Lecture 10: Quantifying uncertainties in Monte Carlo estimates

Professor Ilias Bilionis

Uncertainty Propagation Through a Boundary Value Problem

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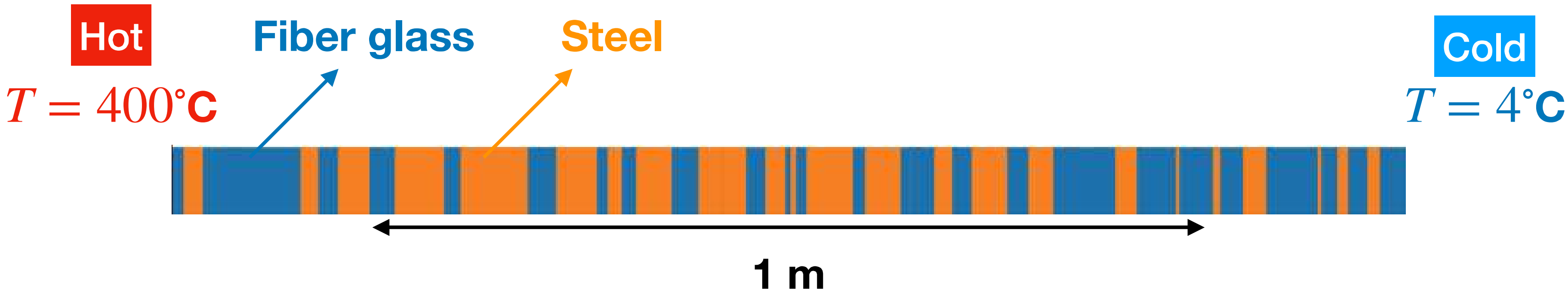
The boundary value problem



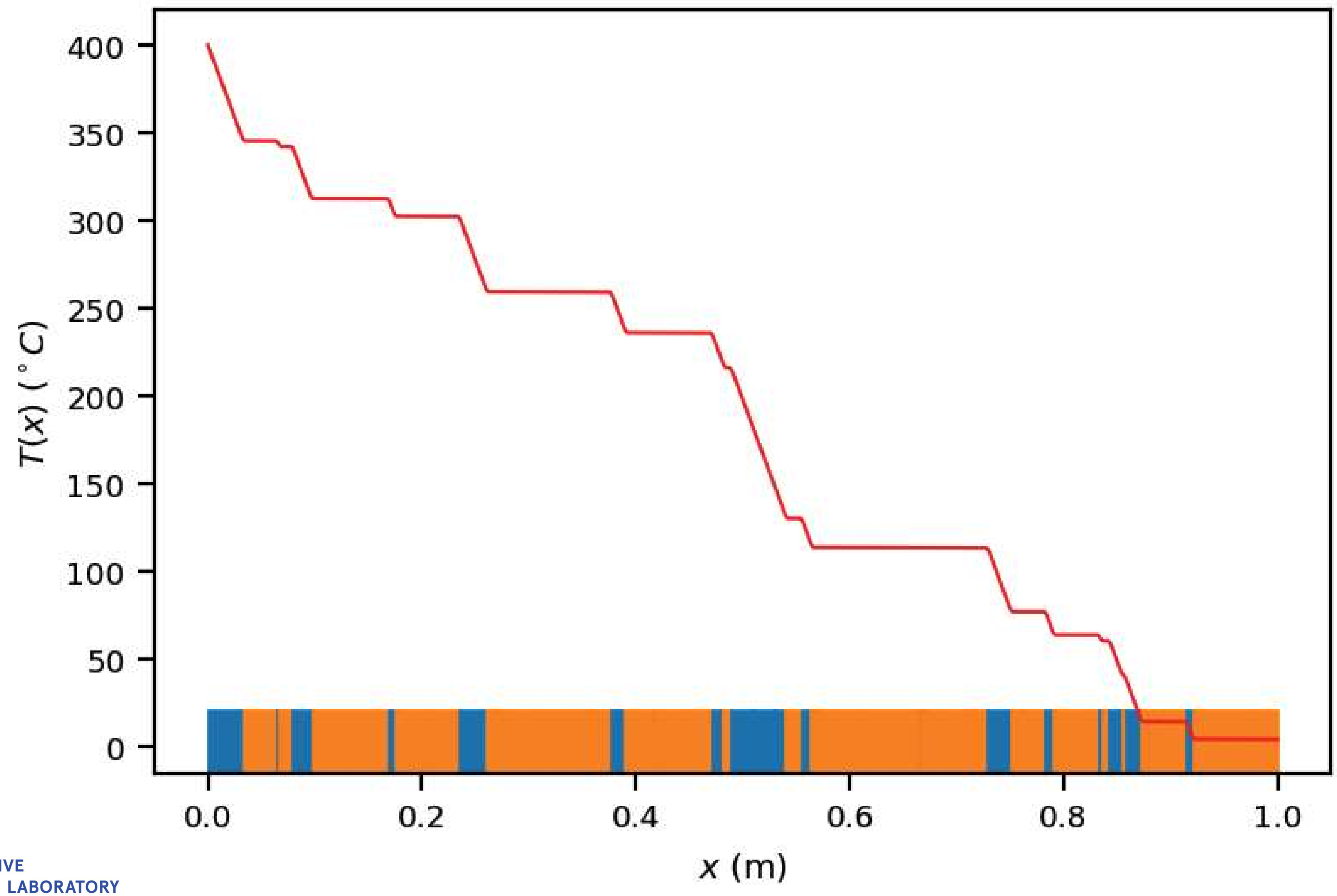
Temperature given by the steady state heat equation:

