Combinational Logic and Introduction to Minispec

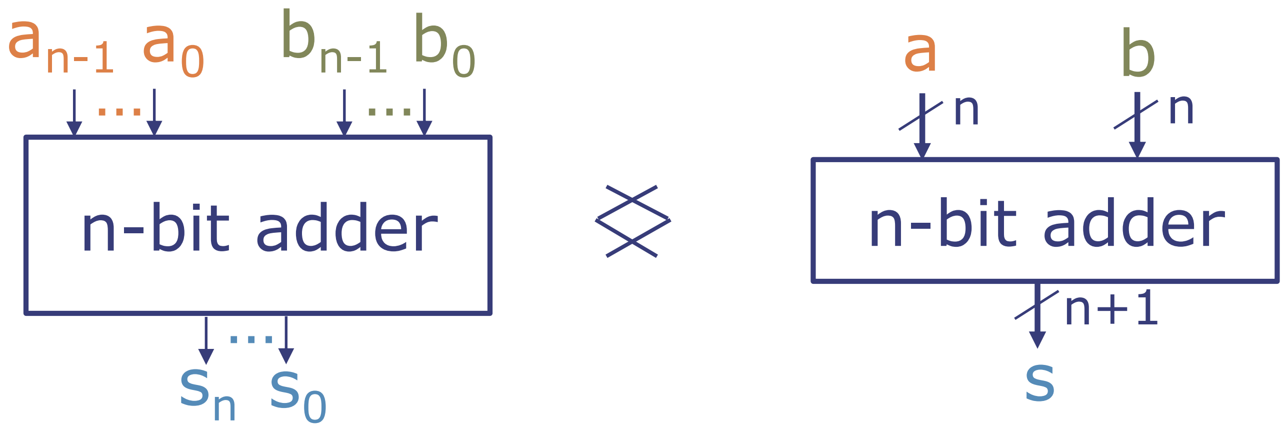
Lecture Goals

- Learn how to design large combinational circuits through three useful examples:
  - Adder
  - Multiplexers
  - Shifter

- Learn how to implement combinational circuits in the Minispec hardware description language (HDL)
  - Design each combinational circuit as a function, which can be simulated or synthesized into gates

Building a Combinational Adder

- Goal: Build a circuit that takes two n-bit inputs a and b and produces (n+1)-bit output s=a+b



- Approach: Implement the binary addition algorithm we have seen (called the standard algorithm)

```
 1110   carry
  1110
+ 0111
------
 10101
```

Formalizing the Standard Algorithm

carry

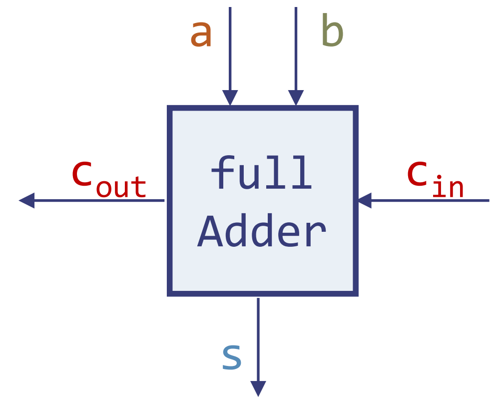
$$\begin{array}{r} 1110 \\ 1110 \\ +\quad 0111 \\ \hline 10101 \end{array} \qquad \Rightarrow \qquad \begin{array}{r} c_4c_3c_2c_1 0 \\ a_3a_2a_1a_0 \\ +\quad b_3b_2b_1b_0 \\ \hline s_4s_3s_2s_1s_0 \end{array}$$

- The i^th step of each addition
  - Takes three 1-bit inputs: a_i, b_i, c_i (carry-in)
  - Produces two 1-bit outputs: s_i, c_{i+1} (carry-out)
  - The 2-bit output $c_{i+1}s_i$ is the binary sum of the three inputs

