

## Computing Depth & Height

→ let  $p$  be position of a node of a tree  $T$ .

the depth of  $p$  is # of ancestors of  $p$ , excluding  $p$  itself.

the depth of  $p$  can be recursively defined as:

- If  $p$  is root, then the depth is 0

- otherwise, depth of  $p$  is one plus the depth of parent  $p$

def (self,  $p$ ):

if self.is-root( $p$ )

return 0

else

return 1 + self.depth(self.parent( $p$ ))

running time for  $T.depth(p)$  is  $O(d_p + 1)$  where  $d$  denotes depth because algo performs constant-time recursive step for each ancestor of  $p$

→ height of a position  $p$  in a tree  $T$  is defined recursively:

- If  $p$  is a leaf, height is zero

- otherwise, height is one more than max heights of  $p$ 's children

~~height of a position  $p$  is the height of the root~~

→ height of a nonempty tree  $T$  is equal to the maximum of the depths of leaf positions

def height(self,  $p$ ):

if self.is-leaf( $p$ )

return 0

else return 1 + max(self.height( $c$ ) for  $c$  in self.children( $p$ ))

~~recursive, progresses in a top-down fashion; eventually called once for each position of  $T$~~

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the root eventually invokes the recursion on each of its children, then their children, etc.

→ let  $T$  be a tree with  $n$  positions, and let  $c_p$  denote the # of children of a position  $p$  of  $T$ . Then, summing over the positions of  $T$ ,  $\sum_p c_p = n - 1$

- each position of  $T$ , with exception of root, is a child of another position and thus contributes one unit to the above sum