Lecture 20: State-space models - Kalman filters

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Derivation of Kalman filter - Predict

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$\underline{0}$: $p(x_0) = N(\mu_0, V_0)$

$\underline{1}$: $p(x_1|u_0) = \int \underbrace{p(x_1|x_0,u_0)}_{G} \underbrace{p(x_0)}_{G} dx_0$

$N(Ax_0+Bu_0, Q) \quad N(\mu_0, V_0)$

$= N(x_1 | A\mu_0 + Bu_0 \quad | AV_0A^T + Q)$

$x_1 = Ax_0 + Bu_0 + z_1, \quad z_1 \sim N(0, Q)$

$\mathbb{E}[x_1] = \mathbb{E}[Ax_0 + Bu_0] = \mathbb{E}[Ax_0] + \mathbb{E}[\overset{Bu_0}{Bu_0}]$

$= A \cdot \mathbb{E}[x_0] + Bu_0 = A\mu_0 + Bu_0$

$\mathbb{C}[x_1] = \mathbb{C}[Ax_0 + Bu_0 + z_1]$

$= \mathbb{C}[Ax_0 + z_1] = \underbrace{\mathbb{C}[Ax_0]}_{AV_0A^T} + \overset{Q}{\mathbb{C}[z_1]} + \overset{0}{\mathbb{C}[Ax_0, z_1]}$

PREDICTIVE SCIENCE LABORATORY

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$\underline{n}$: $p(x_{n-1}|y_{1:n-1}, u_{0:n-2}) = N(x_{n-1}|\mu_{n-1}, V_{n-1})$

Assume you know this!

$p(x_n|y_{1:n-1}, u_{1:n-2}, u_{n-1}) = \int \underbrace{p(x_n|x_{n-1}, u_{n-1})}_{G} \underbrace{p(x_{n-1}|y_{1:n-1}, u_{0:n-2})}_{G} dx_{n-1}$

$= N(x_n | A\mu_{n-1} + Bu_{n-1} | AV_{n-1}A^T + Q)$

$x_n = Ax_{n-1} + Bu_{n-1} + z_n, \quad z_n \sim N(0, Q)$

$\mathbb{E}[x_n] = A\mu_{n-1} + Bu_{n-1}; \quad \mathbb{C}[x_n] = AV_{n-1}A^T + Q$

If $p(x_{n-1}|y_{1:n-1}, u_{0:n-2}) = N(x_{n-1}|\mu_{n-1}, V_{n-1})$

Then <u>Predict</u>:

$$p(x_n|y_{1:n-1}, u_{0:n-2}, u_{n-1}) = N(x_n | A\mu_{n-1} + Bu_{n-1}, AV_{n-1}A^T + Q)$$