

Lecture 8: The Monte Carlo method for estimating expectations

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Estimating the variance

Estimating the variance

- Take a random variable $X \sim p(x)$ and some function $g(x)$.
- We would like to estimate the variance:

$$V = \mathbb{V}[g(X)] = \mathbb{E} \left[\left(g(X) - \mathbb{E}[g(X)] \right)^2 \right] = \mathbb{E} \left[\left(g(X) - \underline{I} \right)^2 \right]$$

- Note that:

$$V = \mathbb{V}[g(X)] = \underline{\mathbb{E} [g^2(X)]} - \underline{I}^2$$

Estimating the variance

- Take X_1, X_2, \dots independent identical copies of X .
- Estimate the mean using a sample average:

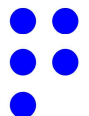
$$\bar{I}_N = \frac{1}{N} \sum_{i=1}^N g(X_i)$$

$$E[g^2(X)]$$

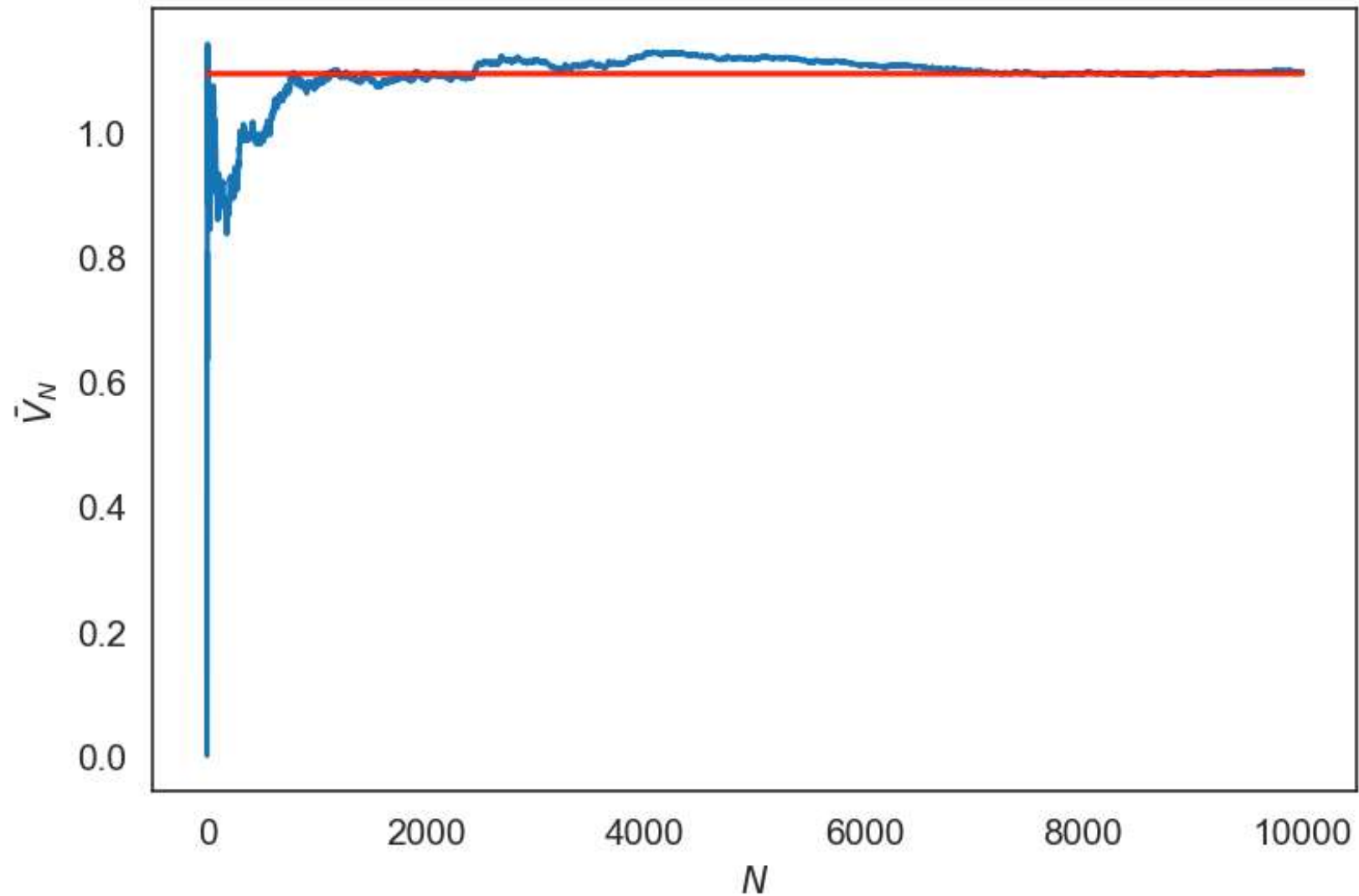
- Estimate the variance by:

$$\bar{V}_N = \frac{1}{N} \sum_{i=1}^N g^2(X_i) - \bar{I}_N^2$$

estimate



Example: 1D variance



Example: 1D variance