$$\frac{d}{dx}\left(c(x)\frac{d}{dx}T(x)\right) = 0$$

with boundary conditions:

$$T(0) = 400^\circ\textbf{C} \textbf{ and } T(1 \textbf{ m}) = 4^\circ\textbf{C}.$$

Solving the boundary value problem for a specific rod



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What do we know about the rod?

- The rod is one meter long.
- The rod is from laminations of two different materials: fiberglass (labeled material 0) and steel (labeled material 1).
- The concentration of the fiberglas is 0.3 and the concentration of steel 0.7.
- The thermal conductivity of the fiberglass 0.045 Wm^{-1}K^{-1} and of the steel 38 Wm^{-1}K^{-1}.
- The rod is made out of D segments and each one of these segments consists of only one of the two types of material. Even though we do not know the exact number of segments, we expect it to be around 100.

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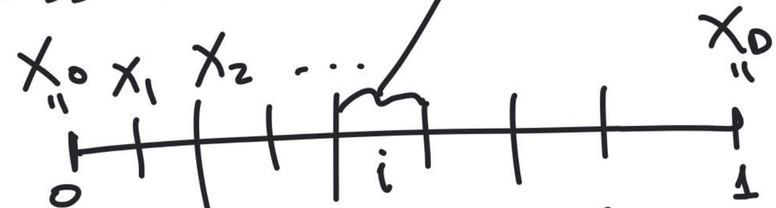
Quantifying our uncertainty about the rod

- The number of segments is: D

$$D \sim \text{Poisson}(100)$$

- The $D+1$ segment coordinates are:



$$(X_1, \dots, X_{D-1}) = \text{sort}(U_1, \dots, U_{D-1})$$

$$U_i \sim U([0,1])$$

- Finally, the material type on each segment is:

