

## UH Random Team Reference

### CONTENTS

1. TEMPLATE	2
1.1. MAIN	2
2. MATH	2
2.1. DISCRETE LOG	2
2.2. NTT	3
2.3. MATRIX POW	3
2.4. FFT	4
2.5. PRIMITIVE ROOT	5
2.6. FWHT	5
2.7. EXTENDED EUCLIDEAN ALGORITHM	8
3. GEOMETRY	8
3.1. BASICS	8
3.2. POLYGON	10
4. DATA STRUCTURE	11
4.1. SEGMENT TREE	11
4.2. DISJOINT SET UNION	12
4.3. SEGMENTTREE PERSISTENT	12
4.4. PBDS	13
4.5. SEGMENT TREE LAZY	14
4.6. AVL	16
4.7. ABI	18
4.8. MOS	18

4.9. SQRT DECOMPOSITION	19
5. STRING	19
5.1. SUFFIX ARRAY	19
5.2. Z FUNCTION	22
5.3. HASHING	22
5.4. AHO CORASIK	23
5.5. KMP PF	23
5.6. TRIE	24
6. GRAPH	25
6.1. TOPOLOGICAL SORT	25
6.2. DIJSKTRA	26
6.3. BRIDGE EDGES	27
6.4. PRIM	28
6.5. KRUSKAL	28
6.6. BELLMAN FORD	29
6.7. DINIC	30
6.8. CENTROID DESCOMPOSITION	31
6.9. LOWER BOUND FLOW	31
6.10. FLOYD WARSHALL	33
6.11. DFS BFS	33
6.12. SCC TARJANS	34
6.13. MIN COST MAX FLOW	35
6.14. ARTICULATION POINT	36
6.15. KUHN	37

## 1. TEMPLATE

## 1.1. MAIN.

```
#include <bits/stdc++.h>

#define MAX(a, b) (a > b) ? a : b
#define MIN(a, b) (a < b) ? a : b
#define int long long
#define vi vector<int>
#define pii pair<int, int>
#define vii vector<pii>

using namespace std;

void solve()
{
}
```

```
int32_t main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    int t;
    cin >> t;

    for (int i = 0; i < t; i++)
    {
        solve();
    }

    return 0;
}
```

## 2. MATH

## 2.1. DISCRETE LOG.

```
int powmod(int a, int b, int m)
{
    int res = 1;
    while (b > 0)
    {
        if (b & 1)
        {
            res = (res * 1ll * a) % m;
        }
        a = (a * 1ll * a) % m;
        b >>= 1;
    }
    return res;
}

int discrete_log(int a, int b, int m)
```

```
{
    a %= m, b %= m;
    int n = sqrt(m) + 1;
    map<int, int> vals;
    for (int p = 1; p <= n; ++p)
        vals[powmod(a, p * n, m)] = p;
    for (int q = 0; q <= n; ++q)
    {
        int cur = (powmod(a, q, m) * 1ll * b) % m;
        if (vals.count(cur))
        {
            int ans = vals[cur] * n - q;
            return ans;
        }
    }
    return -1;
}
```

## 2.2. NTT.

```

using ll = long long;

const ll mod = (119 << 23) + 1, root = 62; // 998244353

ll qp(ll b, ll e)
{
    ll ans = 1;
    for (; e; b = b * b % mod, e /= 2)
        if (e & 1)
            ans = ans * b % mod;
    return ans;
}

void ntt(vector<ll> &a, vector<ll> &rt, vector<ll> &rev, int n)
{
    for (int i = 0; i < n; i++)
        if (i < rev[i])
            swap(a[i], a[rev[i]]);

    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k)
            for (int j = 0; j < k; j++)
            {
                ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
                a[i + j + k] = (z > ai ? ai - z + mod : ai - z);
                ai += (ai + z >= mod ? z - mod : z);
            }
}

vector<ll> convolve(const vector<ll> &a, const vector<ll> &b)

```

```

{
    if (a.empty() || b.empty())
        return {};

    int s = a.size() + b.size() - 1, B = 32 - __builtin_clz(s), n = 1 << B;

    vector<ll> L(a), R(b), out(n), rt(n, 1), rev(n);
    L.resize(n), R.resize(n);

    for (int i = 0; i < n; i++)
        rev[i] = (rev[i / 2] | (i & 1) << B) / 2;

    ll curL = mod / 2, inv = qp(n, mod - 2);
    for (int k = 2; k < n; k *= 2)
    {
        ll z[] = {1, qp(root, curL /= 2)};
        for (int i = k; i < 2 * k; i++)
            rt[i] = rt[i / 2] * z[i & 1] % mod;
    }

    ntt(L, rt, rev, n);
    ntt(R, rt, rev, n);

    for (int i = 0; i < n; i++)
        out[-i & (n - 1)] = L[i] * R[i] % mod * inv % mod;

    ntt(out, rt, rev, n);

    return {out.begin(), out.begin() + s};
}

```

## 2.3. MATRIX POW.

```

const int MAXN = 2;

struct Matrix
{
    ll mat[MAXN][MAXN];
};

Matrix operator*(const Matrix &a, const Matrix &b)
{
    Matrix c;

```

```

    for (int i = 0; i < MAXN; ++i)
        for (int j = 0; j < MAXN; ++j)
            c.mat[i][j] = 0;

    for (int i = 0; i < MAXN; i++)
    {
        for (int k = 0; k < MAXN; k++)
        {
            if (a.mat[i][k] == 0)
                continue;

```

```

        for (int j = 0; j < MAXN; j++)
        {
            c.mat[i][j] += a.mat[i][k] * b.mat[k][j];
        }
    }
    return c;
}
Matrix operator^(Matrix &base, ll e)
{
    Matrix c;
    for (int i = 0; i < MAXN; i++)

```

```

        for (int j = 0; j < MAXN; j++)
            c.mat[i][j] = (i == j);
    while (e)
    {
        if (e & 1ll)
            c = c * base;
        base = base * base;
        e >>= 1;
    }
    return c;
}

```

## 2.4. FFT.

```

struct point
{
    double x, y;
    point(double x = 0, double y = 0) : x(x), y(y) {}
};

point operator+(const point &a, const point &b)
{
    return {a.x + b.x, a.y + b.y};
}
point operator-(const point &a, const point &b)
{
    return {a.x - b.x, a.y - b.y};
}
point operator*(const point &a, const point &b)
{
    return {a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x};
}
point operator/(const point &a, double d) { return {a.x / d, a.y / d}; }

void fft(vector<point> &a, int sign = 1)
{
    int n = a.size(); // n should be a power of two
    double theta = 8 * sign * atan(1.0) / n;
    for (int i = 0, j = 1; j < n - 1; ++j)
    {
        for (int k = n >> 1; k > (i ^ k); k >>= 1)
            ;
        if (j < i)
            swap(a[i], a[j]);
    }
}

```

```

for (int m, mh = 1; (m = mh << 1) <= n; mh = m)
{
    int irev = 0;
    for (int i = 0; i < n; i += m)
    {
        point w = point(cos(theta * irev), sin(theta * irev));
        for (int k = n >> 2; k > (irev ^ k); k >>= 1)
            ;
        for (int j = i; j < mh + i; ++j)
        {
            int k = j + mh;
            point x = a[j] - a[k];
            a[j] = a[j] + a[k];
            a[k] = w * x;
        }
    }
}
if (sign == -1)
    for (auto &p : a)
        p = p / n;
}

vector<point> convolve(vector<point> &a, vector<point> &b)
{
    int n = a.size();
    int m = b.size();
    int k = n + m;
    while (k != (k & -k))
        k += (k & -k);
    while (a.size() < k)
        a.push_back(point(0, 0));
}

```

```

while (b.size() < k)
    b.push_back(point(0, 0));

fft(a, 1);
fft(b, 1);

vector<point> c(k);

```

## 2.5. PRIMITIVE ROOT.

```

int powmod(int a, int b, int p)
{
    int res = 1;
    while (b)
        if (b & 1)
            res = int32_t(res * 111 * a % p), --b;
        else
            a = int32_t(a * 111 * a % p), b >>= 1;
    return res;
}

int generator(int p)
{
    vector<int> fact;
    int phi = p - 1, n = phi;
    for (int i = 2; i * i <= n; ++i)
        if (n % i == 0)
            {

```

## 2.6. FWHT.

```

using ll = long long;

const int mod = 1e9 + 7;

template <const int _mod_>
struct mod_int
{
    static const int mod = _mod_;
    int val;

    mod_int(long long v = 0)
    {
        if (v < 0)

```

```

for (int i = 0; i < k; i++)
    c[i] = a[i] * b[i];

fft(c, -1);

return c;
}

```

```

        fact.push_back(i);
        while (n % i == 0)
            n /= i;
    }
    if (n > 1)
        fact.push_back(n);

    for (int res = 2; res <= p; ++res)
    {
        bool ok = true;
        for (size_t i = 0; i < fact.size() && ok; ++i)
            ok &= powmod(res, phi / fact[i], p) != 1;
        if (ok)
            return res;
    }
    return -1;
}

