$\otimes$: Kronecker Product

→ takes 2 matrices ~~of~~ any size and creates a block matrix

Calculating the Inverse

→ to compute the inverse of $A \in \mathbb{R}^{n \times n}$, we need to find matrix X such that $AX = I_n$.

$$[A \mid I_n] \rightsquigarrow \cdots \rightsquigarrow [I_n \mid A^{-1}]$$

* → if we can bring the augmented equation system into reduced row-echelon form, we can read out the inverse on the RHS of the equation system

VECTOR SPACES

→ Group: consider a set G and operation $\otimes : G \times G \to G$ defined on G.
Then, $G := (G, \otimes)$ is called a group if:

1.) Closure of G under $\otimes$: $\forall x, y \in G : x \otimes y \in G$
2.) Associativity: $\forall x, y, z \in G : (x \otimes y) \otimes z = x \otimes (y \otimes z)$
3.) Neutral Element: $\exists e \in G \; \forall x \in G : x \otimes e = x$ AND $e \otimes x = e$
4.) Inverse Element: $\forall x \in G \; \exists y \in G : x \otimes y = e$ AND $y \otimes x = e$

→ Vector Spaces

- now, we consider also sets that contain outer operations, the multiplication of a vector $x \in G$ by a scalar $\lambda \in \mathbb{R}$.

inner op: $+$
outer op: $\cdot$

→ a real-valued vector space $V = (V, +, \cdot)$ is a set w/ 2 operations

→ $+ : V \times V \to V$

$\cdot : \mathbb{R} \times V \to V$

→ the elements $x \in V$ are vectors. The neutral element of $(V, +)$ is the zero vector $0 = [0, \ldots, 0]^T$, and the inner operation $+$ is called vector addition. The elements λ in $\mathbb{R}$ are scalars and the outer operation $\cdot$ is a multiplication by scalars.

Example: $V = \mathbb{R}^n, n \in \mathbb{N}$ is a vector space:

→ addition: $x + y = (x_1, \ldots, x_n) + (y_1 + \ldots + y_n) = (x_1 + y_1, \ldots, x_n + y_n)$

→ multiplication by scalar: $\lambda x = \lambda(x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n)$