$$M_i \sim \text{Categorical}(0.3, 0.7)$$

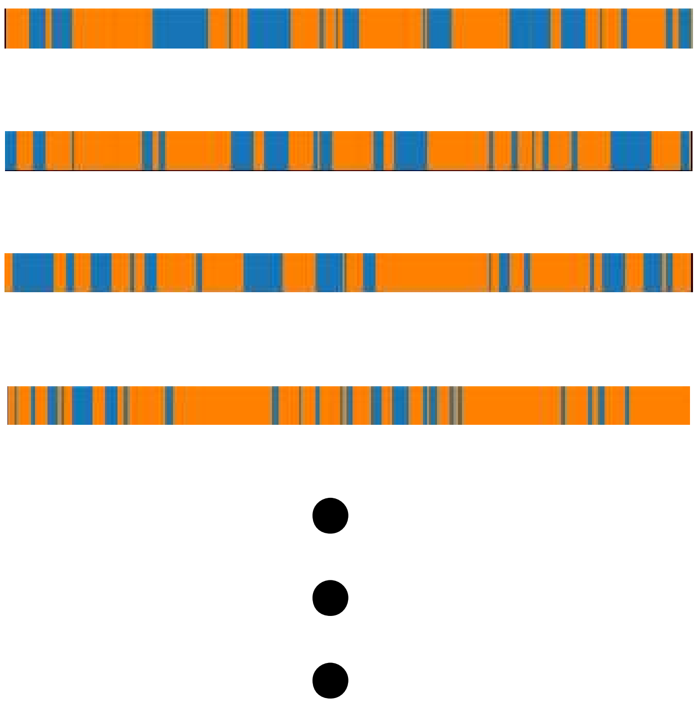
c_0: th. con. of f.g.

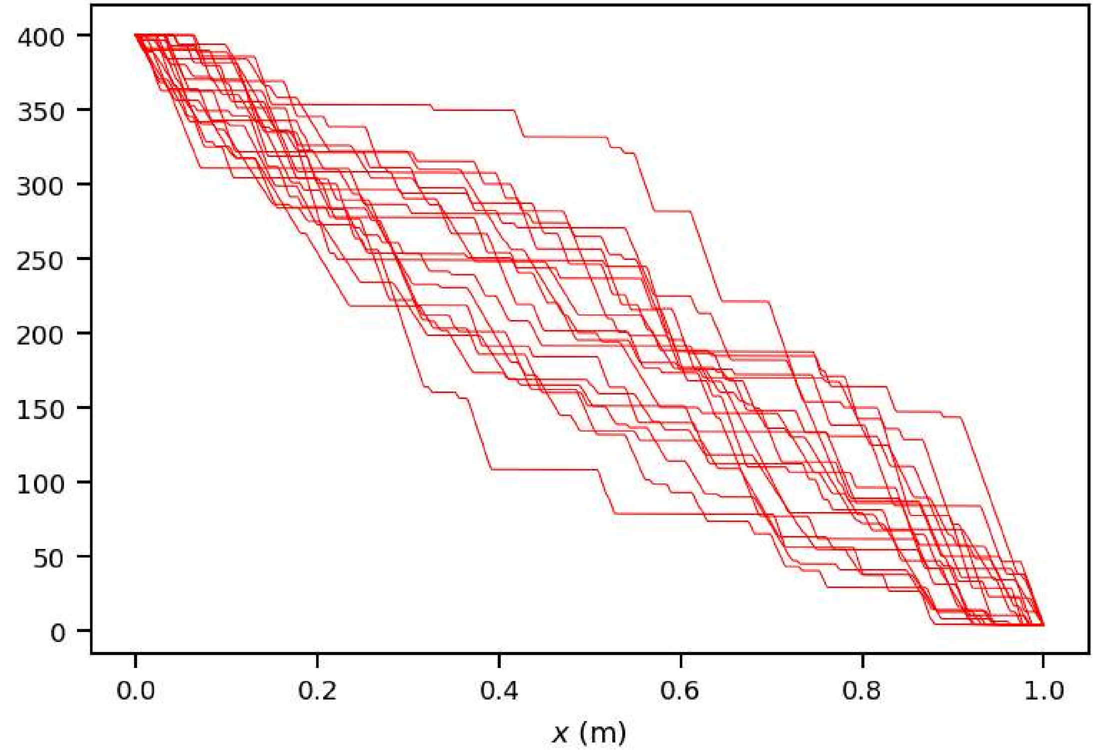
c_1: th. con. of steel

- The thermal conductivity is a random function:

$$c(x) = \sum_{i=1}^{D} 1_{[X_{i-1}, X_i]}(x) \cdot c_{M_i}$$

Now we can sample random rods!





