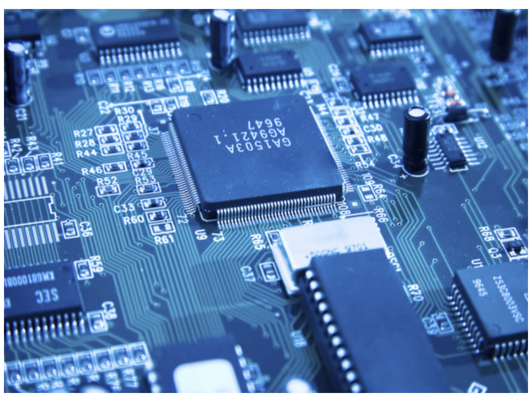
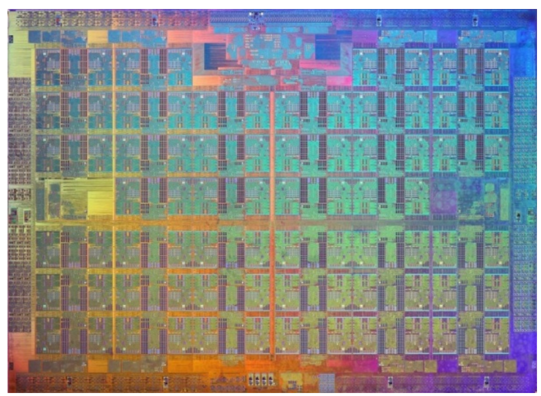
Welcome to 6.004!

Computation Structures





Fall 2019

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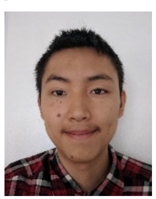
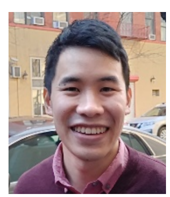


