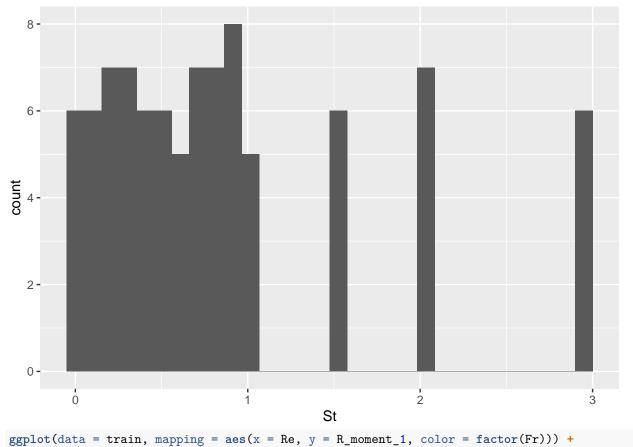
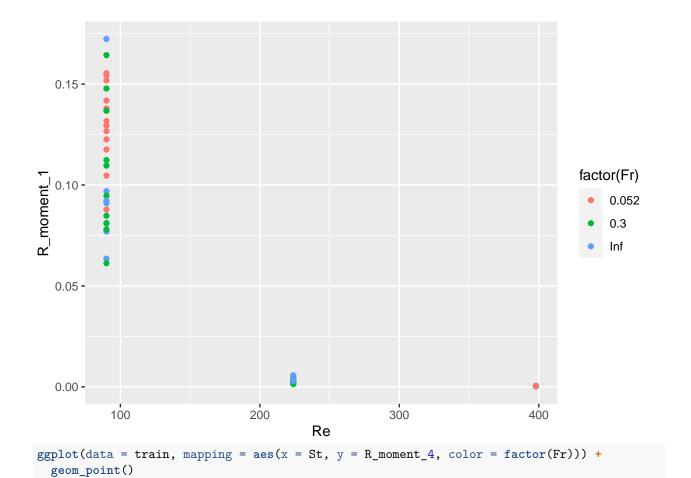
sara-experimental

