Day 07

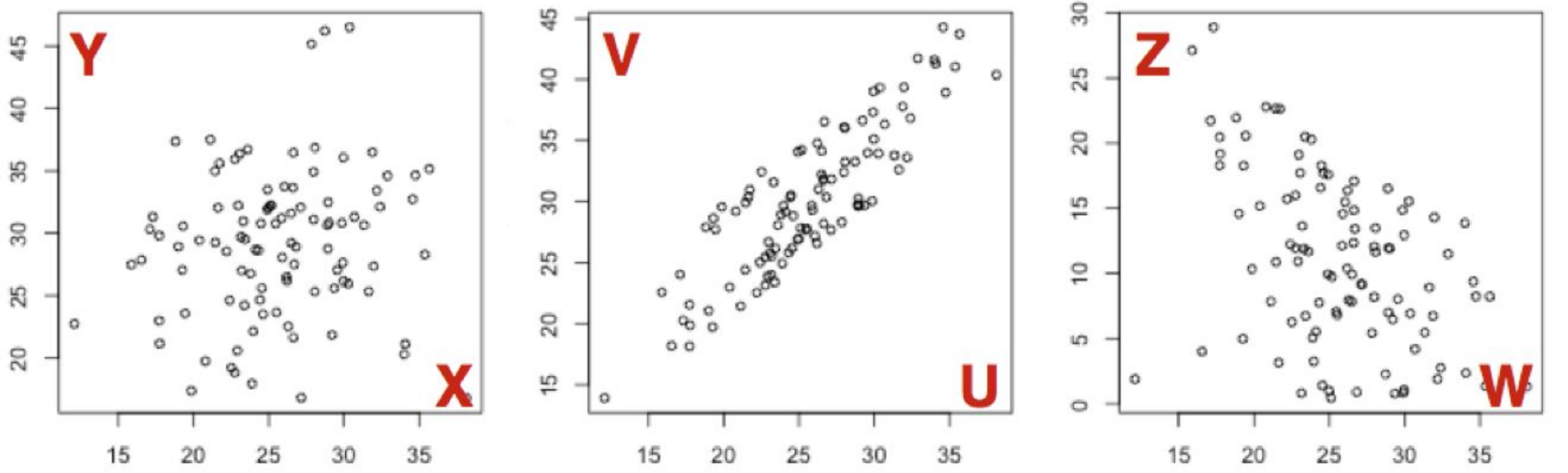
Measuring associations

- Calculating correlation
- Limitations
- False positives
- Visual inference

Calculating correlation

Variables | Attributes / Features

x	10	8	13	9	11	14	6	4	12	7	5
y	8.04	6.95	7.58	8.81	8.33	9.96	7.24	4.26	10.84	4.82	5.68



Correlation coefficient

Pearson Correlation Coefficient

- Measures <u>linear</u> relationship between variables.

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

- n is the sample size
- x_i, y_i are the single samples indexed with *i*
- $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ (the sample mean); and analogously for $\bar{y}$

