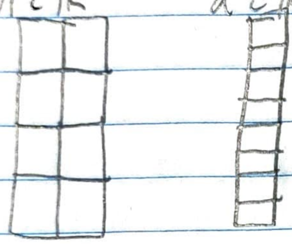
Matrices

Definition: with $m,n \in \mathbb{N}$ a real valued (m,n) matrix A is an $m \cdot n$ tuple of elements $a_{ij}, i=1,\dots,m, j=1,\dots,n$, which is ordered according to a rectangular scheme consisting of m rows and n columns:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \vdots \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad a_{ij} \in \mathbb{R}$$

→ $\mathbb{R}^{m \times n}$ is the set of all real valued (m,n) matrices. $A \in \mathbb{R}^{m \times n}$ can be equivalently represented as $a \in \mathbb{R}^{mn}$ by stacking all n columns of the matrix into a long vector.

$A \in \mathbb{R}^{4 \times 2}$ $\qquad$ $a \in \mathbb{R}^{8}$



→ the sum of two matrices $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{m \times n}$: Addition of Matrices

$$A+B := \begin{bmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ \vdots & & \vdots \\ a_{m1}+b_{m1} & \cdots & a_{mn}+b_{mn} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

→ the elements c_{ij} of the product $C = AB \in \mathbb{R}^{m \times k}$ are computed as

$$c_{ij} = \sum_{i=1}^{n} a_{ij} b_{ij}, \quad i=1,\dots,m, \quad j=1,\dots,k$$

↳ means we multiply element of ith row of A and jth column of B and sum them (dot product)

* → matrices can only be multiplied if neighboring dimensions match

$$\underbrace{A}_{n \times k} \times \underbrace{B}_{k \times m} = \underbrace{C}_{n \times m}$$