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A Robust Nonlinear MPC Framework for Control of Underwater Vehicle Manipulator Systems under High-Level Tasks

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Abstract: Over the last years, the development and control of Autonomous Underwater Vehicles (AUV) with attached robotic manipulators, also called Underwater Vehicle Manipulator System (UVMS), has gained significant research attention. In such applications, feedback controllers which guarantee that the end-effector of the UVMS is fulfilling desired complex tasks should be designed in a way that state and input constraints are taken into consideration. Furthermore, due to their complicated structure, unmodeled dynamics as well as external disturbances may arise. Complex tasks can be conveniently given in the so-called Linear Temporal Logic (LTL). Motivated by this, we develop a combined abstraction and control synthesis framework in which, given the uncertain kinematics/dynamics of the UVMS, a workspace divided into Regions of Interest (RoI) and a desired LTL task, a sequence of feedback control laws that provably guarantee the LTL formula is provided. The feedback control law consists of two parts: an on-line controller which is the outcome of a Finite Horizon Optimal Control Problem (FHOCP); and a backstepping feedback law which is tuned off-line and guarantees that the real trajectory always remains in a bounded hyper-tube centered along the nominal trajectory of the end-effector. The proposed controller falls within the tube-based Nonlinear Model Predictive Control (NMPC) methodology and can handle the rich expressivity of LTL in both safety and reachability specifications. Numerical simulations conducted in MATLAB verify the validity of the proposed framework.

1 Introduction

Most of the autonomous underwater manipulation tasks, such as maintenance of ships, underwater weld inspection, surveying oil/gas searching, require the UVMS to fulfill a sequence of *high-level tasks* imposed by the user (see [1–9]). An example of a high-level task could be "Periodically inspect the ship while avoiding collision with it and with environment". The high-level tasks can be conveniently expressed in temporal-logic languages. Temporal-logic based task planning has gained a significant amount of recent attention, as it provides a fully automated correct-by-design controller synthesis approach for autonomous robots. Moreover, it provides formal high-level languages that can describe planning objectives which are more complex than the well-studied navigation or stabilization tasks [10, 11].

The qualitative specification language that has been used to express the high-level tasks is Linear Temporal Logic (LTL) [10]. Given the dynamics of an autonomous robot and a desired LTL formula, the controller synthesis procedure is given as follows [12]: first, the robot dynamics are abstracted into a discrete representation, the so-called Transition System (TS); second, a product between the TS and an automaton that accepts the runs that satisfy the given LTL formula is computed; and third, once an accepting run in the product is found, it maps into a sequence of feedback controllers of the robot dynamics.

In this paper we aim to employ the aforementioned procedure in order to design feedback control laws which guarantee that a UVMS satisfies a desired LTL specification. Regarding underwater applications, the most challenging part of the LTL control synthesis procedure is the abstraction part due to the fact that the continuous-time controller needs to take into account the following issues:

- the highly nonlinear and complicated dynamics of the UVMS;
- state and input constraints;
- external disturbances, uncertainties as well as unmodeled dynamics;
- the unstructured underwater environment which contains different types of obstacles.

At the same time, regarding practical applications, the UVMS needs to visit specific parts of the workspace, which are called Regions of Interest (RoI)), and execute desired tasks through its end-effector. An example of this procedure is the inspection of ship for damages by a UVMS, in which different parts of the ship are different RoI that the UVMS is required to visit and execute desired actions.

Motivated by the aforementioned discussion, the contribution of this paper is a novel class of feedback controllers that tackle the system and workspace constraints in an efficient way; at the same time we aim to provide an abstraction of robot dynamics into a TS in such a way that the rich LTL expressiveness in both reachability and safety specifications is exploited. In order to address the aforementioned problem in an efficient way, a Nonlinear Model Predictive Control framework is employed [13–16].

One of the main challenges in NMPC is the efficient handling of external disturbances/uncertainties. A promising robust NMPC strategy, originally proposed for discrete-time linear systems in [17] and studied for nonlinear systems in [18–20], is the so called tube-based approach. Then, starting by the kinematic and dynamic modeling of a general uncertain UVMS equipped with an n Degrees of Freedom (DoF) manipulator, given a partitioned workspace into RoI and a desired task written in LTL, a systematic control design methodology for tube-based NMPC which guarantees robust transition between RoI under safety constraints is developed; in particular, the controller consists of two terms: a nominal control input, which is computed on-line and is the outcome of a Finite Horizon Optimal Control Problem (FHOCP) that is repeatedly solved at every sampling time instant, for its nominal system dynamics; and an additive state feedback law which is designed by a backstepping nonlinear control procedure and guarantees that the real trajectory of the closed-loop system will belong to a hyper-tube centered along the nominal trajectory. The volume of the hyper-tube depends on the upper bound of the disturbances, the bounds of the Jacobian matrix as well as the Lipschitz constants of the UVMS dynamics. Secondly, we exploit the aforementioned control design in order to abstract the dynamics of the robot into a TS. Then, by performing the LTL control synthesis procedure, a sequence of control laws that guarantees the satisfaction of the formula is provided.

A preliminary conference version of this paper can be found in [21], in which a robust force/torque NMPC scheme for a UVMS in contact with a compliant surface of the environment is designed. That framework has not considered any high-level tasks and safety with potential obstacles of the environment.

The rest of the paper is structured as follows: Section 2 provides the notation that will be used as well as necessary background knowledge; in Section 3, the problem treated in this paper is formally defined; Section 4 contains the main results of the paper; Section 5 is devoted to a simulation example; and in Section 6, conclusions and future research directions are discussed.

2 Notation and Preliminaries

Some mathematical notations and preliminaries to be used throughout this paper are given below. Define by $\mathbb{N}$ and $\mathbb{R}$ the sets of positive integers and real numbers, respectively. Given a set $\mathcal{S}$, denote by $|\mathcal{S}|$ its cardinality, by $\mathcal{S}^n := \mathcal{S} \times \cdots \times \mathcal{S}$ its n-fold Cartesian product and by $2^{\mathcal{S}}$ the set of all its subsets. Given a vector $z \in \mathbb{R}^n$ define by:

$$\|z\|_2 := \sqrt{z^\top z}, \;\; \|z\|_P := \sqrt{z^\top P z},$$

its Euclidean and weighted norm, with $P \geq 0$. Given vectors z_1, $z_2 \in \mathbb{R}^3$, $S : \mathbb{R}^3 \to \mathfrak{so}(3)$ stands for the skew-symmetric matrix defined according to $S(z_1)z_2 = z_1 \times z_2$ where:

$$\mathfrak{so}(3) := \left\{ S \in \mathbb{R}^{3\times 3} : z^\top S(\cdot) z = 0, \forall z \in \mathbb{R}^3 \right\}.$$

The notation $\lambda_{\min}(P)$ stands for the minimum absolute value of the real part of the eigenvalues of $P \in \mathbb{R}^{n\times n}$; $0_{m\times n} \in \mathbb{R}^{m\times n}$ and $I_n \in \mathbb{R}^{n\times n}$ stand for the $m \times n$ matrix with all entries zeros and the identity matrix, respectively. The notation $\mathrm{diag}\{P_1, \ldots, P_n\}$ stands for the block diagonal matrix with the matrices $P_1, \ldots, P_n$ in the main diagonal;

$$\mathcal{B}(\chi, \mathfrak{r}) := \left\{ y \in \mathbb{R}^3 : \|y - \chi\|_2 \leq \mathfrak{r} \right\},$$

stands for a ball in $\mathbb{R}^3$ with center and radius $\chi \in \mathbb{R}^3$, $\mathfrak{r} > 0$, respectively. Given coordination frames Σ_i, Σ_j, denote by R_i^j the transformation from Σ_i to Σ_j. Given sets $\mathcal{S}_1$, $\mathcal{S}_2 \subseteq \mathbb{R}^n$, $\mathcal{S}_3 \subseteq \mathbb{R}^m$ and the matrix $P \in \mathbb{R}^{n\times m}$, the *Minkowski addition*, the *Pontryagin difference* and the *matrix-set multiplication* are respectively defined by:

$$\begin{aligned} \mathcal{S}_1 \oplus \mathcal{S}_2 &:= \{s_1 + s_2 : s_1 \in \mathcal{S}_1, s_2 \in \mathcal{S}_2\}, \\ \mathcal{S}_1 \ominus \mathcal{S}_2 &:= \{s_1 \in \mathcal{S}_1 : s_1 + s_2 \in \mathcal{S}_1, \forall s_2 \in \mathcal{S}_2\}, \\ P \circ \mathcal{S}_3 &:= \{p : p = Bs, s \in \mathcal{S}_3\}. \end{aligned}$$

Lemma 1. *[20] For any constant $\rho > 0$, vectors z_1, $z_2 \in \mathbb{R}^n$ and matrix $P \in \mathbb{R}^{n\times n}$, $P > 0$ it holds that:*

$$z_1 P z_2 \leq \frac{1}{4\rho} z_1^\top P z_1 + \rho z_2^\top P z_2.$$

Definition 1. *[**?**] A continuous function $\alpha : [0, a) \to \mathbb{R}_{\geq 0}$ belongs to* class $\mathcal{K}$ *if it is strictly increasing and $\alpha(0) = 0$. A continuous function $\boldsymbol{\beta} : [0, a) \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to* class $\mathcal{KL}$ *if:* 1) *for a fixed s, the mapping $\boldsymbol{\beta}(r, s)$ belongs to class $\mathcal{K}$ with respect to r;* 2) *for a fixed r, the mapping $\boldsymbol{\beta}(r, s)$ is strictly decreasing with respect to s; and it holds that $\lim\limits_{s\to\infty} \boldsymbol{\beta}(r, s) = 0$.*

Definition 2. *[18] Consider a dynamical system:*

$$\dot{\chi} = f(\chi, u, d), \; \chi \in \mathcal{X}, \; u \in \mathcal{U}, \; d \in \mathcal{D},$$

with initial condition $\chi(0) \in \mathcal{X}$. A set $\mathcal{X}' \subseteq \mathcal{X}$ is a Robust Control Invariant (RCI) set *for the system, if there exists a feedback control law $u := \kappa(\chi) \in \mathcal{U}$, such that for all $\chi(0) \in \mathcal{X}'$ and for all disturbances $d \in \mathcal{D}$ it holds that $\chi(t) \in \mathcal{X}'$ for all $t \geq 0$, along every solution $\chi(t)$.*

Definition 3. *A nonlinear system $\dot{x} = f(x, u, d), \; x \in \mathcal{X}, \; u \in \mathcal{U}, \; d \in \mathcal{D}$, with initial condition $x(0) \in \mathcal{X}$ is said to be* Input-to-State Stable (ISS) *with respect to $d \in \mathcal{D}$, if there exist functions $\boldsymbol{\beta} \in \mathcal{KL}$, $\gamma \in \mathcal{K}$ such that for any initial condition $x(0) \in \mathcal{X}$ and for any input $u(t) \in \mathcal{U}$, the solution $x(t)$ exists for all $t \in \mathbb{R}_{\geq 0}$ and satisfies:*

$$\|x(t)\|_2 \leq \boldsymbol{\beta}\big(\|x(0)\|_2, t\big) + \gamma\left(\sup_{0 \leq \mathfrak{s} \leq t} \|d(\mathfrak{s})\|_2 \right).$$

An *atomic proposition* is a statement that is either true or false.