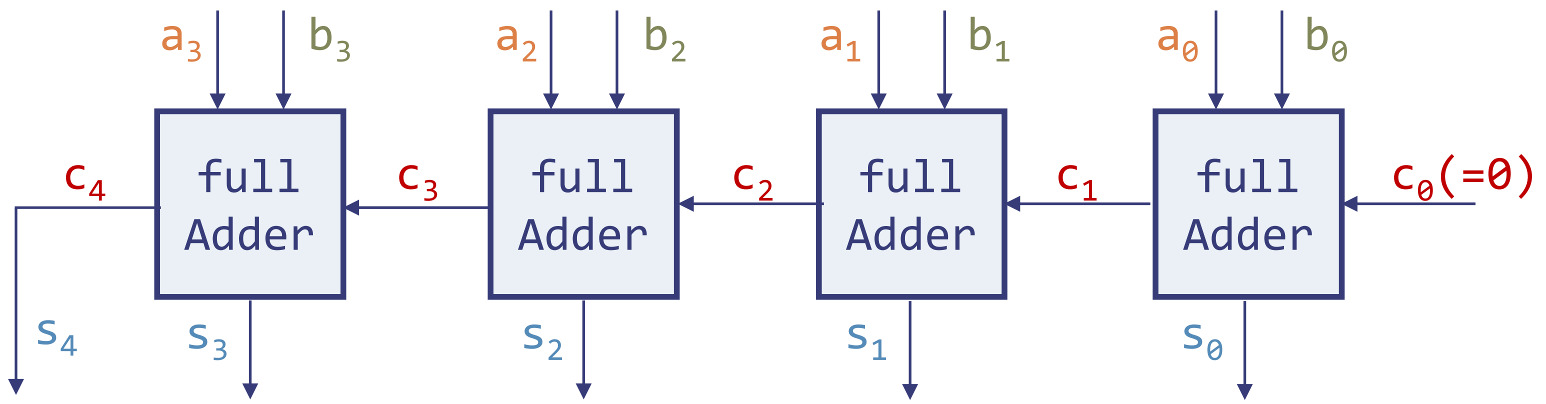
Can you build a circuit that performs a single step with what you've learned so far?

Combinational Logic for an Adder

- First, build a full adder (FA), which
  - Adds three one-bit numbers: a, b, and carry-in
  - Produces a sum bit and a carry-out bit



- Then, cascade FAs to perform binary addition

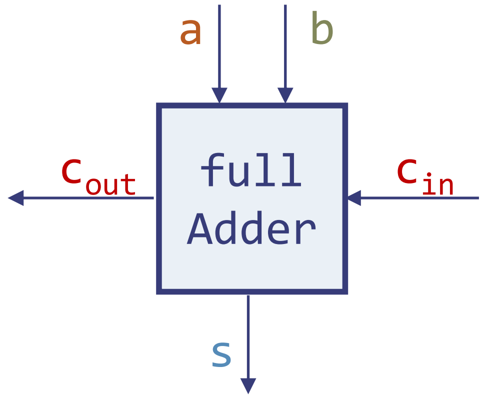


- Result: A ripple-carry adder (simple but slow)

Deriving the Full Adder

Truth table

a	b	c_{in}	c_{out}	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Boolean expressions

$$s = a \oplus b \oplus c_{in}$$

$$c_{out} = a \cdot b + a \cdot c_{in} + b \cdot c_{in}$$

Describing a 32-bit Adder
alternatives

- Truth table with 2^{64} rows and 33 columns
- 32 sets of Boolean equations, where each set describes a FA
- Use some ad-hoc notation to describe recurrences

$$\left.\begin{array}{l} s_k = a_k \oplus b_k \oplus c_k \\ c_{k+1} = a_k \cdot b_k + a_k \cdot c_k + b_k \cdot c_k \end{array}\right\} \quad 0 \le k \le 31$$

- Circuit diagrams: tedious to draw, error-prone

- A hardware description language (HDL), i.e., a programming language specialized to describe hardware
  - Precisely specify the structure and behavior of digital circuits
  - Designs can be automatically simulated or synthesized to hardware
  - Enables building hardware with same principles used to build software (write and compose simple, reusable building blocks)
  - Uses a familiar syntax (functions, variables, control-flow statements, etc.)