```

```

        v = v % mod + mod;
        if (v >= mod)
            v %= mod;
        val = v;
    }

    static int mod_inv(int a, int m = mod)
    {
        int g = m, r = a, x = 0, y = 1;
        while (r != 0)
        {
            int q = g / r;
            g %= r;
            swap(g, r);

```

```

        x -= q * y;
        swap(x, y);
    }
    return x < 0 ? x + m : x;
}

explicit operator int() const { return val; }

mod_int &operator+=(const mod_int &other)
{
    val += other.val;
    if (val >= mod)
        val -= mod;
    return *this;
}

mod_int &operator-=(const mod_int &other)
{
    val -= other.val;
    if (val < 0)
        val += mod;
    return *this;
}

static unsigned fast_mod(uint64_t x, unsigned m = mod)
{
    #if !defined(_WIN32) || defined(_WIN64)
        return x % m;
    #endif
    // Optimized mod for Codeforces 32-bit machines.
    // x must be less than 2^32 * m for this to work, so that x / m fits in
    // a 32-bit integer.
    unsigned x_high = x >> 32, x_low = (unsigned)x;
    unsigned quot, rem;
    asm("divl_%4\n"
        : "=a"(quot), "=d"(rem)
        : "d"(x_high), "a"(x_low), "r"(m));
    return rem;
}

mod_int &operator*=(const mod_int &other)
{
    val = fast_mod((uint64_t)val * other.val);
    return *this;
}

mod_int &operator/=(const mod_int &other) { return *this *= other.inv(); }

```

```

friend mod_int operator+(const mod_int &a, const mod_int &b)
{
    return mod_int(a) += b;
}
friend mod_int operator-(const mod_int &a, const mod_int &b)
{
    return mod_int(a) -= b;
}
friend mod_int operator*(const mod_int &a, const mod_int &b)
{
    return mod_int(a) *= b;
}
friend mod_int operator/(const mod_int &a, const mod_int &b)
{
    return mod_int(a) /= b;
}

mod_int &operator++()
{
    val = val == mod - 1 ? 0 : val + 1;
    return *this;
}

mod_int &operator--()
{
    val = val == 0 ? mod - 1 : val - 1;
    return *this;
}

mod_int operator++(int)
{
    mod_int a = *this;
    ++*this;
    return a;
}
mod_int operator--(int)
{
    mod_int a = *this;
    --*this;
    return a;
}

mod_int operator-() const { return val == 0 ? 0 : mod - val; }
mod_int inv() const { return mod_inv(val); }

bool operator==(const mod_int &other) const { return val == other.val; }
bool operator!=(const mod_int &other) const { return val != other.val; }

```

```

bool operator<(const mod_int &other) const { return val < other.val; }
bool operator>(const mod_int &other) const { return val > other.val; }

template <typename T>
bool operator<(const T &other) const
{
    return val < other;
}

template <typename T>
bool operator>(const T &other) const
{
    return val > other;
}

friend string to_string(const mod_int &m) { return to_string(m.val); }
friend mod_int abs(const mod_int &m) { return mod_int(m.val); }

friend ostream &operator<<(ostream &stream, const mod_int &m)
{
    return stream << m.val;
}
friend istream &operator>>(istream &stream, mod_int &m)
{
    return stream >> m.val;
}
};

// Notar que se uso este tipo de datos entero, que implementa todas las
// operaciones basicos en el sistem residual modulo 10^9+7 para hacer mas
// faciles las implementaciones si se quiere implementarlo, se puede dejar en
// long long, y modificar las operaciones para mantener los resultados teneindo
// en cuenta el modulo a usar.
using T = mod_int<mod>;

void HADAMARD_XOR(vector<T> &a, bool inverse = false)
{
    int n = a.size();
    for (int k = 1; k < n; k <= 1)
    {
        for (int i = 0; i < n; i += 2 * k)
        {
            for (int j = 0; j < k; j++)
            {
                T x = a[i + j];
                T y = a[i + j + k];
                a[i + j] = x + y;
            }
        }
    }
}

```

```

        a[i + j + k] = x - y;
    }
}

if (inverse)
{
    T q = 1 / static_cast<T>(n);

    for (int i = 0; i < n; i++)
    {
        a[i] *= q;
    }
}

void HADAMARD_AND(vector<T> &a, bool inverse = false)
{
    int n = a.size();
    for (int k = 1; k < n; k <= 1)
    {
        for (int i = 0; i < n; i += 2 * k)
        {
            for (int j = 0; j < k; j++)
            {
                T x = a[i + j];
                T y = a[i + j + k];
                if (inverse)
                {
                    a[i + j] = -x + y;
                    a[i + j + k] = x;
                }
                else
                {
                    a[i + j] = y;
                    a[i + j + k] = x + y;
                }
            }
        }
    }
}

void HADAMARD_OR(vector<T> &a, bool inverse = false)
{
    int n = a.size();
    for (int k = 1; k < n; k <= 1)
    {

```

```

    for (int i = 0; i < n; i += 2 * k)
    {
        for (int j = 0; j < k; j++)
        {
            T x = a[i + j];
            T y = a[i + j + k];
            if (inverse)
            {
                a[i + j] = y;
                a[i + j + k] = x - y;
            }
            else
            {
                a[i + j] = x + y;
                a[i + j + k] = x;
            }
        }
    }
}

```

*// Las demas operaciones a nivel de bit tienen una implementacion semejante*

```

template <typename T>
vector<T> FWHT_XOR(vector<T> a, vector<T> b)
{
    bool eq = (a == b);

```

```

    int n = 1;

    while (n < (int)max(a.size(), b.size()))
    {
        n <= 1;

        a.resize(n);
        b.resize(n);

        HADAMARD_XOR(a);

        if (eq)
            b = a;
        else
            HADAMARD_XOR(b);

        for (int i = 0; i < n; i++)
        {
            a[i] *= b[i];
        }

        HADAMARD_XOR(a, true);

        return a;
    }

```

## 2.7. EXTENDED EUCLIDEAN ALGORITHM.

```

int gcd(int a, int b, int &x, int &y)
{
    x = 1, y = 0;
    int x1 = 0, y1 = 1, a1 = a, b1 = b;
    while (b1)
    {
        int q = a1 / b1;

```

```

        tie(x, x1) = make_tuple(x1, x - q * x1);
        tie(y, y1) = make_tuple(y1, y - q * y1);
        tie(a1, b1) = make_tuple(b1, a1 - q * b1);
    }
    return a1;
}

```

## 3. GEOMETRY

### 3.1. BASICS.

```

db DEG_to_RAD(db d) { return d*M_PI / 180.0; }
db RAD_to_DEG(db r) { return r*180.0 / M_PI; }

```

```

db EPS = 1e-9;

struct point {
    db x, y;

```



```

point() { x = y = 0.0; }
point(db _x, db _y) : x(_x), y(_y) {}

bool operator < (const point &other) const {
    if (fabs(x-other.x) > EPS)
        return x < other.x;
    return y < other.y;
}

bool operator == (const point &other) const {
    return (fabs(x-other.x) < EPS) && (fabs(y-other.y) < EPS);
}

db dist(const point &other) {
    return hypot(x-other.x, y-other.y);
}

point rotate(db theta) {
    db rad = DEG_to_RAD(theta);
    return point(x*cos(rad) - y*sin(rad), x*sin(rad) + y*cos(rad));
}

};

struct line {
    db a, b, c;
    line() {}
    line(db _a, db _b, db _c) : a(_a), b(_b), c(_c) {}

    void pointsToLine(const point &p1, const point &p2) {
        if (fabs(p1.x-p2.x) < EPS) {
            a = 1.0;
            b = 0.0;
            c = -p1.x;
        }
        else {
            a = -(db) (p1.y-p2.y) / (p1.x-p2.x);
            b = 1.0;
            c = -(db) (a*p1.x) - p1.y;
        }
    }

    void pointSlopeToLine(point p, db m) {
        a = -m;
        b = 1.0;
        c = -((a * p.x) + (b * p.y));
    }
}

```

```

bool areParallel(const line &other) {
    return (fabs(a-other.a) < EPS) && (fabs(b-other.b) < EPS);
}

bool areSame(const line &other) {
    return areParallel(other) && (fabs(c-other.c) < EPS);
}

bool areIntersect(const line &other, point &p) {
    if (areParallel(other)) return false;
    p.x = (other.b*c - b*other.c) / (other.a*b - a*other.b);
    if (fabs(b) > EPS) p.y = -(a*p.x + c);
    else p.y = -(other.a*p.x + other.c);
    return true;
}

};

struct vec{
    db x, y;
    vec(db _x, db _y) : x(_x), y(_y) {}
    vec(const point &a, const point &b) : x(b.x - a.x), y(b.y - a.y) {}

    vec scale(db s) {
        return vec(x*s, y*s);
    }

    point translate(const point &p) {
        return point(x+p.x, y+p.y);
    }

    db dot(vec a, vec b) { return a.x*b.x + a.y*b.y; }

    db norm_sq(vec v) { return v.x*v.x + v.y*v.y; }

    db angle(const point &a, const point &o, const point &b) {
        vec oa = vec(o, a), ob = vec(o, b);
        return acos(dot(oa, ob) / sqrt(norm_sq(oa) * norm_sq(ob)));
    }

    db cross(vec a, vec b) { return a.x*b.y - a.y*b.x; }

    bool ccw(point p, point q, point r) {
        return cross(vec(p, q), vec(p, r)) > EPS;
    }

    bool collinear(point p, point q, point r) {
        return fabs(cross(vec(p, q), vec(p, r))) < EPS;
    }
}

