

Induction

- showing that for any $n \geq 1$, there is a finite seq. of implications that starts w/ something known to be true and ultimately leads to showing $q(n)$ is true
- specifically: 1.) begin justification by induction by showing $q(n)$ is true for $n=1$
2.) justify the inductive step is true for $n \geq k$

Example

→ Consider fibonacci function $F(n)$. We claim $F(n) < 2^n$.

→ base cases: ($n \leq 2$). $F(1) = 1 < 2 = 2^1$ and $F(2) = 2 < 4 = 2^2$

→ induction: ($n > 2$). Suppose our claim is true for all $n' < n$. Consider $F(n)$.

Since $n > 2$, $F(n) = F(n-2) + F(n-1)$.

Since both $(n-2)$ and $(n-1)$ are less than n , we apply inductive assumption to imply that $F(n) < 2^{n-2} + 2^{n-1}$, since
 $2^{n-2} + 2^{n-1} < 2^{n-1} + 2^{n-1} = 2 \cdot 2^{n-1} = 2^n$

Loop Invariants

→ to prove statement L about a loop is correct, define L in terms of a series of smaller statements L_0, L_1, \dots, L_k where:

- 1.) initial claim (L_0) is true before the loop begins
- 2.) if L_{j-1} is true before iteration j , then L_j will be true after j
- 3.) final statement L_k implies desired statement (L) to be true

def find(S, val): → return index j such that $S[j] = val$ or -1 if nonexistent

$n = \text{len}(S)$

$j = 0$

while $j < n$:

if $S[j] == val$:

return j

$j += 1$

return -1

claim at start:

→ L_j : val is not equal to any of the first j elements of S