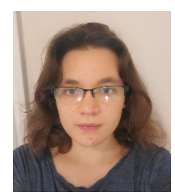
Brian Chen

Felipe Moreno



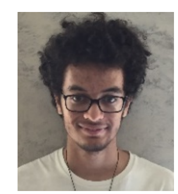
Quan Nguyen

Domenic Nutile



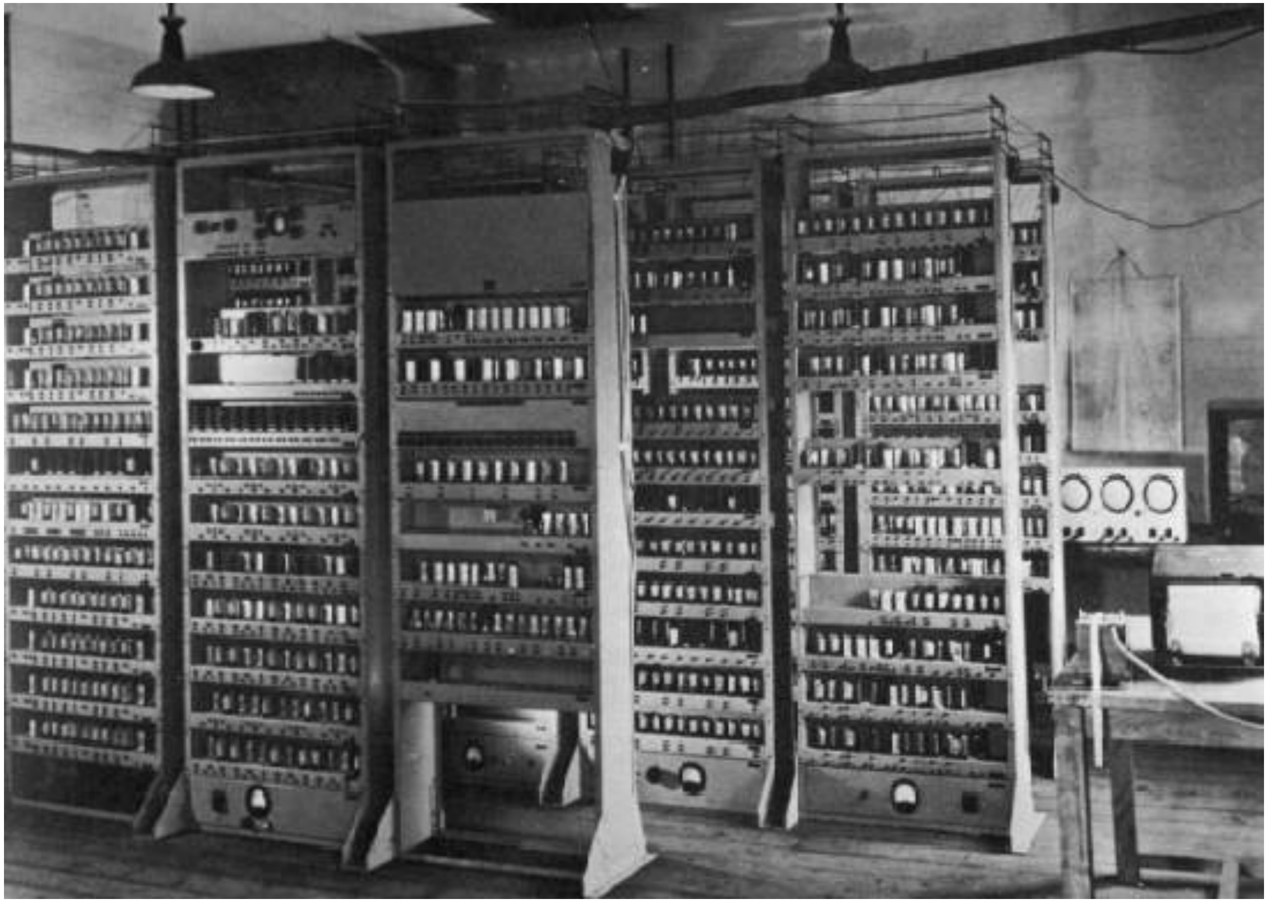
Ileana Rugina

Sebastian Bartlett



Kendall Garner

Computing Devices Then…



ENIAC, 1943   30 tons, 200KW, ~1000 ops/sec

Computing Devices Now

Typical 2019 laptop
1kg, 10W, 10 billion ops/s

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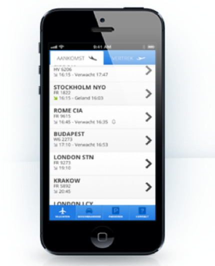

An Introduction to the Digital World

Computer programs

Devices
Materials
Atoms

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Computer programs

Digital design

Combinational and sequential circuits

Devices

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Computer programs

Computer architecture

Processors, caches, pipelining

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Computer programs

Computer systems
Operating systems, virtual memory, I/O

Computer architecture
Processors, caches, pipelining

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An Introduction to the Digital World

Computer programs

Virtual machines

Computer systems

Operating systems, virtual memory, I/O

Instruction set + memory

Computer architecture

Processors, caches, pipelining

Digital circuits

Digital design

Combinational and sequential circuits

Bits, Logic gates

Devices

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The Power of Engineering Abstractions

Good abstractions let us reason about behavior while shielding us from the details of the implementation.

Virtual machines

Instruction set + memory

Digital circuits

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Corollary: implementation technologies can evolve while preserving the engineering investment at higher levels.

Virtual machines

Instruction set + memory

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The Power of Engineering Abstractions

Good abstractions let us reason about behavior while shielding us from the details of the implementation.

Corollary: implementation technologies can evolve while preserving the engineering investment at higher levels.

Leads to hierarchical design:

- Limited complexity at each level ⇒ shorten design time, easier to verify
- Reusable building blocks

Virtual machines

Instruction set + memory

Digital circuits

Bits, Logic gates

Course Outline

- Module 1: Assembly language
  - From high-level programming languages to the language of the computer

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  - Caches and the memory hierarchy
- Module 4: Computer systems
  - Operating system and virtual memory
  - Parallelism and synchronization

Our Focus: Programmable General-Purpose Processors

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- Microprocessors are the most sophisticated digital systems that exist today
  - Understanding them will help you design all kinds of hardware

By the end of the term you will have designed a simple processor from scratch!

We Rely on Modern Design Tools

- We will use RISC-V, a simple and modern instruction set

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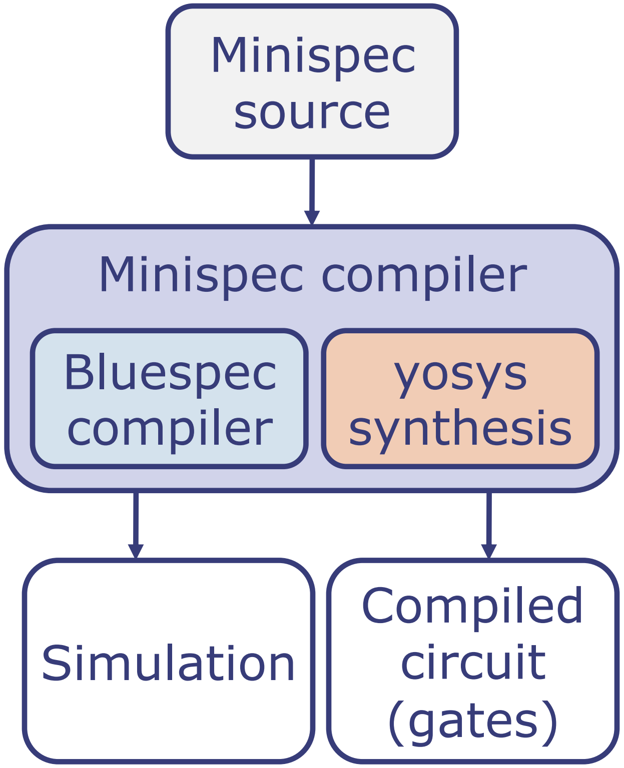
- We will use RISC-V, a simple and modern instruction set
- We will design hardware using Minispec, a new hardware description language built for 6.004
  - Based on Bluespec, but heavily simplified



Course Mechanics

- 2 lectures/week: handouts, videos, and reference materials on website
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- 3 quizzes: Oct 17, Nov 14, Dec 5 (7:30-9:30pm)
  - If you have a conflict, contact us to schedule a makeup

Recitation Mechanics

- 12 recitation sections on Wed & Fri
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- Recitation attendance is mandatory (worth 5% of your grade)
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- Recitations will review lecture material and problems associated with each lecture
  - We recommend you work on the problems before recitation
  - We will post solutions after recitation

Grading

- 80 points from labs,
  20 points from design project,
  90 points from quizzes,
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- Fixed grade cutoffs:
  - A: Points >= 165
  - B: Points >= 145
  - C: Points >= 125
  - F: Points < 125 or not all labs complete

Online and Offline Resources

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*32-083 Combination Lock:*_________________

We Want Your Feedback!