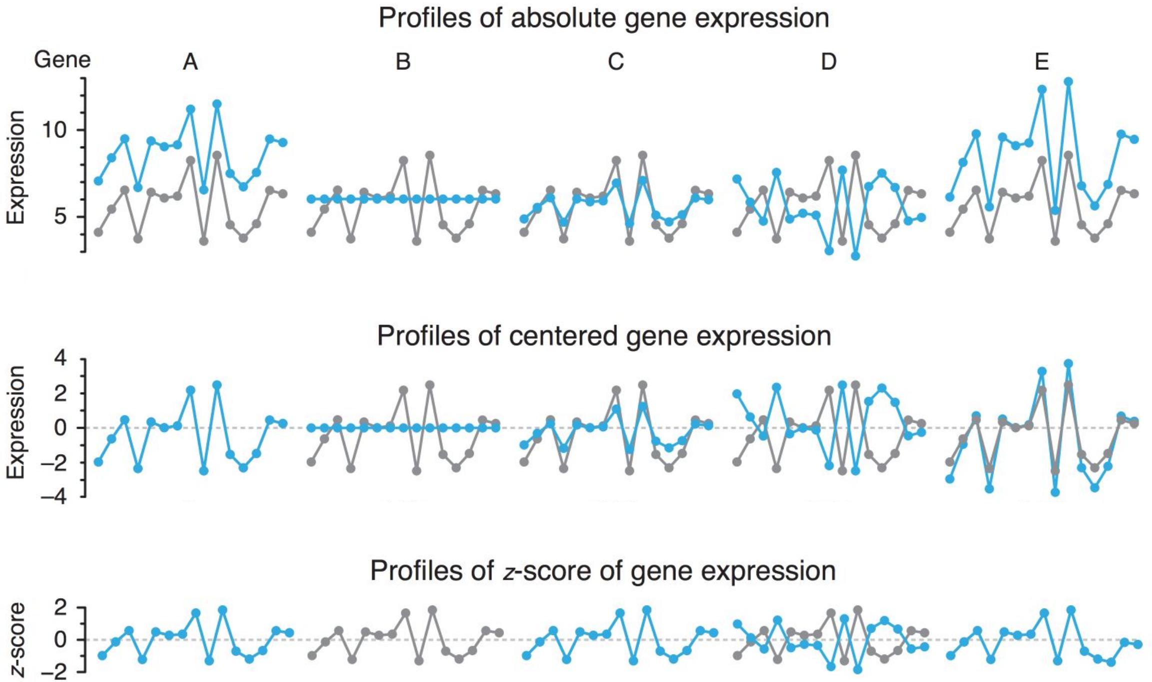
Wikipedia

Correlation coefficient

Pearson Correlation Coefficient

- Captures the relationship between 2 vectors after centering each vector by its mean and scaling by its standard deviation.
- The final quantities for each vector are called z-scores.

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$



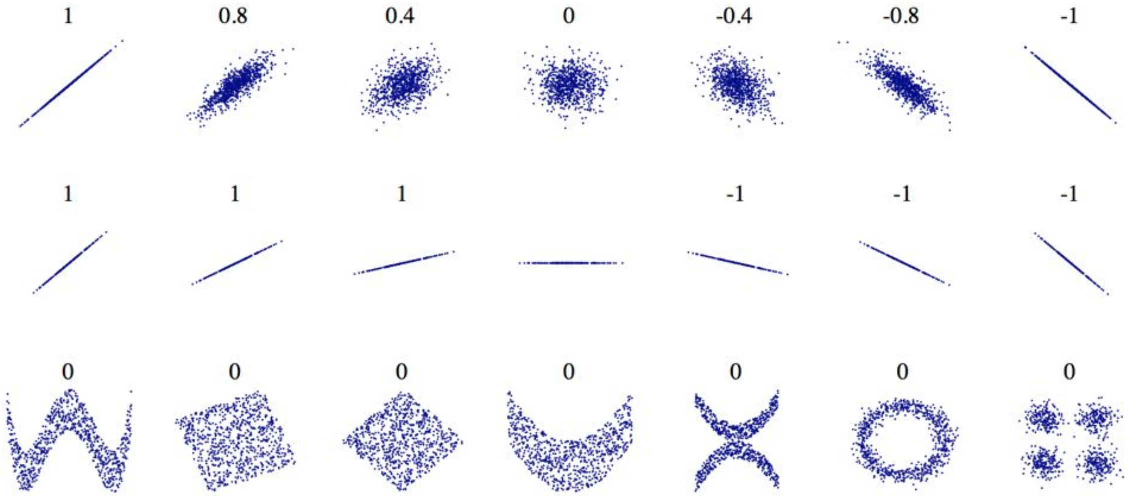
https://www.nature.com/articles/nmeth.4299

Correlation coefficient

Pearson Correlation Coefficient

- Measures 'linear' relationship between variables.

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$



$-1 \leq r \leq +1$

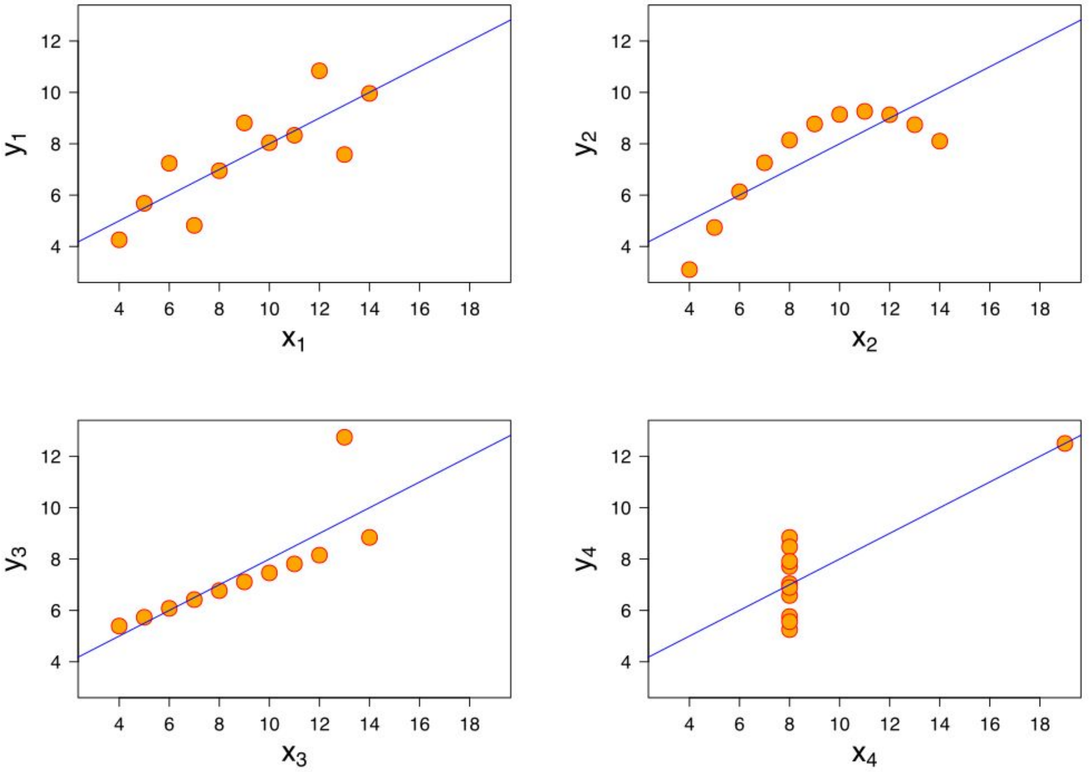
−1 is total −ve correlation | 0 is no correlation | +1 is total +ve correlation

