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Implementation of Antenna Array Beamforming by Using a Novel Neural Network Structure

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Abstract—The present study introduces the implementation of antenna array beamforming based on a new neural network (NN) structure. The NN comprises two hidden layers, which use different interconnectivity patterns. The first one is divided in sublayers, which are equal in number to the inputs of the NN. Each sublayer communicates only with the respective input but is fully interconnected with the second hidden layer. The NN training is performed by using data sets derived by a well-known beamforming technique called minimum variance distortionless response. The trained NN is capable of serving as adaptive beamformer that makes a linear antenna array steer the main lobe towards a desired signal and place nulls towards respective interference signals in the presence of additive zero-mean Gaussian noise. The performance of the trained NN is tested by estimating the mean absolute deviation of main lobe and null directions from their respective desired directions.

Keywords—Adaptive beamforming; antenna array feed; antenna beamforming; direction of arrival; minimum variance distortionless response; neural networks; smart antennas

I. INTRODUCTION

Adaptive beamforming (ABF) is a special category of real time techniques applied on feeding networks of antenna arrays [1]-[16]. The aim of these techniques is to dynamically make an array steer the main lobe peak towards the direction of arrival (DoA) of a desired incoming signal (i.e., signal of interest or SoI) and create a radiation pattern with certain nulls towards the DoA of respective incoming interference signals (i.e., signals of avoidance or SoA). Since the DoA of every signal changes with time, the success of an ABF technique lies in its ability to find at every moment the proper excitation weights of the array that create a main lobe and nulls towards the directions explained above. In this way, the antenna array maximizes the signal to interference-plus-noise ratio (*SINR*) during signal reception.

Several deterministic and evolutionary methods have been proposed so far to be used as ABF techniques [5], [9], [10], [12], [16]. Such a popular technique is the minimum variance distortionless response (MVDR) [17], [18]. This technique is capable of recovering performance degradations caused by mismatches between assumed and actual conditions. Such a mismatch that may happen in practice is an uncertainty of steering vector. However, MVDR suffers from performance degradation due to uncertainty in the interference correlation matrix. Therefore, an ABF technique insensitive to this type of uncertainty would be attractive.

The ABF technique proposed in this study is based on neural networks (NNs) [1], [3], [16]. NNs are already known for their efficiency and their immediate temporal response [19]-[23]. The efficiency is a matter of training, which is an off-line procedure. To perform a decent NN training, we need a considerable training data set. Such a data set consists of a sufficient number of records, where each record is composed of random inputs and desirable outputs. Concerning the ABF problem, the inputs are usually the arrival angles (AAs) of the SoI and all the SoA received by the antenna array. Also, the outputs are appropriate parameters used to form the complex feeding excitations of the antenna array that produce a radiation pattern with a main lobe towards the DoA of the SoI and respective nulls towards the DoA of the incoming interference signals (SoA). After the training, the NN enters into operational mode for real time processing and is expected to provide proper outputs for any combination of inputs.

In a conventional NN, there is always a possibility of performance degradation due to uncertainties in interference correlation. To alleviate this effect, the present study proposes a new NN structure. Due to the specific way the input layer of the NN is connected with the first hidden layer (explained in Section III), uncertainties in interference correlation do not practically affect the ability of the NN to provide proper outputs. The proposed NN is used as a beamformer applied to a

linear array of seven ideal omnidirectional elements, which are uniformly spaced at $\lambda/2$ distance, where λ is the free space wavelength. One SoI and three SoA are considered to be received by the array in the presence of additive zero-mean Gaussian noise with signal to noise ratio (SNR) equal to 10dB. Due to ideal elements used here, no coupling between elements is considered and only the antenna array factor is taken into account to calculate the radiation pattern. The reason of using ideal elements and not realistic ones is to just exhibit the advantage of the new NN structure of becoming insensitive to uncertainties in interference correlation. However, since MVDR is not insensitive to this type of uncertainty (as mentioned above) and the training data set used for the NN is extracted from MVDR, some records of this set may severely degrade the training performance. To avoid such degradation, we filter the data set and keep only records that produce radiation patterns with main lobe and nulls towards the expected directions with the lowest possible deviation.

