

Compact Representation of Systems of Lin. Eq.

$$\begin{cases} 2x_1 + 3x_2 + 5x_3 = 1 \\ 4x_1 - 2x_2 - 7x_3 = 8 \\ 9x_1 + 5x_2 - 3x_3 = 2 \end{cases} \rightarrow \begin{bmatrix} 2 & 3 & 5 \\ 4 & -2 & -7 \\ 9 & 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$$

SOLVING Systems of Linear Equations

Example $\begin{bmatrix} 1 & 0 & 8 & -4 \\ 0 & 1 & 2 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 42 \\ 8 \end{bmatrix}$

→ the goal is to find scalars x_1, \dots, x_4 such that $\sum_{i=1}^4 x_i c_i = b$

PARTICULAR SOLN: $b = \begin{bmatrix} 42 \\ 8 \end{bmatrix} = 42 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 8 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

→ $[42, 8, 0, 0]^T$

→ this isn't the only solution; to capture them all, we must generate 0 in a non-trivial way using the columns of the matrix

↓ express the 3rd column using first 2 columns

$$\begin{bmatrix} 8 \\ 2 \end{bmatrix} = 8 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow 0 = 8c_1 + 2c_2 - 1c_3 + 0c_4$$

$$\begin{bmatrix} 1 & 0 & 8 & -4 \\ 0 & 1 & 2 & 12 \end{bmatrix} \left(\lambda_1 \begin{bmatrix} 8 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right) = \lambda_1 (8c_1 + 2c_2 - c_3) = 0$$

↓ use same reasoning to express 4th column

$$\begin{bmatrix} 1 & 0 & 8 & -4 \\ 0 & 1 & 2 & 12 \end{bmatrix} \left(\lambda_2 \begin{bmatrix} -4 \\ 12 \\ 0 \\ 0 \end{bmatrix} \right) = \lambda_2 (-4c_1 + 12c_2 - c_4) = 0$$

GENERAL SOLUTION $\left\{ x \in \mathbb{R}^4; x = \begin{bmatrix} 42 \\ 8 \\ 0 \\ 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} 8 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} -4 \\ 12 \\ 0 \\ 0 \end{bmatrix}, \lambda_1, \lambda_2 \in \mathbb{R} \right\}$

→ * (general approach:

- 1.) Find a particular solution to $Ax = b$
- 2.) Find all solutions to $Ax = 0$
- 3.) Combine solutions from first 2 steps to obtain general solution

→ Gaussian elimination will help to convert any system of lin. eq. to this particularly simple form which we can apply our steps to