Definition 4. *A Transition System (TS) is a tuple $(\mathcal{S}, \mathcal{S}_0, Act, \longrightarrow, \Pi, L)$ where:*

- *$\mathcal{S}$ is a finite set of states;*
- *$\mathcal{S}_0 \subseteq S$ is a set of initial states;*
- *Act is a set of actions (control inputs);*
- *$\longrightarrow \subseteq \mathcal{S} \times Act \times \mathcal{S}$ is a transition relation;*
- *Π is a finite set of atomic propositions;*
- *$L : \mathcal{S} \to 2^{\Pi}$ is a labeling function function that assigns to each state the atomic propositions that are true in that state.*

For a given state $s \in \mathcal{S}$, define by $\mathrm{Post}(s, \alpha) := \{s' \in \mathcal{S} : (s, \alpha, s') \in \longrightarrow\}$ the set of successors of the state s with action α. An infinite sequence of states of the form $r = s_0 s_1 s_2 \ldots$ is called a *run* of the TS if $s_0 \in \mathcal{S}_0$ and $s_{i+1} \in \mathrm{Post}(s_i, \cdot)$ for all $i \geq 0$. The *trace* of the run $r = s_0 s_1 s_2 \ldots$ is defined by:

$$\mathrm{Trace}(r) := L(s_0) L(s_1) L(s_2) \ldots,$$

In this paper we focus on task specifications φ given in Linear Temporal Logic (LTL). The syntax of LTL (see [10]) over a set of atomic propositions Π is defined by the grammar:

$$\varphi := \top \mid \varpi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \bigcirc \varphi \mid \varphi_1 \, \mathcal{U} \, \varphi_2,$$

where $\varpi \in \Pi$ and $\bigcirc$, $\mathcal{U}$ stand for the next and until operators, respectively; $\neg$ and $\wedge$ are the negation and conjunction operator respectively. The always ($\Box$) and eventually ($\Diamond$) operators can be defined by $\Box := \neg\Diamond\neg\varphi$ and $\Diamond := \top\mathcal{U}\varphi$, respectively. LTL formulas are interpreted on infinite words $r = w_0 w_1 w_2 \ldots$ where $w_i \in 2^{\Pi}$, $\forall i \geq 0$. The satisfaction relation is denoted by $\models$, i.e., for a word w and an LTL formula φ, we write $w \models \varphi$ iff w satisfies φ. The satisfaction relation is defined inductively as follows:

$$\begin{aligned} & w \models \top, \\ & w \models \varpi \Leftrightarrow \varpi \in w_0, \\ & w \models \neg\varphi \Leftrightarrow w \not\models \varphi, \\ & w \models \varphi_1 \wedge \varphi_2 \Leftrightarrow w \models \varphi_1 \text{ and } w \models \varphi_2, \\ & w \models \bigcirc\varphi \Leftrightarrow w_1 w_2 \ldots \models \varphi, \\ & w \models \varphi_1 \mathcal{U} \varphi_2 \Leftrightarrow \exists j \geq 0 \text{ s.t. } w_j w_{j+1} \ldots \models \varphi_2 \text{ and} \\ & \qquad\qquad\qquad \forall i \text{ s.t. } 0 \leq i < j, w_i w_{i+1} \ldots \models \varphi_1. \end{aligned}$$

Given a TS, its infinite run $r = r_0 r_1 r_2 \ldots$ and an LTL formula φ over Π, we say that r satisfies φ, denoted by $r \models \varphi$, if

$$\mathrm{Trace}(r) \models \varphi.$$

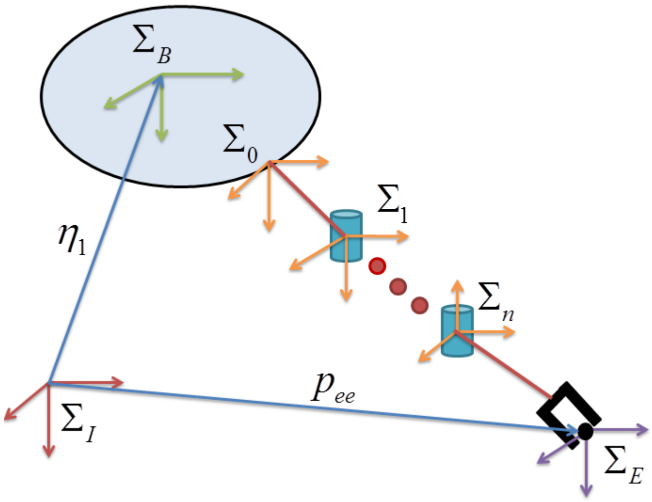

Fig. 1: An AUV Equipped with a n DoF manipulator

Definition 5. *A non-deterministic Büchi Automaton (NBA) is a tuple* $(Q, Q_0, 2^{\Pi}, \delta, F)$ *where*

- *Q is a finite set of states;*
- *$Q_0 \subseteq Q$ is a set of initial states;*
- *2^{Π} is the alphabet;*
- *$\delta : Q \times 2^{\Pi} \to 2^{Q}$ is a transition relation;*
- *$F \subseteq Q$ is a set of accepting states.*

A infinite run $q_0q_1q_2\ldots$ in the NBA is called *accepting* if there exist infinitely many $j \geq 0$ such that $q_j \in F$.

3 Problem Formulation

3.1 Kinematic Model

Consider a UVMS which is composed of an AUV and a n DoF manipulator mounted on the base of the vehicle. The AUV can be considered as a six DoF rigid body with position and orientation vector

$$\eta := [x, y, z \,|\, \phi, \theta, \psi]^{\top} \in \mathbb{R}^6,$$

where the components of the vectors have been named according to SNAME [22] as surge, sway, heave, roll, pitch and yaw respectively. The joint angular position state vector of the manipulator is defined by $q := [q_1, \ldots, q_n]^{\top} \in \mathbb{R}^n$. Define by $\dot{q} := [\dot{q}_1, \ldots, \dot{q}_n]^{\top} \in \mathbb{R}^n$ the corresponding joint velocities.

In order to describe the motion of the combined system, the earth-fixed inertial frame Σ_I, the body-fixed frame Σ_B and the end-effector fixed frame Σ_E are introduced (see Figure 1). Moreover, without loss of generality, the reference frame Σ_0 is chosen to be located at the manipulator's base, and the frames $\Sigma_1, \ldots, \Sigma_n$ are located to the 1-st, $\ldots$, n-th link of the manipulator, respectively, under the Denavit-Hartenberg convention [23]. The translational and rotational kinematic equations for the AUV system (see [1]) are given by:

$$\dot{\eta} = \begin{bmatrix}\dot{\eta}_1\\ \dot{\eta}_2\end{bmatrix} = \mathfrak{J}(\eta_2)\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix},$$

$$\mathfrak{J}(\eta_2) := \begin{bmatrix}\mathfrak{J}_1(\eta_2) & 0_{3\times3}\\ 0_{3\times3} & \mathfrak{J}_2(\eta_2)\end{bmatrix},$$

$$\mathfrak{J}_1(\eta_2) := \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - s_\psi c_\phi & s_\theta c_\phi c_\psi + s_\phi s_\psi\\ s_\psi c_\theta & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\theta s_\psi c_\phi - s_\phi c_\psi\\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix},$$

$$\mathfrak{J}_2(\eta_2) := \begin{bmatrix} 1 & \frac{s_\phi s_\theta}{c_\theta} & \frac{c_\phi s_\theta}{c_\theta}\\ 0 & c_\phi & -s_\phi\\ 0 & \frac{s_\phi}{c_\theta} & \frac{c_\phi}{c_\theta}\end{bmatrix},$$

where:

$$\eta_1 := [x, y, z]^{\top} \in \mathbb{R}^3,\ \ \eta_2 := [\phi, \theta, \psi]^{\top} \in \mathbb{R}^3,$$

denote the position vector and the orientation vector of the frame Σ_B relative to the frame Σ_I, respectively; $\nu_1, \nu_2 \in \mathbb{R}^3$ denote the linear and the angular velocity of the frame Σ_B with respect to Σ_I, respectively; $\mathfrak{J}(\eta_2) \in \mathbb{R}^{6\times6}$ stands for the Jacobian matrix transforming the velocities from Σ_B to Σ_I; $\mathfrak{J}_1(\eta_2), \mathfrak{J}_2(\eta_2) \in \mathbb{R}^{3\times3}$ are the corresponding parts of the Jacobian related to position and orientation, respectively; The notation s_ς and c_ς stand for the trigonometric functions $\sin(\varsigma)$ and $\cos(\varsigma)$ of an angle $\varsigma \in \mathbb{R}$, respectively.

