

‘Sensitivity boosting’ in ice sheets: tipping points and time-scales

Alex Bradley with Ian Hewitt and C.Yao Lai



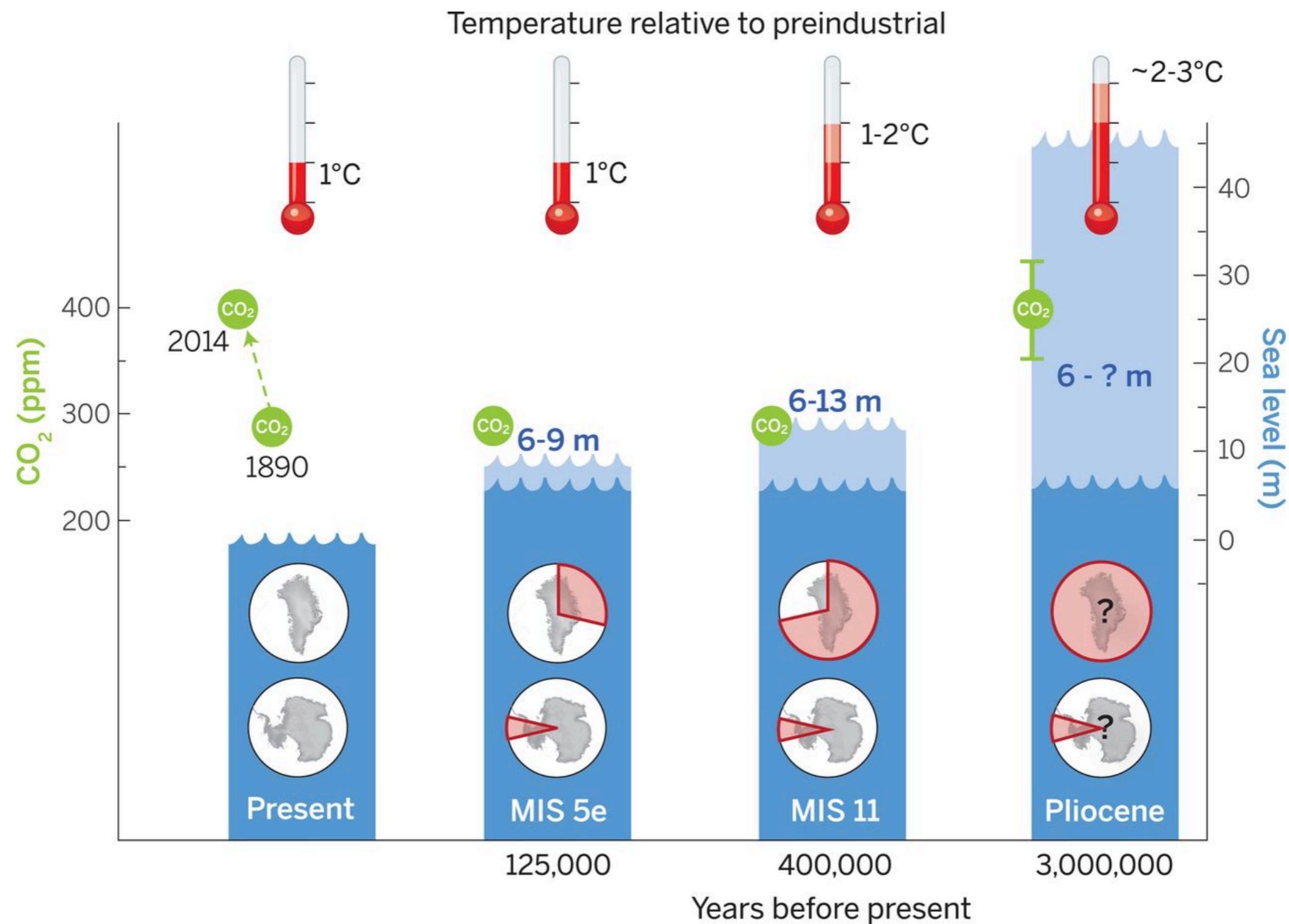
@abraleey



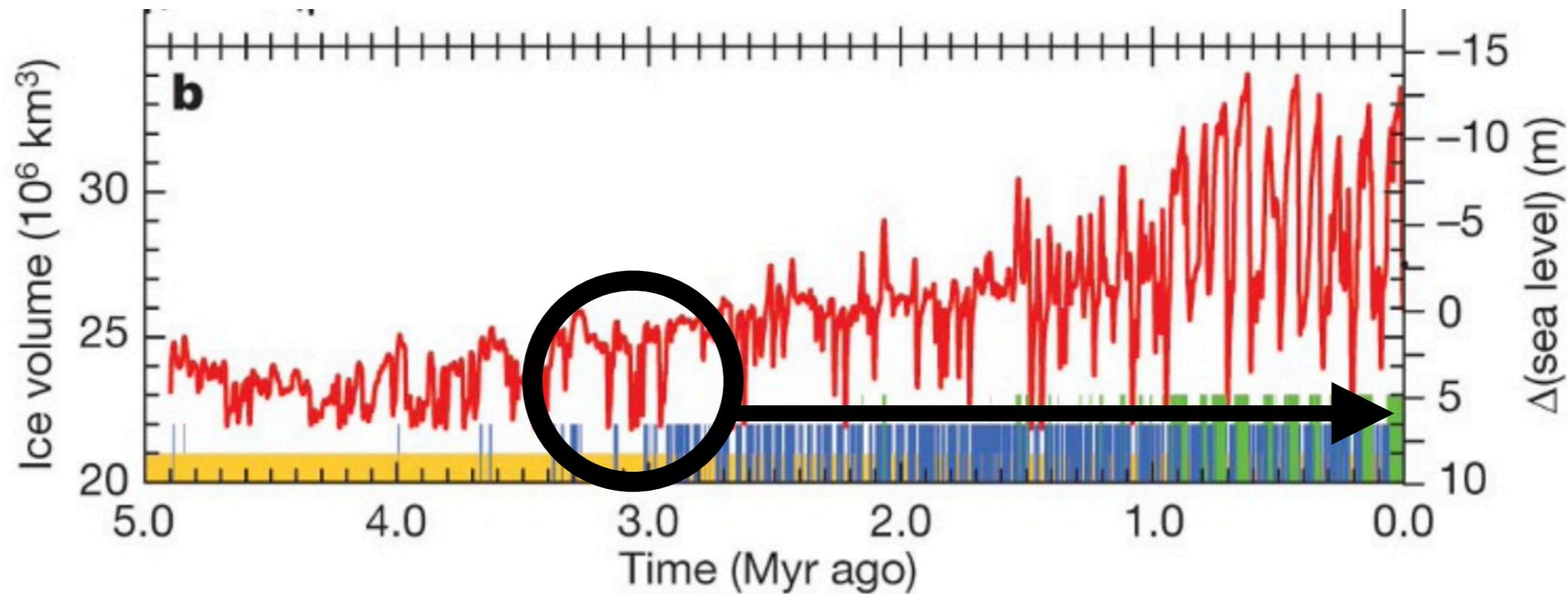
aleey@bas.ac.uk



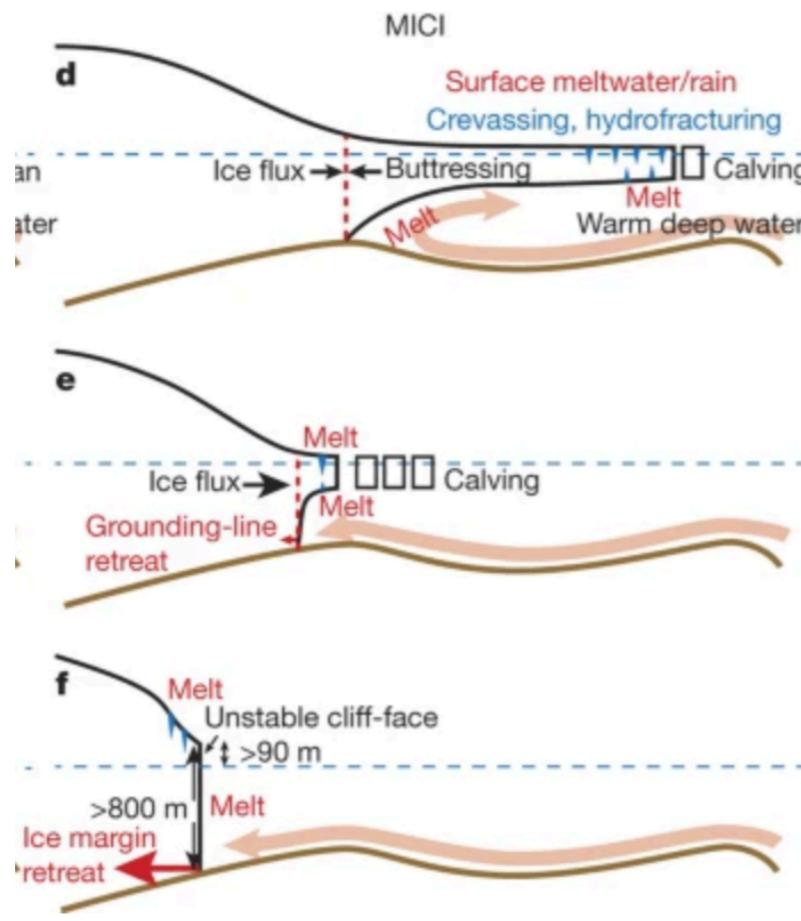
Sea levels have been much higher than today with similar CO₂ and temperatures



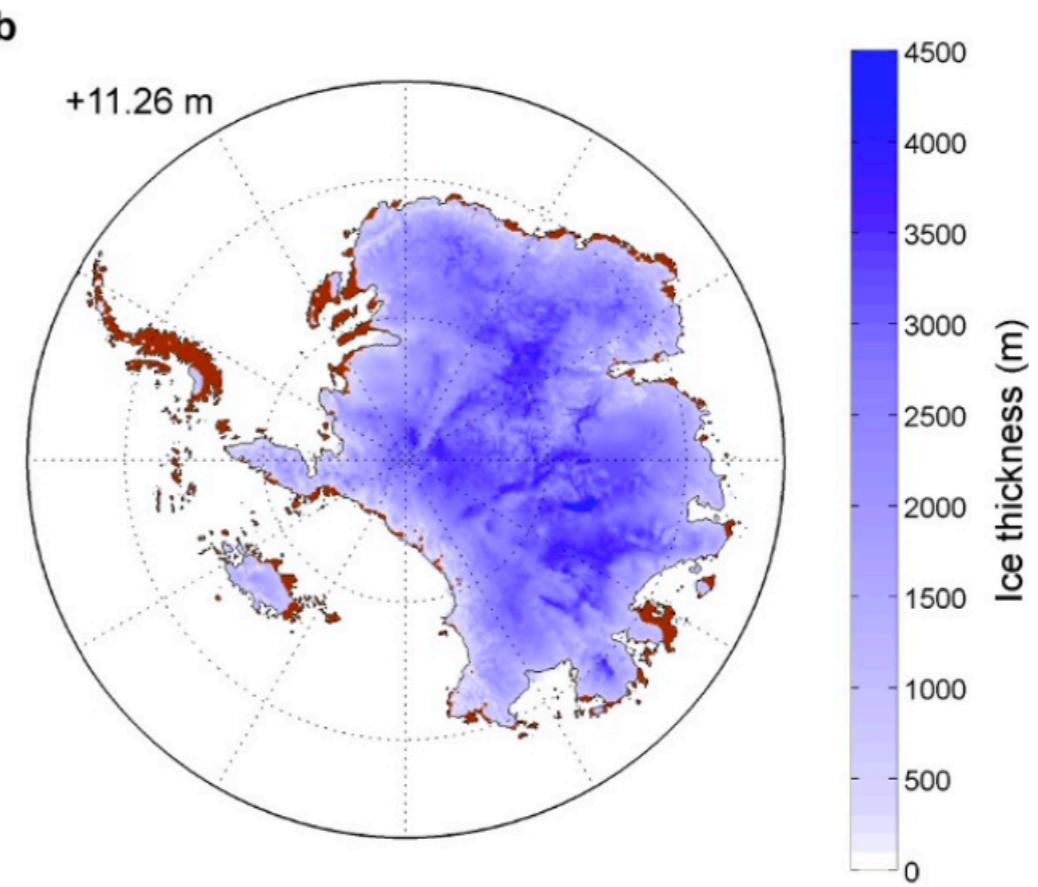
Models struggle to simulate this retreat



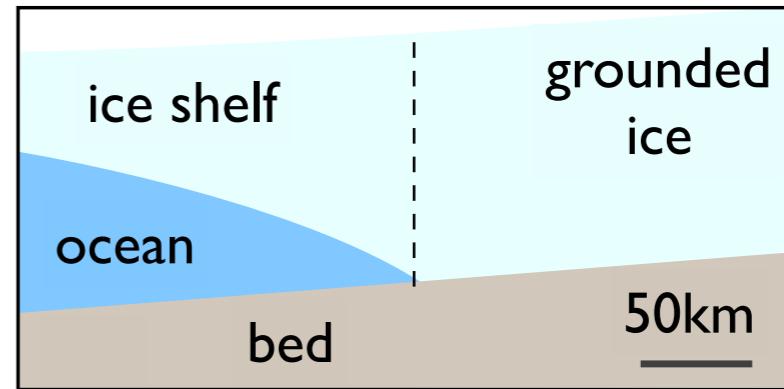
Pollard and DeConto, 2009



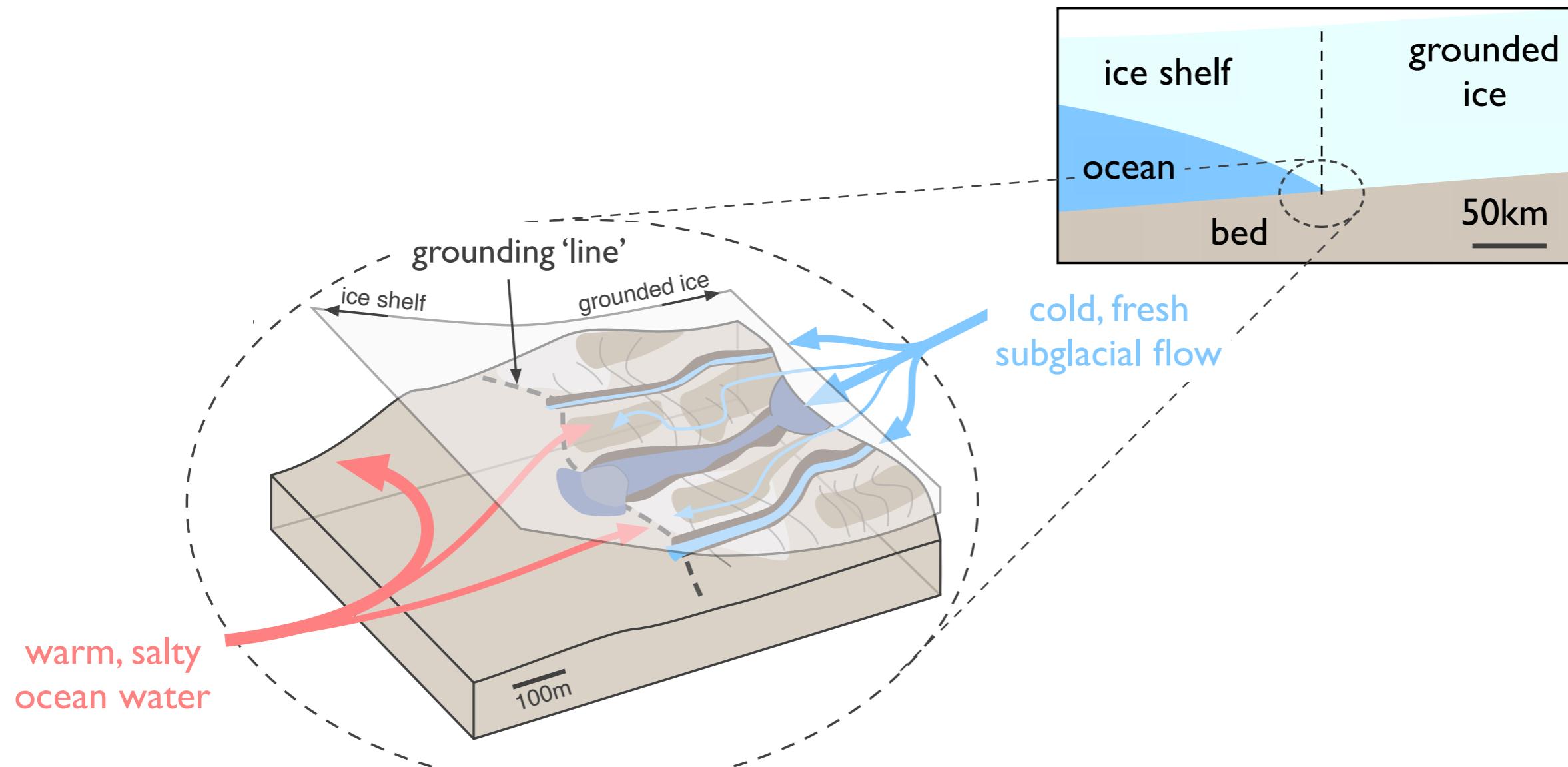
DeConto and Pollard, 2016



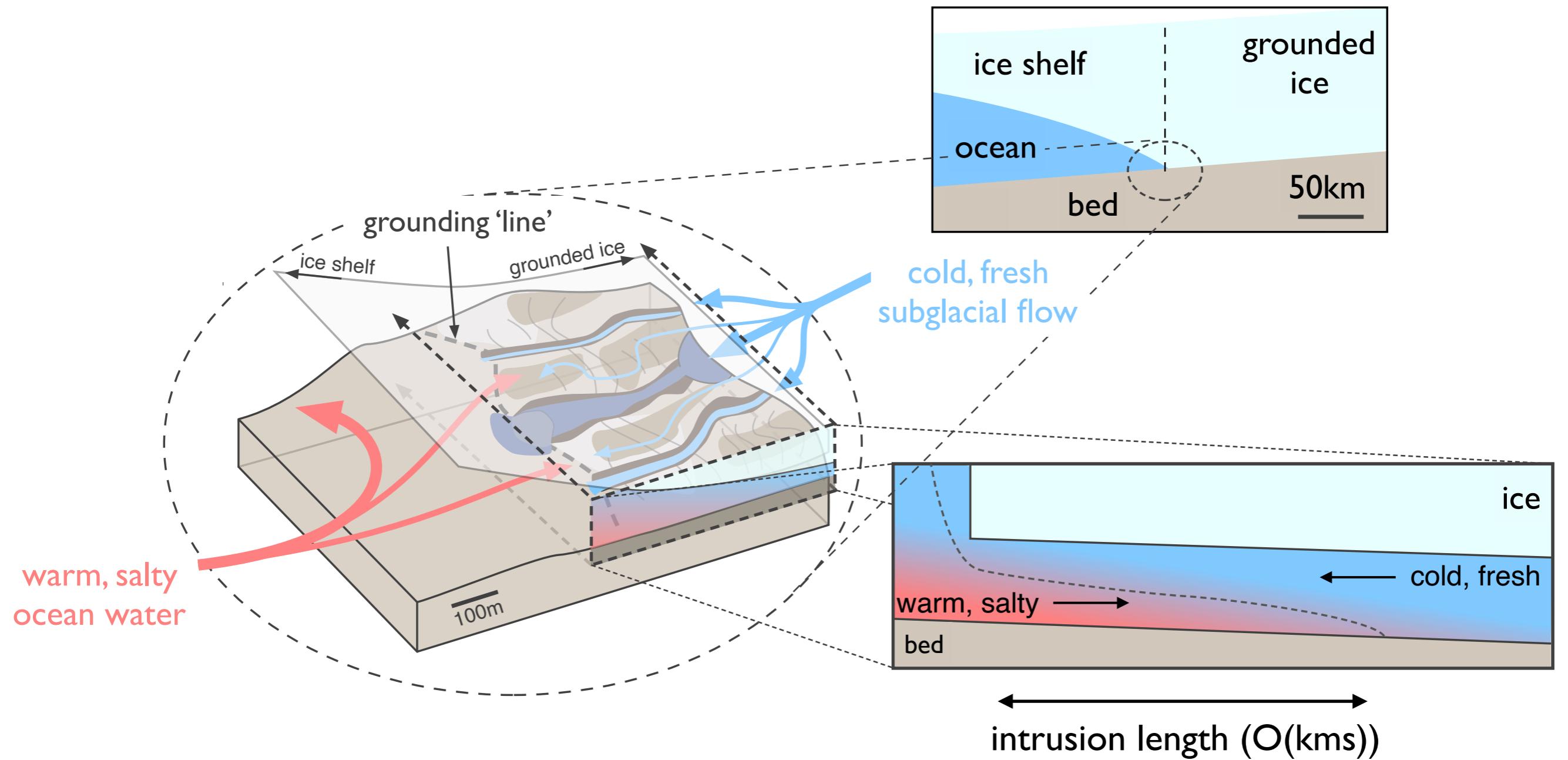
Grounding zone melt boosts ice sheet sensitivity



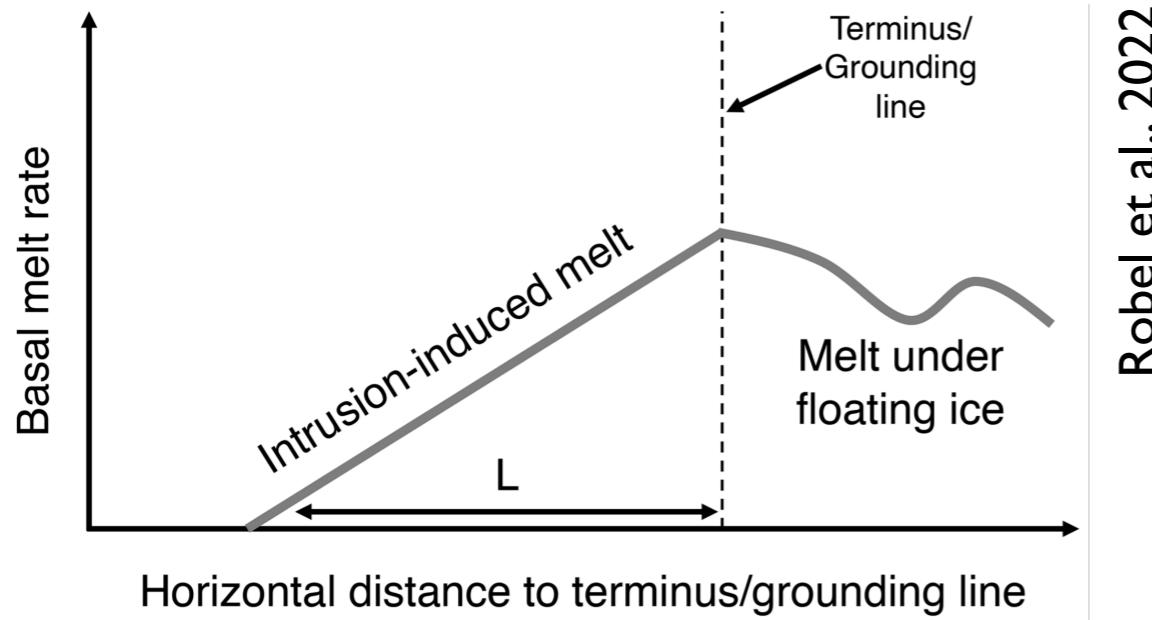
Grounding zone melt boosts ice sheet sensitivity



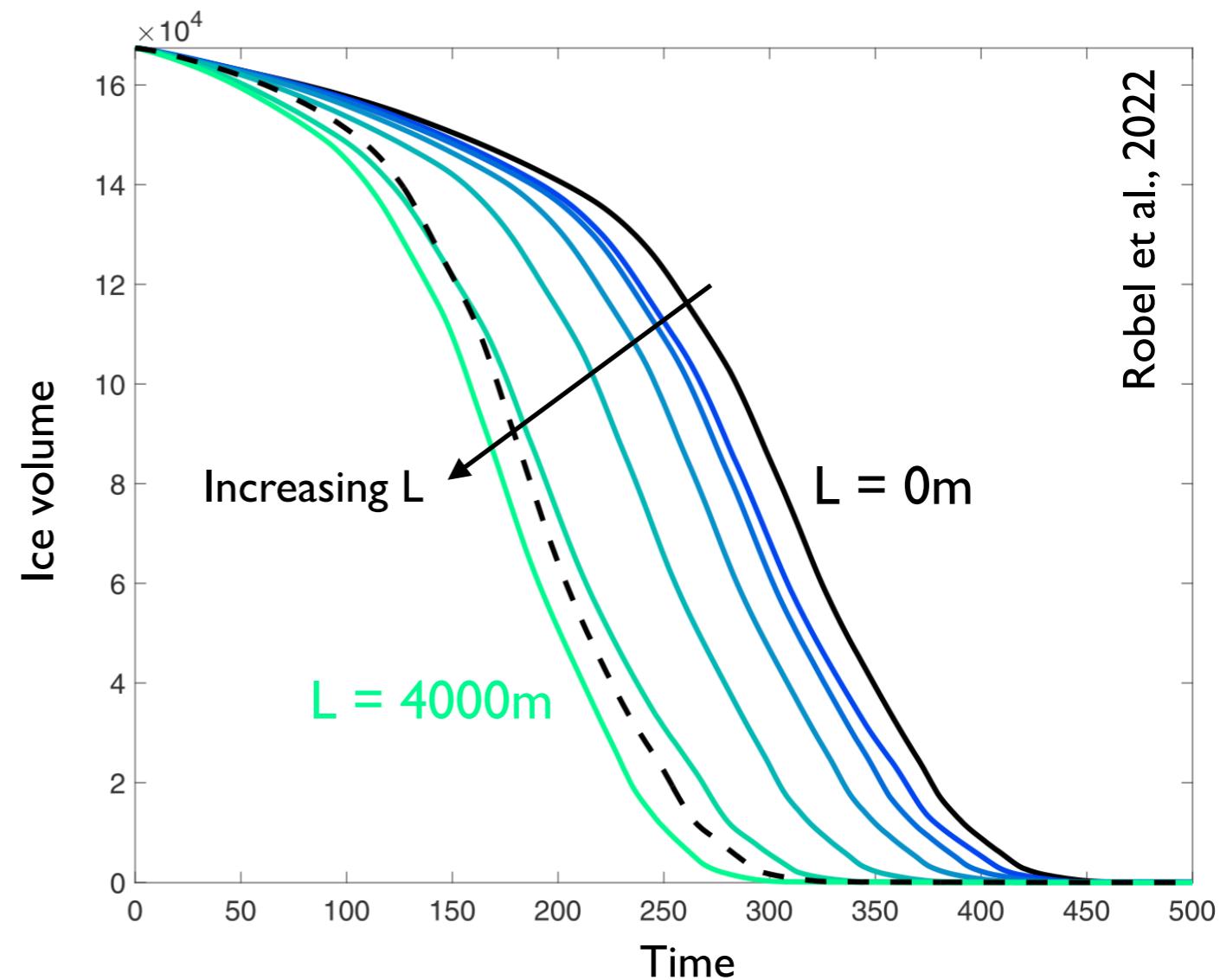
Grounding zone melt boosts ice sheet sensitivity



