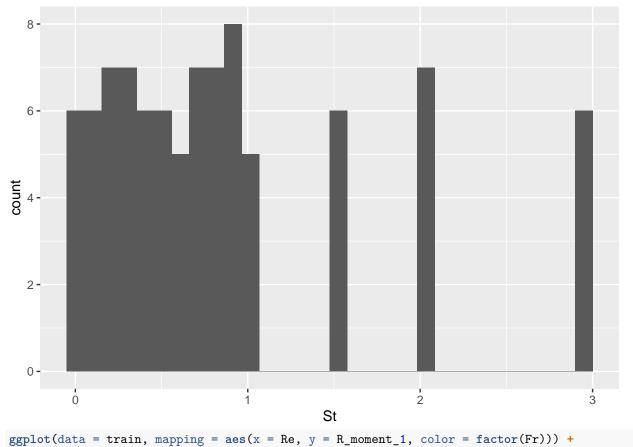
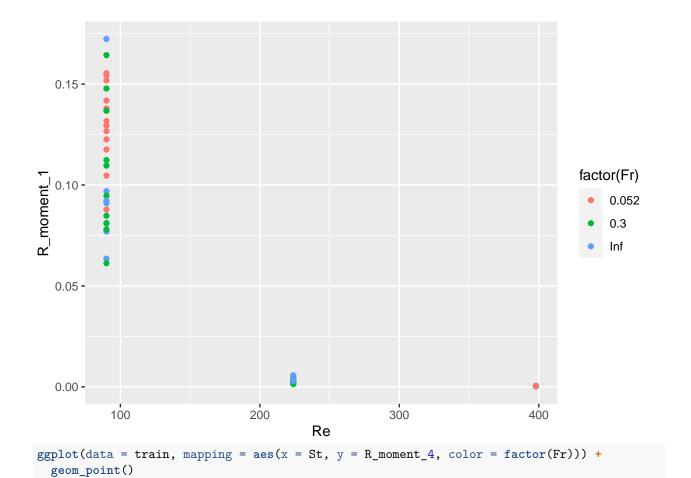
## sara-experimental