Denote by $\mathfrak{q} := \left[\eta_1^{\top}, \eta_2^{\top}, q^{\top}\right]^{\top} \in \mathbb{R}^{6+n}$ the pose configuration vector of the UVMS. Let $\mathfrak{p}$, $\mathfrak{o} \in \mathbb{R}^3$ be the position and orientation vectors of the end-effector with reference to the frame Σ_I, respectively. The vectors $\mathfrak{p}$, $\mathfrak{o}$ depend on the pose $\mathfrak{q}$ and they can be obtained by the the following homogeneous transformation:

$$\begin{aligned}\mathfrak{T}(\mathfrak{q}) &:= \begin{bmatrix} R_E^I(\mathfrak{q}) & \mathfrak{p}(\mathfrak{q})\\ 0_{1\times3} & 1\end{bmatrix}\\ &= T_B^I T_0^B T_1^0 \cdots T_n^{n-1} T_E^n, \end{aligned} \tag{1}$$

where T_i^j is the homogeneous transformation matrix describing the position and orientation of frame Σ_i with reference to the frame Σ_j with $i, j \in \{1, \ldots, n, I, 0, B, E\}$. The end-effector linear velocity $\dot{\mathfrak{p}} \in \mathbb{R}^3$ and the time derivative or the Euler angles $\dot{\mathfrak{o}} \in \mathbb{R}^3$ are related to the body-fixed velocities ν_1, ν_2 and $\dot{q}$ with the following *kinematics model:*

$$\dot{\chi} = J(\mathfrak{q})\zeta, \tag{2}$$

where

$$\chi := \left[\mathfrak{p}^{\top}, \mathfrak{o}^{\top}\right]^{\top} \in \mathbb{R}^6,\ \ \zeta := \left[\nu_1^{\top}, \nu_2^{\top}, \dot{q}^{\top}\right]^{\top} \in \mathbb{R}^{6+n}.$$

In the latter, the stack vector ζ denotes the body-fixed system velocity vector. The Jacobian transformation matrices:

$$J \in \mathbb{R}^{6\times(6+n)},\ \ J_{\text{pos}} \in \mathbb{R}^{3\times(6+n)},\ \ J_{\text{or}} \in \mathbb{R}^{3\times(6+n)},$$

are respectively defined by:

$$J(\mathfrak{q}) := \begin{bmatrix} J_{\text{pos}}(\mathfrak{q})\\ J_{\text{or}}(\mathfrak{q})\end{bmatrix},$$

$$J_{\text{pos}}(\mathfrak{q}) := \left[\mathfrak{J}_1(\eta_2) \,\middle|\, -\mathfrak{J}_1(\eta_2)S(p_{ee}) \,\middle|\, R_0^I J_{e,1}\right],$$

$$J_{\text{or}}(\mathfrak{q}) := \left[0_{3\times3} \,\middle|\, \mathfrak{J}_2(\mathfrak{o})R_B^E \,\middle|\, \mathfrak{J}_2(\mathfrak{o})R_0^E J_{e,2}\right].$$

In the latter, the vector $p_{ee} \in \mathbb{R}^3$ is the local position of the end-effector with reference to the frame Σ_B; the matrices $J_{e,1}$, $J_{e,2} \in \mathbb{R}^{3\times n}$ represent the manipulator Jacobian matrices with respect to the frame Σ_0; and $S(\cdot)$ the skew-symmetric matrix as given in Section 2.

3.2 Dynamic Model

The uncertain nonlinear dynamics of the UVMS are given by [1]:

$$\dot{\zeta} = f(\chi, \zeta) + \mathfrak{u} + d(\mathfrak{q}, \zeta, t), \tag{3}$$

where:

$$f(\chi, \zeta) := -M(\mathfrak{q})^{-1}\Big\{C(\zeta, \mathfrak{q})\zeta + D(\zeta, \mathfrak{q})\zeta + g(\mathfrak{q})\Big\}, \tag{4}$$

where:

- $M(\mathfrak{q}) \in \mathbb{R}^{(6+n)\times(6+n)}$ is the inertia matrix for which it holds that:
$$z^\top M(\mathfrak{q}) z > 0, \;\; \forall z \in \mathbb{R}^{6+n}.$$
- $C(\zeta, \mathfrak{q}) \in \mathbb{R}^{(6+n)\times(6+n)}$ is the matrix of Coriolis and centripetal terms;
- $D(\zeta, \mathfrak{q}) \in \mathbb{R}^{(6+n)\times(6+n)}$ is the matrix of dissipative effects;
- $d(\mathfrak{q}, \zeta, t) \in \mathbb{R}^{6+n}$ is a vector that models the external disturbances, uncertainties and unmodeled dynamics of the system;
- $g(\mathfrak{q}) \in \mathbb{R}^{(6+n)}$ is the vector of gravity and buoyancy effects;
- $\mathfrak{u} \in \mathbb{R}^{6+n}$ denotes the vector of the propulsion forces and moments acting on the vehicle in the frame Σ_B as well as the joint torques.

In order for the kinematic model (2) to be well-posed, state constraints are imposed such that:

$$\mathcal{Q} := \Big\{\mathfrak{q} \in \mathbb{R}^{6+n} : \lambda_{\min}\left[\tfrac{J^+(\mathfrak{q}) + J^+(\mathfrak{q})^\top}{2}\right] \geq \underline{J}, \\ \|J(\mathfrak{q})\|_2 \leq \overline{J}, \; \|\dot{J}(\mathfrak{q})\|_2 \leq \widetilde{J}\Big\}, \tag{5}$$

where:

$$J^+(\mathfrak{q}) := J(\mathfrak{q})J(\mathfrak{q})^\top, \;\; \underline{J} > 0, \;\; \overline{J} > 0, \;\; \widetilde{J} > 0,$$

We further consider that the UVMS is in the presence of velocity and control input constraints captured by the sets:

$$\zeta \in \mathcal{Z} \subseteq \mathbb{R}^{6+n}, \;\; u \in \mathcal{U} \subseteq \mathbb{R}^{6+n},$$

respectively. According to (1), the constraints $\mathfrak{q} \in \mathcal{Q}$ impose also constraints on the vector $\chi \in \mathcal{X} \subseteq \mathbb{R}^6$, where the set $\mathcal{X}$ can be computed by the transformation $\mathfrak{T}(\mathfrak{q})$, as given in (1). Note also that the function f given in (4) is assumed to be continuously differentiable in the set $\mathcal{X} \times \mathcal{Z}$. Furthermore, we consider bounded disturbances $d \in \mathcal{D}$ where:

$$\mathcal{D} := \Big\{d \in \mathbb{R}^{6+n} : \|d(\mathfrak{q}, \zeta, t)\|_2 \leq \widetilde{d}, \;\forall (\mathfrak{q}, \zeta) \in \mathcal{Q} \times \mathcal{Z}, \;\forall t \in \mathbb{R}_{\geq 0}\Big\},$$

with $\widetilde{d} > 0$.

For the kinematics/dynamics (2),(3), define the corresponding *nominal kinematics / dynamics* by:

$$\dot{\overline{\chi}} = J(\overline{\mathfrak{q}})\overline{\zeta}, \tag{6a}$$

$$\dot{\overline{\zeta}} = f(\overline{\chi}, \overline{\zeta}) + \overline{u}, \tag{6b}$$

where $d(\cdot) \equiv 0$, $\overline{\mathfrak{q}} \in \mathcal{Q}$, $\overline{\chi} \in \mathcal{X}$, $\overline{\zeta} \in \mathcal{Z}$ and $\overline{u} \in \mathcal{U}$. Define the stack vector $\overline{\xi} := [\overline{\chi}, \overline{\zeta}]^\top \in \mathbb{R}^{12+n}$ and consider the *linear nominal system:*

$$\dot{\overline{\xi}} = A\overline{\xi} + B\overline{u}, \tag{7}$$

with:

$$A \in \mathbb{R}^{(12+n)\times(12+n)}, \;\; B \in \mathbb{R}^{(12+n)\times(6+n)},$$

which is the outcome of the Jacobian linearization of the nominal dynamics (6a),(6b) around the equilibrium point $\xi = 0$. Due to the dimension of the control input ($6 + n > 6$), the linear system (7) is stabilizable.

Assume that the UVMS is operating in a bounded workspace $\mathcal{W} \subseteq \mathbb{R}^3$ in which there exist $\mathfrak{m} \in \mathbb{N}$ RoI labeled by $\mathcal{M} := \{1, \dots, \mathfrak{m}\}$. Without loss of generality, the RoI are modeled by balls, i.e., $\mathcal{R}_m := \mathcal{B}(y_m, \gamma_m)$, $m \in \mathcal{M}$ where $y_m \in \mathcal{W}$ and $\gamma_m > 0$ stand for the center and the radius of the m-th RoI, respectively. Define also the union of all RoI by:

$$\mathcal{R} := \bigcup_{m \in \mathcal{M}} \mathcal{R}_m. \tag{8}$$

Due to the fact that we are interested in imposing safety specifications, at each time $t \geq 0$, the UVMS occupies a ball $\mathcal{B}(\eta_1(t), \beta)$ that covers its volume with radius $\beta > 0$. Assume that:

$$\min_{m \in \mathcal{M}}\{\gamma_m\} > \beta,$$

which means that the RoI have sufficiently larger volume than the volume of the UVMS.

3.3 Objectives

The control objective is to control the UVMS with kinematics/dynamics as in (2), (3) such that it navigates between RoI of the workspace so that it obeys a high-level specification over atomic tasks. Atomic tasks are captured through a given set of atomic propositions Π. Each RoI is labeled with atomic propositions that hold true there. Define the labeling function:

$$L : \mathcal{R} \to 2^\Pi, \tag{9}$$

with $\mathcal{R}$ as given in (8), which maps each RoI with atomic propositions that hold true there. Note that some of the RoI can be assigned with labels that indicate *unsafe regions*, i.e., the UVMS is required to avoid visiting them (*safety specifications*).

Definition 6. *Given a trajectory $\mathfrak{p}(t)$ of the UVMS's end-effector, its corresponding* behavior *that is given by an infinite sequence of the form:*

$$\mathfrak{b} := (\mathfrak{p}(\mathfrak{t}_0), \varpi_0)(\mathfrak{p}(\mathfrak{t}_1), \varpi_1)(\mathfrak{p}(\mathfrak{t}_2), \varpi_2) \dots,$$

is well-defined, if the following conditions hold:

1. $\mathfrak{t}_0 < \mathfrak{t}_1 < \mathfrak{t}_2 < \dots$*;*
2. $\varpi_j \in 2^\Pi$*, for every* $j \geq 0$*;*
3. $\mathcal{B}(\eta_1(\mathfrak{t}_j), \beta) \subsetneq \mathcal{R}_j$ *and* $\varpi_j \in L(\mathcal{R}_j)$*, with* $\mathcal{R}_j \in \mathcal{R}$*, for every* $j \geq 0$*;*
4. $\mathcal{B}(\eta_1(\mathfrak{t}_j), \beta) \subsetneq \mathcal{R}_j$ *and* $\mathcal{B}(\eta_1(\mathfrak{t}_{j+1}), \beta) \subsetneq \mathcal{R}_{j+1}$ *with* $\mathcal{R}_j \neq \mathcal{R}_{j+1}$ *for every* $j \geq 0$*.*

Definition 7. *A trajectory of the end-effector of the UVMS $\mathfrak{p}(t)$ satisfies an LTL formula φ over a set of atomic propositions Π formally, written as:*

$$\mathfrak{p}(t) \models \varphi, \;\; \forall t \geq 0, \tag{10}$$

if and only if there exists a well-defined behavior $\mathfrak{b} = (\mathfrak{p}(\mathfrak{t}_0), \varpi_0)(\mathfrak{p}(\mathfrak{t}_1), \varpi_1)(\mathfrak{p}(\mathfrak{t}_2), \varpi_2) \dots$, according to Definition 6, for which it holds that:

$$\varpi_0 \varpi_1 \varpi_2 \dots \models \varphi,$$

according to LTL semantics given in Section 2.