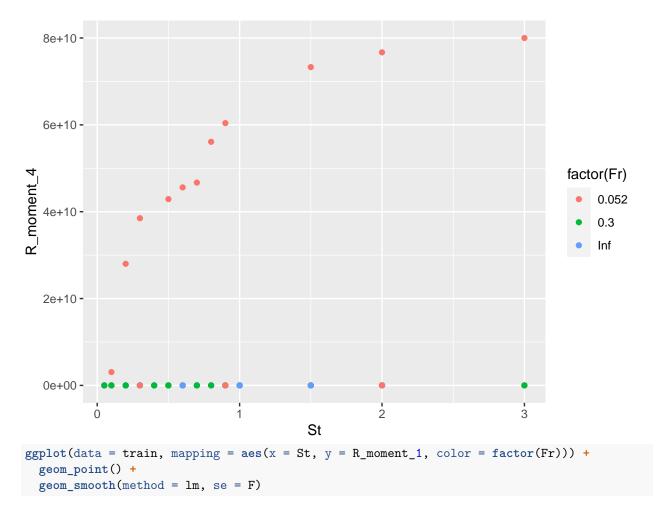
Sara Shao

10/7/2021

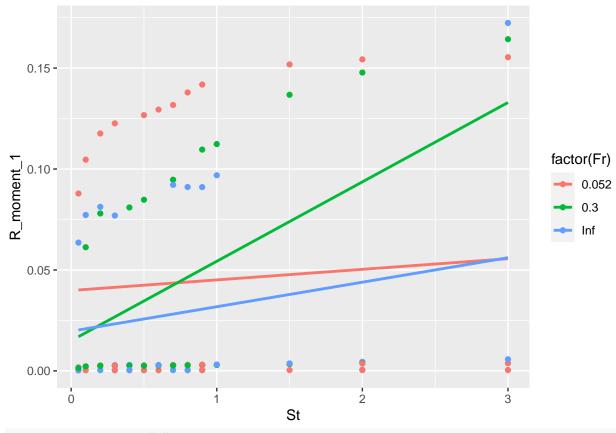
```
library(tidyverse)
## -- Attaching packages ------ tidyverse 1.3.0 --
## v tibble 3.0.6
                     v purrr
                               0.3.4
## v tidyr
            1.1.2
                      v dplyr
                               1.0.4
            1.4.0
## v readr
                    v forcats 0.5.1
## -- Conflicts ----- tidyverse_conflicts() --
## x lubridate::as.difftime() masks base::as.difftime()
## x lubridate::date() masks base::date()
## x dplyr::filter()
                           masks stats::filter()
## x readr::guess_encoding() masks rvest::guess_encoding()
## x lubridate::intersect() masks base::intersect()
## x dplyr::lag()
                         masks stats::lag()
## x purrr::pluck() masks rvest::pluck()
## x lubridate::setdiff() masks base::setdiff()
## x lubridate::union()
                            masks base::union()
train <- read.csv('data-train.csv')</pre>
head(train)
                Fr R_moment_1 R_moment_2 R_moment_3 R_moment_4
## 1 0.10 224 0.052 0.00215700 0.1303500 14.37400 1586.5000
## 2 3.00 224 0.052 0.00379030 0.4704200 69.94000 10404.0000
## 3 0.70 224
             Inf 0.00290540 0.0434990 0.82200
                                                     15.5510
             Inf 0.06352800 0.0906530
                                        0.46746
## 4 0.05 90
                                                      3.2696
## 5 0.70 398 Inf 0.00036945 0.0062242
                                        0.12649
                                                      2.5714
## 6 2.00 90 0.300 0.14780000 2.0068000
                                         36.24900
                                                    671.6700
ggplot(data = train, mapping = aes(x = St)) +
 geom_histogram()
```







$geom_smooth()$ using formula 'y ~ x'



```
train_data <- train %>%
    mutate(Fr = as.ordered(Fr)) %>%
    mutate(Re = as.ordered(Re))

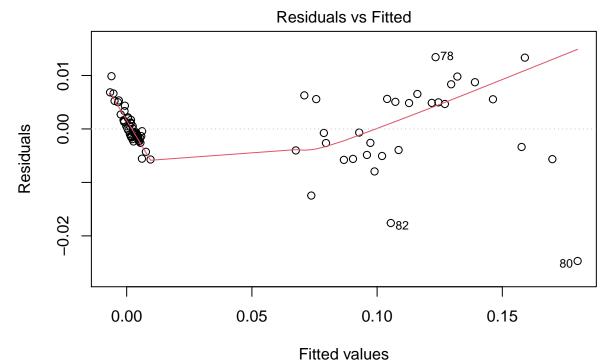
lm_R1 <- lm(R_moment_1 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

lm_R2 <- lm(R_moment_2 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

lm_R3 <- lm(R_moment_3 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

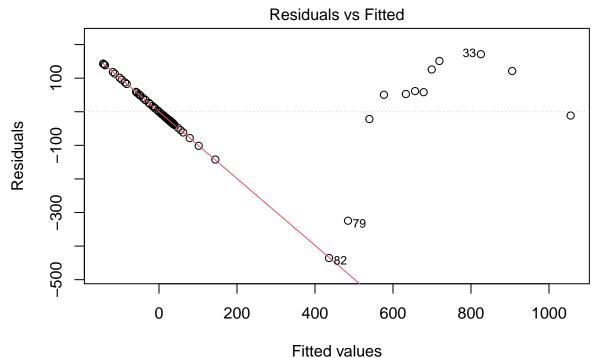
lm_R4 <- lm(R_moment_4 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

plot(lm_R1, 1)</pre>
```



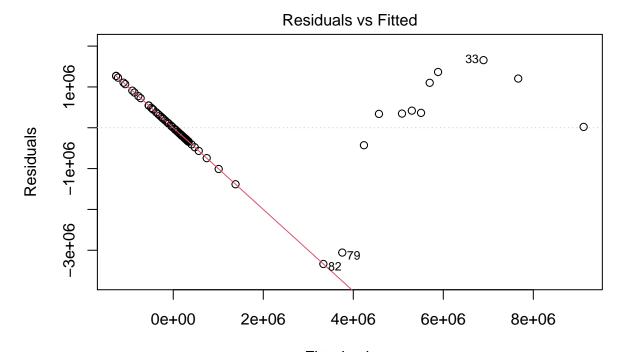
Im(R_moment_1 \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)

plot(lm_R2, 1)



