2018 6.058 PSET 3

Alexander Sludds

IAP 2018

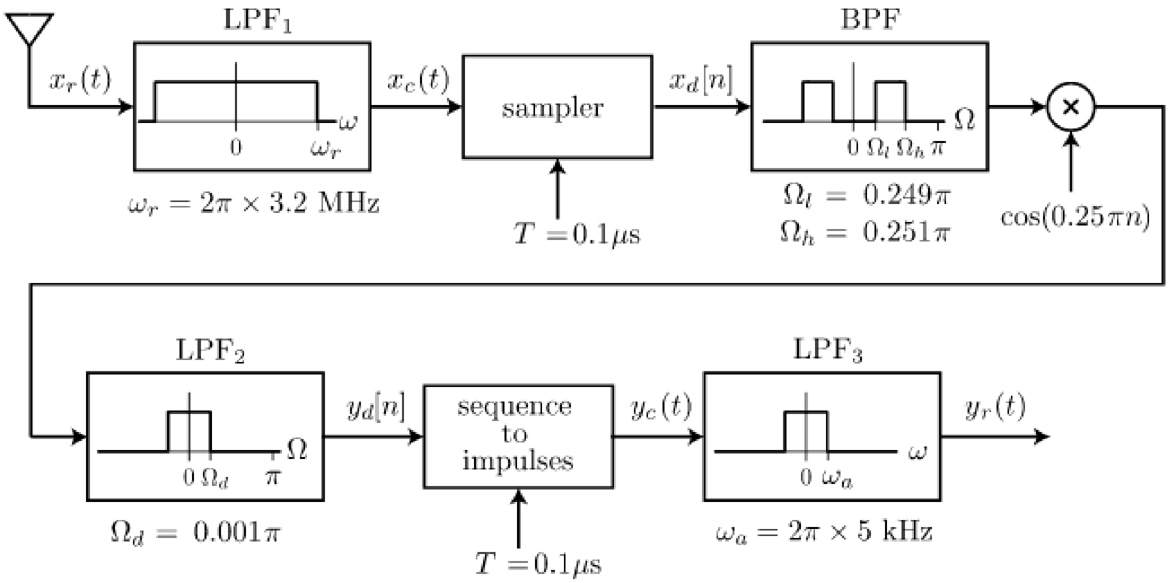
Problem 1

What is the transfer function of a system defined by the differential equation:

$$\frac{dy}{dt} + 10y(t) = \frac{d^2x}{dt^2} + 100\frac{dx}{dt} + 10x(t)$$

Problem 2

Commercial AM radio stations broadcast radio frequencies within a limited range: $2\pi(f_c - 5khz) < \omega < 2\pi(f_c + 5khz)$ where $f_c = n \times 10khz$ where n is an integer between 54 and 160. The system shown below is intended to decode one of the AM radio signals using Discrete Time signal processing methods. Assume all of the filters are ideal.



sampler: $x_d[n] = x_c(nT)$

sequence to impulses: $y_c(t) = \sum_{n=-\infty}^{\infty} y_d[n]\delta(t - nT)$

Determine the center frequency f_c for the AM station that this receiver will detect

Problem 3

Given the above problem, state if the below statements are true or false:

Increasing the cutoff frequency ω_r of LPF1 by a factor of 1.5 will cause aliasing.
Decreasing the cutoff frequency ω_r of LPF1 by a factor of 2 will have no effect on the ouput $y_r(t)$.
Halving the sampling interval T would have no effect on the output $y_r(t)$.
Doubling the sampling interval T would have no effect on the output $y_r(t)$.
Increasing the cutoff frequency Ω_d of LPF2 will change $y_r(t)$ by adding signals from unwanted radio stations.
Increasing the cutoff frequency Ω_d of LPF2 will change $y_r(t)$ because aliasing will occur.
Doubling the cutoff frequency Ω_d of LPF2 will have no effect on $y_r(t)$.
Halving the cutoff frequency Ω_d of LPF2 will have no effect on $y_r(t)$

Problem 4

Let H_C represent a causal continuous time system. The system is described by

$$\frac{dy}{dt} + 3y(t) = x(t)$$

where $y(t)$ is the output signal and $x(t)$ is the input signal.

