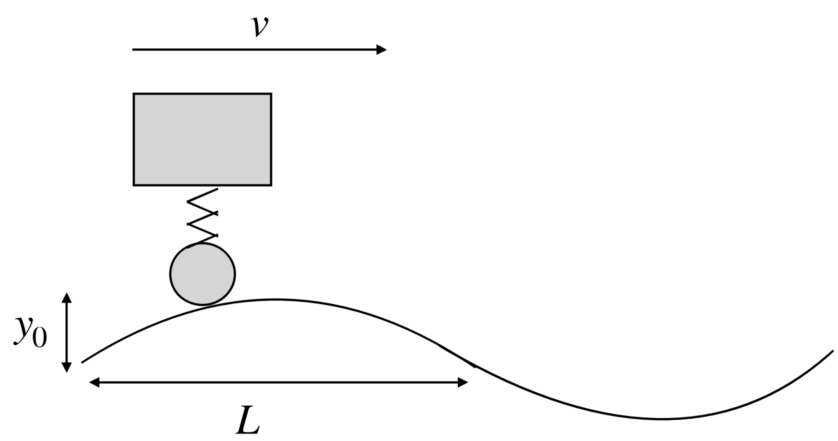
Example: Driving a trailer on a bumpy road

- m: mass
- k: spring constant
- v: velocity
- y_0: amplitude of road roughness
- L: "wavelength" of road roughness



<u>Dynamics</u>

$$\omega = \frac{2\pi v}{L}$$

Angular velocity

$$X = \left| \frac{k y_0}{k - m\omega^2} \right|$$

Amplitude

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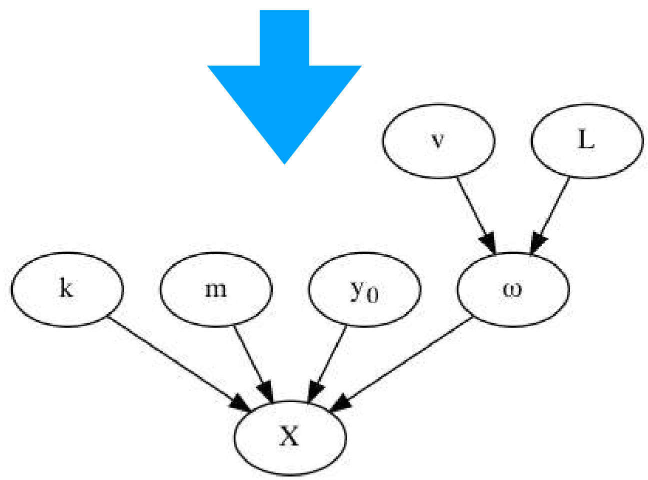
Example: Driving a trailer on a bumpy road

$$\omega = \frac{2\pi v}{L} \qquad X = \left|\frac{k y_0}{k - m\omega^2}\right|$$



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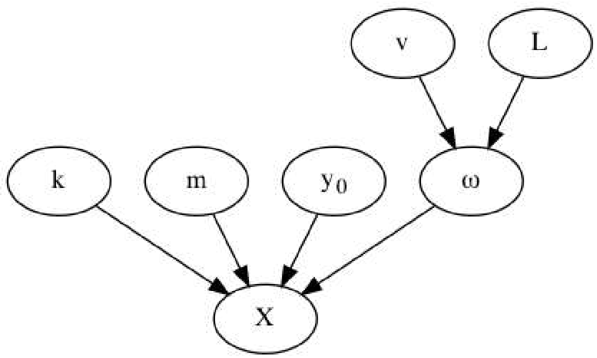
Example: Driving a trailer on a bumpy road



Variable	Type	Values
k	Manufacturing uncertainty	[159,999, 160,001] N/m
v	Operating condition	[80, 150] km/hour
m	Loading condition	[100, 200] kg
y	Road condition	[0, 100] mm
L	Road condition	[1, 2] m

Our state of knowledge about the problem.

