Case Study: Modeling Liquid Mechanics

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2021-10-13

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.0 --
```

```
## v tibble  3.0.6     v purrr   0.3.4
## v tidyr   1.1.2     v dplyr   1.0.4
## v readr   1.4.0     v forcats 0.5.1
```

```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x lubridate::as.difftime() masks base::as.difftime()
## x lubridate::date()        masks base::date()
## x dplyr::filter()          masks stats::filter()
## x readr::guess_encoding()  masks rvest::guess_encoding()
## x lubridate::intersect()   masks base::intersect()
## x dplyr::lag()             masks stats::lag()
## x purrr::pluck()           masks rvest::pluck()
## x lubridate::setdiff()     masks base::setdiff()
## x lubridate::union()       masks base::union()
```

```
library(mgcv)
```

```
## Loading required package: nlme
```

```
##
## Attaching package: 'nlme'
```

```
## The following object is masked from 'package:dplyr':
##
##     collapse
```

```
## This is mgcv 1.8-31. For overview type 'help("mgcv-package")'.
```

```
test_data <- read.csv("data-test.csv")
train_data <- read.csv("data-train.csv")
```

```
train_data
```

```
##      St  Re    Fr R_moment_1 R_moment_2 R_moment_3 R_moment_4
## 1  0.10 224 0.052 0.00215700 1.3035e-01 1.4374e+01 1.5865e+03
## 2  3.00 224 0.052 0.00379030 4.7042e-01 6.9940e+01 1.0404e+04
## 3  0.70 224   Inf 0.00290540 4.3499e-02 8.2200e-01 1.5551e+01
## 4  0.05  90   Inf 0.06352800 9.0653e-02 4.6746e-01 3.2696e+00
## 5  0.70 398   Inf 0.00036945 6.2242e-03 1.2649e-01 2.5714e+00
## 6  2.00  90 0.300 0.14780000 2.0068e+00 3.6249e+01 6.7167e+02
## 7  0.20  90   Inf 0.08127300 3.2450e-01 3.0363e+00 3.2976e+01
## 8  3.00 224   Inf 0.00574730 1.1966e-01 2.7480e+00 6.3159e+01
## 9  0.90 224   Inf 0.00302150 4.5244e-02 8.4530e-01 1.5809e+01
```

```
## 10 0.60 398 0.052 0.00031431 4.4672e-03 8.2060e-02 1.5077e+00
## 11 0.90  90   Inf 0.09102700 5.9539e-01 7.2454e+00 9.5166e+01
## 12 0.30 398   Inf 0.00036022 6.2830e-03 1.3546e-01 2.9211e+00
## 13 2.00 224   Inf 0.00447250 8.0804e-02 1.6668e+00 3.4408e+01
## 14 1.00 224 0.052 0.00312380 3.6478e-01 5.3322e+01 7.7958e+03
## 15 0.50  90 0.052 0.12670000 6.8596e+02 5.4300e+06 4.2900e+10
## 16 0.60 224 0.300 0.00257400 3.6621e-02 6.7102e-01 1.2309e+01
## 17 0.10  90   Inf 0.07722700 2.2120e-01 1.8833e+00 2.0190e+01
## 18 1.00  90 0.300 0.11236000 1.1261e+00 1.7335e+01 2.8261e+02
## 19 0.70 224 0.052 0.00285610 3.1273e-01 4.4529e+01 6.3423e+03
## 20 0.20  90 0.052 0.11760000 5.1774e+02 3.8100e+06 2.8000e+10
## 21 0.10  90 0.300 0.06125200 6.9867e-02 2.4338e-01 1.1379e+00
## 22 0.50 398   Inf 0.00036800 6.3559e-03 1.3341e-01 2.8013e+00
## 23 0.20 224   Inf 0.00269160 3.9016e-02 7.6384e-01 1.4978e+01
## 24 1.50 398 0.052 0.00038321 5.9338e-03 1.1156e-01 2.1004e+00
## 25 0.90 398   Inf 0.00038344 6.4432e-03 1.2925e-01 2.5935e+00
## 26 0.50 224 0.052 0.00274240 3.0355e-01 4.3911e+01 6.3530e+03
## 27 0.10 398 0.052 0.00027479 3.2549e-03 5.8006e-02 1.0344e+00
## 28 0.40 224 0.052 0.00268090 2.8897e-01 4.1585e+01 5.9861e+03
## 29 0.30  90 0.052 0.12261000 6.2727e+02 4.9100e+06 3.8500e+10
## 30 0.05 224 0.052 0.00173740 1.6633e-03 2.0228e-02 3.6438e-01
## 31 1.50 224 0.052 0.00341630 4.0300e-01 5.8417e+01 8.4710e+03
## 32 0.80  90   Inf 0.09107400 6.1825e-01 7.4973e+00 9.7048e+01
## 33 1.50  90 0.052 0.15181000 9.9690e+02 8.5500e+06 7.3300e+10
## 34 0.05 398   Inf 0.00022202 1.0055e-03 1.0857e-02 1.1782e-01
## 35 0.80 224   Inf 0.00298090 4.4580e-02 8.3764e-01 1.5759e+01
## 36 0.90 224 0.300 0.00280490 4.1143e-02 7.5132e-01 1.3729e+01
## 37 0.40 224   Inf 0.00292630 4.6261e-02 9.2914e-01 1.8681e+01
## 38 0.80 398 0.052 0.00033341 4.9036e-03 9.1143e-02 1.6948e+00
## 39 1.50 224 0.300 0.00341050 5.4101e-02 1.0222e+00 1.9340e+01
## 40 0.20 224 0.052 0.00257870 2.6830e-01 3.9080e+01 5.6959e+03
## 41 0.30 224   Inf 0.00283770 4.3589e-02 8.6962e-01 1.7373e+01
## 42 0.30 224 0.300 0.00250630 3.5881e-02 6.8596e-01 1.3132e+01
## 43 2.00 224 0.300 0.00381230 6.1927e-02 1.1844e+00 2.2705e+01
## 44 1.00  90   Inf 0.09691800 6.7696e-01 8.2384e+00 1.0602e+02
## 45 0.80 224 0.052 0.00295750 3.3361e-01 4.8161e+01 6.9539e+03
## 46 1.00 224 0.300 0.00289530 4.2300e-02 7.6755e-01 1.3941e+01
## 47 0.70  90 0.300 0.09471100 6.9751e-01 9.1793e+00 1.3187e+02
## 48 0.30 224 0.052 0.00256750 2.6547e-01 3.7665e+01 5.3451e+03
## 49 0.40 224 0.300 0.00262070 3.9502e-02 7.6851e-01 1.4966e+01
## 50 0.10 224 0.300 0.00221530 2.4475e-02 4.2167e-01 7.2842e+00
## 51 3.00  90   Inf 0.17234000 2.2386e+00 4.0454e+01 7.6198e+02
## 52 1.00 224   Inf 0.00309680 4.6454e-02 8.6381e-01 1.6077e+01
## 53 2.00 398   Inf 0.00053647 1.0022e-02 2.1023e-01 4.4109e+00
## 54 0.80  90 0.052 0.13793000 8.2524e+02 6.8000e+06 5.6100e+10
## 55 0.40 398 0.052 0.00029691 4.1375e-03 7.6124e-02 1.4014e+00
## 56 0.50 398 0.052 0.00030716 4.3494e-03 8.0143e-02 1.4770e+00
## 57 0.70  90   Inf 0.09217600 5.6482e-01 6.7191e+00 8.8723e+01
## 58 2.00  90 0.052 0.15433000 1.0269e+03 8.8700e+06 7.6700e+10
## 59 0.90  90 0.300 0.10962000 1.0319e+00 1.5797e+01 2.6136e+02
## 60 0.30  90   Inf 0.07694500 3.2652e-01 3.4052e+00 4.1042e+01
## 61 0.50 224 0.300 0.00250710 3.5152e-02 6.4378e-01 1.1801e+01
## 62 0.50  90 0.300 0.08477300 4.9728e-01 6.0317e+00 8.3287e+01
## 63 0.80 398   Inf 0.00037399 6.2457e-03 1.2542e-01 2.5193e+00
```