Wikipedia

Anscombe's quartet: "calculation are exact; graphs are rough!"

11 data points

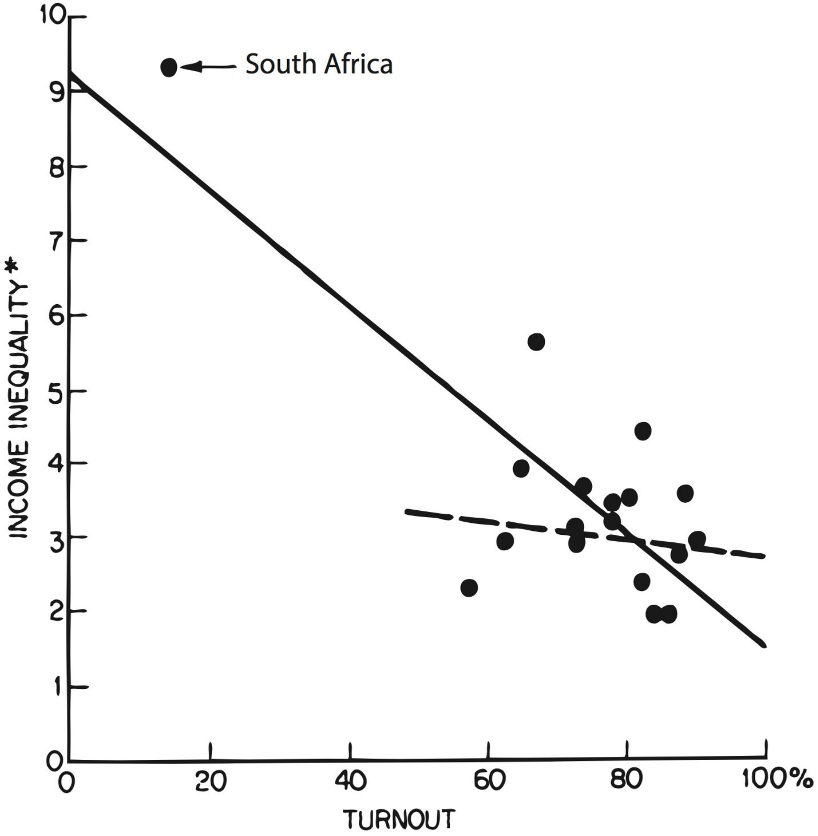
- Mean (x) = 9
- Var (x) = 11
- Mean (y) = 7.50
- Var (y) ~ 4.12
- Cor (x, y) = 0.816
- Linear regression line:
    - y = 3.00 + 0.500x



Anscombe, F. J. (1973). "Graphs in Statistical Analysis". American Statistician 27 (1): 17–21.

Wikipedia

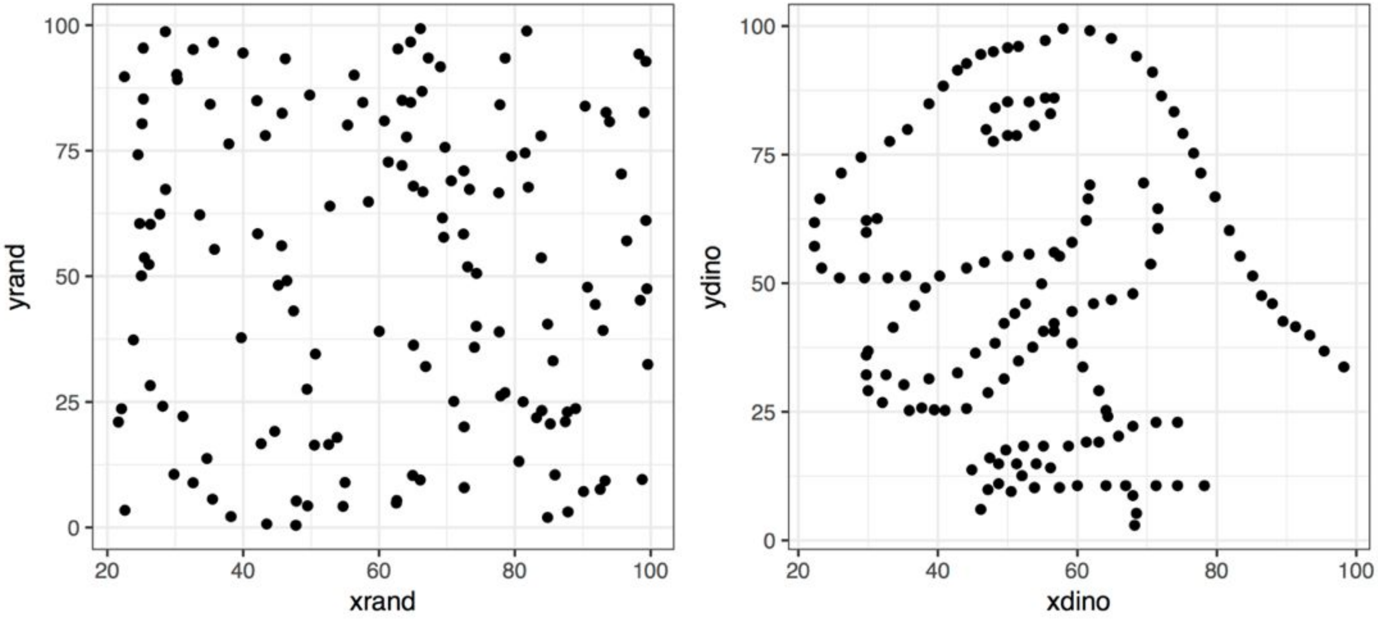
What does a correlation coefficient tell you about the data?



