Lecture 9: Monte Carlo estimates of various statistics

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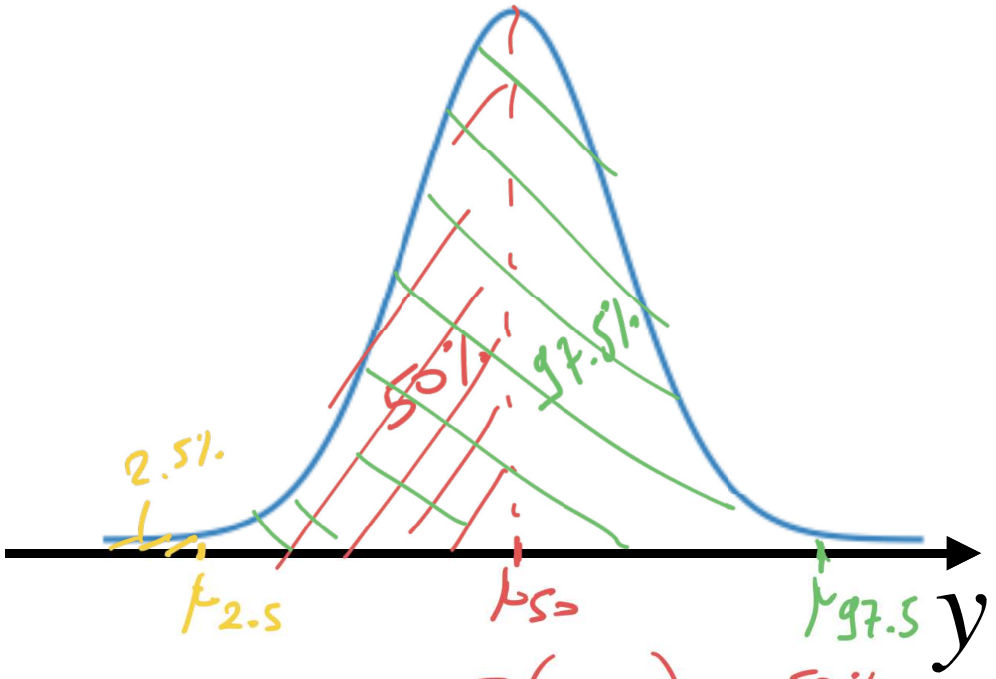
Estimating predictive quantiles

What are the predictive quantiles?

- Take a random variable $X \sim p(x)$ and some function $g(x)$.
- Let $Y = g(X)$ and $F(y)$ be the CDF of Y.
- The q-predictive quantile of Y is the value μ_q such that:



$$p(Y \leq \mu_q) = \frac{q}{100}$$

$$F(\mu_q) = \frac{q}{100}$$

$F(\mu_{50}) = 50\%$

median

$F(\mu_{97.5}) = 97.5\%$

Estimating the predictive quantiles

- We need to find:

$$F(\mu_q) = \frac{q}{100}$$

- We can turn this into a root finding problem:

$$F(\mu_q) - \frac{q}{100} = 0$$

- However, $F(y)$ is known and we will use an estimate.

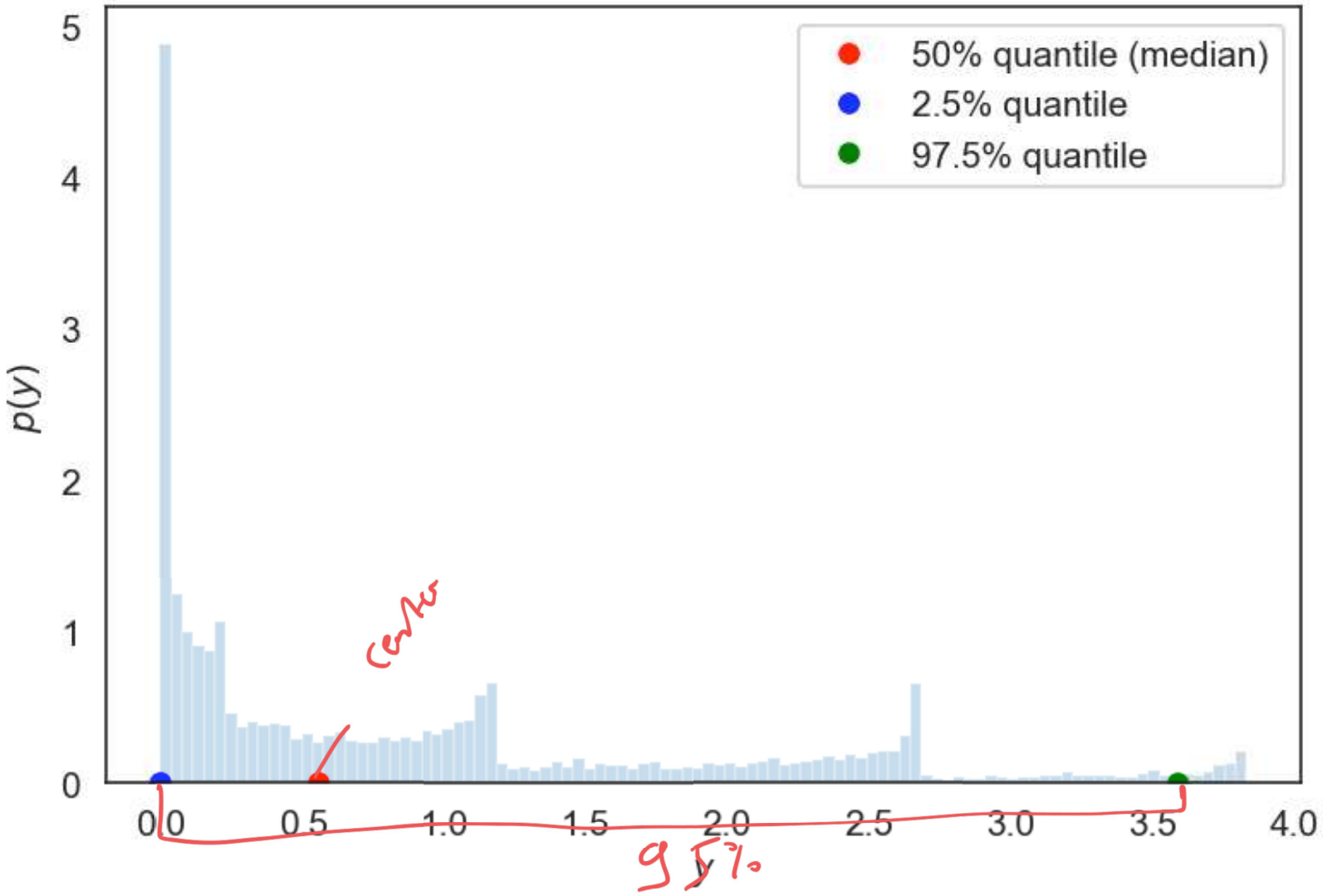
Estimating the predictive quantiles

- Take $X_1, X_2, \ldots$ independent identical copies of X.
- Find the sampling estimate of the CDF of $Y = g(X)$, say $\bar{F}_N(y)$.
- Numerically solve the root finding problem:

$$\bar{F}_N(\mu_q) - \frac{q}{100} = 0$$

- Obtain estimate of the quantile $\bar{\mu}_{q,N}$.

Example: 1D - Predictive Quantiles



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Example: 1D Box Plots