```
## 64 0.20 398   Inf 0.00033521 5.4505e-03 1.1408e-01 2.3884e+00
## 65 0.70  90 0.052 0.13173000 7.3694e+02 5.8700e+06 4.6700e+10
## 66 2.00 398 0.052 0.00039644 6.1040e-03 1.1209e-01 2.0593e+00
## 67 0.70 224 0.300 0.00260870 3.6438e-02 6.5445e-01 1.1765e+01
## 68 0.60 224 0.052 0.00279390 3.0594e-01 4.3745e+01 6.2554e+03
## 69 0.30 398 0.052 0.00030066 4.3488e-03 8.3446e-02 1.6023e+00
## 70 0.90 224 0.052 0.00305410 3.5419e-01 5.1795e+01 7.5758e+03
## 71 0.80 224 0.300 0.00268160 3.7714e-02 6.7549e-01 1.2112e+01
## 72 0.20 224 0.300 0.00246950 3.4818e-02 6.7088e-01 1.2939e+01
## 73 3.00 398 0.052 0.00040188 5.4492e-03 9.1871e-02 1.5565e+00
## 74 0.90  90 0.052 0.14184000 8.7019e+02 7.2500e+06 6.0400e+10
## 75 0.40 398   Inf 0.00036977 6.4986e-03 1.3933e-01 2.9880e+00
## 76 0.20  90 0.300 0.07798500 2.5598e-01 2.0965e+00 2.0849e+01
## 77 0.60  90 0.052 0.12946000 7.1816e+02 5.7200e+06 4.5600e+10
## 78 1.50  90 0.300 0.13678000 1.8254e+00 3.2833e+01 6.0903e+02
## 79 0.10  90 0.052 0.10464000 1.6015e+02 6.9900e+05 3.0700e+09
## 80 3.00  90 0.052 0.15538000 1.0443e+03 9.1400e+06 8.0000e+10
## 81 2.00 224 0.052 0.00363470 4.4512e-01 6.5387e+01 9.6105e+03
## 82 0.05  90 0.052 0.08786800 5.3449e-01 2.2205e+01 1.5679e+03
## 83 0.40  90 0.300 0.08095700 3.9996e-01 4.3303e+00 5.3618e+01
## 84 0.90 398 0.052 0.00034145 5.0555e-03 9.4083e-02 1.7522e+00
## 85 3.00  90 0.300 0.16433000 2.3317e+00 4.4516e+01 8.8779e+02
## 86 0.05 224   Inf 0.00153380 2.5653e-04 3.0407e-04 5.4466e-04
## 87 0.05 224 0.300 0.00135380 1.0303e-04 5.1400e-05 4.1600e-05
## 88 0.60 224   Inf 0.00291710 4.4317e-02 8.5282e-01 1.6431e+01
## 89 1.50 224   Inf 0.00370310 6.0910e-02 1.1829e+00 2.2990e+01
```

