→ Identity Matrix: in $\mathbb{R}^{n\times n}$, we define an identity matrix as an $n\times n$ matrix containing 1 on the diagonal and 0 everywhere else.

$$I_n := \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{n\times n}$$

MATRIX PROPERTIES

→ Associativity: $\forall A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{n\times p}, C \in \mathbb{R}^{p\times q}: (AB)C = A(BC)$

→ Distributivity: $\forall A, B \in \mathbb{R}^{m\times n}, C, D \in \mathbb{R}^{n\times p}: (A+B)C = AC + BC$
$A(C+D) = AC + AD$

→ Mult. of Identity Matrix: $\forall A \in \mathbb{R}^{m\times n}: I_m A = A I_n = A$

Inverse and Transpose

→ Matrix Inverse: consider square matrix $A \in \mathbb{R}^{n\times n}$. Let matrix $B \in \mathbb{R}^{n\times n}$ have the property $AB = I_n = BA$.

→ not every matrix has an inverse. If it does exist, A is regular/invertible, otherwise singular/non-invertable.

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

↓

$A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 4 & 5 \\ 6 & 7 & 7 \end{bmatrix}, B = \begin{bmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{bmatrix}$ are inverse since $AB = I = BA$.

→ Matrix Transpose: for $A \in \mathbb{R}^{m\times n}$ the matrix $B \in \mathbb{R}^{n\times m}$ with $b_{ij} = a_{ji}$ is called the transpose of A. ($B = A^T$)

→ in general, A^T can be obtained by writing the columns of A as rows of A^T.

→ Symmetric Matrix: A matrix is symmetric if $A = A^T$. Only (n,n) matrices can be.

→ the sum of symmetric matrices $A, B \in \mathbb{R}^{n\times n}$ is always symmetric