- Your input is crucial to fine-tune this offering of the course and improve future versions
- Periodic informal surveys
- Any time: Email us or post on Piazza

Binary Number Encoding and Arithmetic

Digital Information Encoding

- Digital systems represent and process information using discrete symbols or digits

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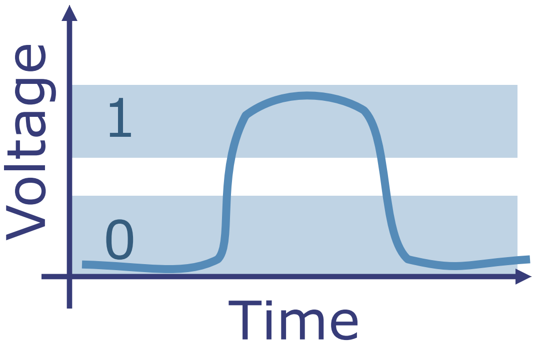

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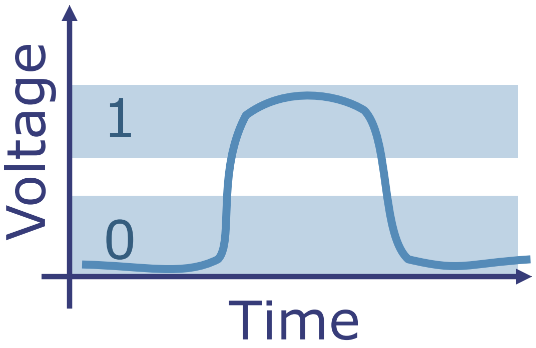

- We can implement operations like +, >, AND, etc. on binary numbers in hardware very efficiently

Encoding Positive Integers

It is straightforward to encode positive integers as a sequence of bits. Each bit is assigned a weight. Ordered from right to left, these weights are increasing powers of 2. The value of an N-bit number is given by the following formula:

$$v = \sum_{i=0}^{N-1} 2^i b_i$$

2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	1	1	1	1	0	1	0	0	0	0

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Smallest number? *Largest number?*

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Smallest number? *0*

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Smallest number? 0 *Largest number?* 2^N-1

Hexadecimal Notation

Long strings of bits are tedious and error-prone to transcribe, so we often use a higher-radix notation, choosing the radix so that it's simple to recover the original bit string.

A popular choice is to transcribe numbers in base-16, called hexadecimal. Each group of 4 adjacent bits is represented as a single hexadecimal digit.

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$\underbrace{0\ 0\ 0\ 0}_{0}$

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$$\underbrace{0111}_{7}\;\underbrace{1101}_{D}\;\underbrace{0000}_{0}$$

0b011111010000 = 0x7D0

Binary Addition and Subtraction

- Addition and subtraction in base 2 are performed just like in base 10

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Base 10

$$\begin{array}{r} 14 \\ +\ \ 7 \\ \hline \end{array}$$

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$$
\begin{array}{r}
{\color{red}\mathit{1}} \\
14 \\
+\;\;7 \\
\hline
{\color{red}\mathit{1}}
\end{array}
$$

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$$\begin{array}{r} \color{red}{\mathit{1}} \\ 14 \\ +\ \ 7 \\ \hline \color{red}{\mathit{1}} \end{array}$$

carry

Binary Addition and Subtraction

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$$\begin{array}{r} \color{red}{1} \\ 14 \\ +\ \ 7 \\ \hline \color{red}{21} \end{array}$$

carry

Binary Addition and Subtraction

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Base 10	Base 2
$\begin{array}{r} {\color{red}1} \\ 14 \\ +\ \ 7 \\ \hline {\color{red}21} \end{array}$ *carry*	$\begin{array}{r} 1110 \\ +\ \ 111 \\ \hline \end{array}$

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carry

$$\begin{array}{r} \color{red}{1} \\ 14 \\ +\ \ 7 \\ \hline \color{red}{21} \end{array}$$

Base 2

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Binary Addition and Subtraction

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Base 10	Base 2
1 *carry*	*1*
14	1110
+ 7	+ 111
21	*01*

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14	1110
+ 7	+ 111
21	*101*

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1 carry	*111*
14	1110
+ 7	+ 111
21	*0101*

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1 ← *carry*	*111*
14	1110
+ 7	+ 111
21	*10101*

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Binary Addition and Subtraction

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Base 10	Base 2
1 *carry*	*111*
14	1110
+ 7	+ 111
21	*10101*

-1
14
− 7
7

Binary Addition and Subtraction

- Addition and subtraction in base 2 are performed just like in base 10

Base 10

carry

$$\begin{array}{r} 1 \\ 14 \\ +\ \ 7 \\ \hline 21 \end{array}$$

borrow

$$\begin{array}{r} -1 \\ 14 \\ -\ \ 7 \\ \hline 7 \end{array}$$

Base 2

$$\begin{array}{r} 111 \\ 1110 \\ +\ \ 111 \\ \hline 10101 \end{array}$$

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Base 10	Base 2

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$$\begin{array}{r} \scriptstyle 1 \quad \textit{carry} \\ 14 \\ +\ \ 7 \\ \hline 21 \end{array}$$

