Distribution	Parameters	Possible Description	Range Ω_X	$\mathbb{E}[X]$	Var(X)	PMF $(p_X(k) \text{ for } k \in \Omega_X)$
Uniform (disc)	$X \sim \text{Unif}(a, b)$ for $a, b \in \mathbb{Z}$ and $a \le b$	Equally likely to be any $integer$ in $[a, b]$	$\{a,\ldots,b\}$	$\frac{a+b}{2}$	$\frac{(b-a)(b-a+2)}{12}$	$\frac{1}{b-a+1}$
Bernoulli	$X \sim \mathrm{Ber}(p)$ for $p \in [0, 1]$	Takes value 1 with prob p and 0 with prob $1-p$	{0,1}	p	p(1-p)	$p^k \left(1 - p\right)^{1 - k}$
Binomial	$X \sim \operatorname{Bin}(n, p)$ for $n \in \mathbb{N}$, and $p \in [0, 1]$	Sum of n iid $Ber(p)$ rvs. # of heads in n independent coin flips with $P(head) = p$.	$\{0,1,\dots,n\}$	np	np(1-p)	$\binom{n}{k}p^k\left(1-p\right)^{n-k}$
Poisson	$X \sim \text{Poi}(\lambda)$ for $\lambda > 0$	# of events that occur in \mathbf{one} unit of time independently with rate λ per unit time	$\{0,1,\ldots\}$	λ	λ	$e^{-\lambda} \frac{\lambda^k}{k!}$
Geometric	$X \sim \text{Geo}(p)$ for $p \in [0, 1]$	# of independent Bernoulli trials with parameter p up to and including first success	$\{1,2,\ldots\}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$(1-p)^{k-1}p$
Hypergeometric	$X \sim \operatorname{HypGeo}(N, K, n)$ for $n, K \leq N$ and $n, K, N \in \mathbb{N}$	# of successes in n draws (w/o replacement) from N items that contain K successes in total	$\{\max(0, n+K-N), \dots, \min(n,K)\}$	$n\frac{K}{N}$	$n\frac{K(N-K)(N-n)}{N^2(N-1)}$	$\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$
Negative Binomial	$X \sim \text{NegBin}(r, p)$ for $r \in \mathbb{N}, p \in [0, 1]$	Sum of r iid $Geo(p)$ rvs. # of independent flips until r^{th} head with $P(head) = p$	$\{r,r+1,\ldots\}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\binom{k-1}{r-1} p^r (1-p)^{k-r}$
Multinomial	$\mathbf{X} \sim \mathrm{Mult}_r(n, \mathbf{p})$ for $r, n \in \mathbb{N}$ and $\mathbf{p} = (p_1, p_2,, p_r),$ $\sum_{i=1}^r p_i = 1$	Generalization of the Binomial distribution, n trials with r categories each with probability p_i	$k_i \in \{0, \dots, n\},\$ $i \in \{1, \dots, r\}$ and $\Sigma k_i = n$	$\mathbb{E}[\mathbf{X}] = n\mathbf{p} = \begin{bmatrix} np_1 \\ \vdots \\ np_r \end{bmatrix}$	$\begin{aligned} Var(X_i) &= np_i(1-p_i) \\ Cov(X_i, X_j) &= \\ -np_ip_j, i \neq j \end{aligned}$	$\binom{n}{k_1,\dots,k_r}\prod_{i=1}^r p_i^{k_i}$
Multivariate Hypergeometric	$\mathbf{X} \sim \text{MVHG}_r(N, \mathbf{K}, n)$ for $r, n \in \mathbb{N}$, $\mathbf{K} \in \mathbb{N}^r$ and $N = \sum_{i=1}^r K_i$	Generalization of the Hypergeometric distribution, n draws from r categories each with K_i successes (w/out replacement)	$k_i \in \{0, \dots, K_i\},$ $i \in \{1, \dots, r\}$ and $\Sigma k_i = n$	$\mathbb{E}[\mathbf{X}] = n\frac{\mathbf{K}}{N} = \begin{bmatrix} n\frac{K_1}{N} \\ \vdots \\ n\frac{K_r}{N} \end{bmatrix}$	$\begin{aligned} & \operatorname{Var}(X_i) = \\ & n \frac{K_i}{N} \cdot \frac{N - K_i}{N} \cdot \frac{N - n}{N - 1} \\ & \operatorname{Cov}(X_i, X_j) = \\ & - n \frac{K_i}{N} \frac{K_j}{N} \cdot \frac{N - n}{N - 1}, i \neq j \end{aligned}$	$\frac{\prod_{i=1}^{r} \binom{K_i}{k_i}}{\binom{N}{n}}$