Introduction to Minispec

A simple HDL based on Bluespec

Combinational Logic as Functions

- In Minispec, combinational circuits are described using functions

Return type

Function name

Input arguments

```
function Bool inv(Bool x);
    Bool result = !x;
    return result;
endfunction
```

Statement(s), including a return statement

- All values have a fixed type, which is known statically (e.g., result is of type Bool)
- Note: Types Start With An Uppercase Letter, variable and function names are lowercase

Bool Type and Operations

- Values of type Bool can be True or False
- Bool supports Boolean and comparison operations:

```
Bool a = True;
Bool b = False;

Bool x = !a;      // False since a == True
Bool y = a && b; // False since b == False
Bool z = a || b; // True since a == True

Bool n = a != b; // True; equivalent to XOR
Bool e = a == b; // False; equivalent to XNOR
```

- Bool is the simplest type, but working with many single-bit values is tedious
  - Need a type that represents multi-bit values!

Bit#(n) Type and Operations

- Bit#(n) represents an n-bit value
- Bit#(n) supports the following basic operations:
  - Bitwise logical: ~ (negation), & (AND), | (OR), ^ (XOR)

```
Bit#(4) a = 4'b0011;  // 4-bit binary 3
Bit#(4) b = 4'b0101;  // 4-bit binary 5
Bit#(4) x = ~a;       // 4'b1100
Bit#(4) y = a & b;    // 4'b0001
Bit#(4) z = a ^ b;    // 4'b0110
```

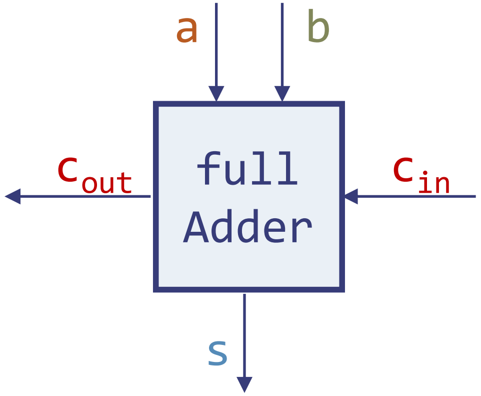
  - Bit selection

```
Bit#(1) l = a[0];   // 1'b1 (least significant)
Bit#(3) m = a[3:1]; // 3'b001
```