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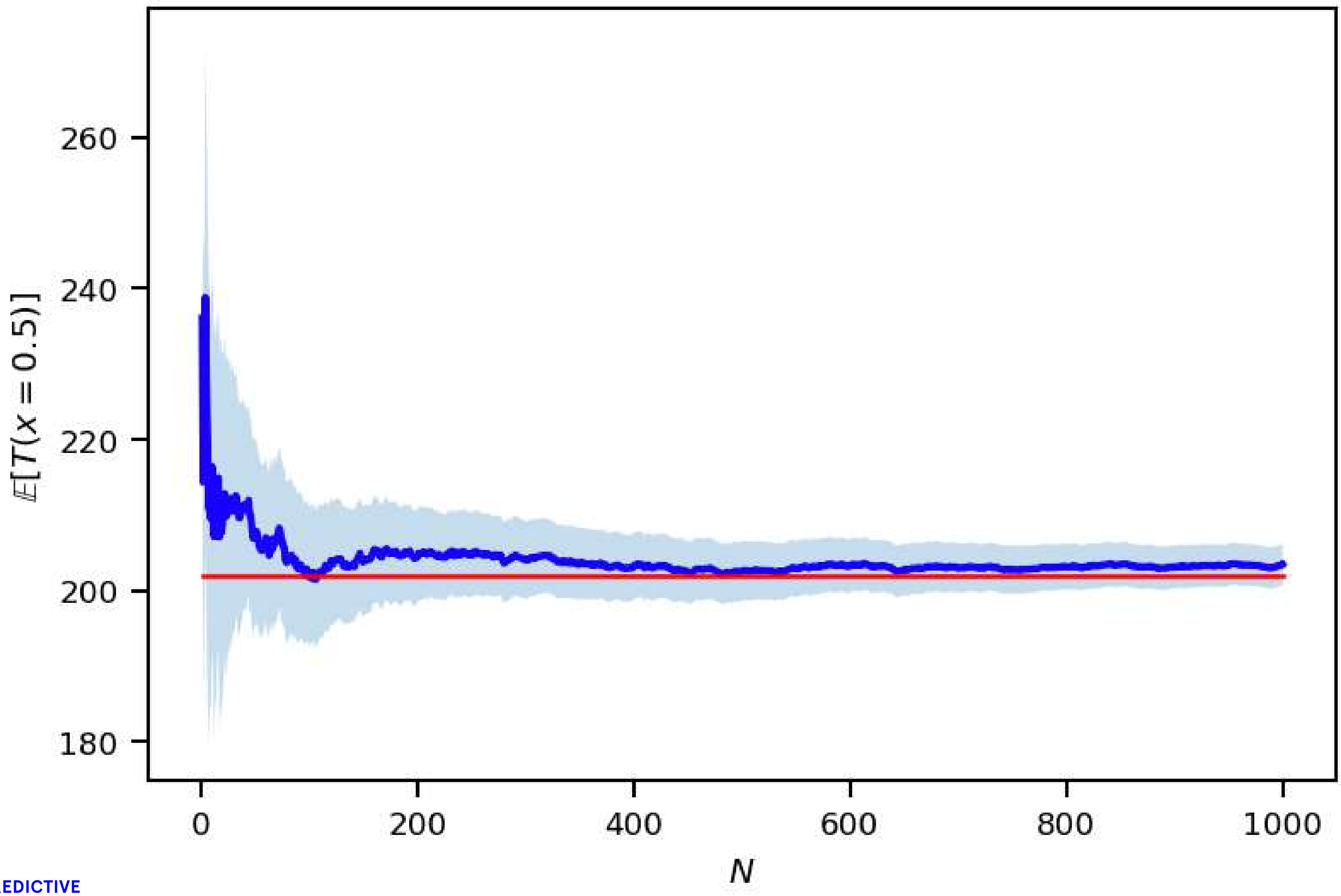
Estimating the expected temperature at the center



