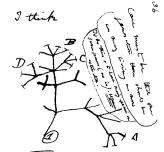
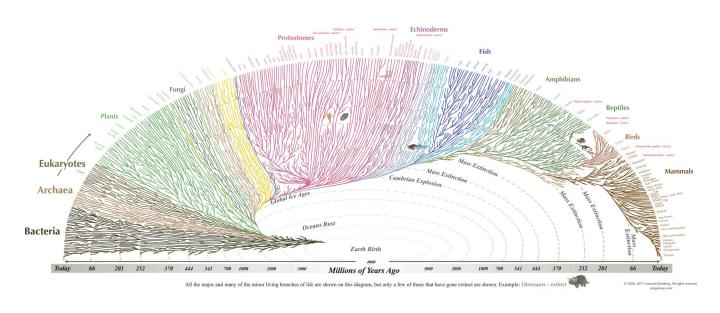
Week 4: Sequence alignment

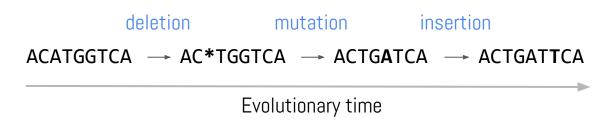
- Global alignment
 - Dynamic programming
 - Needleman-Wunsch algorithm
- Local alignment
 - Smith-Waterman algorithm
 - BLAST

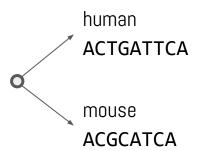
Sequence evolution



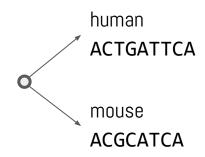
Then betwee A & B. change by & celetion. C & B. The frint prediction, B & D rather present his trackers. Then formed. - Kenny William







Sequence alignment



Sequences can be aligned by allowing for gaps and mismatches.

ACTGATTCA

ACTGATTCA

ACTG-ATTCA

ACGCA-TCA

AC-GCATCA

AC-GCAT-CA

Which alignment is correct?

A scoring scheme:

- Match: 2
- Mismatch: -3
- Gap: -2

We will come back to this!

$$2+2-3-3+2-2+2+2+2$$
 $2+2-2+2-3-3+2+2+2$ $2+2-2+2-2+2+2+2+2$ $= 4$ $= 8$

Alignment is gap placement.

How many possible alignments?

Solve a given complex problem by:

- 1. Breaking it into **subproblems** and
- 2. Storing the results of subproblems to avoid computing the same results again.

Two key properties of a problem that suggest that the given problem can be solved using DP.

- 1. Overlapping Subproblems
 - Given problem can be recursively broken down into subproblems that can be related to each other. That is, total no. of subproblems is polynomial.
- 2. Optimal Substructure
 - The optimal solution can be produced by combining optimal solutions of subproblems.



Richard Bellman

Optimal decision processes, involved time series & planning - thus 'dynamic' & 'programming'.

"It's impossible to use the word dynamic in a pejorative sense"; DP was "something not even a Congressman could object to."

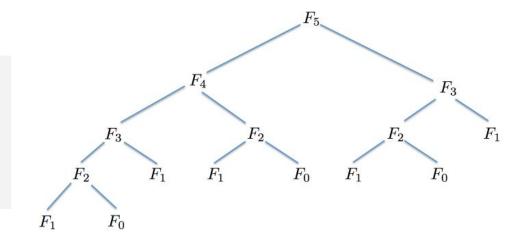
Hemachandra/Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,

$$F_0 := 0; F_1 := 1;$$

 $F_n = F_{n-1} + F_{n-2}, \text{ for all } n \ge 2.$

A trivial algorithm for computing F_n :

```
naive_fib(n):
   if n ≤ 1: return n
   else: return naive_fib(n - 1) +
        naive_fib(n - 2)
```



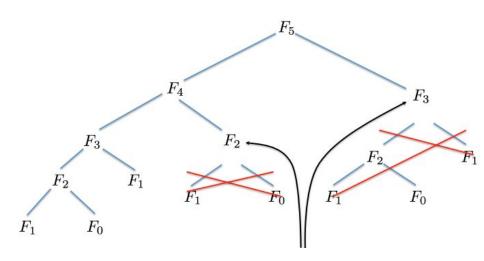
Hemachandra/Fibonacci numbers: $F_0 := 0$; $F_1 := 1$; $F_n = F_{n-1} + F_{n-2}$, for all $n \ge 2$.

