CS 109 Quiz 5 (40 points):

1. [12 points] True or False (2 points correct, 1 point blank, 0 points guess). Note that true means **always** true.
   a. If $Y = X^2$, then $\rho(X,Y) = 1$.
   b. If $Cov(X,Y) = 0$, then $X \perp Y$.
   c. If $Y = -0.5X$, then $\rho(X,Y) = -0.5$.
   d. If X, Y are random variables, then $E\big[E[X|Y]\big] = E[Y]$.
   e. $Var(X + X) = 2Var(X)$.
   f. For a continuous random variable X, $0 \leq f_X(x) \leq 1$ for all x.
2. [18 points] Definitions (3 points each).
   a. Suppose $Z = X + Y$, where $X \perp Y$. A summation formula for $P(Z = z)$ in terms of p_X and p_Y: $p_Z(z) =$
   b. In general, a formula for $Var(X - Y)$:
   c. If X, Y are discrete, $E[X|Y = y] =$
   d. The PMF and variance of $Bin(n, p)$.
   e. If X, Y are discrete, a formula for $P(X = Y)$ involving possibly $p_{X,Y}$ and at most one summation.
   f. If X, Y are continuous, a formula for $P(X = Y)$ involving possibly $f_{X,Y}$ and at most one integral.
3. [10 points] Short answer.
   a. Suppose there are K kit kats in a bag of N total candies. You are allowed to reach in grab n candies to eat $(n \leq N)$. Let X be the number of kit kats you get. What is $p_X(k)$? We say $X \sim HypGeo(N, K, n)$.
   b. Suppose there are N cards in a deck, K of which are Aces. Let Y be the number of cards you draw up to and including your k^{th} ace $(k \leq K)$. What is $p_Y(n)$? We say $Y \sim NegHypGeo(N, K, k)$.

Here is a nice chart:

	Fixed Number of Trials	Fixed Number of Successes
With replacement	$Binomial$	$Negative\ Binomial$
Without replacement	$Hypergeometric$	$Negative\ Hypergeometric$

1. There are n people at a party, and they leave their hat with a hat-check person. Unfortunately, the hat-check person's shift ended before the party ended, and someone else came to replace her. Now when each person leaves, they get a random hat back. Let X be the number of people who get their own hat back.

a. Compute $E[X]$.
b. Compute $Var(X)$.

2. Let $X \sim Unif(-1,1)$ be discrete; that is, takes on values in $\{-1,0,+1\}$ with probability $1/3$ each. Let $Y = X^2$. Show that $Cov(X, Y) = 0$, but that X and Y are not independent.

3. Let $X \sim Geo(p)$. Use the Law of Total Expectation, conditioning on the first flip being Heads or Tails, to derive

a. $E[X]$.
b. $E[X^2]$ and hence $Var(X)$.

4. Suppose the number of radioactive particles emitted in an hour are $X \sim Poi(\lambda)$. You have a device which fails to record each particle emission with probability p, independent of other particles. Let Y be the number of particles actually observed by the device. What is $E[Y]$?

5. Let $(X_1, \dots, X_r) \sim Multinomial(n, p_1, \dots, p_r)$.

a. What distribution does X_i follow marginally? What is $E[X_i]$ and $Var(X_i)$?
b. What is $Cov(X_i, X_j)$ for $i \neq j$?