http://socviz.co/lookatdata.html

What does a correlation coefficient tell you about the data?

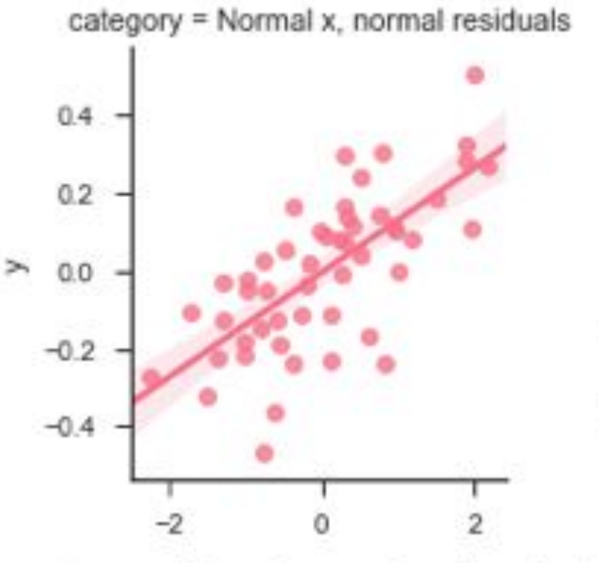
Correlation = -0.06

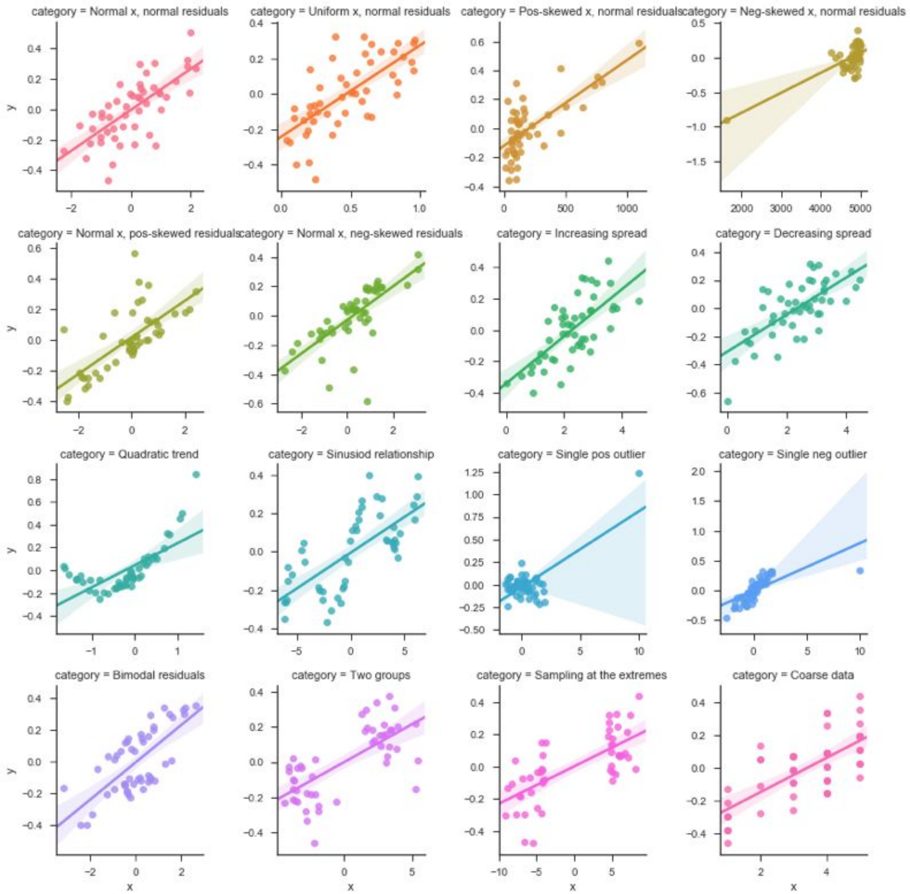


https://www.autodeskresearch.com/publications/samestats

What does a correlation coefficient tell you about the data?

