RBE 502 Homework 2

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```
clc; clear;
```

Question 1: State Space Form

$$m * \ddot{z} + \lambda * \dot{z} + k * z = 0$$

$$\vec{x} = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$$

$$\dot{\vec{x}} = \begin{bmatrix} x_2 \\ \frac{-\lambda * x_2 - k * x_1}{m} \end{bmatrix}$$

Question 2: Dynamic Equation in State Space Form

Given F = u:

$$m * \ddot{z} + \lambda * \dot{z} + k * z = u$$

$$\vec{x} = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$$

$$\dot{\vec{x}} = \begin{bmatrix} x_2 \\ \frac{u - \lambda * x_2 - k * x_1}{m} \end{bmatrix}$$

Question 3: Is the System Controllable?

Symbolically:

$$\dot{x_1} = x_2$$

$$\dot{x_2} = \frac{u}{m} - \frac{\lambda * x_2}{m} - \frac{k * x_1}{m}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{-k}{m} & \frac{-/lambda}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

After substituting values: $k = 2, m = 5, \lambda = 1$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{-2}{5} & \frac{-1}{5} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{5} \end{bmatrix} u$$

The system is controllable, because the system is reachable, since the matrix $\begin{bmatrix} B & A * B \end{bmatrix}$ is full rank, as proven below

```
A = [   0,    1;
     -0.4, -0.2];
B = [0; 0.2];

matrix_rank = rank([B, A*B])
```

```
matrix_rank =

     2
```

Question 4: Design a Setpoint Controller

Since system is stable at the origin, we can ommit $-kx$ from the input function

$$\dot{x} = A * x_e + B(k_r * r)$$

$A * x_e + B(k_r * r) = 0$ && $r = c * x_e$

$$x_e = -B * k_r * r * A^{-1}$$

$$r = -c * A^{-1} * B * k_r * r$$

$$k_r = -(c * A^{-1} * B)^{-1}$$

```
c = [1, 0];
y_r = 5;

k_r = -inv(c * inv(A) * B)

u = k_r * y_r
```

```
k_r =

     2


u =
```

```
10
```

Plugging u back into the equation above, we get:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{-2}{5} & \frac{-1}{5} \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Question 5: Implement a Setpoint Controller in Matlab

Test 1:

```
x0 = 0;
v0 = 0;
[v_out, a_out] = Controller(x0, v0);
disp("Test 1:");
disp("Output Velocity: " + v_out);
disp("Output Acceleration: " + a_out);
```

```
Test 1:
Output Velocity: 0
Output Acceleration: 2
```

Test 2:

```
x0 = -10;
v0 = 16;
[v_out, a_out] = Controller(x0, v0);
disp("Test 2:");
disp("Output Velocity: " + v_out);
disp("Output Acceleration: " + a_out);
```

```
Test 2:
Output Velocity: 16
Output Acceleration: 2.8
```

Test 3: Test to make sure controller stabilizes around $x_0 = 5$

```
x0 = 5;
v0 = 0;
[v_out, a_out] = Controller(x0, v0);
disp("Test 3:");
disp("Output Velocity: " + v_out);
disp("Output Acceleration: " + a_out);
```

```
Test 3:
Output Velocity: 0
Output Acceleration: 0
```

Test 4:

```
x0 = 5;
```

```
v0 = 5;
[v_out, a_out] = Controller(x0, v0);
disp("Test 4:");
disp("Output Velocity: " + v_out);
disp("Output Acceleration: " + a_out);


function [v, a] = Controller(x0, v0)
    out = [0, 1; -0.4, -0.2] * [x0; v0] + [0; 2];
    v = out(1);
    a = out(2);
end
```

```
Test 4:
Output Velocity: 5
Output Acceleration: -1
```

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