

Lecture 7: Basic Sampling

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Inverse sampling

Inverse Sampling

- Consider an arbitrary univariate continuous random variable X with CDF $F(x)$, How do you sample from it?

- Draw a uniform number u . $\sim U([0,1])$

- Set:

$$x = F^{-1}(u)$$

inverse of the CDF

- and you get your sample!

Why does inverse sampling work?

- Let $U \sim U([0,1])$ be a uniform random variable.
- For any CDF $F(x)$ define the random variable:

$$X = F^{-1}(U)$$

- The CDF of X is:

$$\begin{aligned} p(X \leq x) &= p(F^{-1}(U) \leq x) = p(F(F^{-1}(U)) \leq F(x)) \\ &= p(U \leq F(x)) = F_U(F(x)) = F(x) \end{aligned}$$

Example: The exponential distribution

- Take an exponential random variable as an example:

$$X \sim \text{Exp}(r)$$

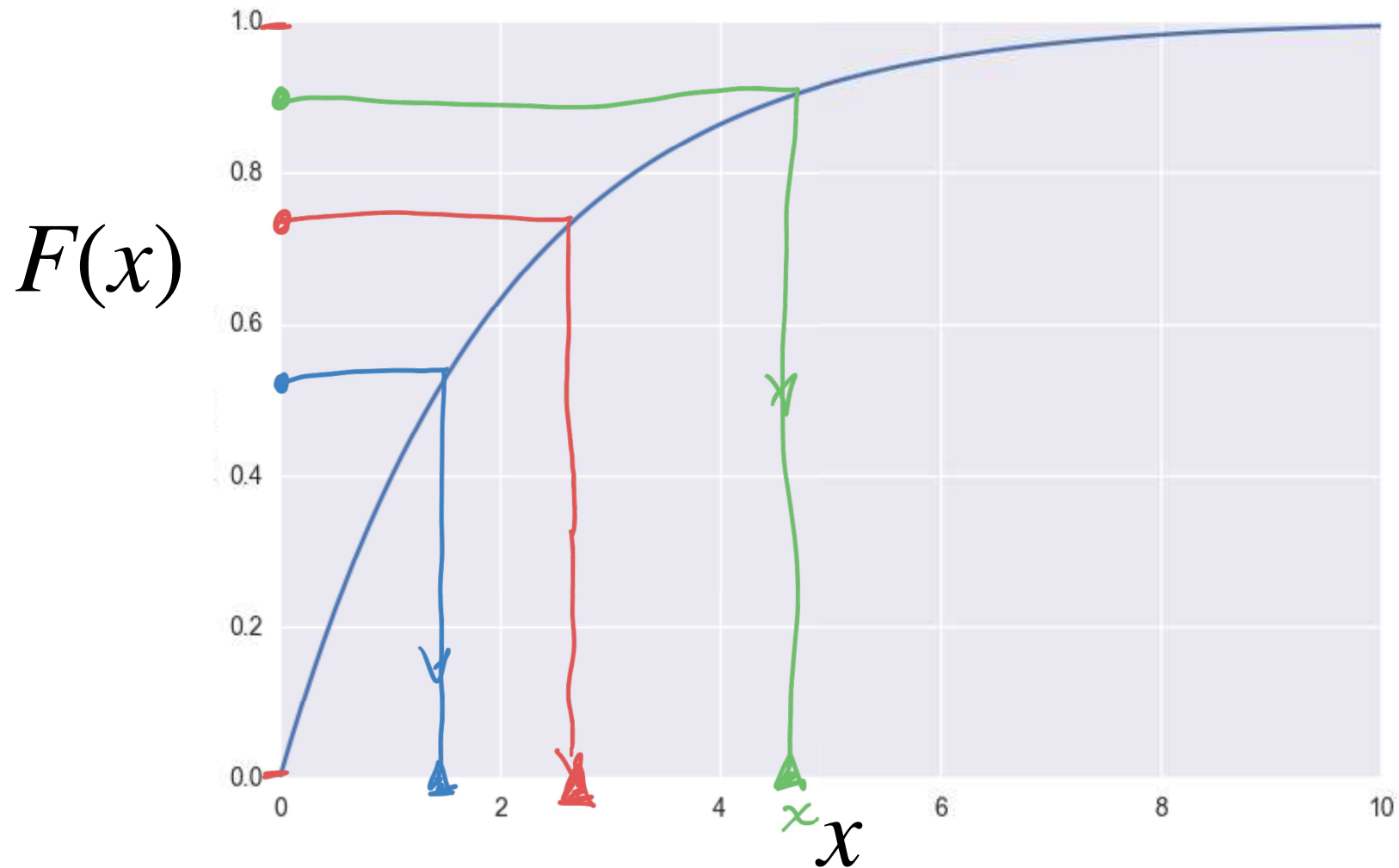
- The CDF is:

$$F(x) = 1 - e^{-rx}$$

- The inverse of the CDF is:

$$F^{-1}(u) = -\frac{\ln(1 - u)}{r}$$

The Exponential Distribution



Inverse Sampling for Exponential