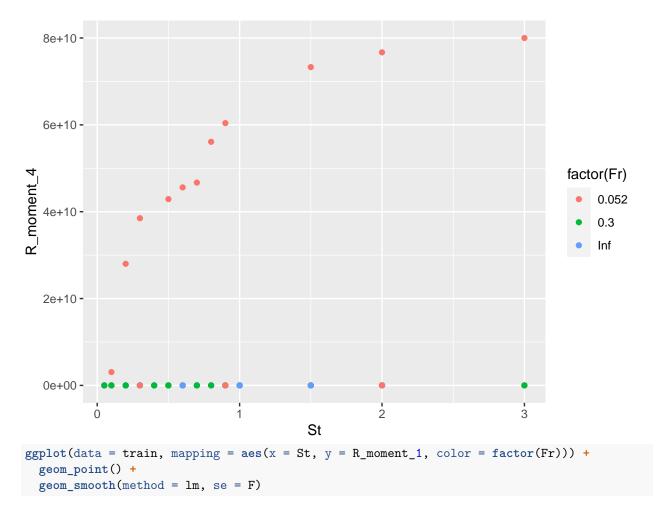
## Sara Shao

## 10/7/2021

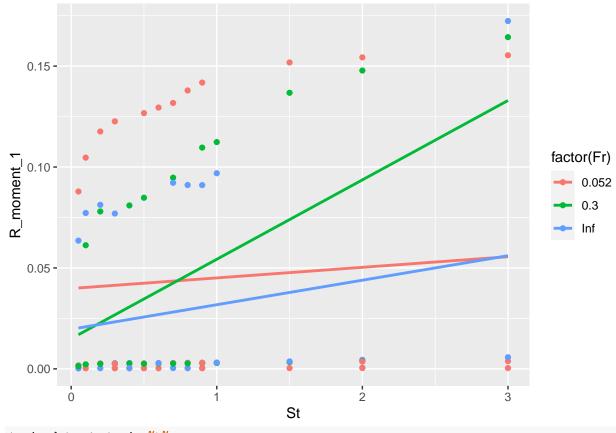
```
library(tidyverse)
## -- Attaching packages ------ tidyverse 1.3.0 --
## v tibble 3.0.6
                     v purrr
                               0.3.4
## v tidyr
            1.1.2
                      v dplyr
                               1.0.4
            1.4.0
## v readr
                    v forcats 0.5.1
## -- Conflicts ----- tidyverse_conflicts() --
## x lubridate::as.difftime() masks base::as.difftime()
## x lubridate::date() masks base::date()
## x dplyr::filter()
                           masks stats::filter()
## x readr::guess_encoding() masks rvest::guess_encoding()
## x lubridate::intersect() masks base::intersect()
## x dplyr::lag()
                         masks stats::lag()
## x purrr::pluck() masks rvest::pluck()
## x lubridate::setdiff() masks base::setdiff()
## x lubridate::union()
                            masks base::union()
train <- read.csv('data-train.csv')</pre>
head(train)
                Fr R_moment_1 R_moment_2 R_moment_3 R_moment_4
## 1 0.10 224 0.052 0.00215700 0.1303500 14.37400 1586.5000
## 2 3.00 224 0.052 0.00379030 0.4704200 69.94000 10404.0000
## 3 0.70 224
             Inf 0.00290540 0.0434990 0.82200
                                                     15.5510
             Inf 0.06352800 0.0906530
                                        0.46746
## 4 0.05 90
                                                      3.2696
## 5 0.70 398 Inf 0.00036945 0.0062242
                                        0.12649
                                                      2.5714
## 6 2.00 90 0.300 0.14780000 2.0068000
                                         36.24900
                                                    671.6700
ggplot(data = train, mapping = aes(x = St)) +
 geom_histogram()
```







##  $geom_smooth()$  using formula 'y ~ x'



```
train_data <- train %>%
   mutate(Fr = as.factor(Fr)) %>%
   mutate(Re = as.factor(Re))

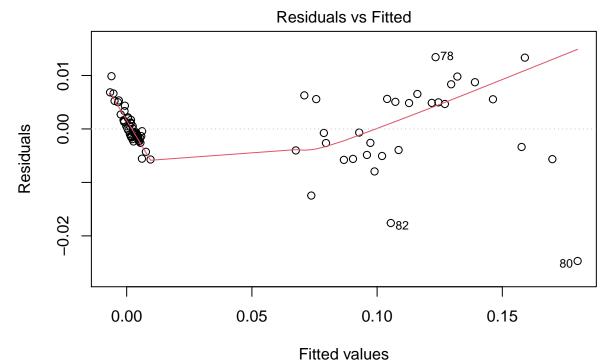
lm_R1 <- lm(R_moment_1 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

lm_R2 <- lm(R_moment_2 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

lm_R3 <- lm(R_moment_3 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

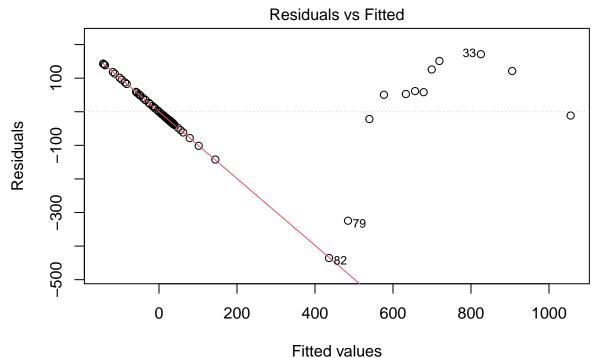
lm_R4 <- lm(R_moment_4 ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)

plot(lm_R1, 1)</pre>
```



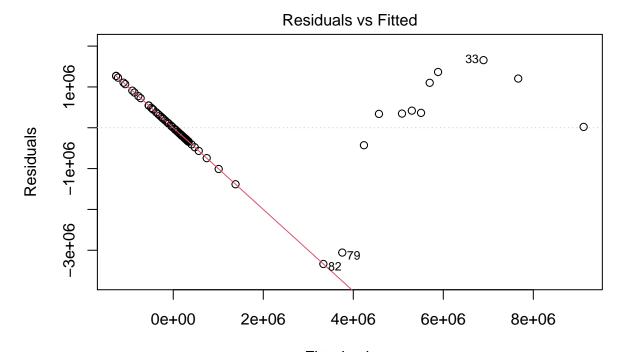
Im(R\_moment\_1  $\sim$  log(St) + Re + Fr + St \* Fr + Fr \* Re + St \* Re)

plot(lm\_R2, 1)