II. BEAMFORMING PROBLEM DEFINITION

A linear array of M ideal omnidirectional elements receives $N+1$ monochromatic signals ($N < M$) at a wavelength λ , which specifically are a SoI $s_0(k)$ with AA θ_0 and N SoA $s_n(k)$ with AAs θ_n ($n=1, \dots, N$) (see Fig. 1), where index k indicates the k -th time sample. Each AA is defined by the DoA of the respective signal and the array axis. The mean power P_{s_0} of the SoI is considered as reference power for all signals:

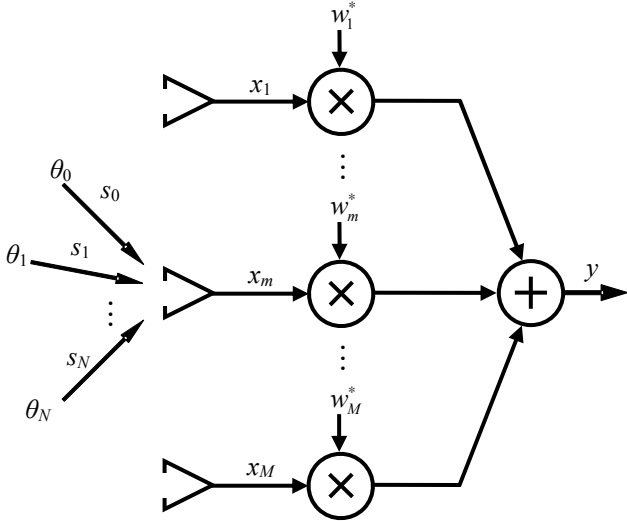


Fig. 1. Adaptive beamformer block diagram.

$$P_{s_0} = E[|s_0(k)|^2] = 1 \text{ Watt} \quad (1)$$

where $E[\cdot]$ denotes the mean value. The signal $x_m(k)$ received at the input of m -th element ($m=1, \dots, M$) includes an additive zero-mean Gaussian noise signal $n_m(k)$ with variance σ^2 . Actually, the variance expresses the mean power of noise signals and, since $P_{s_0}=1$, it can be calculated by the value of SNR in dB as follows:

$$\sigma^2 = 10^{-\text{SNR}(\text{dB})/10} \quad (2)$$

Also, we may consider that $n_1(k), \dots, n_M(k)$ are uncorrelated to each other. Then, the noise correlation matrix can be simplified as follows:

$$\bar{R}_{nn} = E[\bar{n}(k)\bar{n}^H(k)] = \sigma^2 I \quad (3)$$

where

$$\bar{n}(k) = [n_1(k) \ \cdots \ n_M(k)]^T \quad (4)$$

is the noise vector, I is the identity matrix, and superscripts T and H represent respectively the transpose and the Hermitian transpose operation.

Thus, the input signals of the array elements are expressed by the following vector:

$$\bar{x}(k) = \bar{a}_0 s_0(k) + \bar{A} \bar{s}(k) + \bar{n}(k) \quad (5)$$

where

$$\bar{x}(k) = [x_1(k) \ \cdots \ x_M(k)]^T \quad (6)$$

$$\bar{s}(k) = [s_1(k) \ \cdots \ s_N(k)]^T \quad (7)$$

$$\bar{A} = [\bar{a}_1 \ \cdots \ \bar{a}_N] \quad (8)$$

$$\bar{a}_n = [1 \ e^{j\beta d \cos \theta_n} \ \cdots \ e^{j(M-1)\beta d \cos \theta_n}]^T, \quad n = 0, 1, \dots, N \quad (9)$$

are, respectively, the input vector, the SoA vector, the array steering matrix, and the array steering vector that corresponds to AA θ_n . Also, β is the free space wavenumber ($\beta=2\pi/\lambda$) and d is the distance between adjacent elements of array ($d=\lambda/2$ for the problem studied here). The input vector can be considered as the sum of a desired and an undesired component:

$$\bar{x}(k) = \bar{x}_d(k) + \bar{x}_u(k) \quad (10)$$

where

$$\bar{x}_d(k) = \bar{a}_0 s_0(k) \quad (11)$$

$$\bar{x}_u(k) = \bar{A} \bar{s}(k) + \bar{n}(k) \quad (12)$$

The array output is expressed as follows:

$$y(k) = \bar{w}^H \bar{x}(k) \quad (13)$$

where

$$\bar{w} = [w_1 \ \cdots \ w_M]^T \quad (14)$$

is the excitation weight vector. From (10) and (13), it is obvious that the output can be considered as the sum of a desired and an undesired component:

$$y(k) = y_d(k) + y_u(k) = \bar{w}^H \bar{x}_d(k) + \bar{w}^H \bar{x}_u(k) \quad (15)$$