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$$\begin{array}{r} 1110 \\ -\ \ 111 \\ \hline \end{array}$$

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- Addition and subtraction in base 2 are performed just like in base 10

Base 10	Base 2
1 carry	*1 1 1*
14	1110
+ 7	+ 111
2 1	*1 0 1 0 1*
-1 borrow	-1-1-1
14	1110
– 7	– 111
0 7	*1 1 1*

Binary Addition and Subtraction

- Addition and subtraction in base 2 are performed just like in base 10

Base 10

carry

$$\begin{array}{r} 1 \\ 14 \\ +\ \ 7 \\ \hline 21 \end{array}$$

borrow

$$\begin{array}{r} -1 \\ 14 \\ -\ \ 7 \\ \hline 07 \end{array}$$

Base 2

$$\begin{array}{r} 111 \\ 1110 \\ +\ \ 111 \\ \hline 10101 \end{array}$$

$$\begin{array}{r} -1-1-1 \\ 1110 \\ -\ \ 111 \\ \hline 0111 \end{array}$$

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- Addition and subtraction in base 2 are performed just like in base 10

Base 10	Base 2	
$$\begin{array}{rr} & 1 \text{ — carry} \\ & 14 \\ + & 7 \\ \hline & 21 \end{array}$$	$$\begin{array}{rr} & 111 \\ & 1110 \\ + & 111 \\ \hline & 10101 \end{array}$$	
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Binary Addition and Subtraction

- Addition and subtraction in base 2 are performed just like in base 10

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1 *carry*	*111*	
14	1110	
+ 7	+ 111	
21	*10101*	
-1 *borrow*	*-1-1-1*	
14	1110	011
− 7	− 111	− 101
07	*0111*	*0*

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1 (*carry*) 14 + 7 *21*	*111* 1110 + 111 *10101*	
-1 (*borrow*) 14 − 7 *07*	*-1-1-1* 1110 − 111 *0111*	011 − 101 *10*

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- Addition and subtraction in base 2 are performed just like in base 10

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$\begin{array}{r} \color{red}{1} \\ 14 \\ +\ \ 7 \\ \hline \color{red}{21} \end{array}$ carry	$\begin{array}{r} \color{red}{111\ } \\ 1110 \\ +\ \ \ 111 \\ \hline \color{red}{10101} \end{array}$	
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14	1110	
+ 7	+ 111	
21	10101	
borrow		
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Base 10	Base 2	
1 (*carry*) 14 + 7 *21*	*111* 1110 + 111 *10101*	*What does this mean?* *-2^3 + 0b110* *We need a way to represent negative numbers!*
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- If we use a fixed number of bits, addition and other operations may produce results outside the range that the output can represent (up to 1 extra bit for addition)
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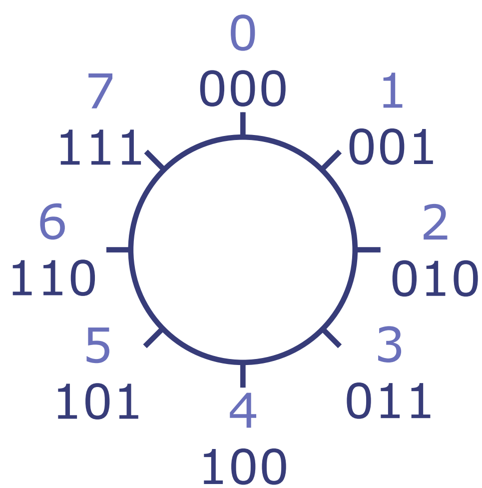

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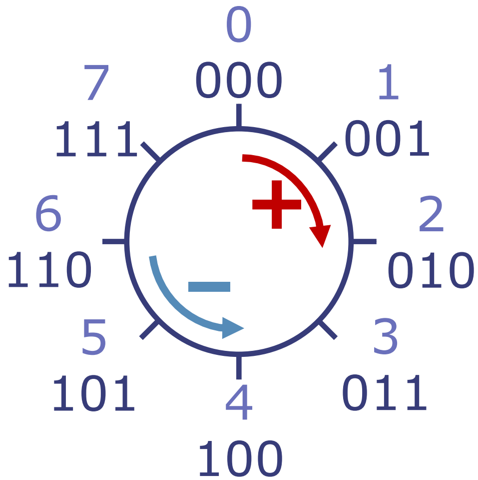

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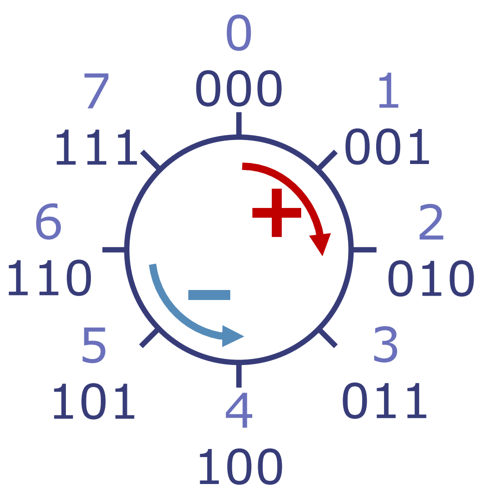
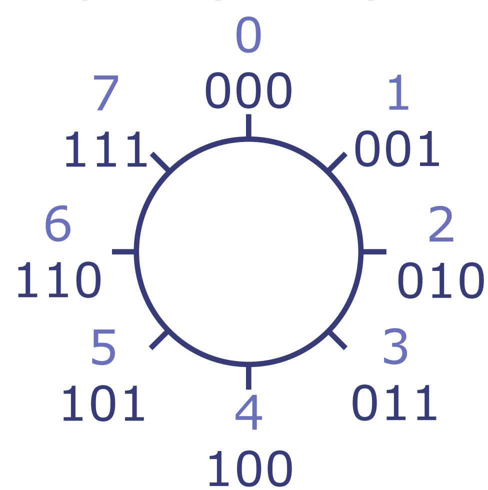


Example: (3 – 5) mod 2^3 ?