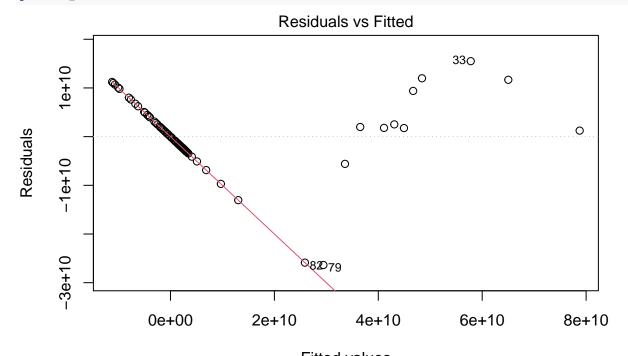
Im(R\_moment\_2  $\sim$  log(St) + Re + Fr + St \* Fr + Fr \* Re + St \* Re)

plot(lm\_R3, 1)



Fitted values  $Im(R\_moment\_3 \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$ 

plot(lm\_R4, 1)



Fitted values  $Im(R\_moment\_4 \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$ 

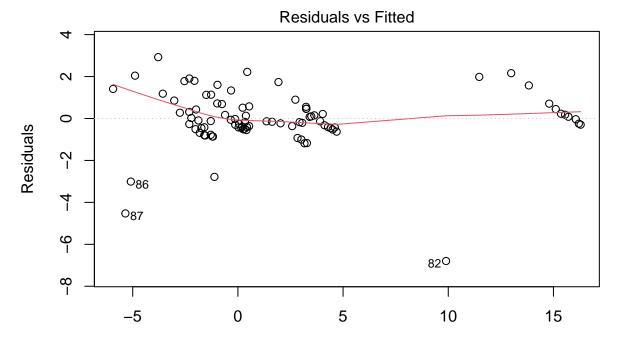
 $lm1 \leftarrow lm(log(R_moment_1) \sim log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data) \\ summary(lm1)$ 