Never recompute a subproblem F(k), $k \le n$, if it has been computed before.

Memoization: Remembering previously computed values.

Improved algorithm for computing F_n :

```
memo = \{ \}
fib(n):
    if n in memo: return memo[n]
    else if n = 0: return 0
    else if n = 1: return 1
    else: f = fib(n - 1) + fib(n - 2)
    memo[n] = f
     return f
```



These values are already computed and stored in memo when runtime processes these nodes of the recursion.

- 1. Overlapping Subproblems
- 2. Optimal Substructure

DP ≈ recursion + memoization (reuse)

- Remember (memoize) previously solved "subproblems"; e.g., in Fibonacci, we memoized the solutions to the subproblems F_{ϱ} , F_{1} , \cdot • F_{n-1} , while unraveling the recursion.
- If we encounter a subproblem that has already been solved, reuse solution.
- Runtime ≈ (no. of subproblems) * (time per subproblem)

- 1. Scoring function: substitution matrix & gap penalty
- 2. Matrix initialization & filling
- 3. Traceback

Step 1

A scoring scheme:

- Match: 1

- Mismatch: -2

- Gap: -1

	_	G	С	A	Т
_					
G					
A					
Т					

- 1. Scoring function: substitution matrix & gap penalty
- 2. Matrix initialization & filling
- 3. Traceback

$$M(0, j) = j*p$$
 $Step 2$
 $M(i, 0) = i*p$
 $M(i, j) = MAX(M(i-1, j) + p, top$
 $M(i, j-1) + p, left$
 $M(i-1, j-1) + S(A_i, B_j)$ diagonal

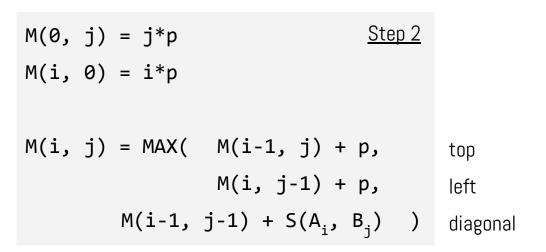
	_	G	С	A	Т
_					
G					
A					
Т					

- 1. Scoring function: substitution matrix & gap penalty
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$$M(0, j) = j*p$$
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 $M(i-1, j-1) + S(A_i, B_j)$ diagonal

	_	G	С	A	Т
_	0	-1	-2	-3	-4
G	-1				
A	-2				
Т	-3				

- 1. Scoring function: substitution matrix & gap penalty
- 2. Matrix initialization & filling
- 3. Traceback



	_	G	С	A	т
_	0	-1	-2	-3	-4
G	-1	?			
A	-2				
т	-3				

	_	G	С	A	Т
_	0	-1	-2	-3	-4
G	-1	-2			
A	-2				
Т	-3				

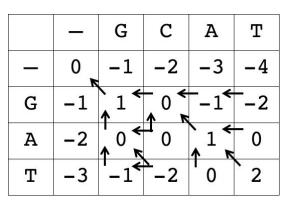
	_	G	С	A	Т
_	0	-1	-2	-3	-4
G	-1←	2			
A	-2				
т	-3				

	_	G	С	A	Т
_	0 ,	-1	-2	-3	-4
G	-1	1			
A	-2				
Т	-3				

	_	G	С	A	Т
_	0 ,	-1	-2	-3	-4
G	-1	1			
A	-2				
Т	-3				

- 1. Scoring function: substitution matrix & gap penalty
- 2. Matrix initialization & filling
- 3. Traceback

```
M(0, j) = j*p Step 2
M(i, 0) = i*p
M(i, j) = MAX( M(i-1, j) + p, top M(i, j-1) + p, left M(i-1, j-1) + S(A<sub>i</sub>, B<sub>j</sub>) ) diagonal
```



