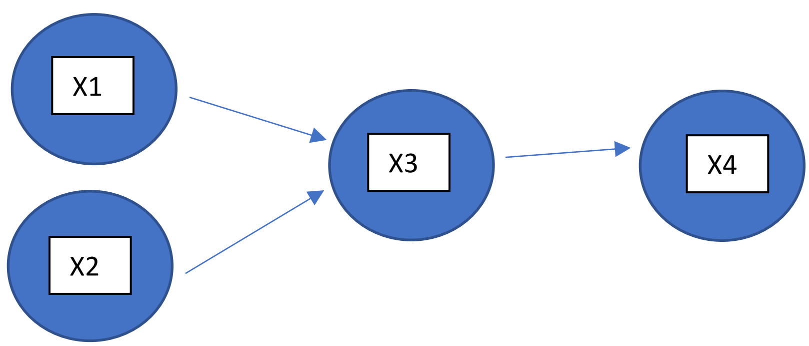
CS 109 Quiz 6 (33 points):

1. [33 points] Definitions (3 points each).
   a. For random variables X, Y, $Cov(X, Y) =$
   b. Events A and B are conditionally independent given C: $P(A, B|C) =$
   c. If X, Y are continuous, $E[X|Y = y] =$
   d. Suppose Y is a discrete rv that somehow influences X. Give a formula for $E[X]$ which uses the law of total expectation conditioning on Y. $E[X] =$
   e. If X is discrete and Y is continuous, then using Bayes Theorem (watch your p's and f's), $p_{X|Y}(x|y) =$
   f. If X, Y are discrete, then in terms of $p_{X,Y}$, $E[g(X, Y)] =$
   g. Suppose X_1, X_2, X_3, X_4 are discrete random variables. We will abuse notation and write $p(x_1, x_2, x_3, x_4)$ to mean $P(X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4 = x_4)$. In your answers, feel free to do the same. For example, $p(x_1, x_3) = p(X_1 = x_1, X_3 = x_3)$. Decompose into as many terms as possible.
      a. If there is **no assumption** of independence, then $p(x_1, x_2, x_3, x_4) =$
      b. If X_1, X_2, X_3, X_4 are assumed **independent**, then $p(x_1, x_2, x_3, x_4) =$
      c. Given this Bayesian network, $p(x_1, x_2, x_3, x_4) =$



   h. If $X_1, \ldots, X_n$ are iid with mean μ and variance σ^2, and $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, then
      a. $E[\bar{X}_n] =$
      b. $Var(\bar{X}_n) =$
      c. The Central Limit Theorem says that, as $n \to \infty$, the distribution of $\bar{X}_n$ is (give distribution and parameter(s)):