# Lecture 26: Physics-informed deep neural networks

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# Physics-informed regularization: Solving high-dimensional stochastic partial differential equations problems

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# Example: Elliptic SPDE

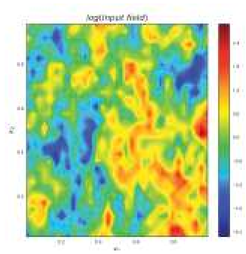
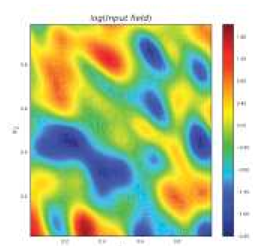
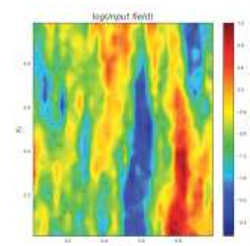
$$\nabla(a(\mathbf{x})\nabla u(\mathbf{x})) = 0,$$

$$\mathbf{x} = (x_1, x_2) \in \Omega = [0,1]^2,$$

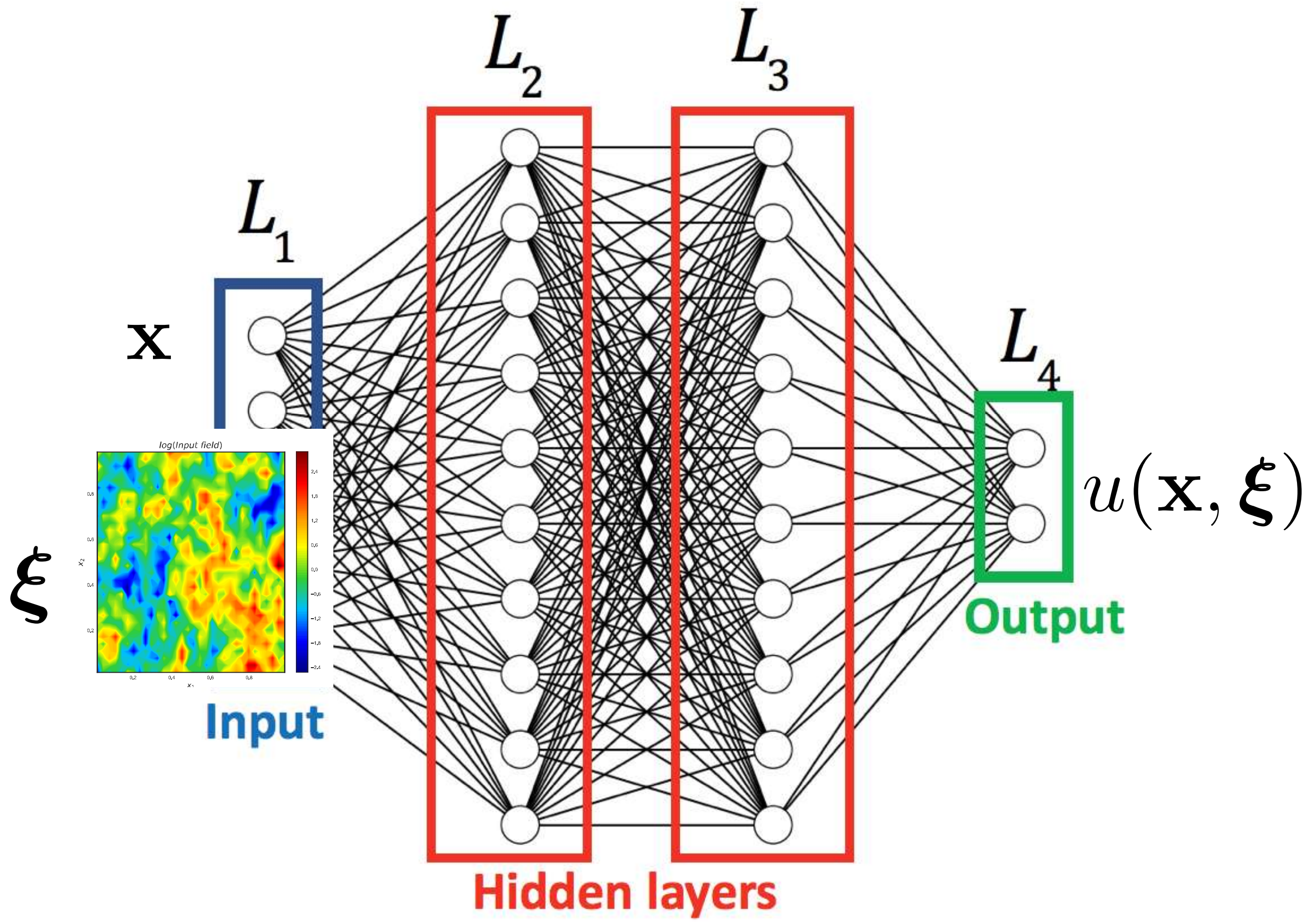
$$u = 0, \forall x_1 = 1,$$

$$u = 1, \forall x_1 = 0,$$

$$\frac{\partial u}{\partial n} = 0, \forall x_2 = 1.$$