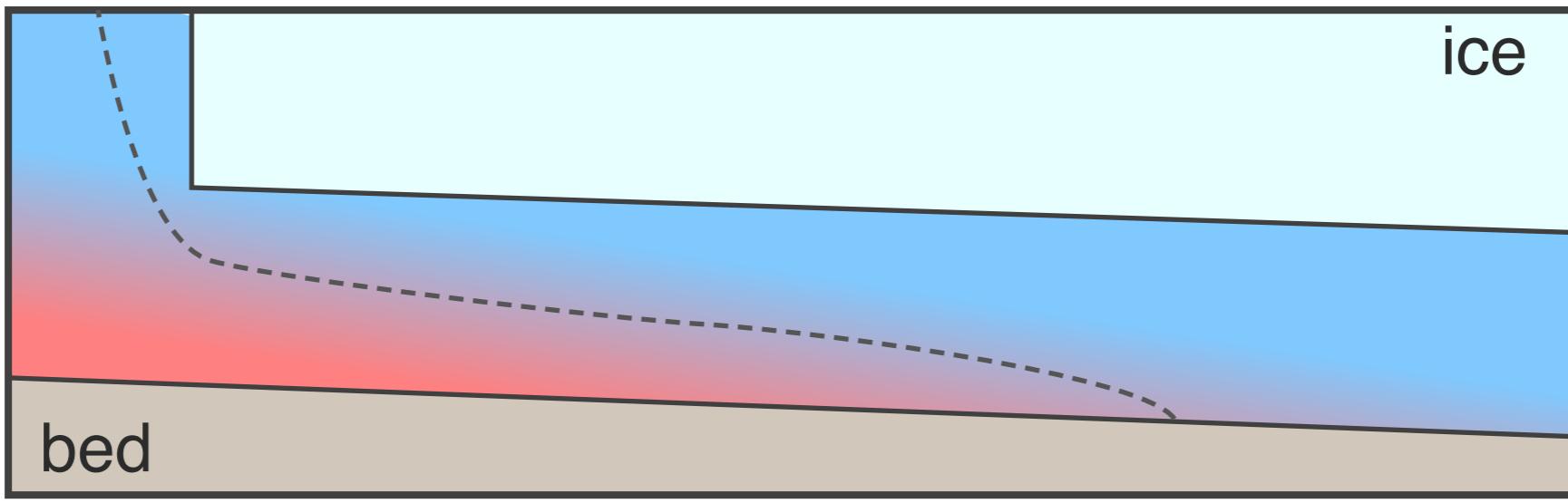
Grounding zone melt boosts ice sheet sensitivity



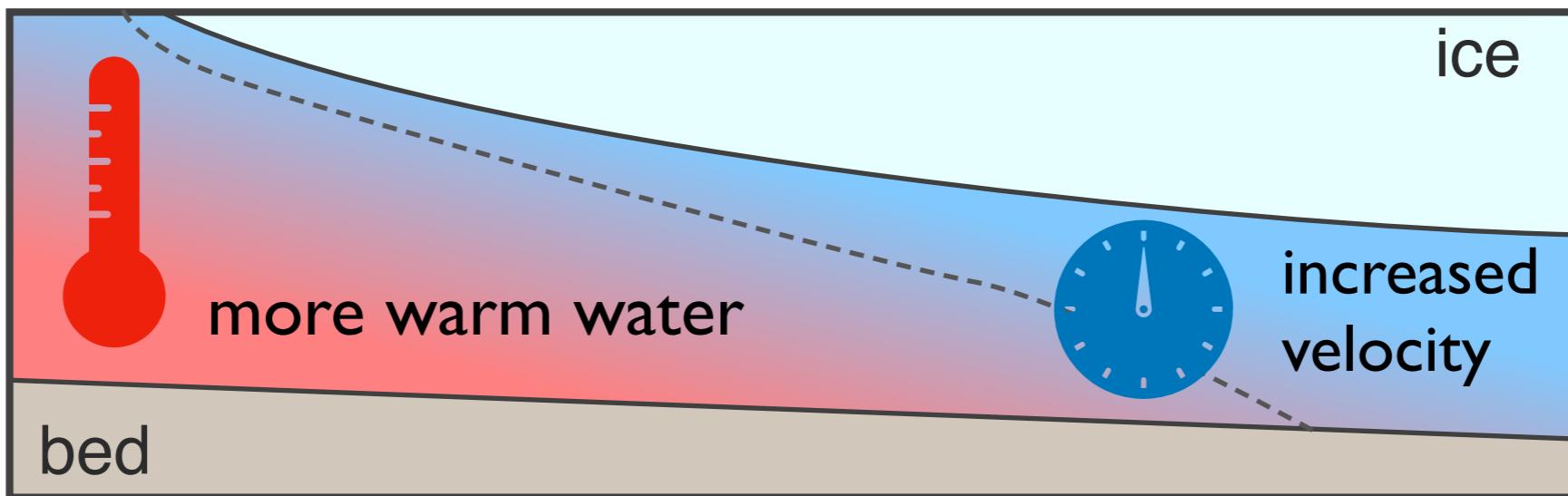
Significant seawater intrusion has dramatic consequences for ice dynamics

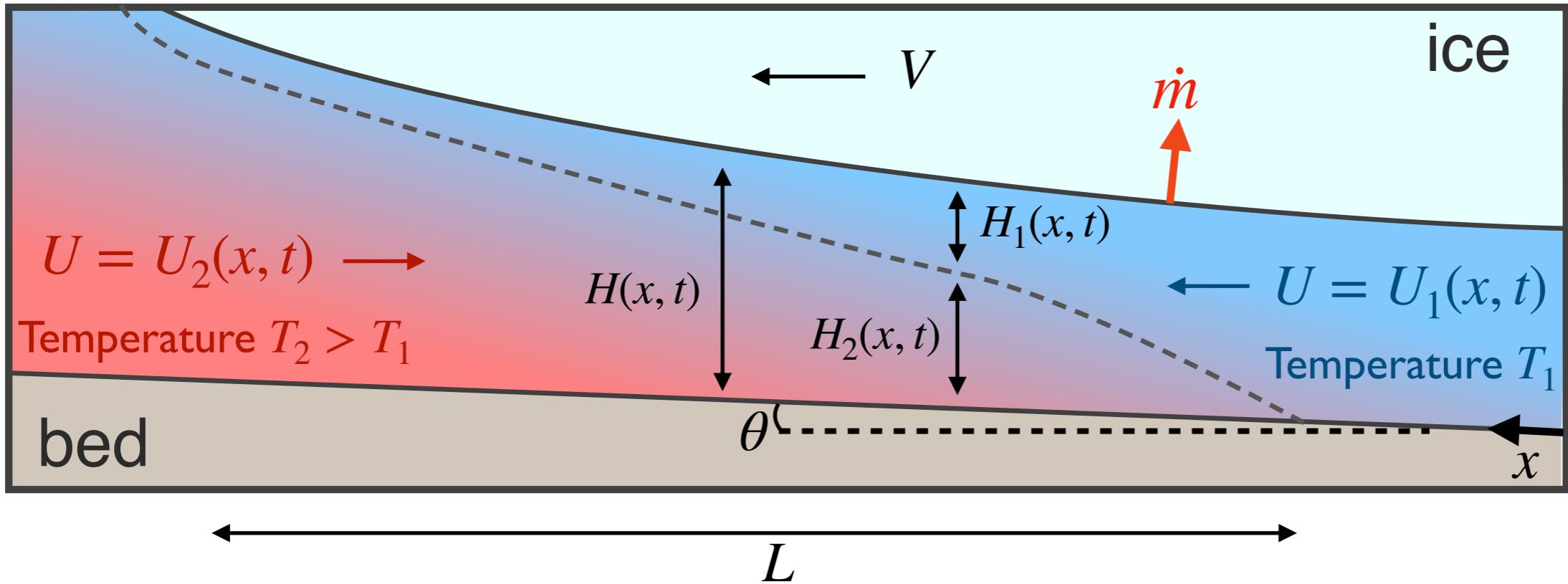


Melt feedbacks on seawater intrusions

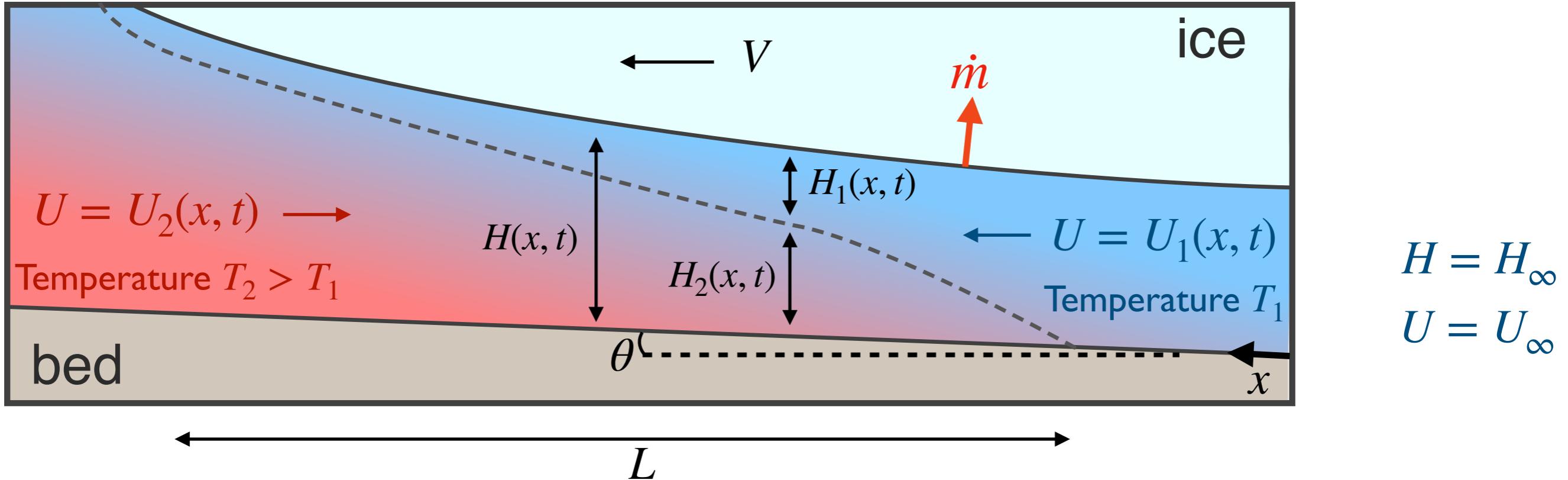


↓
melt response





$$H = H_\infty$$
$$U = U_\infty$$



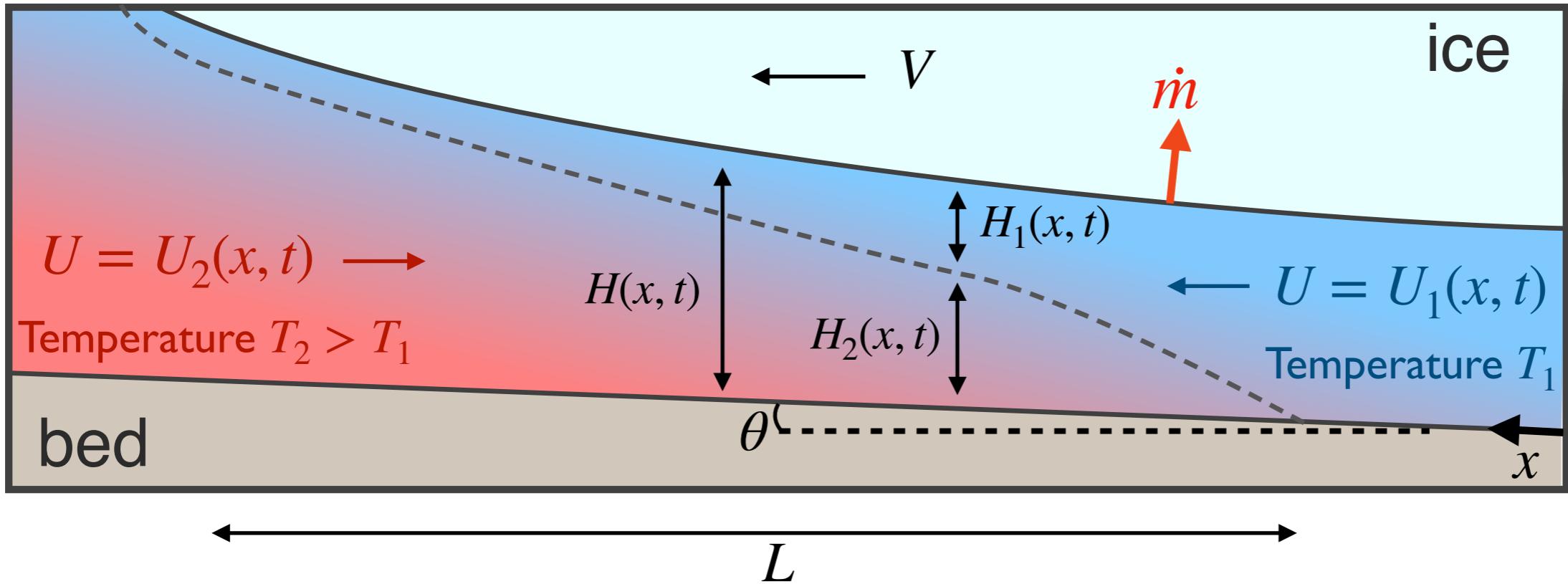
Momentum Conservation:

inertia	barotropic pressure gradient	interfacial drag	wall drag	gravitational driving
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$$\frac{\partial U_1}{\partial t} + U_1 \frac{\partial U_1}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{C_i |U_1 - U_2| (U_1 - U_2)}{H_1} + \frac{C_d U_1^2}{H_1} = 0$$

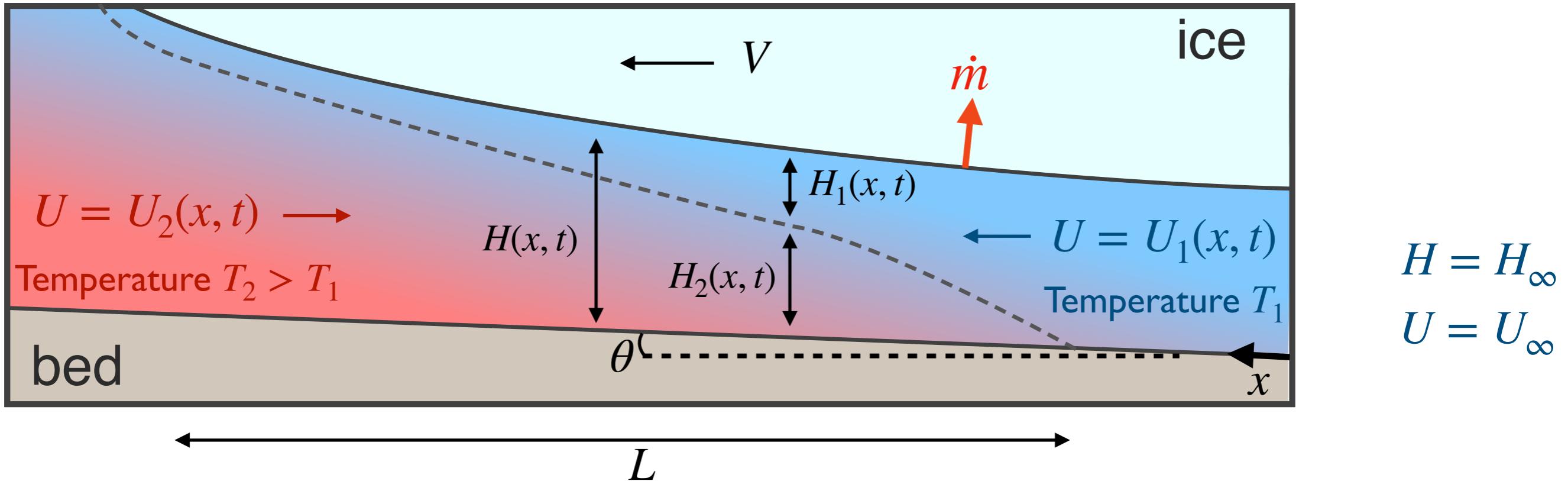
$$\frac{\partial U_2}{\partial t} + U_2 \frac{\partial U_2}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{C_i |U_1 - U_2| (U_1 - U_2)}{H_2} + \frac{C_d U_2^2}{H_2} + g' \left(\frac{\partial H_2}{\partial x} + \tan \theta \right) = 0$$

$$(\text{Fr}^2 - 1) \frac{\partial H_1}{\partial x} = \text{Fr}^2 \left(C_d + C_i \frac{H}{H - H_1} \right) - \left(\tan \theta + \frac{\partial H}{\partial x} \right)$$



Momentum Conservation:

$$(\text{Fr}^2 - 1) \frac{\partial H_1}{\partial x} = \text{Fr}^2 \left(C_d + C_i \frac{H}{H - H_1} \right) - \left(\tan \theta + \frac{\partial H}{\partial x} \right)$$



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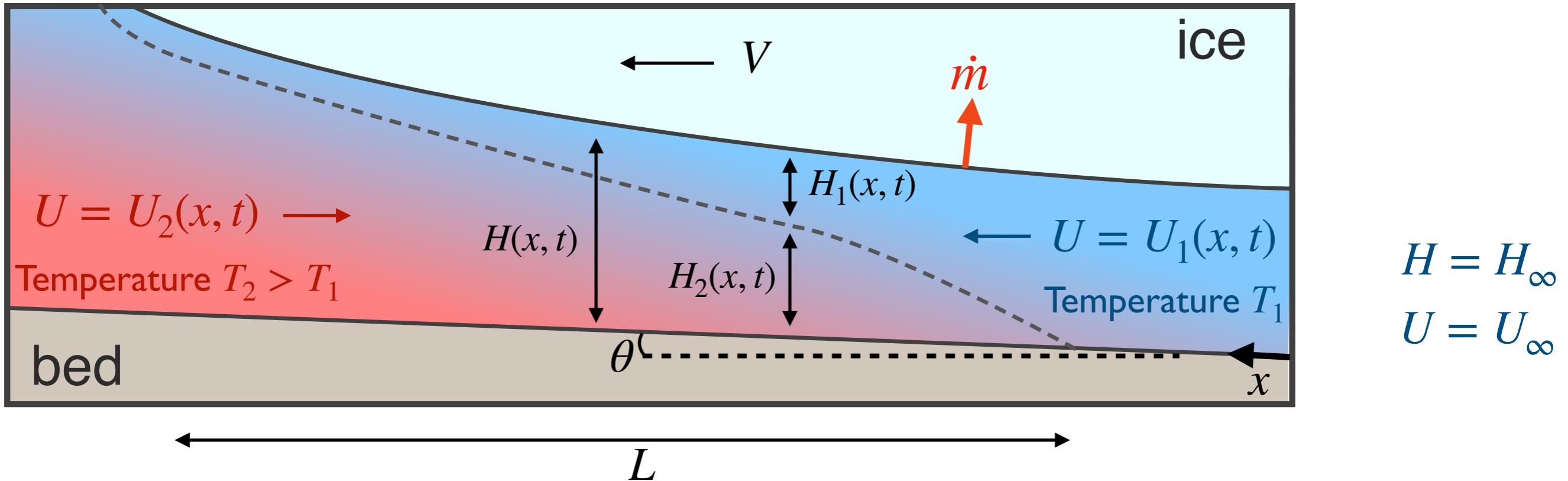
Melting: $\dot{m} = \frac{StC}{L} u^* \Delta T$

thermal driving
boundary layer velocity $u^* = U_1$

$$\begin{aligned} \Delta T &= T - T_f \\ T &= \frac{H_1}{H} T_1 + \left(1 - \frac{H_1}{H} \right) T_2 \end{aligned}$$

T_f : local freezing point

$$\dot{m} = \frac{StC}{L} U_1 \left[\frac{H_1}{H} T_1 + \left(1 - \frac{H_1}{H} \right) \right]$$



Momentum Conservation:

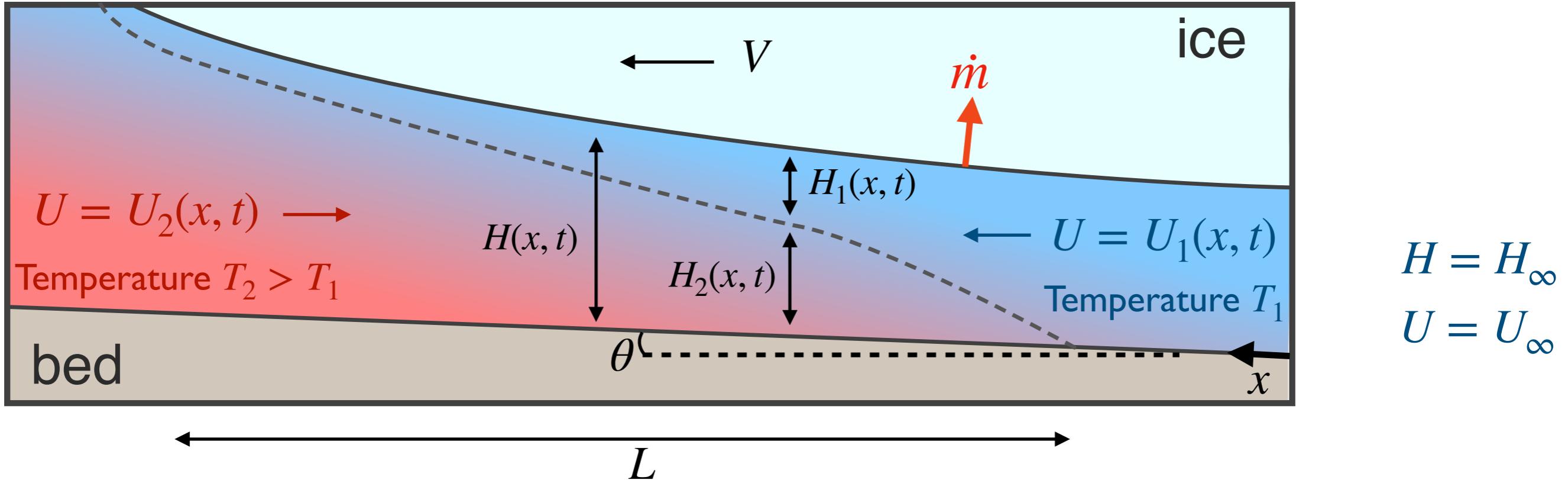
$$(Fr^2 - 1) \frac{\partial H_1}{\partial x} = Fr^2 \left(C_d + C_i \frac{H}{H - H_1} \right) - \left(\tan \theta + \frac{\partial H}{\partial x} \right)$$

Melting:

$$\dot{m} = \frac{StC}{L} U_1 \left[\frac{H_1}{H} T_1 + \left(1 - \frac{H_1}{H} \right) \right]$$

$$H = H_\infty$$

$$U = U_\infty$$



Momentum Conservation:

$$(Fr^2 - 1) \frac{\partial H_1}{\partial x} = Fr^2 \left(C_d + C_i \frac{H}{H - H_1} \right) - \left(\tan \theta + \frac{\partial H}{\partial x} \right)$$

Melting:

$$\dot{m} = \frac{StC}{L} U_1 \left[\frac{H_1}{H} T_1 + \left(1 - \frac{H_1}{H} \right) \right]$$

Kinematic Condition:

$$\frac{\partial H}{\partial t} + V \frac{\partial H}{\partial x} = \dot{m}$$

Momentum conservation:

$$\left(\frac{F^2}{h_1^3} - 1 \right) \frac{\partial h_1}{\partial x} = \frac{F^2}{h_1^3} \left(1 + C \frac{h}{h - h_1} \right) - \left(S + \frac{\partial h}{\partial x} \right)$$

Melting + kinematic:

$$\frac{\partial h}{\partial t} + \frac{1}{M} \frac{\partial h}{\partial x} = \frac{1}{h_1} \left(1 - \frac{h_1}{h} \right)$$

$$S = \frac{\tan \theta}{C_d}$$

dimensionless bed slope

$$F = \frac{U_\infty}{\sqrt{g' H_\infty}}$$

upstream Froude number

$$C = \frac{C_i}{C_d}$$

dimensionless drag

$$M = \frac{u_\infty}{V} \frac{St}{c_d} \frac{T_2 - T_1}{L/c}$$

dimensionless melt

+ boundary condition: $h_1^{3/2} = F^{2/3}$ at $x = 0$

$T_2 = 1.9^\circ C$ ($M = 0.38$)

$t = 0.0$



70m (no melt feedback, Robel et al. 2022)

110m → with melt feedback

‘bounded intrusion’

$$M < M_c$$

$T_2 = 2^\circ C$ ($M = 0.4$)

$t = 0.0$



70m

$L \rightarrow \infty$

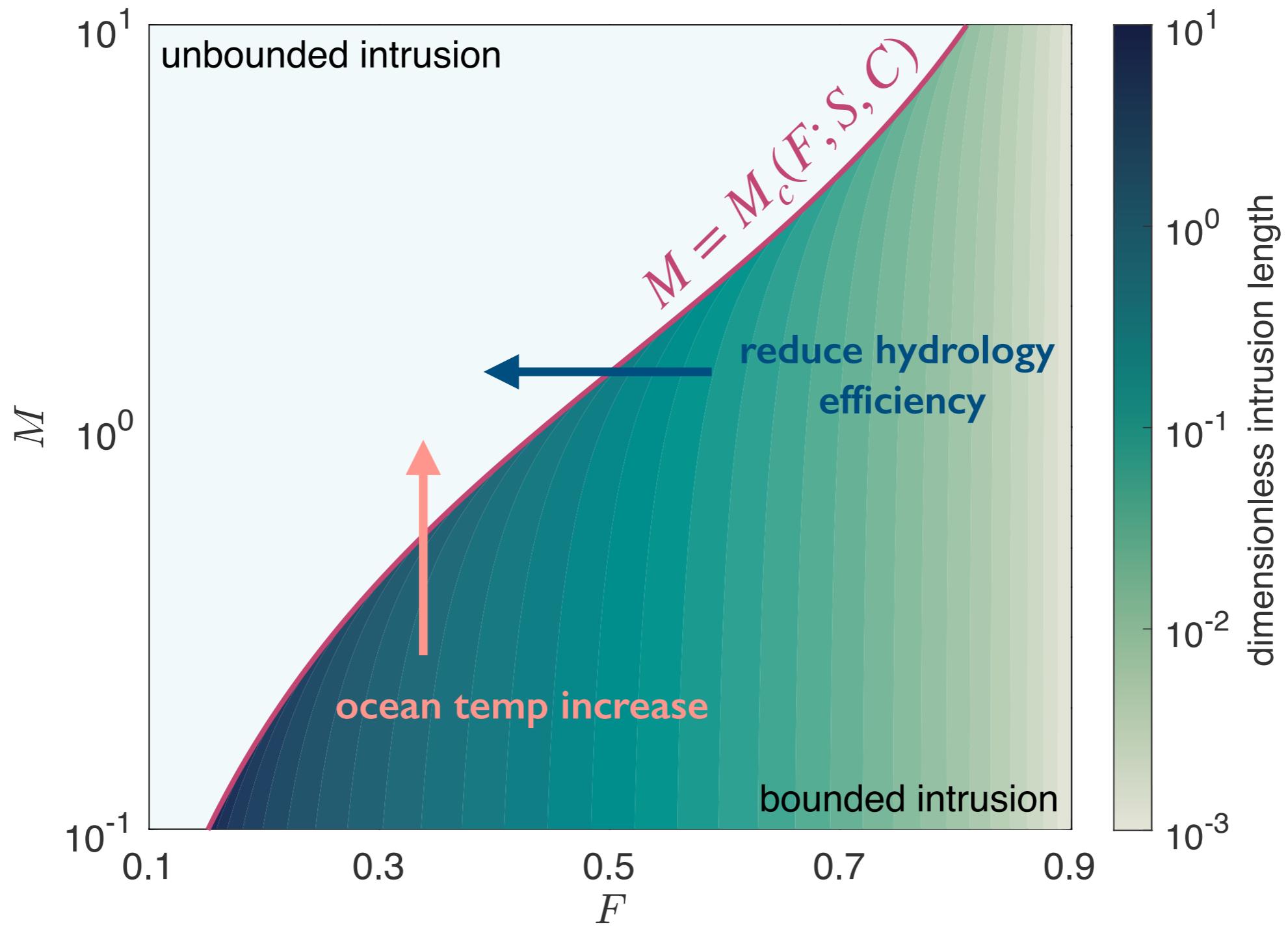
‘unbounded intrusion’

