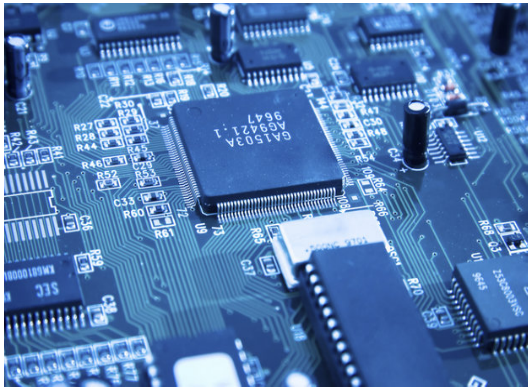
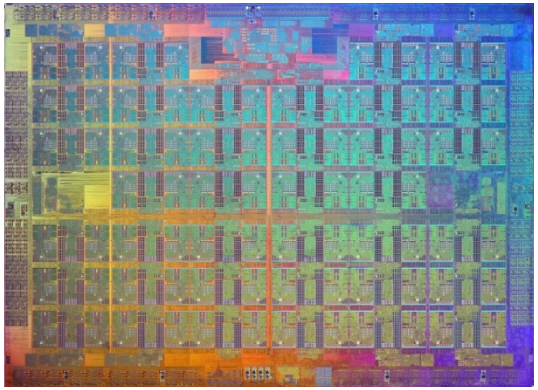
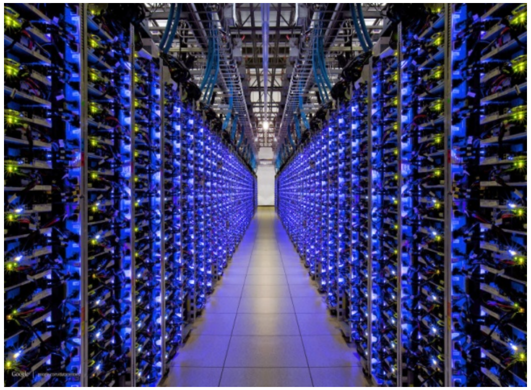
Welcome to 6.004!

Computation Structures







Fall 2019

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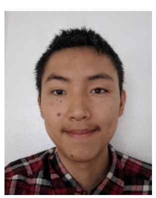

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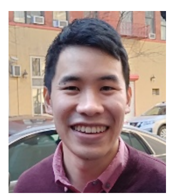

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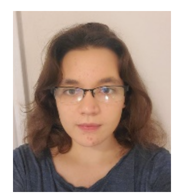