The mean power values of $y_d(k)$ and $y_u(k)$ are respectively given by:

$$P_{yd} = E[|y_d(k)|^2] = \bar{w}^H \bar{a}_0 \bar{a}_0^H \bar{w} \quad (16)$$

$$P_{yu} = E[|y_u(k)|^2] = \bar{w}^H \bar{A} \bar{R}_{ss} \bar{A}^H \bar{w} + \sigma^2 \bar{w}^H \bar{w} \quad (17)$$

where

$$\bar{R}_{ss} = E[\bar{s}(k) \bar{s}^H(k)] \quad (18)$$

is the correlation matrix of SoA. Finally, (16) and (17) can be used to extract the value of $SINR$:

$$SINR = \frac{P_{yd}}{P_{yu}} = \frac{\bar{w}^H \bar{a}_0 \bar{a}_0^H \bar{w}}{\bar{w}^H \bar{A} \bar{R}_{ss} \bar{A}^H \bar{w} + \sigma^2 \bar{w}^H \bar{w}} \quad (19)$$

III. MINIMUM VARIANCE DISTORTIONLESS RESPONSE

The MVDR beamformer searches for the optimum \bar{w} that minimizes P_{yu} , while the desired output signal $y_d(k)$ is maintained, resulting thus in a maximum $SINR$ value [17], [18]. The optimum \bar{w} is given by:

$$\bar{w}_{MVDR} = \frac{\bar{R}_{uu}^{-1} \bar{a}_0}{\bar{a}_0^H \bar{R}_{uu}^{-1} \bar{a}_0} \quad (20)$$

where

$$\bar{R}_{uu} = E[\bar{x}_u(k) \bar{x}_u^H(k)] = \bar{A} \bar{R}_{ss} \bar{A}^H + \sigma^2 I \quad (21)$$

is the correlation matrix of $x_u(k)$. It must be noted that (20) results in conjugate values for array elements, which are symmetrical with respect to the middle element/elements of the array. Concerning the beamforming problem studied here, the array consists of seven elements ($M=7$) and therefore the excitation weights derived from MVDR will have conjugate values for elements, which are symmetrical with respect to the fourth element, i.e., $w_1 = w_7^*$, $w_2 = w_6^*$ and $w_3 = w_5^*$.

IV. NOVEL NEURAL NETWORK DESCRIPTION

A lot of problems have been solved so far by using NNs [1], [3], [16], [19]-[23]. One of the earliest implementations of NNs has been the *multi-layer perceptron* (MLP) in the form of Feed Forward Neural Networks (FFNNs) [19]. An increasing efficacy of this simple design has been documented in the past years with the emergence of Deep Learning, where FFNNs are typically used.

A FFNN is typically comprised of three layers: an input layer that contains all the necessary inputs, a hidden layer which is the host of the bulk of the neurons and the output layer which provides the appropriate output values. Each neuron has an activation function. Each activation function can be either linear, hyperbolic tangent or logistic function and is chosen depending on the problem to which the NN is applied.

The NN training is an off-line and iterative process, which is necessary for improving NN performance. To perform training, a data set that comprises a sufficient number of records with random inputs and desirable outputs is necessary. At every iteration, all these input values travel through the FFNN and then respective outputs are computed, resulting in a mean square error (MSE) between produced and desired outputs. Then, the value of MSE travels back through the network according to a method called *backpropagation*. In this way, the neurons are corrected at every iteration. As MSE decreases, the NN is considered to improve its performance. At the end of the training process, MSE has reached its lowest value and the NN is considered to be finally trained.

The NN structure proposed in this study (shown in Fig. 2) is an adaptation to the particular problem of adaptive beamforming. The first hidden layer is divided in sublayers, which are equal in number to the inputs of the NN. Each sublayer communicates only with the respective input, but is fully interconnected with the second hidden layer, which is connected to the output layer as usual. This provides a more modular approach to the design of the NN in order to adapt better to the problem in question. The transfer function used in this study is the hyperbolic tangent and the Levenberg-Marquardt is used as training function. The proposed NN structure has actually been inspired by convolutional NNs (CNNs), which have recently become very popular due to their performance in image processing [24], [25]. However, what is proposed here is a simplified version of CNN structure tailored to the needs of an adaptive beamformer.

