Discrete Distributions

Distribution	Parameters	Possible Description	Range Ω_X	$\mathbb{E}[\mathbf{X}]$	$\mathbf{Var}(\mathbf{X})$	PMF ($p_X(k)$ for $k \in \Omega_X$)
Uniform (disc)	$X \sim \text{Unif}(a,b)$ for $a, b \in \mathbb{Z}$ and $a \leq b$	Equally likely to be any *integer* in $[a,b]$	$\{a, \dots, b\}$	$\frac{a+b}{2}$	$\frac{(b-a)(b-a+2)}{12}$	$\frac{1}{b-a+1}$
Bernoulli	$X \sim \text{Ber}(p)$ for $p \in [0,1]$	Takes value 1 with prob p and 0 with prob $1-p$	$\{0,1\}$	p	$p(1-p)$	$p^k (1-p)^{1-k}$
Binomial	$X \sim \text{Bin}(n,p)$ for $n \in \mathbb{N}$, and $p \in [0,1]$	Sum of n iid Ber(p) rvs. # of heads in n independent coin flips with $P(\text{head}) = p$.	$\{0, 1, \dots, n\}$	np	$np(1-p)$	$\binom{n}{k} p^k (1-p)^{n-k}$
Poisson	$X \sim \text{Poi}(\lambda)$ for $\lambda > 0$	# of events that occur in **one** unit of time independently with rate λ per unit time	$\{0, 1, \dots\}$	λ	λ	$e^{-\lambda} \frac{\lambda^k}{k!}$
Geometric	$X \sim \text{Geo}(p)$ for $p \in [0,1]$	# of independent Bernoulli trials with parameter p *up to and* ***including*** *first success*	$\{1, 2, \dots\}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$(1-p)^{k-1} p$
Hypergeometric	$X \sim \text{HypGeo}(N,K,n)$ for $n, K \leq N$ and $n, K, N \in \mathbb{N}$	# of successes in n draws (w/o replacement) from N items that contain K successes in total	$\{\max(0, n+K-N), \dots, \min(n,K)\}$	$n\frac{K}{N}$	$n\frac{K(N-K)(N-n)}{N^2(N-1)}$	$\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$
Negative Binomial	$X \sim \text{NegBin}(r,p)$ for $r \in \mathbb{N}, p \in [0,1]$	Sum of r iid Geo(p) rvs. # of independent flips until r^{th} head with $P(\text{head}) = p$	$\{r, r+1, \dots\}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\binom{k-1}{r-1} p^r (1-p)^{k-r}$
Multinomial	$\mathbf{X} \sim \text{Mult}_r(n, \mathbf{p})$ for $r, n \in \mathbb{N}$ and $\mathbf{p} = (p_1, p_2, ..., p_r)$, $\sum_{i=1}^r p_i = 1$	Generalization of the Binomial distribution, n trials with r categories each with probability p_i	$k_i \in \{0, \dots, n\}$, $i \in \{1, \dots, r\}$ and $\Sigma k_i = n$	$\mathbb{E}[\mathbf{X}] = n\mathbf{p} = \begin{bmatrix} np_1 \\ \vdots \\ np_r \end{bmatrix}$	$Var(X_i) = np_i(1-p_i)$ $Cov(X_i, X_j) = -np_i p_j, i \neq j$	$\binom{n}{k_1, \dots, k_r} \prod_{i=1}^r p_i^{k_i}$
Multivariate Hypergeometric	$\mathbf{X} \sim \text{MVHG}_r(N, \mathbf{K}, n)$ for $r, n \in \mathbb{N}$, $\mathbf{K} \in \mathbb{N}^r$ and $N = \sum_{i=1}^r K_i$	Generalization of the Hypergeometric distribution, n draws from r categories each with K_i successes (w/out replacement)	$k_i \in \{0, \dots, K_i\}$, $i \in \{1, \dots, r\}$ and $\Sigma k_i = n$	$\mathbb{E}[\mathbf{X}] = n\frac{\mathbf{K}}{N} = \begin{bmatrix} n\frac{K_1}{N} \\ \vdots \\ n\frac{K_r}{N} \end{bmatrix}$	$Var(X_i) = n\frac{K_i}{N} \cdot \frac{N-K_i}{N} \cdot \frac{N-n}{N-1}$ $Cov(X_i, X_j) = -n\frac{K_i}{N}\frac{K_j}{N} \cdot \frac{N-n}{N-1}, i \neq j$	$\frac{\prod_{i=1}^r \binom{K_i}{k_i}}{\binom{N}{n}}$

Continuous Distributions							
Distribution	**Parameters**	**Possible Description**	**Range** Ω_X	$\mathbb{E}[\mathbf{X}]$	**Var**$(\mathbf{X})$	**PDF** $f_X(x)$ **for** $x \in \Omega_X$	**CDF** $(\mathbf{F_X}(\mathbf{x}) = \mathbb{P}(\mathbf{X} \leq \mathbf{x}))$
Uniform	$X \sim \text{Unif}(a, b)$ for $a < b$	Equally likely to be any real number in $[a, b]$	$[a, b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{1}{b-a}$	$\begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } x \geq b \end{cases}$
Exponential	$X \sim \text{Exp}(\lambda)$ for $\lambda > 0$	Time until first event in Poisson process	$[0, \infty)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\lambda e^{-\lambda x}$	$\begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$
Normal	$X \sim \mathcal{N}(\mu, \sigma^2)$ for $\mu \in \mathbb{R}$, and $\sigma^2 > 0$	Standard bell curve	$(-\infty, \infty)$	μ	σ^2	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$
Gamma	$X \sim \text{Gam}(r, \lambda)$ for $r, \lambda > 0$	Sum of r iid Exp(λ) rvs. Time to r^{th} event in Poisson process. Conjugate prior for Exp, Poi parameter λ	$[0, \infty)$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	$\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$	Note: $\Gamma(r) = (r-1)!$ for integers r.
Beta	$X \sim \text{Beta}(\alpha, \beta)$ for $\alpha, \beta > 0$	Conjugate prior for Ber, Bin, Geo, NegBin parameter p	$(0, 1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$	
Dirichlet	$\mathbf{X} \sim$ Dir$(\alpha_1, \alpha_2, \ldots, \alpha_r)$ for $\alpha_i, r > 0$ and $r \in \mathbb{N}, \alpha_i \in \mathbb{R}$	Generalization of Beta distribution. Conjugate prior for Multinomial parameter $\mathbf{p}$	$x_i \in (0,1)$; $\sum_{i=1}^r x_i = 1$	$\mathbb{E}[X_i] = \frac{\alpha_i}{\sum_{j=1}^r \alpha_j}$		$\frac{1}{B(\alpha)} \prod_{i=1}^r x_i^{a_i - 1}$, $x_i \in (0,1), \sum_{i=1}^r x_i = 1$	
Multivariate Normal	$\mathbf{X} \sim \mathcal{N}_n(\boldsymbol{\mu}, \mathbf{\Sigma})$ for $\boldsymbol{\mu} \in \mathbb{R}^n$ and $\mathbf{\Sigma} \in \mathbb{R}^{n \times n}$	Generalization of Normal distribution	$\mathbb{R}^n$	$\boldsymbol{\mu}$	$\mathbf{\Sigma}$	$\frac{1}{(2\pi)^{n/2}\lvert\Sigma\rvert^{1/2}} \cdot$ $\exp(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu}))$	