Taller Matemáticas - mayo

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1. Productos Notables

a. $\left(\frac{2}{7}mn+\frac{1}{3}np\right)^3$

Solución

Es una suma elevada a la 3 (o de grado 3) de la forma

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

$$donde\begin{cases} a=\frac{2}{7}mn \\ \\ b=\frac{1}{3}np \end{cases}$$

$$\begin{aligned}
\left(\frac{2}{7}mn+\frac{1}{3}np\right)^3 &= \left(\frac{2}{7}mn\right)^3 &+& \quad 3\left(\frac{2}{7}mn\right)^2\left(\frac{1}{3}np\right) &+& \quad 3\left(\frac{2}{7}mn\right)\left(\frac{1}{3}np\right)^2 &+& \quad \left(\frac{1}{3}np\right)^3 \\
&= \left(\frac{2^3}{7^3}m^3n^3\right) &+& \quad 3\left(\frac{2^2}{7^2}m^2n^2\right)\left(\frac{1}{3}np\right) &+& \quad 3\left(\frac{2}{7}mn\right)\left(\frac{1^2}{3^2}n^2p^2\right) &+& \quad \left(\frac{1^3}{3^3}n^3p^3\right) \\
&= \left(\frac{2\cdot2\cdot2}{7\cdot7\cdot7}m^3n^3\right) &+& \quad 3\left(\frac{2\cdot2}{7\cdot7}m^2n^2\right)\left(\frac{1}{3}np\right) &+& \quad 3\left(\frac{2}{7}mn\right)\left(\frac{1\cdot1}{3\cdot3}n^2p^2\right) &+& \quad \left(\frac{1\cdot1\cdot1}{3\cdot3\cdot3}n^3p^3\right) \\
&= \left(\frac{8}{343}m^3n^3\right) &+& \quad 3\left(\frac{4}{49}m^2n^2\right)\left(\frac{1}{3}np\right) &+& \quad 3\left(\frac{2}{7}mn\right)\left(\frac{1}{9}n^2p^2\right) &+& \quad \left(\frac{1}{27}n^3p^3\right) \\
&= \frac{8}{343}m^3n^3 &+& \quad \frac{\cancel{3}\cdot4}{49}m^2n^2\left(\frac{1}{\cancel{3}}np\right) &+& \quad \frac{\cancel{3}\cdot2}{7}mn\left(\frac{1}{\cancel{9}}n^2p^2\right) &+& \quad \frac{1}{27}n^3p^3 \\
&= \frac{8}{343}m^3n^3 &+& \quad \frac{4}{49}m^2n^2np &+& \quad \frac{2}{7}mn\left(\frac{1}{3}n^2p^2\right) &+& \quad \frac{1}{27}n^3p^3 \\
&= \frac{8}{343}m^3n^3 &+& \quad \frac{4}{49}m^2n^2np &+& \quad \frac{2}{21}mnn^2p^2 &+& \quad \frac{1}{27}n^3p^3 \\
&= \frac{8}{343}m^3n^3 &+& \quad \frac{4}{49}n^3m^2p &+& \quad \frac{2}{21}n^3p^2m &+& \quad \frac{1}{27}n^3p^3
\end{aligned}$$

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$$\left(\frac{2}{7}mn+\frac{1}{3}np\right)^3 = \frac{8}{343}m^3n^3+\frac{4}{49}n^3m^2p+\frac{2}{21}n^3p^2m+\frac{1}{27}n^3p^3$$

b. $\left(\frac{1}{2}pq^2+\frac{2}{3}p^2q\right)^2$

Solución

Es una suma elevada a la 2 (o de grado 2) de la forma

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$donde \begin{cases} a = \frac{1}{2}pq^2 \\ \\ b = \frac{2}{3}p^2q \end{cases}$$

