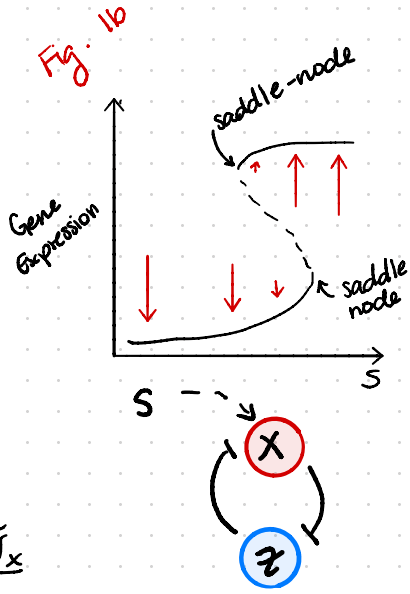
## Problem 2

a) $$\frac{d\tilde{x}}{dt} = \frac{\tilde{\alpha}_x + \beta_x S}{1 + S + (\tilde{z}/\tilde{z}_x)^{n_{zx}}} - \tilde{\delta}_x \tilde{x}$$

$$\frac{d\tilde{z}}{dt} = \frac{\tilde{\alpha}_z}{1 + (\tilde{x}/\tilde{x}_z)^{n_{xz}}} - \tilde{\delta}_z \tilde{z}$$

Fig. 1b




b) $$\delta_z = \frac{\tilde{\delta}_z}{\tilde{\delta}_x} \qquad t = \tilde{t}\,\tilde{\delta}_x$$ ← correction: paper states $t = \tilde{t}\tilde{\delta}_x$

$$\alpha_x = \frac{\tilde{\alpha}_x}{\tilde{\alpha}_z} \qquad \beta_x = \frac{\tilde{\beta}_x}{\tilde{\alpha}_z} \qquad z_x = \frac{\tilde{z}_x \tilde{\delta}_x}{\tilde{\alpha}_z}$$

$$x_z = \frac{\tilde{x}_z \tilde{\delta}_x}{\tilde{\alpha}_z}$$

$$X = \frac{\tilde{x}\tilde{\delta}_x}{\tilde{\alpha}_z} \qquad Z = \frac{\tilde{z}\tilde{\delta}_x}{\tilde{\alpha}_z}$$

Plugging in:

$$\left(\frac{\tilde{\alpha}_z \tilde{\delta}_x}{\tilde{\delta}_x}\right)\frac{dx}{dt} = \frac{\alpha_x \tilde{\alpha}_z + \beta_x \tilde{\alpha}_z S}{1 + S + \left[\frac{z \tilde{\alpha}_z \tilde{\delta}_x}{\tilde{\delta}_x \tilde{\alpha}_z z_x}\right]^{n_{zx}}} - \frac{\tilde{\delta}_x x \tilde{\alpha}_z}{\tilde{\delta}_x}$$

$\div \alpha_z$

$$\boxed{\frac{dx}{dt} = \frac{\alpha_x + \beta_x S}{1 + S + (z/z_x)^{n_{zx}}} - x}$$

$$\left(\frac{\tilde{\alpha}_z \tilde{\delta}_x}{\tilde{\delta}_x}\right)\frac{dz}{dt} = \frac{\tilde{\alpha}_z}{1 + \left[\frac{x \tilde{\alpha}_z \tilde{\delta}_x}{\tilde{\delta}_x x_z \tilde{\alpha}_z}\right]^{n_{xz}}} - \frac{\delta_z z \tilde{\alpha}_z \tilde{\delta}_x}{\tilde{\delta}_x}$$

$\div \alpha_z$

$$\frac{dz}{dt} = \frac{1}{1 + (x/x_z)^{n_{x_z}}} - \delta_z z$$