Correlation = 0.7

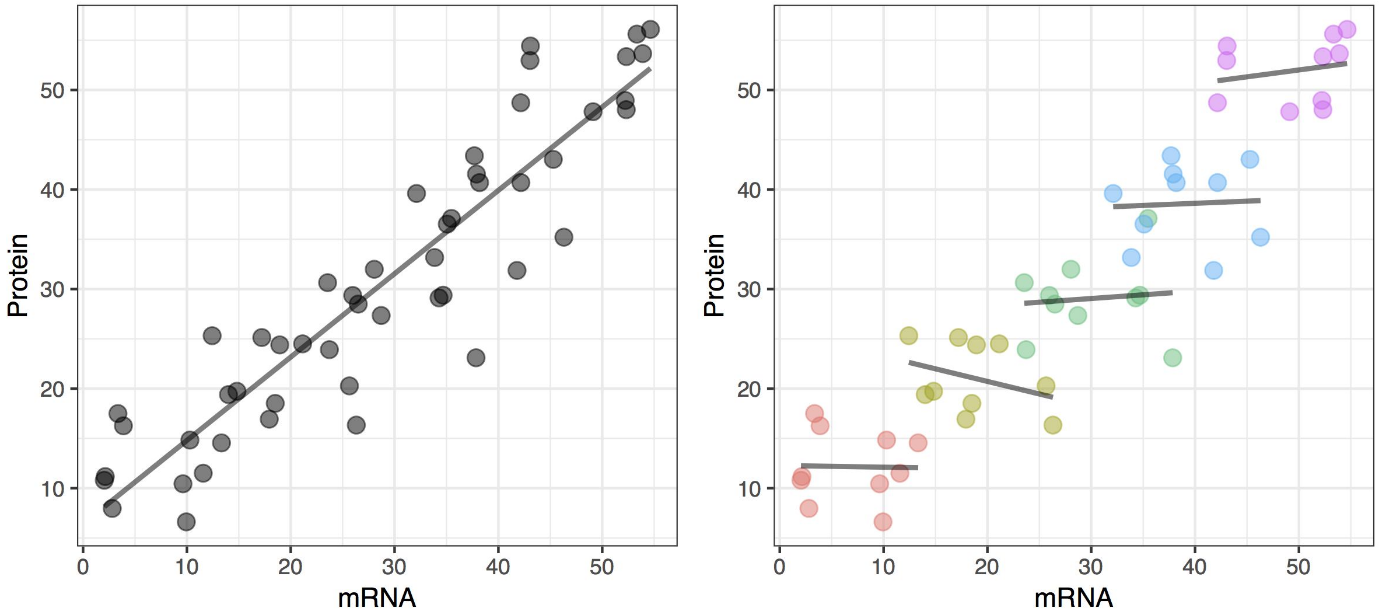




Wikipedia

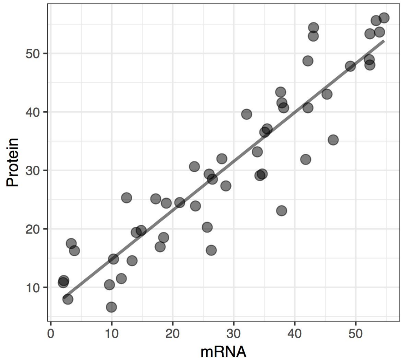
What does a correlation coefficient tell you about the data?

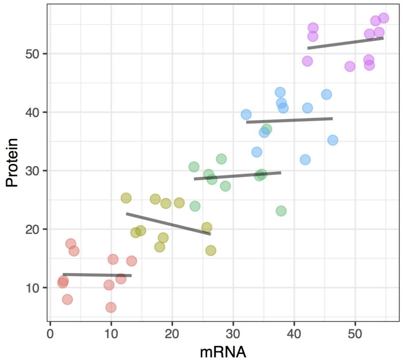
Simpson's Paradox



What does a correlation coefficient tell you about the data?