```

```

}

db distToLine(point p, point a, point b) {
    vec ap = vec(a, p), ab = vec(a, b);
    db u = dot(ap, ab) / norm_sq(ab);
    point c = ab.scale(u).translate(a);
    return c.dist(p);
}

db distToLineSegment(point p, point a, point b) {
    vec ap = vec(a, p), ab = vec(a, b);
    db u = dot(ap, ab) / norm_sq(ab);
    if (u < 0.0) {
        point c = point(a.x, a.y);
        return c.dist(p);
    }
    if (u > 1.0) {
        point c = point(b.x, b.y);
        return c.dist(p);
    }
    return distToLine(p, a, b);
}
};

```

### 3.2. POLYGON.

```

struct polygon {
    vector<point> P;
    polygon(const vector<point> &P) : P(_P) {}

    db perimeter() {
        db ans = 0.0;
        for (int i = 0; i < (int)P.size()-1; ++i)
            ans += P[i].dist(P[i+1]);
        return ans;
    }

    db area() {
        db ans = 0.0;
        for (int i = 0; i < (int)P.size()-1; ++i)
            ans += (P[i].x*P[i+1].y - P[i+1].x*P[i].y);
        return fabs(ans) / 2.0;
    }
}

```

```

struct circle {
    point c;
    db r;
    circle(const point &c, db _r) : c(_c), r(_r) {}

    int inside(const point &p) {
        db dist = c.dist(p);
        return dist < r ? 1 : (fabs(dist-r) < EPS ? 0 : -1);
    }

    point inCircle(point p1, point p2, point p3) {
        line l1, l2;
        double ratio = p1.dist(p2) / p1.dist(p3);
        point p = vec(p2, p3).scale(ratio / (1+ratio)).translate(p2);
        l1.pointsToLine(p1, p);
        ratio = p2.dist(p1) / p2.dist(p3);
        p = vec(p1, p3).scale(ratio / (1+ratio)).translate(p1);
        l2.pointsToLine(p2, p);
        point c;
        l1.areIntersect(l2, c);
        return c;
    }
};

```

```

bool isConvex(const vector<point> &P) {
    int n = (int)P.size();
    if (n <= 3) return false;
    bool firstTurn = vec().ccw(P[0], P[1], P[2]);
    for (int i = 1; i < n-1; ++i)
        if (vec().ccw(P[i], P[i+1], P[(i+2) == n ? 1 : i+2]) != firstTurn)
            return false;
    return true;
}

int insidePolygon(point pt) {
    int n = (int)P.size();
    if (n <= 3) return -1;
    bool on_polygon = false;
    for (int i = 0; i < n-1; ++i)
        if (fabs(pt.dist(P[i]) + pt.dist(P[i+1]) - P[i].dist(P[i+1])) < EPS)
            on_polygon = true;
    if (on_polygon) return 0;
    double sum = 0.0;
}

```

```

for (int i = 0; i < n-1; ++i) {
    if (vec().ccw(pt, P[i], P[i+1]))
        sum += vec().angle(P[i], pt, P[i+1]);
    else
        sum -= vec().angle(P[i], pt, P[i+1]);
}

```

```

}
return fabs(sum) > M_PI ? 1 : -1;
}
};

```

## 4. DATA STRUCTURE

### 4.1. SEGMENT TREE.

```

class SegmentTree
{
private:
    vi values;

    vi p_values;
    int n;

    int left(int p) { return p << 1; };
    int right(int p) { return (p << 1) + 1; }

    int simple_node(int index) { return values[index]; }

    int prop(int x, int y) { return x + y; }

    void build(int p, int l, int r)
    {
        if (l == r)
        {
            p_values[p] = simple_node(l);
            return;
        }

        build(left(p), l, (l + r) / 2);
        build(right(p), (l + r) / 2 + 1, r);

        p_values[p] = prop(p_values[left(p)], p_values[right(p)]);
    }

    void set(int p, int l, int r, int i, int v)
    {
        if (l == r)
        {
            values[l] = v;
            p_values[p] = simple_node(l);
        }
    }
}

```

```

        return;
    }

    if (i <= (l + r) / 2)
        set(left(p), l, (l + r) / 2, i, v);
    else
        set(right(p), (l + r) / 2 + 1, r, i, v);

    p_values[p] = prop(p_values[left(p)], p_values[right(p)]);
}

int query(int p, int l, int r, int lq, int rq)
{
    if (lq <= l && r <= rq)
        return p_values[p];

    int l1 = l, r1 = (l + r) / 2;
    int l2 = (l + r) / 2 + 1, r2 = r;

    if (l1 > rq || lq > r1)
        return query(right(p), l2, r2, lq, rq);
    if (l2 > rq || lq > r2)
        return query(left(p), l1, r1, lq, rq);

    int lt = query(left(p), l1, r1, lq, rq);
    int rt = query(right(p), l2, r2, lq, rq);

    return prop(lt, rt);
}

public:
    SegmentTree(vi &a)
    {
        values = a;
        n = a.size();
        p_values.assign(4 * n, 0);
    }
}

```

```

    build(1, 0, n - 1);
}

int query(int i, int j) { return query(1, 0, n - 1, i, j); }

```

```

void set(int i, int v) { set(1, 0, n - 1, i, v); }

int get(int i) { return values[i]; }
};

```

## 4.2. DISJOINT SET UNION.

```

struct dsu
{
    vi p, rank, sz;
    int num_sets;
    dsu(int n)
    {
        p.assign(n + 1, 0);
        iota(p.begin(), p.end(), 0);
        rank.assign(n + 1, 0);
        sz.assign(n + 1, 1);
        num_sets = n;
    }
    int get(int x)
    {
        if (p[x] < 0)
            return x;
    }

```

```

        return p[x] = get(p[x]);
    }
    bool is_same_set(int a, int b) { return get(a) == get(b); }
    int size_of(int a) { return sz[get(a)]; }
    void unite(int a, int b)
    {
        a = get(a);
        b = get(b);
        if (rank[a] > rank[b])
            swap(a, b);
        p[a] = b;
        if (rank[a] == rank[b])
            rank[b]++;
        sz[b] += sz[a];
        num_sets--;
    }
};

```

## 4.3. SEGMENTTREE PERSISTENT.

```

struct PersistentImplicitST
{
    struct node
    {
        ll l, r;
        ll sum;
        node *ls, *rs;
        node(ll liml, ll limr) : l(liml), r(limr), ls(0), rs(0), sum(0) {}
    };

    typedef node *pnode;
    vector<pnode> root;
    int versionCount;

    PersistentImplicitST(ll liml, ll limr)
    {
        root.push_back(new node(liml, limr));
    }

```

```

        versionCount = 1;
    }

    pnode clone(pnode p)
    {
        if (!p)
            return 0;
        pnode ans = new node(p->l, p->r);
        ans->sum = p->sum;
        ans->ls = p->ls;
        ans->rs = p->rs;
        return ans;
    }

    ll sum(pnode p)
    {
        if (!p)

```

```

    return 0;
    return p->sum;
}

void update(ll pos, ll v)
{
    root.push_back(update(root.back(), pos, v));
}

void update(int ver, ll pos, ll v)
{
    root.push_back(update(root[ver], pos, v));
}

pnode update(pnode p, ll pos, ll v)
{
    ll l = p->l;
    ll r = p->r;
    if (l > pos || r < pos)
        return p;

    p = clone(p);
    if (l == r && l == pos)
    {
        p->sum += v;
        return p;
    }

    ll mid = (l + r) >> 1;
    if (pos <= mid)
    {
        if (!p->ls)
            p->ls = new node(l, mid);
        p->ls = update(p->ls, pos, v);
    }
}

```

#### 4.4. PBDS.

```

#include <bits/extc++.h> // pbds
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
            tree_order_statistics_node_update>
    ost;

int main()
{

```

```

    else
    {
        if (!p->rs)
            p->rs = new node(mid + 1, r);
        p->rs = update(p->rs, pos, v);
    }
    p->sum = sum(p->ls) + sum(p->rs);
    return p;
}

ll query(ll L, ll R)
{
    return query(root.back(), L, R);
}

ll query(int ver, ll L, ll R)
{
    return query(root[ver], L, R);
}

ll query(pnode p, ll L, ll R)
{
    if (!p)
        return 0;
    ll l = p->l;
    ll r = p->r;
    if (L <= l && r <= R)
        return p->sum;
    if (l > R || L > r)
        return 0;

    ll mid = (l + r) >> 1;
    return query(p->ls, L, R) + query(p->rs, L, R);
}
};