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The uncertainty propagation problem

Having quantified our uncertainty about all unknowns, propagate this uncertainty through the causal model to characterize our uncertainty about a quantity of interest.

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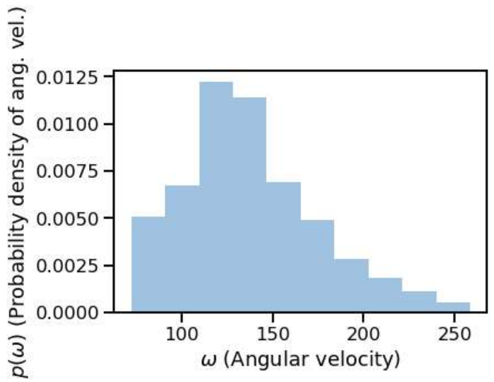
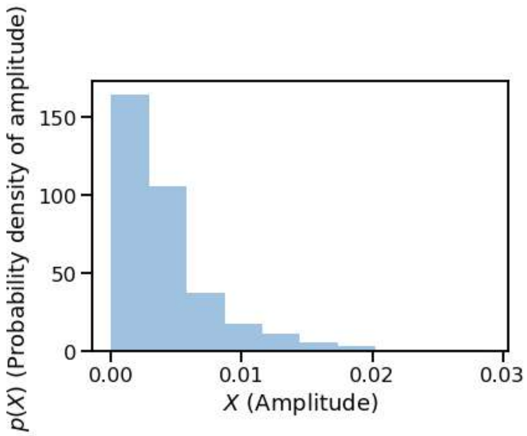
The Monte Carlo solution to the uncertainty propagation problem

- Sample random inputs many times.
- Evaluate model outputs at these inputs.
- Estimate any statistics of interest.





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The model calibration problem

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The model calibration problem

- The model calibration problem is the inverse of the uncertainty propagation problem.
- That is why such problems are also called **inverse problems**.
- We observe a quantity that is predicted by the model and we want to characterize how this observation changes our state of knowledge about the model parameters.

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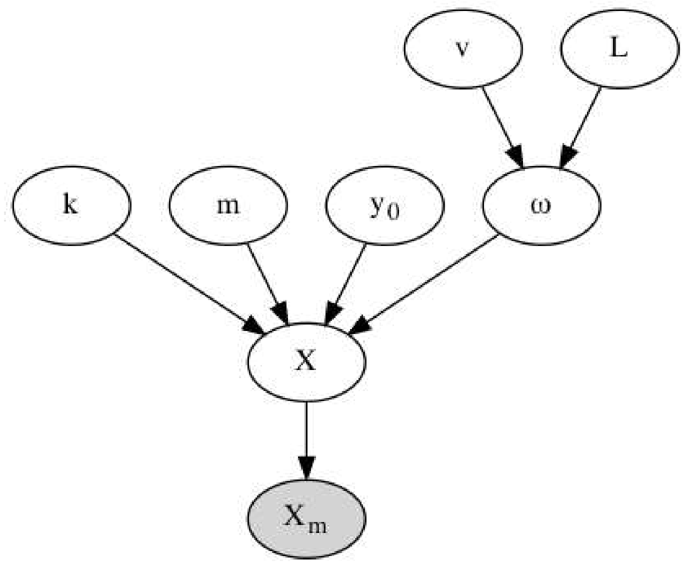
Example: Driving a trailer on a bumpy road

- m: mass
- k: spring constant
- v: velocity
- y_0: amplitude of road roughness
- L: “wavelength” of road roughness
- X_m: the measurement



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The formal solution to the model calibration problem

- Quantify our **prior** state of knowledge about all the model parameters.
- Use Bayes' rule to condition the prior knowledge on the observations to get the **posterior** state of knowledge.
- Create a practical procedure that characterizes our posterior state of knowledge.