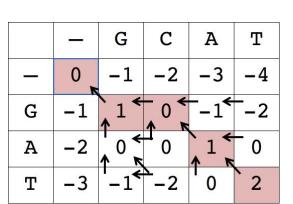
- 1. Scoring function: substitution matrix & gap penalty
- 2. Matrix initialization & filling
- 3. Traceback

Align GCAT with GAT

GCAT G-AT

top left diagonal

Step 3



- 1. Scoring function: substitution matrix & gap penalty
- 2. Matrix initialization & filling
- 3. Traceback

Align ATGCT with ATTACA

M(0,	j)	= j*p	
M(i,	0)	= i*p	
M(i,	j)	= MAX(M(i-1, j) + p,
			M(i, j-1) + p,
		M(i-1,	$j-1) + S(A_i, B_j)$

top

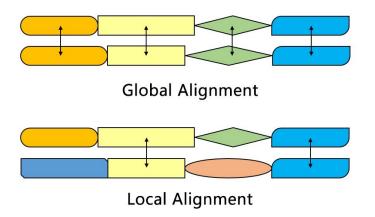
left

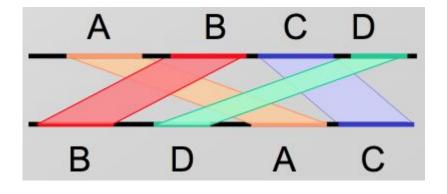
diagonal

2	_	A	Т	Т	A	С	A
_							
Α							
Т							
G							
С							
T							

Global & local alignment

A local alignment of strings s and t is an alignment of a substring of s with a substring of t.

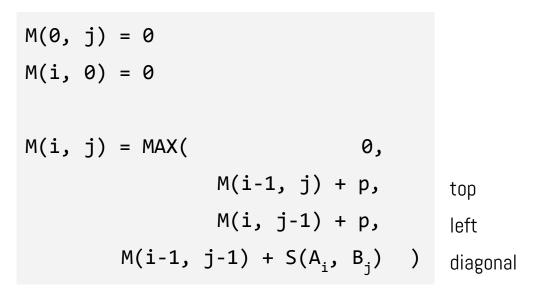




Smith-Waterman algorithm

Similar to Needleman-Wunsch, with 3 changes:

- First row/column set to 0.
- No negative scores, set to 0.
- Backtrack from cell with highest score, stop at 0.



	-	G	С	A	Т
_					
G					
С					
Т					

Smith-Waterman algorithm

Similar to Needleman-Wunsch, with 3 changes:

- First row/column set to 0.
- No negative scores, set to 0.
- Backtrack from cell with highest score, stop at 0.

$$M(0, j) = 0$$
 $M(i, 0) = 0$
 $M(i, j) = MAX($
 $M(i-1, j) + p, top$
 $M(i, j-1) + p, left$
 $M(i-1, j-1) + S(A_i, B_j)$ diagonal

Align GCAT with GCT

GC GC

	ı	G	С	A	т
_	0	0	0	0	0
G	0	1	0	0	0
С	0	0	2	1	0
Т	0	0	1	1	2



Margaret Dayhoff
Applying math & computational techniques to the sequencing of proteins and nucleic acids.

- 1965: First collection of protein seqs.
- Single-letter code for amino acids.
- 1966: 'Evolutionary trees'.
- 1978: First AA similarity-scoring matrix.
- 1980: Launched the Protein Information Resource, the first online database system that could be accessed by telephone line.

Substitution matrix: A collection of scores for aligning nucleotides or amino acids with one another.