Simpson's Paradox

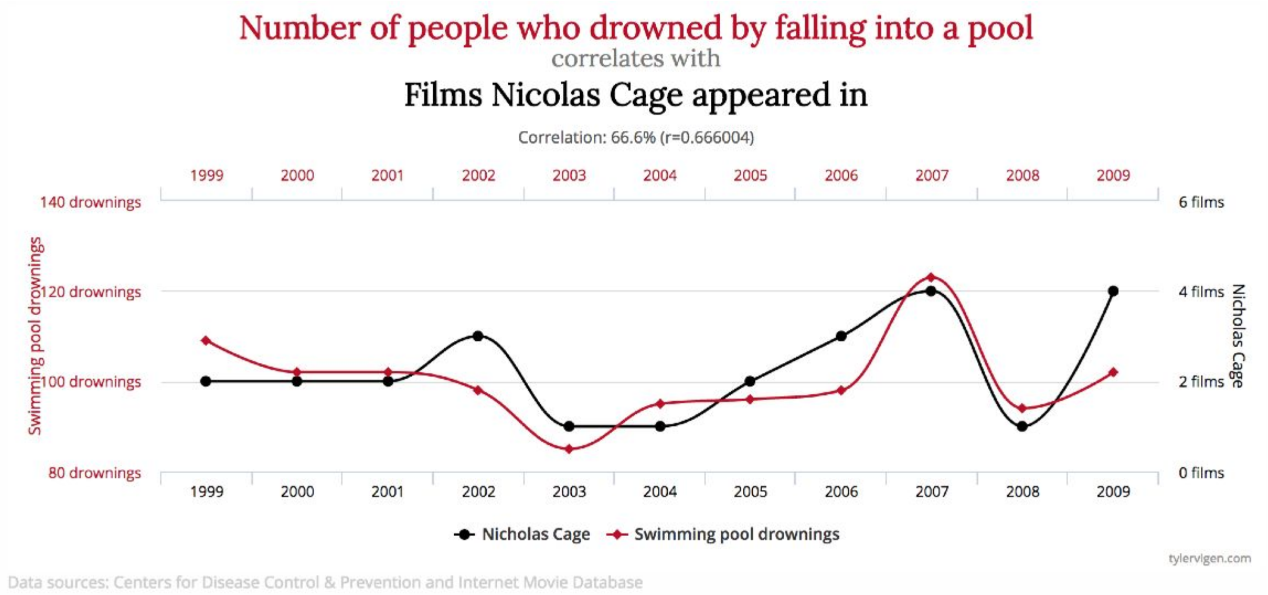




Success rates of kidney stone removal surgeries			
Treatment	**Diameter < 2 cm**	**Dia. ≥ 2 cm**	**Overall**
Open surgery	93%	73%	78%
Percutaneous nephrolithotomy	87%	69%	83%

Spurious correlations

What does Nicholas Cage have to do with people drowning in swimming pools?



Checkout https://www.google.com/trends/correlate

Spurious correlations

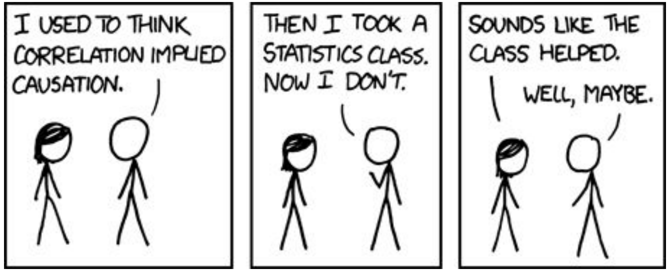
Simulate fluctuations in correlation coefficients

- Repeat 10,000: Calculate correlation coefficients of $n = 10$ samples of two independent uniformly distributed variables between (0, 1). Plot a histogram.
- Mark statistically significant coefficients ($\alpha = 0.05$).
- Plot the samples with the three largest and smallest correlation coefficients (statistically significant).
- Vary sample size $n = \{5, 10, 20, 50\}$.

Correlation does not imply causation

There is a significant correlation between annual chocolate consumption and number of Nobel laureates for different countries ($r(20)=.79$; $p<0.001$) → chocolate intake provides nutritional ground for sprouting Nobel laureates.

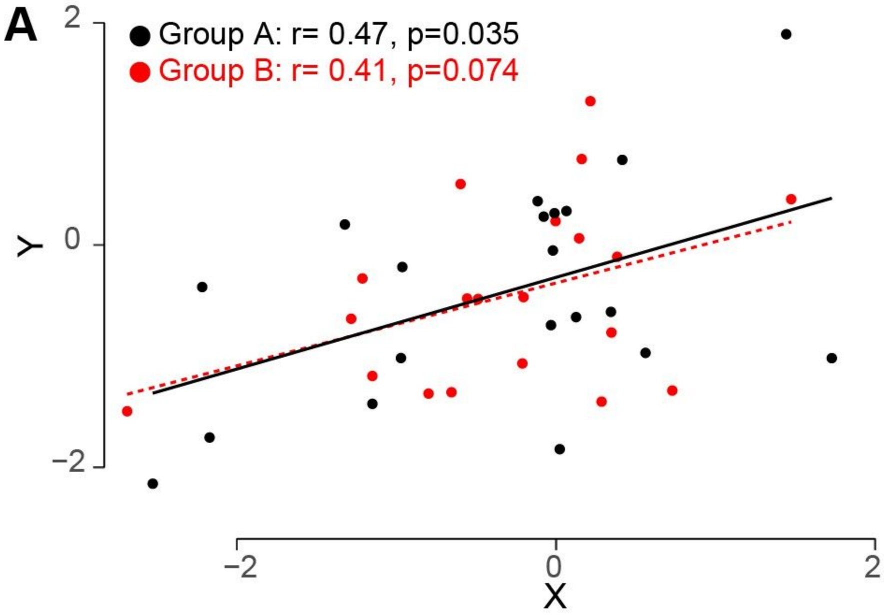
- Correlation can occur by random chance.
- Confounding variables could lead to correlation.
- Even when there is causation, there might not be obvious correlation.