```

```

int n = 9;
int A[] = {2, 4, 7, 10, 15, 23, 50, 65, 71}; // as in Chapter 2
ost tree;
for (int i = 0; i < n; ++i) // O(n log n)
    tree.insert(A[i]);
// O(log n) select
cout << *tree.find_by_order(0) << "\n"; // 1-smallest = 2
cout << *tree.find_by_order(n - 1) << "\n"; // 9-smallest/largest = 71

```

```

cout << *tree.find_by_order(4) << "\n"; // 5-smallest = 15
// O(log n) rank
cout << tree.order_of_key(2) << "\n"; // index 0 (rank 1)
cout << tree.order_of_key(71) << "\n"; // index 8 (rank 9)

```

#### 4.5. SEGMENT TREE LAZY.

```

class SegmentTreeLazy
{
private:
    vi values;
    vector<bool> lazy;
    vi l_values;
    vi p_values;
    int n;

    int left(int p) { return p << 1; };

    int right(int p) { return (p << 1) + 1; }

    int simple_node(int index) { return values[index]; }

    int prop(int x, int y) { return x + y; }

    int prop_lazy(int x, int y) { return x + y; }

    int prop_lazy_up(int x, int y, int s) { return x + y * s; }

    void update_lazy(int p, int l, int r)
    {
        if (l == r)
        {
            values[l] = prop_lazy(values[l], l_values[p]);
        }

        p_values[p] = prop_lazy_up(p_values[p], l_values[p], r - l + 1);
    }

    void propagate_lazy(int p, int l, int r)
    {
        lazy[p] = false;

        if (l == r)
            return;

        l_values[left(p)] = lazy[left(p)]

```

```

cout << tree.order_of_key(15) << "\n"; // index 4 (rank 5)
return 0;
}

```

```

        ? prop_lazy(l_values[left(p)], l_values[p])
        : l_values[p];
    l_values[right(p)] = lazy[right(p)]
        ? prop_lazy(l_values[right(p)], l_values[p])
        : l_values[p];

    lazy[left(p)] = true;
    lazy[right(p)] = true;
}

void build(int p, int l, int r)
{
    if (l == r)
    {
        p_values[p] = simple_node(l);
        return;
    }

    build(left(p), l, (l + r) / 2);
    build(right(p), (l + r) / 2 + 1, r);

    p_values[p] = prop(p_values[left(p)], p_values[right(p)]);
}

void set(int p, int l, int r, int i, int v)
{
    if (lazy[p])
    {
        update_lazy(p, l, r);
        propagate_lazy(p, l, r);
    }

    if (l == r)
    {
        values[l] = v;
        p_values[p] = simple_node(l);
        return;
    }

```

```

    if (i <= (l + r) / 2)
        set(left(p), l, (l + r) / 2, i, v);
    else
        set(right(p), (l + r) / 2 + 1, r, i, v);

    p_values[p] = prop(p_values[left(p)], p_values[right(p)]);
}

int query(int p, int l, int r, int lq, int rq)
{
    if (lazy[p])
    {
        update_lazy(p, l, r);
        propagate_lazy(p, l, r);
    }

    if (lq <= l && r <= rq)
        return p_values[p];

    int l1 = l, r1 = (l + r) / 2;
    int l2 = (l + r) / 2 + 1, r2 = r;

    if (l1 > rq || lq > r1)
        return query(right(p), l2, r2, lq, rq);
    if (l2 > rq || lq > r2)
        return query(left(p), l1, r1, lq, rq);

    int lt = query(left(p), l1, r1, lq, rq);
    int rt = query(right(p), l2, r2, lq, rq);

    return prop(lt, rt);
}

void set_rank(int p, int l, int r, int lq, int rq, int value)
{
    if (lazy[p])
    {
        update_lazy(p, l, r);
        propagate_lazy(p, l, r);
    }

    if (l > rq || lq > r)
        return;

    if (lq <= l && r <= rq)
    {
        lazy[p] = true;
        l_values[p] = value;
    }

```

```

        update_lazy(p, l, r);
        propagate_lazy(p, l, r);
        return;
    }

    set_rank(left(p), l, (l + r) / 2, lq, rq, value);
    set_rank(right(p), (l + r) / 2 + 1, r, lq, rq, value);

    p_values[p] = prop(p_values[left(p)], p_values[right(p)]);
}

int get(int p, int l, int r, int i)
{
    if (lazy[p])
    {
        update_lazy(p, l, r);
        propagate_lazy(p, l, r);
    }

    if (l == r)
        return values[i];

    if (i <= (l + r) / 2)
        return get(left(p), l, (l + r) / 2, i);

    return get(right(p), (l + r) / 2 + 1, r, i);
}

public:
    SegmentTreeLazy(vi &a)
    {
        values = a;
        n = a.size();
        p_values.assign(4 * n, 0);
        lazy.assign(4 * n, false);
        l_values.assign(4 * n, 0);
        build(1, 0, n - 1);
    }

    int query(int i, int j) { return query(1, 0, n - 1, i, j); }

    void set(int i, int v) { set(1, 0, n - 1, i, v); }

    void set_rank(int i, int j, int v) { set_rank(1, 0, n - 1, i, j, v); }

    int get(int i) { return get(1, 0, n - 1, i); }
};

```

## 4.6. AVL.

```

struct avl {
    int key;
    int height;
    int size;
    avl *left;
    avl *right;

    avl(int k) {
        key = k;
        height = 1;
        size = 1;
        left = NULL;
        right = NULL;
    }

    int getBalance() {
        int leftHeight = 0;
        int rightHeight = 0;

        if (left != NULL)
            leftHeight = left->height;

        if (right != NULL)
            rightHeight = right->height;

        return leftHeight - rightHeight;
    }

    void updateSize() {
        int leftSize = 0;
        int rightSize = 0;

        if (left != NULL)
            leftSize = left->size;

        if (right != NULL)
            rightSize = right->size;

        size = leftSize + rightSize + 1;
    }

    void updateHeight() {
        int leftHeight = 0;
        int rightHeight = 0;

```

```

        if (left != NULL)
            leftHeight = left->height;

        if (right != NULL)
            rightHeight = right->height;

        height = max(leftHeight, rightHeight) + 1;
    }

    avl *rotateLeft() {
        avl *newRoot = right;
        right = newRoot->left;
        newRoot->left = this;
        updateHeight();
        newRoot->updateHeight();
        return newRoot;
    }

    avl *rotateRight() {
        avl *newRoot = left;
        left = newRoot->right;
        newRoot->right = this;
        updateHeight();
        newRoot->updateHeight();
        return newRoot;
    }

    avl *balance() {
        updateHeight();
        updateSize();
        int balance = getBalance();

        if (balance == 2) {
            if (left->getBalance() < 0)
                left = left->rotateLeft();
            return rotateRight();
        }

        if (balance == -2) {
            if (right->getBalance() > 0)
                right = right->rotateRight();
            return rotateLeft();
        }
    }

```



```

    return this;
}

avl *insert(int k) {
    if (k < key) {
        if (left == NULL)
            left = new avl(k);
        else
            left = left->insert(k);
    }
    else {
        if (right == NULL)
            right = new avl(k);
        else
            right = right->insert(k);
    }

    return balance();
}

avl *findMin() {
    if (left == NULL)
        return this;
    else
        return left->findMin();
}

avl *removeMin() {
    if (left == NULL)
        return right;
    left = left->removeMin();
    return balance();
}

avl *remove(int k) {
    if (k < key)
        left = left->remove(k);
    else if (k > key)
        right = right->remove(k);
    else {
        avl *leftChild = left;
        avl *rightChild = right;

        delete this;

        if (rightChild == NULL)
            return leftChild;
    }
}

```

```

        avl *min = rightChild->findMin();
        min->right = rightChild->removeMin();
        min->left = leftChild;
        return min->balance();
    }

    return balance();
}

int getRank(int k) {
    if (k < key) {
        if (left == NULL)
            return 0;
        else
            return left->getRank(k);
    }
    else if (k > key) {
        if (right == NULL)
            return 1 + left->size;
        else
            return 1 + left->size + right->getRank(k);
    }
    else
        return left->size;
}

int getKth(int k) {
    if (k < left->size)
        return left->getKth(k);
    else if (k > left->size)
        return right->getKth(k - left->size - 1);
    else
        return key;
}

static avl *join(avl *left, avl *right) {
    if (left->height < right->height) {
        right->left = join(left, right->left);
        return right->balance();
    }
    else if (left->height > right->height) {
        left->right = join(left->right, right);
        return left->balance();
    }
    else {
        avl *min = right->findMin();

```

```

        min->right = right->removeMin();
        min->left = left;
        return min->balance();
    }
}

pair<avl *, avl *> split(int k) {
    if (k < key) {
        pair<avl *, avl *> p = left->split(k);
        left = p.second;
    }
}