##

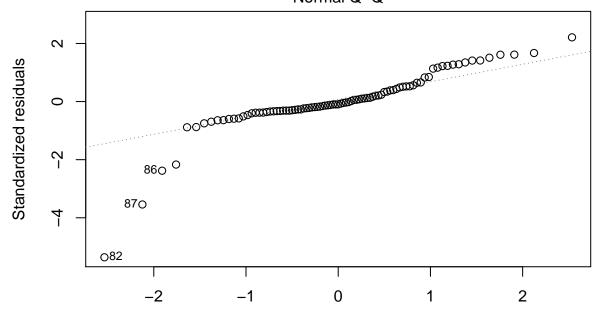
```
## Call:
## lm(formula = log(R_moment_1) ~ log(St) + Re + Fr + St * Fr +
      Fr * Re + St * Re, data = train data)
##
## Residuals:
##
                          Median
                                        3Q
        Min
                    10
                                                 Max
## -0.211809 -0.042926 -0.006391 0.038831 0.171243
## Coefficients: (1 not defined because of singularities)
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.95494
                           0.03716 -52.606 < 2e-16 ***
                                    10.003 1.89e-15 ***
## log(St)
               0.14567
                           0.01456
## Re224
               -3.84790
                           0.03653 -105.330 < 2e-16 ***
## Re398
              -5.98498
                           0.04408 -135.783 < 2e-16 ***
## Fr0.3
                                     -9.637 9.23e-15 ***
               -0.41718
                           0.04329
## FrInf
               -0.43145
                           0.03943
                                    -10.942 < 2e-16 ***
## St
              -0.00182
                                     -0.069
                           0.02631
                                               0.945
## Fr0.3:St
               0.14612
                           0.02916
                                      5.011 3.51e-06 ***
                0.13488
                                      5.522 4.60e-07 ***
## FrInf:St
                           0.02442
## Re224:Fr0.3 0.25492
                           0.04471
                                      5.701 2.22e-07 ***
## Re398:Fr0.3
                     NΑ
                                NΑ
                                         NA
                                                  NΑ
## Re224:FrInf 0.37513
                           0.04533
                                      8.275 3.58e-12 ***
## Re398:FrInf 0.48589
                                      9.808 4.39e-15 ***
                           0.04954
## Re224:St
               0.02473
                           0.02376
                                      1.041
                                               0.301
## Re398:St
              -0.01258
                           0.03065
                                     -0.411
                                               0.683
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.07645 on 75 degrees of freedom
## Multiple R-squared: 0.999, Adjusted R-squared: 0.9988
## F-statistic: 5797 on 13 and 75 DF, p-value: < 2.2e-16
lm2 <- lm(log(R_moment_2) ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)</pre>
summary(lm2)
##
## Call:
## lm(formula = log(R_moment_2) ~ log(St) + Re + Fr + St * Fr +
##
      Fr * Re + St * Re, data = train_data)
##
## Residuals:
      Min
                1Q Median
                                30
                                       Max
## -3.6442 -0.2697 -0.0561 0.3429 1.8016
## Coefficients: (1 not defined because of singularities)
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                7.55038
                            0.41301 18.282 < 2e-16 ***
                                     9.274 4.50e-14 ***
## log(St)
                 1.50086
                            0.16184
## Re224
                -7.37089
                            0.40601 -18.155 < 2e-16 ***
## Re398
                            0.48987 -22.316 < 2e-16 ***
               -10.93201
## Fr0.3
                            0.48112 -13.823 < 2e-16 ***
                -6.65031
## FrInf
                -6.49869
                            0.43823 -14.830
                                             < 2e-16 ***
## St
                            0.29241 -2.494
                -0.72933
                                              0.0148 *
## Fr0.3:St
                0.04758
                            0.32411
                                      0.147
                                              0.8837
## FrInf:St
               -0.01113
                            0.27145 -0.041
                                             0.9674
```

```
## Re224:Fr0.3 4.50066
                           0.49695
                                     9.057 1.16e-13 ***
                                        NΑ
## Re398:Fr0.3
                     NA
                                NΑ
                                                 NΑ
## Re224:FrInf
                4.51871
                           0.50382
                                     8.969 1.71e-13 ***
## Re398:FrInf
                6.79875
                           0.55058 12.348 < 2e-16 ***
## Re224:St
               -0.17530
                           0.26403
                                    -0.664
                                             0.5088
               -0.66556
## Re398:St
                           0.34062 - 1.954
                                             0.0544 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8497 on 75 degrees of freedom
## Multiple R-squared: 0.9553, Adjusted R-squared: 0.9476
## F-statistic: 123.4 on 13 and 75 DF, p-value: < 2.2e-16
lm3 <- lm(log(R_moment_3) ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)
summary(lm3)
##
## Call:
## lm(formula = log(R_moment_3) \sim log(St) + Re + Fr + St * Fr +
##
      Fr * Re + St * Re, data = train_data)
##
## Residuals:
      Min
               1Q Median
                               30
                                      Max
## -6.7949 -0.4431 -0.1224 0.5575 2.9257
## Coefficients: (1 not defined because of singularities)
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 17.03012
                          0.69563 24.481 < 2e-16 ***
## log(St)
                                    8.665 6.47e-13 ***
                2.36189
                           0.27259
## Re224
              -10.99422
                           0.68384 -16.077 < 2e-16 ***
## Re398
                           0.82509 -19.695 < 2e-16 ***
              -16.25017
## Fr0.3
              -12.76155
                           0.81035 -15.748 < 2e-16 ***
## FrInf
              -12.43166
                           0.73811 -16.842 < 2e-16 ***
## St
               -1.18717
                          0.49251 - 2.410
                                            0.0184 *
## Fr0.3:St
               -0.02117 0.54590 -0.039
                                            0.9692
               -0.13003 0.45721 -0.284
## FrInf:St
                                            0.7769
## Re224:Fr0.3 8.53024
                           0.83702 10.191 8.37e-16 ***
## Re398:Fr0.3
                     NA
                                NA
                                        NA
                                                 NA
## Re224:FrInf
                8.46417
                           0.84860
                                    9.974 2.14e-15 ***
## Re398:FrInf 12.91606
                           0.92735 13.928 < 2e-16 ***
## Re224:St
               -0.34559
                           0.44472 - 0.777
                                             0.4395
## Re398:St
               -1.12784
                           0.57371 -1.966
                                           0.0530 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.431 on 75 degrees of freedom
## Multiple R-squared: 0.9459, Adjusted R-squared: 0.9365
## F-statistic: 100.8 on 13 and 75 DF, p-value: < 2.2e-16
lm4 <- lm(log(R_moment_4) ~ log(St) + Re + Fr + St*Fr + Fr*Re + St*Re, data = train_data)</pre>
summary(lm4)
##
## Call:
## lm(formula = log(R_moment_4) ~ log(St) + Re + Fr + St * Fr +
```

