→ **Row-Echelon Form**

1.) all rows which contain only zeros are at the bottom of the matrix
2.) if a row is nonzero, the pivot of said row must be to the right of pivot above

→ reduced row echelon:

1.) in row echelon
2.) every pivot is 1
3.) the pivot is the only nonzero in its column

→ **Minus-1 Trick**

- used to read out solutions x of a homogeneous system of lin. eq. $Ax = 0$
- assume that A is in reduced row-echelon form w/o any rows of just 0

$$A = \begin{bmatrix} 0 & \cdots & 0 & 1 & * & \cdots & * & 0 & * & \cdots & * & 0 & * & \cdots & * \\ \vdots & & \vdots & 0 & 0 & \cdots & 0 & 1 & * & \cdots & * & \vdots & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & \vdots & & \vdots & 0 & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & 0 & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & * & \cdots & * \end{bmatrix}$$

→ $*$ can be any arbitrary real #
→ first nonzero entry per row must be 1 and rest in column must be 0

→ the columns $j_1, \ldots, j_k$ with pivots are standard unit vectors $e_1, \ldots, e_k \in \mathbb{R}^k$
→ we extend this matrix to be $n \times n$ by adding $n-k$ rows of form:

$$[\,0 \,\cdots\, 0 \;\; -1 \;\; 0 \,\cdots\, 0\,]$$

- so that the diagonal of augmented matrix $\tilde{A}$ contains either 1 or -1.
- then, columns of $\tilde{A}$ contain the -1 as pivots are solutions of $Ax = 0$, which is called the kernel or null space

Example: (already in row-echelon form)

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & -4 \end{bmatrix} \xrightarrow[\text{add rows}]{\text{make } n \times n} \tilde{A} = \begin{bmatrix} 1 & 3 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{array}{l} \\ \leftarrow \text{added} \\ \\ \\ \leftarrow \text{added} \end{array}$$

using form of places where pivots on diagonal are missing

Solutions