```
train_data <- train_data %>%
  mutate(Fr = as.factor(Fr)) %>%
  mutate(Re = as.factor(Re))

train_data
```

```
##      St  Re    Fr R_moment_1 R_moment_2 R_moment_3 R_moment_4
## 1  0.10 224 0.052 0.00215700 1.3035e-01 1.4374e+01 1.5865e+03
## 2  3.00 224 0.052 0.00379030 4.7042e-01 6.9940e+01 1.0404e+04
## 3  0.70 224   Inf 0.00290540 4.3499e-02 8.2200e-01 1.5551e+01
## 4  0.05  90   Inf 0.06352800 9.0653e-02 4.6746e-01 3.2696e+00
## 5  0.70 398   Inf 0.00036945 6.2242e-03 1.2649e-01 2.5714e+00
## 6  2.00  90   0.3 0.14780000 2.0068e+00 3.6249e+01 6.7167e+02
## 7  0.20  90   Inf 0.08127300 3.2450e-01 3.0363e+00 3.2976e+01
## 8  3.00 224   Inf 0.00574730 1.1966e-01 2.7480e+00 6.3159e+01
## 9  0.90 224   Inf 0.00302150 4.5244e-02 8.4530e-01 1.5809e+01
## 10 0.60 398 0.052 0.00031431 4.4672e-03 8.2060e-02 1.5077e+00
## 11 0.90  90   Inf 0.09102700 5.9539e-01 7.2454e+00 9.5166e+01
## 12 0.30 398   Inf 0.00036022 6.2830e-03 1.3546e-01 2.9211e+00
## 13 2.00 224   Inf 0.00447250 8.0804e-02 1.6668e+00 3.4408e+01
## 14 1.00 224 0.052 0.00312380 3.6478e-01 5.3322e+01 7.7958e+03
## 15 0.50  90 0.052 0.12670000 6.8596e+02 5.4300e+06 4.2900e+10
## 16 0.60 224   0.3 0.00257400 3.6621e-02 6.7102e-01 1.2309e+01
## 17 0.10  90   Inf 0.07722700 2.2120e-01 1.8833e+00 2.0190e+01
## 18 1.00  90   0.3 0.11236000 1.1261e+00 1.7335e+01 2.8261e+02
## 19 0.70 224 0.052 0.00285610 3.1273e-01 4.4529e+01 6.3423e+03
## 20 0.20  90 0.052 0.11760000 5.1774e+02 3.8100e+06 2.8000e+10
## 21 0.10  90   0.3 0.06125200 6.9867e-02 2.4338e-01 1.1379e+00
```

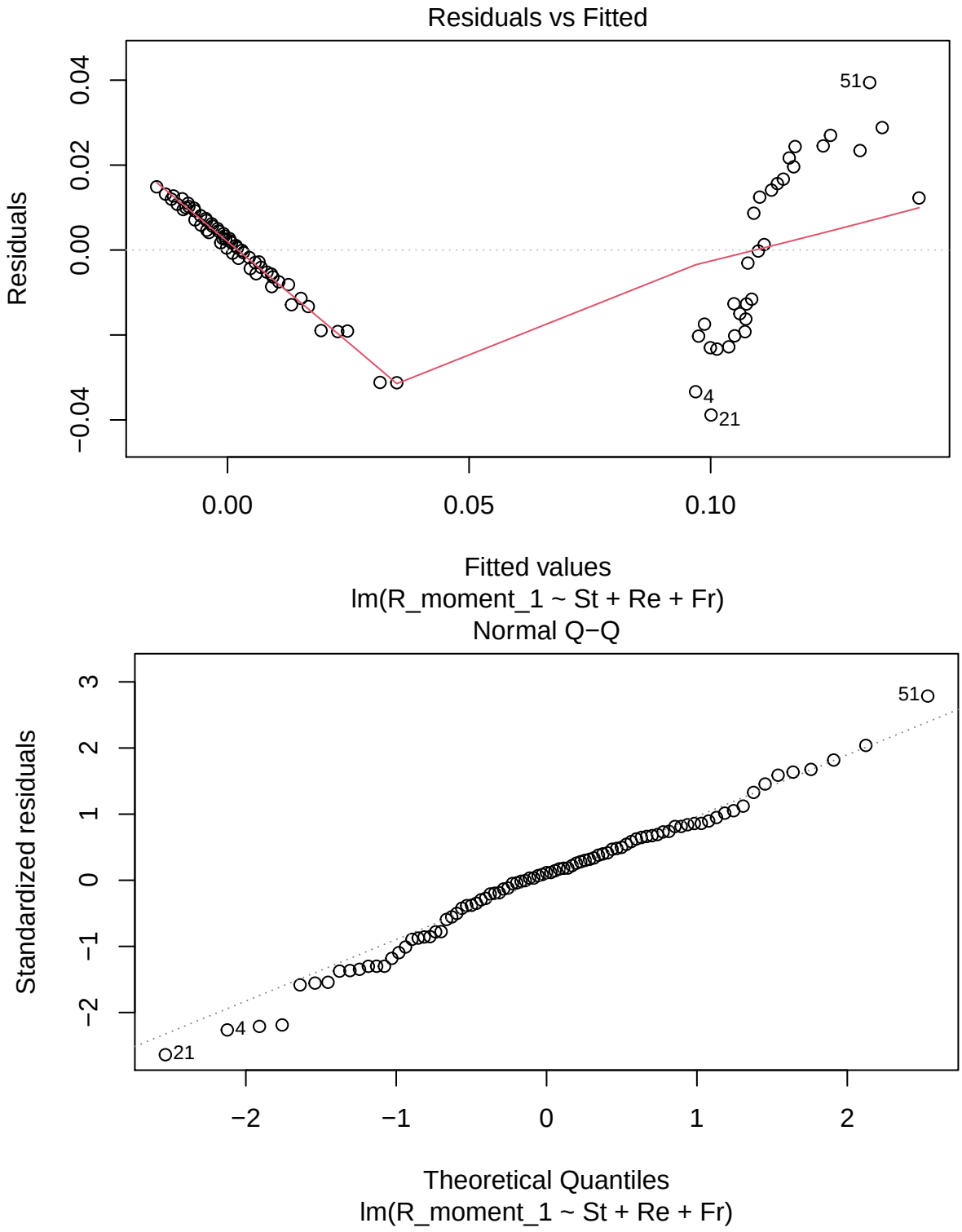
```
## 22 0.50 398   Inf 0.00036800 6.3559e-03 1.3341e-01 2.8013e+00
## 23 0.20 224   Inf 0.00269160 3.9016e-02 7.6384e-01 1.4978e+01
## 24 1.50 398 0.052 0.00038321 5.9338e-03 1.1156e-01 2.1004e+00
## 25 0.90 398   Inf 0.00038344 6.4432e-03 1.2925e-01 2.5935e+00
## 26 0.50 224 0.052 0.00274240 3.0355e-01 4.3911e+01 6.3530e+03
## 27 0.10 398 0.052 0.00027479 3.2549e-03 5.8006e-02 1.0344e+00
## 28 0.40 224 0.052 0.00268090 2.8897e-01 4.1585e+01 5.9861e+03
## 29 0.30  