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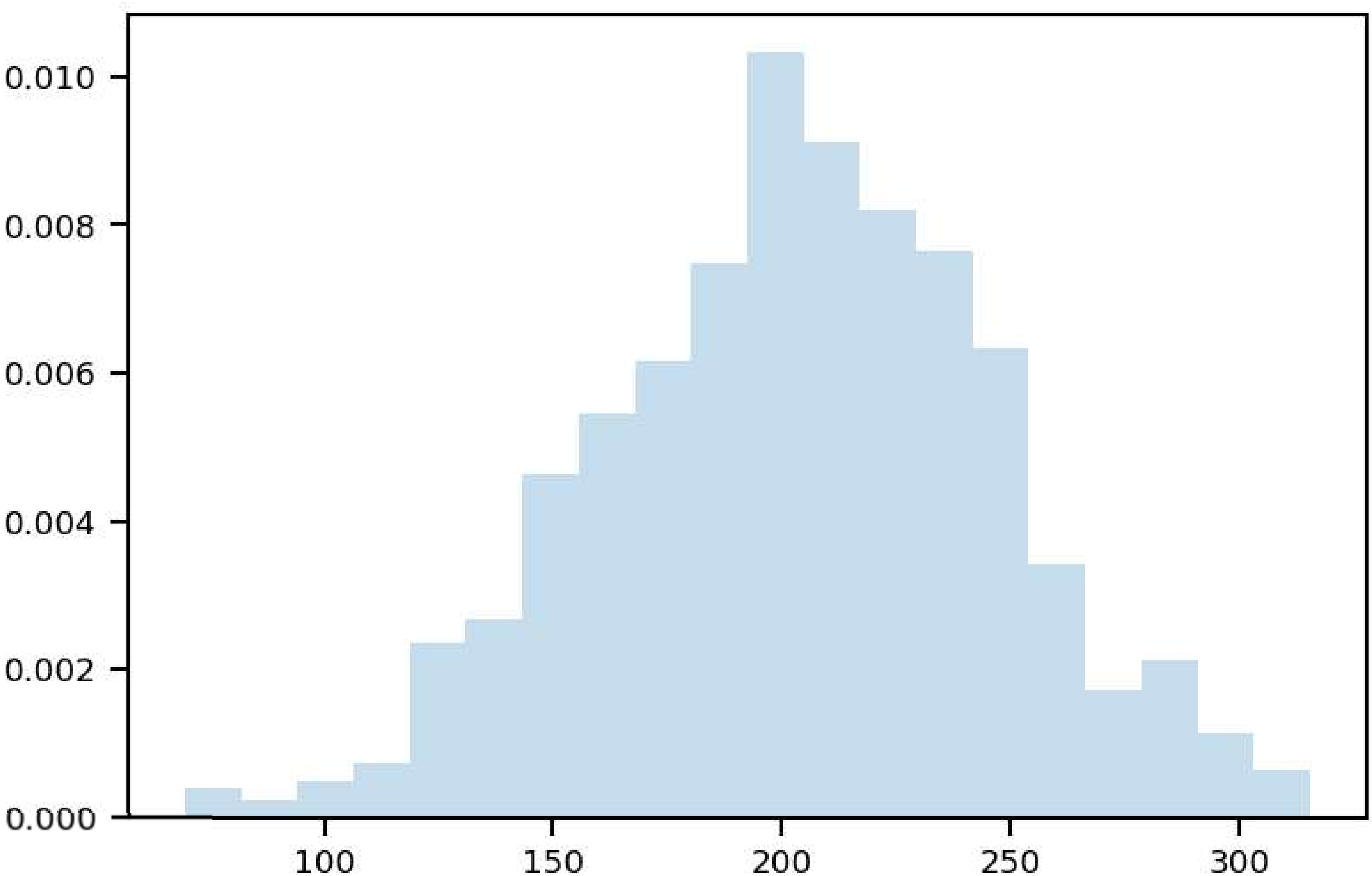
Estimating the probability density of the temperature at the center