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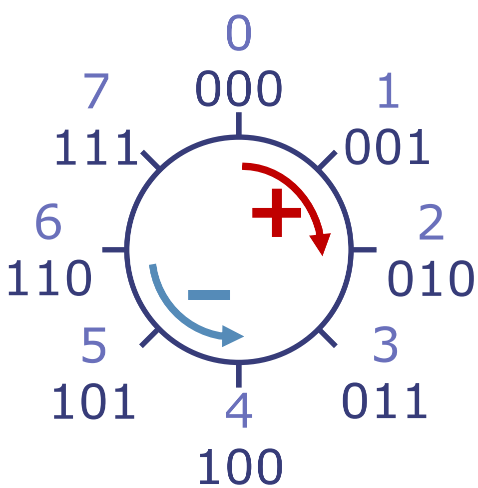

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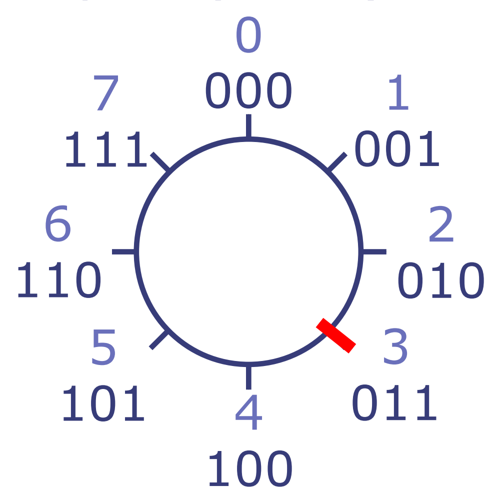

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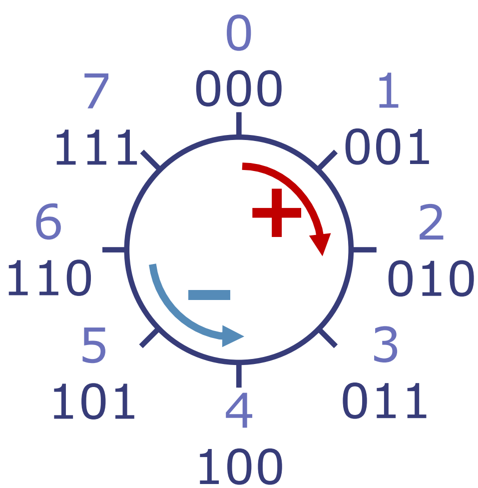

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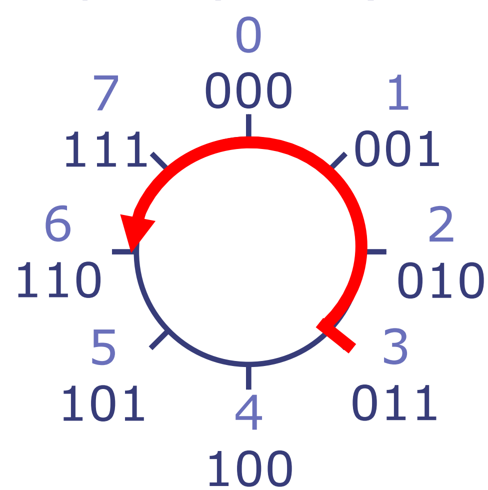

Encoding Negative Integers

Attempt #1: Use a sign-magnitude representation for decimal numbers, encoding the sign of the number (using “+” and “-”) separately from its magnitude (using decimal digits).

We could use the same approach for binary representations:

1	1	1	1	1	1	0	1	0	0	0	0

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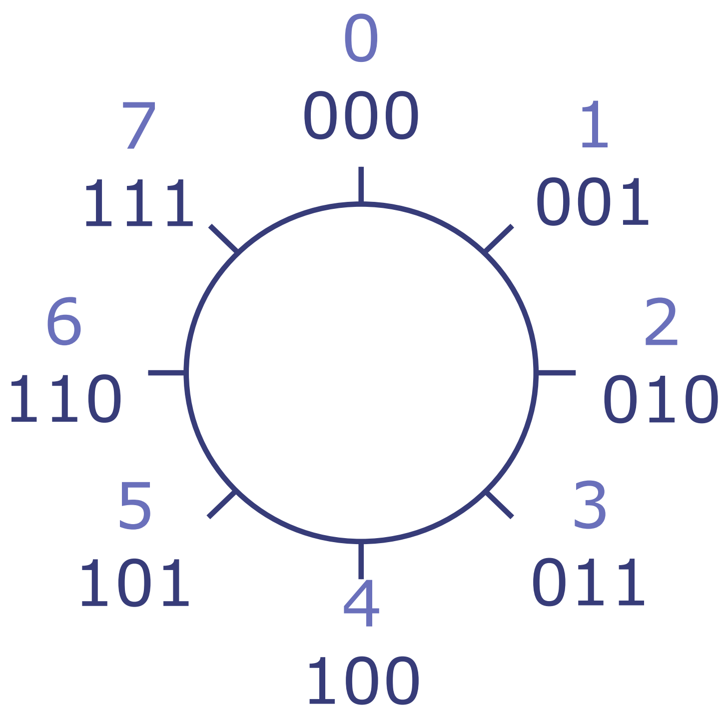
What issues might this encoding have?