$$M > M_c$$

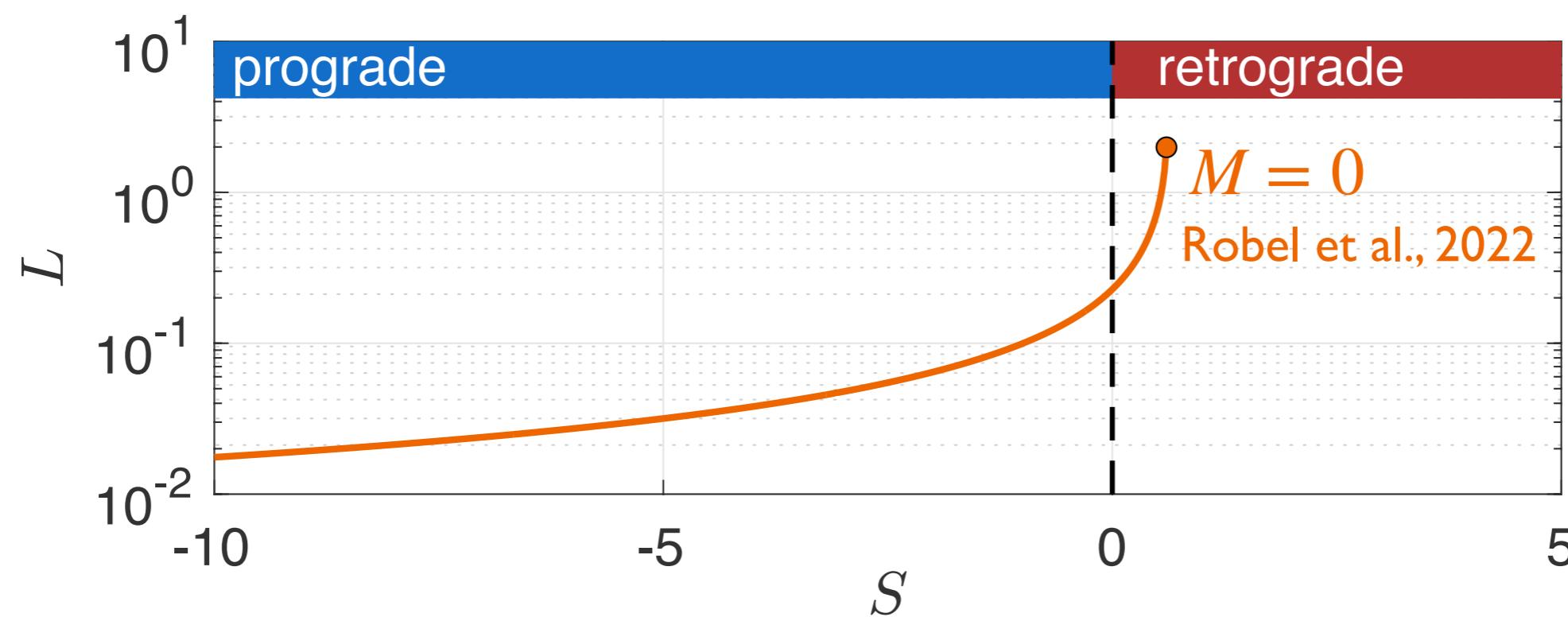
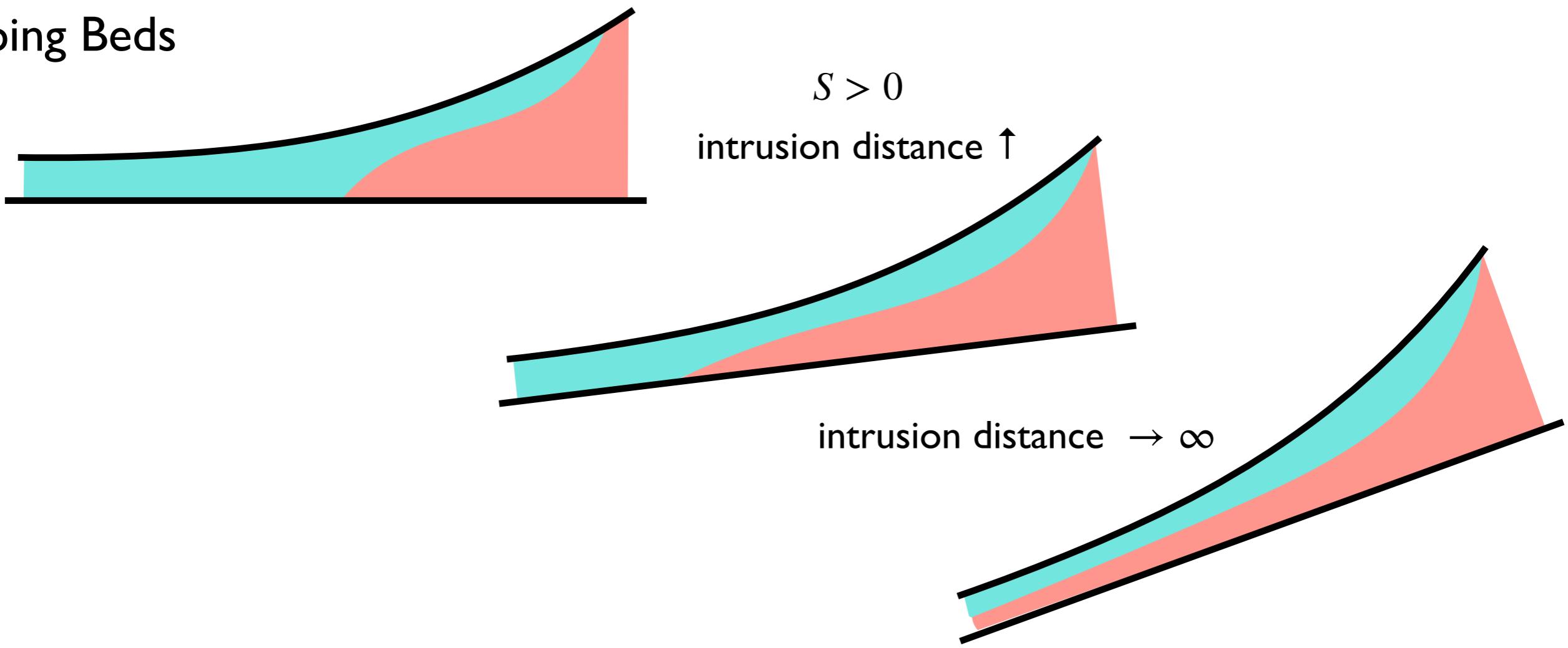
A small change in ocean temperature leads to a large response in grounding zone melting, with significant implications for ice dynamics

$$M = \frac{u_\infty}{V} \frac{\text{St}}{c_d} \frac{T_2 - T_1}{L/c}$$

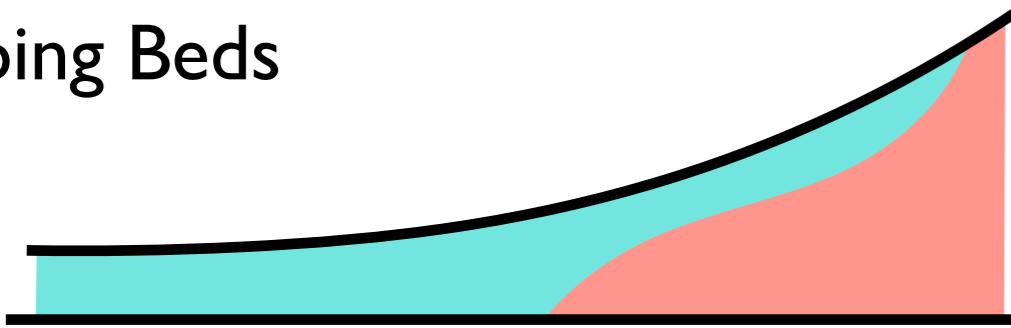
V : ice velocity
 u_∞ : upstream meltwater velocity
 $T_2 - T_1$: ocean forcing



Sloping Beds

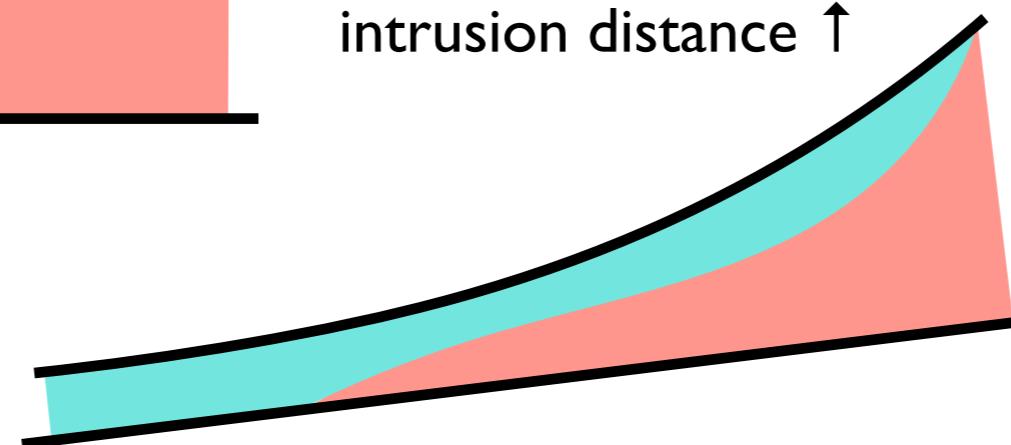


Sloping Beds

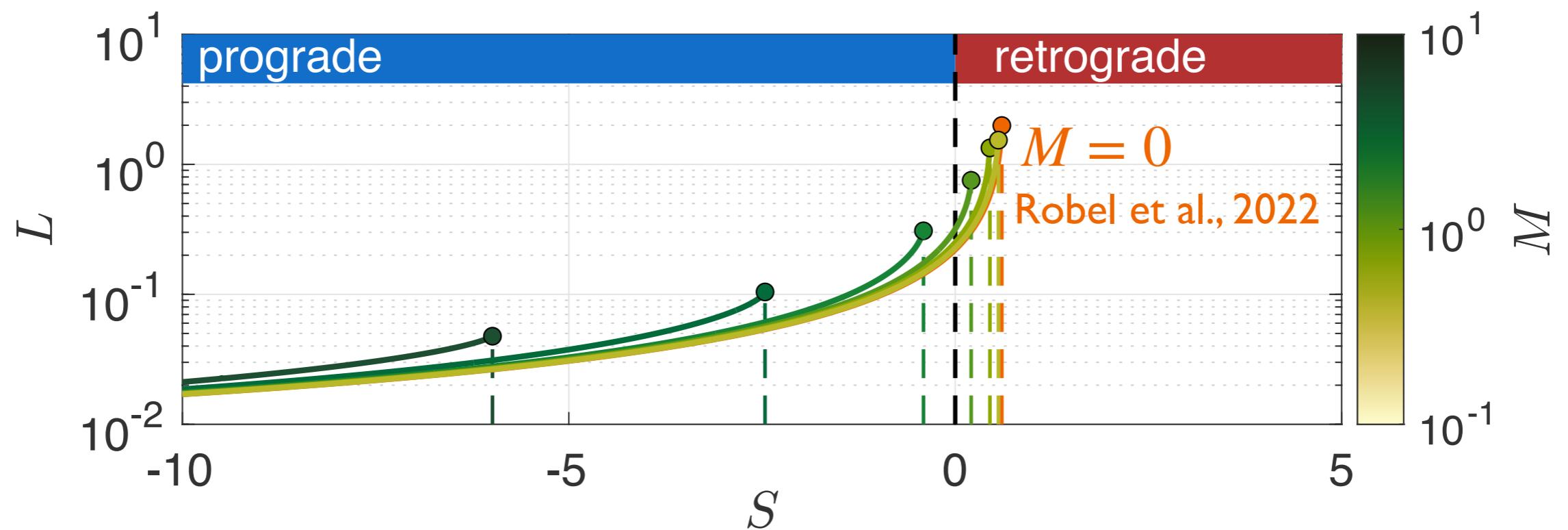
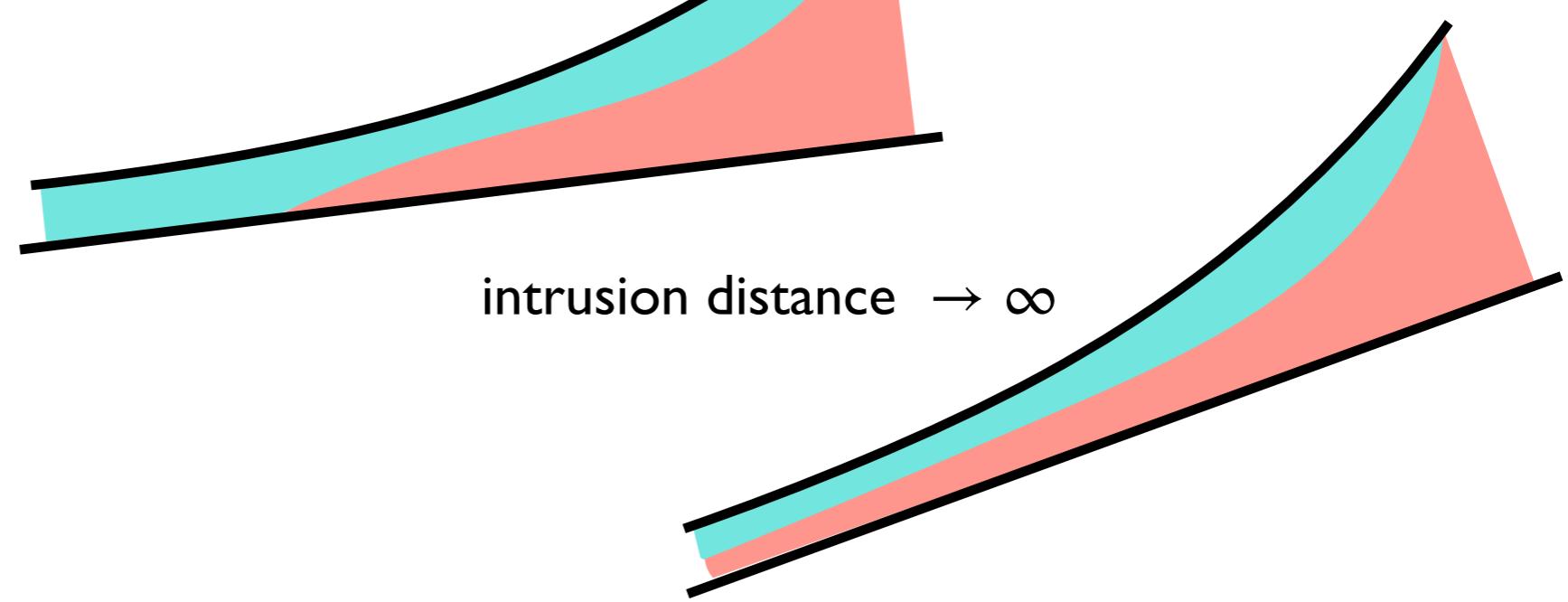


$$S > 0$$

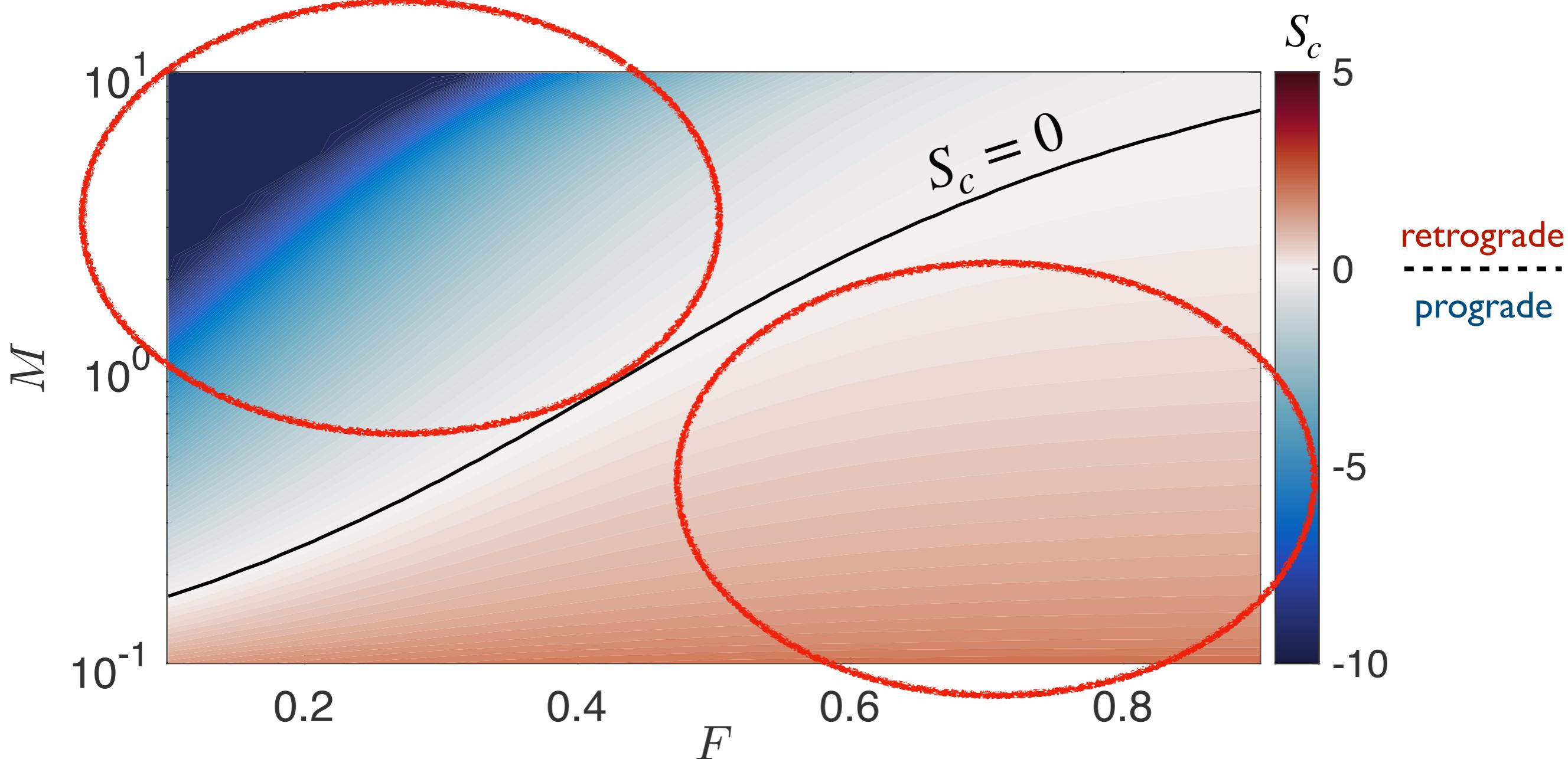
intrusion distance \uparrow



intrusion distance $\rightarrow \infty$



prograde bedslopes can also have unbounded
intrusion - less stable than we think?



melt feedback makes unbounded intrusion easier on
retrograde bedslopes - candidate mechanism to explain
warm period retreat?

F is poorly constrained, plot as a function of M, S

$$S = \frac{\tan \theta}{C_d}$$

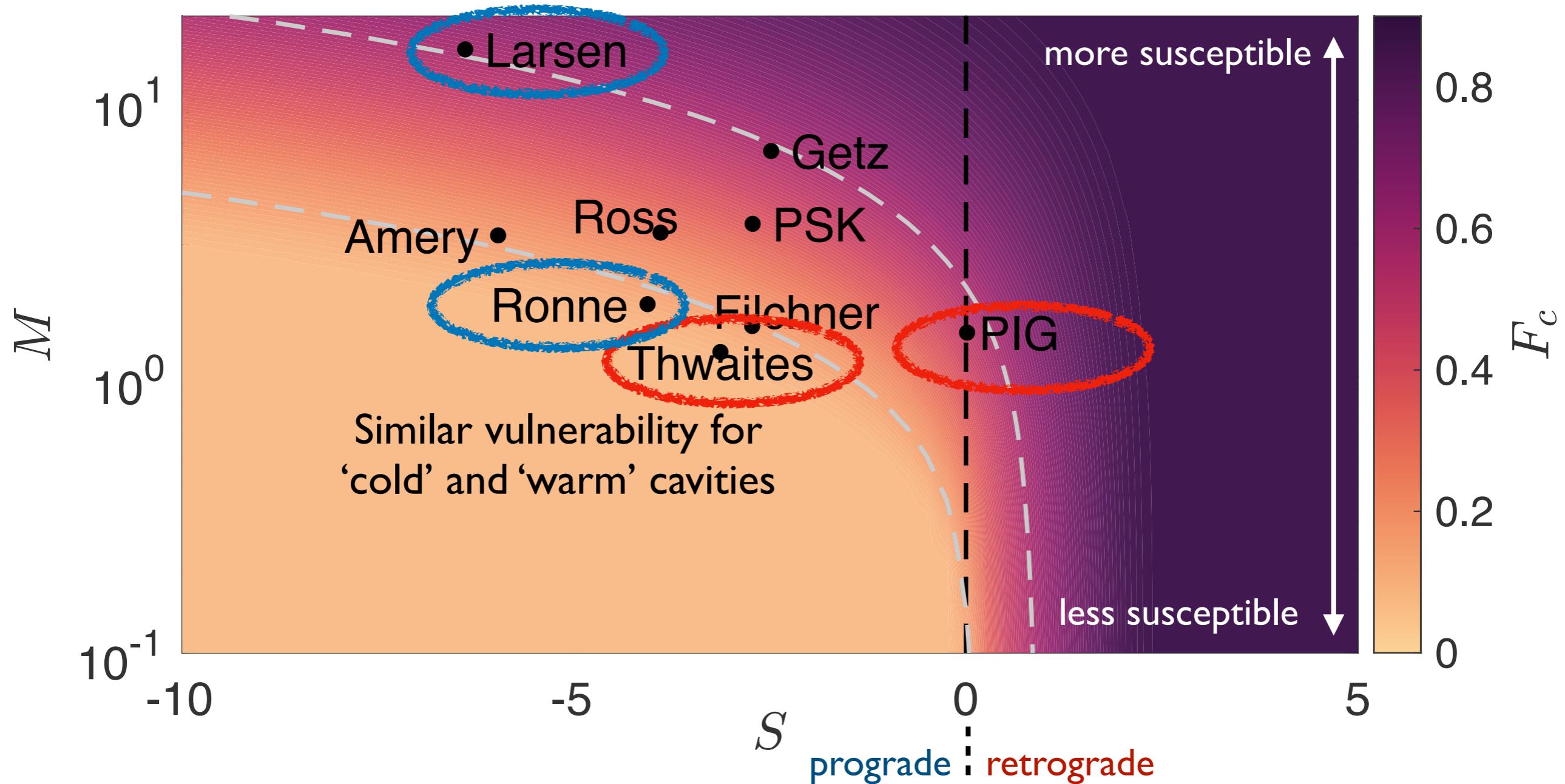
dimensionless bed slope

$$F = \frac{U_\infty}{\sqrt{g' H_\infty}}$$

upstream Froude number

$$M = \frac{u_\infty}{V} \frac{St}{c_d} \frac{T_2 - T_1}{L/c}$$

dimensionless melt



F is poorly constrained, plot as a function of M, S

$$S = \frac{\tan \theta}{C_d}$$

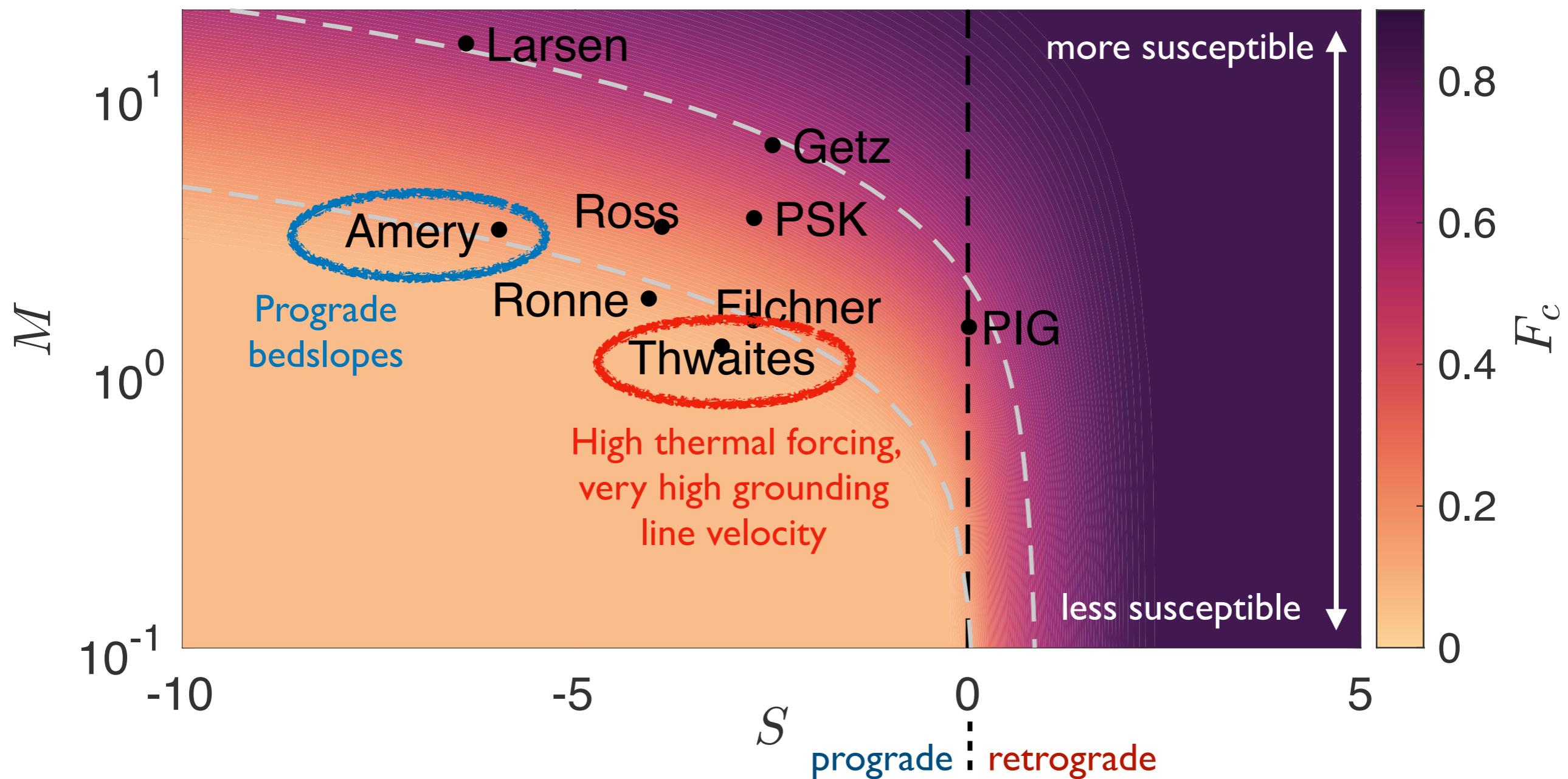
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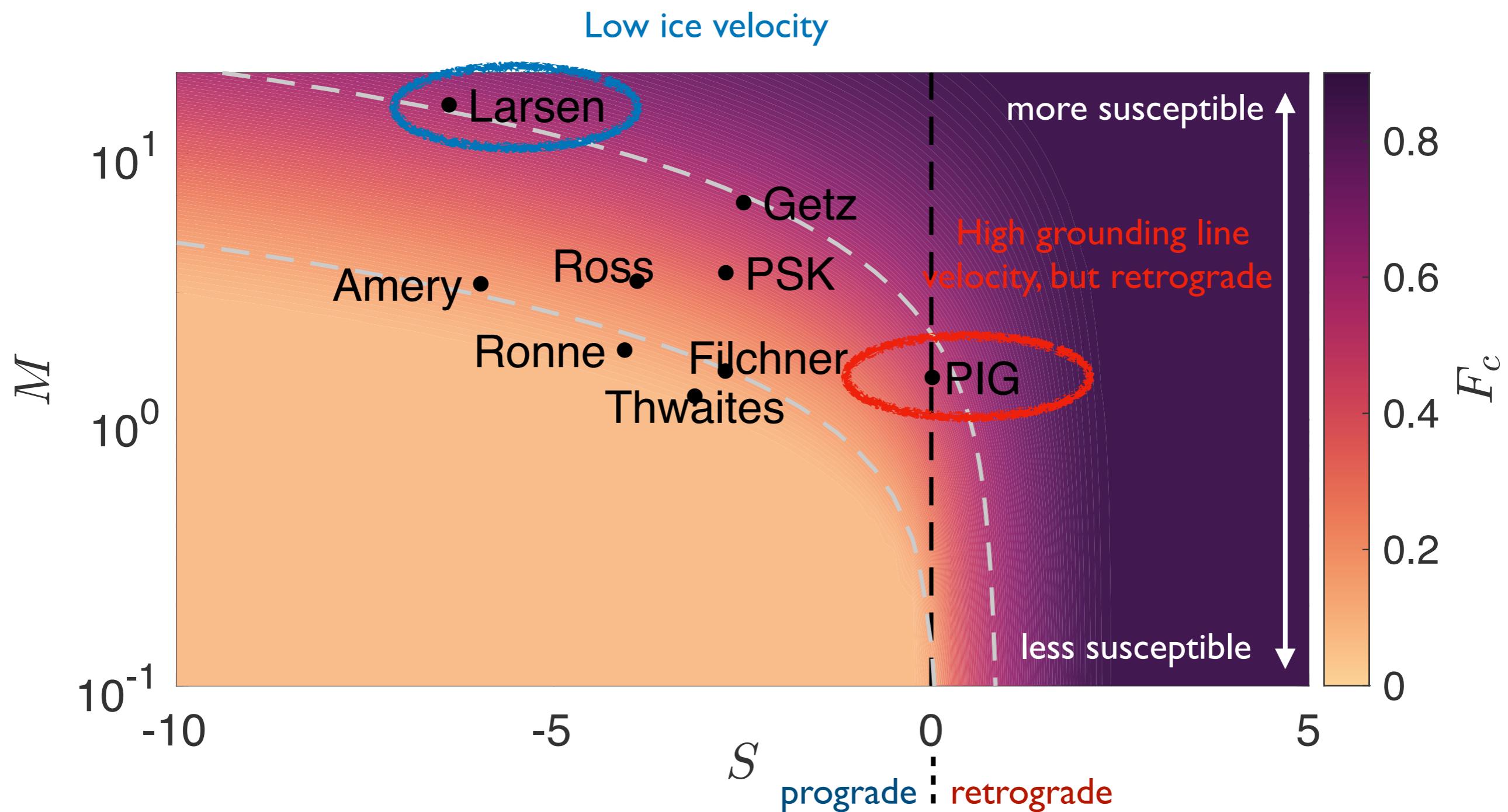
dimensionless bed slope