```

#### 4.7. ABI.

```

class Abi
{
private:
    vi p;
    int _size;

    int ls_one(int i) { return i & (-i); }

public:
    Abi(int n)
    {
        _size = n;
        p.assign(n + 1, 0);
    }

    int rsq(int k)
    {
        int sum = 0;

        for (int i = k; i > 0; i -= ls_one(i))
    }
}

```

#### 4.8. MOS.

```

void remove(int idx); // TODO: remove value at idx from data structure
void add(int idx); // TODO: add value at idx from data structure
int get_answer(); // TODO: extract the current answer of the data structure

int block_size;

struct Query

```

```

        return {p.first, join(this, left)};
    }
    else {
        pair<avl *, avl *> p = right->split(k);
        right = p.first;
        return {join(this, right), p.second};
    }
}
};

```

```

    {
        sum += p[i];
    }

    return sum;
}

int sum(int a, int b) { return rsq(b) - rsq(a - 1); }

void adjust_sum(int k, int v)
{
    for (int i = k; i < p.size(); i += ls_one(i))
        p[i] += v;
}

int size()
{
    return _size;
}
};

```

```

{
    int l, r, idx;
    bool operator<(Query other) const
    {
        return make_pair(l / block_size, r) <
               make_pair(other.l / block_size, other.r);
    }
};

```

```
vector<int> mo_s_algorithm(vector<Query> queries)
{
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());

    // TODO: initialize data structure

    int cur_l = 0;
    int cur_r = -1;
    // invariant: data structure will always reflect the range [cur_l, cur_r]
    for (Query q : queries)
    {
        while (cur_l > q.l)
        {
            cur_l--;
            add(cur_l);
        }
        while (cur_r < q.r)
        {
```

```
            cur_r++;
            add(cur_r);
        }
        while (cur_l < q.l)
        {
            remove(cur_l);
            cur_l++;
        }
        while (cur_r > q.r)
        {
            remove(cur_r);
            cur_r--;
        }
        answers[q.idx] = get_answer();
    }
    return answers;
}
```

#### 4.9. Sqrt Decomposition.

```
struct sqd {
    int n;
    int b;
    vi a;
    vi bsum;
    sqd(vi &a) {
        n = a.size();
        b = sqrt(n);
        this->a = a;
        bsum.assign(b + 1, 0);
        for (int i = 0; i < n; i++)
            bsum[i / b] += a[i];
    }
    void update(int i, int v) {
```

```
        bsum[i / b] += v - a[i];
        a[i] = v;
    }
    int query(int l, int r) {
        int sum = 0;
        for (int i = l; i <= r; i++)
            if (i % b == 0 && i + b - 1 <= r) {
                sum += bsum[i / b];
                i += b - 1;
            } else
                sum += a[i];
        return sum;
    }
};
```

### 5. STRING

#### 5.1. SUFFIX ARRAY.

```
class SuffixArray
{
```

```
public:
    SuffixArray(string s)
    {
```

```

    n = s.size() + 1;
    s_value = s + "$";

    ra.assign(n, 0);
    sa.assign(n, 0);
    temp_ra.assign(n, 0);
    temp_sa.assign(n, 0);

    construct_sa();
    build_lcp();
}

int size() { return n; }

int get_int(int i) { return sa[i]; }

int cant_match(string p)
{
    pii ans = matching(p);

    if (ans.first == -1 && ans.second == -1)
        return 0;

    return ans.second - ans.first + 1;
}

int get_lcp(int i) { return plcp[sa[i]]; }

int cant_substr() { return v_cant_substr; }

string get_str(int i) { return s_value.substr(sa[i], n - sa[i] - 1); }

private:
string s_value;
int n;
int v_cant_substr;

vi ra;
vi sa;
vi c;
vi temp_ra;
vi temp_sa;
vi phi;
vi plcp;

void counting_sort(int k)
{

```

```

    int sum = 0;
    int maxi = max((int)300, n);

    c.assign(maxi, 0);

    for (int i = 0; i < n; i++)
        c[i + k < n ? ra[i + k] : 0]++;

    for (int i = 0; i < maxi; i++)
    {
        int tx = c[i];
        c[i] = sum;
        sum += tx;
    }

    for (int i = 0; i < n; i++)
        temp_sa[c[sa[i] + k < n ? ra[sa[i] + k] : 0]++] = sa[i];

    for (int i = 0; i < n; i++)
        sa[i] = temp_sa[i];
}

void construct_sa()
{
    int k, r;

    for (int i = 0; i < n; i++)
    {
        ra[i] = s_value[i];
        sa[i] = i;
    }

    for (k = 1; k < n; k <= 1)
    {
        counting_sort(k);
        counting_sort(0);

        temp_ra[sa[0]] = r = 0;

        for (int i = 1; i < n; i++)
            temp_ra[sa[i]] = (ra[sa[i]] ==
                ra[sa[i - 1]] && ra[sa[i] + k] == ra[sa[i - 1] + k]) ? r : ++r;

        for (int i = 0; i < n; i++)
            ra[i] = temp_ra[i];

        if (ra[sa[n - 1]] == n - 1)

```

```

        break;
    }
}

pii matching(string p)
{
    int l = 0;
    int r = n - 1;
    int p_size = p.size();

    string comp;

    while (l < r)
    {
        int m = (l + r) / 2;

        comp = s_value.substr(sa[m], min(n - sa[m], p_size));

        if (comp >= p)
            r = m;
        else
            l = m + 1;
    }

    comp = s_value.substr(sa[l], min(n - sa[l], p_size));

    if (comp != p)
        return {-1, -1};

    int ans_l = 1;

    l = 0;
    r = n - 1;

    while (l < r)
    {
        int m = (l + r) / 2;

        comp = s_value.substr(sa[m], min(n - sa[m], p_size));

        if (comp > p)
            r = m;
        else
            l = m + 1;
    }

```

```

    }

    comp = s_value.substr(sa[r], min(n - sa[r], p_size));

    if (comp != p)
        r--;

    int ans_r = r;

    return {ans_l, ans_r};
}

void build_lcp()
{
    phi.assign(n, 0);
    plcp.assign(n, 0);

    phi[0] = -1;

    for (int i = 1; i < n; i++)
        phi[sa[i]] = sa[i - 1];

    int l = 0;
    int q = 0;
    for (int i = 0; i < n; i++)
    {
        if (phi[i] == -1)
        {
            plcp[i] = 0;
            continue;
        }

        while (s_value[i + 1] == s_value[phi[i] + 1])
            l++;

        plcp[i] = l;
        q += l;
        l = max(l - 1, (int)0);
    }

    v_cant_substr = n * (n - 1) / 2 - q;
}
};

```

## 5.2. Z FUNCTION.

```
// Z[i] is the length of the longest substring
// starting from S[i] which is also a prefix of S.
vi z_function(string s)
{
    int n = (int)s.length();
    vi z(n);

    for (int i = 1, l = 0, r = 0; i < n; ++i)
    {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}

// suff[i] = length of the longest common suffix of s and s[0..i]
```

## 5.3. HASHING.

```
struct custom_hash
{
    static uint64_t splitmix64(uint64_t x)
    {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const
    {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};

const int MAXN = 5e5 + 5;
const int MOD = 1e9 + 7;
```

```
vi suffixes(const string &s)
{
    int n = s.length();

    vi suff(n, n);

    for (int i = n - 2, g = n - 1, f; i >= 0; --i)
    {
        if (i > g && suff[i + n - 1 - f] != i - g)
            suff[i] = min(suff[i + n - 1 - f], i - g);
        else
        {
            for (g = min(g, f = i); g >= 0 && s[g] == s[g + n - 1 - f]; --g)
                ;
            suff[i] = f - g;
        }
    }

    return suff;
}
```

```
int mod_pow(int b, int e)
{
    int res = 1;
    while (e)
    {
        if (e & 1)
            res = res * b % MOD;
        b = b * b % MOD;
        e >>= 1;
    }
    return res % MOD;
}

typedef unordered_map<ll, ll, custom_hash> safe_map;

int dx[] = {1, 0, -1, 0, 1, -1, -1, 1};
int dy[] = {0, 1, 0, -1, 1, 1, -1, -1};

int dp[MAXN];
vi ady_list[MAXN];
```

```

int n, m, t, k;
int f[MAXN];
int comb(int n, int k)
{
    int fk = mod_pow(f[k], MOD - 2);

```

#### 5.4. AHO CORASIK.

```

struct Vertex
{
    int next[K];
    bool output = false;
    int p = -1;
    char pch;
    int link = -1;
    int go[K];

    Vertex(int p = -1, char ch = '$') : p(p), pch(ch)
    {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};

vector<Vertex> t(1);

void add_string(string const &s)
{
    int v = 0;
    for (char ch : s)
    {
        int c = ch - 'a';
        if (t[v].next[c] == -1)
        {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
}

```

#### 5.5. KMP PF.

```

vi prefix_function(string p)
{

```

```

    int fnk = mod_pow(f[n - k], MOD - 2);
    int x = ((f[n] * fk) % MOD * fnk % MOD);

    return x;
}