- The scores represent the relative ease with which one nucleotide or amino acid may mutate into or substitute for another.
- Purely statistical, nothing directly to do with structure/biochemistry.

```
Cys Gln Glu Gly His Ile Leu Lys Met Phe Pro Ser Thr Trp
```

Substitution matrix: Each scores is a <u>log-odds score</u> equal to the logarithm of the ratio of the likelihoods of two hypotheses: the residues can substitute for one another or not.

$$s(a,b) = \frac{1}{\lambda} \log \frac{p_{ab}}{f_a f_b}$$

- \bullet p_{ah} : likelihood of these two residues being correlated because they're homologous.
 - \circ p_{ab} are the target frequencies: the probability that we expect to observe residues *a* and *b* aligned in homologous sequence alignments.
- $f_a f_h$: likelihood of these two residues being uncorrelated and unrelated, occurring independently.
 - \circ f_a and f_b are background frequencies: the probabilities that we expect to observe amino acids *a* and *b* on average in any protein sequence.
- λ: a scaling factor, usually set to something that lets helps round off all the terms in the score matrix to sensible integers.

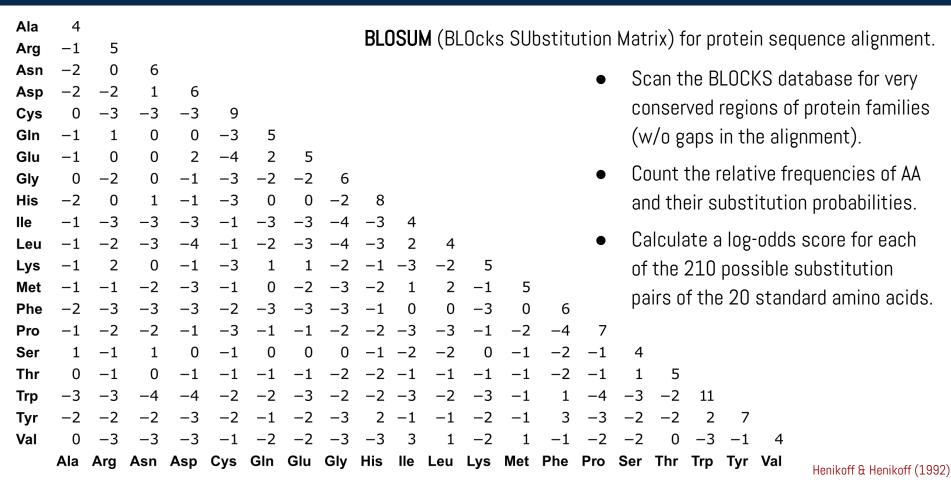
Substitution matrix: Each scores is a log-odds score equal to the logarithm of the ratio of the likelihoods of two hypotheses: the residues can substitute for one another or not.

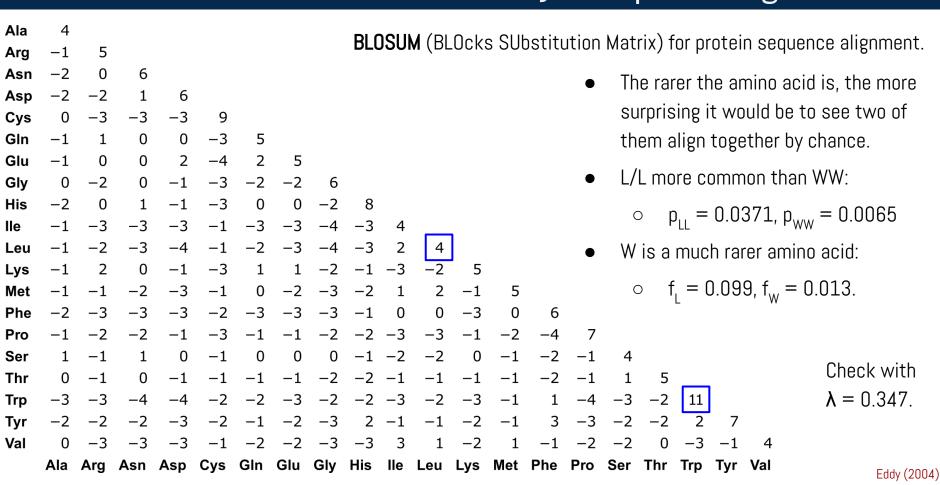
$$s(a,b) = \frac{1}{\lambda} \log \frac{p_{ab}}{f_a f_b}$$