$$F = \frac{U_\infty}{\sqrt{g' H_\infty}}$$

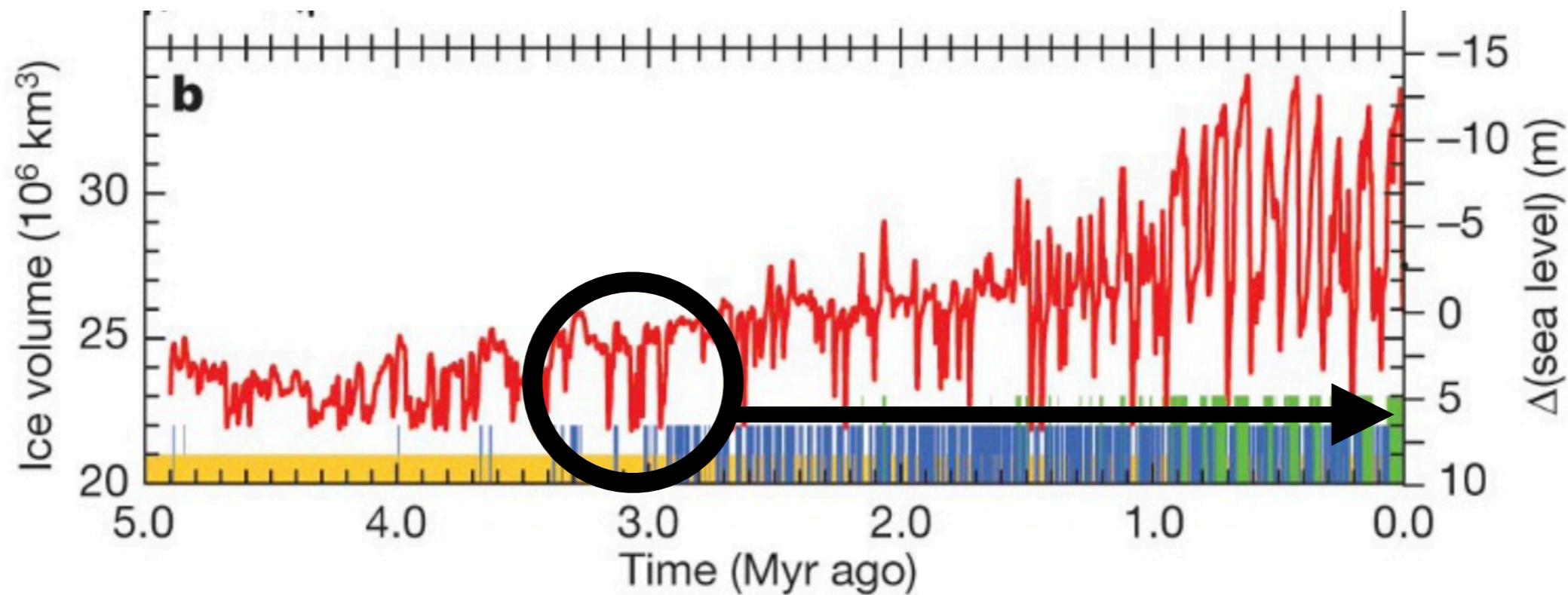
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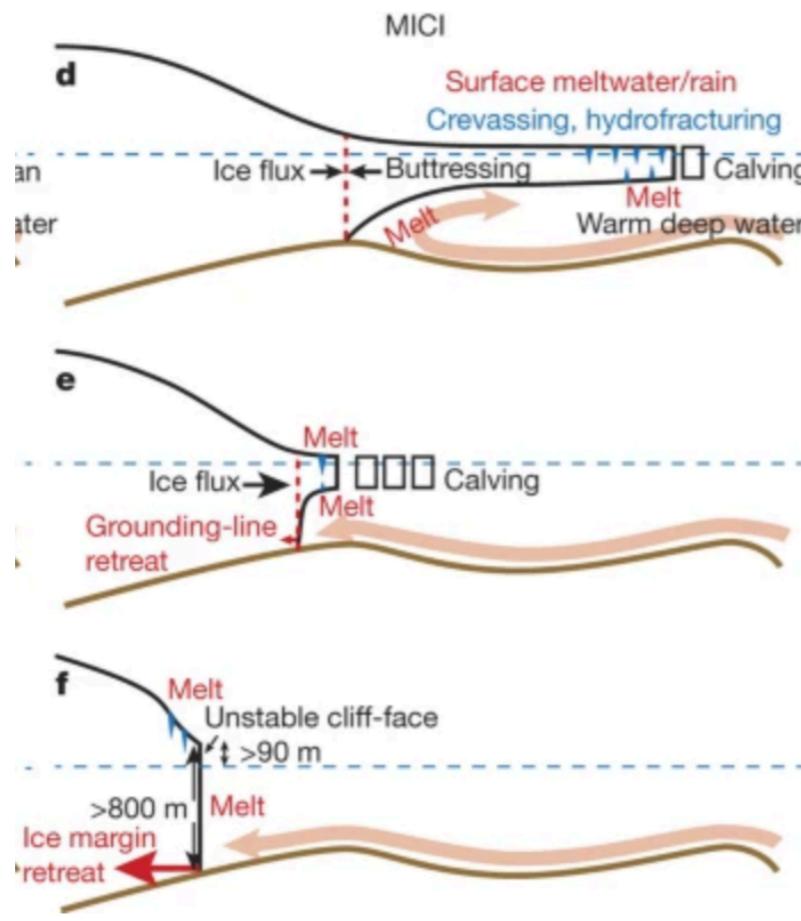
dimensionless melt



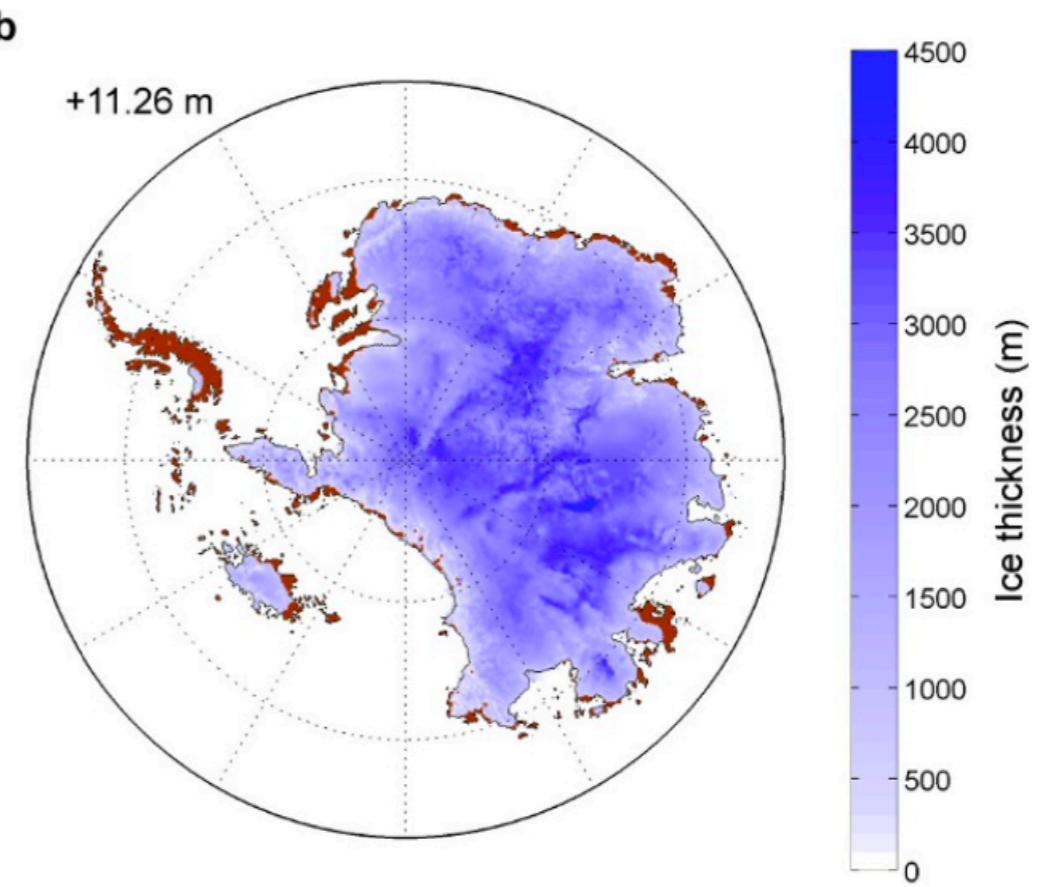
Models struggle to simulate this retreat



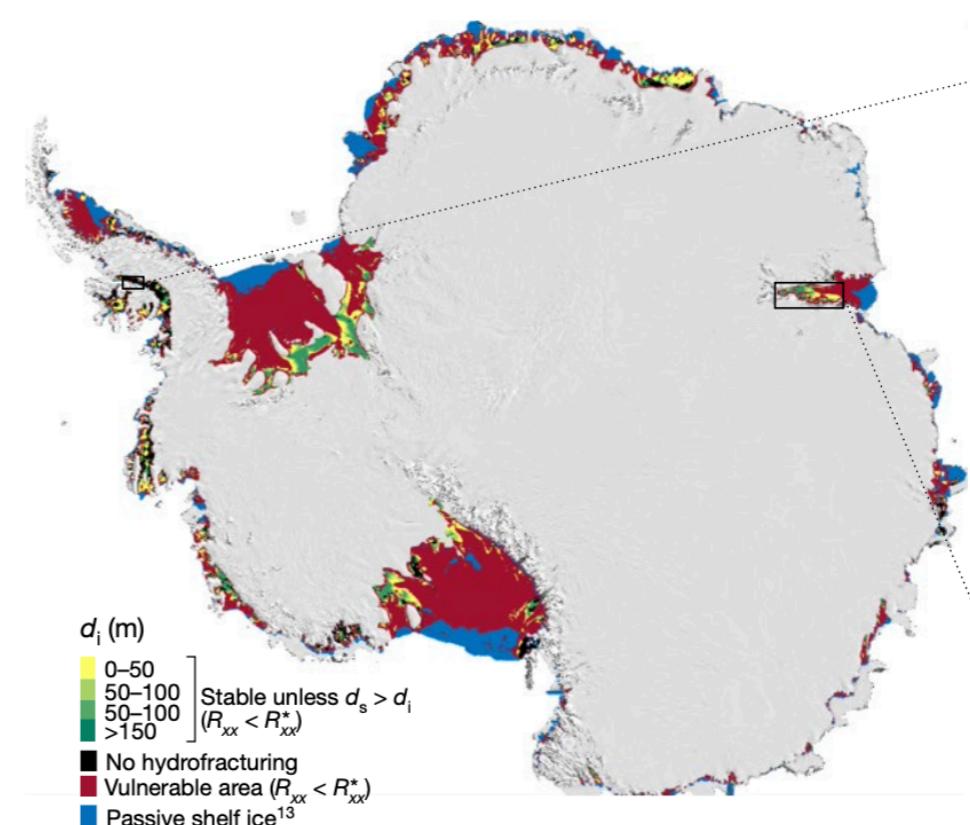
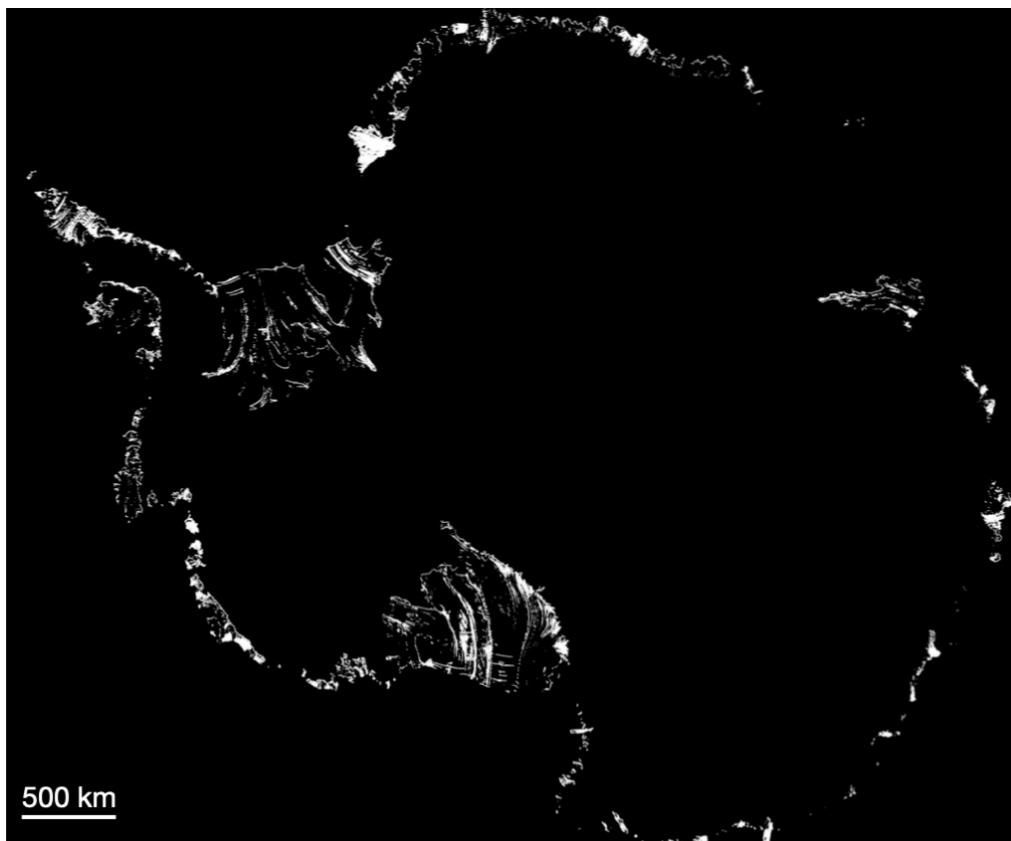
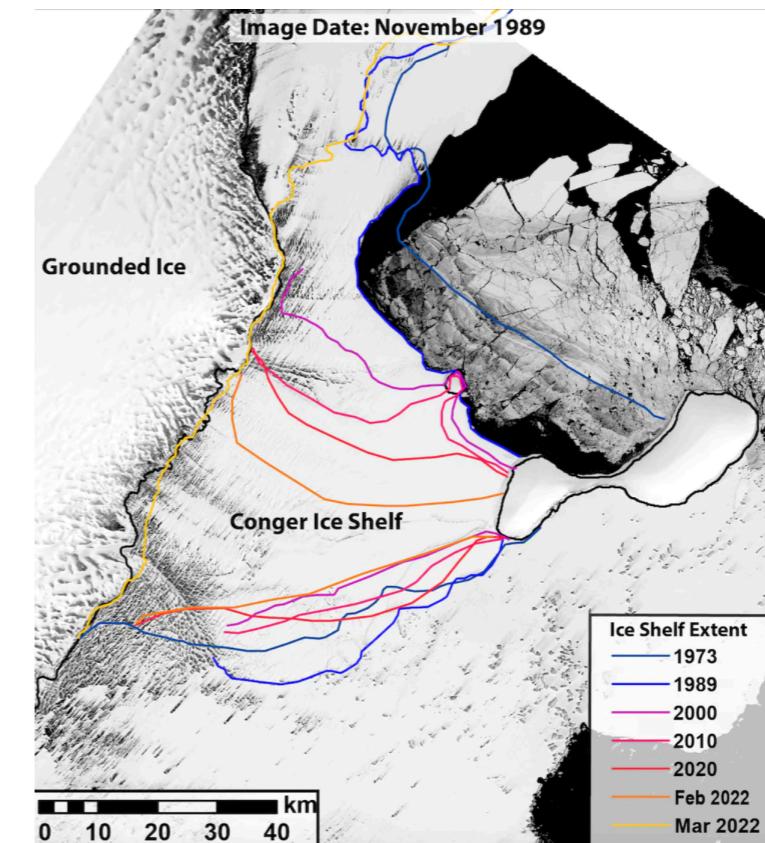
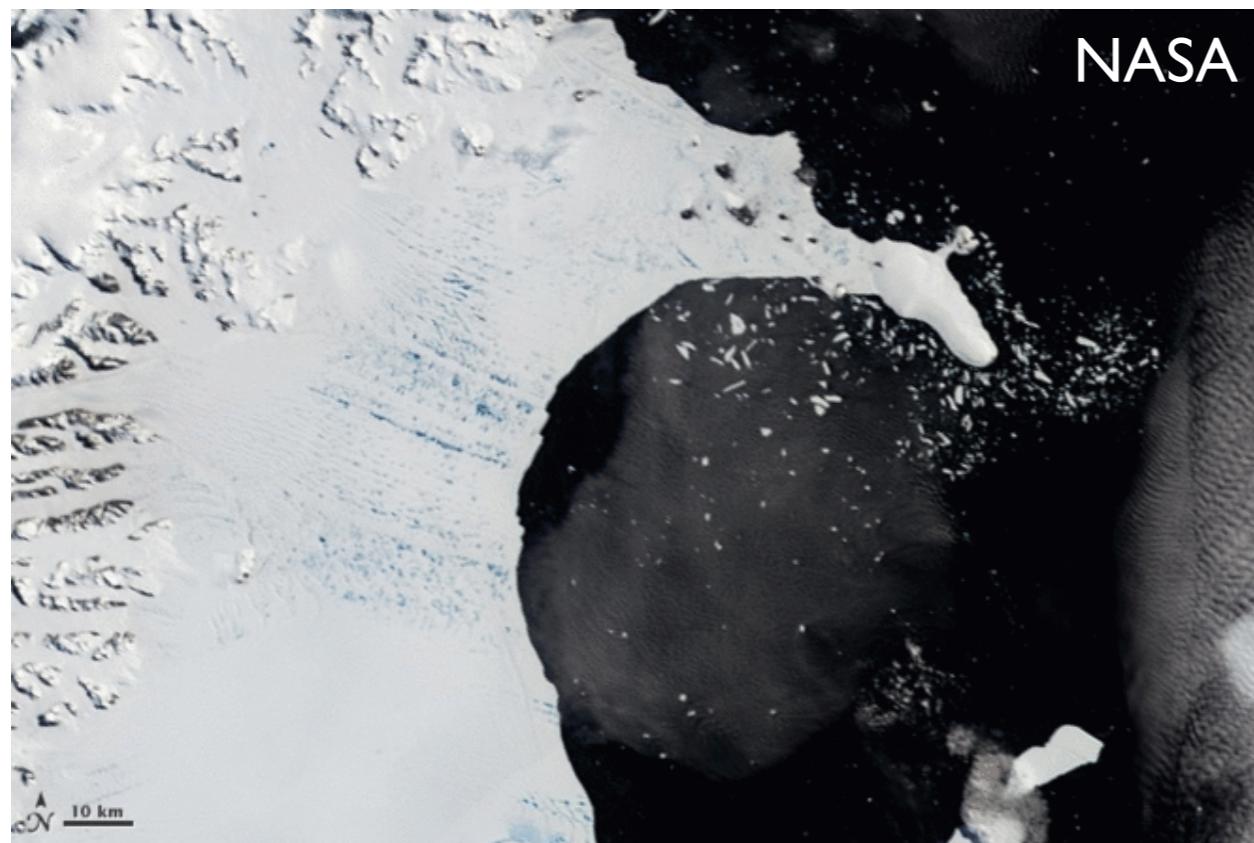
Pollard and DeConto, 2009



DeConto and Pollard, 2016

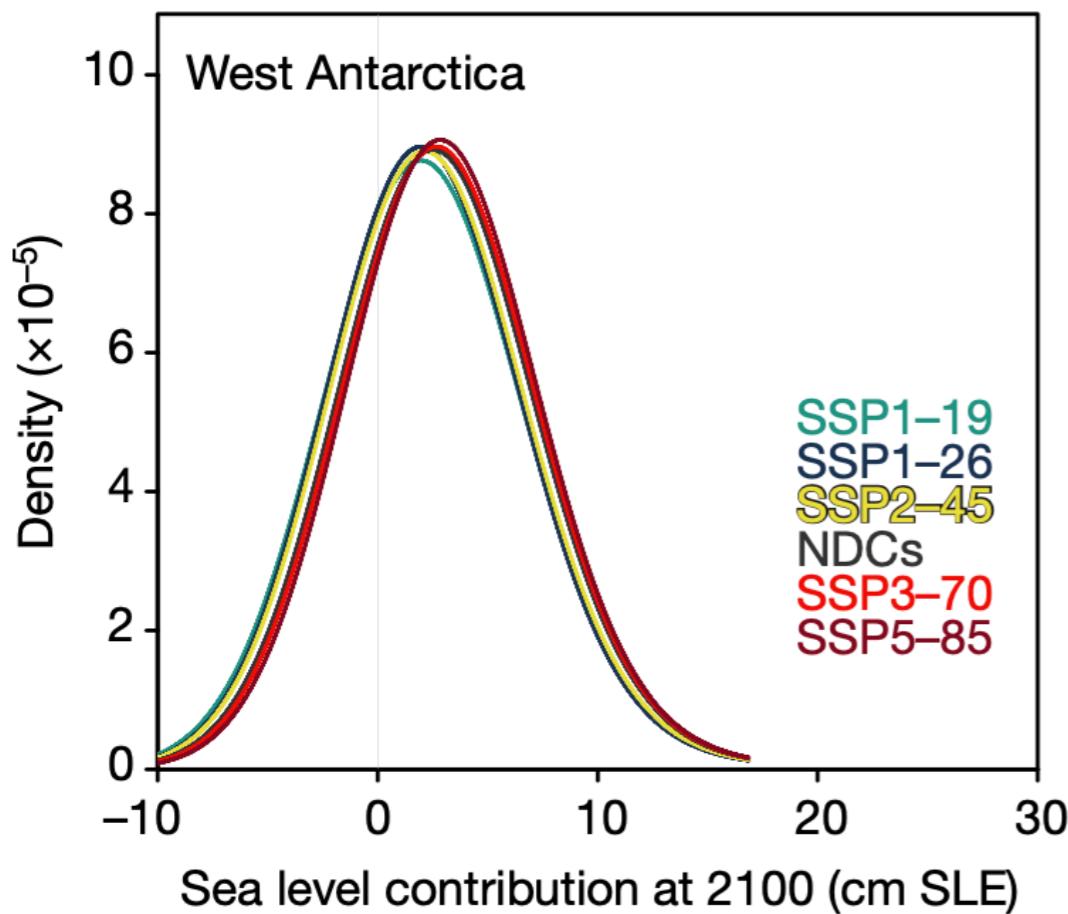


MICI questioned, but still ice shelf collapse still real possibility

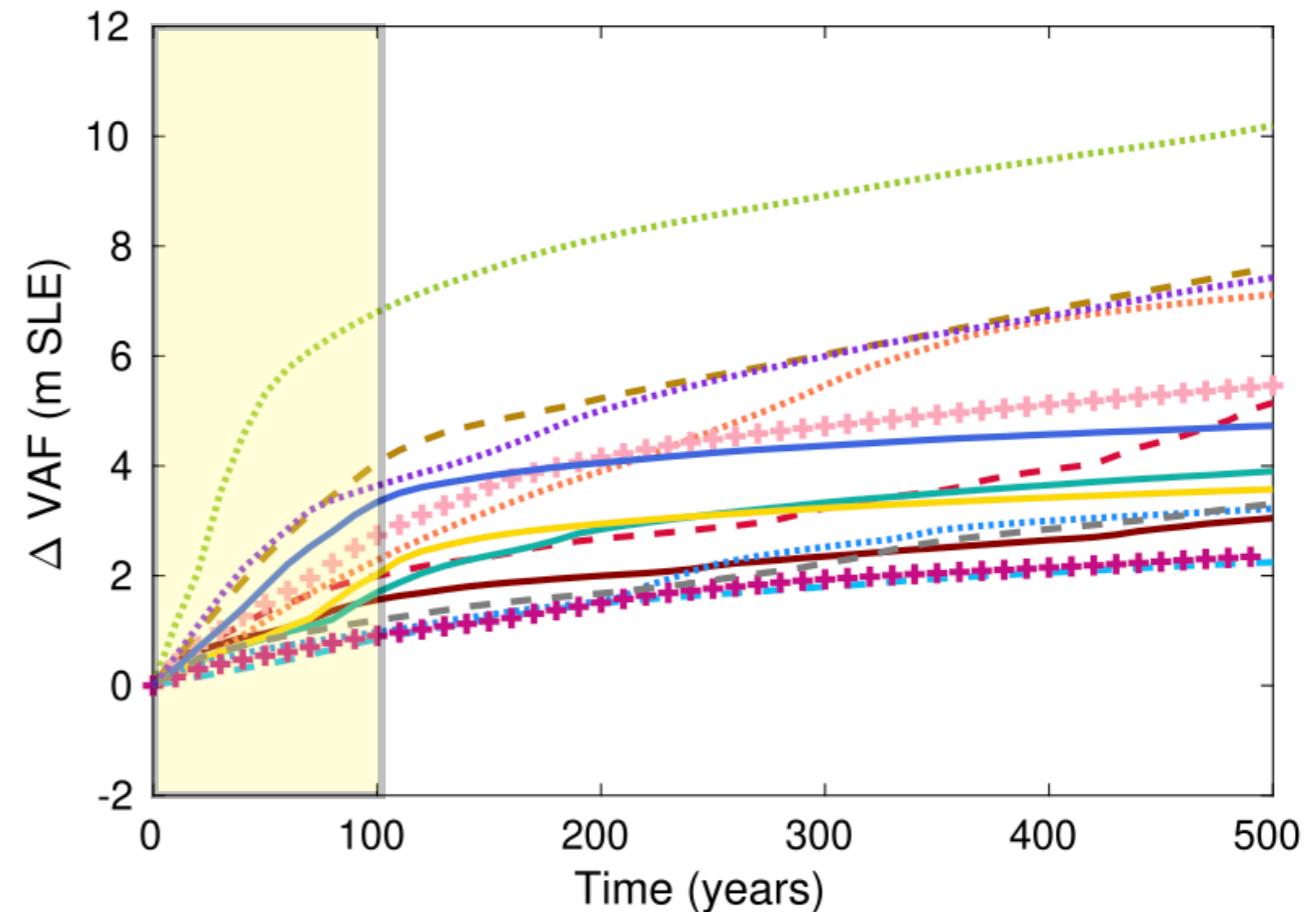


When ice shelves collapse — the biggest uncertainty in SLR projections?