  - Concatenation

```
Bit#(8) c = {a, b}; // 8'b00110101
```

Full Adder in Minispec

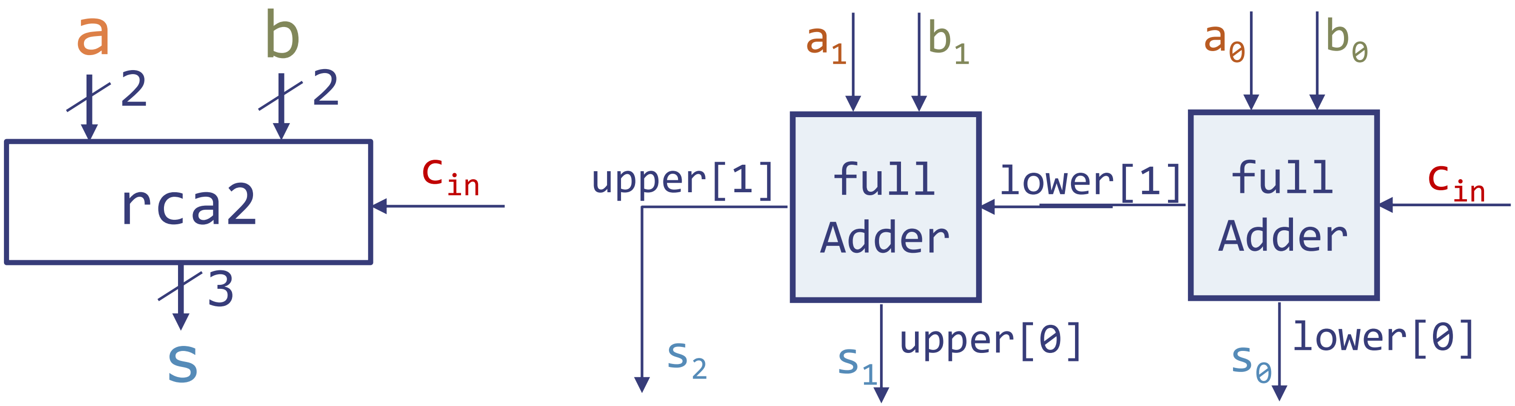


$$s = a \oplus b \oplus c_{in}$$

$$c_{out} = a \cdot b + a \cdot c_{in} + b \cdot c_{in}$$

```
function Bit#(2) fullAdder(Bit#(1) a, Bit#(1) b, Bit#(1) cin);
    Bit#(1) s = a ^ b ^ cin;
    Bit#(1) cout = (a & b) | (a & cin) | (b & cin);
    return {cout, s};
endfunction
```

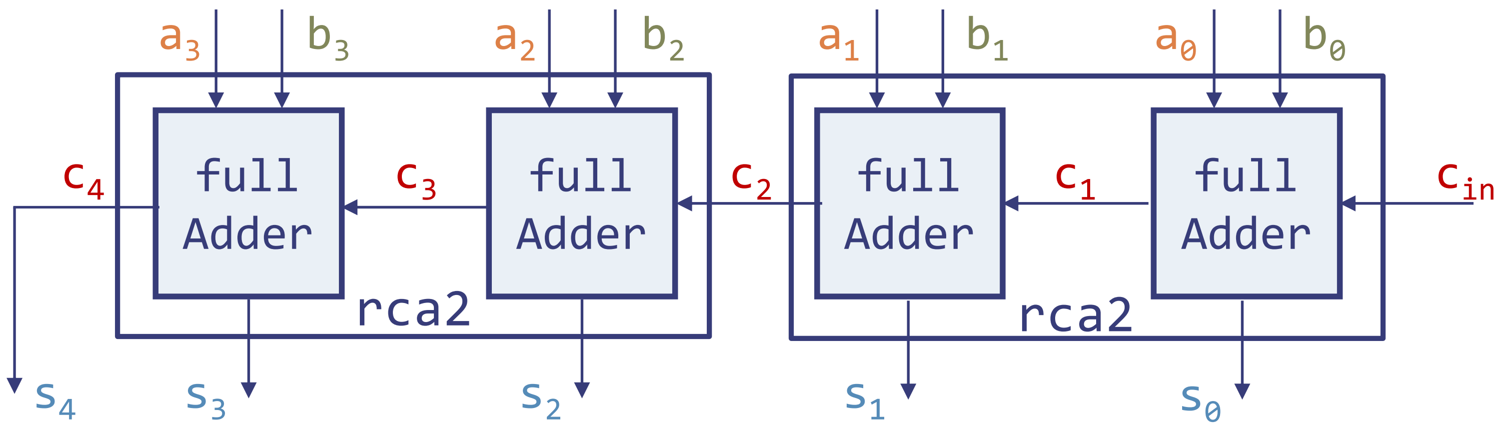
2-bit Ripple-Carry Adder




```
function Bit#(3) rca2(Bit#(2) a, Bit#(2) b, Bit#(1) cin);
    Bit#(2) lower = fullAdder(a[0], b[0], cin);
    Bit#(2) upper = fullAdder(a[1], b[1], lower[1]);
    return {upper, lower[0]};
endfunction
```

