Problem 1

(50pt) Consider a Dubins' car dynamical system

$$\begin{aligned}\dot{x} &= v\cos\theta\\ \dot{y} &= v\sin\theta\\ \dot{\theta} &= w\end{aligned}$$

where v and w are linear and angular velocities of the vehicle.

- (10pt) Design a dynamically feasible trajectory to drive the system from the initial state $[x(0), y(0), \theta(0)]^\intercal = [10, 20, \pi/4]^\intercal$ to a final state $[x(T), y(T), \theta(T)]^\intercal = [0, 0, \pi/3]^\intercal$ with $T = 15$. Recall the equations to be used: initial and final position and orientation, initial velocity and final velocity (assuming both zeros or your own choice), and the constraint of non-slip.
- (5pt) Now, consider a new set of states $\mathbf{z} = [z_1, z_2, z_3, z_4]^\intercal$, with $z_2 = \dot{x}$ and $z_4 = \dot{y}$. Let's define two new inputs a_x and a_y and let $\dot{z}_2 = a_x$ and $\dot{z}_4 = a_y$. Can you write down the first order ODE that describes the system with state variable $\mathbf{z} = [z_1, z_2, z_3, z_4]^\intercal$ and input variables $\mathbf{u} = [a_x, a_y]^\intercal$? That is, $\dot{\mathbf{z}} = A\mathbf{z} + B\mathbf{u}$ and find out the A and B matrices. Is the linear system reachable?
- (10pt) Given the trajectory x^d, y^d you computed in the first step, you shall obtain the desired trajectories for $z_1^d, z_2^d, z_3^d, z_4^d$ as well as a_x^d and a_y^d. Design a trajectory tracking controller in the linear system $\dot{\mathbf{z}} = A\mathbf{z} + B\mathbf{u}$ to track the desired trajectory in $\mathbf{z}^d, \mathbf{u}^d$. Given the gain and the equation of the controller in the linear system.
- (5 pt) Now, consider a dubins car with augmented state vector $\mathbf{x}_a = [x, y, v, \theta]^\intercal$ where v the linear velocity is augmented into the state vector. Consider a new input vector $[\dot{v} = a, w]$ where the linear acceleration a becomes one of your new inputs. Given this new state vector, derive the first order ODE expressing the dynamics of the system:

$$\dot{\mathbf{x}}_a = f(\mathbf{x}_a, a, w) \tag{1}$$

- (5pt) Express the linear acceleration a and angular velocity w as functions of a_x and a_y and the state variables x, y, θ, v.
- (10pt) Recall during the lecture, we show that if you give a and w computed above into the nonlinear system obtained in (1), the system should track the desired trajectory with asymptotally convergence. This controller uses *dynamic feedback linearization.* Use these guided questions and the referred paper to implement this controller in matlab to validate the theory. plot the trajectory in 2D (just x y. omit the orientation)