90 0.052 0.12261000 6.2727e+02 4.9100e+06 3.8500e+10
## 30 0.05 224 0.052 0.00173740 1.6633e-03 2.0228e-02 3.6438e-01
## 31 1.50 224 0.052 0.00341630 4.0300e-01 5.8417e+01 8.4710e+03
## 32 0.80  90   Inf 0.09107400 6.1825e-01 7.4973e+00 9.7048e+01
## 33 1.50  90 0.052 0.15181000 9.9690e+02 8.5500e+06 7.3300e+10
## 34 0.05 398   Inf 0.00022202 1.0055e-03 1.0857e-02 1.1782e-01
## 35 0.80 224   Inf 0.00298090 4.4580e-02 8.3764e-01 1.5759e+01
## 36 0.90 224   0.3 0.00280490 4.1143e-02 7.5132e-01 1.3729e+01
## 37 0.40 224   Inf 0.00292630 4.6261e-02 9.2914e-01 1.8681e+01
## 38 0.80 398 0.052 0.00033341 4.9036e-03 9.1143e-02 1.6948e+00
## 39 1.50 224   0.3 0.00341050 5.4101e-02 1.0222e+00 1.9340e+01
## 40 0.20 224 0.052 0.00257870 2.6830e-01 3.9080e+01 5.6959e+03
## 41 0.30 224   Inf 0.00283770 4.3589e-02 8.6962e-01 1.7373e+01
## 42 0.30 224   0.3 0.00250630 3.5881e-02 6.8596e-01 1.3132e+01
## 43 2.00 224   0.3 0.00381230 6.1927e-02 1.1844e+00 2.2705e+01
## 44 1.00  90   Inf 0.09691800 6.7696e-01 8.2384e+00 1.0602e+02
## 45 0.80 224 0.052 0.00295750 3.3361e-01 4.8161e+01 6.9539e+03
## 46 1.00 224   0.3 0.00289530 4.2300e-02 7.6755e-01 1.3941e+01
## 47 0.70  90   0.3 0.09471100 6.9751e-01 9.1793e+00 1.3187e+02
## 48 0.30 224 0.052 0.00256750 2.6547e-01 3.7665e+01 5.3451e+03
## 49 0.40 224   0.3 0.00262070 3.9502e-02 7.6851e-01 1.4966e+01
## 50 0.10 224   0.3 0.00221530 2.4475e-02 4.2167e-01 7.2842e+00
## 51 3.00  90   Inf 0.17234000 2.2386e+00 4.0454e+01 7.6198e+02
## 52 1.00 224   Inf 0.00309680 4.6454e-02 8.6381e-01 1.6077e+01
## 53 2.00 398   Inf 0.00053647 1.0022e-02 2.1023e-01 4.4109e+00
## 54 0.80  90 0.052 0.13793000 8.2524e+02 6.8000e+06 5.6100e+10
## 55 0.40 398 0.052 0.00029691 4.1375e-03 7.6124e-02 1.4014e+00
## 56 0.50 398 0.052 0.00030716 4.3494e-03 8.0143e-02 1.4770e+00
## 57 0.70  90   Inf 0.09217600 5.6482e-01 6.7191e+00 8.8723e+01
## 58 2.00  90 0.052 0.15433000 1.0269e+03 8.8700e+06 7.6700e+10
## 59 0.90  90   0.3 0.10962000 1.0319e+00 1.5797e+01 2.6136e+02
## 60 0.30  90   Inf 0.07694500 3.2652e-01 3.4052e+00 4.1042e+01