- Functions are inlined: Each function call creates a new instance (copy) of the called circuit
  - Allows composing simple circuits to build larger ones

4-bit Ripple-Carry Adder



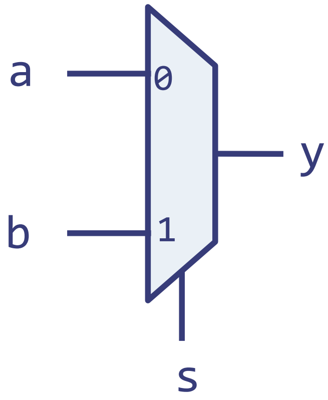

```
function Bit#(5) rca4(Bit#(4) a, Bit#(4) b, Bit#(1) cin);
    Bit#(3) lower = rca2(a[1:0], b[1:0], cin);
    Bit#(3) upper = rca2(a[3:2], b[3:2], lower[2]);
    return {upper, lower[1:0]};
endfunction
```

- Composing functions lets us build larger circuits, but writing very large circuits this way is tedious
  - Next lecture: Writing an n-bit adder in a single function

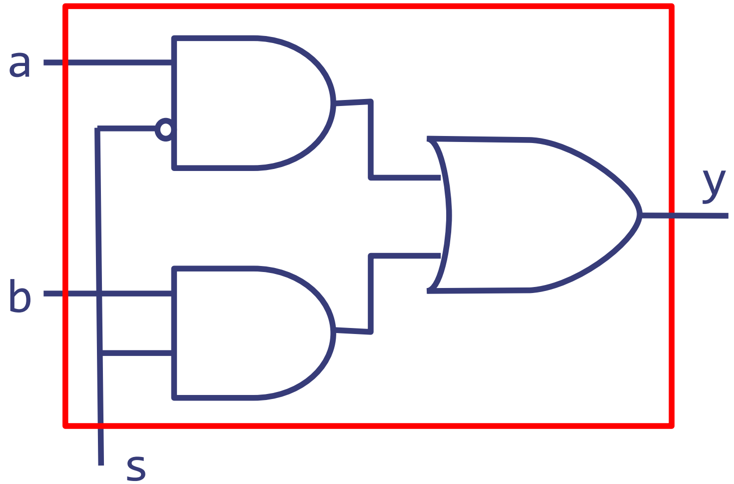
Multiplexers

2-way Multiplexer

- A 2-way multiplexer or mux selects between two inputs a and b based on a single-bit input s (select input)



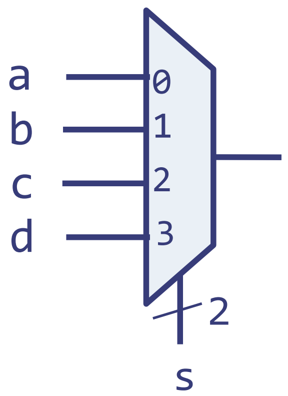
- Gate-level implementation: $y = a \cdot \overline{s} + b \cdot s$



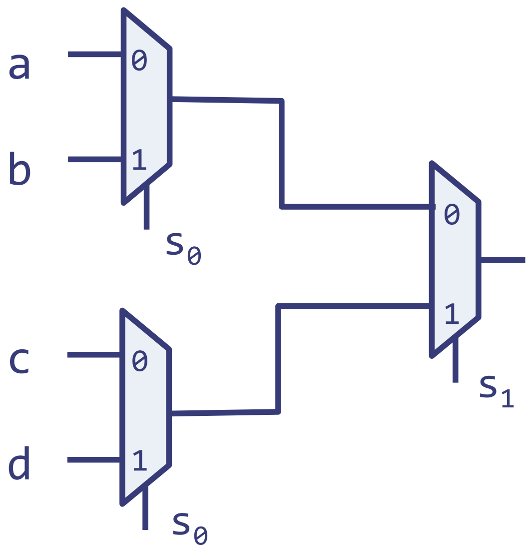
  - If *a* and *b* are *n*-bit wide then this structure is replicated *n* times; s is the same input for all the replicated structures