xkcd.com/552

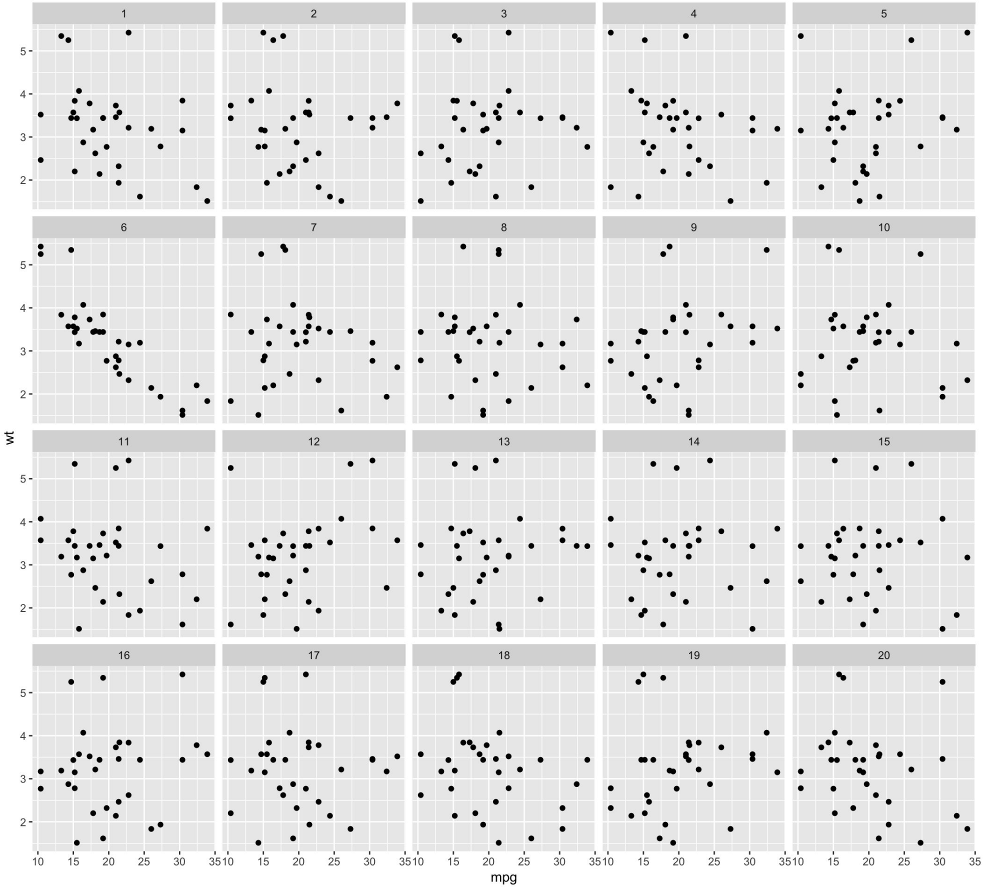
Interpreting comparison b/w two correlation w/o comparing them

Conclusion regarding the impact of an intervention based on correlation in treatment group vs. correlation in control group.



https://elifesciences.org/articles/48175

Spurious correlations – But it *looks* associated!