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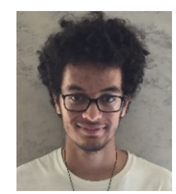
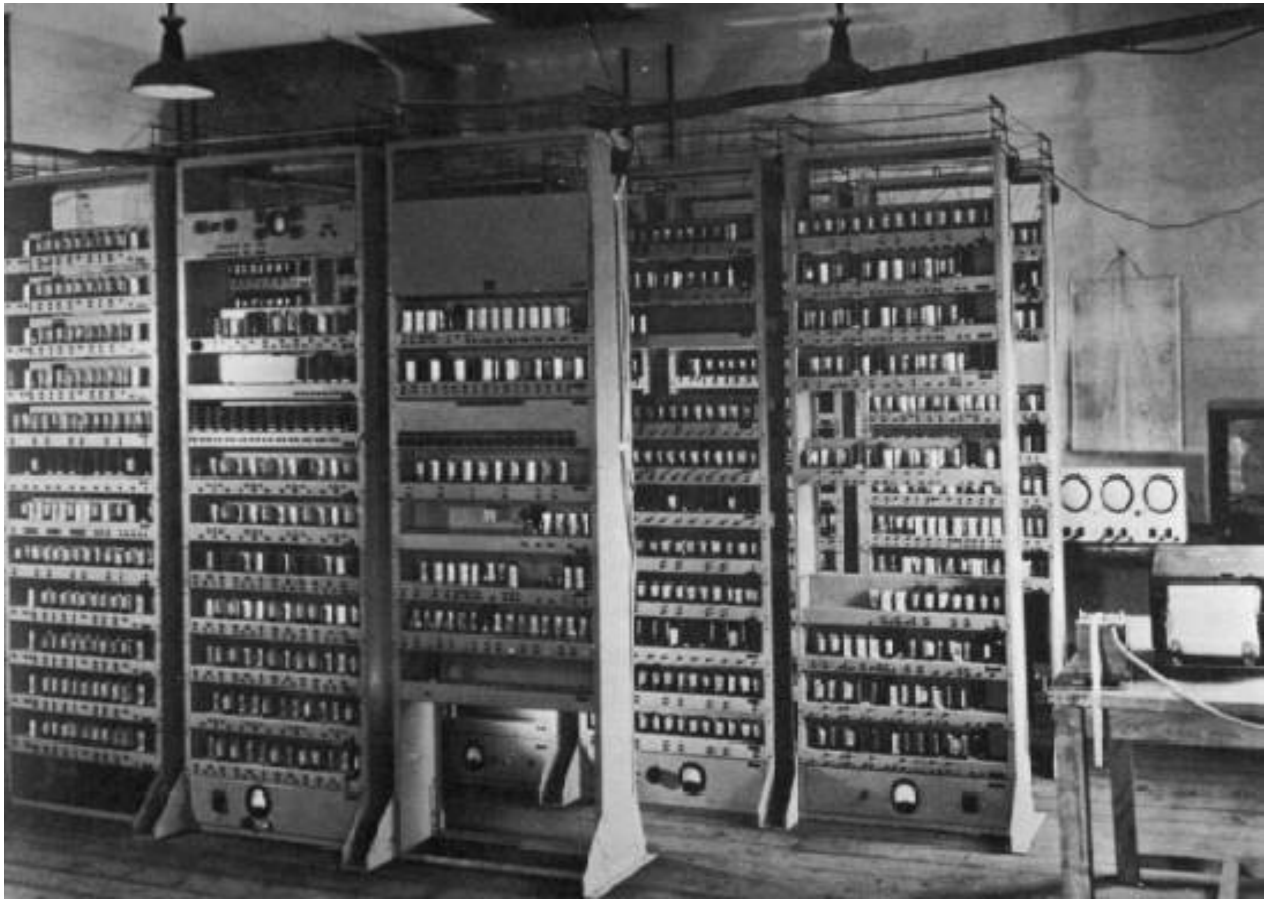


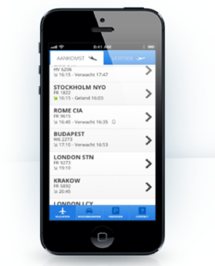
Kendall Garner

Computing Devices Then…



ENIAC, 1943   30 tons, 200KW, ~1000 ops/sec

Computing Devices Now



Typical 2019 laptop
1kg, 10W, 10 billion ops/s

An Introduction to the Digital World

Computer programs

→ *Virtual machines*

Computer systems
Operating systems, virtual memory, I/O

→ *Instruction set + memory*

Computer architecture
Processors, caches, pipelining

→ *Digital circuits*

Digital design
Combinational and sequential circuits

→ *Bits, Logic gates*

Devices
Materials
Atoms

The Power of Engineering Abstractions

Good abstractions let us reason about behavior while shielding us from the details of the implementation.

Corollary: implementation technologies can evolve while preserving the engineering investment at higher levels.

Leads to hierarchical design:

- Limited complexity at each level ⇒ shorten design time, easier to verify
- Reusable building blocks

Virtual machines

Instruction set + memory

Digital circuits

Bits, Logic gates

Course Outline

- Module 1: Assembly language
  - From high-level programming languages to the language of the computer
- Module 2: Digital design
  - Combinational and sequential circuits
- Module 3: Computer architecture
  - Simple and pipelined processors
  - Caches and the memory hierarchy
- Module 4: Computer systems
  - Operating system and virtual memory
  - Parallelism and synchronization