3.4 Problem Statement

The problem treated in this paper can be now formalized as follows:

Problem 1. *Consider a UVMS composed of an AUV and an attached manipulator with n DoF, which is operating in a bounded workspace $\mathcal{W} \subseteq \mathbb{R}^3$, governed by kinematics and dynamics given in* (2) *and* (3)*, respectively. The workspace $\mathcal{W}$ contains the RoI $\mathcal{R}_m$, modeled by the balls $\mathcal{B}(y_m, \gamma_m)$, $m \in \mathcal{M}$. Assume a high-level task φ expressed in LTL over the set of atomic propositions Π and labeling function L as in* (9)*. The system is in the presence of state and input constraints as well as bounded disturbances which are respectively given by:*

$$\mathfrak{q} \in \mathcal{Q},\ \chi \in \mathcal{X},\ \zeta \in \mathcal{Z},\ \mathfrak{u} \in \mathcal{U},\ d \in \mathcal{D}. \tag{11}$$

Then, design a feedback control law $\mathfrak{u} := \kappa(\chi, \zeta)$ *such that the end-effector trajectory in the workspace $\mathfrak{p}(t)$ fulfills the LTL specification φ, i.e.,*

$$\mathfrak{p}(t) \models \varphi,\ \ \forall t \geq 0,$$

according to (10) *of Definition* 7*. Moreover, the UVMS is required to remain in the workspace for all times.*

4 Main Results

In this section, we propose a novel feedback control law that solves Problem 1 in a systematic way as follows:

1. Due to the fact that it is required to design a feedback control law that guarantees the transition of the UVMS between RoI under state and input constraints given by (11), as well as safety specifications, we utilize a Nonlinear Model Predictive Control (NMPC) framework [13–15]. Furthermore, since the UVMS is under the presence of disturbances/uncertainties $d \in \mathcal{D}$, we provide a novel robust analysis, the so-called tube-based NMPC approach. In particular, first, the error states and the corresponding transformed constraints sets are defined in Section 4.1. Then, the proposed feedback control law consists of two parts: an on-line control law which is the outcome of a solution to a Finite Horizon Optimal Control Problem (FHOCP) for the nominal system dynamics (see Section 4.3); and a state feedback law which is designed off-line and guarantees that the real system trajectories always lie within a hyper-tube centered along the nominal trajectories (see Section 4.2).
2. The dynamics (2),(3) are abstracted into a TS, exploiting the fact that the runs of the TS project into well-defined behaviors according to Definition 6 (Section 4.4).
3. By invoking ideas from correct by construction formal methods based control synthesis [10, 12], a procedure that gives a sequence of control laws that serve as solution to Problem 1 is provided (Section 4.5).

4.1 Errors and Constraints

Consider that the UVMS with kinematics/dynamics as in (2),(3) is occupying a RoI $\mathcal{R}_{\mathrm{s}} \in \mathcal{R}$. The feedback control law needs to guarantee that the UVMS navigates towards a desired RoI $\mathcal{R}_{\mathrm{d}} \in \mathcal{R}$, $\mathcal{R}_{\mathrm{s}} \neq \mathcal{R}_{\mathrm{d}}$ without intersecting with any other RoI of the workspace, due to the fact that safety specifications are required. Define the vector $\chi_{\mathrm{d}} := [p_{\mathrm{d}}^\top, 0, 0, 0] \in \mathbb{R}^6$, where $p_d \in \mathbb{R}^3$ stands for the center of the desired RoI $\mathcal{R}_{\mathrm{d}}$. Define the error state $e := \chi - \chi_{\mathrm{d}} \in \mathbb{R}^6$. Then, the *uncertain error kinematics/dynamics* are given by:

$$\dot{e} = J(\mathfrak{q})\zeta, \tag{12a}$$

$$\dot{\zeta} = f(e + \chi_{\mathrm{d}}, \zeta) + \mathfrak{u} + d(\mathfrak{q}, \zeta, t), \tag{12b}$$

and the corresponding *nominal error kinematics/dynamics* by:

$$\dot{\overline{e}} = J(\overline{\mathfrak{q}})\overline{\zeta}, \tag{13a}$$

$$\dot{\overline{\zeta}} = f(\overline{e} + \chi_{\mathrm{d}}, \overline{\zeta}) + \overline{u}, \tag{13b}$$

By recalling that $\mathcal{B}(\mathfrak{p}(t), \beta)$ stands for the ball covering the UVMS at time t, define the set that captures the *state-transition constraints* by:

$$\widetilde{X} := \{\chi \in \mathcal{X} : \mathcal{B}(\mathfrak{p}, \beta) \subsetneq \mathcal{W},\ \ \mathcal{B}(\mathfrak{p}, \beta) \cap \{\mathcal{R}\backslash\{\mathcal{R}_{\mathrm{s}}, \mathcal{R}_{\mathrm{d}}\}\} = \emptyset\}.$$

The two constraints refer to the fact that the UVMS needs to remain in the workspace for all times and the fact that it should not intersect with any other RoI except $\mathcal{R}_{\mathrm{s}}$ and $\mathcal{R}_{\mathrm{d}}$. In order to translate the constraints for the state $\chi \in \widetilde{\mathcal{X}}$ to constraints that are dictated regarding the error e, the constraints set

$$\mathcal{E} := \widetilde{\mathcal{X}} \oplus (-\chi_{\mathrm{d}}),$$

is introduced, with $\oplus$ as given in Section 2.

4.2 Feedback Control Design

Consider the feedback law:

$$\mathfrak{u} := \overline{u}(\overline{e}, \overline{\zeta}) + \kappa(e, \zeta, \overline{e}, \overline{\zeta}), \tag{14}$$

which consists of a nominal control law $\overline{u}(\overline{e}, \overline{\zeta}) \in \mathcal{U}$ and a state feedback law $\kappa(\cdot)$. The control action $\overline{u}(\overline{e}, \overline{\zeta})$ will be the outcome of a FHOCP for the nominal kinematics/dynamics (13a),(13b) which is solved on-line at each sampling time. The state feedback law $\kappa(\cdot)$ is used to guarantee that the real trajectories $e(t)$, $\zeta(t)$, which are the solution to (12a),(12b), always remain within a bounded hyper-tube centered along the nominal trajectories $\overline{e}(t)$, $\overline{\zeta}(t)$ which are the solution to (13a),(13b). Define by:

$$\mathfrak{e} := e - \overline{e} \in \mathbb{R}^6,$$

$$\mathfrak{z} := \zeta - \overline{\zeta} \in \mathbb{R}^{6+n},$$

the deviation between the real states of the uncertain system (12a),(12b) and the states of the nominal system (13a),(13b), respectively, with $\mathfrak{e}(0) = \mathfrak{z}(0) = 0$. It will be proved hereafter that the trajectories $\mathfrak{e}(t)$, $\mathfrak{z}(t)$ remain invariant in certain compact sets. The dynamics of the states $\mathfrak{e}$, $\mathfrak{z}$ are written as:

$$\dot{\mathfrak{e}} = \mathfrak{b}(\chi, \overline{\chi}, \zeta) + J(\overline{\mathfrak{q}})\mathfrak{z}, \tag{15a}$$

$$\dot{\mathfrak{z}} = \mathfrak{l}(e, \overline{e}, \zeta, \overline{\zeta}) + (\mathfrak{u} - \overline{u}) + d(\mathfrak{q}, \zeta, t), \tag{15b}$$

where the functions $\mathfrak{b}$, $\mathfrak{l}$ are defined by:

$$\mathfrak{b}(\chi, \overline{\chi}, \zeta) := \mathfrak{c}(\chi, \zeta) - \mathfrak{c}(\overline{\chi}, \zeta),$$

$$\mathfrak{l}(e, \overline{e}, \zeta, \overline{\zeta}) := f(e + \chi_{\mathrm{d}}, \zeta) - f(\overline{e} + \chi_{\mathrm{d}}, \overline{\zeta}),$$

with

$$\mathfrak{c}(\chi, \zeta) := J(\mathfrak{q})\zeta.$$

Since the aforementioned functions are continuously differentiable, the following hold:

$$\begin{aligned}
\|\mathfrak{b}(\chi, \overline{\chi}, \zeta)\|_2 &= \|\mathfrak{c}(\chi, \zeta) - \mathfrak{c}(\overline{\chi}, \zeta)\|_2 \\
&\leq L_{\mathfrak{c}}\|\chi - \overline{\chi}\|_2 \\
&= L_{\mathfrak{c}}\|\mathfrak{e}\|_2, \\
\|\mathfrak{l}(e, \overline{e}, \zeta, \overline{\zeta})\|_2 &\leq \|f(e + \chi_{\mathrm{d}}, \zeta) - f(\overline{e} + \chi_{\mathrm{d}}, \zeta)\|_2 \\
&\quad + \|f(\overline{e} + \chi_{\mathrm{d}}, \zeta) - f(\overline{e} + \chi_{\mathrm{d}}, \overline{\zeta})\|_2 \\
&\leq L_1\|e - \overline{e}\|_2 + L_2\|\zeta - \overline{\zeta}\|_2 \\
&\leq L\left(\|\mathfrak{e}\|_2 + \|\mathfrak{z}\|_2\right).
\end{aligned}$$

The constant $L_{\mathfrak{c}}$ stands for the Lipschitz constant of function $\mathfrak{c}$ with respect to the variable χ; L_1, L_2 stand for the Lipschitz constants of function h with respect to the variables χ and ζ, respectively, and

$$L := \max\{L_1, L_2\}.$$

Lemma 2. *The state feedback law designed by:*

$$\kappa(e,\overline{e},\zeta,\overline{\zeta}) \coloneqq -k(e-\overline{e}) - k\sigma J(\overline{q})^\top(\zeta-\overline{\zeta}), \tag{16}$$

where k, $\sigma > 0$ are chosen such that the following hold:

$$\underline{\sigma} > 0,\ \ \sigma \coloneqq \frac{L_\mathfrak{c}+\underline{\sigma}}{\underline{J}},\ \ \rho > \tfrac{\Lambda_1}{4\underline{\sigma}},\ \ k > \rho\Lambda_1 + \Lambda_2, \tag{17a}$$

$$\Lambda_1 \coloneqq \left[L + \overline{J} + \sigma\left(L_\mathfrak{c} + \widetilde{J}\right)\right], \Lambda_2 \coloneqq \left(L + \sigma\overline{J}^2\right), \tag{17b}$$

renders the sets:

$$\Omega_1 \coloneqq \left\{\mathfrak{e}\in\mathbb{R}^6 : \|\mathfrak{e}\|_2 \le \tfrac{\widetilde{d}}{\min\{\alpha_1,\alpha_2\}}\right\}, \tag{18a}$$

$$\Omega_2 \coloneqq \left\{\mathfrak{z}\in\mathbb{R}^{6+n} : \|\mathfrak{z}\|_2 \le \tfrac{2\widetilde{d}}{\overline{J}\min\{\alpha_1,\alpha_2\}}\right\}, \tag{18b}$$