## 61 0.50 224   0.3 0.00250710 3.5152e-02 6.4378e-01 1.1801e+01
## 62 0.50  90   0.3 0.08477300 4.9728e-01 6.0317e+00 8.3287e+01
## 63 0.80 398   Inf 0.00037399 6.2457e-03 1.2542e-01 2.5193e+00
## 64 0.20 398   Inf 0.00033521 5.4505e-03 1.1408e-01 2.3884e+00
## 65 0.70  90 0.052 0.13173000 7.3694e+02 5.8700e+06 4.6700e+10
## 66 2.00 398 0.052 0.00039644 6.1040e-03 1.1209e-01 2.0593e+00
## 67 0.70 224   0.3 0.00260870 3.6438e-02 6.5445e-01 1.1765e+01
## 68 0.60 224 0.052 0.00279390 3.0594e-01 4.3745e+01 6.2554e+03
## 69 0.30 398 0.052 0.00030066 4.3488e-03 8.3446e-02 1.6023e+00
## 70 0.90 224 0.052 0.00305410 3.5419e-01 5.1795e+01 7.5758e+03
## 71 0.80 224   0.3 0.00268160 3.7714e-02 6.7549e-01 1.2112e+01
## 72 0.20 224   0.3 0.00246950 3.4818e-02 6.7088e-01 1.2939e+01
## 73 3.00 398 0.052 0.00040188 5.4492e-03 9.1871e-02 1.5565e+00
## 74 0.90  90 0.052 0.14184000 8.7019e+02 7.2500e+06 6.0400e+10
## 75 0.40 398   Inf 0.00036977 6.4986e-03 1.3933e-01 2.9880e+00
```

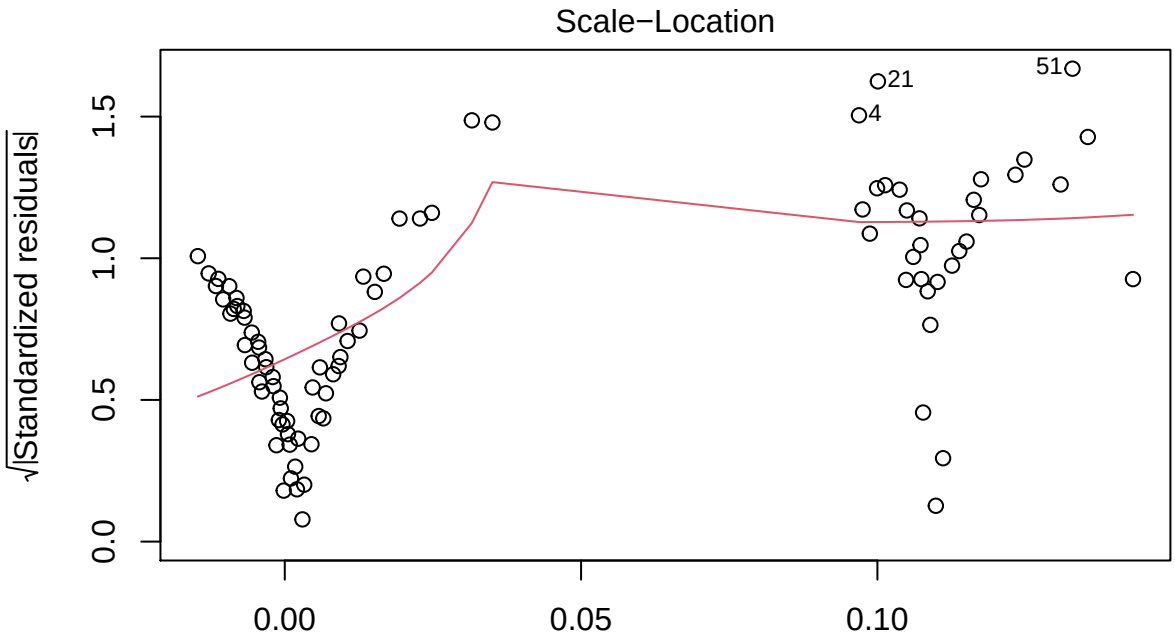
```
## 76 0.20  90   0.3 0.07798500 2.5598e-01 2.0965e+00 2.0849e+01
## 77 0.60  90 0.052 0.12946000 7.1816e+02 5.7200e+06 4.5600e+10
## 78 1.50  90   0.3 0.13678000 1.8254e+00 3.2833e+01 6.0903e+02
## 79 0.10  90 0.052 0.10464000 1.6015e+02 6.9900e+05 3.0700e+09
## 80 3.00  90 0.052 0.15538000 1.0443e+03 9.1400e+06 8.0000e+10
## 81 2.00 224 0.052 0.00363470 4.4512e-01 6.5387e+01 9.6105e+03
## 82 0.05  90 0.052 0.08786800 5.3449e-01 2.2205e+01 1.5679e+03
## 83 0.40  90   0.3 0.08095700 3.9996e-01 4.3303e+00 5.3618e+01
## 84 0.90 398 0.052 0.00034145 5.0555e-03 9.4083e-02 1.7522e+00
## 85 3.00  90   0.3 0.16433000 2.3317e+00 4.4516e+01 8.8779e+02
## 86 0.05 224   Inf 0.00153380 2.5653e-04 3.0407e-04 5.4466e-04
## 87 0.05 224   0.3 0.00135380 1.0303e-04 5.1400e-05 4.1600e-05
## 88 0.60 224   Inf 0.00291710 4.4317e-02 8.5282e-01 1.6431e+01
## 89 1.50 224   Inf 0.00370310 6.0910e-02 1.1829e+00 2.2990e+01
```

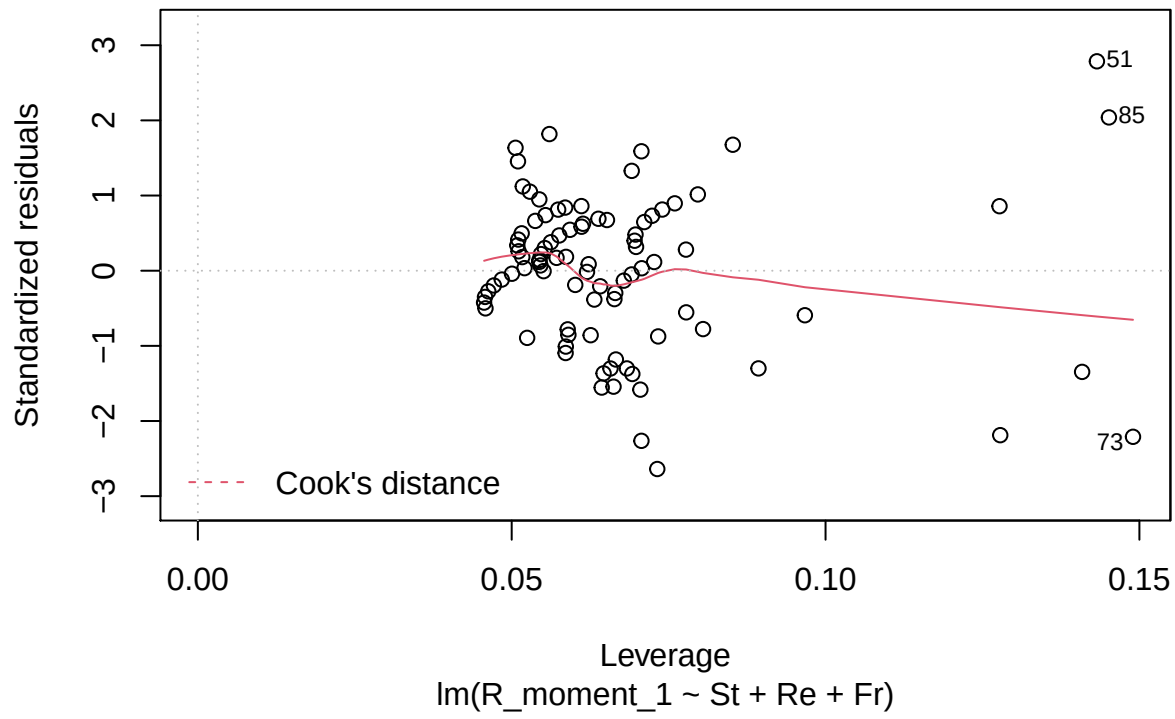
```
lm1 <- lm(R_moment_1 ~ St + Re + Fr, data = train_data)
summary(lm1)
```

