Vector Subspaces

→ sets contained in the original space w/ the property that if we perform vector space operations within this subspace, it will never leave it.

Definition

→ let $V = (V, +, \cdot)$ be a vector space of $U \subseteq V$, $U \neq 0$. Then $U(U, +, \cdot)$ is a vector subspace of V if U is a vector space w/ vector space operations $+$ and $\cdot$ restricted to $U \times U$ and $\mathbb{R} \times U$.

$U \subseteq V$ means U is subspace of V

→ if $U \subseteq V$ and V is a vector space, then U inherits properties:

- Abelian group properties
- distributivity & associativity
- neutral element

→ to determine if $(U, +, \cdot)$ is a subspace of V, we still need to show:

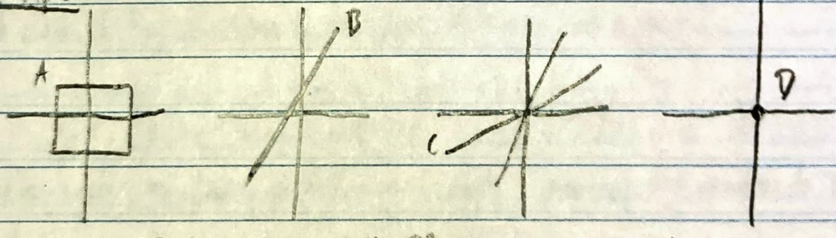
1.) $U \neq \emptyset$, in particular $0 \in U$

2.) Closure of U:

a.) outer operation: $\forall \lambda \in \mathbb{R}\ \forall x \in U: \lambda x \in U$

b.) inner operation: $\forall x, y \in U: x + y \in U$

Example



→ only example D is a subspace of $\mathbb{R}^2$. A & C violate closure property; B does not contain 0.

→ the solution set of a homogeneous system of lin eq. $Ax = 0$ with n unknowns $x = [x_1, \dots, x_n]^T$ is a subspace of $\mathbb{R}^n$.

→ inhomogeneous system $Ax = b$, $b \neq 0$ is not a subspace of $\mathbb{R}^n$

→ intersection of arbitrarily many subspaces is a subspace itself