Generating Set & Span

→ consider vector space $V = (V, +, \cdot)$ and set of vectors $A = \{x_1, \dots, x_k\} \subseteq V$.
If every vector $v \in V$ can be expressed as a linear combination of $x_1, \dots, x_k$, A is called a generating set of V.

→ If A spans the vector space V, we can write $V = \text{span}[A]$ or $V = \text{span}[x_1, \dots, x_k]$

Generating sets are sets of vectors that span vector subspaces where every vector can be represented as a linear combination of the vectors in the generating set.

Basis: consider $V(V, +, \cdot)$ and $A \subseteq V$. A generating set A of V is minimal if there exists no smaller set $\tilde{A} \subseteq A \subseteq V$ that spans V. Every linearly independent generating set of V is minimal and called a basis of V.

→ Let $V = (V, +, \cdot)$ and $B \subseteq V$, $B \neq \emptyset$. Then, the following are equivalent:
- B is a basis of V
- B is a minimal generating set
- B is a maximal linearly independent set of vectors in V; adding any other vector to this set will make it linearly dependent
- Every vector $x \in V$ is a linear combination of vectors from B, and every linear combination is unique.

$$x = \sum_{i=1}^{k} \lambda_i b_i = \sum_{i=1}^{k} \psi_i b_i$$

→ Every V possesses a basis B. All bases have the same # of elements, the basis vectors.
  → we consider only finite-dimensional vector spaces
  → thus, the dimension of V is the number of basis vectors in V
  → we write $\dim(V)$
  → if $U \subseteq V$ is a subspace of V, then $\dim(U) \leq \dim(V)$ and $\dim(U) = \dim(V)$ only if $U = V$.

→ A basis of subspace $U = \text{span}[x_1, \dots, x_m] \subseteq \mathbb{R}^n$ can be found by:
1.) write spanning vectors as columns of matrix A
2.) determine row-echelon form
3.) spanning vectors associated w/ pivot columns are a basis of U.