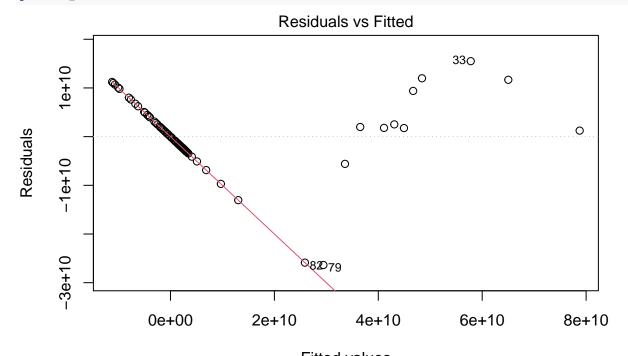
Im(R_moment_2 \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)

plot(lm_R3, 1)



Fitted values $Im(R_moment_3 \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$

plot(lm_R4, 1)



Fitted values $Im(R_moment_4 \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$

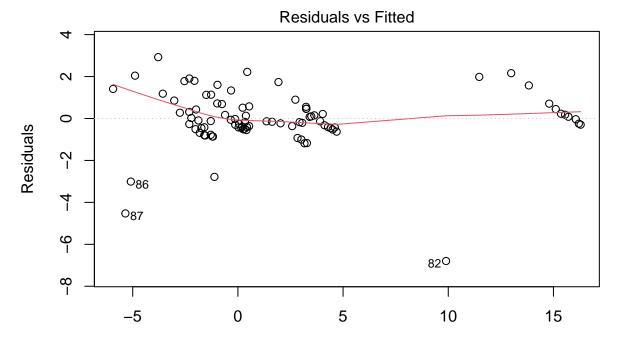
 $lm1 \leftarrow lm(log(R_moment_1) \sim log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data) \\ summary(lm1)$