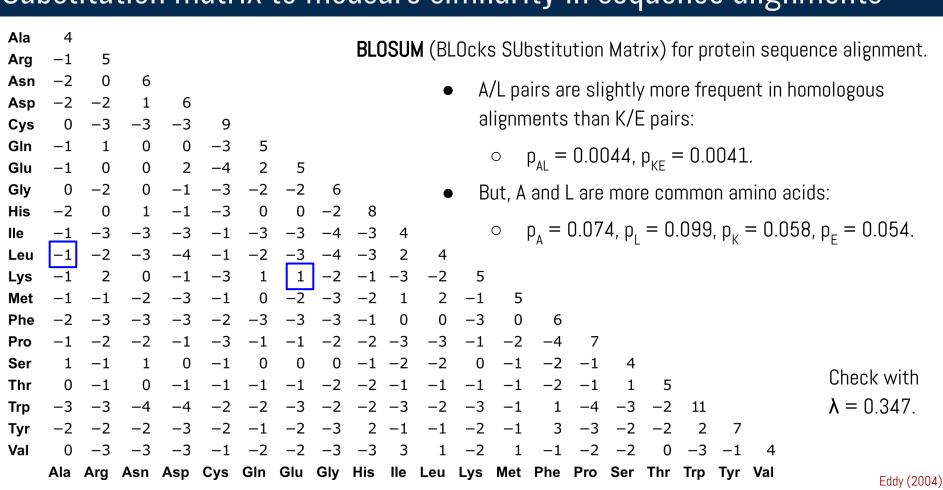
- p_{ab}: likelihood of these two residues being correlated because they're homologous.
- $\mathbf{f}_{a}\mathbf{f}_{b}$: likelihood of these two residues being uncorrelated and unrelated, occurring independently.
- λ: a scaling factor

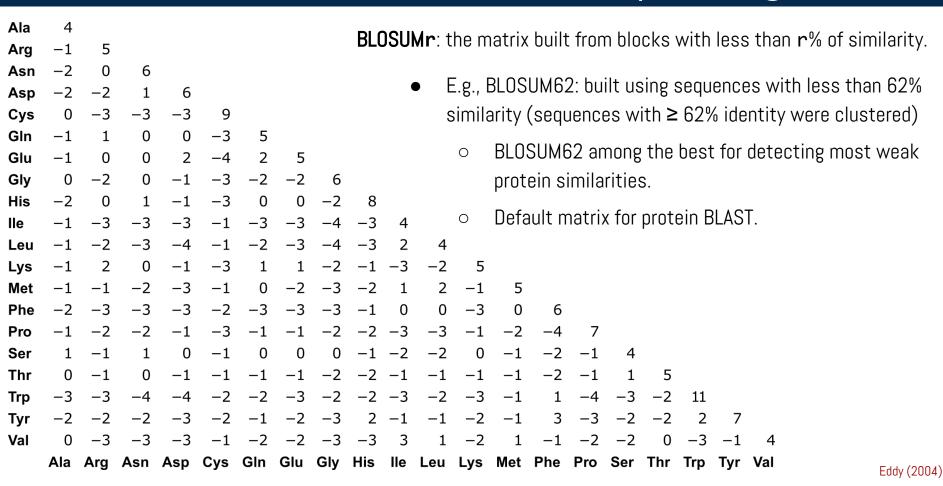
Assuming that each aligned residue pair is statistically independent of the others (biologically dubious, but mathematically convenient):

- The score of an alignment ("alignment score") = sum of individual log-odds scores for each aligned residue pair.