4-way Multiplexer

- A 4-way multiplexer selects between four inputs based on the value of a 2-bit input s



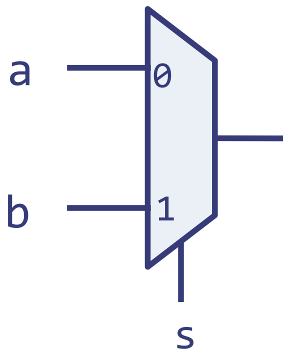
  - Typically implemented using 2-way multiplexers



- An n-way multiplexer can be implemented with a tree of n-1 2-way multiplexers

Multiplexers in Minispec

- 2-way mux → Conditional operator



Minispec

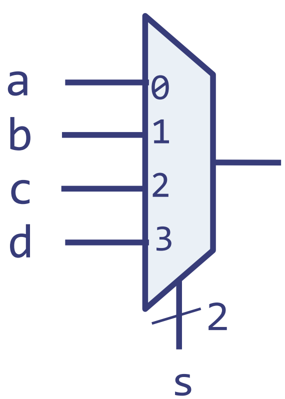
```
s? b : a
```

Python

```
b if s else a
```

s has type Bool; True is treated as 1 and False as 0

- N-way mux → Case expression



Minispec

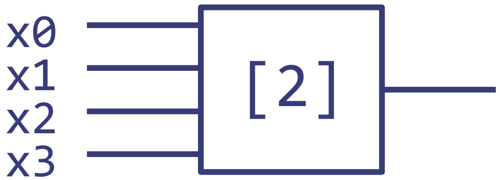
```
case (s)
    0 :  a;
    1 :  b;
    2 :  c;
    3 :  d;
endcase
```

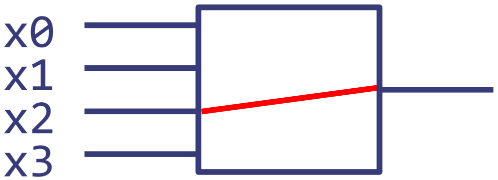
s has type Bit#(2)

Selecting a Wire: x[i]

assume x is 4 bits wide

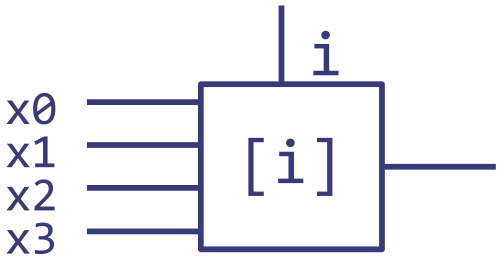
- Constant selector: e.g., x[2]

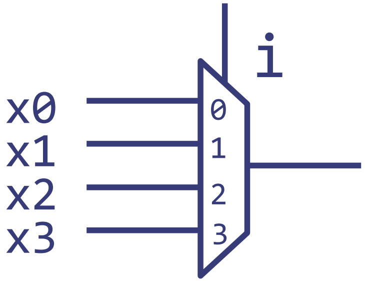




no hardware; x[2] is just the name of a wire

- Dynamic selector: x[i]

