Lecture 4: Continuous Random Variables

Professor Ilias Bilionis

Expectation and variance of a continuous random variable

**PREDICTIVE
SCIENCE LABORATORY**

Expectation of a continuous random variable

- The expectation of a continuous random variable is:

$$\mathbb{E}[X] = \int x \, p(x) \, dx$$

where the integral is over all the possible values.

- Its properties are the same as the expectation of a discrete random variable.

$$\mathbb{E}[X + c] = \mathbb{E}[X] + c \; ; \; \mathbb{E}[\lambda X] = \lambda \mathbb{E}[X]$$

Expectation of a function of a continuous random variable

- The expectation of a function of the random variable is:
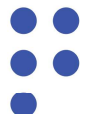
$$\mathbb{E}[f(X)] = \int f(x)\, p(x)\, dx$$

Variance of a continuous random variable
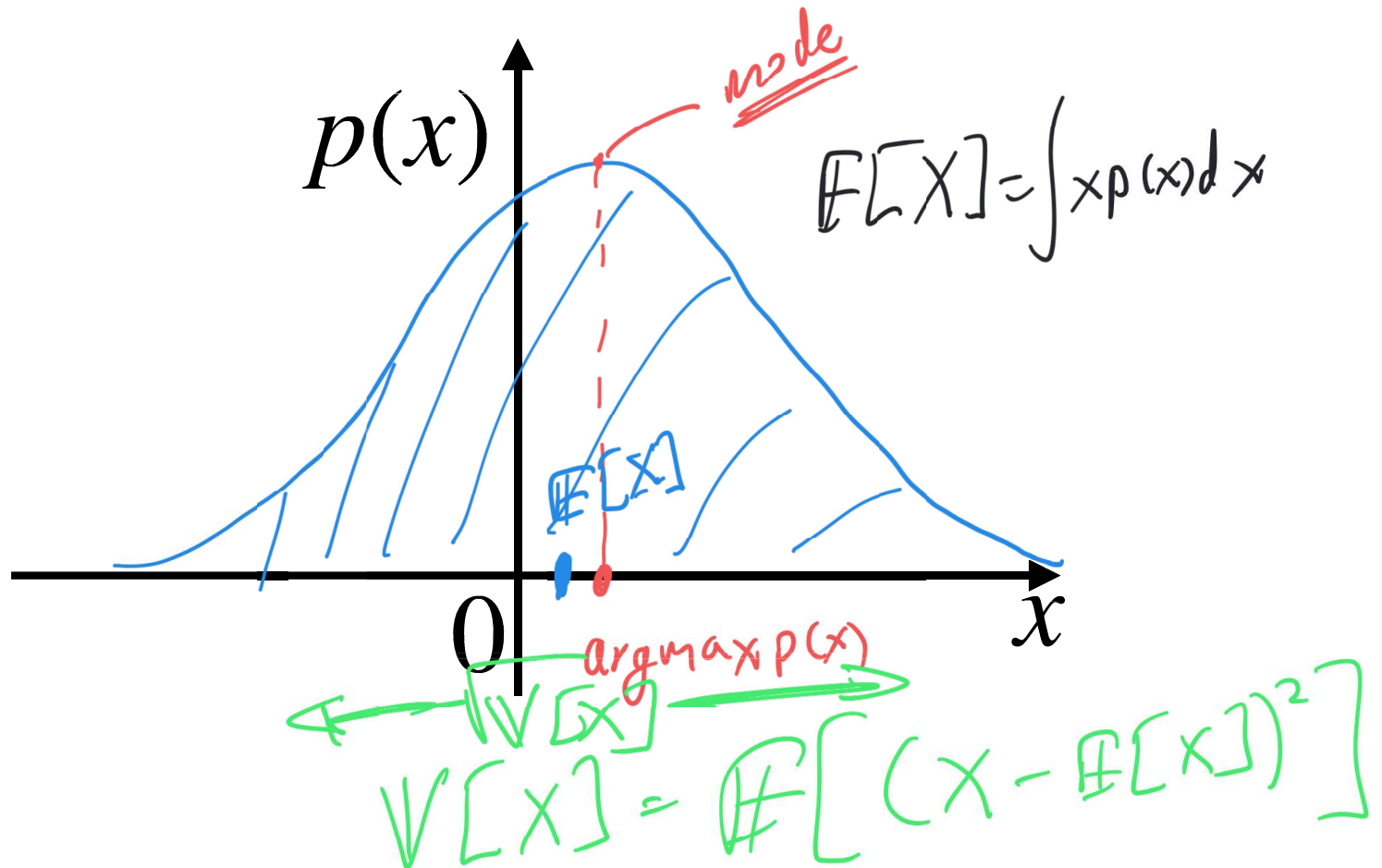
- The variance of a continuous random variable is:

$$\mathbb{V}[X] = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right].$$

- Its properties are the same as the variance of a discrete random variable.

$$V[X+c] = V[X] \; ; \; V[\lambda X] = \lambda^2 V[X]$$
$$V[X] = \mathbb{E}[X^2] - \left(\mathbb{E}[X]\right)^2$$

Interpretation of expectation and variance for a good random variable

$$\mathbb{E}[X] = \int x \, p(x) \, dx$$

mode

$$\mathbb{E}[X]$$

argmax $p(x)$

$$\mathbb{V}[X]$$

$$\mathbb{V}[X] = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$

Careful with how you interpret expectation and variance for a bad random variable