```
##
## Call:
## lm(formula = R_moment_1 ~ St + Re + Fr, data = train_data)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.038834 -0.008614  0.001702  0.009854  0.039423
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.106488   0.003971  26.818  < 2e-16 ***
## St           0.012213   0.002078   5.877 8.42e-08 ***
## Re224       -0.108091   0.003682 -29.353  < 2e-16 ***
## Re398       -0.111553   0.004632 -24.081  < 2e-16 ***
## Fr0.3       -0.007623   0.004245  -1.796  0.07618 .
## FrInf       -0.010210   0.003787  -2.696  0.00849 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01529 on 83 degrees of freedom
## Multiple R-squared:  0.9293, Adjusted R-squared:  0.9251
## F-statistic: 218.2 on 5 and 83 DF,  p-value: < 2.2e-16
```

```
plot(lm1)
```


Residuals vs Fitted
Residuals
0.04
0.02
0.00
−0.04
51
4
21
0.00
0.05
0.10
Fitted values
lm(R_moment_1 ~ St + Re + Fr)
Normal Q−Q
Standardized residuals
3
2
1
0
−1
−2
51
4
21
−2
−1
0
1
2
Theoretical Quantiles
lm(R_moment_1 ~ St + Re + Fr)





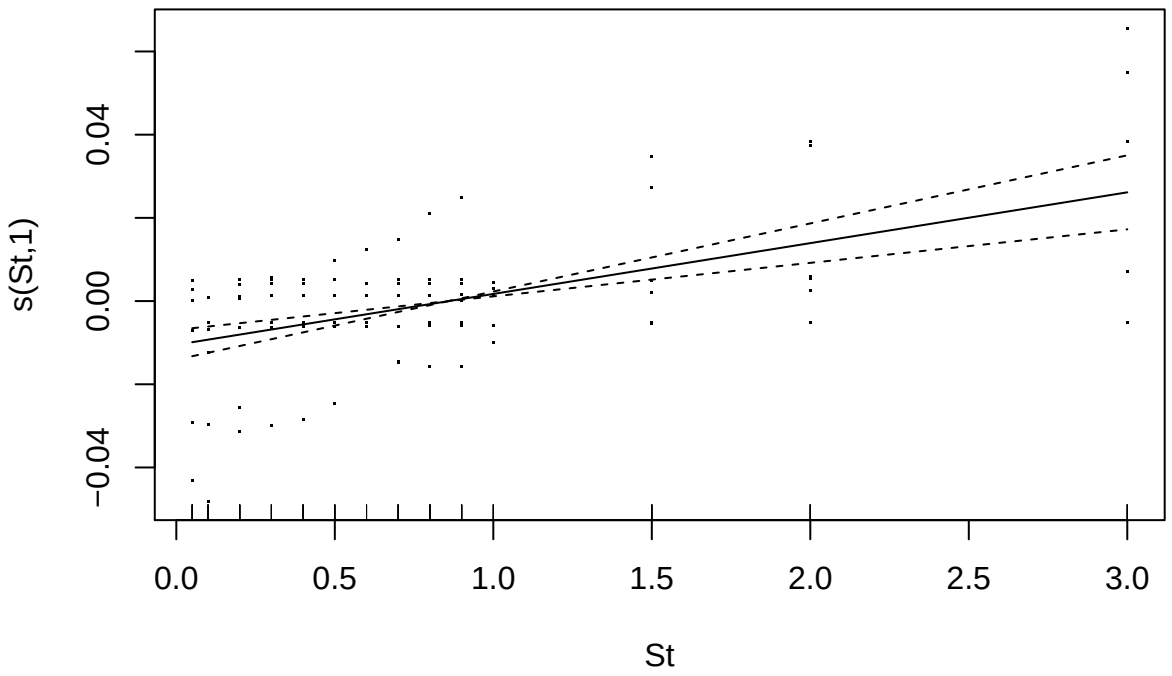

```
gam1 <- gam(R_moment_1 ~ s(St) + Re + Fr, data = train_data)