RCI sets for the error dynamics (15a), (15b), *according to Definition* 2, *where the constants α_1, $\alpha_2 > 0$ are defined by:*

$$\alpha_1 \coloneqq \underline{\sigma} - \tfrac{\Lambda_1}{4\rho},\ \alpha_2 \coloneqq k - \rho\Lambda_1 - \Lambda_2. \tag{19}$$

Proof: A backstepping control methodology will be used [24]. The state $\mathfrak{z}$ in (15a) can be seen as virtual input to be designed such that the Lyapunov function

$$\mathfrak{L}_1(\mathfrak{e}) \coloneqq \frac{1}{2}\|\mathfrak{e}\|_2^2,$$

for the system (15a) is always decreasing. The time derivative of $\mathfrak{L}_1$ along the trajectories of system (15a) is given by:

$$\dot{\mathfrak{L}}(\mathfrak{e}) = \mathfrak{e}^\top J(\overline{\mathfrak{q}})\mathfrak{z} + \mathfrak{e}^\top \mathfrak{b}(\cdot) \le \mathfrak{e}^\top J(\overline{\mathfrak{q}})\mathfrak{z} + L_\mathfrak{c}\|\mathfrak{e}\|_2^2. \tag{20}$$

Design the virtual control input as

$$\mathfrak{z} \equiv -\sigma J(\overline{\mathfrak{q}})^\top\mathfrak{e},$$

with $\underline{J}$, σ as given in (5), (17a), respectively. Then, by employing (5), (20) becomes:

$$\begin{aligned}\dot{\mathfrak{L}}(\mathfrak{e}) &\le -\sigma\mathfrak{e}^\top J^+(\overline{\mathfrak{q}})\mathfrak{e} + L_\mathfrak{c}\|\mathfrak{e}\|_2^2\\ &\le -\sigma\lambda_{\min}\left[\tfrac{J^+(\overline{\mathfrak{q}})+J^+(\overline{\mathfrak{q}})^\top}{2}\right]\|\mathfrak{e}\|_2^2 + L_\mathfrak{c}\|\mathfrak{e}\|_2^2\\ &\le -\sigma\underline{J}\|\mathfrak{e}\|_2^2 + L_\mathfrak{c}\|\mathfrak{e}\|_2^2\\ &= -\underline{\sigma}\|\mathfrak{e}\|_2^2.\end{aligned} \tag{21}$$

Define the backstepping auxiliary error state

$$\mathfrak{r} \coloneqq \mathfrak{z} + \sigma J(\overline{\mathfrak{q}})^\top\mathfrak{e} \in \mathbb{R}^{6+n},$$

and the the stack vector

$$\mathfrak{y} \coloneqq [\mathfrak{e}^\top, \mathfrak{r}^\top]^\top \in \mathbb{R}^{12+n}.$$

Consider the Lyapunov function

$$\mathfrak{L}(\mathfrak{y}) = \tfrac{1}{2}\|\mathfrak{y}\|^2.$$

Its time derivative along the trajectories of the system (15a),(15b) is given by:

$$\begin{aligned}\dot{\mathfrak{L}}(\mathfrak{y}) &= \mathfrak{e}^\top\dot{\mathfrak{e}} + \mathfrak{r}^\top\left[\dot{\mathfrak{z}} + \sigma J(\overline{\mathfrak{q}})^\top\dot{\mathfrak{e}} + \sigma\dot{J}(\overline{\mathfrak{q}})^\top\mathfrak{e}\right]\\ &= [\mathfrak{e} + \sigma J(\overline{\mathfrak{q}})\mathfrak{r}]^\top\dot{\mathfrak{e}} + \mathfrak{r}^\top\dot{\mathfrak{z}} + \sigma\mathfrak{r}^\top\dot{J}(\overline{\mathfrak{q}})^\top\mathfrak{e}\\ &= -\sigma\mathfrak{e}^\top J^+(\overline{\mathfrak{q}})\mathfrak{e} + \mathfrak{e}^\top\mathfrak{b}(\cdot) + \sigma\mathfrak{r}^\top J(\overline{\mathfrak{q}})^\top\mathfrak{b}(\cdot) + \mathfrak{e}^\top J(\overline{\mathfrak{q}})\mathfrak{r}\\ &+ \sigma\mathfrak{r}^\top J^+(\overline{\mathfrak{q}})\mathfrak{r} + \sigma\mathfrak{r}^\top\dot{J}(\overline{\mathfrak{q}})^\top\mathfrak{e} + \mathfrak{r}^\top\mathfrak{l}(\cdot) + \mathfrak{r}^\top(\mathfrak{u}-\overline{u}) + \mathfrak{r}^\top d(\cdot).\end{aligned} \tag{22}$$

By invoking (21) as well as the following:

$$\begin{aligned}\sigma\mathfrak{r}^\top J(\overline{\mathfrak{q}})^\top\mathfrak{b}(\cdot) &\le \sigma\|\mathfrak{r}\|_2\|J(\overline{\mathfrak{q}})\|_2\|\mathfrak{b}(\cdot)\|_2 \le \sigma L_\mathfrak{c}\overline{J}\|\mathfrak{e}\|_2\|\mathfrak{r}\|_2,\\ \mathfrak{e}^\top J(\overline{\mathfrak{q}})\mathfrak{r} &\le \|\mathfrak{e}\|_2\|J(\overline{\mathfrak{q}})\|_2\|\mathfrak{r}\|_2 \le \overline{J}\|\mathfrak{e}\|_2\|\mathfrak{r}\|_2,\\ \sigma\mathfrak{r}^\top J^+(\overline{\mathfrak{q}})\mathfrak{r} &\le \sigma\|\mathfrak{r}\|_2^2\|J^+(\overline{\mathfrak{q}})\|_2 \le \sigma\|\mathfrak{r}\|_2^2\|J(\overline{\mathfrak{q}})\|_2\left\|J^\top(\overline{\mathfrak{q}})\right\|_2\\ &\le \sigma\overline{J}^2\|\mathfrak{r}\|_2^2,\\ \sigma\mathfrak{r}^\top\dot{J}(\overline{\mathfrak{q}})^\top\mathfrak{e} &\le \sigma\|\mathfrak{e}\|_2\|\dot{J}(\overline{\mathfrak{q}})\|_2\|\mathfrak{r}\|_2 \le \sigma\widetilde{J}\|\mathfrak{e}\|_2\|\mathfrak{r}\|_2\\ \mathfrak{r}^\top\mathfrak{l}(\cdot) &\le L\|\mathfrak{e}\|_2\|\mathfrak{r}\|_2 + L\|\mathfrak{r}\|_2^2,\\ \mathfrak{r}^\top d(\cdot) &\le \|\mathfrak{r}\|_2\|d(\cdot)\|_2 \le \|\mathfrak{y}\|_2\widetilde{d},\end{aligned}$$

(22) becomes:

$$\begin{aligned}\dot{\mathfrak{L}}(\mathfrak{y}) \le -\underline{\sigma}\|\mathfrak{e}\|_2^2 + \Lambda_1\|\mathfrak{e}\|_2\|\mathfrak{r}\|_2&\\ + \Lambda_2\|\mathfrak{r}\|_2^2 + \mathfrak{r}^\top(\mathfrak{u}-\overline{u}) + \|\mathfrak{y}\|_2\widetilde{d}.&\end{aligned} \tag{23}$$

with Λ_1, Λ_2 given in (17b). By using Lemma 1 for $n = P = 1$, we get:

$$\|\mathfrak{e}\|_2\|\mathfrak{r}\|_2 \le \tfrac{1}{4\rho}\|\mathfrak{e}\|_2^2 + \rho\|\mathfrak{r}\|_2^2,$$

with ρ designed so that (17a) holds. Combining the latter with (23) it yields:

$$\begin{aligned}\dot{\mathfrak{L}}(\mathfrak{y}) \le -\left(\underline{\sigma} - \tfrac{\Lambda_1}{4\rho}\right)\|\mathfrak{e}\|_2^2 + \left(\rho\Lambda_1 + \Lambda_2\right)\|\mathfrak{r}\|_2^2&\\ + \mathfrak{r}^\top(\mathfrak{u}-\overline{u}) + \|\mathfrak{y}\|_2\widetilde{d}.&\end{aligned}$$

By designing $\mathfrak{u} - \overline{u} = -k\mathfrak{r} = -k\mathfrak{e} - k\sigma J(\overline{\mathfrak{q}})^\top\mathfrak{z}$, which is compatible with (14) and the same as in (16), we have:

$$\begin{aligned}\dot{\mathfrak{L}}(\mathfrak{y}) &\le -\left(\underline{\sigma} - \tfrac{\Lambda_1}{4\rho}\right)\|\mathfrak{e}\|_2^2 - \left(k - \rho\Lambda_1 - \Lambda_2\right)\|\mathfrak{r}\|_2^2 + \|\mathfrak{y}\|_2\widetilde{d}\\ &\le -\min\{\alpha_1,\alpha_2\}\|\mathfrak{y}\|_2^2 + \|\mathfrak{y}\|_2\widetilde{d}\\ &= -\|\mathfrak{y}\|_2\left[\min\{\alpha_1,\alpha_2\}\|\mathfrak{y}\|_2 - \widetilde{d}\right],\end{aligned}$$

as α_1 and α_2 given in (19). Thus, it holds that:

$$\dot{\mathfrak{L}}(\mathfrak{y}) < 0,\ \text{when}\ \|\mathfrak{y}\|_2 > \tfrac{\widetilde{d}}{\min\{\alpha_1,\alpha_2\}}.$$

Taking the latter into consideration and the fact that $\mathfrak{y}(0)$, we have that:

$$\|\mathfrak{y}(t)\| \le \tfrac{\widetilde{d}}{\min\{\alpha_1,\alpha_2\}}, \forall t \ge 0.$$

Moreover, the following inequalities hold:

$$\begin{aligned}\|\mathfrak{e}\|_2 \le \|\mathfrak{y}\|_2 &\Rightarrow \|\mathfrak{e}(t)\|_2 \le \tfrac{\widetilde{d}}{\min\{\alpha_1,\alpha_2\}}, \forall t \ge 0,\\ \left|\|\mathfrak{e}\|_2 - \left\|J^\top\mathfrak{z}\right\|_2\right| &\le \left\|\mathfrak{e} + J^\top\mathfrak{z}\right\|_2 = \|\mathfrak{z}\|_2 \le \|\mathfrak{y}\|_2\\ &\Rightarrow \|\mathfrak{z}(t)\|_2 \le \tfrac{2\widetilde{d}}{\overline{J}\min\{\alpha_1,\alpha_2\}},\ \ \forall t \ge 0,\end{aligned}$$

which concludes the proof. □

Remark 1. *According to Lemma* 2, *the volume of the tube which is centered along the nominal trajectories $\overline{e}(t)$, $\overline{\zeta}(t)$, that are solution of system* (13a),(13b), *depends on the parameters $\widetilde{d}$, $\overline{J}$, $\underline{J}$, $\widetilde{J}$, L and $L_\mathfrak{c}$. By tuning the parameters ρ and k from* (17a) *appropriately, the volume of the tube can be adjusted. Figure* 2 *shows a graphical representation of the tube-based approach.*

