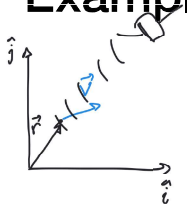
Example: Object tracking



$$m\ddot{\vec{r}} = \vec{F} = u_x\hat{i} + u_y\hat{j} \quad (*)$$

$$\frac{d\vec{r}}{dt} = \vec{v} \quad (1)$$

$$(*) \Rightarrow \frac{d\vec{v}}{dt} = \frac{u_x}{m}\hat{i} + \frac{u_y}{m}\hat{j} \quad (2)$$

Timestep Δt:

$$(1) \Rightarrow \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \vec{v} \Rightarrow \vec{r}(t+\Delta t) = \vec{r}(t) + \vec{v}\cdot\Delta t \quad (3)$$

$$(2) \Rightarrow \vec{v}(t+\Delta t) = \vec{v}(t) + \frac{\vec{F}}{m}\Delta t = \vec{v}(t) + \frac{u_x\Delta t}{m}\hat{i} + \frac{u_y\Delta t}{m}\hat{j} \quad (4)$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j}; \quad \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$(3) \Rightarrow \begin{aligned} r_x(t+\Delta t) &= r_x(t) + \Delta t \cdot v_x(t) \quad (5)\\ r_y(t+\Delta t) &= r_y(t) + \Delta t \cdot v_y(t) \quad (6)\end{aligned}$$

$$(4) \Rightarrow \begin{aligned} v_x(t+\Delta t) &= v_x(t) + \frac{\Delta t}{m}u_x(t) \quad (7)\\ v_y(t+\Delta t) &= v_y(t) + \frac{\Delta t}{m}u_y(t) \quad (8)\end{aligned}$$

$$\Rightarrow x_{n+1} = A\cdot x_n + Bu_n$$

$$x_n = \begin{pmatrix} r_x(n\Delta t)\\ r_y(n\Delta t)\\ v_x(n\Delta t)\\ v_y(n\Delta t)\end{pmatrix} \in \mathbb{R}^4, \ d=4; \quad A = \begin{pmatrix} 1 & 0 & \Delta t & 0\\ 0 & 1 & 0 & \Delta t\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$u_n = \begin{pmatrix} u_x(n\Delta t)\\ u_y(n\Delta t)\end{pmatrix} \in \mathbb{R}^2, \ m=2; \quad B = \begin{pmatrix} 0 & 0\\ 0 & 0\\ \Delta t/m & 0\\ 0 & \Delta t/m\end{pmatrix} \quad (4\times 2)$$

$$x_{n+1} = Ax_n + Bu_n + z_n$$

process noise

z_n ? — capture all uncertainties including inclination

$$z_n \sim N(0, Q), \quad Q = \begin{pmatrix} \varepsilon & 0 & 0 & 0\\ 0 & \varepsilon & 0 & 0\\ 0 & 0 & \sigma_{q_1}^2 & 0\\ 0 & 0 & 0 & \sigma_{q_2}^2\end{pmatrix} \quad (4\times 4)$$

$$y_n = Cx_n + w_n$$

measurement noise

GPS measurement of location $\mathbb{R}^2$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{pmatrix} \quad (2\times 4), \quad w_n \sim N(0, R)$$

p.d. $R = \begin{pmatrix} \sigma_r^2 & 0\\ 0 & \sigma_r^2\end{pmatrix}$ (2×2)

Newton's Laws

Discretized in time

Mapped discret. motion eq. to linear dynamical system formalism

$$x_{n+1} = Ax_n + Bu_n + z_n$$

$$y_n = Cx_n + w_n$$

$\Downarrow$

Kalman Filter