summary(gam1)
```

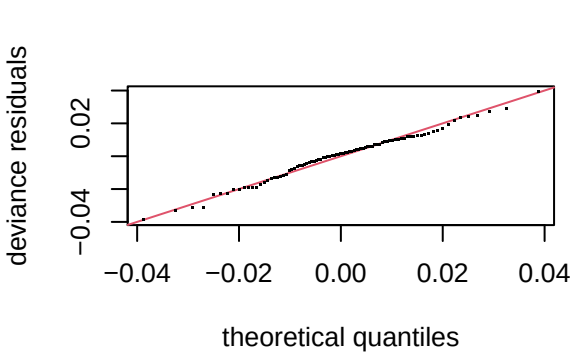
```
##
```

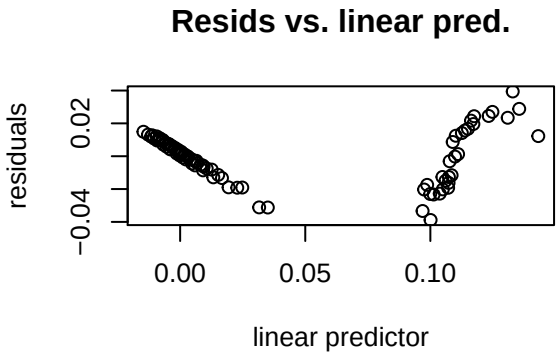
```
## Family: gaussian
## Link function: identity
##
## Formula:
## R_moment_1 ~ s(St) + Re + Fr
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.116986   0.003443  33.976  < 2e-16 ***
## Re224       -0.108091   0.003682 -29.353  < 2e-16 ***
## Re398       -0.111553   0.004632 -24.081  < 2e-16 ***
## Fr0.3       -0.007623   0.004245  -1.796  0.07618 .
## FrInf       -0.010210   0.003787  -2.696  0.00849 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##       edf Ref.df     F  p-value
## s(St)   1      1 34.53 6.48e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.925   Deviance explained = 92.9%
## GCV = 0.00025062  Scale est. = 0.00023373  n = 89
```

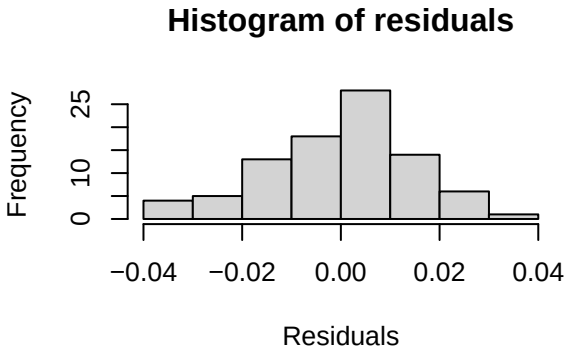
```
plot.gam(gam1, residuals = TRUE)
```

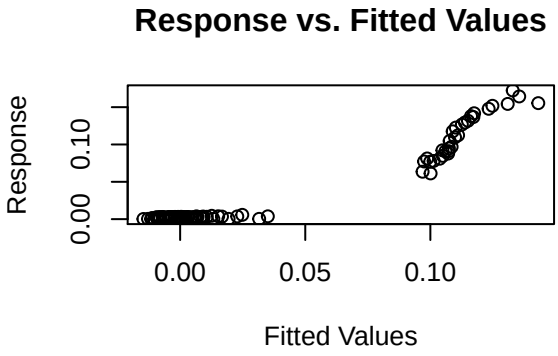



```
gam.check(gam1)
```



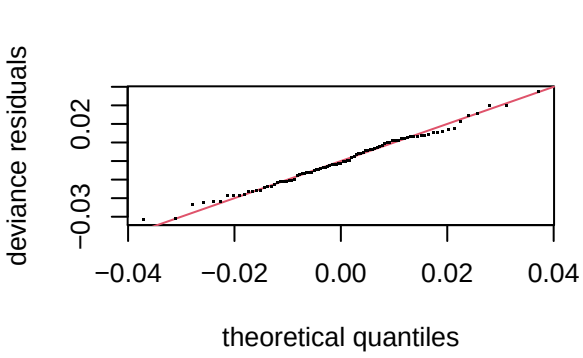


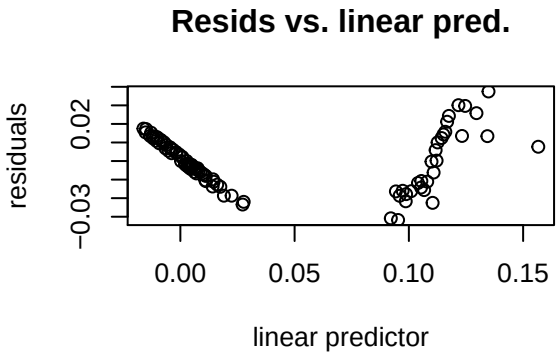


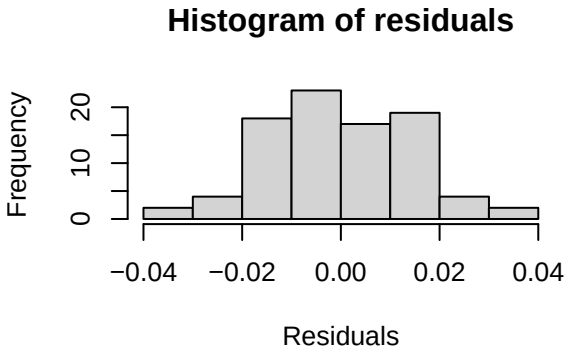


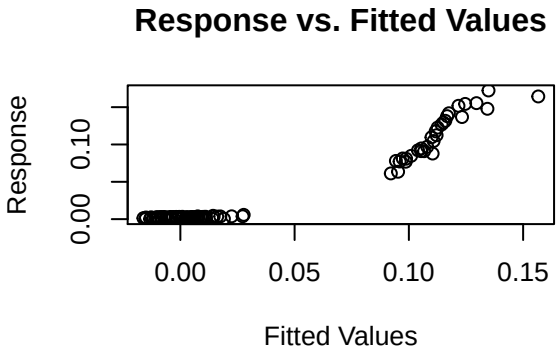

```
##
## Method: GCV   Optimizer: magic
## Smoothing parameter selection converged after 5 iterations.
## The RMS GCV score gradient at convergence was 7.915501e-08 .
## The Hessian was positive definite.
## Model rank =  14 / 14
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##       k' edf k-index p-value
## s(St)  9   1    1.08    0.78
```