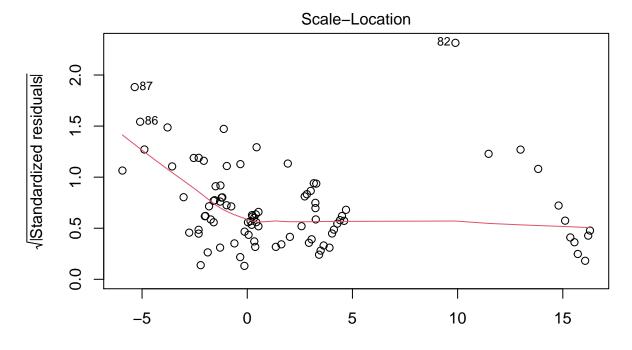
```
##
      Fr * Re + St * Re, data = train_data)
##
## Residuals:
##
      Min
               1Q Median
                               ЗQ
                                      Max
## -9.6675 -0.6183 -0.1392 0.7410 3.8875
##
## Coefficients: (1 not defined because of singularities)
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 26.4189
                           0.9472 27.891 < 2e-16 ***
                                   8.377 2.28e-12 ***
## log(St)
                3.1095
                           0.3712
## Re224
              -14.6485
                           0.9312 -15.731 < 2e-16 ***
## Re398
                           1.1235 -19.269 < 2e-16 ***
              -21.6490
## Fr0.3
                           1.1034 -17.018 < 2e-16 ***
              -18.7784
## FrInf
                           1.0051 -18.202 < 2e-16 ***
              -18.2947
               -1.5753
## St
                           0.6706 -2.349
                                           0.0215 *
## Fr0.3:St
               -0.1004
                           0.7433
                                   -0.135
                                            0.8930
## FrInf:St
               -0.2494
                           0.6226
                                   -0.401
                                            0.6899
## Re224:Fr0.3 12.4829
                           1.1398
                                   10.952
                                          < 2e-16 ***
## Re398:Fr0.3
                    NA
                               NA
                                       NA
                                                NA
## Re224:FrInf 12.3551
                           1.1555
                                   10.692
                                           < 2e-16 ***
## Re398:FrInf 18.9591
                           1.2628
                                   15.014
                                          < 2e-16 ***
## Re224:St
               -0.4931
                           0.6056
                                   -0.814
                                            0.4180
## Re398:St
                           0.7812 -1.960
               -1.5312
                                            0.0537 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.949 on 75 degrees of freedom
## Multiple R-squared: 0.9457, Adjusted R-squared: 0.9363
## F-statistic: 100.6 on 13 and 75 DF, p-value: < 2.2e-16
plot(lm3)
```



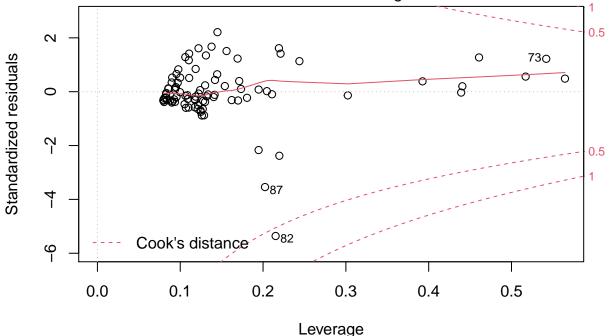
Fitted values  $Im(log(R\_moment\_3) \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re) \\ Normal Q-Q$ 



Theoretical Quantiles Im(log(R\_moment\_3) ~ log(St) + Re + Fr + St \* Fr + Fr \* Re + St \* Re)



Fitted values  $Im(log(R_moment_3) \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$ Residuals vs Leverage



 $Im(log(R_moment_3) \sim log(St) + Re + Fr + St * Fr + Fr * Re + St * Re)$ 

```
set.seed(123)
errors <- c()

for(i in c(1:nrow(train_data))){
   subset <- train_data[-c(i),]
   model <- lm(log(R_moment_1) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)</pre>
```

```
prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_1 - prediction)^2
  errors[i] <- error
mean(errors)
## [1] 2.720867e-05
set.seed(123)
errors <- c()
for(i in c(1:nrow(train data))){
  subset <- train_data[-c(i),]</pre>
  model <- lm(log(R_moment_2) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)
  prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_2 - prediction)^2</pre>
  errors[i] <- error</pre>
}
mean(errors)
## [1] 6312.875
set.seed(123)
errors <- c()
for(i in c(1:nrow(train_data))){
  subset <- train_data[-c(i),]</pre>
  model <- lm(log(R_moment_3) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)
  prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_3 - prediction)^2</pre>
  errors[i] <- error</pre>
mean(errors)
## [1] 777311609607
set.seed(123)
errors <- c()
for(i in c(1:nrow(train_data))){
  subset <- train_data[-c(i),]</pre>
  model <- lm(log(R_moment_4) ~ log(St) + Re + Fr + St*Fr + Re*Fr + St*Re, data = subset)</pre>
  prediction <- exp(predict(model, train_data[c(i),]))</pre>
  error <- (train_data[c(i),]$R_moment_4 - prediction)^2
  errors[i] <- error
mean(errors)
```

## [1] 7.403514e+19