##

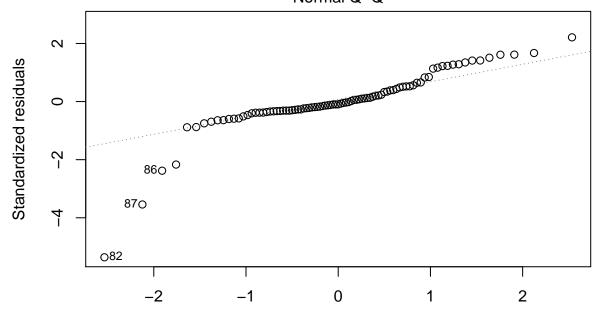
```
## Call:
## lm(formula = log(R_moment_1) ~ log(St) + Re + Fr + St * Fr +
       Fr * Re + St * Re, data = train data)
##
## Residuals:
##
                          Median
                                        3Q
        Min
                    10
                                                 Max
## -0.211809 -0.042926 -0.006391 0.038831 0.171243
## Coefficients: (1 not defined because of singularities)
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.349488
                           0.027649 -193.480 < 2e-16 ***
                                     10.003 1.89e-15 ***
## log(St)
                0.145668
                           0.014562
## Re.L
               -4.028476
                           0.027949 -144.139 < 2e-16 ***
                           0.022457
## Re.Q
                0.644476
                                      28.698 < 2e-16 ***
## Fr.L
               -0.102135
                           0.019793
                                      -5.160 1.95e-06 ***
## Fr.Q
                0.109493
                           0.026874
                                       4.074 0.000113 ***
## St
               0.095896
                           0.021136
                                       4.537 2.13e-05 ***
## Fr.L:St
               0.095376
                           0.017271
                                       5.522 4.60e-07 ***
## Fr.Q:St
               -0.064244
                           0.022042
                                     -2.915 0.004692 **
## Re.L:Fr.L
               0.242947
                           0.024770
                                       9.808 4.39e-15 ***
## Re.Q:Fr.L
               -0.076314
                           0.022588
                                    -3.379 0.001159 **
## Re.L:Fr.Q
               -0.077781
                           0.046306
                                      -1.680 0.097169 .
## Re.Q:Fr.Q
                                          NA
                                                   NA
                      NA
                                 NA
               -0.008897
                           0.021672
                                      -0.411 0.682581
## Re.L:St
## Re.Q:St
               -0.025325
                           0.018376
                                      -1.378 0.172252
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.07645 on 75 degrees of freedom
## Multiple R-squared: 0.999, Adjusted R-squared: 0.9988
## F-statistic: 5797 on 13 and 75 DF, p-value: < 2.2e-16
lm2 <- lm(log(R_moment_2) ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)</pre>
summary(lm2)
##
## Call:
## lm(formula = log(R_moment_2) ~ log(St) + Re + Fr + St * Fr +
##
       Fr * Re + St * Re, data = train_data)
##
## Residuals:
       Min
                1Q Median
                                30
                                       Max
## -3.6442 -0.2697 -0.0561 0.3429 1.8016
## Coefficients: (1 not defined because of singularities)
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) -0.300240
                           0.307283 -0.977 0.33167
                                     9.274 4.50e-14 ***
## log(St)
                1.500864
                           0.161841
## Re.L
               -4.269814
                           0.310614 -13.746 < 2e-16 ***
## Re.Q
                                     4.401 3.52e-05 ***
               1.098368
                           0.249584
## Fr.L
                           0.219976 -8.763 4.20e-13 ***
               -1.927717
                                      3.170 0.00221 **
## Fr.Q
               0.946857
                           0.298671
## St
                                    -4.246 6.16e-05 ***
               -0.997463
                           0.234905
## Fr.L:St
               -0.007867
                           0.191944 -0.041 0.96742
## Fr.Q:St
               -0.043393
                           0.244968 -0.177 0.85988
```

```
## Re.L:Fr.L
              3.399373
                          0.275290 12.348 < 2e-16 ***
             -0.646249
## Re.Q:Fr.L
                          0.251039 -2.574 0.01202 *
                          0.514630 -5.029 3.27e-06 ***
## Re.L:Fr.Q
              -2.588037
## Re.Q:Fr.Q
                     NA
                                NΑ
                                       NA
                                                 NΑ
## Re.L:St
              -0.470624
                          0.240856
                                   -1.954 0.05443
## Re.Q:St
              -0.128587
                          0.204226 -0.630 0.53085
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.8497 on 75 degrees of freedom
## Multiple R-squared: 0.9553, Adjusted R-squared: 0.9476
## F-statistic: 123.4 on 13 and 75 DF, p-value: < 2.2e-16
lm3 <- lm(log(R_moment_3) ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)
summary(lm3)
##
## Call:
## lm(formula = log(R_moment_3) \sim log(St) + Re + Fr + St * Fr +
##
      Fr * Re + St * Re, data = train_data)
##
## Residuals:
      Min
               1Q Median
                               30
                                      Max
## -6.7949 -0.4431 -0.1224 0.5575 2.9257
## Coefficients: (1 not defined because of singularities)
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.54701
                        0.51756
                                  8.785 3.81e-13 ***
## log(St)
                                   8.665 6.47e-13 ***
              2.36189
                          0.27259
                          0.52317 -9.362 3.06e-14 ***
## Re.L
              -4.89792
## Re.Q
              1.52364
                          0.42038
                                   3.624 0.000525 ***
## Fr.L
              -3.75114
                          0.37051 -10.124 1.12e-15 ***
## Fr.Q
              1.83513
                          0.50306
                                   3.648 0.000486 ***
## St
              -1.72871
                         0.39565 -4.369 3.95e-05 ***
## Fr.L:St
              -0.09194
                        0.32329 -0.284 0.776889
## Fr.Q:St
             -0.03580
                          0.41260 -0.087 0.931098
## Re.L:Fr.L
              6.45803
                          0.46368 13.928 < 2e-16 ***
## Re.Q:Fr.L
              -1.15825
                          0.42283 -2.739 0.007689 **
## Re.L:Fr.Q
              -4.96308
                          0.86680 -5.726 2.01e-07 ***
## Re.Q:Fr.Q
                    NA
                               NA
                                       NA
                                               NA
## Re.L:St
              -0.79750
                          0.40568 -1.966 0.053018
## Re.Q:St
              -0.17827
                          0.34398 -0.518 0.605816
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.431 on 75 degrees of freedom
## Multiple R-squared: 0.9459, Adjusted R-squared: 0.9365
## F-statistic: 100.8 on 13 and 75 DF, p-value: < 2.2e-16
lm4 <- lm(log(R_moment_4) ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)</pre>
summary(lm4)
##
## Call:
## lm(formula = log(R_moment_4) ~ log(St) + Re + Fr + St * Fr +
```