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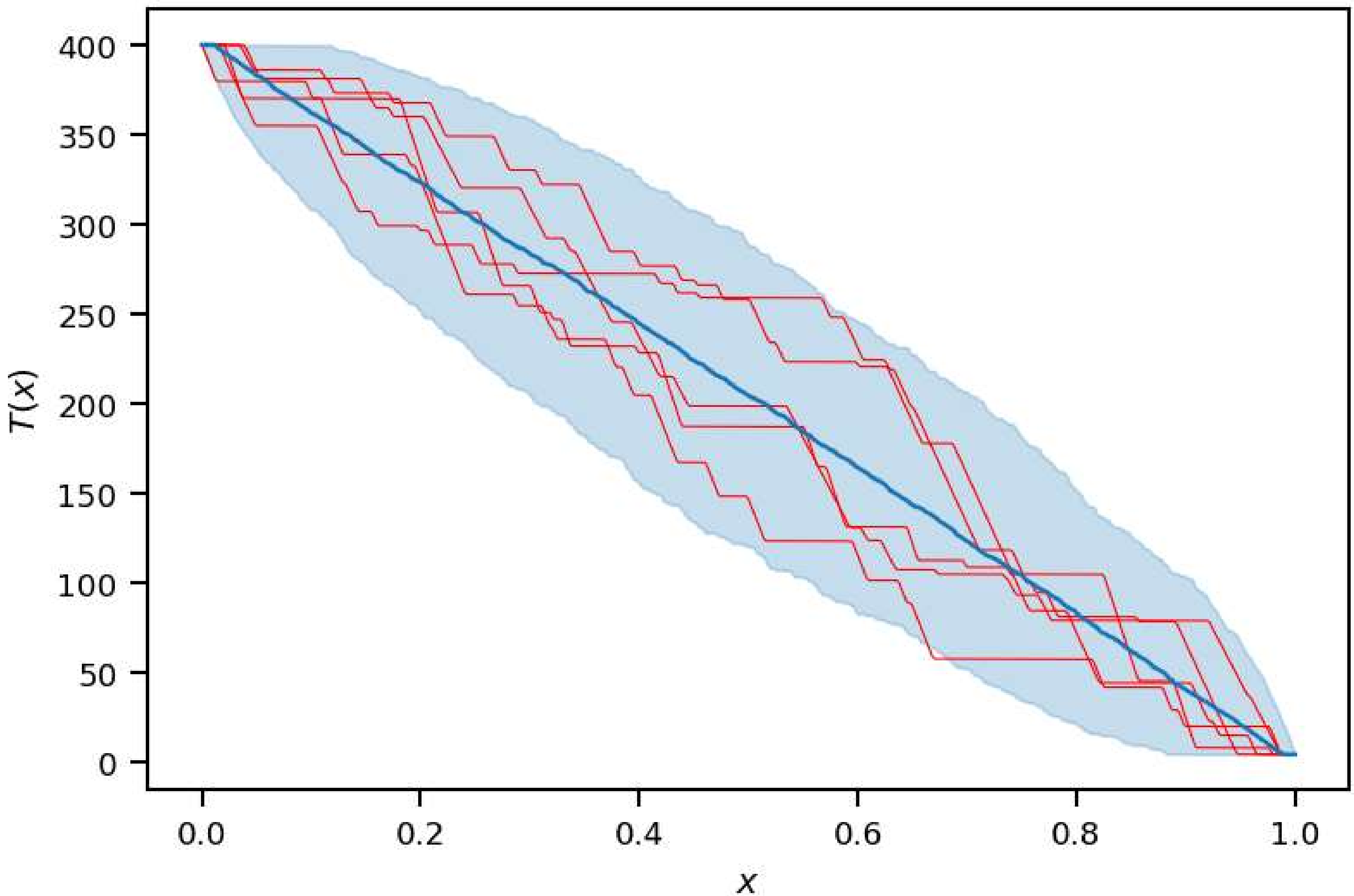
95% Predictive quantiles



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