What are the poles of this system?

Problem 5

You will start designing a causal discrete time version of the continuous system from problem 4. Let $x[n] = x(nT)$ and $y[n] = y(nT)$ where T is the sampling interval. We define the derivative to be

$$\frac{dy}{dt} = \frac{y(t+T) - y(t)}{T}$$

What is the expression for the poles of H_D, the discrete time system.

What is the range of values of T over which H_D is stable?

Problem 6

We shall now consider a more complicated continuous time system H_C described by

$$\frac{d^2y}{dt^2} + 100y(t) = x(t)$$

Determine the poles of this sytem

Problem 7

We will attempt to create a discrete time system for the above continuous time one. Given that second derivatives are approximated as

$$\frac{d^2y}{dt^2} = \frac{\frac{dy(t+T)}{dt} - \frac{dy(t)}{dt}}{T}$$

Determine the poles of the discrete time system.

What are the range of values of T over which the discrete system system is stable?

Problem 8

Consider the system $H(s) = \frac{1}{s+10}$. Suppose we apply some proportional control to it with a constant of K_p. What values of the proportionality constant lead to a stable system? Why don't we just crank the proportionality constant ALL the way up? To understand this final statement better add a noise source $N(s)$ just after the input. What is the transfer function between the noise source and the output when there is proportional feedback?

Problem 9

Write a closed loop transfer function for the system $H(s) = \frac{30}{(s+1)(s^2+16)}$ after a PD controller has been added to it. Can the system be stabilized with just a proportional controller?

Problem 10

Draw the magnitude bode plot for the system $H(s) = \frac{(s+1)(s+1000)}{(s+10)(s+100)}$

Problem 11

Consider a QAM (quadrature amplitude modulation) constellation which takes on 64 separate values, each value corresponding to one of 000000,000001,...,111111. We send one of these QAM packets every $\frac{1}{1000}$th of a second. How many bits of information do we send per second?

Problem 12

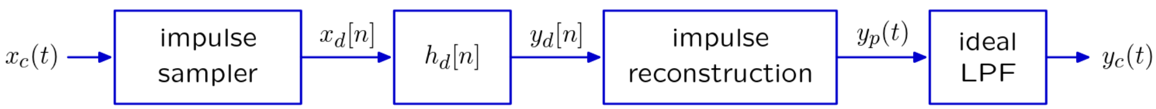
Our QAM transmitter has the ability to generate cosines and sines which have a magnitude range of -10 to 10. The signal will be subject to some amount of "random" noise upon transmission; however, we can be extremely confident that even after being perturbed by noise the signal will still fall within a circle of radius 1 about the QAM point. Given these constraints, what is the maximum number of QAM points per unit area that we may have without regions of overlapping error? Can you generalize this in two dimensions for an arbitrary radius r?

Extra Credit:

Can you create a generalized formula in 3-dimensions?

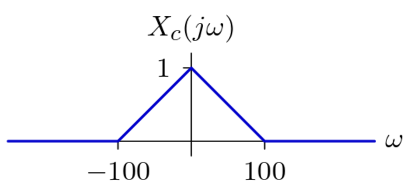
Problem 13

Sampling and reconstruction allow us to process CT signals using digital electronics as shown in the following figure.



The "impulse sampler" and "impulse reconstuction" use sampling interval $T = \pi/100$. The unit-sample function $h_d[n]$ represents the unit-sample response of an ideal DT low-pass filter with gain of 1 for frequencies in the range $-\frac{\pi}{2} < \Omega < \frac{\pi}{2}$. The "ideal LPF" passes frequencies in the range $-100 < \omega < 100$. It also has a gain of T throughout its pass band.

Assume that the Fourier transform of the input $x_c(t)$ is $X(j\omega)$ shown below.



Determine $Y_c(j\omega)$.