$\log a(x) =$  or  or  ...

# Representing the solution of the PDE with a DNN

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# Loss function based on integrated squared residual

$$\begin{cases} -\nabla\cdot\left[a(x;\xi)\nabla u(x,\xi)\right] = f(x), & x\in B \\ u|_{\partial B} = g \\ \xi \sim p(\xi) \end{cases}$$

$$u(x,\xi) = N(x,\xi;\theta)$$

$$L(\theta) = \mathbb{E}_{\xi}\left[\int_B \left\{\nabla\cdot\left[a(x,\xi)\nabla u(x,\xi)\right] + f(x)\right\}^2 dx + \lambda\int_{\partial B}\left[u(x,\xi) - g(x)\right]^2 dx\right]$$

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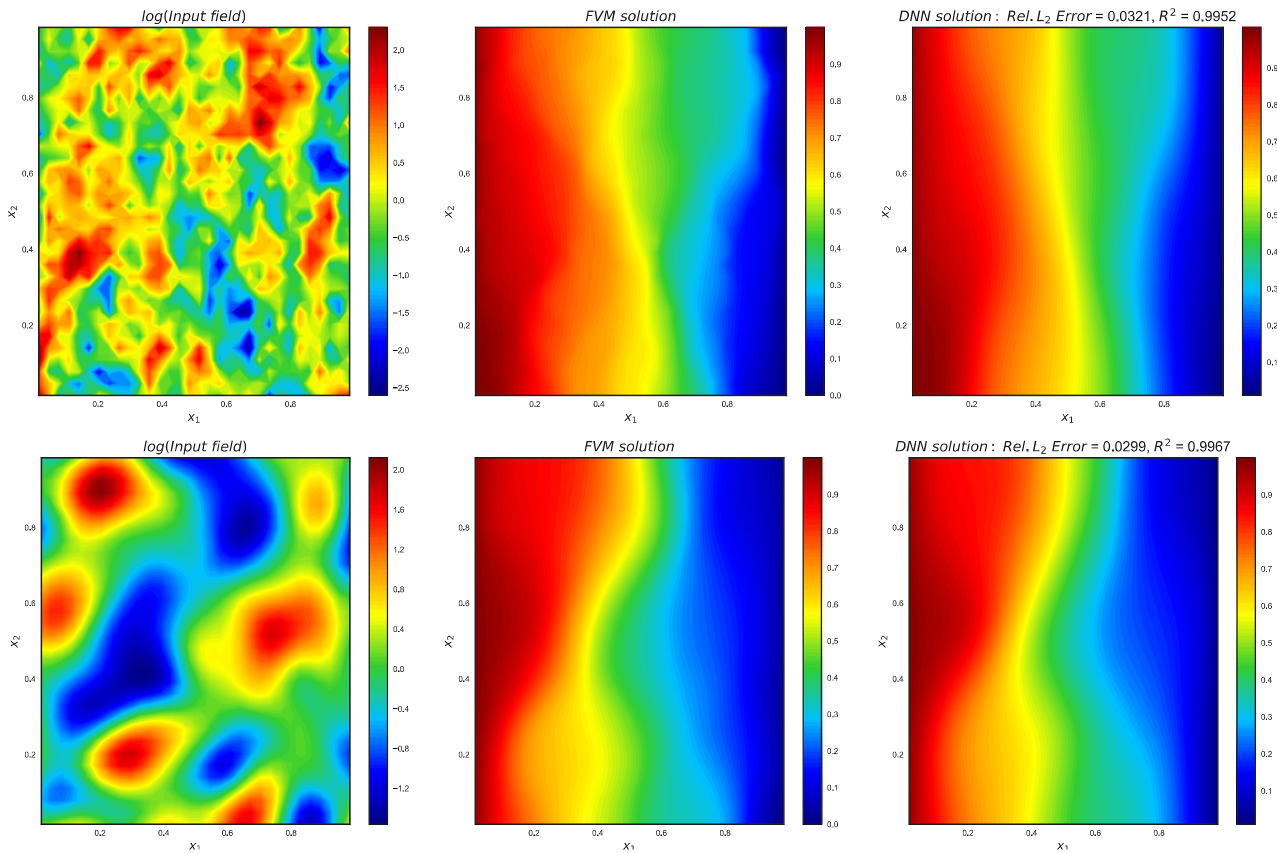
# One network for all kinds of random fields