```
gam2 <- gam(R_moment_1 ~ s(St, by = Fr) + Re + Fr, data = train_data)
gam.check(gam2)
```









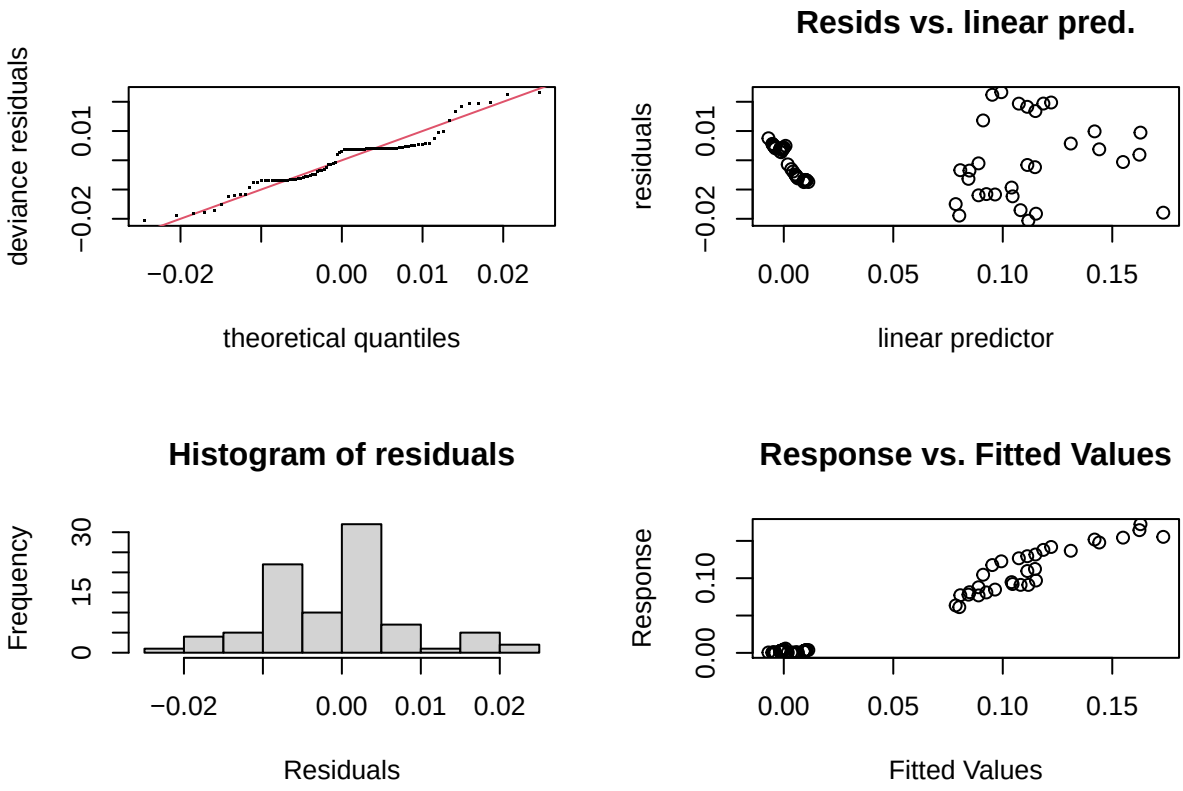

```
##
## Method: GCV   Optimizer: magic
## Smoothing parameter selection converged after 6 iterations.
## The RMS GCV score gradient at convergence was 3.3883e-08 .
## The Hessian was positive definite.
## Model rank =  32 / 32
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                 k'  edf k-index p-value
## s(St):Fr0.052 9.00 1.25    1.14    0.91
## s(St):Fr0.3   9.00 1.00    1.14    0.85
## s(St):FrInf   9.00 1.00    1.14    0.87
```

```
summary(gam2)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## R_moment_1 ~ s(St, by = Fr) + Re + Fr
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.116626   0.003303  35.310  < 2e-16 ***
## Re224       -0.107127   0.003559 -30.097  < 2e-16 ***
## Re398       -0.110509   0.004454 -24.811  < 2e-16 ***
## Fr0.3       -0.007625   0.004065  -1.876  0.06427 .
```

```
## FrInf       -0.010491   0.003627  -2.892  0.00492 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                 edf Ref.df      F  p-value
## s(St):Fr0.052 1.253  1.461  2.927 0.052203 .
## s(St):Fr0.3   1.000  1.000 26.863 1.39e-06 ***
## s(St):FrInf   1.000  1.000 14.649 0.000249 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.931   Deviance explained = 93.7%
## GCV = 0.00023613  Scale est. = 0.00021423  n = 89
```

```
gam3 <- gam(R_moment_1 ~ s(St, by = Re) + Re + Fr, data = train_data)
gam.check(gam3)
```




```
##
## Method: GCV   Optimizer: magic
## Smoothing parameter selection converged after 9 iterations.
## The RMS GCV score gradient at convergence was 3.672427e-08 .
## The Hessian was positive definite.
## Model rank =  32 / 32
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##               k'  edf k-index p-value
## s(St):Re90  9.00 2.19    1.01    0.49
```

```
## s(St):Re224 9.00 1.00    1.01    0.46
## s(St):Re398 9.00 1.00    1.01    0.50
```

```
summary(gam3)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## R_moment_1 ~ s(St, by = Re) + Re + Fr
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.117714   0.002181  53.977  < 2e-16 ***
## Re224       -0.107963   0.002330 -46.335  < 2e-16 ***
## Re398       -0.112463   0.002932 -38.359  < 2e-16 ***
## Fr0.3       -0.010869   0.002696  -4.031 0.000126 ***
## FrInf       -0.010412   0.002414  -4.313 4.57e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                edf Ref.df      F p-value
## s(St):Re90   2.185  2.655 80.049  <2e-16 ***
## s(St):Re224 1.000  1.000  0.104   0.748
## s(St):Re398 1.000  1.000  0.291   0.591
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =   0.97   Deviance explained = 97.3%
## GCV = 0.00010386  Scale est. = 9.314e-05  n = 89
```