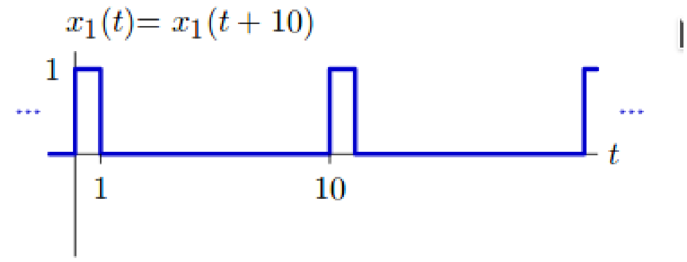
6.058 PSET 2

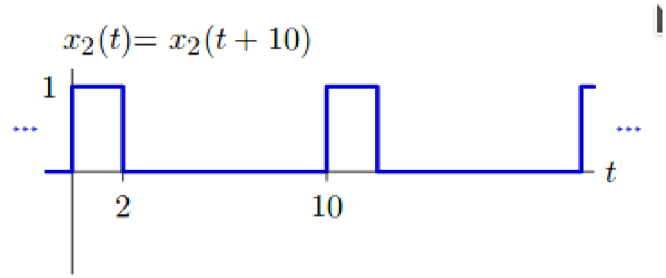
IAP 2018

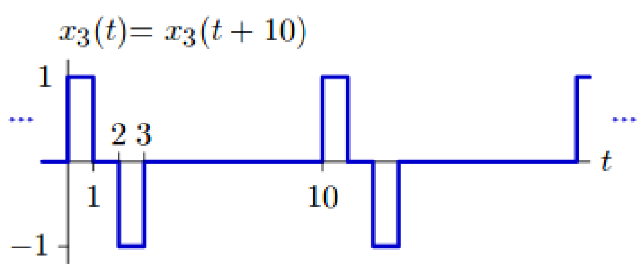
Due Night of January 14th

Problem 1

Determine the Fourier Series coefficients for the following signals. Show calculations. The first fourier series is difficult, but the rest can be found using that information.

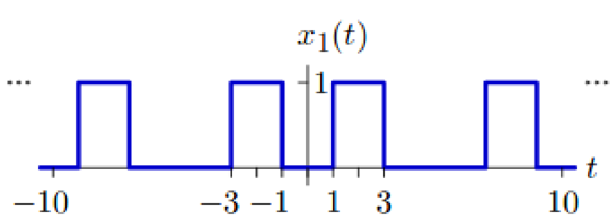






Problem 2

Find the fourier series of



assuming that the signal is periodic with a period of 10.

Problem 3

Find the Fourier Transform of $\frac{\sin(\omega t)}{\omega t}$.

Problem 4

Find the Fourier Transform of $\frac{\sin(\omega t)}{\omega t} * \frac{\sin(\omega t)}{\omega t}$. (Properties of the Fourier Transform can help you). NOTE: Is this a convolution or a multiplication?

Problem 5

Prove the following property of the Fourier Transform:

$$F(\frac{d}{dt}y(t)) = j\omega Y(j\omega)$$

where $F(x)$ denotes taking the Fourier Transform of x

Problem 6

Suppose we have a system modeled by the equation

$$\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 7y(t) = \frac{d}{dt}x(t) + 5x(t)$$

Find $\frac{Y(j\omega)}{X(j\omega)} = H(j\omega)$. (HINT: Problem 5 might help)

Problem 7

Calculate the Discrete Fourier Transform of $\delta[n-k]$ (the delta function shifted by k).

Problem 8

Find the unit impulse response $h[n]$ of the following system, and express it as a linear combination of shifted delta functions.

$$y[n] = 4x[n-1] - 3x[n-2] + 2x[n-3] - x[n-4]$$

Problem 9

Use your solutions to problems 7 and 8 to calculate the DTFT of the system described in problem 8 (you shouldn't have to perform many computations).

Problem 10 (Extra Credit)

Consider some normalized Gaussian distribution g(x):

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-x^2}{2\sigma^2}}$$

Prove that the Fourier transform of g(x) is another Gaussian, specified by

$$G(\omega) = e^{\frac{-\omega^2\sigma^2}{2}}$$