Edwards et al. 2021 — CMIP



Sun et al. 2020 — ABUMIP (no ice shelves)

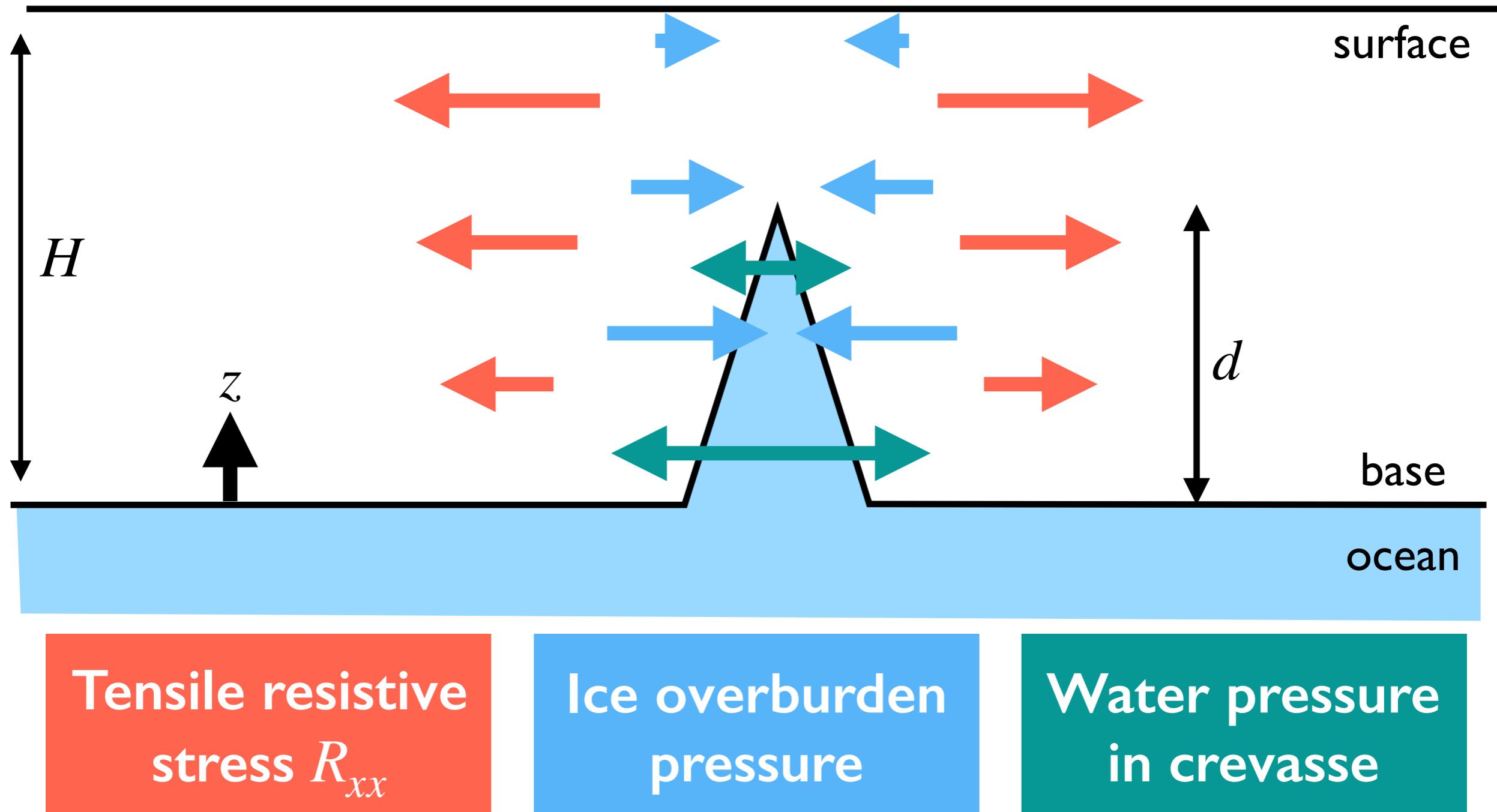


What is SLR
by 2100?



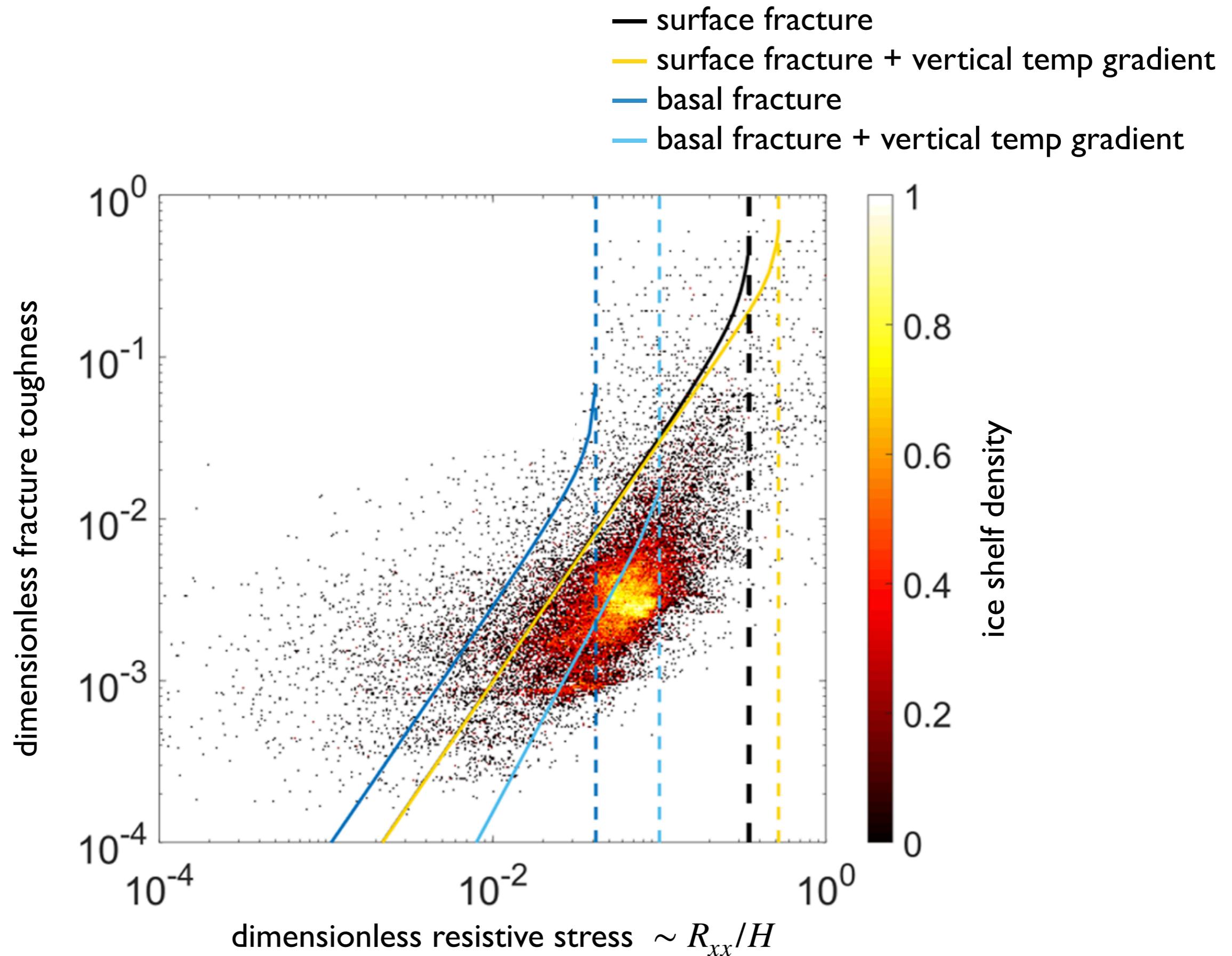
Which ice shelves collapse before
2100 and when?

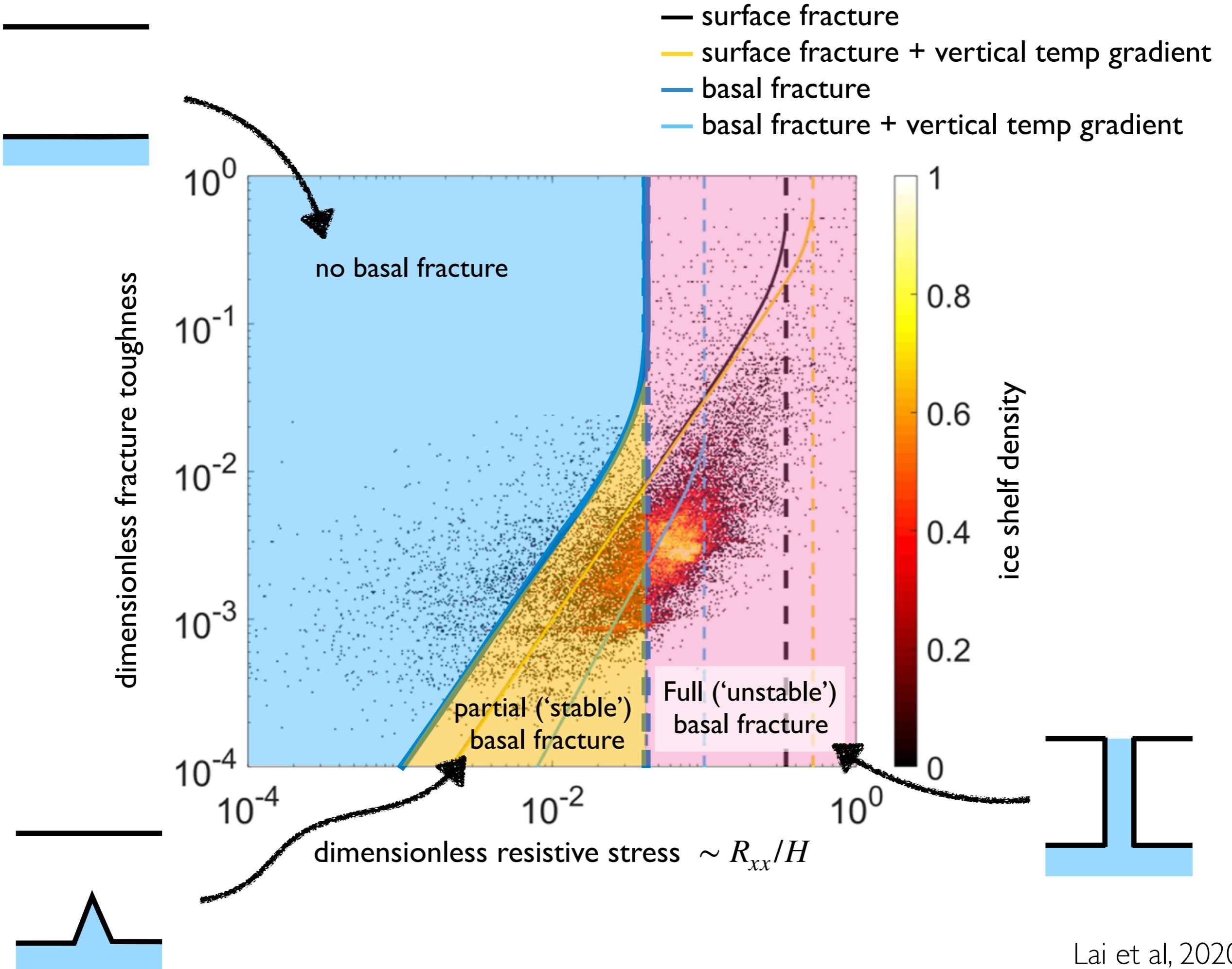
Lai et al. results based on LEFM

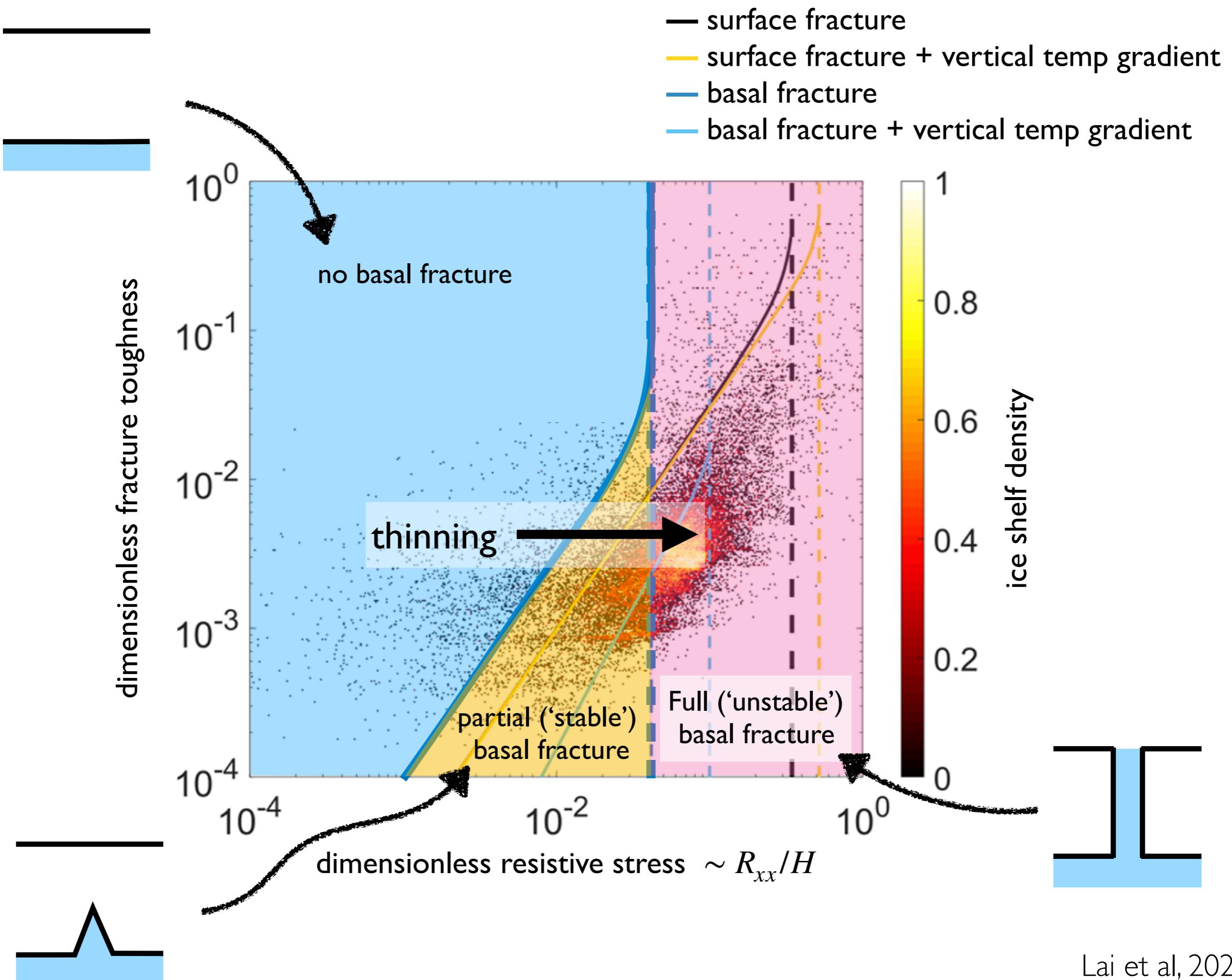


Stress intensity = tensile resistive stress + water pressure + ice overburden

Crevasse depth set by **stress intensity = ice fracture toughness $F \approx 150$ MPa**

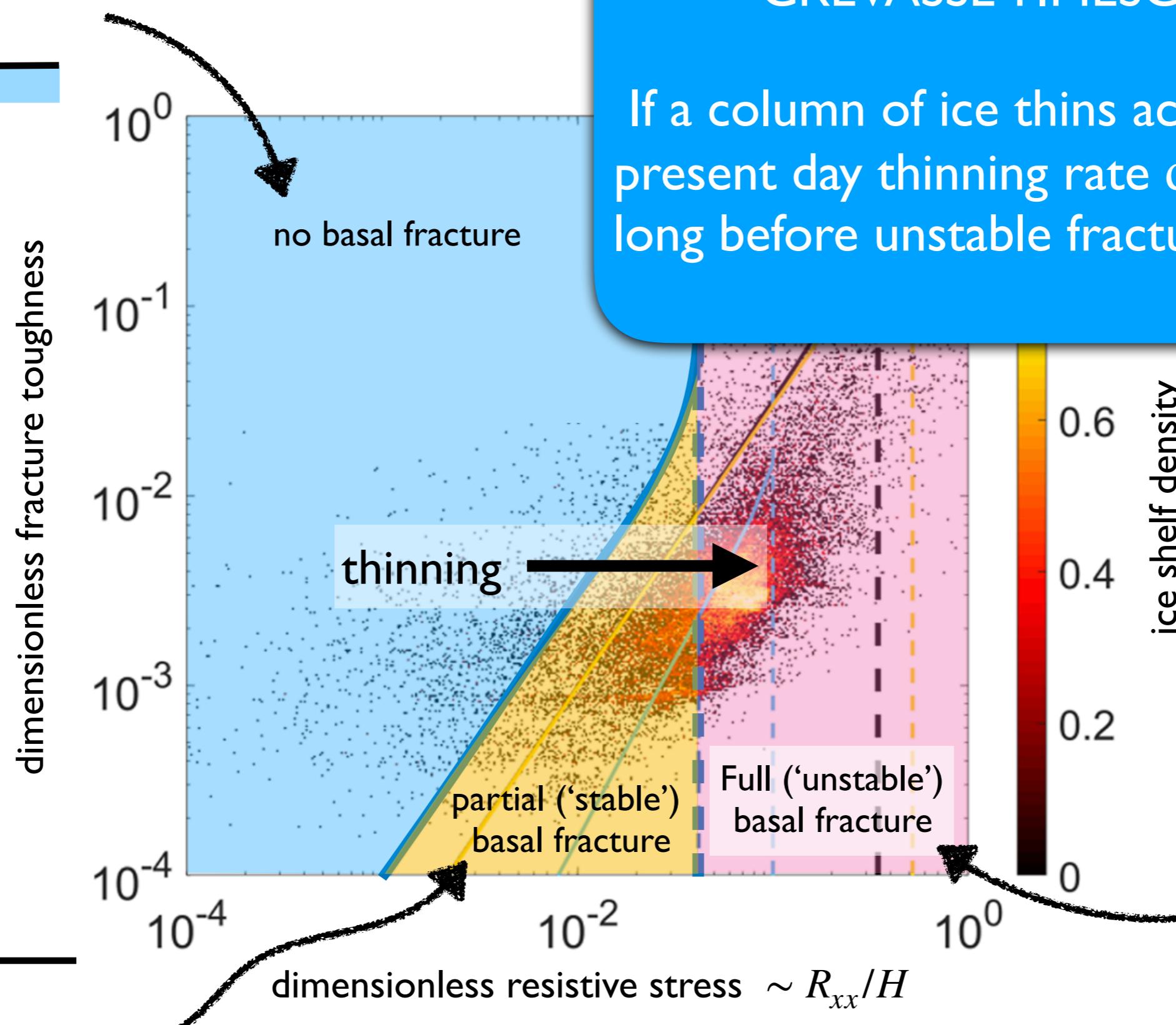


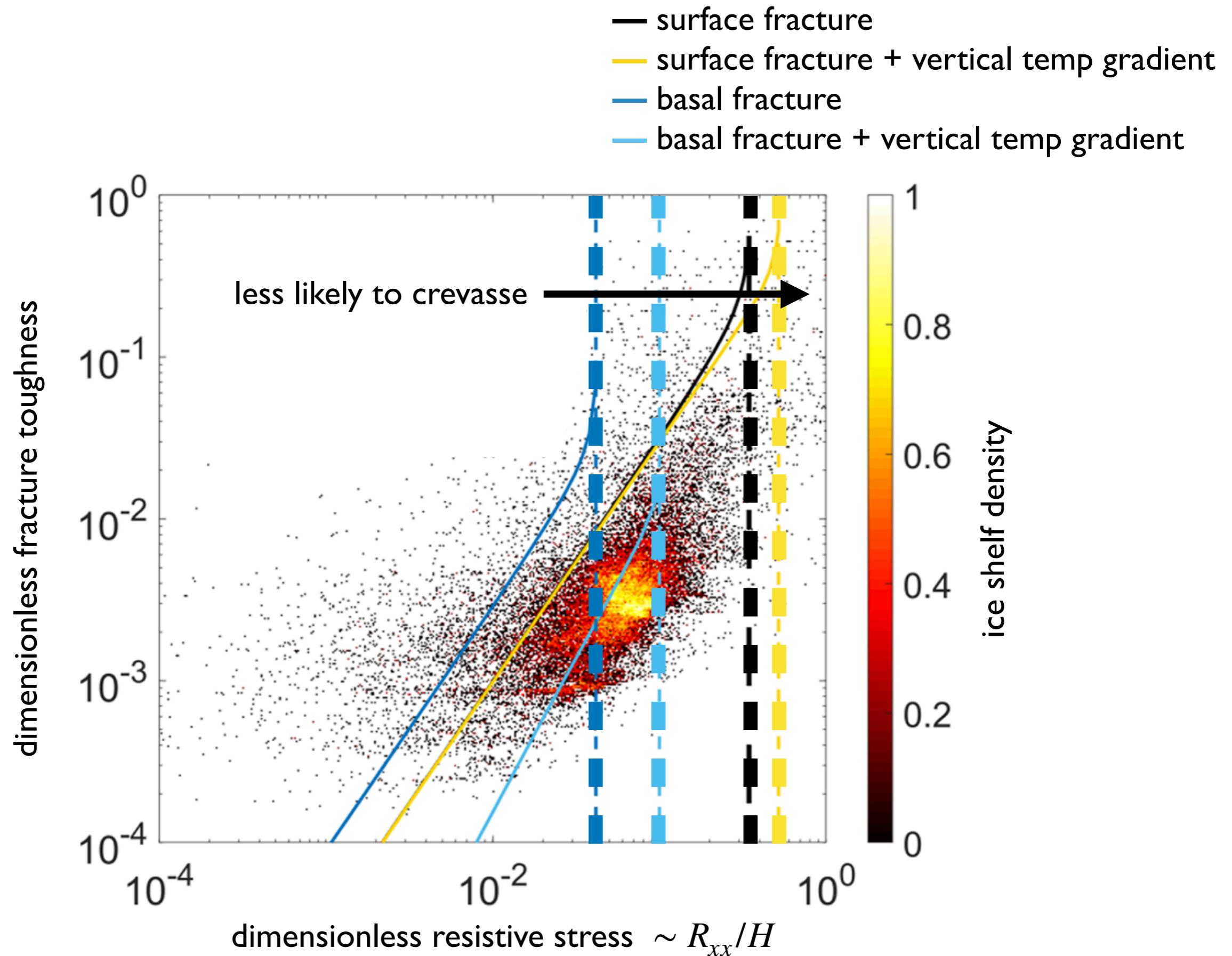




'CREVASSÉ TIMESCALE'

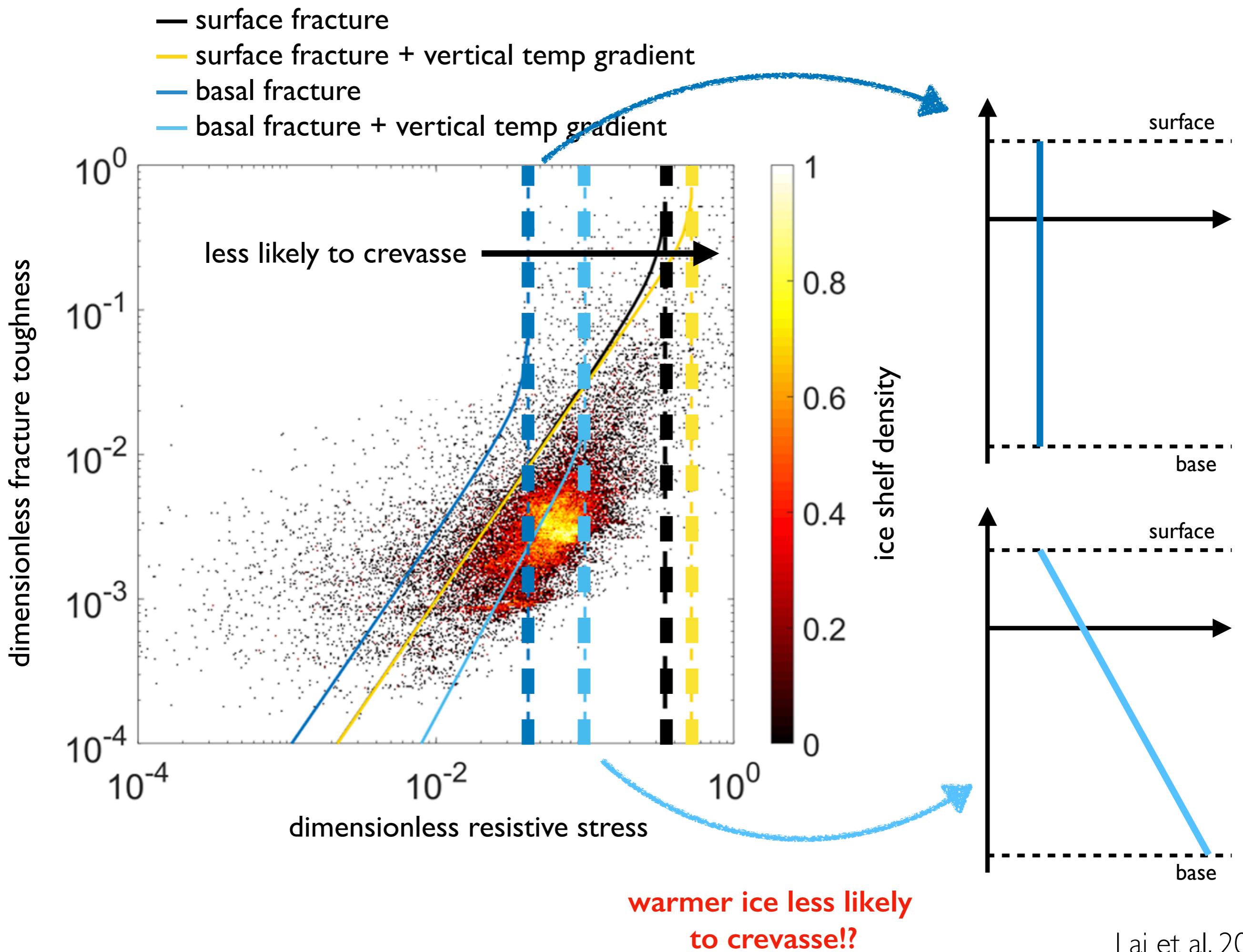
If a column of ice thins according to present day thinning rate dH/dt , how long before unstable fracture occurs?