```

```

    }
    t[v].output = true;
}

int go(int v, char ch);

int get_link(int v)
{
    if (t[v].link == -1)
    {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}

int go(int v, char ch)
{
    int c = ch - 'a';
    if (t[v].go[c] == -1)
    {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}

```

```

vi pf(p.size());

```

```

pf[0] = 0;
int k = 0;

for (int i = 1; i < p.size(); i++)
{
    while (k > 0 && p[k] != p[i])
        k = pf[k - 1];

    if (p[k] == p[i])
        k++;

    pf[i] = k;
}

return pf;
}

vi kmp(string t, string p)
{
    vi result;

```

```

vi pf = prefix_function(p);
int k = 0;

for (int i = 0; i < t.size(); i++)
{
    while (k > 0 && p[k] != t[i])
        k = pf[k - 1];

    if (p[k] == t[i])
        k++;

    if (k == p.size())
    {
        result.push_back(i - (p.size() - 1));
        k = pf[k - 1];
    }
}

return result;
}

```

## 5.6. TRIE.

```

class Trie
{
private:
    int cant_string;
    int cant_string_me;
    int cant_node;
    char value;
    Trie *children[alphabet];

public:
    Trie(char a)
    {
        cant_string = 0;
        cant_node = 1;
        cant_string_me = 0;
        value = a;

        for (int i = 0; i < alphabet; i++)
            children[i] = NULL;
    }

    pair<Trie *, int> search(string s)
    {

```

```

        Trie *node = this;
        int i = 0;

        while (i < s.size() && node->children[s[i] - first_char] != NULL)
        {
            node = node->children[s[i] - first_char];

            i++;
        }

        return {node, i};
    }

    void insert(string s)
    {
        int q = s.size() - search(s).second;

        Trie *node = this;

        for (int i = 0; i < s.size(); i++)
        {
            node->cant_node += q;

```



```

    if (node->children[s[i] - first_char] == NULL)
    {
        node->children[s[i] - first_char] = new Trie(s[i]);
        q--;
    }

    node = node->children[s[i] - first_char];
    node->cant_string_me++;
}

node->cant_string++;
}

void eliminate(string s)
{
    if (!contains(s))
        return;

    Trie *node = this;
    int q = 0;

    for (int i = 0; i < s.size(); i++)
    {
        if (node->children[s[i] - first_char] == NULL)
        {
            node->children[s[i] - first_char] = new Trie(s[i]);
        }

        if (node->children[s[i] - first_char]->cant_string_me == 1)
        {
            node->children[s[i] - first_char] = NULL;

            q = s.size() - i;

```

```

        break;
    }

    node = node->children[s[i] - first_char];
    node->cant_string_me--;

    if (i == s.size() - 1)
        node->cant_string--;
}

node = this;

for (int i = 0; i < s.size() - q + 1; i++)
{
    node->cant_node -= q;
    node = node->children[s[i] - first_char];
}

bool contains(string s)
{
    auto q = search(s);
    return q.second == s.size() && q.first->cant_string >= 1;
}

int cant_words_me() { return cant_string_me; }

int cant_words() { return cant_string; }

Trie *get(char a) { return children[a - first_char]; }

int size() { return cant_node; }
};

```

## 6. GRAPH

### 6.1. TOPOLOGICAL SORT.

```

vector<int> topoSort(int V, vector<int> adj[])
{
    vector<int> in(V);
    vector<int> resp;

    for (int i = 0; i < V; i++)
    {
        for (int j = 0; j < adj[i].size(); j++)

```

```

        {
            in[adj[i][j]]++;
        }
    }

    queue<int> q;

    for (int i = 0; i < V; i++)

```

```

{
    if (in[i] == 0)
        q.push(i);
}

while (q.size() != 0)
{
    int n = q.front();
    q.pop();

    for (int i = 0; i < adj[n].size(); i++)
    {

```

```

        in[adj[n][i]]--;

        if (in[adj[n][i]] == 0)
            q.push(adj[n][i]);
    }

    resp.push_back(n);
}

return resp;
}

```

## 6.2. DIJKSTRA.

```

int infinite = (int)1e9;

//  $O(V^2)$ 
vector<int> dijkstral(int V, vector<vector<int>> adj[], int S)
{
    vector<int> d;

    d.assign(V, infinite);
    d[S] = 0;

    vector<bool> mask;

    mask.assign(V, false);

    for (int i = 0; i < V; i++)
    {
        int m = infinite;
        int act = -1;

        for (int j = 0; j < V; j++)
        {
            if (mask[j])
                continue;

            if (m > d[j])
            {
                m = d[j];
                act = j;
            }
        }
    }
}

```

```

        for (int j = 0; j < adj[act].size(); j++)
        {
            if (d[act] + adj[act][j][1] < d[adj[act][j][0]])
            {
                d[adj[act][j][0]] = d[act] + adj[act][j][1];
            }
        }

        mask[act] = true;
    }

    return d;
}

//  $O((V+E) \log(E))$ 
vi dijkstra2(int V, vii adj[], int S)
{
    vector<int> dist(V, infinite);
    dist[S] = 0;

    priority_queue<pair<int, int>> q;
    q.emplace(0, S);

    while (!q.empty())
    {
        auto [d, u] = q.top();
        q.pop();

        if (d > dist[u])
            continue;
    }
}

```

```

    for (auto &[v, w] : adj[u])
    {
        if (dist[u] + w >= dist[v])
            continue;

        dist[v] = dist[u] + w;
        q.emplace(dist[v], v);
    }

    return dist;
}

void solve()
{
}

```

### 6.3. BRIDGE EDGES.

```

vector<bool> visited;
vector<int> t;
vector<int> low;
set<pair<int, int>> bridges;

void dfs_bridges(vector<int> adj[], int n, int p, int q)
{
    t[n] = q;
    low[n] = q++;
    visited[n] = true;

    int j = 0;

    for (int i = 0; i < adj[n].size(); i++)
    {
        if (!visited[adj[n][i]])
        {
            dfs_bridges(adj, adj[n][i], n, q);
            low[n] = min(low[adj[n][i]], low[n]);
            j++;
        }
        else if (adj[n][i] != p)
        {
            low[n] = min(t[adj[n][i]], low[n]);
        }
    }
}

```

```

int main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    int t;
    cin >> t;

    for (int i = 0; i < t; i++)
    {
        solve();
    }

    return 0;
}

}

if (t[n] == low[n] && p != -1)
{
    bridges.insert({min(n, p), max(n, p)});
}

}

set<pair<int, int>> bridge_edges(int V, vector<int> adj[])
{
    visited.assign(V, false);
    t.assign(V, -1);
    low.assign(V, -1);
    bridges = set<pair<int, int>>();

    for (int i = 0; i < V; i++)
    {
        if (!visited[i])
        {
            dfs_bridges(adj, i, -1, 1);
        }
    }

    return bridges;
}

```

## 6.4. PRIM.

```

int spanningTreePrim(int V, vector<vector<int>> adj[])
{
    priority_queue<pair<int, int>> q;

    vector<bool> mask;
    mask.assign(V, false);
    mask[0] = true;

    int cost = 0;

    for (int i = 0; i < adj[0].size(); i++)
    {
        q.push({-adj[0][i][1], adj[0][i][0]});
    }

    while (q.size() != 0)
    {
        auto aux = q.top();
        q.pop();

```

```

        int k = aux.second;
        if (mask[k])
            continue;

        mask[k] = true;
        cost += abs(aux.first);

        for (int i = 0; i < adj[k].size(); i++)
        {
            if (!mask[adj[k][i][0]])
            {
                q.push({-adj[k][i][1], adj[k][i][0]});
            }
        }
    }

    return cost;
}

```

## 6.5. KRUSKAL.

```

class udfs
{
private:
    vector<int> p, rank, sizeSet;
    int disjointSet;

public:
    udfs(int n)
    {
        p.assign(n, 0);
        rank.assign(n, 0);
        sizeSet.assign(n, 1);
        disjointSet = n;
        for (int i = 0; i < n; i++)
        {
            p[i] = i;
        }
    }

    int find(int n)

```

```

{
    if (n == p[n])
        return n;
    p[n] = find(p[n]);
    return p[n];
}

bool isSameSet(int i, int j) { return find(i) == find(j); }

void unionSet(int i, int j)
{
    if (!isSameSet(i, j))
    {
        disjointSet--;
        int x = find(i);
        int y = find(j);
        if (rank[x] > rank[y])
        {
            p[y] = x;
            sizeSet[x] += sizeSet[y];

```

```

    }
    else
    {
        p[x] = y;
        sizeSet[y] += sizeSet[x];
        if (rank[x] == rank[y])
            rank[y]++;
    }
}

int numDisjoinset() { return disjoinSet; }

int sizeofSet(int i) { return sizeSet[find(i)]; }
};

// Function to find sum of weights of edges of the Minimum Spanning Tree.
int spanningTreeKruskal(int V, vector<vector<int>>> adj[])
{
    ufds dsu(V);

    vector<pair<int, pair<int, int>>> a;

    for (int i = 0; i < V; i++)