The case studied here considers four incoming signals, i.e., one SoI and three SoA, which result in a NN with four inputs, i.e., AA θ_0 of SoI and AAs θ_1 , θ_2 and θ_3 of the three respective SoA. Therefore, the first hidden layer must be composed of four sublayers (one sublayer per input). After testing a number of layouts in terms of NN performance, 12 nodes per sublayer were chosen. Additionally, a second hidden layer of 24 nodes was chosen in the NN structure to interconnect the sublayers with the outputs. Since the NN is trained by a data set acquired from the application of MVDR, it is obvious that the training process will be based upon the concept of conjugate complex weights for elements, which are symmetrical with respect to the fourth element of the array. We also may consider that all the weights are normalized with respect to the fourth element of the array, i.e., $w_4=1$. Then, three amplitudes $|w_1|$, $|w_2|$ and $|w_3|$, and three phases φ_1 , φ_2 and φ_3 , i.e., six excitation parameters in total, are sufficient to describe the array excitation:

$$\begin{aligned}
w_1 &= |w_1| \exp(j\varphi_1) & w_7 &= |w_1| \exp(-j\varphi_1) \\
w_2 &= |w_2| \exp(j\varphi_2) & w_6 &= |w_2| \exp(-j\varphi_2) \\
w_3 &= |w_3| \exp(j\varphi_3) & w_5 &= |w_3| \exp(-j\varphi_3) \\
w_4 &= 1
\end{aligned} \quad (22)$$

Since the values of these parameters must be extracted by the NN, it is obvious that the output layer must be composed of six outputs. All these details that concern the proposed NN structure are illustrated in Fig. 2. It is also obvious that each record of the NN training set must contain four random values of angles $\theta_0, \theta_1, \theta_2$ and θ_3 and the respective desired values of $|w_1|, |w_2|, |w_3|, \varphi_1, \varphi_2$ and φ_3 , extracted from MVDR.

The inputs and the outputs are all suitably normalized to optimize the training of the NN. To test its performance, we calculate the mean absolute divergence between the requested directions of the main lobe and the nulls and the respective directions derived from the NN output.

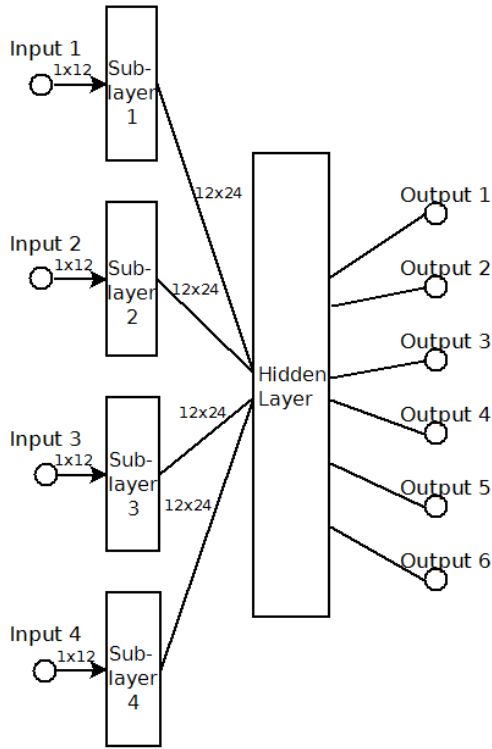


Fig. 2. Novel neural network structure.

V. SIMULATION RESULTS

One thousand iterations were used to train the proposed NN. The variation of MSE with the number of iterations is illustrated in Fig. 3. At the end of the training process, MSE has become equal to 5×10^{-5} . This value is considered quite low and therefore the NN can be assumed to be sufficiently trained.

To test the performance of the proposed NN, a set of 1000 records (test records) were used. Each one of these records has the structure of a training record (i.e., contain four random values of $\theta_0, \theta_1, \theta_2$ and θ_3 and the respective desired values of

$|w_1|, |w_2|, |w_3|, \varphi_1, \varphi_2$ and φ_3 , extracted from MVDR) but does not coincide with any training record. The values of $\theta_0, \theta_1, \theta_2$ and θ_3 of each test record are applied to the NN and the respective outputs are used to construct the array excitation weights (w_1, \dots, w_7). These weights are then used to derive the radiation pattern and extract the actual main lobe direction (θ_0) as well as the actual null directions ($\theta_1, \theta_2, \theta_3$). In this way, an angular absolute deviation between desired and actual values of $\theta_0, \theta_1, \theta_2$ and θ_3 are calculated for each test record. The mean value of this deviation derived from the whole set of 1000 test records does not exceed 0.6 degrees. The radiation pattern, which is produced from the proposed NN-based beamformer and corresponds to one of the test records with $\theta_0=35^\circ, \theta_1=90^\circ, \theta_2=120^\circ$ and $\theta_3=150^\circ$, is displayed in Fig. 4. The radiation pattern has been normalized with respect to the main lobe peak. The agreement between the desired AAs and the respective directions of main lobe and nulls is evident.