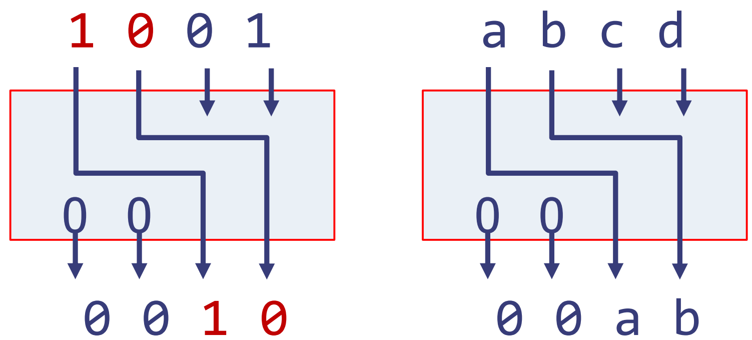




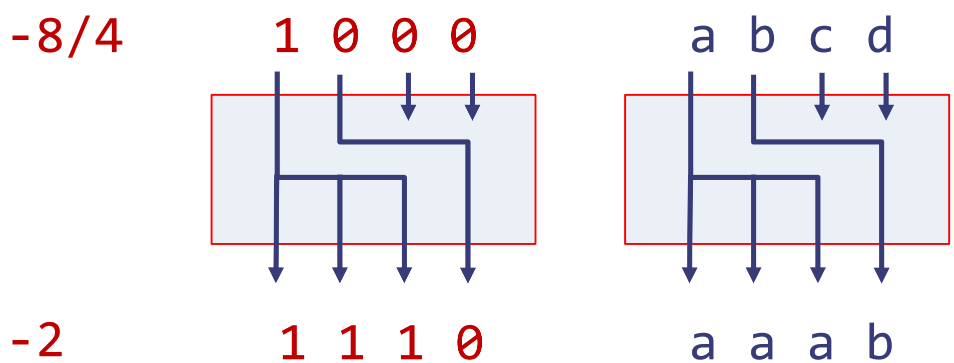
4-way mux

Shift operators

Fixed-size shifts



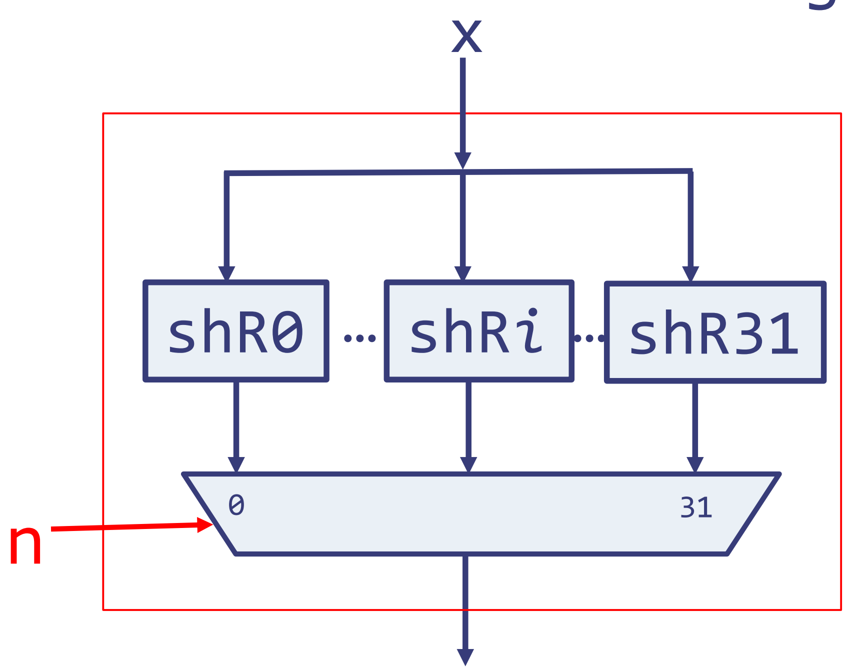
- Fixed size shift operation is cheap in hardware
  - Just wire the circuit appropriately
- Arithmetic shifts are similar



useful for multiplication and division by 2^n

Logical right shift by *n*

- Suppose we want to build a shifter that right-shifts a value x by *n* where n is between 0 and 31
- One way to do this is by selecting from 32 different fixed-size shifters using a mux



How many 2-way one-bit muxes are needed to implement this structure?