```

```

{
    for (int j = 0; j < adj[i].size(); j++)
    {
        a.push_back({adj[i][j][1], {i, adj[i][j][0]}});
    }
}

sort(a.begin(), a.end());

int cost = 0;

for (int i = 0; i < a.size(); i++)
{
    if (!dsu.isSameSet(a[i].second.first, a[i].second.second))
    {
        cost += a[i].first;

        dsu.unionSet(a[i].second.first, a[i].second.second);
    }
}

return cost;
}

```

## 6.6. BELLMAN FORD.

```

int infinite = (int)1e9;

vector<int> bellman_ford(int V, vector<vector<int>>> &edges, int S)
{
    vector<int> d;
    d.assign(V, infinite);
    d[S] = 0;

    for (int i = 0; i < V - 1; i++)
    {
        for (int j = 0; j < edges.size(); j++)
        {
            if (d[edges[j][0]] + edges[j][2] < d[edges[j][1]])
            {
                d[edges[j][1]] = d[edges[j][0]] + edges[j][2];
            }
        }
    }
}

```

```

    }
}

for (int j = 0; j < edges.size(); j++)
{
    if (d[edges[j][0]] + edges[j][2] < d[edges[j][1]])
    {
        vector<int> resp(1);
        resp[0] = -1;

        return resp;
    }
}

return d;
}

```

## 6.7. DINIC.

```

template <typename flow_type>
struct dinic
{
    struct edge
    {
        size_t src, dst, rev;
        flow_type flow, cap;
    };

    int n;
    vector<vector<edge>> adj;

    dinic(int n) : n(n), adj(n), level(n), q(n), it(n) {}

    void add_edge(size_t src, size_t dst, flow_type cap, flow_type rcap = 0)
    {
        adj[src].push_back({src, dst, adj[dst].size(), 0, cap});
        if (src == dst)
            adj[src].back().rev++;
        adj[dst].push_back({dst, src, adj[src].size() - 1, 0, rcap});
    }

    vector<int> level, q, it;

    bool bfs(int source, int sink)
    {
        fill(level.begin(), level.end(), -1);
        for (int qf = level[q[0] = sink] = 0, qb = 1; qf < qb; ++qf)
        {
            sink = q[qf];
            for (edge &e : adj[sink])
            {
                edge &r = adj[e.dst][e.rev];
                if (r.flow < r.cap && level[e.dst] == -1)
                    level[q[qb++] = e.dst] = 1 + level[sink];
            }
        }
        return level[source] != -1;
    }
};

```

```

flow_type augment(int source, int sink, flow_type flow)
{
    if (source == sink)
        return flow;
    for (; it[source] != adj[source].size(); ++it[source])
    {
        edge &e = adj[source][it[source]];
        if (e.flow < e.cap && level[e.dst] + 1 == level[source])
        {
            flow_type delta = augment(e.dst, sink, min(flow, e.cap - e.flow));
            if (delta > 0)
            {
                e.flow += delta;
                adj[e.dst][e.rev].flow -= delta;
                return delta;
            }
        }
    }
    return 0;
}

flow_type max_flow(int source, int sink)
{
    for (int u = 0; u < n; ++u)
        for (edge &e : adj[u])
            e.flow = 0;
    flow_type flow = 0;
    flow_type oo = numeric_limits<flow_type>::max();

    while (bfs(source, sink))
    {
        fill(it.begin(), it.end(), 0);
        for (flow_type f; (f = augment(source, sink, oo)) > 0;)
            flow += f;
    } // level[u] = -1 => source side of min cut
    return flow;
};

```

## 6.8. CENTROID DESCOMPOSITION.

```

const int MAXN = 2e5 + 5;

vi ady[MAXN];

bitset<MAXN> is_centroid;

int sz[MAXN], ct_par[MAXN];
void centroid_dfs(int node, int parent)
{
    sz[node] = 1;
    for (int &nxt : ady[node])
    {
        if (is_centroid[nxt] || nxt == parent)
            continue;
        centroid_dfs(nxt, node);
        sz[node] += sz[nxt];
    }
}

int get_centroid(int node, int parent, int tree_sz)
{
    for (int nxt : ady[node])
    {
        if (is_centroid[nxt] || nxt == parent)
            continue;
        if (sz[nxt] * 2 > tree_sz)
            return get_centroid(nxt, node, tree_sz);
    }
    return node;
}

void centroid_decomp(int node, int parent = -1)
{
    centroid_dfs(node, -1);

```

```

    int tree_sz = sz[node];
    int centroid = get_centroid(node, -1, tree_sz);
    is_centroid[centroid] = 1;
    ct_par[centroid] = parent;

    for (int &child : ady[centroid])
    {
        if (is_centroid[child])
            continue;
        centroid_decomp(child, centroid);
    }
}

void solve()
{
}

int32_t main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    int t;
    cin >> t;

    for (int i = 0; i < t; i++)
    {
        solve();
    }

    return 0;
}

```

## 6.9. LOWER BOUND FLOW.

```

template <typename T>
struct dinic
{
    struct edge
    {
        int src, dst;
        T low, cap, flow;
    };

```

```

        int rev;
    };

    int n;
    vector<vector<edge>> adj;

    dinic(int n) : n(n), adj(n + 2) {}

```

```

void add_edge(int src, int dst, T low, T cap)
{
    adj[src].push_back({src, dst, low, cap, 0, (int)adj[dst].size()});
    if (src == dst)
        adj[src].back().rev++;
    adj[dst].push_back({dst, src, 0, 0, 0, (int)adj[src].size() - 1});
}

vector<int> level, iter;

T augment(int u, int t, T cur)
{
    if (u == t)
        return cur;
    for (int &i = iter[u]; i < (int)adj[u].size(); ++i)
    {
        edge &e = adj[u][i];
        if (e.cap - e.flow > 0 && level[u] > level[e.dst])
        {
            T f = augment(e.dst, t, min(cur, e.cap - e.flow));
            if (f > 0)
            {
                e.flow += f;
                adj[e.dst][e.rev].flow -= f;
                return f;
            }
        }
    }
    return 0;
}

int bfs(int s, int t)
{
    level.assign(n + 2, n + 2);
    level[t] = 0;
    queue<int> Q;
    for (Q.push(t); !Q.empty(); Q.pop())
    {
        int u = Q.front();
        if (u == s)
            break;
        for (edge &e : adj[u])
        {
            edge &erev = adj[e.dst][e.rev];
            if (erev.cap - erev.flow > 0 && level[e.dst] > level[u] + 1)
            {
                Q.push(e.dst);
            }
        }
    }
}

```

```

        level[e.dst] = level[u] + 1;
    }
}
return level[s];
}

const T oo = numeric_limits<T>::max();

T max_flow(int source, int sink)
{
    vector<T> delta(n + 2);

    for (int u = 0; u < n; ++u) // initialize
        for (auto &e : adj[u])
        {
            delta[e.src] -= e.low;
            delta[e.dst] += e.low;
            e.cap -= e.low;
            e.flow = 0;
        }

    T sum = 0;
    int s = n, t = n + 1;

    for (int u = 0; u < n; ++u)
    {
        if (delta[u] > 0)
        {
            add_edge(s, u, 0, delta[u]);
            sum += delta[u];
        }
        else if (delta[u] < 0)
            add_edge(u, t, 0, -delta[u]);
    }

    add_edge(sink, source, 0, oo);
    T flow = 0;

    while (bfs(s, t) < n + 2)
    {
        iter.assign(n + 2, 0);
        for (T f; (f = augment(s, t, oo)) > 0;)
            flow += f;
    }

    if (flow != sum)

```



```

    return -1; // no solution

    for (int u = 0; u < n; ++u)
        for (auto &e : adj[u])
        {
            e.cap += e.low;
            e.flow += e.low;
            edge &erev = adj[e.dst][e.rev];
            erev.cap -= e.low;
            erev.flow -= e.low;
        }

    adj[sink].pop_back();

```

## 6.10. FLOYD WARSHALL.

```

int infinite = (int)1e8;

void shortest_distance(vector<vector<int>> &matrix)
{
    for (int k = 0; k < matrix.size(); k++)
    {
        for (int i = 0; i < matrix.size(); i++)
        {
            for (int j = 0; j < matrix[0].size(); j++)
            {
                matrix[i][j] = min(matrix[i][j], matrix[i][k] + matrix[k][j]);
            }
        }
    }
}

```

## 6.11. DFS BFS.

```

void dfs_g(int n, int c, vi adj[], vector<bool> &visited, vi &cc)
{
    visited[n] = true;
    cc[n] = c;

    for (int i = 0; i < adj[n].size(); i++)
    {
        if (!visited[adj[n][i]])
            dfs_g(adj[n][i], c, adj, visited, cc);
    }
}