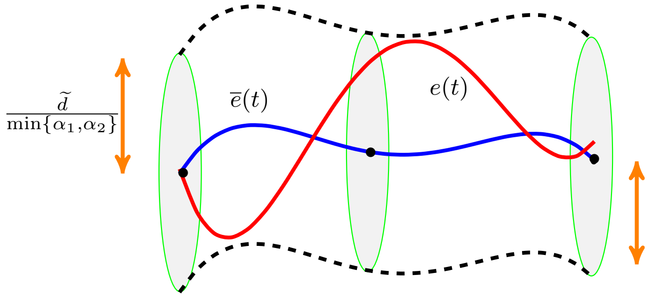

Fig. 2: The hyper-tube centered along the trajectory $\overline{e}(t)$ (depicted by blue line) with radius $\frac{\widetilde{d}}{\min\{\alpha_1,\alpha_2\}}$. Under the proposed control law, the real trajectory $e(t)$ (depicted with red line) lies inside the hyper-tube for all times, i.e., $\|\mathfrak{e}(t)\| \leq \frac{\widetilde{d}}{\min\{\alpha_1,\alpha_2\}}, \forall t \in \mathbb{R}_{\geq 0}$.

By using (14), the closed-loop system is written as:

$$\dot{e} = J(\mathfrak{q})\zeta, \tag{24a}$$

$$\dot{\zeta} = f(e+\chi_{\mathrm{d}}, \zeta) + \overline{u}(\overline{e}, \overline{\zeta}) - k(e-\overline{e}) - k\sigma J(\overline{q})^\top(\zeta - \overline{\zeta}) + d(\mathfrak{q}, \zeta, t), \tag{24b}$$

Assumption 1. *It holds that:*

$$\inf_{\substack{y_m, y_{m'} \in \mathcal{R} \\ m, m' \in \mathcal{M}, \\ m \neq m'}} \|y_m - y_{m'}\|_2 > 2\beta + \frac{\widetilde{d}}{\sqrt{\min\{\alpha_1,\alpha_2\}}},$$

i.e., there is sufficient space between any pairs of RoIs such that the UVMS can be navigated without intersecting them.

The aforementioned assumption is required in order for the navigation problem to have a solution.

4.3 On-line Optimal Control

Consider a sequence of sampling times $\{t_k\}$, $k \in \mathbb{N}$, with a constant sampling period $0 < h < T$, where T is a prediction horizon. For the sampling times it holds that $t_{k+1} := t_k + h$, $\forall k \in \mathbb{N}$. At each sampling time t_k, a FHOCP is solved for the nominal system (13a)-(13b) as follows:

$$\min_{\overline{u}(\cdot)} \left\{ \|\overline{\xi}(t_k+T)\|_P^2 + \int_{t_k}^{t_k+T} \left[\|\overline{\xi}(\mathfrak{s})\|_Q^2 + \|\overline{u}(\mathfrak{s})\|_R^2 \right] d\mathfrak{s} \right\} \tag{25a}$$

subject to:

$$\dot{\overline{\xi}}(\mathfrak{s}) = g(\overline{\xi}(\mathfrak{s}), \overline{u}(\mathfrak{s})), \ \ \overline{\xi}(t_k) = \xi(t_k), \tag{25b}$$

$$\overline{\xi}(\mathfrak{s}) \in \overline{\mathcal{E}} \times \overline{\mathcal{Z}}, \ \ \overline{u}(\mathfrak{s}) \in \overline{\mathcal{U}}, \ \ \forall \mathfrak{s} \in [t_k, t_k+T], \tag{25c}$$

$$\overline{\xi}(t_k+T) \in \mathcal{F}, \tag{25d}$$

where:

$$\xi := \left[e^\top, \zeta^\top\right]^\top \in \mathbb{R}^{12+n},$$

$$g(\xi, u) := \begin{bmatrix} J(\mathfrak{q})\zeta \\ f(e+\chi_{\mathrm{d}}, \zeta) + u \end{bmatrix},$$

and $Q, P \in \mathbb{R}^{(12+n)\times(12+n)}$ and $R \in \mathbb{R}^{(6+n)\times(6+n)}$ are positive definite gain matrices. We will explain hereafter the sets $\overline{\mathcal{E}}, \overline{\mathcal{V}}, \overline{\mathcal{U}}$ and $\mathcal{F}$.

In order to guarantee that while the FHOCP (25a)-(25d) is solved for the nominal dynamics (13a)-(13b), the real states e, ζ and control input $\mathfrak{u}$ satisfy the corresponding state $\mathcal{E}$, $\mathcal{Z}$ and input constraints $\mathcal{U}$, respectively, the following modification is performed:

$$\mathbb{E} := \mathcal{E} \ominus \Omega_1,$$
$$\mathbb{Z} := \mathcal{Z} \ominus \Omega_2,$$
$$\mathbb{U} := \mathcal{U} \ominus \left[\Lambda \circ \overline{\Omega}\right],$$

with:

$$\Lambda := \mathrm{diag}\{-kI_6, -k\sigma \overline{J} I_{6+n}\} \in \mathbb{R}^{(12+n)\times(12+n)},$$
$$\overline{\Omega} := \Omega_1 \times \Omega_2,$$

the operators $\ominus$, $\circ$ as defined in Section 2, and Ω_1, Ω_2 as given in (18a), (18b), respectively. Intuitively, the sets $\mathcal{E}$, $\mathcal{Z}$ and $\mathcal{U}$ are tightened accordingly, in order to guarantee that while the nominal states $\overline{e}$, $\overline{\zeta}$ and the nominal control input $\overline{u}$ are calculated, the corresponding real states e, ζ and real control input $\mathfrak{u}$ satisfy the state and input constraints $\mathcal{E}$, $\mathcal{Z}$ and $\mathcal{U}$, respectively. This constitutes a standard constraints set modification technique adopted in tube-based NMPC frameworks (for more details see [18]). Define the *terminal set* by:

$$\mathcal{F} := \left\{\overline{\xi} \in \mathbb{E} \times \mathbb{Z} : \|\overline{\xi}\|_P \leq \epsilon\right\}, \ \ \epsilon > 0,$$

which is used to enforce the stability of the system [14]. In particular, due to the fact that the linearized nominal dynamics (7) are stabilizable, it can be proven that (see [14, Lemma 1, p. 4]) there exists a *local controller*

$$u_{\mathrm{loc}} := \mathfrak{K}\,\overline{\xi} \in \mathbb{U}, \ \ \mathfrak{K} \in \mathbb{R}^{(6+n)\times(6+n)}, \ \ \mathfrak{K} > 0,$$

which guarantees that:

$$\frac{d}{dt}\left(\|\overline{\xi}\|_P^2\right) \leq -\|\overline{\xi}\|_{\widetilde{Q}}^2, \ \ \forall \overline{\xi} \in \mathcal{F},$$

with

$$\widetilde{Q} := Q + \mathfrak{K}^\top R \mathfrak{K}.$$

Theorem 1. *Suppose that Assumption* 1 *holds. Let the UVMS with dynamics as in* (2), (3) *occupy RoI $\mathcal{R}_{\mathrm{s}}$ and p_{d} be the center of a desired RoI $\mathcal{R}_{\mathrm{d}}$. Suppose also that the FHOCP* (25a)-(25d) *is feasible at time $t = 0$. Then, the feedback control law* (14) *renders the closed-loop system* (24a)-(24b) *Input to State Stable (ISS) with respect to the disturbance $d(\mathfrak{q}, \zeta, t)$.*

Proof: The proof of the theorem consists of two parts:

Feasibility Analysis: It can be shown that recursive feasibility is established and it implies subsequent feasibility. The proof of this part is similar to the feasibility proof of [20, Theorem 2, Sec. 4, p. 12].

Convergence Analysis: Recall that:

$$e = \chi - \chi_{\mathrm{d}}, \ \ \mathfrak{e} = e - \overline{e}, \ \ \mathfrak{z} = \zeta - \overline{\zeta}.$$

Then, we get:

$$\|\chi(t) - \chi_{\mathrm{d}}\|_2 \leq \|\overline{e}(t)\|_2 + \|\mathfrak{e}(t)\|_2,$$
$$\|\zeta(t)\|_2 \leq \|\overline{\zeta}(t)\|_2 + \|\mathfrak{z}(t)\|_2,$$

which, by using the fact that:

$$\|\overline{e}\|_2 \leq \|\overline{\xi}\|_2, \ \ \|\overline{\zeta}\|_2 \leq \|\overline{\xi}\|_2,$$

Algorithm 1 Implementation of feedback control law $u(t)$

Step 0: At time $t_0 := 0$, set $\xi(0) = \overline{\xi}(0)$ where $\xi(0)$ is the current state.
Step 1: At time t_k and current state $(\xi(t_k), \overline{\xi}(t_k))$, solve the FHOCP (25a)-(25d) to obtain the nominal control action $\overline{u}(t_k)$ and the actual control action $\mathfrak{u}(t_k) = \overline{u}(t_k) + \kappa(\xi(t_k), \overline{\xi}(t_k))$.
Step 2: Apply the control $\mathfrak{u}(t_k)$ to the system (12a), (12b), during the sampling interval $[t_k, t_{k+1})$, where $t_{k+1} = t_k + h$.
Step 3: Measure the state $\xi(t_{k+1})$ at the next time instant t_{k+1} of the system (12a), (12b) and compute the successor state $\overline{\xi}(t_{k+1})$ of the nominal system (13a), (13b) under the nominal control action $\overline{u}(t_k)$.
Step 4: Set $(\xi(t_k), \overline{\xi}(t_k)) \leftarrow (\xi(t_{k+1}), \overline{\xi}(t_{k+1}))$, $\;t_k \leftarrow t_{k+1}$;
Go to Step 1.

as well as the bounds from (18a), (18b) the latter inequalities become:

$$\|\chi(t) - \chi_{\mathrm{d}}\|_2 \leq \|\overline{\xi}(t)\|_2 + \frac{\widetilde{d}}{\min\{\alpha_1, \alpha_2\}}, \tag{26a}$$

$$\|\zeta(t)\|_2 \leq \|\overline{\xi}(t)\|_2 + \frac{2\widetilde{d}}{\overline{J}\min\{\alpha_1, \alpha_2\}}, \;\; \forall t \geq 0. \tag{26b}$$

