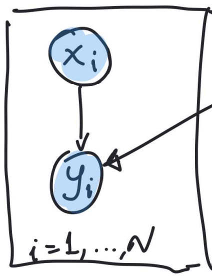
The logistic regression model

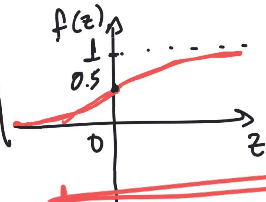
Given: $x_{1:N} = (x_1, \dots, x_N)$, $y_{1:N} = (y_1, \dots, y_N)$; $y_i \in \{0, 1\}$

Find: $p(y \mid x, x_{1:N}, y_{1:N}) = ?$



Likelihood:

$$p(y_i = 1 \mid x_i, \underline{w}) = \overset{?}{f}(w_0 + w_1 x_i)$$



$$f(z) = \mathrm{sigm}(z) = \frac{\exp\{z\}}{1 + \exp\{z\}}$$

Important

$$\boxed{p(y_i = 1 \mid x_i, \underline{w}) = \mathrm{sigm}(w_0 + w_1 x_i)}$$

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$$p(y_i = 0 \mid x_i, \underline{w}) = 1 - p(y_i = 1 \mid x_i, \underline{w})$$
$$= 1 - \mathrm{sigm}(w_0 + w_1 x_i)$$

$$p(y_i \mid x_i, \underline{w}) = \underbrace{\left[\mathrm{sigm}(w_0 + w_1 x_i)\right]^{y_i}}_{\text{activated when } y_i = 1} \cdot \underbrace{\left[1 - \mathrm{sigm}(w_0 + w_1 x_i)\right]^{1-y_i}}_{\text{activated when } y_i = 0}$$

$$p(y_{1:N} \mid x_{1:N}, \underline{w}) = \prod_{i=1}^{N} p(y_i \mid x_i, \underline{w})$$
$$= \prod_{i=1}^{N} \left[\mathrm{sigm}(w_0 + w_1 x_i)\right]^{y_i} \cdot \left[1 - \mathrm{sigm}(w_0 + w_1 x_i)\right]^{1-y_i}$$