$$\begin{aligned} \left(\frac{1}{2}pq^2 + \frac{2}{3}p^2q\right)^2 &= \left(\frac{1}{2}pq^2\right)^2 &+& \quad 2\left(\frac{1}{2}pq^2\right)\left(\frac{2}{3}p^2q\right) &+& \quad \left(\frac{2}{3}p^2q\right)^2 \\ &= \frac{1^2}{2^2}p^2q^{(2\cdot2)} &+& \quad \frac{\cancel{2}\cdot1\cdot2}{\cancel{2}\cdot3}pq^2p^2q &+& \quad \frac{2^2}{3^2}p^{(2\cdot2)}q^2 \\ &= \frac{1}{2\cdot2}p^2q^4 &+& \quad \frac{2}{3}p^{(1+2)}q^{(2+1)} &+& \quad \frac{4}{3\cdot3}p^4q^2 \\ &= \frac{1}{4}p^2q^4 &+& \quad \frac{2}{3}p^3q^3 &+& \quad \frac{4}{9}p^4q^2 \end{aligned}$$

c. $\left(\frac{2}{5}xn + \frac{1}{3}ny\right)^2$

Solución

Es una suma elevada a la 2 (o de grado 2) de la forma

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$donde \begin{cases} a = \frac{2}{5}xn \\ \\ b = \frac{1}{3}ny \end{cases}$$

$$\begin{aligned} \left(\frac{2}{5}xn + \frac{1}{3}ny\right)^2 &= \left(\frac{2}{5}xn\right)^2 &+& \quad 2\left(\frac{2}{5}xn\right)\left(\frac{1}{3}ny\right) &+& \quad \left(\frac{1}{3}ny\right)^2 \\ &= \frac{2^2}{5^2}x^2n^2 &+& \quad \frac{2\cdot2\cdot1}{5\cdot3}xnny &+& \quad \frac{1^2}{3^2}n^2y^2 \\ &= \frac{2\cdot2}{5\cdot5}x^2n^2 &+& \quad \frac{4}{15}n^2xy &+& \quad \frac{1\cdot1}{3\cdot3}n^2y^2 \\ &= \frac{4}{25}x^2n^2 &+& \quad \frac{4}{15}n^2xy &+& \quad \frac{1}{9}n^2y^2 \end{aligned}$$

d. $\left(\frac{5}{12}mq^3 - \frac{1}{9}p^2q\right)^2$

Solución

Es una resta elevada a la 2 (o diferencia de grado 2) de la forma

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$donde \begin{cases} a = \frac{5}{12}mq^3 \\ \\ b = \frac{1}{9}p^2q \end{cases}$$

$$
\begin{aligned}
\left(\frac{5}{12}mq^3-\frac{1}{9}p^2q\right)^2 &= \left(\frac{5}{12}mq^3\right)^2 && - && 2\left(\frac{5}{12}mq^3\right)\left(\frac{1}{9}p^2q\right) && + && \left(\frac{1}{9}p^2q\right)^2 \\
&= \frac{5^2}{12^2}m^2q^{(3\cdot 2)} && - && \frac{\cancel{2}\cdot 5\cdot 1}{\cancel{12}\cdot 9}mq^3p^2q && + && \frac{1^2}{9^2}p^{(2\cdot 2)}q^2 \\
&= \frac{5\cdot 5}{12\cdot 12}m^2q^6 && - && \frac{1\cdot 5\cdot 1}{6\cdot 9}mq^{(3+1)}p^2 && + && \frac{1\cdot 1}{9\cdot 9}p^4q^2 \\
&= \frac{25}{144}m^2q^6 && - && \frac{5}{54}q^4p^2m && + && \frac{1\cdot 1}{81}p^4q^2
\end{aligned}
$$

e. $\left(\frac{3}{7}xnm+\frac{2}{9}nmy\right)^2$

f. $\left(\frac{3}{7}xnm+\frac{2}{9}nmy\right)\left(\frac{3}{7}xnm-\frac{2}{9}nmy\right)$

g. $\left(\frac{2}{5}xn+\frac{1}{3}ny\right)\left(\frac{2}{5}xn+\frac{1}{3}ny\right)$

h. $(2a-3b+4c)^2$

i. $\left(2+3x-5x^2\right)^2$