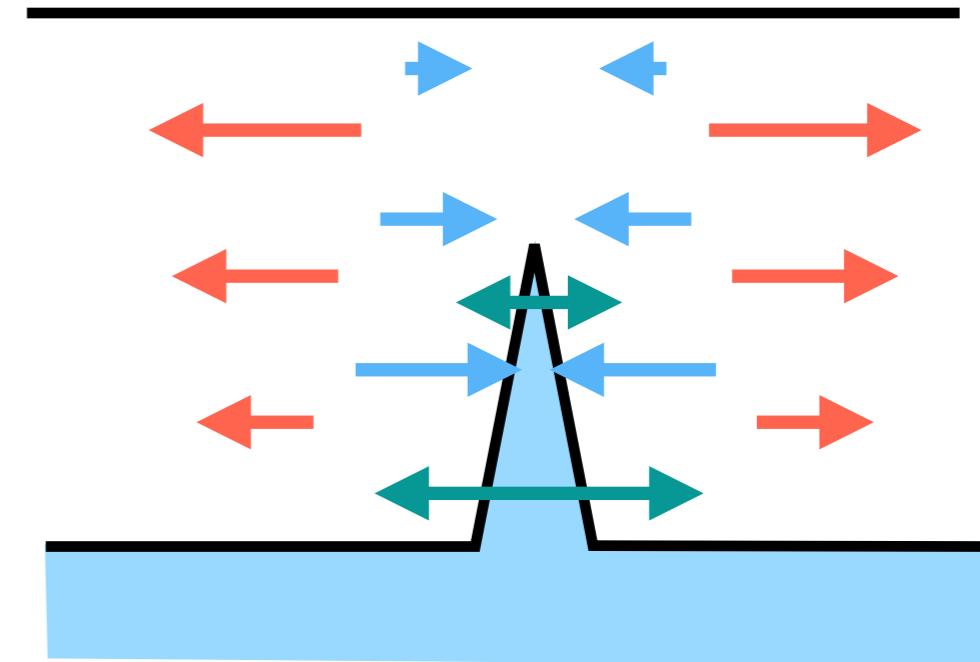
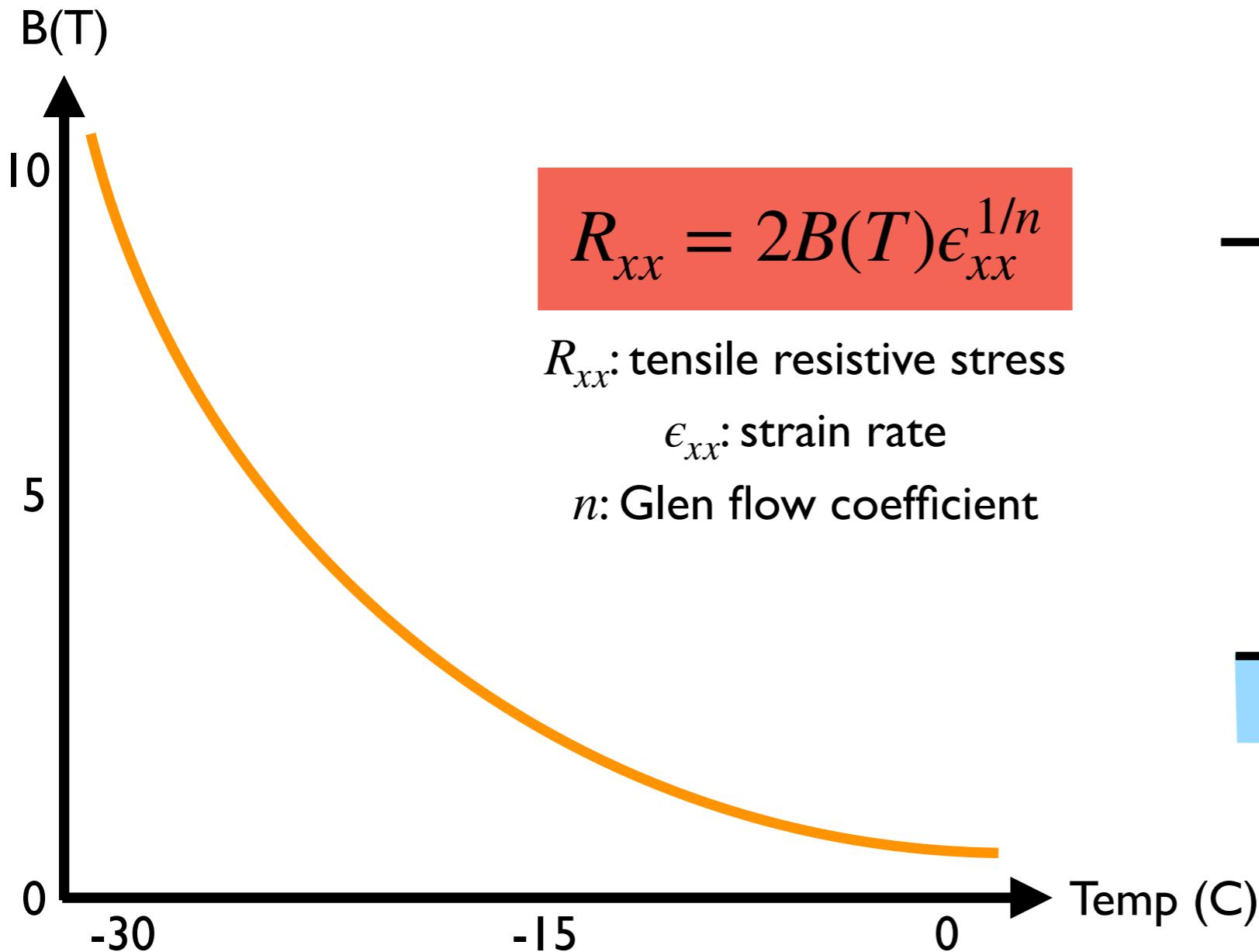


(1) surface crevasses less likely than basal crevasses(!)

(2) temperature profile really matters

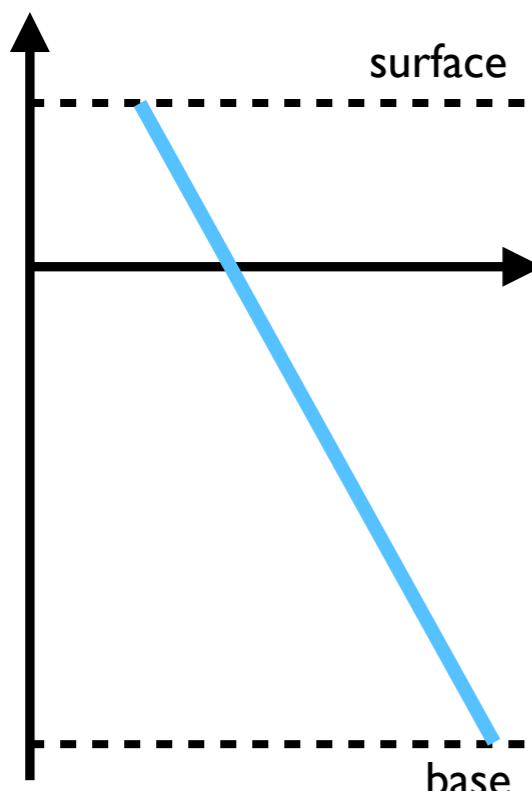


Why is warmer ice less likely to crevasse?



~ order of magnitude variation in viscosity
across expected range of temperatures

Stress intensity = tensile resistive stress + water pressure + ice overburden

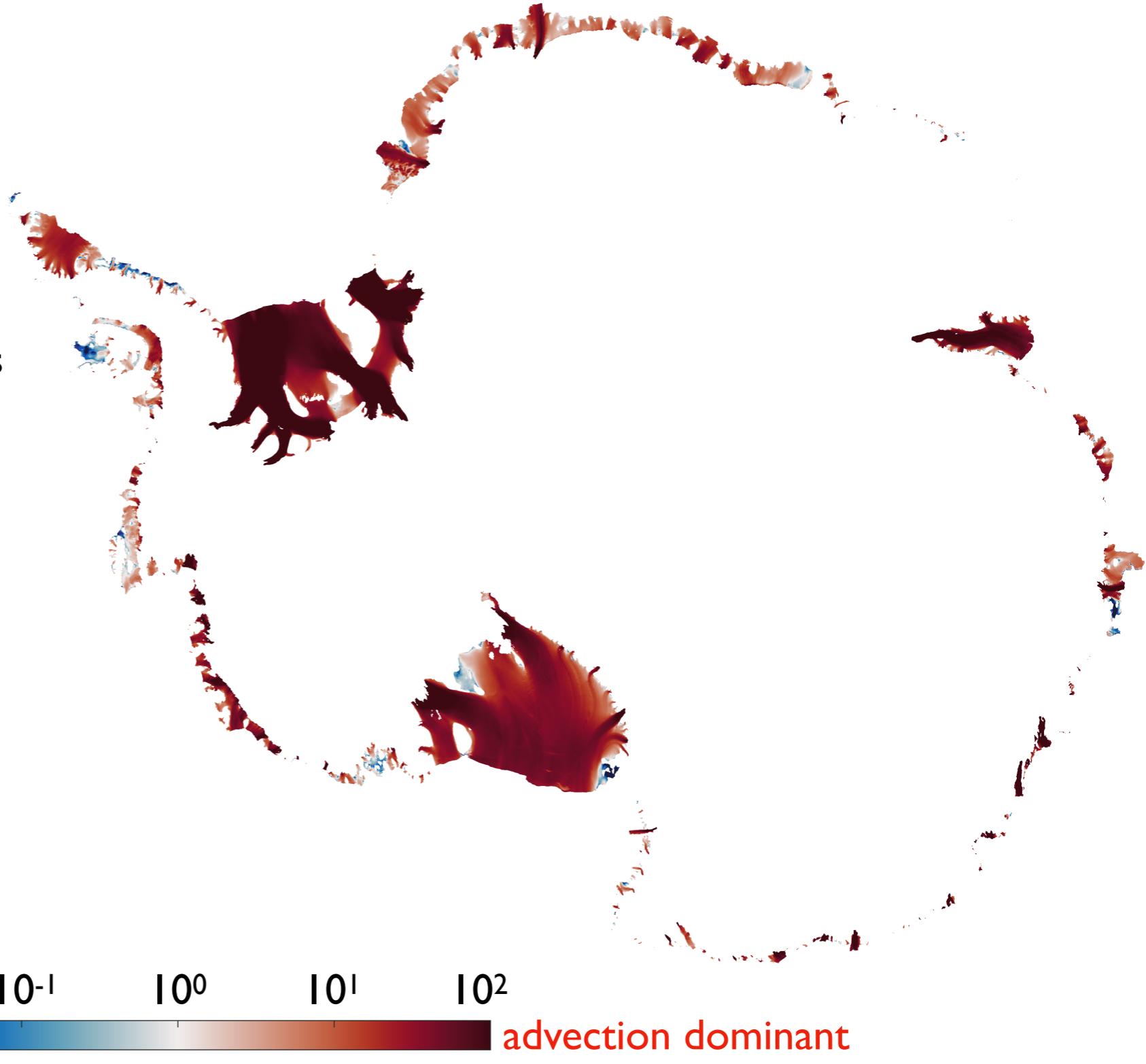


Linear profile based on diffusion dominated heat transfer...

$$\mathbf{u} \cdot \nabla T = \nabla \cdot (\kappa \nabla T)$$

...but really advection dominates

$$Pe = \frac{H^2 |\mathbf{u}|}{L \kappa_i}$$



Flowline description of ice shelf temperature (Sergienko et al., 2013)

appropriate for advection dominant flow

JOURNAL OF GEOPHYSICAL RESEARCH: EARTH SURFACE, VOL. 118, 10,621–10,636, 2013

Alternative ice shelf equilibria determined by ocean environment

O. V. Sergienko,¹ D. N. Goldberg,² and C. M. Little³

Received 18 October 2012; revised 30 January 2013; accepted 7 March 2013; published 10 June 2013.

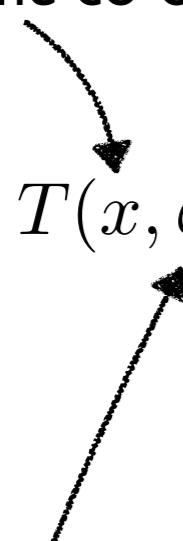
Alternative ice shelf equilibria determined by ocean environment

[OV Sergienko, DN Goldberg...](#) - Journal of Geophysical ..., 2013 - Wiley Online Library

Dynamic and thermodynamic regimes of ice shelves experiencing weak (1 m year^{-1}) to strong ($\sim 10 \text{ m year}^{-1}$) basal melting in cold (bottom temperature close to the in situ freezing point) and warm oceans (bottom temperature more than half of a degree warmer than the in situ freezing point) are investigated using a 1-D coupled ice/ocean model complemented with a newly derived analytic expression for the steady state temperature distribution in ice shelves. This expression suggests the existence of a basal thermal ...

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along flow line co-ordinate



basal temperature (freezing)

$$T(x, \zeta) = T_g [\xi(x, \zeta)] + \{T_f(x) - T_g [\xi(x, \zeta)]\} \exp\left(-\frac{\dot{m}H}{\kappa}\zeta\right)$$

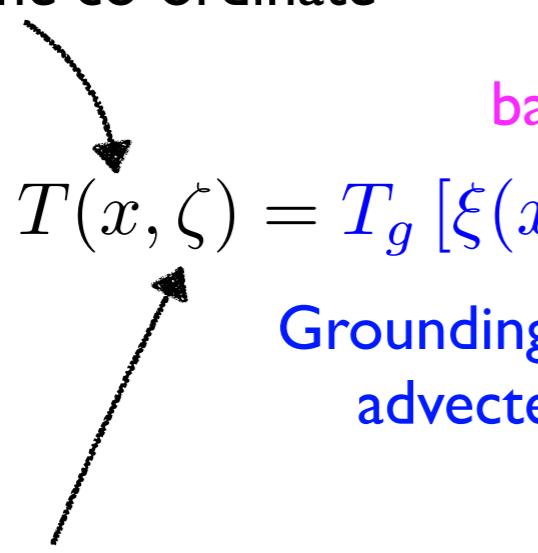
Grounding line temperature
adveected downstream

diffusive part, restricted to
boundary layer of width $\ell = \frac{\kappa}{\dot{m}H}$

dimensionless vertical co-ordinate

Flowline description of ice shelf temperature (Sergienko et al., 2013)

along flow line co-ordinate


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basal temperature (freezing)
Grounding line temperature
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boundary layer of width $\ell = \frac{\kappa}{\dot{m}H}$

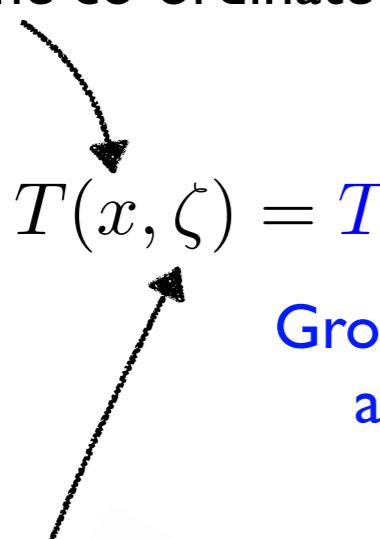
dimensionless vertical co-ordinate

Flowline description of ice shelf temperature (Sergienko et al., 2013)

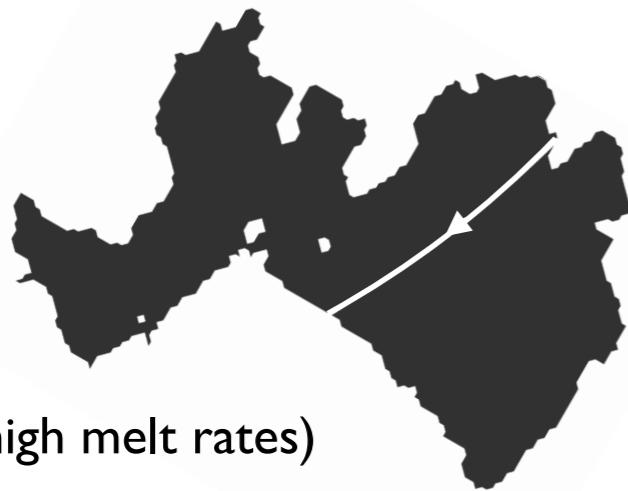
along flow line co-ordinate

$$T(x, \zeta) = T_g [\xi(x, \zeta)] + \{T_f(x) - T_g [\xi(x, \zeta)]\} \exp\left(-\frac{\dot{m}H}{\kappa}\zeta\right)$$

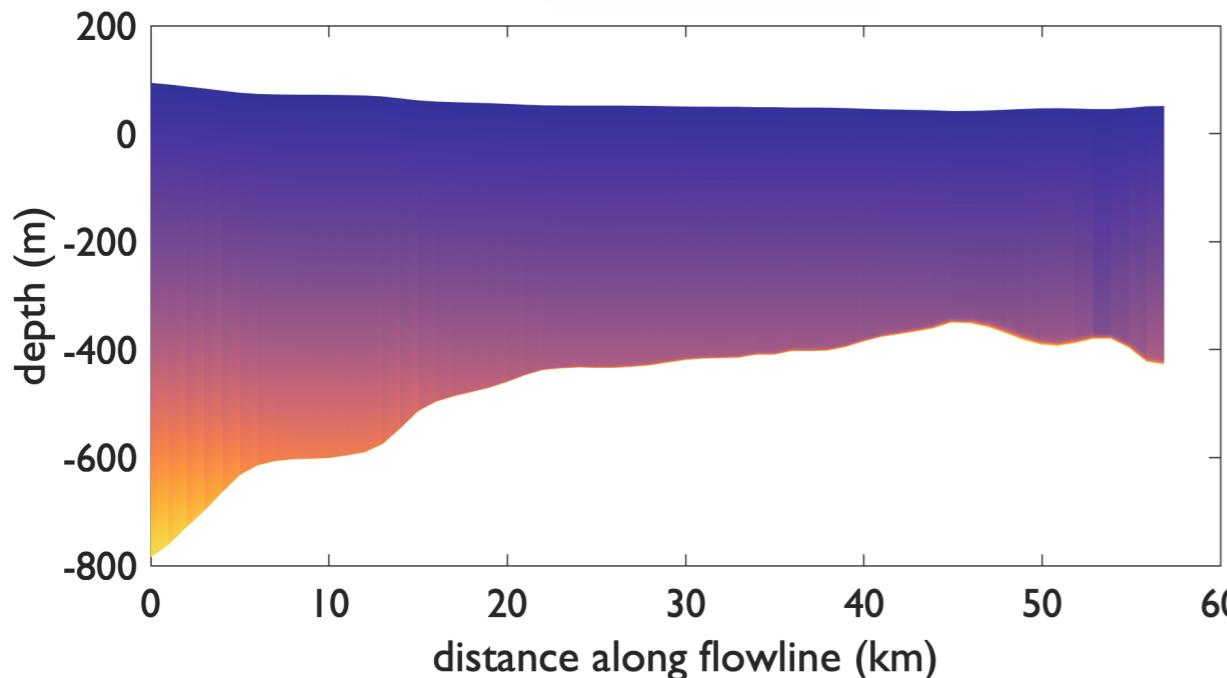
basal temperature (freezing)
 Grounding line temperature advectioned downstream
 diffusive part, restricted to boundary layer of width $\ell = \frac{\kappa}{\dot{m}H}$



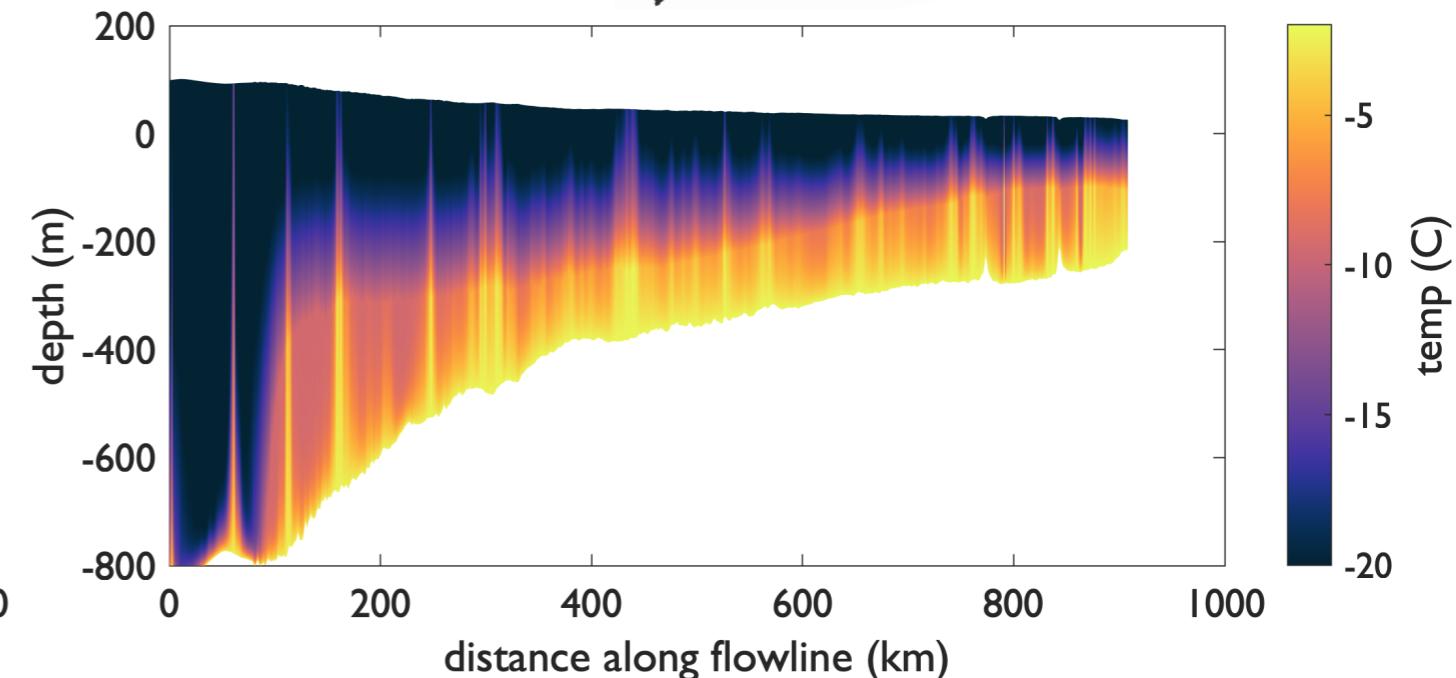
dimensionless vertical co-ordinate



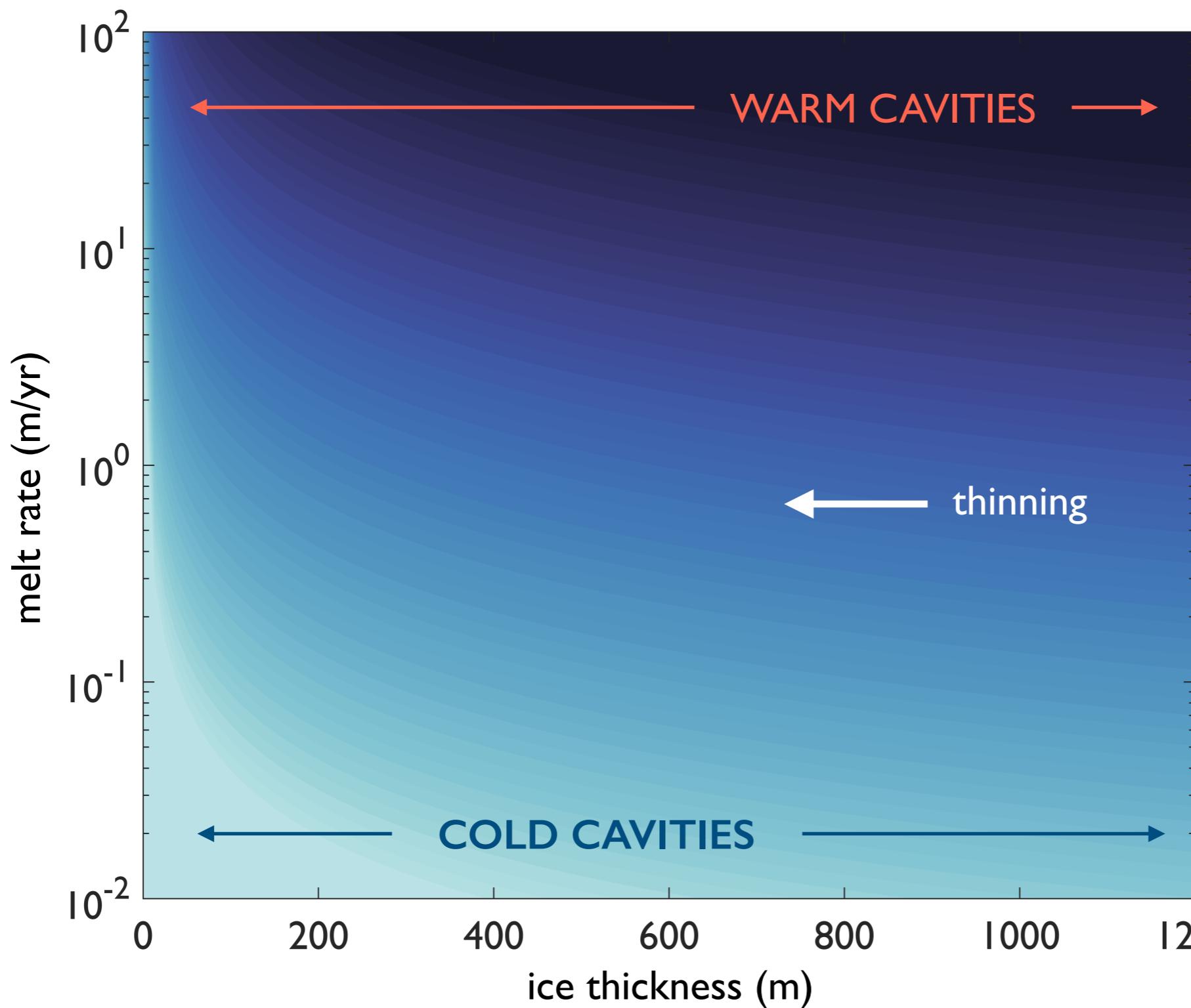
Pine Island (high melt rates)



Ross (low melt rates)



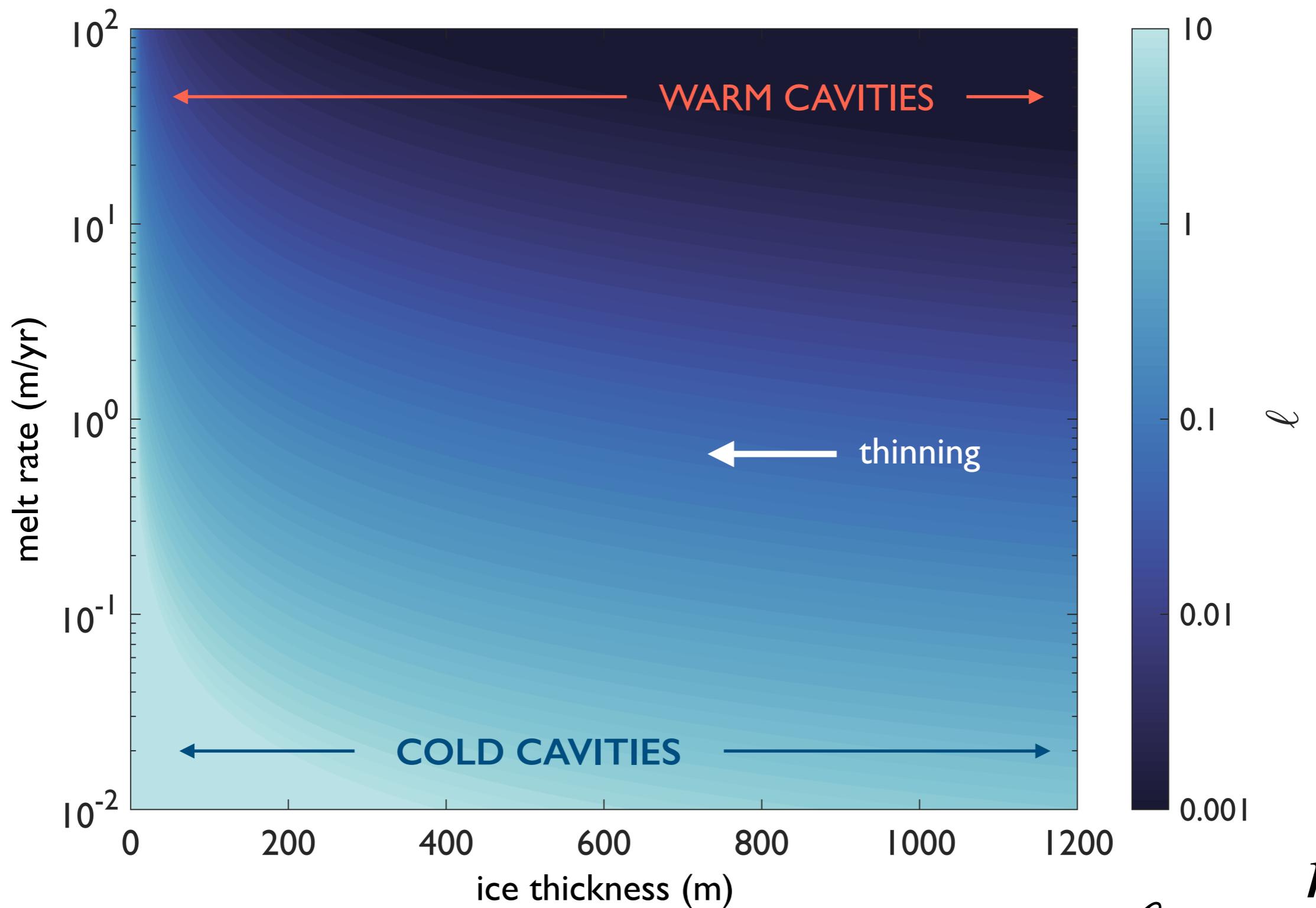
(thick ice) high melting: low ℓ
→ little temperature correction