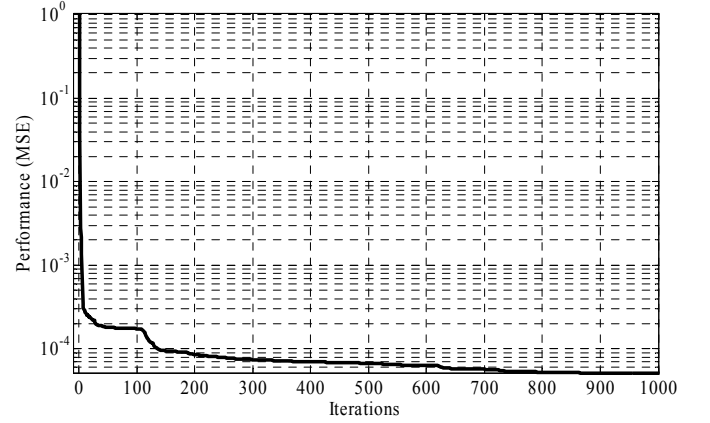


Fig. 3. Variation of MSE vs. iterations during the NN training process.

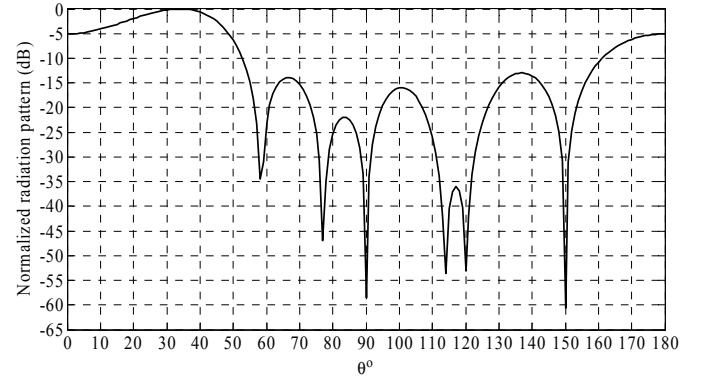


Fig. 4. Normalized radiation pattern produced by the proposed NN-based beamformer with $\theta_0=35^\circ, \theta_1=90^\circ, \theta_2=120^\circ$ and $\theta_3=150^\circ$.

To better evaluate the performance of the proposed NN, we made a comparison with a conventional NN. In order to have a fair comparison, the conventional NN contains the same number of nodes per layer (i.e., 4 inputs, 48 nodes for the first hidden layer, 24 nodes for the second hidden layer and 6 outputs), and is trained by applying the same 1000-iteration training process and using the same training data set, like the proposed NN. At the end of the training process, the conventional NN achieved MSE not lower than 10^{-4} , which is

worse than the respective MSE value achieved by the proposed NN. Also, the same set of 1000 test records were used for the conventional NN resulting in mean absolute deviation of main lobe and null directions nearly equal to 1.2 degrees, which is twice the respective deviation derived from the proposed NN. It is therefore obvious that, under the same training process, the proposed NN has better performance than a conventionally structured NN.

VI. CONCLUSION

A novel NN structure has been introduced to be used as an adaptive beamformer of an antenna array. In this structure, the first hidden layer is divided into sublayers, and a direct connection between every input and a respective sublayer is performed in order to alleviate the performance degradation due to uncertainties in interference correlation. The proposed NN structure has been trained by a filtered data set extracted by MVDR. Then, the NN is applied to a linear antenna array composed of seven equally-spaced elements, which are considered to receive one desired and three interference signals in the presence of additive zero-mean Gaussian noise with SNR equal to 10dB. A comparison with a conventional NN structure has shown that the proposed NN produces radiation patterns with main lobe closer to the direction of a desired incoming signal and at least three nulls produced with lower deviation from the directions of the respective interference signals.

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