→ Consider V with k linearly independent vectors $b_1, .., b_k$ and m lin. combinations

$$x_1 = \sum_{i=1}^{k} \lambda_{i1} b_i$$
$$\vdots$$
$$x_m = \sum_{i=1}^{k} \lambda_{im} b_i$$

→ defining $B = [b_1, ..., b_k]$ as matrix whose columns are linearly independent vectors $b_1, ..., b_k$ we can write:

$$x_j = B\lambda_j, \ \lambda_j = \begin{bmatrix} \lambda_{1j} \\ \vdots \\ \lambda_{kj} \end{bmatrix}, \ j = 1, ..., m$$

→ in a vector space, m linear combinations of k vectors $x_1, ..., x_k$ are linearly <u>dependent</u> if $m > k$.

<u>Example</u>: consider the linearly independent vectors $b_1, b_2, b_3, b_4 \in \mathbb{R}^n$

$$\left.\begin{aligned} x_1 &= b_1 - 2b_2 + b_3 - b_4 \\ x_2 &= -4b_1 - 2b_2 + 0 + 4b_4 \\ x_3 &= 2b_1 + 3b_2 - b_3 - 3b_4 \\ x_4 &= 17b_1 - 10b_2 + 11b_3 + b_4 \end{aligned}\right\} \text{are vectors } x_1 - x_4 \in \mathbb{R}^n \text{ linearly independent?}$$

$$\begin{bmatrix} 1 & -4 & 2 & 17 \\ -2 & -2 & 3 & -10 \\ 1 & 0 & -1 & 11 \\ -1 & 4 & -3 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & -15 \\ 0 & 0 & 1 & -18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

→ last column of reduced row echelon form is non-trivially solvable; last column is not a pivot column. $x_4 = -7x_1 - 15x_2 - 18x^3$, meaning the vectors are linearly dependent.

BASIS & RANK

→ in a vector space V, we are particularly interested in sets of vectors A which have property any vector $v \in V$ can be obtained by a linear combination of vectors in A. These are <u>special vectors</u>.