Karumuri, S.; Tripathy, R.; Bilionis, I.; Panchal, J. Simulator-Free Solution of High-Dimensional Stochastic Elliptic Partial Differential Equations Using Deep Neural Networks. *Journal of Computational Physics* **2020**, *404*, 109120. https://doi.org/10.1016/j.jcp.2019.109120.

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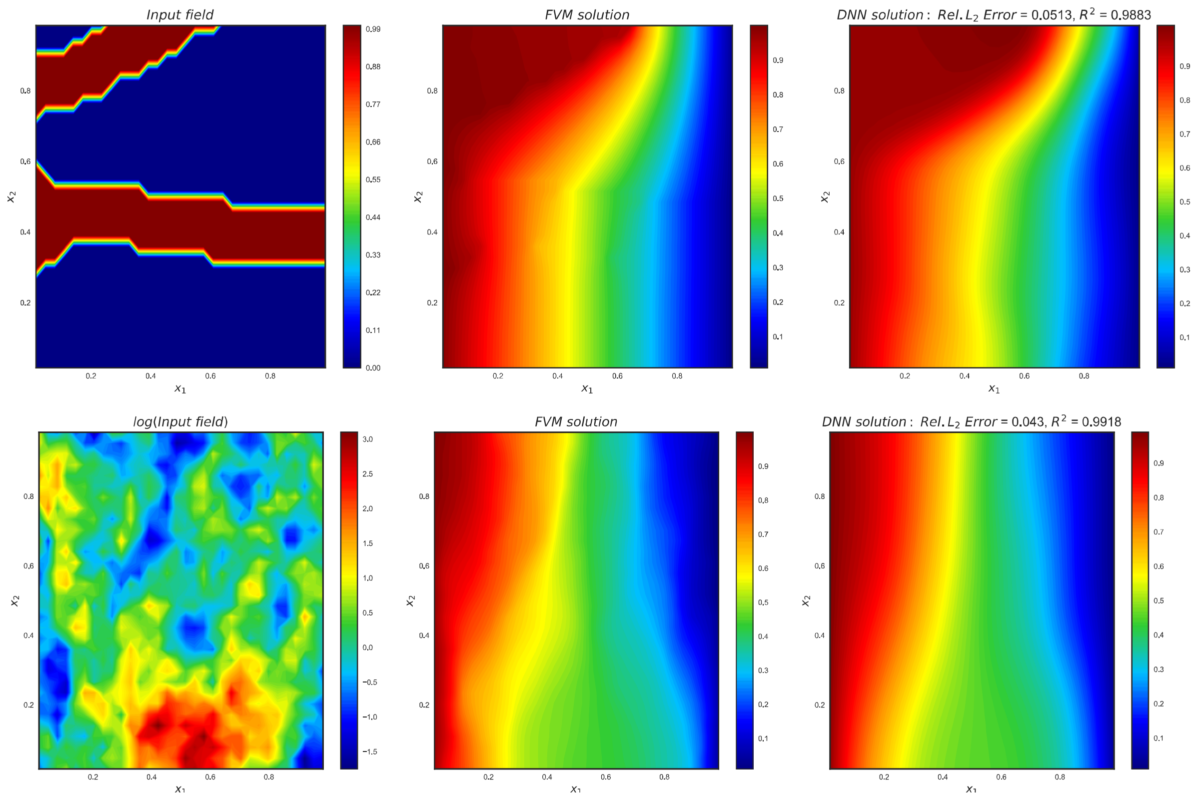


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