

Lecture 3: Discrete Random Variables

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Expectation of a discrete random variable

Expectation of a random variable

- The expectation of a random variable is:

$$\mathbb{E}[X] := \sum_x xp(x)$$

- You can think of the expectation as the value of the random variable that one should "expect" to get.
- However, take this interpretation with a grain of salt because it may be a value that the random variable has a zero probability of getting...

Properties of the expectation

- For any function $g(x)$:

$$\mathbb{E}[g(X)] = \sum_x g(x)p(x)$$

Properties of the expectation

- Take any constant c :

$$\mathbb{E}[X + c] = \mathbb{E}[X] + c$$

Proof:

$$\begin{aligned}\mathbb{E}[X + c] &= \sum_x (x + c) p(x) = \sum_x x p(x) + \sum_x c p(x) \\ &= \mathbb{E}[X] + c \cdot \sum_x p(x) \quad \downarrow \\ &= \mathbb{E}[X] + c \cdot 1 \quad \square\end{aligned}$$

Properties of the expectation

- For any λ real number:

$$\mathbb{E}[\lambda X] = \lambda \mathbb{E}[X]$$

Proof: $\mathbb{E}[\lambda X] = \sum_x \lambda x p(x) = \lambda \cdot \sum_x x \cdot$