Since only the nominal system dynamics (13a)-(13b) are used for the online computation of the control action $\overline{u}(s) \in \overline{\mathcal{U}}$, $s \in [t_k, t_k + T]$ through the FHOCP (25a)-(25d), by invoking nominal NMPC stability results found on [14**?**], it can be proven that the NMPC control law $\overline{u}$ renders the closed loop trajectories of the nominal system (13a)-(13b) asymptotically ultimated bounded in the sets $\mathcal{F}$. Then, from [**?** , Lemma 4.5, p. 150], there exists a class $\mathcal{KL}$ function β, such that:

$$\|\overline{\xi}(t)\| \leq \boldsymbol{\beta}(\|\overline{\xi}(0)\|_2, t), \;\; \forall t \in \mathbb{R}_{\geq 0}. \tag{27}$$

By combining (26a)-(26b) with (27) we get:

$$\|\chi(t) - \chi_{\mathrm{d}}\|_2 \leq \boldsymbol{\beta}(\|\overline{\xi}(0)\|_2, t) + \frac{\widetilde{d}}{\min\{\alpha_1, \alpha_2\}},$$
$$\|\zeta(t)\|_2 \leq \boldsymbol{\beta}(\|\overline{\xi}(0)\|_2, t) + \frac{2\widetilde{d}}{\overline{J}\min\{\alpha_1, \alpha_2\}},$$

for every $t \in \mathbb{R}_{\geq 0}$. The latter inequalities leads to the conclusion of the proof. ☐

Remark 2. *A major advantage of tube-based approach compared to other robust NMPC approaches, is that the FHOCP is solved only for the nominal system dynamics, thus the complexity is the same with the complexity of solving a nominal sampled-data NMPC (see [18] for more details)*

Algorithm 1 depicts the procedure of how the proposed control law is calculated and applied to the system. Theorem 1 implies that the UVMS with kinematics/dynamics as in (2), (3), starting at a RoI $\mathcal{R}_{\mathrm{s}}$, is driven by the controller (14) towards a desired RoI $\mathcal{R}_{\mathrm{d}}$, while all constraints imposed to the system are satisfied, i.e., the UVMS does not intersect with other RoI and always remains in the workspace $\mathcal{W}$.

4.4 Discrete System Abstraction

The abstraction that captures the dynamics of the robot into a TS is given through he following definition:

Definition 8. *The motion of the UVMS in the workspace $\mathcal{W}$ is modeled by the TS $\mathcal{T} = (\mathcal{S}, \mathcal{S}_0, \Pi, \longrightarrow, \Sigma, L)$ where:*

- $S = \mathcal{R} = \bigcup_{m \in \mathcal{M}} \mathcal{R}_m$ *is the set of states of the UVMS that contains all the RoI of the workspace $\mathcal{W}$;*
- $S_0 \in S$ *is a set of initial states defined by the UVMS' s initial position in the workspace;*
- Act *is the set of actions containing the union of all feedback controllers* (14) *which can navigate the robot between RoI;*
- $\longrightarrow \subseteq \mathcal{S} \times \mathrm{Act} \times \mathcal{S}$ *is the transition relation. We say that* $(\mathcal{R}_{\mathrm{s}}, u, \mathcal{R}_{\mathrm{d}}) \in \longrightarrow$, *with* $\mathcal{R}_s$, $\mathcal{R}_d \in \mathcal{R}$ *with* $\mathcal{R}_s \neq \mathcal{R}_d$ *if there exists a feedback control law* $u \in$ *Act as in* (14) *which can drive the UVMS from the region* $\mathcal{R}_{\mathrm{s}}$ *to the region* $\mathcal{R}_{\mathrm{d}}$ *without intersecting with any other RoI of the workspace;*
- L *is the labeling function as given in* (9)*;* Π *is the set of atomic propositions imposed by Problem* 1.

By construction, each behavior

$$\mathfrak{b} := (\mathfrak{p}(\mathfrak{t}_0), \varpi_0)(\mathfrak{p}(\mathfrak{t}_1), \varpi_1)(\mathfrak{p}(\mathfrak{t}_2), \varpi_2)\ldots,$$

produced by the TS $\mathcal{T}$, it is associated with the trajectory $\mathfrak{p}(t)$ of the system (2)-(3) and it is well-defined, as given in Definition 6. Hence, according to Definition 7, if we find a run of $\mathcal{T}$ satisfying the given LTL formula φ, we also find a desired behavior of the original system, and hence a trajectory $\mathfrak{p}(t)$ that is a solution to Problem 1.

4.5 Control Synthesis

There are several well-established methods to find an infinite run of $\mathcal{T}$ such that $\mathrm{Trace}(r) \models \varphi$. In this paper, we adopt the method from [10], which is based on checking the emptiness of the product BA which will be defined hereafter.

Definition 9. *Let $\mathcal{T}$ be the TS from Definition* 8 *and $\mathcal{A}_\varphi$ a BA that accepts the runs that satisfy the given formula φ. Then, the product automaton $\mathcal{A}_p := \mathcal{T} \otimes \mathcal{A}_\varphi$ is a tuple $(Q', Q'_0, Act, \delta', F')$ where:*

- $Q' := \mathcal{S} \times Q$;
- $Q'_0 := \{(s_0, q_0) : q_0 \in Q_0\}$;
- $\delta' \subseteq Q' \times Act \times Q'$. *We say that* $((s, \hat{q}), \cdot, (s', \hat{q}')) \in \delta'$ *if* $(s, \cdot, s') \in \longrightarrow$ *and* $\hat{q}' \in \delta(\hat{q}, L(s))$;
- $F' := \{(s, q_f) : s \in \mathcal{S} \text{ and } q_f \in F\}$;

Figure 3 depicts a framework under which a sequence of feedback control laws $u(\chi, \zeta)$ that guarantee the satisfaction of the LTL formula φ can be computed. First, a BA $\mathcal{A}_\varphi$ that accepts a run satisfying the specification formula φ is constructed. The related software tool can be found in [25]. Second, the product automaton $\mathcal{A}_p := \mathcal{T} \otimes \mathcal{A}_\varphi$ according to Definition 9 is constructed. By performing graph search to the product automaton $\mathcal{A}_p$, a run that satisfies the LTL formula φ and maps into a sequence of feedback control laws $u(\chi, \zeta)$ of the form (14) can be found.

Proposition 1. *The solution that it is obtained from the aforementioned controller synthesis procedure provides a sequence of feedback control laws $u(\chi, v)$ as in* (14) *that guarantees the satisfaction of the formula φ of a UVMS governed by kinematics/dynamics as in* (2)-(3)*, thus, providing a solution to Problem* 1.

5 Numerical Simulations

In order to illustrate the proposed approach, consider the Girona 500 AUV depicted in Figure 4 equipped with an ARM 5E Micro manipulator from [7, 26]. The manipulator consists of $n = 4$ revolute joints with limits:

$$-0.52 \leq q_1 \leq 1.46, 0.1471 \leq q_2 \leq 1.3114,$$
$$-1.297 \leq q_3 \leq 0.73, \;\; -3.14 \leq q_4 \leq 3.14.$$

The UVMS is operating in a workspace $\mathcal{W} = \{x, y, z \in \mathbb{R} : -5 \leq x, y, z \leq 5\} \subseteq \mathbb{R}^3$. The initial states are set to:

$$\chi(0) = \left[\mathfrak{p}(0)^\top, \mathfrak{o}(0)^\top\right]^\top = [-1.0, 1.3, -1.0, 0.0, -\tfrac{\pi}{8}, \tfrac{\pi}{12}]^\top.$$

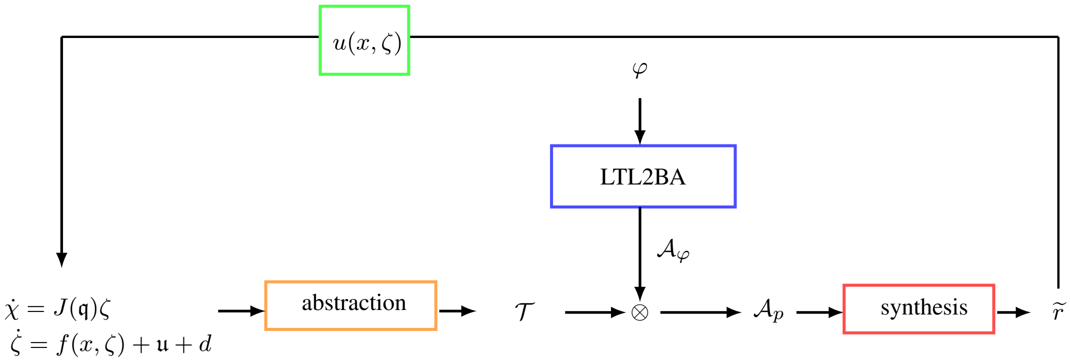

Fig. 3: A graphical illustration of the combined abstraction and controller synthesis framework.

	$d_i(m)$	q_i	$a_i(m)$	α_i(rad)
1	0	q_1	0.1	$-\frac{\pi}{2}$
2	0	q_2	0.26	0
3	0	q_3	0.09	$\frac{\pi}{2}$
4	0.29	q_4	0	0
E	Rot$(y, -\frac{\pi}{2})$			

Table 1 Denavit-Hantenberg Parameters of the ARM 5E Micro

According to (1), the transformation matrices which lead to the forward kinematics are given by:

$$T_B^I = \begin{bmatrix} \mathfrak{J}_1(\eta_2) & \eta_1 \\ 0_{1\times 3} & 1 \end{bmatrix}, \ T_0^B = \begin{bmatrix} I_{3\times 3} & [0.53, 0, 0.36]^\top \\ 0_{1\times 3} & 1 \end{bmatrix},$$

and $T_i^{i-1}, i = 1, \dots, 4$ are given by the Denavit-Hantenberg parameters which can be calculated from Table 1. By imposing the constraints

$$-\pi + \epsilon \le \phi - \epsilon, \ \psi \le \pi, \quad -\tfrac{\pi}{2} + \epsilon \le \theta \le \tfrac{\pi}{2} - \epsilon,$$

$\epsilon = 0.1$, according to (5) we get $\underline{J} = 0.5095$ and $L_{\mathfrak{c}} = 2\sqrt{2}$. We apply the methodology of this paper by considering the following disturbed kinematic model:

$$\dot{\chi} = J(\mathfrak{q})\zeta + d(\mathfrak{q}, t),$$

with:

$$d(\mathfrak{q}, t) = 0.2\sin(t) I_6 \Rightarrow \|d(\mathfrak{q}, t)\|_2 \le 0.2 = \widetilde{d},$$

in which the vector ζ stands for the virtual control input to be designed. The input constraints are set to

$$\|\nu_1\|_2 \le 2, \ \|\nu_2\|_2 \le 2, \ \|\dot{q}\|_2 \le 2.$$

Then, by using (15a) and (20) and designing the control gain $\sigma = 3.084$, the resulting RCI is

$$\Omega = \left\{ \mathfrak{e} \in \mathbb{R}^6 : \|\mathfrak{e}\|_2 \le \frac{\widetilde{w}}{\sigma \underline{J} + L_{\mathfrak{c}}} = 0.3 \right\}.$$