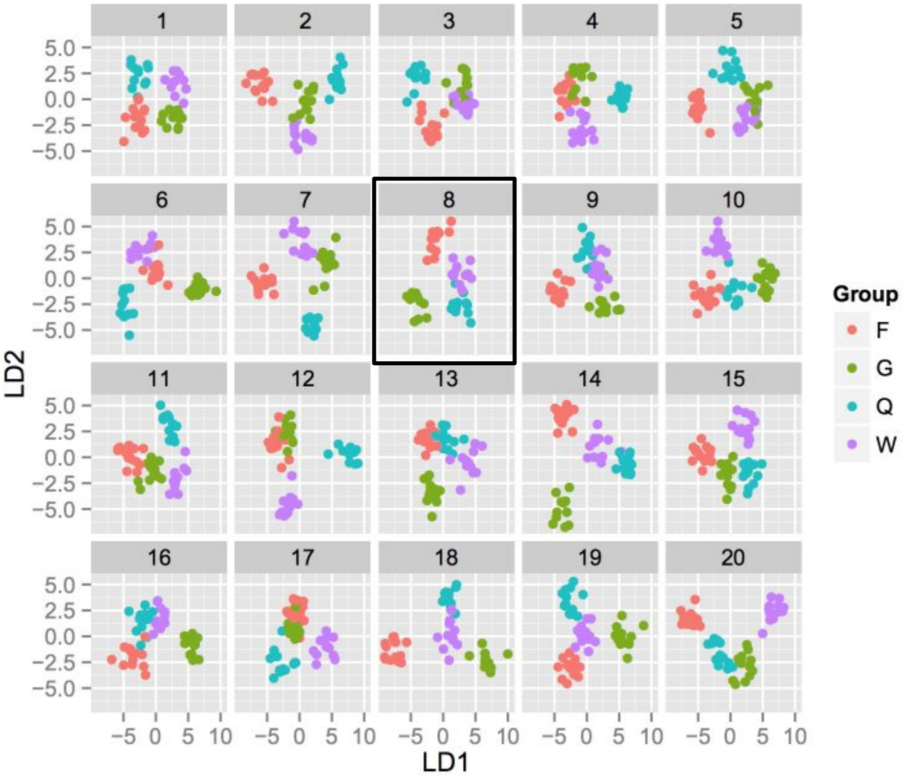


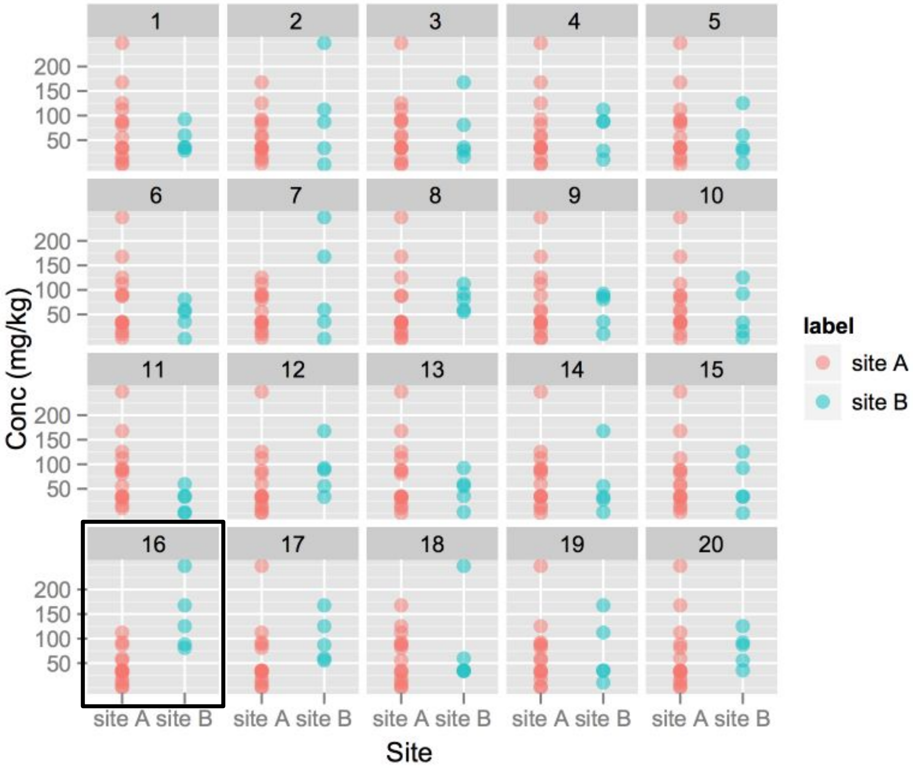
Create a <u>lineup</u> for visual inference

- Place the plot of the real data amongst a set of null plots to create a lineup; Null plots are generated in a way consistent with the null hypothesis.
- If the observer can pick the real data as different from the others, this puts weight on the statistical significance of the structure in the plot.

https://lib.dr.iastate.edu/cgi/viewcontent.cgi?article=4995&context=etd

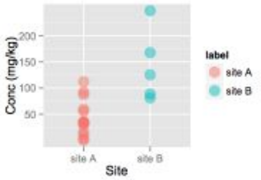
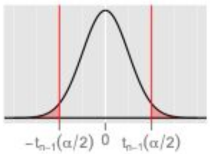
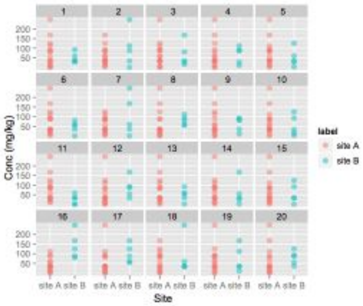
Spurious correlations – But it *looks* associated!





https://lib.dr.iastate.edu/cgi/viewcontent.cgi?article=4995&context=etd

Spurious correlations – But it *looks* associated!

	Mathematical Inference	Visual Inference
Hypothesis	$H_0 : \mu_1 = \mu_2$ vs $H_a : \mu_1 \neq \mu_2$	$H_0 : \mu_1 = \mu_2$ vs $H_a : \mu_1 \neq \mu_2$
	$\downarrow$	$\downarrow$
Test Statistic	$T(y) = \frac{\bar{y}_1 - \bar{y}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$	$T(y) =$ 
	$\downarrow$	$\downarrow$
Sampling Distribution	$f_{T(y)}(t)$; 	$f_{T(y)}(t)$; 
	$\downarrow$	$\downarrow$
Reject H_0 if	observed T is extreme	observed plot is identifiable