Continuous Distributions										
Distribution	Parameters	Possible Description	Range Ω_X	$\mathbb{E}[X]$	Var(X)	PDF $f_X(x)$ for $x \in \Omega_X$	$\mathbf{CDF} \\ (F_X(x) = \mathbb{P}(X \le x))$			
Uniform	$X \sim \text{Unif}(a, b)$ for $a < b$	Equally likely to be any real number in $[a, b]$	[a,b]	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{1}{b-a}$	$\begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \le x < b \\ 1 & \text{if } x \ge b \end{cases}$			
Exponential	$X \sim \operatorname{Exp}(\lambda)$ for $\lambda > 0$	Time until first event in Poisson process	$[0,\infty)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\lambda e^{-\lambda x}$	$\begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \ge 0 \end{cases}$			
Normal	$X \sim \mathcal{N}(\mu, \sigma^2)$ for $\mu \in \mathbb{R}$, and $\sigma^2 > 0$	Standard bell curve	$(-\infty,\infty)$	μ	σ^2	$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$			
Gamma	$X \sim \operatorname{Gam}(r, \lambda)$ for $r, \lambda > 0$	Sum of r iid $\operatorname{Exp}(\lambda)$ rvs. Time to r^{th} event in Poisson process. Conjugate prior for Exp , Poi parameter λ	$[0,\infty)$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	$\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$	Note: $\Gamma(r) = (r-1)!$ for integers r .			
Beta	$X \sim \text{Beta}(\alpha, \beta)$ for $\alpha, \beta > 0$	Conjugate prior for Ber, Bin, Geo, NegBin parameter p	(0,1)	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{\left(\alpha+\beta\right)^{2}\left(\alpha+\beta+1\right)}$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}\left(1-x\right)^{\beta-1}$				
Dirichlet	$\mathbf{X} \sim $ Dir $(\alpha_1, \alpha_2, \dots, \alpha_r)$ for $\alpha_i, r > 0$ and $r \in \mathbb{N}, \alpha_i \in \mathbb{R}$	Generalization of Beta distribution. Conjugate prior for Multinomial parameter p	$x_i \in (0,1);$ $\sum_{i=1}^{r} x_i = 1$	$\mathbb{E}[X_i] = \frac{\alpha_i}{\sum_{j=1}^r \alpha_j}$		$\frac{\frac{1}{B(\alpha)} \prod_{i=1}^{r} x_i^{a_i - 1}}{x_i \in (0, 1), \sum_{i=1}^{r} x_i},$ $x_i \in (0, 1), \sum_{i=1}^{r} x_i = 1$				
Multivariate Normal	$\mathbf{X} \sim \mathcal{N}_n(oldsymbol{\mu}, oldsymbol{\Sigma})$ for $oldsymbol{\mu} \in \mathbb{R}^n$ and $oldsymbol{\Sigma} \in \mathbb{R}^{n imes n}$	Generalization of Normal distribution	\mathbb{R}^n	μ	Σ	$\frac{\frac{1}{(2\pi)^{n/2} \Sigma ^{1/2}}\cdot}{\exp(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu}))}$				

Probability & Statistics with Applications to Computing

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