```

```

adj[source].pop_back();

while (bfs(source, sink) < n + 2)
{
    iter.assign(n + 2, 0);
    for (T f; (f = augment(source, sink, oo)) > 0;)
        flow += f;
} // level[u] == n + 2 ==> s-side

return flow;
}
};

```

```

void find_path_k(vector<vector<bool>> &matrix, int k)
{
    for (int x = 0; x < k; x++)
    {
        for (int i = 0; i < matrix.size(); i++)
        {
            for (int j = 0; j < matrix[0].size(); j++)
            {
                matrix[i][j] = matrix[i][j] || (matrix[i][x] && matrix[x][j]);
            }
        }
    }
}

```

```

void dfs_t(int n, int p, int d, vi adj[], vi &deep)
{
    deep[n] = d;

    for (int i = 0; i < adj[n].size(); i++)
    {
        if (p != adj[n][i])
            dfs_t(adj[n][i], n, d + 1, adj, deep);
    }
}

```

```

vi bfs(int node, int n, vi adj[])
{
    vi result(n);
    vector<bool> visited;
    visited.assign(n, false);

    queue<int> q;
    visited[node] = true;

    q.push(node);

    while (q.size() != 0)
    {
        int w = q.front();

```

```

        q.pop();

        for (int i = 0; i < adj[w].size(); i++)
        {
            if (!visited[adj[w][i]])
            {
                q.push(adj[w][i]);
                result[adj[w][i]] = result[w] + 1;
                visited[adj[w][i]] = true;
            }
        }
    }

    return result;
}

```

## 6.12. SCC TARJANS.

```

stack<int> q;
vector<bool> mask;
vector<int> cc_list;

void g_transp(int V, vector<int> adj[], vector<int> new_adj[])
{
    for (int i = 0; i < V; i++)
    {
        for (int j = 0; j < adj[i].size(); j++)
        {
            new_adj[adj[i][j]].push_back(i);
        }
    }
}

void dfs_visit(int n, vector<int> adj[], int cc)
{
    mask[n] = true;

    for (int i = 0; i < adj[n].size(); i++)
    {
        if (!mask[adj[n][i]])
            dfs_visit(adj[n][i], adj, cc);
    }

    if (cc == -1)
        q.push(n);
    else

```

```

    {
        cc_list[n] = cc;
    }
}

void tarjans(int V, vector<int> adj[])
{
    vector<int> new_adj[V];
    g_transp(V, adj, new_adj);

    mask.assign(V, false);
    cc_list.assign(V, -1);

    for (int i = 0; i < V; i++)
    {
        if (mask[i])
            continue;

        dfs_visit(i, adj, -1);
    }

    for (int i = 0; i < V; i++)
        mask[i] = false;

    int ind = 0;

    while (q.size() != 0)
    {

```

```

int act = q.top();

q.pop();

if (!mask[act])
{

```

```

    dfs_visit(act, new_adj, ind);
    ind++;
}
}
}

```

### 6.13. MIN COST MAX FLOW.

```

template <typename flow_type, typename cost_type>
struct min_cost_max_flow
{
    struct edge
    {
        size_t src, dst, rev;
        flow_type flow, cap;
        cost_type cost;
    };

    int n;
    vector<vector<edge>> adj;

    min_cost_max_flow(int n) : n(n), adj(n), potential(n), dist(n), back(n) {}

    void add_edge(size_t src, size_t dst, flow_type cap, cost_type cost)
    {
        adj[src].push_back({src, dst, adj[dst].size(), 0, cap, cost});
        if (src == dst)
            adj[src].back().rev++;
        adj[dst].push_back({dst, src, adj[src].size() - 1, 0, 0, -cost});
    }

    vector<cost_type> potential;

    inline cost_type rcost(const edge &e)
    {
        return e.cost + potential[e.src] - potential[e.dst];
    }

    void bellman_ford(int source)
    {
        for (int k = 0; k < n; ++k)
            for (int u = 0; u < n; ++u)
                for (edge &e : adj[u])
                    if (e.cap > 0 && rcost(e) < 0)

```

```

        potential[e.dst] += rcost(e);
    }

    const cost_type oo = numeric_limits<cost_type>::max();

    vector<cost_type> dist;
    vector<edge *> back;

    cost_type dijkstra(int source, int sink)
    {
        fill(dist.begin(), dist.end(), oo);

        typedef pair<cost_type, int> node;
        priority_queue<node, vector<node>, greater<node>> pq;

        for (pq.push({dist[source] = 0, source}); !pq.empty();)
        {
            node p = pq.top();
            pq.pop();

            if (dist[p.second] < p.first)
                continue;
            if (p.second == sink)
                break;

            for (edge &e : adj[p.second])
                if (e.flow < e.cap && dist[e.dst] >
                    dist[e.src] + rcost(e))
                {
                    back[e.dst] = &e;
                    pq.push({dist[e.dst] =
                        dist[e.src] + rcost(e), e.dst});
                }
        }

        return dist[sink];
    }
}

```

```

pair<flow_type, cost_type> max_flow(int source, int sink)
{
    flow_type flow = 0;
    cost_type cost = 0;

    for (int u = 0; u < n; ++u)
        for (edge &e : adj[u])
            e.flow = 0;

    potential.assign(n, 0);
    dist.assign(n, 0);
    back.assign(n, nullptr);

    bellman_ford(source); // remove negative costs

    while (dijkstra(source, sink) < oo)
    {

```

```

        for (int u = 0; u < n; ++u)
            if (dist[u] < dist[sink])
                potential[u] += dist[u] - dist[sink];

        flow_type f = numeric_limits<flow_type>::max();

        for (edge *e = back[sink]; e; e = back[e->src])
            f = min(f, e->cap - e->flow);
        for (edge *e = back[sink]; e; e = back[e->src])
            e->flow += f, adj[e->dst][e->rev].flow -= f;

        flow += f;
        cost += f * (potential[sink] - potential[source]);
    }
    return {flow, cost};
};

```

#### 6.14. ARTICULATION POINT.

```

vector<bool> visited;
vi t;
vi low;
vector<bool> art;

void dfs_art(vi adj[], int n, int p, int q)
{
    t[n] = q;
    low[n] = q++;
    visited[n] = true;

    int j = 0;

    for (int i = 0; i < adj[n].size(); i++)
    {
        if (!visited[adj[n][i]])
        {
            dfs_art(adj, adj[n][i], n, q);
            low[n] = min(low[adj[n][i]], low[n]);
            j++;

            if (low[adj[n][i]] >= t[n] && p != -1)
            {
                art[n] = true;
            }
        }
    }
}

```

```

    }
    else if (adj[n][i] != p)
    {
        low[n] = min(t[adj[n][i]], low[n]);
    }
}

if (p == -1)
{
    art[n] = j >= 2;
}

}

void articulationPoints(int V, vi adj[])
{
    visited.assign(V, false);
    t.assign(V, -1);
    low.assign(V, -1);
    art.assign(V, false);

    for (int i = 0; i < V; i++)
    {
        if (!visited[i])
        {
            dfs_art(adj, i, -1, 1);
        }
    }
}

```

```

    }
}

```

## 6.15. KUHN.

```

const int MAXN = 2 * 1e5 + 5;

int n, m;

vi mt;
vi ady[MAXN];
vector<bool> used;
vector<bool> visitedx;
vector<bool> visitedy;

vector<bool> coverx;
vector<bool> covery;

bool try_kuhn(int v)
{
    if (used[v])
        return false;
    used[v] = true;
    for (int to : ady[v])
    {
        if (mt[to] == -1 || try_kuhn(mt[to]))
        {
            mt[to] = v;
            return true;
        }
    }
    return false;
}

void augm_path(int n, bool isX)
{
    if (isX)
        visitedx[n] = true;
    else
        visitedy[n] = true;

    if (!isX)
    {
        if (!visitedx[mt[n]])
            augm_path(mt[n], true);
    }
}

```

```

}

```

```

    return;
}

for (auto i : ady[n])
    if (!visitedy[i])
        augm_path(i, false);
}

int32_t main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    cin >> n >> m;
    forl(i, 1, m)
    {
        int a, b;
        cin >> a >> b;
        ady[a].pb(b);
    }
    int max_matching = 0;

    mt.assign(m + 1, -1);
    int ans = 0;
    for (int v = 1; v <= n; v++)
    {
        used.assign(n + 1, false);
        if (try_kuhn(v))
            max_matching++;
    }

    // vertex cover

    visitedx.assign(n + 1, false);
    visitedy.assign(m + 1, false);

    used.assign(n + 1, false);
    for (int v = 1; v <= m; v++)
        if (mt[v] != -1)
            used[mt[v]] = true;
}

```

```
for (int v = 1; v <= n; v++)
    if (!used[v] && !visitedx[v])
        augm_path(v, true);

coverx.assign(n + 1, false);
covery.assign(m + 1, false);

for (int v = 1; v <= n; v++)
    if (!visitedx[v])
```

```
        coverx[v] = true;

for (int v = 1; v <= m; v++)
    if (visitedy[v])
        covery[v] = true;

cout << max_matching << '\n';
return 0;
}
```