(thin ice) low melting: high ℓ
→ large temperature correction

$$\ell = \frac{\kappa_i}{\dot{m}H}$$

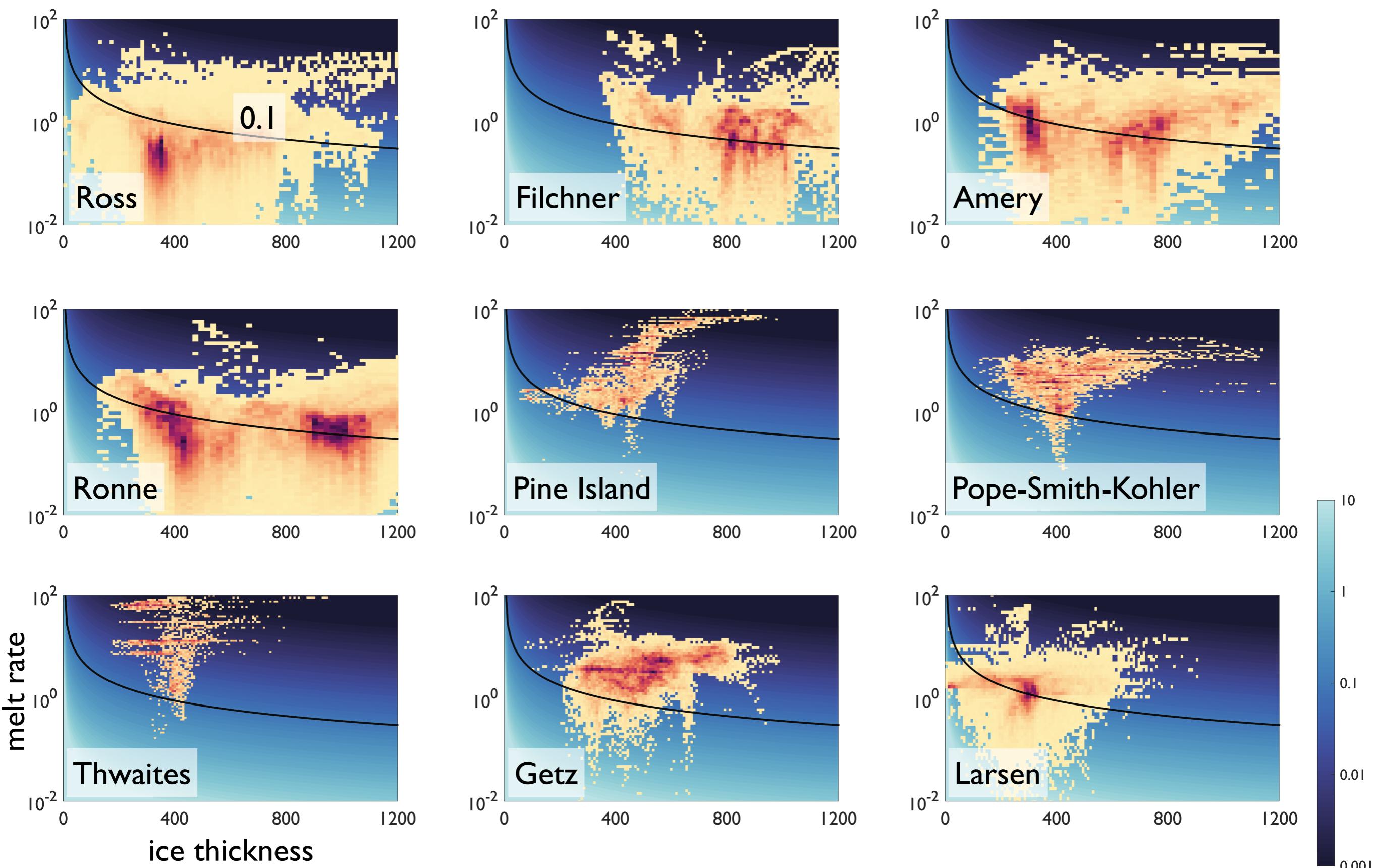
(thick ice) high melting: low ℓ
→ little temperature correction + high thinning rates



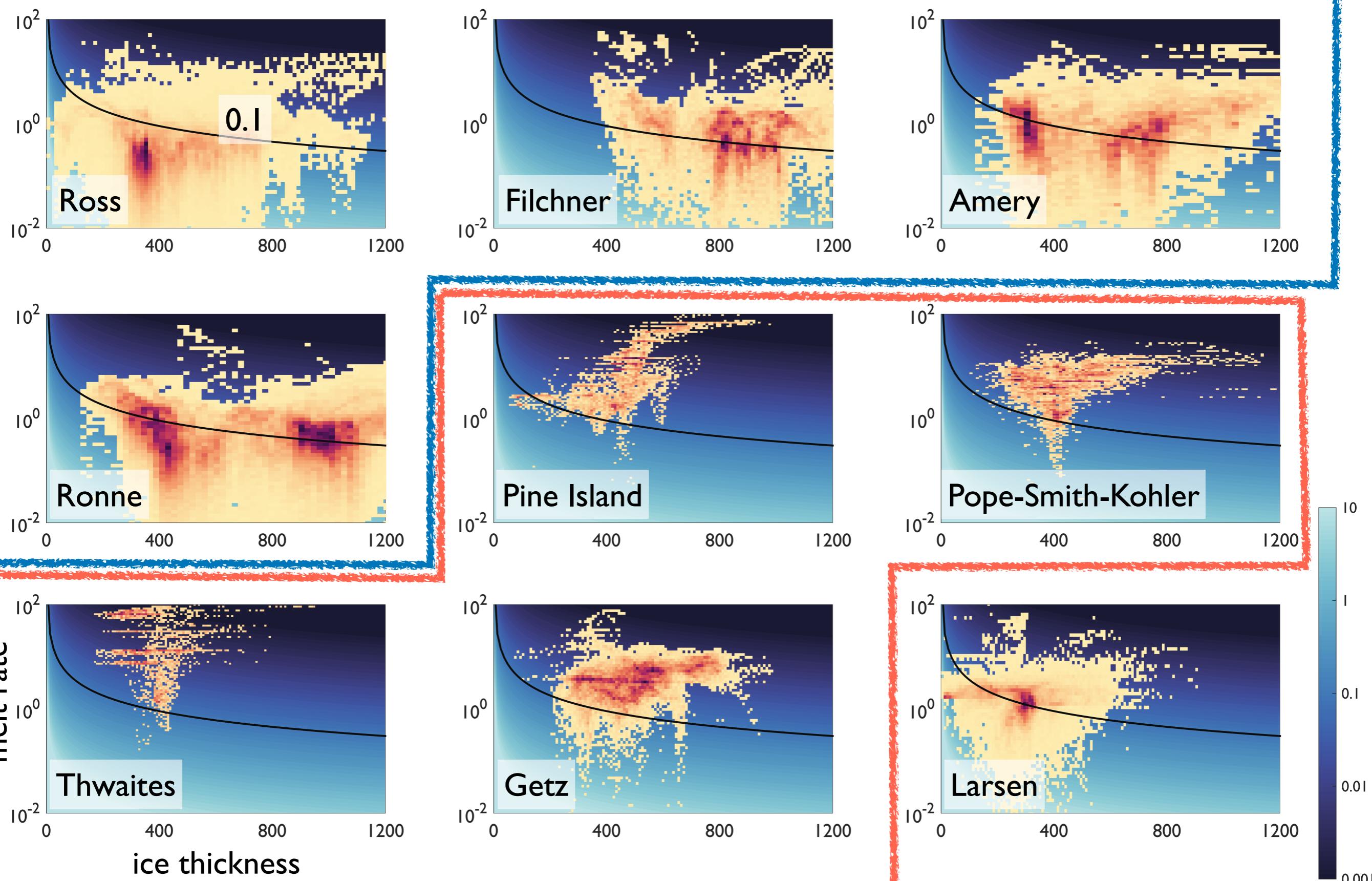
(thin ice) low melting: high ℓ
→ large temperature correction + low thinning rates

$$\ell = \frac{\kappa_i}{\dot{m}H}$$

What does cold/warm cavity mean?

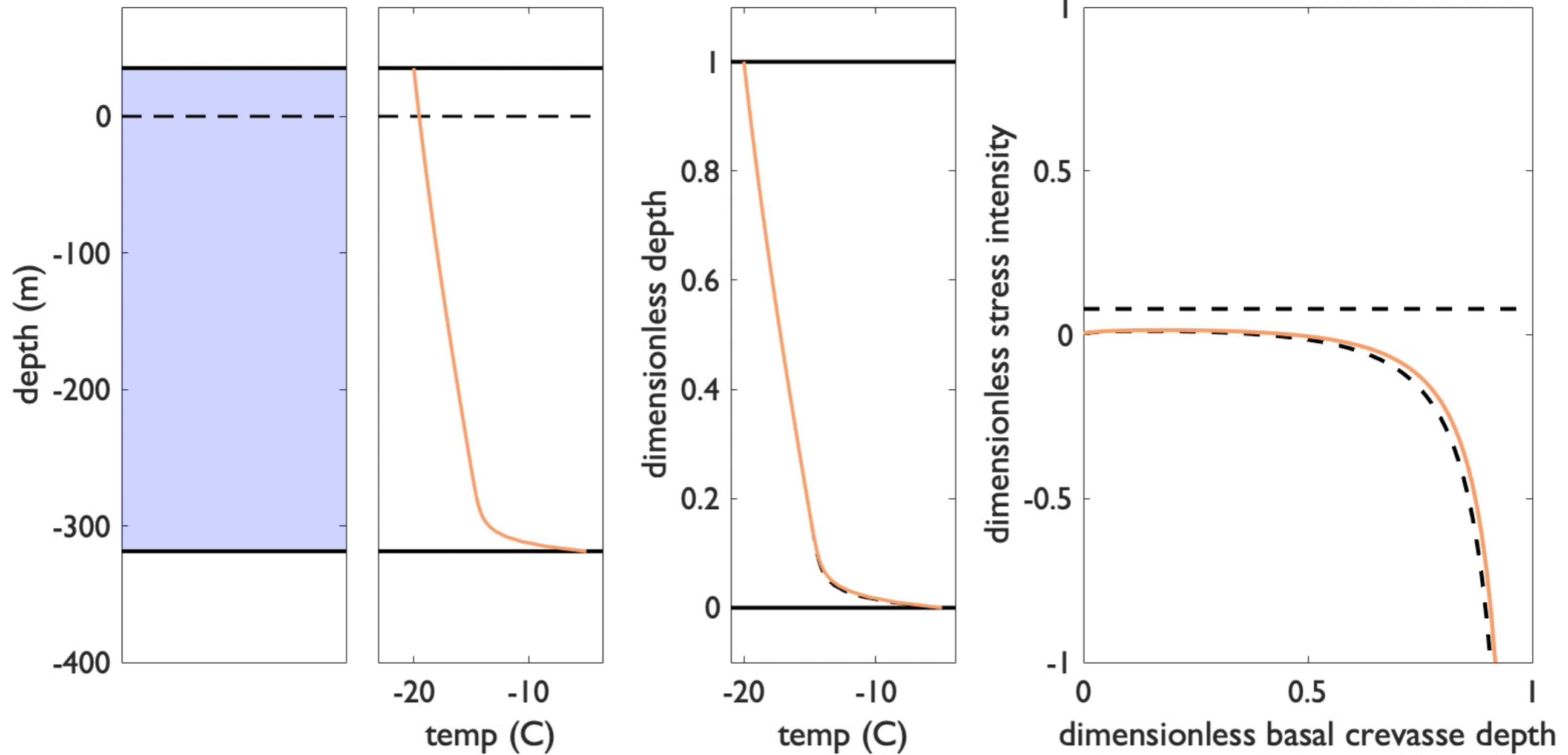


What does cold/warm cavity mean?



'Crevasse timescale'

$t = 9.2\text{ yrs}$, $H = 354\text{ m}$

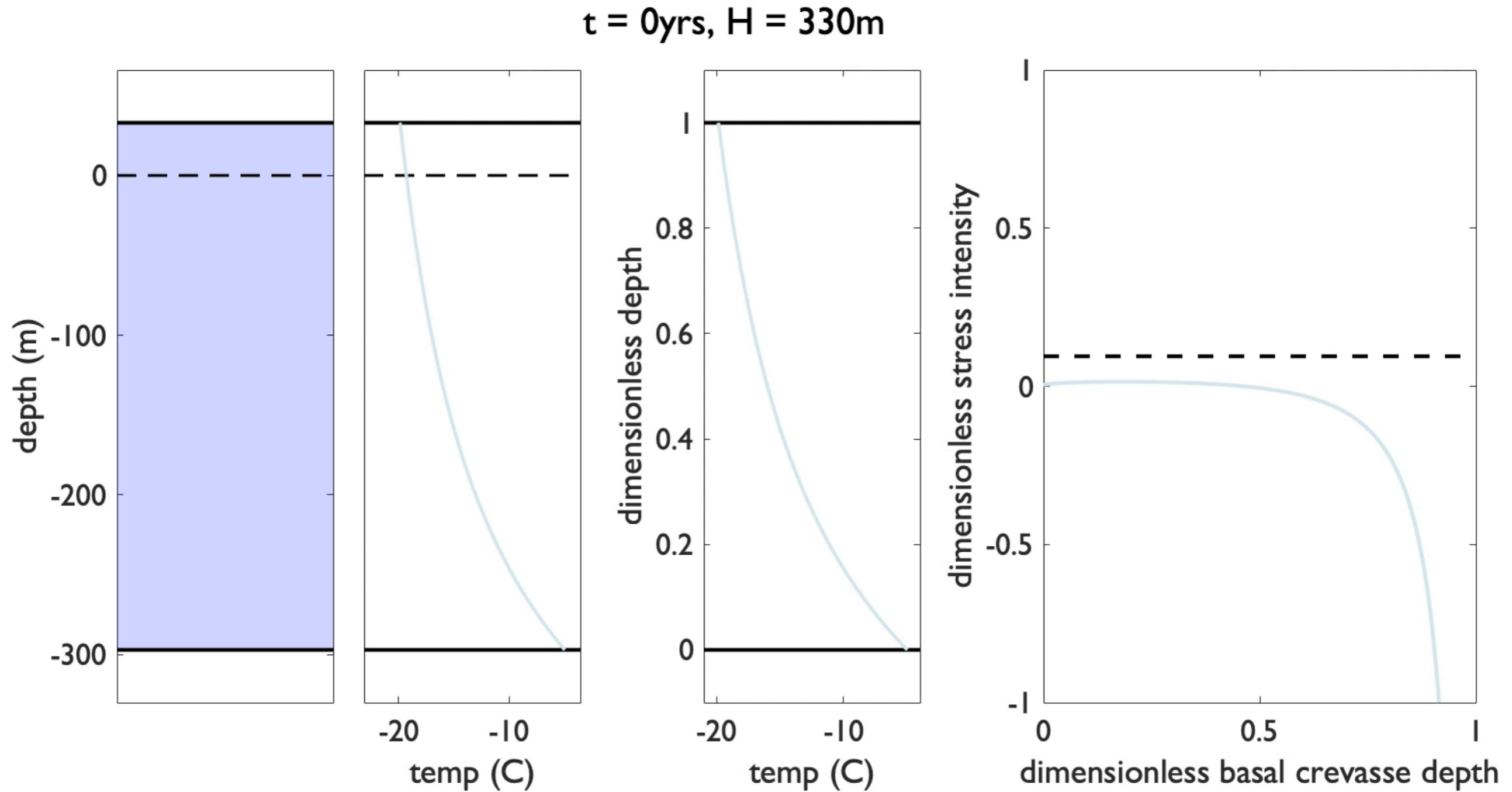


high melt rate \rightarrow little *absolute* change in temp profile

Crevasse timescale $\tau \approx 38$ years

No melt change (dashed): $\tau_0 \approx 36$ years

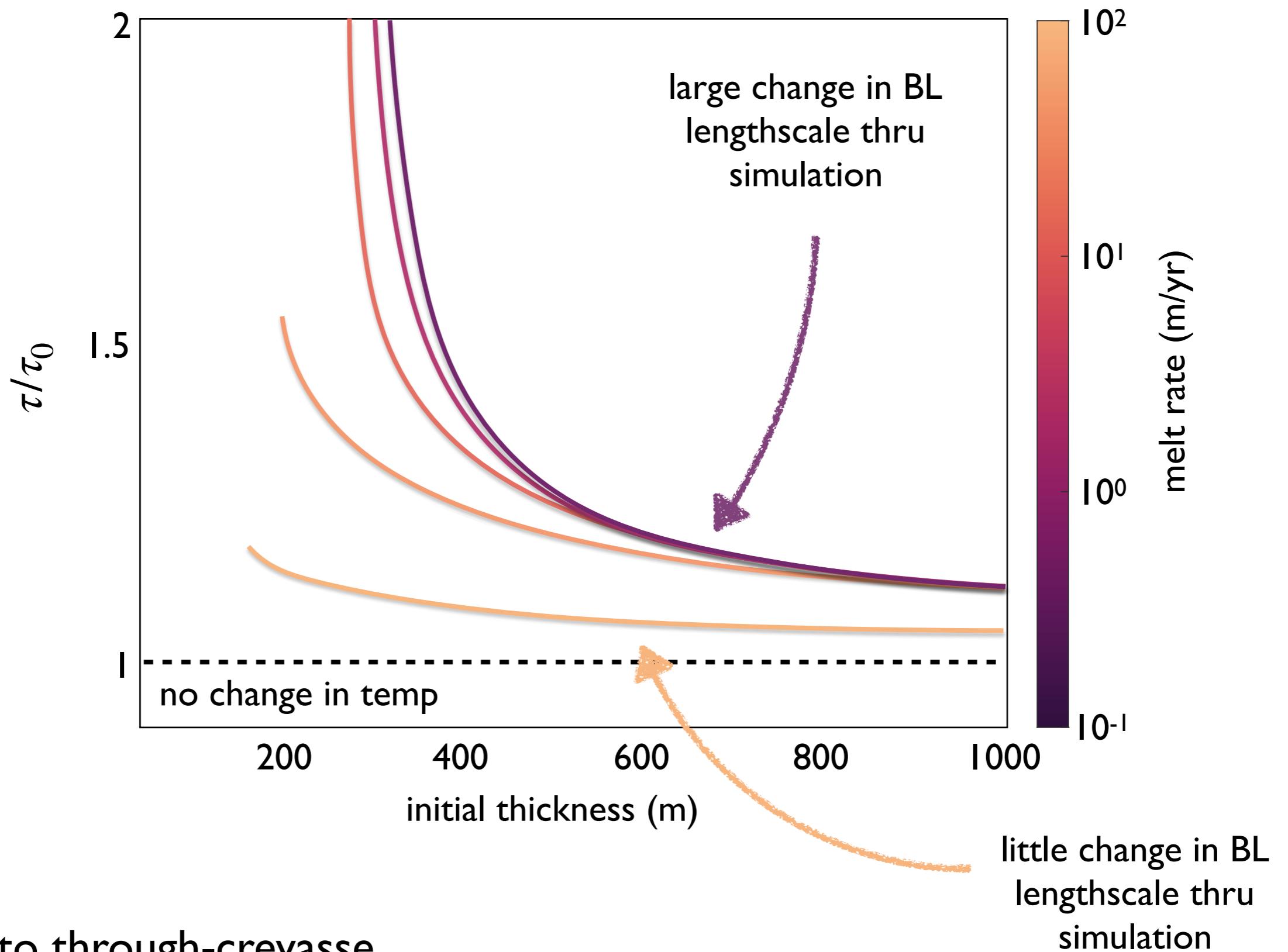
'Crevasse timescale'



low melt rate \rightarrow large *absolute* change in temp profile

Crevasse timescale $\tau \approx 308$ years

No melt change (dashed): $\tau_0 \approx 200$ years



τ : time to through-crevasse

τ_0 : time to through-crevasse if
temp profile does not change

COLD WATER (LOW MELT RATE) CAVITY ICE SHELVES

- Low thinning rates (high τ_0)
- Large adjustment temp profile (high τ/τ_0)
- Low strain rates (high τ)

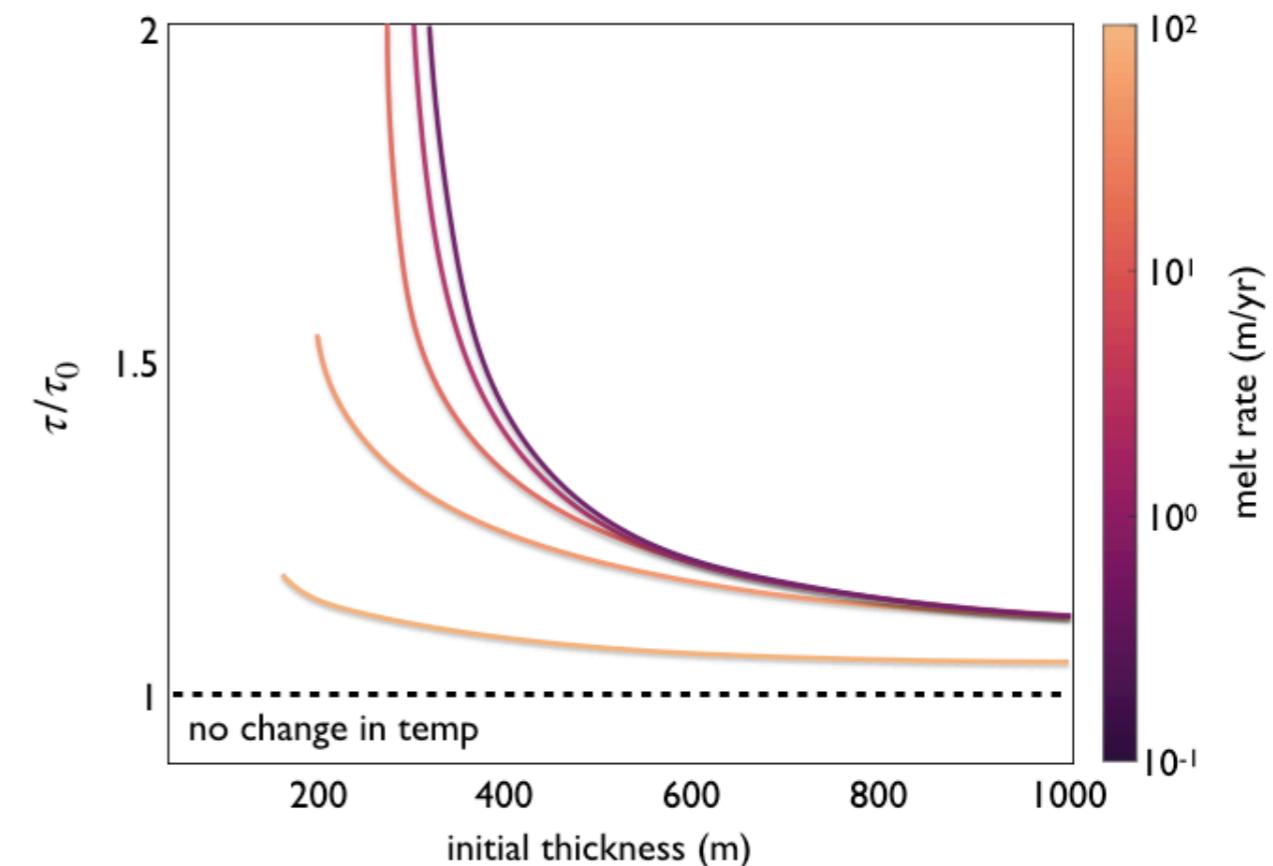
WARM WATER (HIGH MELT RATE) CAVITY ICE SHELVES

- Low thinning rates (high τ_0)
- Large adjustment temp profile (high τ/τ_0)
- High strain rates (low τ)

EXPECT LARGE DIFFERENCE IN
'CREVASSÉ TIMESCALE'

τ : time to through-crevasse

τ_0 : time to through-crevasse if
temp profile does not change

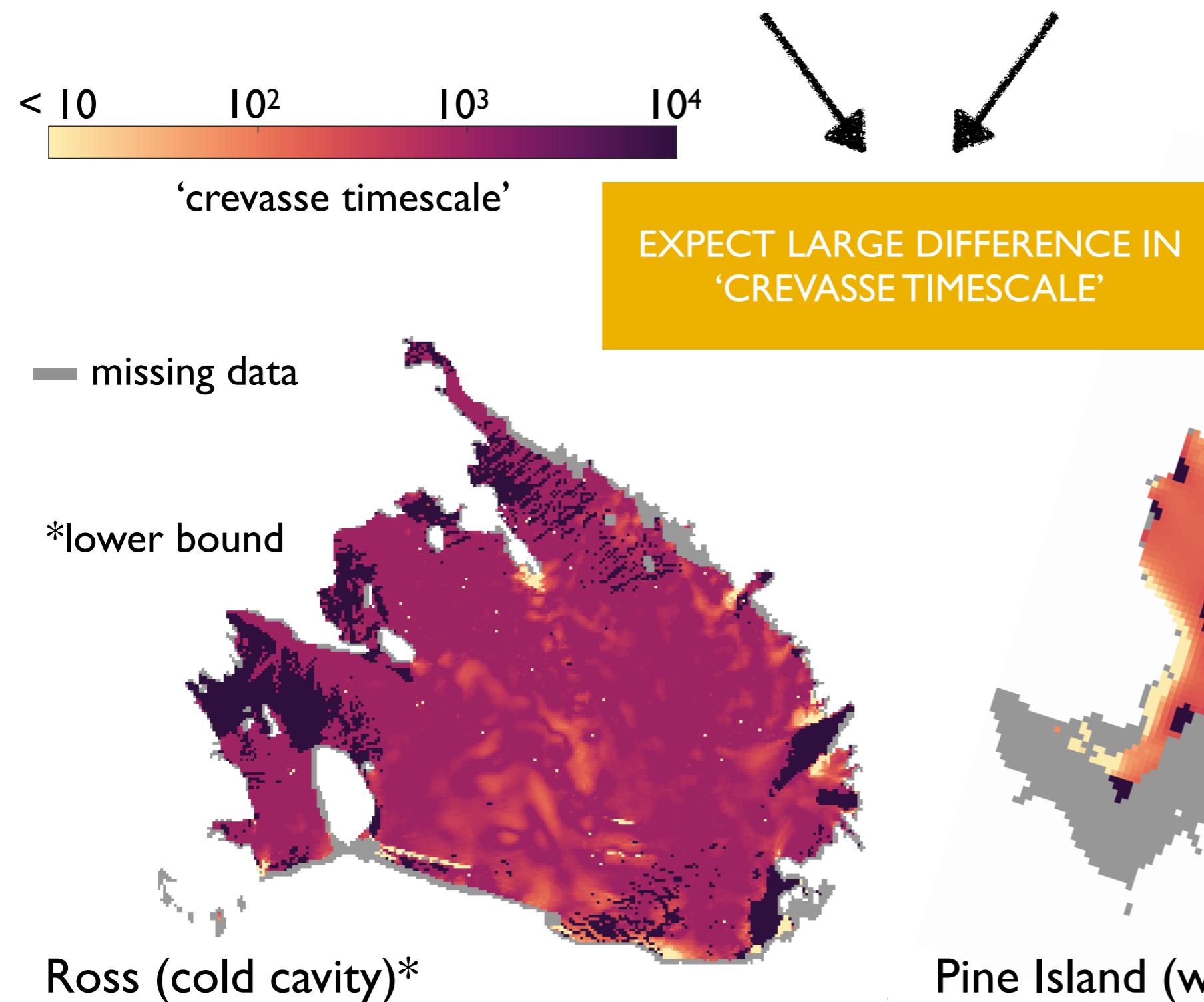


COLD WATER (LOW MELT RATE) CAVITY ICE SHELVES

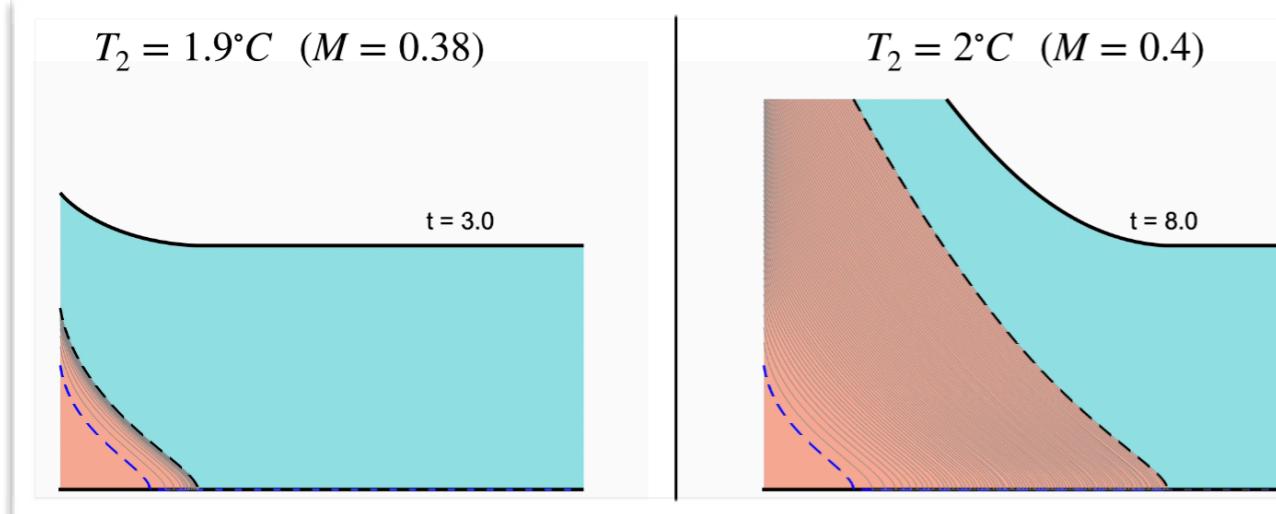
- Low thinning rates (high τ_0)
- Large adjustment temp profile (high τ/τ_0)
- Low strain rates (high τ)