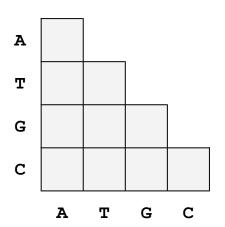








Substitution matrix for DNA



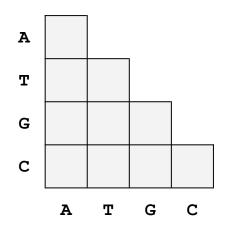
Making-up an arbitrary matrix by fixing the p_{ab} values \rightarrow directly describes what homologous alignments are expected to look like.

• The resulting score matrix is optimal for detecting alignments that match these target frequencies.

Say, the matrix should be optimized for finding 88% identity alignments.

- Assume that all mismatches are equiprobable, and composition of both alignments and background sequences is uniform at 25% for each nucleotide (\mathbf{f}_a , $\mathbf{f}_b = 0.25$ for all a,b). Then,
 - Four identities: $p_{aa} = 0.22$
 - 12 types of mismatch: $p_{ab} = 0.01$.
- If we set $\lambda = 1$, this gives +1.26 for a match and -1.83 for a mismatch.
- Setting $\lambda = 0.25$ and round off: we have a new scoring system of +4/-7.

Substitution matrix for DNA



Given a scoring matrix, we can back calculate target frequencies if two conditions are met:

- 1. It must have at least one positive score, and
- 2. The expected score for random sequence alignments must be negative.

True for most score matrices:

- These properties are necessary to make local sequence alignment algorithms like BLAST and Smith-Waterman work.
- Both conditions are met by definition for matrices derived as log-odds scores, except for the useless case of $p_{ab} = f_a f_b$ for all a,b.

Examples:

- FASTA & WU-BLASTN: arbitrary +5/-4 scoring system;
 Optimal for detecting alignments that are 65% identical.
- NCBI BLASTN: +1/-2 scoring system; Optimal for detecting alignments that are 95% identical.

 $s(a,b) = \frac{1}{\lambda} \log \frac{Pab}{f f}$

How do we scale this up to search an entire sequence database?

Given a query sequence, and a large set of target sequences (millions), which target sequences (if any) are related to the query?

- Individual alignments need not be perfect: Once initial matches are found, they can fine-tune them later.
- Must be very fast.

Exploit the nature of the problem (most sequences will be unrelated to the query):

- If any match with % identity ≤ 90 is going to be rejected, can ignore sequences which don't have a stretch of 10 nucleotides in a row.
- Pre-screen sequences for common long stretches.
- Pre-process the database offline and index k-mers.

BLAST

TITLE CITED BY YEAR

1990

Basic local alignment search tool

SF Altschul, W Gish, W Miller, EW Myers, DJ Lipman Journal of molecular biology 215 (3), 403-410

> U.S. National Library of Medicine NCBI National Center for Biotechnology Information Sign in to NCB BLAST ® Recent Results Saved Strategies Help **Basic Local Alignment Search Tool** IgBLAST 1.8.0 released BLAST finds regions of similarity between biological sequences. The program A new version of IgBLAST is now available. compares nucleotide or protein sequences to sequence databases and Wed, 15 Nov 2017 16:00:00 EST More BLAST news... calculates the statistical significance. Learn more Web BLAST blastx translated nucleotide ▶ protein **Protein BLAS Nucleotide BLAST** tblastn protein ▶ protein nucleotide ▶ nucleotide protein ▶ translated nucleotide **BLAST Genomes** Enter organism common name, scientific name, or tax id

> > Human

Mouse

Rat

Microbes

https://www.ncbi.nlm.nih.gov/BLAST/

Upcoming project deadline: Project topic due on Jan 31

- <u>Discuss with me</u> and any other PI; Read recent papers.
- Briefly describe a project idea:
 - o Title
 - Project advisor (if someone outside class)
 - 250-word abstract addressing the following 4 Qs:
 - What is the problem?
 - How is it addressed currently & what are the limitations?
 - What is your approach to addressing it & why is likely to be successful?
 - If successful, why does it matter (what is the impact)?
- Submit a PDF.