Two representations for 0 (+0, -0)
Addition and subtraction use different algorithms and are more complex than with unsigned numbers

Deriving a Better Encoding

Can you simply relabel some of the digits to represent negative numbers while retaining the nice properties of modular arithmetic?



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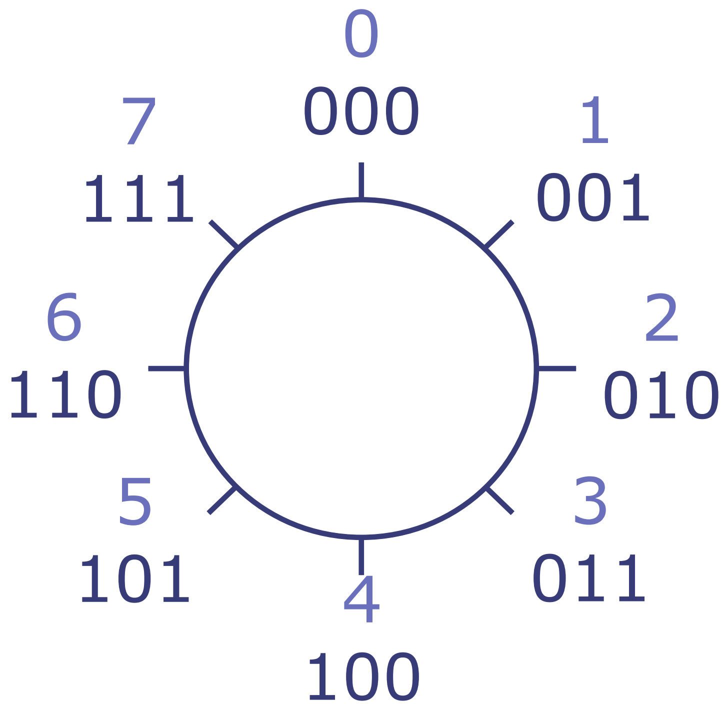

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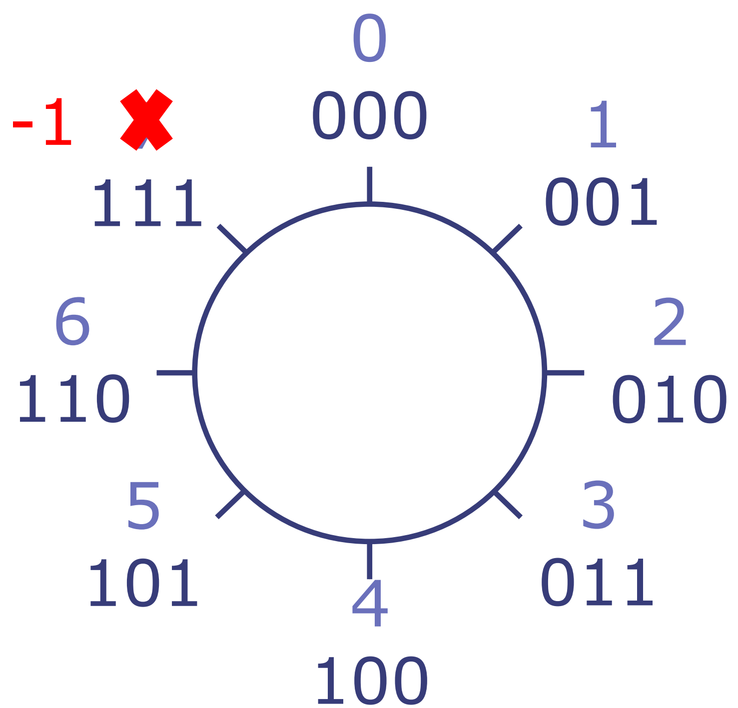

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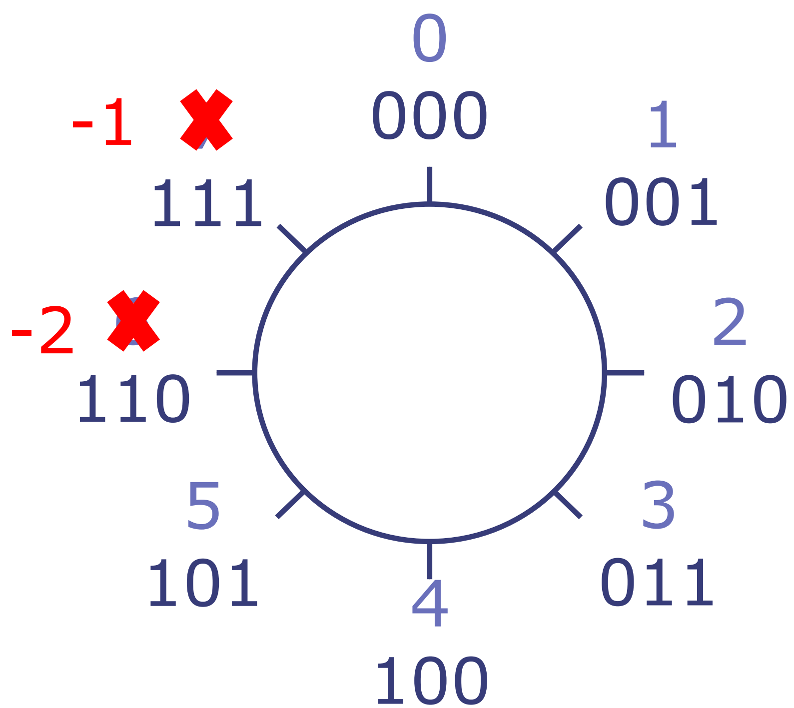