WARM WATER (HIGH MELT RATE) CAVITY ICE SHELVES

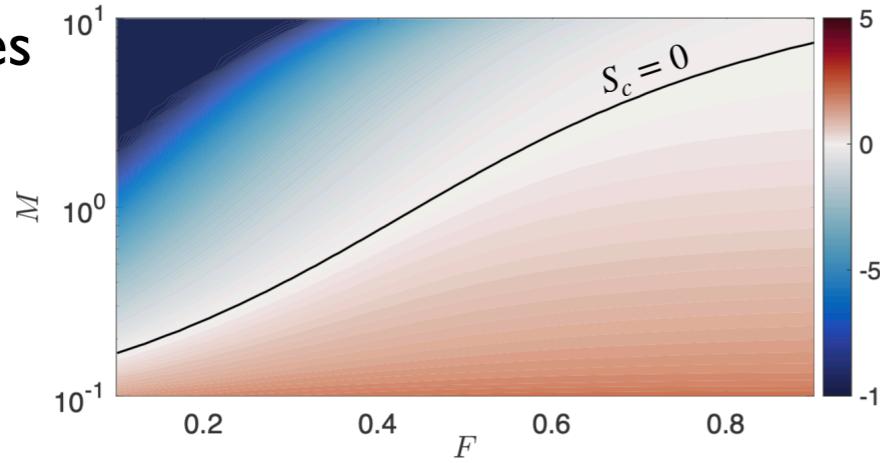
- Low thinning rates (high τ_0)
- Large adjustment temp profile (high τ/τ_0)
- High strain rates (low τ)



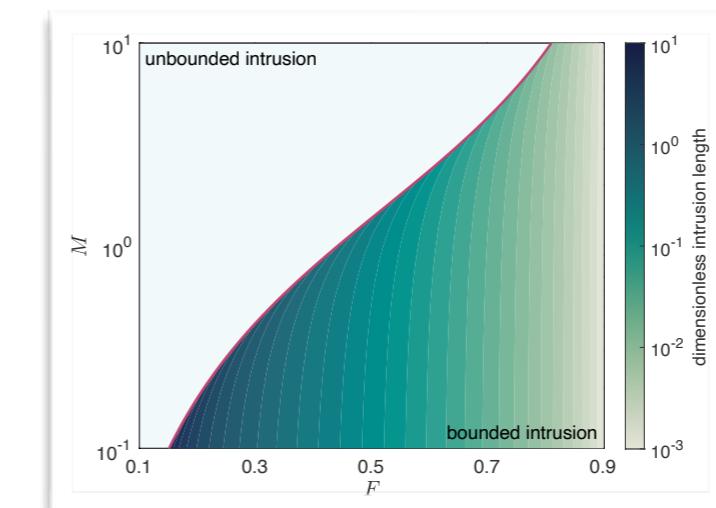
Melt feedback result in grounding zone tipping points



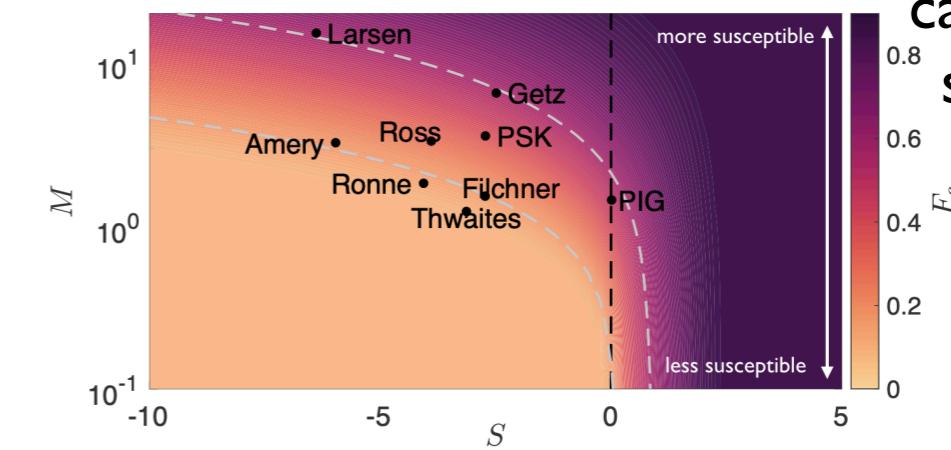
Prograde slopes
vulnerable,
retrograde
enhanced



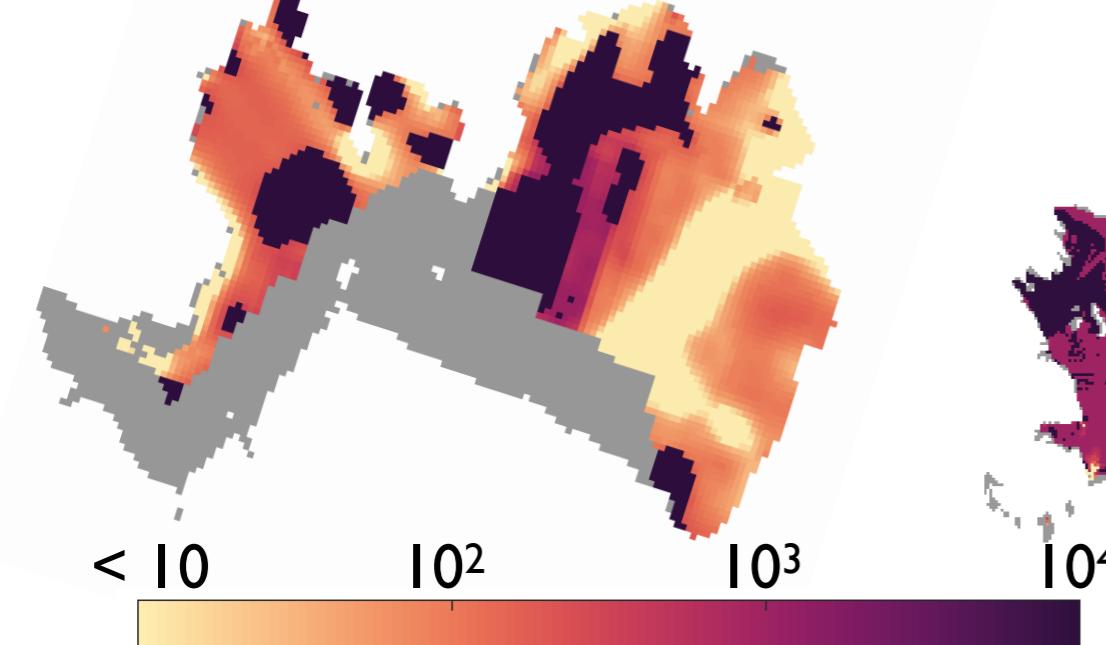
Tipping point is ‘generic’



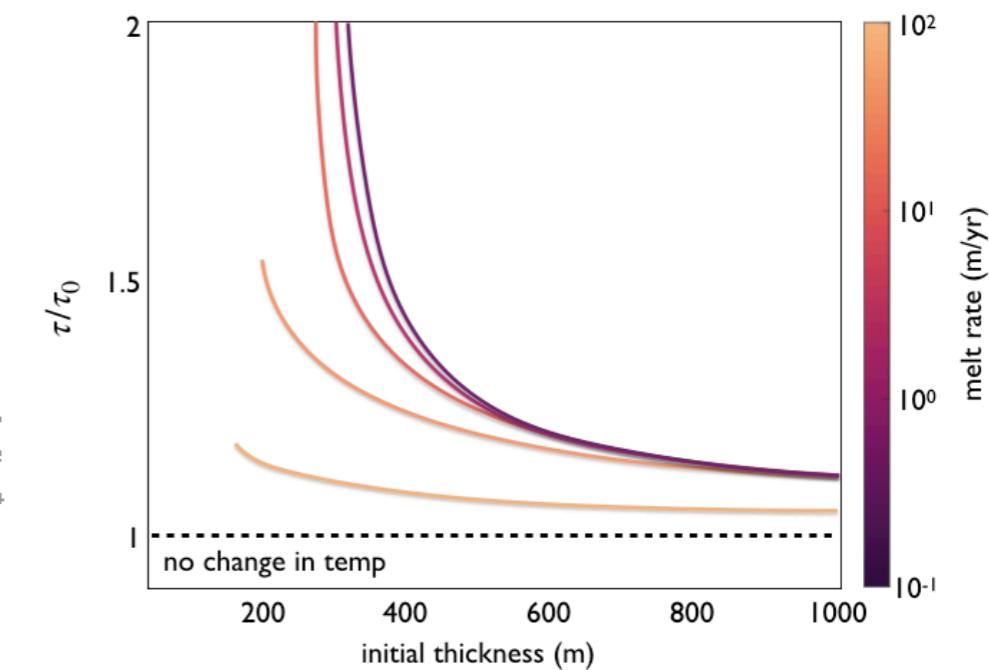
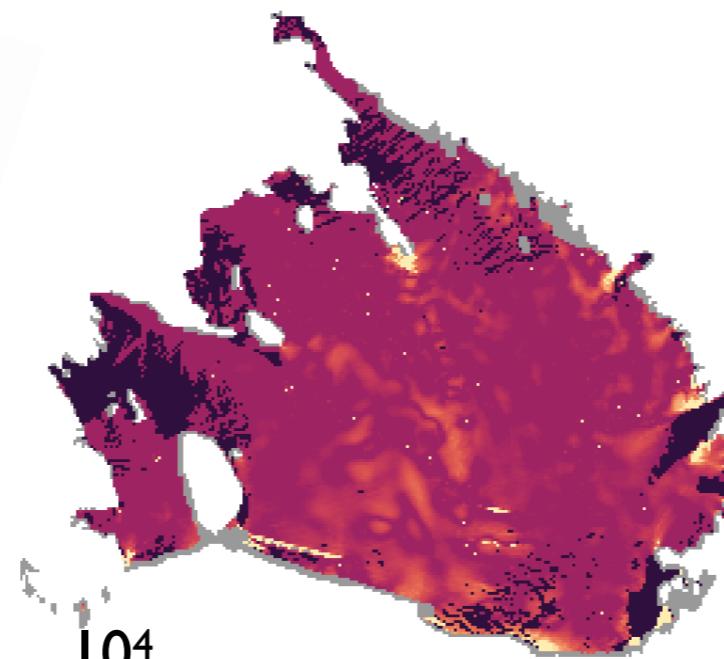
Cold and warm
cavity shelves
susceptible



Very different ‘collapse’ timescales for
cold and warm water ice shelves



Temp profile change, thinning and strain rates all contribute



‘Sensitivity boosting’ in ice sheets: tipping points and time-scales

Alex Bradley with Ian Hewitt and C.Yao Lai



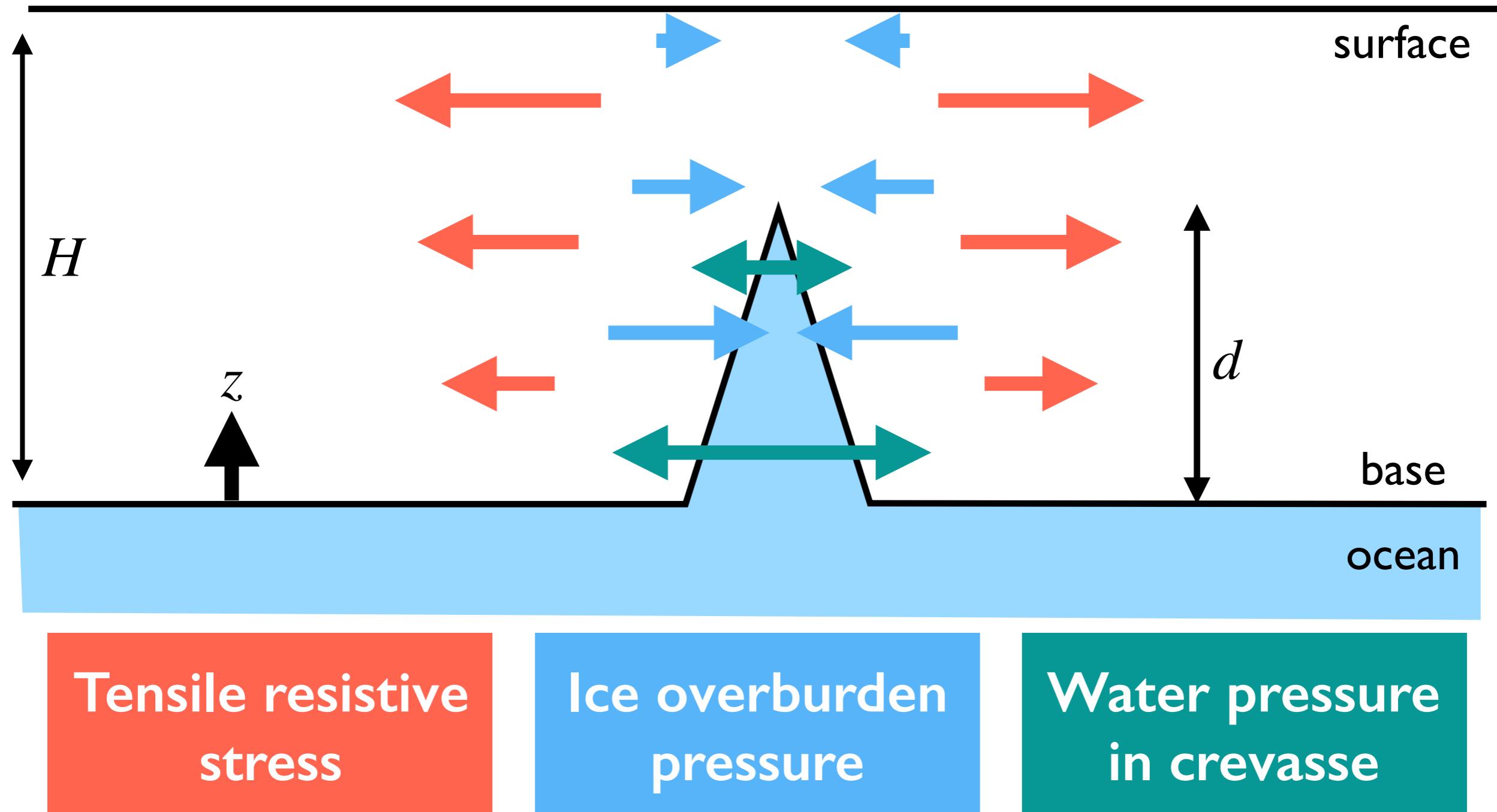
@abraleey



aleey@bas.ac.uk

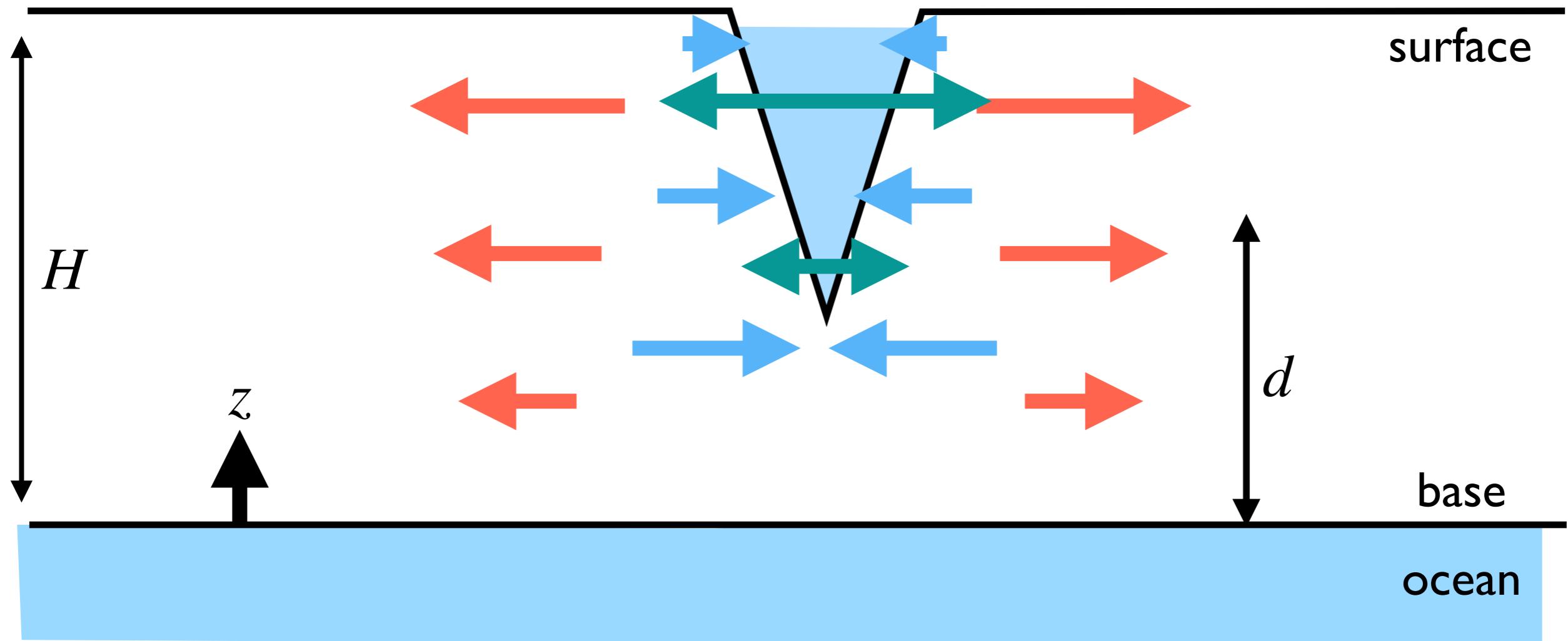


Basal vs surface crevassing?



Stress intensity = tensile resistive stress + water pressure - ice overburden

Basal vs surface crevassing?

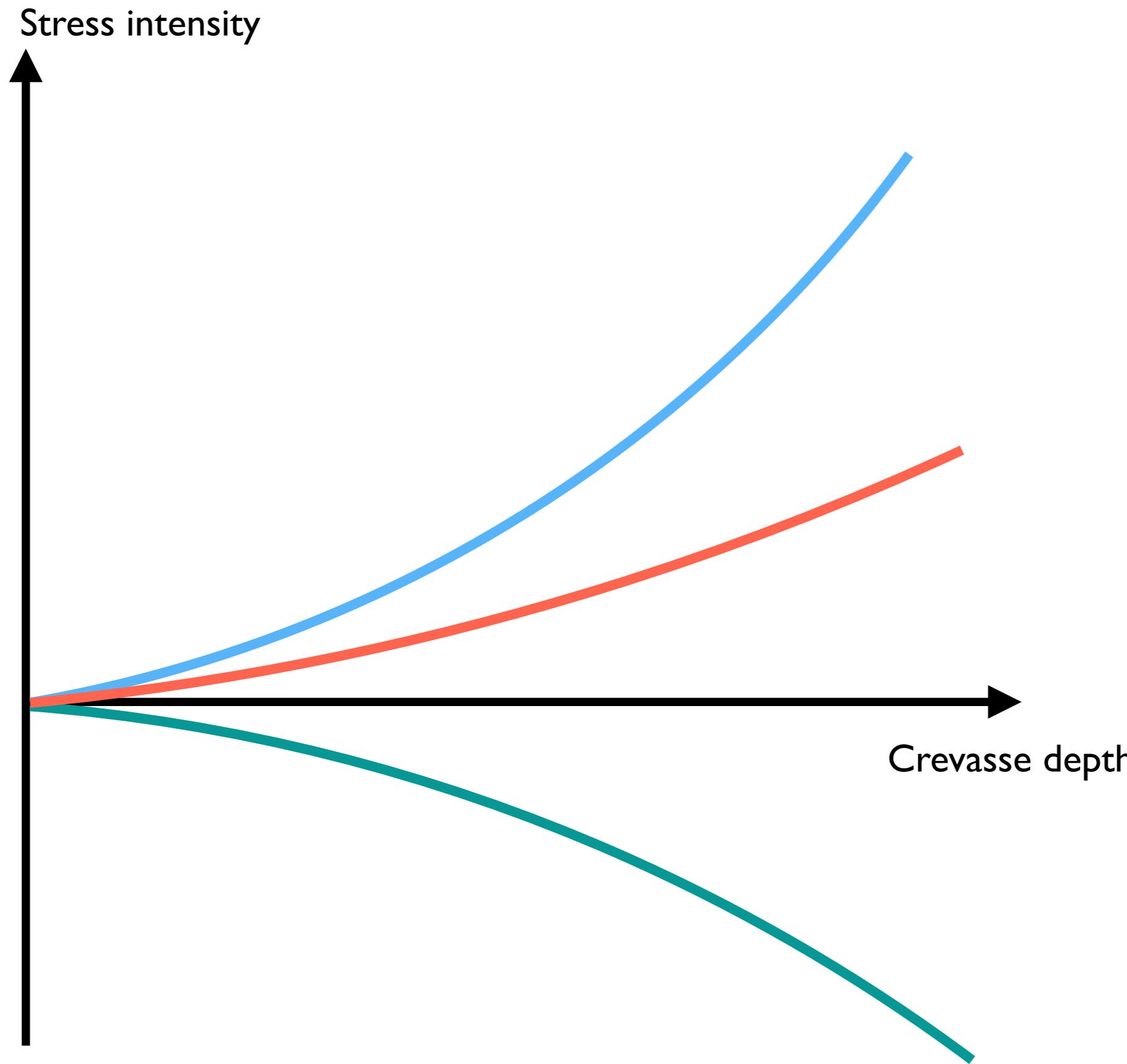


Tensile resistive
stress

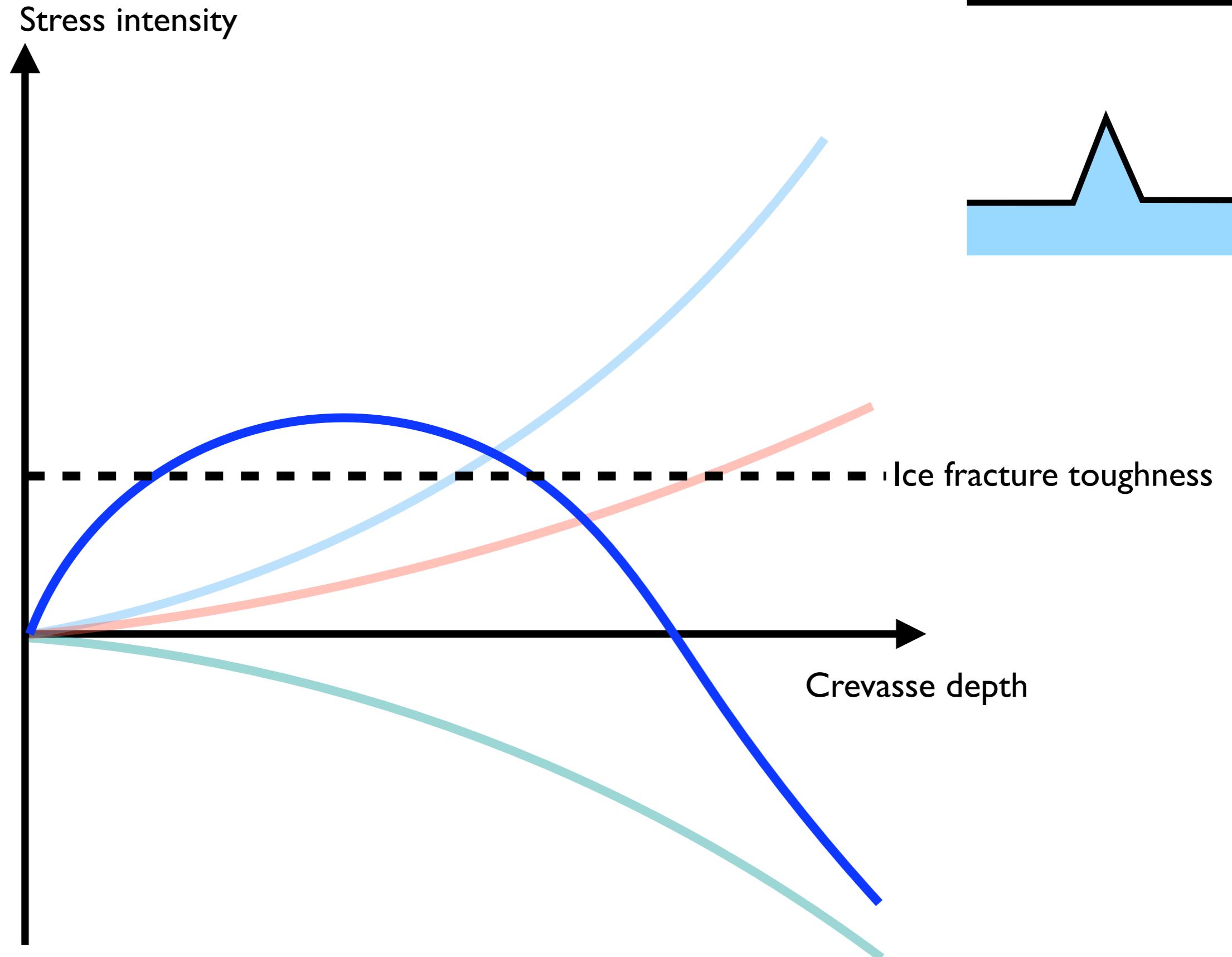
Ice overburden
pressure

Water pressure
in crevasse

Stress intensity = tensile resistive stress + water pressure - ice overburden



Stress intensity = tensile resistive stress + water pressure + ice overburden



Stress intensity = tensile resistive stress + water pressure + ice overburden