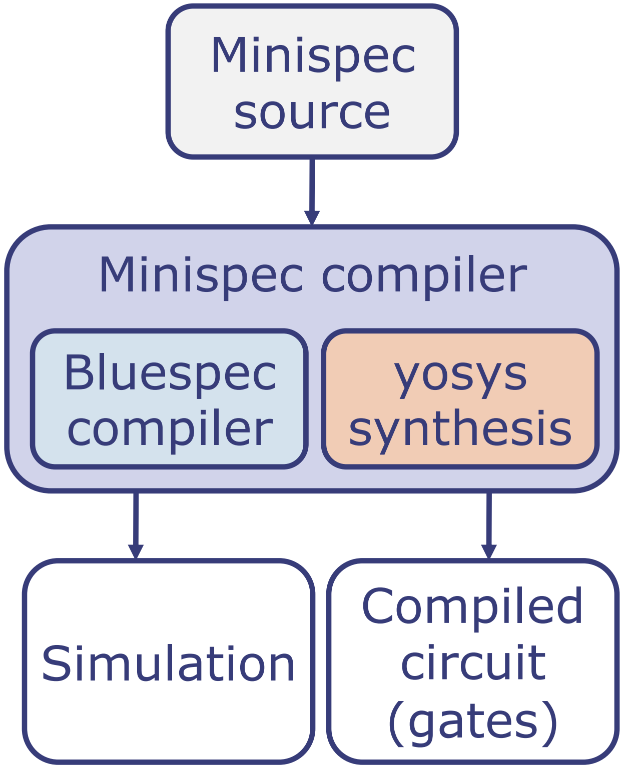
Our Focus: Programmable General-Purpose Processors

- Microprocessors are the basic building block of computer systems
  - Understanding them is crucial even if you do not plan to work as a hardware designer
- Microprocessors are the most sophisticated digital systems that exist today
  - Understanding them will help you design all kinds of hardware

By the end of the term you will have designed a simple processor from scratch!

We Rely on Modern Design Tools

- We will use RISC-V, a simple and modern instruction set
- We will design hardware using Minispec, a new hardware description language built for 6.004
  - Based on Bluespec, but heavily simplified



Course Mechanics

- 2 lectures/week: handouts, videos, and reference materials on website
- 2 recitations/week: work through tutorial problems using skills and concepts from previous lecture
- 7 mandatory lab exercises
  - Online submission + check-off meetings in lab
  - Due throughout the term (7 free late days meant to give you flexibility, cover short illnesses, etc.; see website)
- One open-ended design project
  - Due at the end of the term
- 3 quizzes: Oct 17, Nov 14, Dec 5 (7:30-9:30pm)
  - If you have a conflict, contact us to schedule a makeup

Recitation Mechanics

- 12 recitation sections on Wed & Fri
  - If you have a conflict with your assigned section, choose a different one—no need to let us know
- Recitation attendance is mandatory (worth 5% of your grade)
  - Everyone has 4 excused absences
- Recitations will review lecture material and problems associated with each lecture
  - We recommend you work on the problems before recitation
  - We will post solutions after recitation

Grading

- 80 points from labs,
  20 points from design project,
  90 points from quizzes,
  10 points from recitation attendance

- Fixed grade cutoffs:
  - A: Points >= 165
  - B: Points >= 145
  - C: Points >= 125
  - F: Points < 125 or not all labs complete

Online and Offline Resources

- The course website has up-to-date information, handouts, and references to supplemental reading: http://6004.mit.edu
- We use Piazza extensively
  - Fastest way to get your questions answered
  - All course announcements are made on Piazza
- We will hold regular office hours in the lab (room 32-083) to help you with lab assignments, infrastructure, and any other questions

*32-083 Combination Lock:*___________________

We Want Your Feedback!

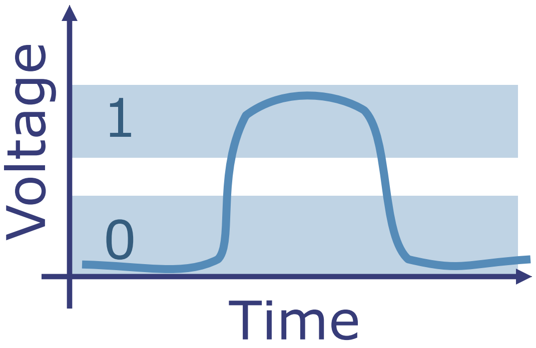
- Your input is crucial to fine-tune this offering of the course and improve future versions
- Periodic informal surveys
- Any time: Email us or post on Piazza

Binary Number Encoding and Arithmetic

Digital Information Encoding

- Digital systems represent and process information using discrete symbols or digits

- These are typically binary digits (bits): 0 and 1



- We can implement operations like +, >, AND, etc. on binary numbers in hardware very efficiently

Encoding Positive Integers

It is straightforward to encode positive integers as a sequence of bits. Each bit is assigned a weight. Ordered from right to left, these weights are increasing powers of 2. The value of an N-bit number is given by the following formula:

$$v = \sum_{i=0}^{N-1} 2^i b_i$$

2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	1	1	1	1	0	1	0	0	0	0

What value does 011111010000 encode?