The optimization horizon and the sampling time are set to $T = 0.7$ sec and $h = 0.1$ sec, respectively. The NMPC gains are set to

$$Q = P = 0.5 I_6, \ \ R = 0.5 I_{10}.$$

In the workspace we have $\mathfrak{m} = 14$ RoI with radius $\gamma_m = 0.6$, $\forall m \in \mathcal{M}$ from which 6 of them stand for unsafe regions that the robot is prohibited to visit (depicted with red color in Figure 5). The RoI are located in the following positions on the workspace $\mathcal{W}$:

$$\begin{aligned}
\mathcal{R}_1 &: [-3.8, 3.8, 4.0], \ \ \mathcal{R}_2 : [-3.0, 2.0, 2.5], \\
\mathcal{R}_3 &: [-1.2, 2.0, 0.5], \ \ \mathcal{R}_4 : [0.0, 2.8, 2.0], \\
\mathcal{R}_5 &: [2.0, 2.6, 3.0], \ \ \mathcal{R}_6 : [0.0, 0.0, 0.0], \\
\mathcal{R}_7 &: [3.4, 1.6, 1.5], \ \ \mathcal{R}_8 : [-1.2, -2.0, 1.2], \\
\mathcal{R}_9 &: [3.0, 0.0, -0.5], \ \ \mathcal{R}_{10} : [2.5, -2.0, 1.0], \\
\mathcal{R}_{11} &: [-2.7, -3.0, -0.5], \ \ \mathcal{R}_{12} : [0.7, -3.0, 2.5].
\end{aligned}$$

The unsafe regions are placed in specific positions such that the UVMS needs to maneuver to avoid them. The desired LTL formula is set to:

$$\varphi = \Box\{\neg \text{obs}\} \wedge \Diamond \{\text{goal}_1 \wedge \bigcirc \text{goal}_2\}, \tag{28}$$

over the set of atomic propositions

$$\Pi = \{\text{obs}, \text{goal}_1, \text{goal}_2\},$$

and labeling function:

$$\begin{aligned}
&L(\mathcal{R}_5) = \{\text{goal}_1\}, \ \ \mathrm{L}(\mathcal{R}_9) = \{\text{goal}_2\}, \\
&L(\mathcal{R}_i) = \{\text{obs}\}, \ \ \mathrm{i} \in \{2, 4, 6, 7, 8, 10\}, \\
&L(\mathcal{R}_j) = \emptyset, \ \ j \in \{1, 3, 11, 12, 13, 14\}.
\end{aligned}$$

The formula intuitively means “Eventually reach goal 1 and then directly move to goal 2, while always avoid any obstacles of the environment". Figures 5, 6 depicts the workspace with RoI, unsafe regions, the nominal trajectory of the UVMS (orange color) and the real trajectory of the UVMS (black dashed color). According to Figures 5, 6 the robot never visits the unsafe RoI, eventually visits RoI $\mathcal{R}_5$ and next visits $\mathcal{R}_9$. Thus,

$$\mathfrak{p}(t) \models \varphi, \ \ \forall t \ge 0,$$

according to (10) of Definition 7. The error signals for the transition between RoI $\mathcal{R}_1$-$\mathcal{R}_3$ are depicted in Figure 7. The control effort for the transition from $\mathcal{R}_1$ to $\mathcal{R}_3$ is presented in Figure 8. It can be observed that the desired task is performed while all the state/input constraints are satisfied.

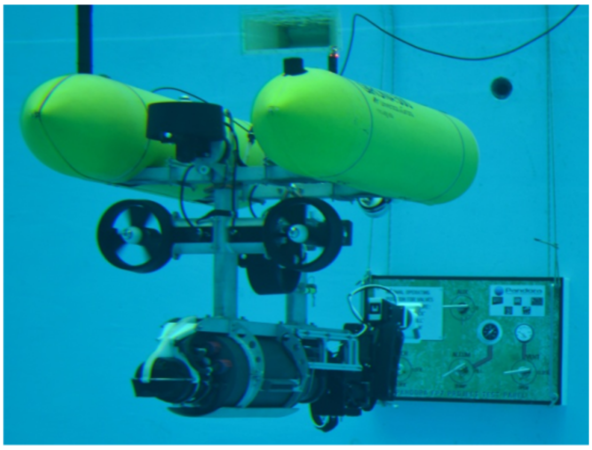

Fig. 4: The GIRONA-UVMS composed of Girona500 AUV and ARM 5E Micro manipulator [7].

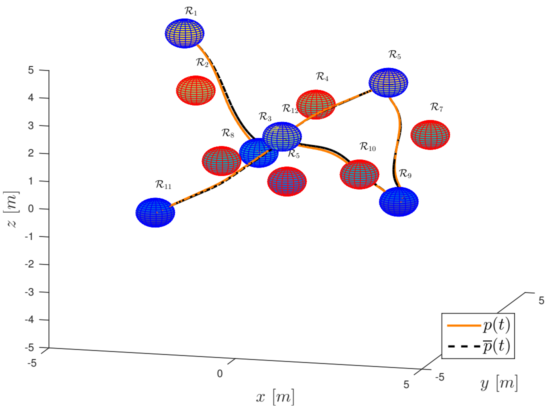

Fig. 5: The evolution of the trajectory of the UVMS in the workspace $\mathcal{W}$. The RoI and unsafe regions are depicted with blue and red color, respectively. The real and the nominal trajectories $\chi(t)$ and $\overline{\chi}(t)$, respectively, are depicted with orange and dashed black color. The UVMS successfully satisfies the task φ given in (28).

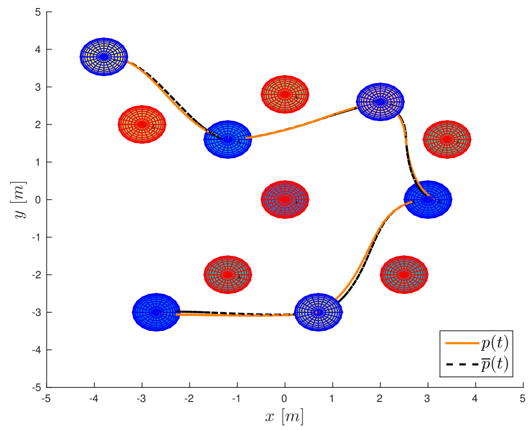

Fig. 6: A planar view of the workspace indicating that the UVMS is not intersecting with any unsafe RoI.

6 Conclusion

This paper addresses the problem of control of a UVMS under high-level tasks given in LTL. First, a tube-based NMPC framework that guarantees the robust transition of the UVMS between RoI of workspace under state, input and safety constraints, is provided. Then, by utilizing ideas from previous works, an abstraction and

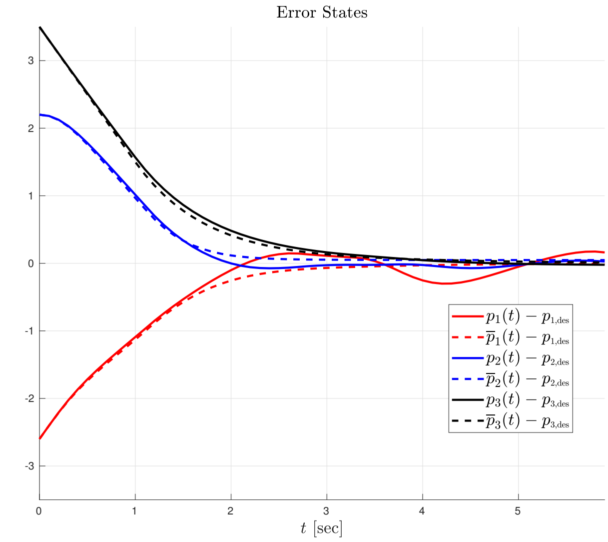

Fig. 7: The evolution of the real position errors of the end-effector $\mathfrak{p}_1(t) - \mathfrak{p}_{1,\mathrm{d}}$, $\mathfrak{p}_2(t) - \mathfrak{p}_{2,\mathrm{d}}$, $\mathfrak{p}_3(t) - \mathfrak{p}_{3,\mathrm{d}}$ depicted with solid lines as well as the corresponding nominal position errors $\overline{\mathfrak{p}}_1(t) - \mathfrak{p}_{1,\mathrm{d}}$, $\overline{\mathfrak{p}}_2(t) - \mathfrak{p}_{2,\mathrm{d}}$, $\overline{\mathfrak{p}}_3(t) - \mathfrak{p}_{3,\mathrm{d}}$ depicted with dashed lines, for the transition of the UVMS between RoI $\mathcal{R}_1 - \mathcal{R}_3$.

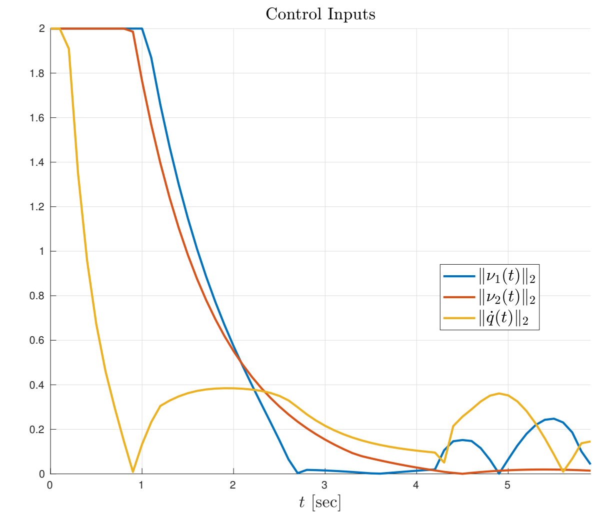

Fig. 8: The virtual control input signals $\|\nu_1(t)\|_2$, $\|\nu_2(t)\|_2$ and $\|\dot{q}(t)\|_2$ of the kinematic model (2) for the transition of the UVMS between RoI $\mathcal{R}_1$-$\mathcal{R}_3$. It holds that $\|\nu_1\|_2 \leq 2$, $\|\nu_2\|_2 \leq 2$ and $\|\dot{q}\|_2 \leq 2$.

controller synthesis framework that gives a sequence of controllers that provably satisfy the given task is presented. Simulation results verify the efficiency of the proposed approach. Future research will be devoted towards extending the current framework into underwater object transportation of multiple UVMSs under event-triggered control communication.

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