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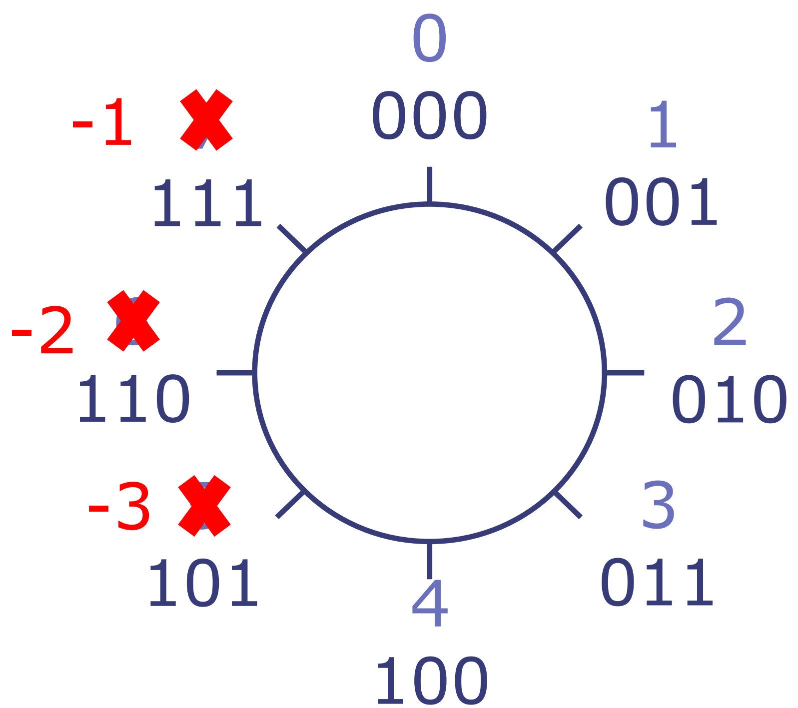

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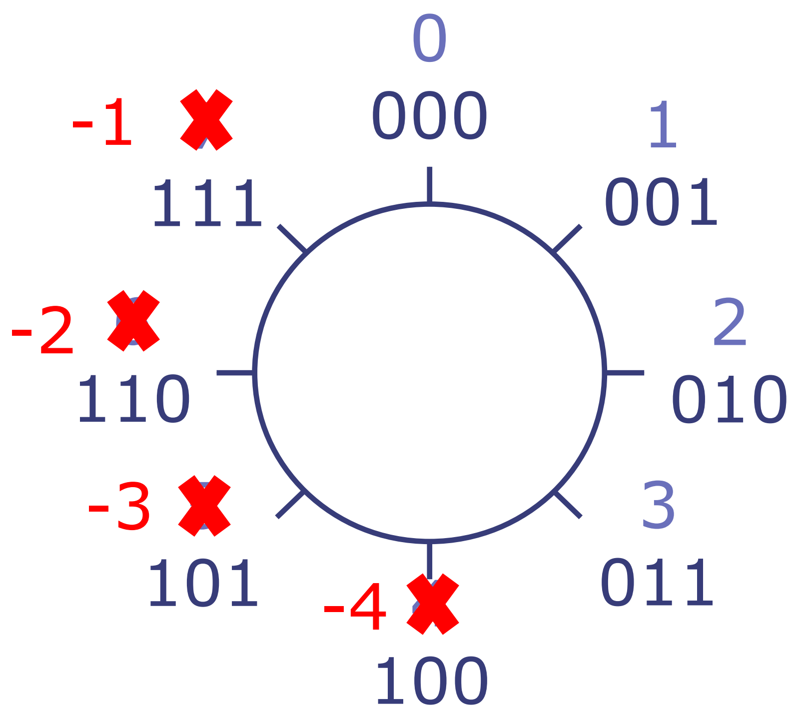