$V = 0*2^{11} + 1*2^{10} + 1*2^9 + \ldots$

$= 1024 + 512 + 256 + 128 + 64 + 16 = 2000$

Smallest number? 0

Largest number? 2^N-1

Hexadecimal Notation

Long strings of bits are tedious and error-prone to transcribe, so we often use a higher-radix notation, choosing the radix so that it's simple to recover the original bit string.

A popular choice is to transcribe numbers in base-16, called hexadecimal. Each group of 4 adjacent bits is represented as a single hexadecimal digit.

Hexadecimal - base 16

0000	0	1000	8
0001	1	1001	9
0010	2	1010	A
0011	3	1011	B
0100	4	1100	C
0101	5	1101	D
0110	6	1110	E
0111	7	1111	F

2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	1	1	1	1	0	1	0	0	0	0

$$\underbrace{0111}_{7}\;\underbrace{1101}_{D}\;\underbrace{0000}_{0}$$

0b011111010000 = 0x7D0

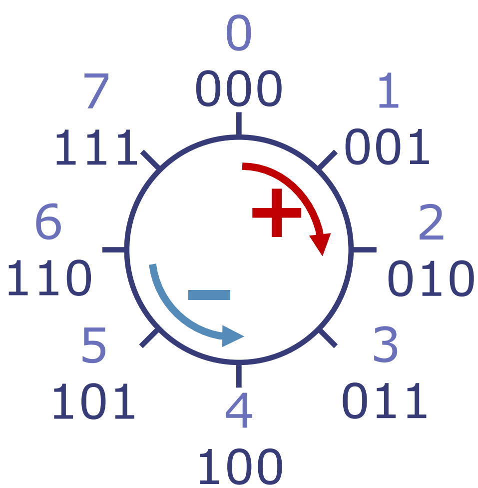
Binary Addition and Subtraction

- Addition and subtraction in base 2 are performed just like in base 10

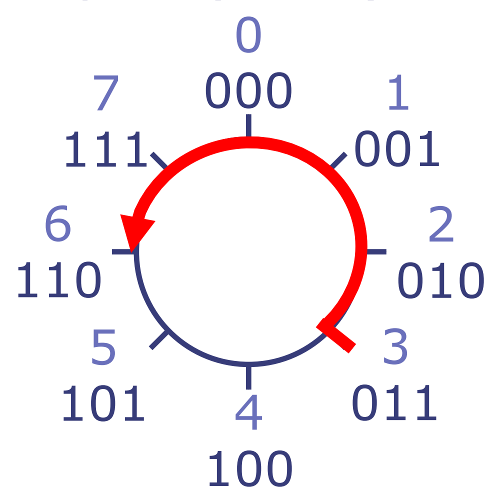
Base 10	Base 2	
1 *carry*	*111*	*What does this mean?*
14	1110	$-2^3 + 0b110$
+ 7	+ 111	*We need a way to represent negative numbers!*
21	*10101*	
-1 *borrow*	*-1-1-1*	*-1*
14	1110	011
− 7	− 111	− 101
07	*0111*	*???110*

Binary Modular Arithmetic

- If we use a fixed number of bits, addition and other operations may produce results outside the range that the output can represent (up to 1 extra bit for addition)
  - This is known as an overflow
- Common approach: Ignore the extra bit
  - Gives rise to modular arithmetic: With N-bit numbers, equivalent to following all operations with mod 2^N
  - Visually, numbers "wrap around":



Example: (3 – 5) mod 2^3 ?



Encoding Negative Integers

Attempt #1: Use a sign-magnitude representation for decimal numbers, encoding the sign of the number (using “+” and “-”) separately from its magnitude (using decimal digits).

