6.058 PSET 1

IAP 2018

Due Night of January 10th. Upon completion send finished pset to charlesf@mit.edu

Problem 1

An RC circuit has the impulse response

$$\frac{1}{RC}e^{-\frac{n}{RC}}u[n]$$

Find the unit step response of this RC circuit. Hint: An RC circuit is made of linear components.

Note: This problem gives an amplitude that is not equal to 1, this is because we are approximating this continuous time system as a discrete time system which causes approximation errors.

Problem 2

Prove that the following system is LTI:

$$y[n] + 4y[n-1] = 3x[n] - 2x[n-1]$$

Problem 3

Given the system from Problem 2, find the unit step response.

Problem 4

In general, we can write $\frac{d}{dt}x(t)$ in discrete time as $\frac{x[n]-x[n-1]}{T}$ where T is the sampling rate/ sampling frequency.

Write the following differential equation as it's discrete difference equation varient:

$$\frac{d}{dt}y(t) + 3y(t) = \frac{d^2}{dt^2}x(t) + 2\frac{d}{dt}x(t) + 5x(t)$$

Problem 5

Prove the following:

$$h[n] = s[n] - s[n-1]$$

where $h[n]$ is the impulse response and $s[n]$ is the unit step response.

Also prove the continuous time version of this problem:

$$h(t) = \frac{d}{dt}s(t)$$

Problem 6

Prove that the following is true where x_{even} and x_{odd} are the even and odd parts of the signal. Note it does not matter if the signal is discrete or continuous.

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_{even}^2[n] + \sum_{n=-\infty}^{\infty} x_{odd}^2[n]$$

Problem 7

Find the convolution of the following two signals

$$x[n] = 2^n u[-n] \quad h[n] = u[n]$$

Problem 8

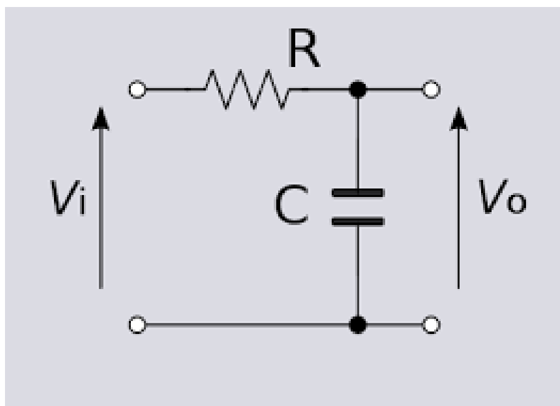
What is the cross-correlation of $\sin(7x)$ and $\sin(5x)$? What is the cross-correlation of $\sin(5x)$ and $\sin(5x)$? Why the different result?

Problem 9

If I have an audio track with a bunch of musical instruments playing, usually the most dominant with be the drum. What will result when we perform the cross-correlation on this piece of music? How does this benefit us and what information does it contain?

RC Circuis

Problem 1 has an equation for how an RC circuit works. It is pretty important that you understand where this equation comes form. The circuit looks a bit like this:



Wikipedia (and possibly my Tech degree) informs me that the equations that govern a resistor and a capacitor are:

$$V_r = I_r R \quad I_c = C\frac{dV_c}{dt}$$

We see that since current has nowhere else to flow (current flows in loops) we must have $I_c = I_r$ or $\frac{V_r}{R} = C\frac{dV_c}{dt}$
We also have that $V_i = V_r + V_c$ since the input voltage is equal to the voltage across the resistor and the capacitor.
Combining equations and manipulating we find that $\frac{dt}{RC} = \frac{dV_c}{V_i - V_c}$.
Solving we find that $\frac{t}{RC} = -ln(V_i - V_c) + K$. We note that the output voltage is the same as the voltage across the capacitor and that at $t = 0^-$ we have that $V_i = 0$ and $V_c = 0$ since no current is flowing.
Another important fact to note is that a capacitor's current responses instantly to a change in voltage. However, the voltage itself can not instantly change, it must be a continuous function.
We must have that $\frac{t}{RC} = \ln(\frac{V_i - V_{0+}}{V_i - V_o})$ where V_{0+} is the voltage across the capacitor instantly after the impulse is sent into the circuit and V_o is the output voltage of the circuit (also the same as $V_c(t)$.
We shall model the impulse in the following way: we shall take the limit of the above equation as time goes to zero, and also set $V_i = \frac{1}{t}$ since this diverges as t goes to zero.
Our equation that we are considering now is

$$\lim_{t \to 0^+} \frac{1}{t}(1 - e^{\frac{-t}{RC}}) = V_o$$

Using L'hopital's rule (the rule you learned and never had a practical use for until this moment) we find that this solves to

$$\frac{1}{RC} = V_o(t = 0^+)$$

So, going back to the equation

$$V_o = V_i(1 - e^{\frac{-t}{RC}})$$

We find that an expression for the output voltage is

$$V_o = \frac{1}{RC}(1 - e^{\frac{-t}{RC}})$$