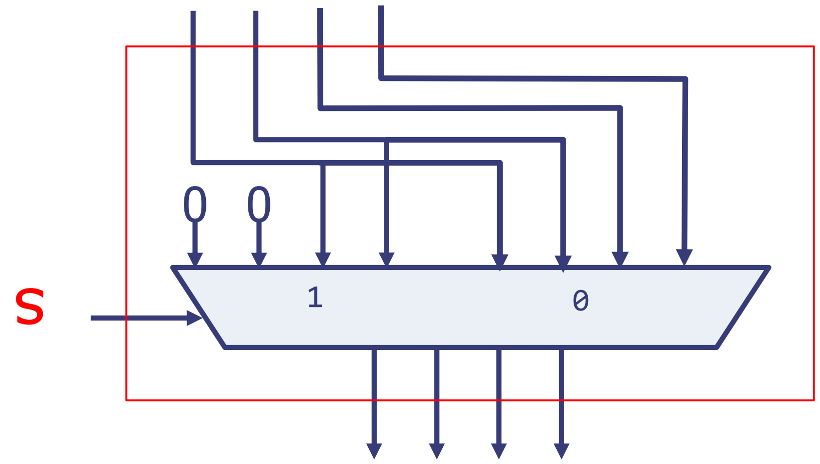
n*(n-1)

Can we do better?

Barrel shifter
An efficient circuit to perform logical right shift by *n*

- Shift by *n* can be broken down into log *n* steps of fixed-length shifts of size 1, 2, 4, …
  - For example, we can perform shift 5 (=4+1) by doing shifts of size 4 and 1
  - Thus, 8’b01100111 shift 5 can be performed in two steps:
    - $8'b01100111 \xRightarrow[\text{shift 4}]{} 8'b00000110 \xRightarrow[\text{shift 1}]{} 8'b00000011$
- For a 32-bit number, a 5-bit *n* can specify all the needed shifts
  - $3_{10} = 00011_2$, $5_{10} = 00101_2$, $21_{10} = 10101_2$
  - The bit encoding of *n* tells us which shifters are needed; if the value of the i^{th} (least significant) bit is 1 then we need to shift by 2^i bits

Conditional operation: Shift versus no-shift

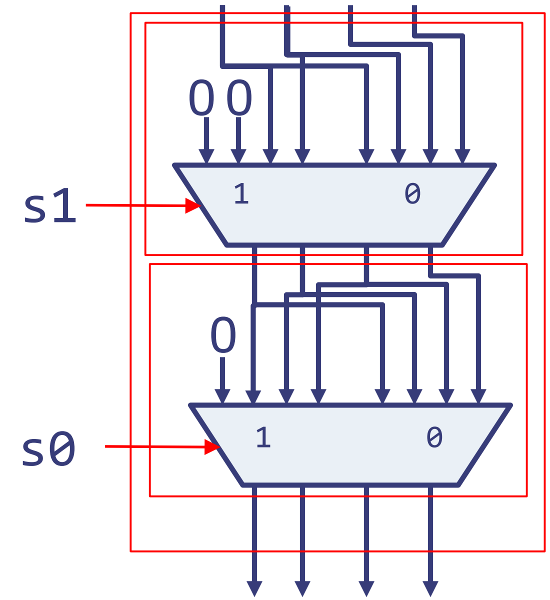


- We need a mux to select the appropriate wires: if s is 1 the mux selects the wires on the left, otherwise it selects the wires on the right

```
(s==0)?{a,b,c,d}:{2'b0,a,b};
```

Barrel shifter implementation

- A barrel shifter for an n-bit number uses a cascade of log *n* muxes, each performing a conditional fixed-size shift of sizes 1, 2, 4, …



- Example: A barrel shifter for 4-bit numbers can be expressed as two conditional expressions:

```
function Bit#(4) barrelShifter(Bit#(4) x, Bit#(2) s);
    Bit#(4) r1 = (s[1] == 0)?  x : {2'b00, x[3:2]};
    Bit#(4) r0 = (s[0] == 0)? r1 : {1'b0, r1[3:0]};
    return r0;
endfunction
```

Thank you!

Next lecture:
Complex combinational circuits
and advanced Minispec