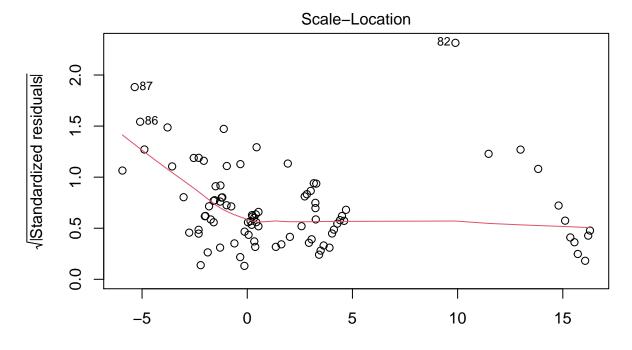
```
##
       Fr * Re + St * Re, data = train_data)
##
## Residuals:
##
      Min
                1Q Median
                               ЗQ
                                      Max
## -9.6675 -0.6183 -0.1392 0.7410 3.8875
##
## Coefficients: (1 not defined because of singularities)
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 9.28287
                          0.70476 13.172 < 2e-16 ***
## log(St)
               3.10948
                          0.37119
                                    8.377 2.28e-12 ***
## Re.L
               -5.63270
                          0.71240 -7.907 1.80e-11 ***
## Re.Q
                          0.57242
                                    3.404 0.001070 **
               1.94835
## Fr.L
              -5.55550
                          0.50452 -11.012 < 2e-16 ***
## Fr.Q
                                    3.964 0.000167 ***
               2.71540
                          0.68500
## St
              -2.36662
                          0.53876 -4.393 3.62e-05 ***
## Fr.L:St
              -0.17635
                          0.44023
                                   -0.401 0.689864
## Fr.Q:St
              -0.01987
                                   -0.035 0.971880
                          0.56184
## Re.L:Fr.L
              9.47956
                          0.63138
                                   15.014 < 2e-16 ***
                          0.57576
## Re.Q:Fr.L
              -1.66019
                                   -2.883 0.005130 **
## Re.L:Fr.Q
              -7.28081
                           1.18031
                                   -6.169 3.21e-08 ***
## Re.Q:Fr.Q
                     NA
                               NA
                                       NA
                                                NA
## Re.L:St
              -1.08269
                           0.55241
                                   -1.960 0.053716 .
                          0.46840 -0.475 0.636226
## Re.Q:St
              -0.22245
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.949 on 75 degrees of freedom
## Multiple R-squared: 0.9457, Adjusted R-squared: 0.9363
## F-statistic: 100.6 on 13 and 75 DF, p-value: < 2.2e-16
plot(lm3)
```



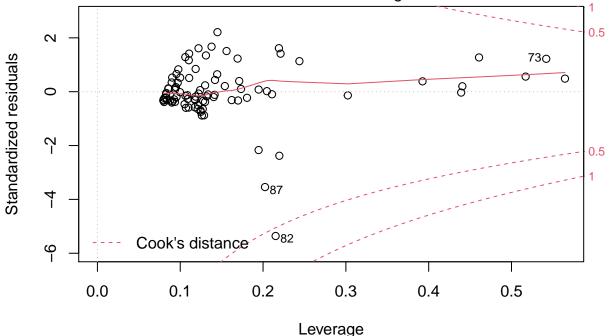
Fitted values $Im(log(R_moment_3) \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re) \\ Normal Q-Q$



Theoretical Quantiles Im(log(R_moment_3) ~ log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)



Fitted values $Im(log(R_moment_3) \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$ Residuals vs Leverage



 $Im(log(R_moment_3) \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$

```
set.seed(123)
errors <- c()

for(i in c(1:nrow(train_data))){
   subset <- train_data[-c(i),]
   model <- lm(log(R_moment_1) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)</pre>
```

```
prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_1 - prediction)^2
  errors[i] <- error
mean(errors)
## [1] 2.720867e-05
set.seed(123)
errors <- c()
for(i in c(1:nrow(train data))){
  subset <- train_data[-c(i),]</pre>
  model <- lm(log(R_moment_2) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)
  prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_2 - prediction)^2</pre>
  errors[i] <- error</pre>
}
mean(errors)
## [1] 6312.875
set.seed(123)
errors <- c()
for(i in c(1:nrow(train_data))){
  subset <- train_data[-c(i),]</pre>
  model <- lm(log(R_moment_3) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)
  prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_3 - prediction)^2</pre>
  errors[i] <- error</pre>
mean(errors)
## [1] 777311609607
set.seed(123)
errors <- c()
for(i in c(1:nrow(train_data))){
  subset <- train_data[-c(i),]</pre>
  model <- lm(log(R_moment_4) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)</pre>
  prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_4 - prediction)^2
  errors[i] <- error
mean(errors)
```

[1] 7.403514e+19