Lecture 3: Discrete Random Variables

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Variance of a discrete random variable

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Expectation of a random variable

- The variance of a random variable is:

$$= \sum_x (x - \mathbb{E}[X])^2 p(x)$$

$$\mathbb{V}[X] := \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$

- You can think of the variance as the spread of the random variable around its expectation.

- However, do not take this too literally for discrete random variables.

Properties of the variance

- Take any constant c:

$$\mathbb{V}[X+c] = \mathbb{V}[X]$$

Proof:

$$\mathbb{V}[X+c] = \mathbb{E}\left[\left(X+\cancel{c}-\underbrace{\mathbb{E}[X+c]}_{\mathbb{E}[X]+\cancel{c}}\right)^2\right]$$

$$= \mathbb{E}\left[\left(X-\mathbb{E}[X]\right)^2\right] \quad \blacksquare$$

Properties of the variance

- Take any constant λ:

$$\mathbb{V}[\lambda X] = \lambda^2 \mathbb{V}[X]$$

Proof:

$$\mathbb{V}[\lambda X] = \mathbb{E}[(\lambda X - \underbrace{\mathbb{E}[\lambda X]}_{\lambda \mathbb{E}[X]})^2]$$

$$= \mathbb{E}[(\lambda X - \lambda \mathbb{E}[X])^2]$$

$$= \mathbb{E}[\lambda^2 (X - \mathbb{E}[X])^2]$$

$$= \lambda^2 \mathbb{E}[(X$$

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Properties of the variance

- It holds that:

$$\boxed{\mathbb{V}[X] = \mathbb{E}\left[X^2\right] - \left(\mathbb{E}[X]\right)^2}$$

Proof: $\mathbb{V}[X] = \mathbb{E}\left[\left(X - \mathbb{E}[X]\right)^2\right]$

$= \mathbb{E}\left[X^2 - 2X\,\mathbb{E}[X] + \left(\mathbb{E}[X]\right)^2\right]$

$= \mathbb{E}\left[X^2 - 2X\,\mathbb{E}[X]\right] + \left(\mathbb{E}[X]\right)^2$

$= \mathbb{E}\left[X^2\right] - 2\left(\mathbb{E}[X]\right)^2 + \left(\mathbb{E}[X]\right)^2$ ∎

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