CS 109 Quiz 4 (31 points):

1. [10 points] True or False (2 points correct, 1 point blank, 0 points guess). Note that true means **always** true.

For all parts, suppose $X_1, \dots X_n$ are iid (independent and identically distributed) $Ber(p)$.

a. $(\sum_{i=1}^{n} X_i) \sim Bin(n, p)$.
b. $X_1^2 \sim Ber(p^2)$.
c. $X_1 X_2 \sim Ber(p^2)$.
d. Let $Y_i = X_i X_{i+1}$ for $i = 1, \dots, n-1$. Then, $(\sum_{i=1}^{n-1} Y_i) \sim Bin(n-1, p^2)$.
e. Let $Y_i = X_i X_{i+1}$ for $i = 1, \dots, n-1$. Then, $E[\sum_{i=1}^{n-1} Y_i] = (n-1)p^2$.

2. [21 points] Definitions (3 points each).

a. Cite the Chain Rule. $\Pr(A \cap B \cap C) =$.
b. A formula for $f_{X|Y}(x|y)$ in terms of $f_{Y|X}(y|x)$.
c. $E[g(X)]$ for a continuous random variable X.
d. A formula for $\Pr(a \leq X \leq b, c \leq Y \leq d)$ depending on the joint cdf $F_{X,Y}$. (Hint: has 4 terms, draw a picture)
e. Joint PDF of X, W given joint PDF $f_{W,X,Y,Z}$: $f_{X,W}(x, w) =$.
f. The PMF and expectation of a $Geo(p)$ random variable.
g. The variance of a $Ber(p)$ random variable.

1. You are playing table tennis, and the score is tied at 20-20 (called a deuce). The game continues until someone wins by two points. You win a point with probability p, independently of other points. What is the probability you win the game?

2. You have been hooked on the popular trading card game, Digimon, and are trying to collect all n distinct cards. Each pack you buy has one random card, each equally likely, and costs $\$8$. How much money can Digimon Co expect to extort from you?

3.

a. Show that if $X \sim Exp(\lambda)$, then for any $s, t \geq 0$, $P(X > s + t | X > t) = P(X > s)$. That is, that the exponential distribution is **memoryless**.

b. Suppose X and Y are iid $Exp(\lambda)$, and let $Z = X + Y$. This is like the waiting time until the second event, similar to a negative binomial distribution. We would write $Z \sim Gamma(2, \lambda)$. What is $F_Z(z)$?

4. Let $X \sim Unif(0,1)$. At each time step $t = 1,2, \dots, T$, you draw $Y_t \sim Unif(0,1)$ and compare it to X. You stop at time T when $Y_t > X$ for the **first** time. What is $E[T]$?

5. Consider a continuous joint distribution, (X, Y), where $X \in [0,1]$ is the proportion of the time until the midterm that you actually study for it, and $Y \in [0,1]$ is your percentage score on the exam. Set up but **DO NOT EVALUATE** any of your answers. Take care in setting up the limits of integration. The joint PDF is:

$$f_{X,Y}(x,y) = \begin{cases} ce^{-(y-x)}, & x, y \in [0,1] \text{ and } y \geq x \\ 0, & y < x \end{cases}$$

a. Sketch the joint range $\Omega_{X,Y}$, and interpret it in English.

b. Write an expression that we could evaluate to find c.

c. Write an expression for the marginal PDF $f_Y(y)$. Carefully define it $\forall y \in \mathbb{R}$.

d. Write an expression that we could evaluate to find $P(Y \geq 0.9)$.

e. Write an expression that we could evaluate to find $E[Y]$.

f. Write an expression that we could evaluate to find $f_{X|Y}(x|y)$.

g. Are X and Y independent?