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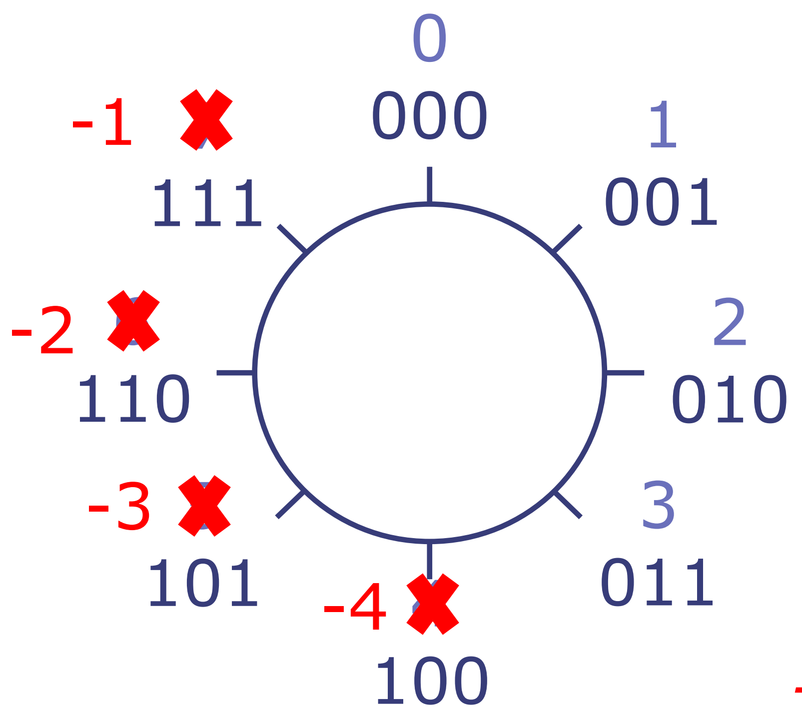

This is called two's complement encoding

Two's Complement Encoding

In two's complement encoding, the high-order bit of the N-bit representation has negative weight:

$$v = -2^{N-1} b_{N-1} + \sum_{i=0}^{N-2} 2^i b_i$$

N bits

-2^{N-1}	2^{N-2}	⋯	⋯	⋯	2^3	2^2	2^1	2^0

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- To compute A-B, we can simply use addition and compute A+(-B)
  - Result: Same circuit can add and subtract!

Two's Complement Example

Compute 3 – 6 using 4-bit 2's complement addition

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- *3: 0011*
-

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- *3: 0011*
- *6: 0110*
-

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$$\begin{array}{r} 0011 \\ +\ 1010 \\ \hline \end{array}$$

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- *3: 0011*
- *6: 0110*
- *-6: 1010*

$$
\begin{array}{r}
0011 \\
+\ \ 1010 \\
\hline
\mathit{1}
\end{array}
$$

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- *3: 0011*
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- *3: 011*
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- *6: 0110*
- *-6: 1010*

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\begin{array}{r}
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0011 \\
+\ \ 1010 \\
\hline
{\color{red}1101}
\end{array}
$$

Compute 3 – 2 using 3-bit 2's complement addition

- *3: 011*
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- *-2: 110*

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- *6: 0110*
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$$\begin{array}{r} \scriptstyle 1 \\ 0011 \\ +\quad 1010 \\ \hline 1101 \end{array}$$

Compute 3 – 2 using 3-bit 2's complement addition

- *3: 011*
- *2: 010*
- *-2: 110*

$$\begin{array}{r} \scriptstyle 11 \\ 011 \\ +\quad 110 \\ \hline 1001 \end{array}$$

Two's Complement Example

Compute 3 – 6 using 4-bit 2's complement addition

- *3: 0011*
- *6: 0110*
- *-6: 1010*

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Keep only last 3 bits

Two’s Complement Example

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What does this 1 mean?

Keep only last 3 bits

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```
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   0011
+  1010
  -----
   1101
```

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```
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  -----
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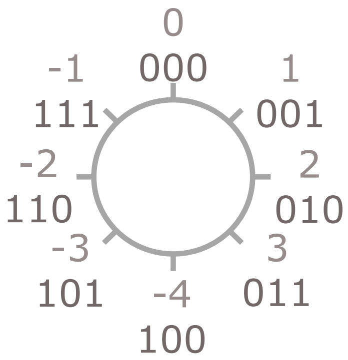
Keep only last 3 bits



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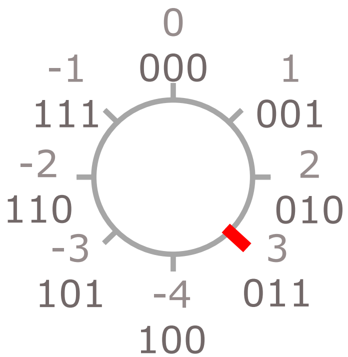

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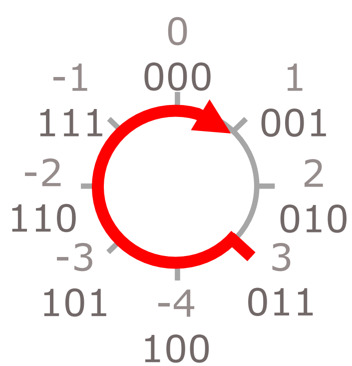

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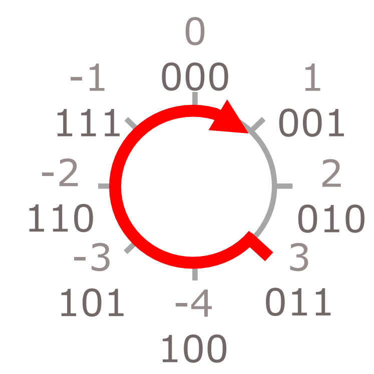

Zero crossing

Summary

- Digital systems encode information using binary for efficiency and reliability

- We can encode unsigned integers using strings of bits; long addition and subtraction are done as in decimal

- Two’s complement allows encoding negative integers while preserving the simplicity of unsigned arithmetic

Thank you!

Next lecture:
Introduction to assembly and RISC-V