Lecture 25: Deep neural networks continued

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Regularization through parameter penalties

Regularization terms in loss functions

$$J(\theta) = \underbrace{L(\theta)}_{\text{loss funct.}} + \lambda \underbrace{R(\theta)}_{\text{regul. term}} + \mu R_2(\theta) + \dots$$

λ: hyper-parameter
regularizing parameter.

$$+ \quad R(\theta) = \|\theta\|_2^2 = \sum_i \theta_i^2$$

$$+ \quad R(\theta) = \|\theta\|_1 = \sum_i |\theta_i|$$

$\vdots$

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Bayesian interpretation of regularization

$$\max_\theta p(y_{1:n} \mid x_{1:n}, \theta) \Rightarrow \min_\theta L(\theta)$$

prior over weights $p(\theta)$ $\Rightarrow p(\theta \mid x_{1:n}, y_{1:n}) \propto p(y_{1:n} \mid x_{1:n}, \theta)\, p(\theta)$

MAP of θ: $\max_\theta \log p(\theta \mid x_{1:n}, y_{1:n})$

$$J(\theta) = -\log p(\theta \mid x_{1:n}, y_{1:n}) = \underbrace{-\log p(y_{1:n} \mid x_{1:n}, \theta)}_{L(\theta)} \underbrace{- \log p(\theta)}_{\lambda R(\theta)}$$

Gaussian prior: $\theta \sim N(0, \lambda^{-1})$

$$-\log p(\theta) = -\log N(\theta \mid 0, \lambda^{-1})$$

$$= -\lambda \|\theta\|_2^2 + \text{const.}$$

$$\Rightarrow R(\theta) = \|\theta\|_2^2$$

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