We could use the same approach for binary representations:

1	1	1	1	1	1	0	1	0	0	0	0

“0” for “+”
“1” for “-”

=-2000

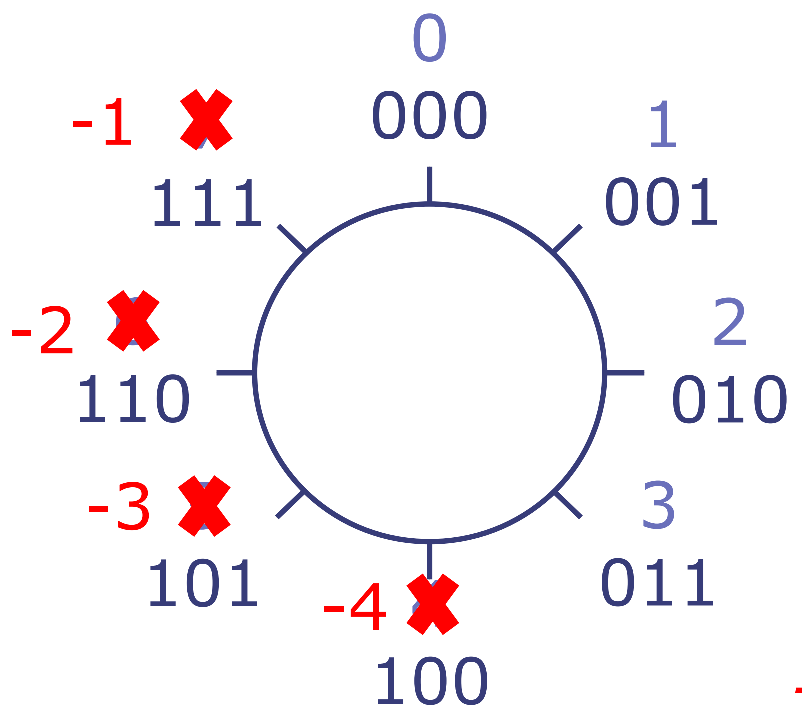
What issues might this encoding have?

Two representations for 0 (+0, -0)
Addition and subtraction use different algorithms and are more complex than with unsigned numbers

Deriving a Better Encoding

Can you simply relabel some of the digits to represent negative numbers while retaining the nice properties of modular arithmetic?

Yes!



This is called two's complement encoding

Two's Complement Encoding

In two's complement encoding, the high-order bit of the N-bit representation has negative weight:

$$v = -2^{N-1}b_{N-1} + \sum_{i=0}^{N-2} 2^i b_i$$

N bits

-2^{N-1}	2^{N-2}	⋯	⋯	⋯	2^3	2^2	2^1	2^0

- Negative numbers have "1" in the high-order bit
- *Most negative number?* 10…0000 -2^{N-1}
- *Most positive number?* 01…1111 $+2^{N-1} - 1$
- *If all bits are 1?* 11…1111 -1

Two's Complement and Arithmetic

- To negate a number (i.e., compute –A given A), we invert all the bits and add one

  Why does this work?

$$-A + A = 0 = -1 + 1$$

$$-A = \underbrace{(-1 - A)}_{} + 1$$

$$\begin{array}{r} 1 \ldots 1\ 1 \\ -\ A_{n-1} \ldots A_1 A_0 \\ \hline \overline{A_{n-1}} \ldots \overline{A_1}\,\overline{A_0} \end{array}$$

- To compute A-B, we can simply use addition and compute A+(-B)
  - Result: Same circuit can add and subtract!

Two's Complement Example

Compute 3 – 6 using 4-bit 2's complement addition

- *3: 0011*
- *6: 0110*
- *-6: 1010*

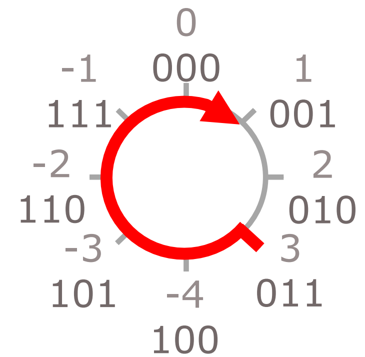
$$
\begin{array}{r}
\textit{1} \\
0011 \\
+\ \ 1010 \\
\hline
\textit{1101}
\end{array}
$$

Compute 3 – 2 using 3-bit 2's complement addition

- *3: 011*
- *2: 010*
- *-2: 110*

$$
\begin{array}{r}
\textit{11} \\
011 \\
+\ \ 110 \\
\hline
\textit{1}\underbrace{\textit{001}}_{\text{Keep only last 3 bits}}
\end{array}
$$

What does this 1 mean?



Zero crossing

Summary

- Digital systems encode information using binary for efficiency and reliability

- We can encode unsigned integers using strings of bits; long addition and subtraction are done as in decimal

- Two’s complement allows encoding negative integers while preserving the simplicity of unsigned arithmetic

Thank you!

Next lecture:
Introduction to assembly and RISC-V