CMPT 440 – Spring 2019: Quantum Finite Automata

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Theoretical Background

Although the original idea of quantum finite automata was briefly discussed in the early 1970s, it was popularized in the late 1990s and early 2000s by Moore and Crutchfield. According to their findings published in "Quantum automata and quantum grammars" a quantum finite-state automaton (QFA) is a real-time quantum automaton where H, s_{init}, and the U_a all have a finite dimensionality n (Moore and Crutchfield (2000)). To better understand this definition, it is important to know what a real-time quantum automaton is. This is defined earler in their paper as the 5-tuple Q = {H, s_{init}, H_{accept}, A, U_a } where H is a Hilbert space containing all the possible state vectors, s_{init} is the initial state vector contained within H, H_{accpet} is the accepting state vector and is associated with an operator P_{accept} that projects onto it, A is the input alphabet, and U_a is the unitary transition matrix for each symbol in the alphabet. Each state vector has a probability that is used in the process P_{accept} to determine if it is in H_{accept}. By reducing the scope of the automaton to a finite amount, a QFA can be created. QFAs accept non-regular languages as opposed to regular languages accepted by DFAs and NFAs (Moore and Crutchfield (2000)).

An Example

Since QFAs do not accept regular languages, they can be used to define whether a language is regular or not depending on if a QFA can accept it without error. This is proposed in "On the Power of Quantum Finite State Automata" by Kondacs and Watrous when they state that if a language L is recognized by a bounded error, then the language must be regular (Kondacs and Watrous (1997)). Using a slightly modified definition of a QFA (including possible rejection states), a definition for a norm value of the projection operator is derived and states that no error probability (denoting it will not be accepted) will be bounded beyond a 1/2 difference and if it is it will produce a bounding error. In order to prove that regular languages will be caught in a bounding error, a proof for a non-regular language shows it is not bounded more than 1/2, resulting in a non-bounding error. It is important to note that although the definition used to prove this is modified from the definition presented in the Theoretical Background section, this is because the definition in Kondac's work was adapted from a definition for 2 way QFAs which requires halting states to properly function (Kondacs and Watrous (1997)). Both definitions state the same requirements for a proper 1 way QFA.

References

A. Kondacs and J. Watrous. On the power of quantum finite state automata. In *Proceedings 38th Annual Symposium on Foundations of Computer Science*, pages 66–75. IEEE, 1997.

C. Moore and J. P. Crutchfield. Quantum automata and quantum grammars. *Theoretical Computer Science*, 237(1-2):275–306, 2000.