CMP9780, EGR3031 & BME3002

Module Information and Introduction to Signal Processing

Key Information

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  - Isaac Newton Building, INB2217
  - Office Hours
    - Wed 12:00 – 14:00

Key Information

12 Week Study (Delivery (12 Lectures/Workshops)

Runs over Semester A

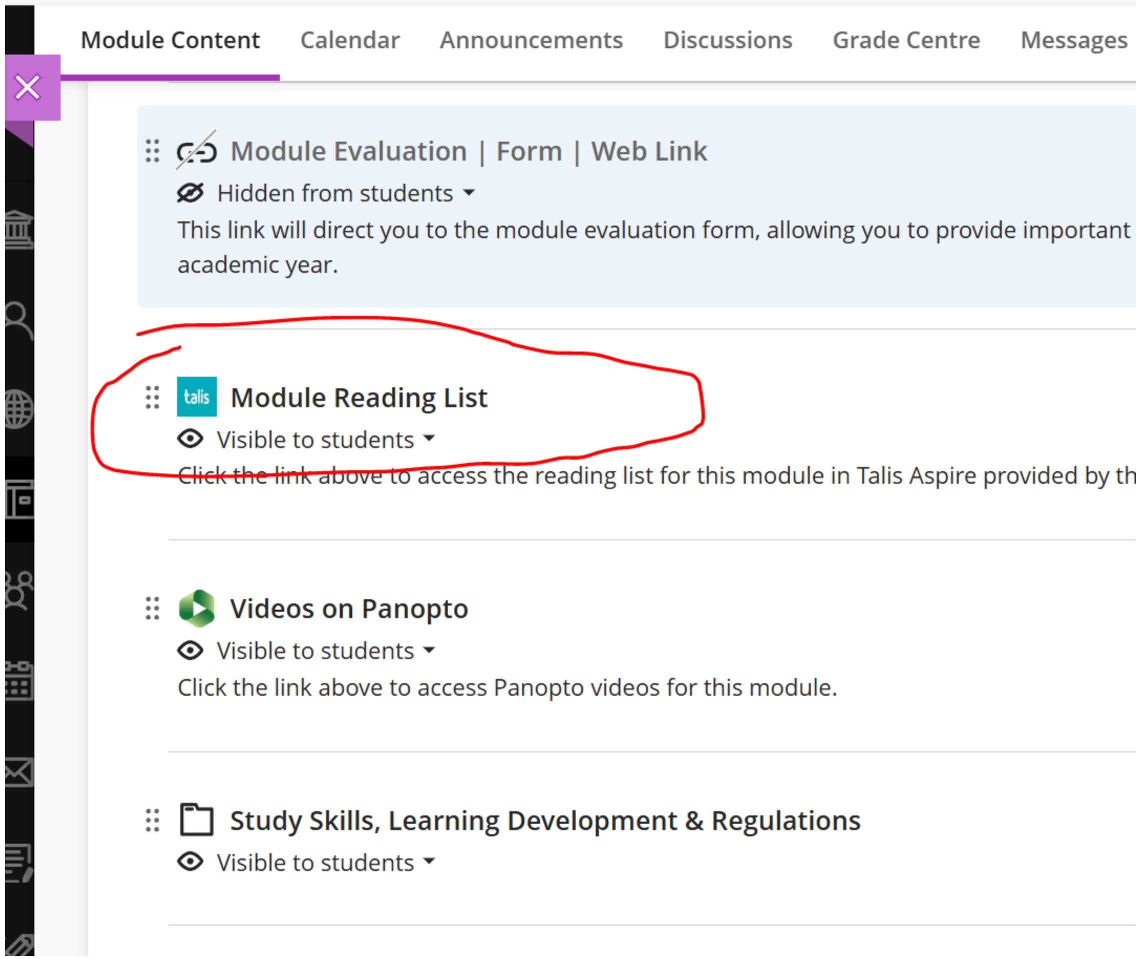
Lectures, Workshops and directed activity sessions

Blackboard support

All lecture notes will appear on a week-by-week basis

Key Information

- **Assessment Strategy**
  - **CMP9870: Applied Signal and Image Processing:**
    - **Signal Processing (50%)**
    - **Image Processing (50%)**
  - **Signal Processing And System Identification**
    - **Coursework 25 % LO2 and LO4**
    - **Time Constrained Assessment 75 % LO1 and LO3**
  - **Biomedical Imaging And Signal Processing**
    - **Time Constrained Assessment 40 %**
    - **Coursework 60 %**
- **Dates on *Blackboard***
- **Read List**
  - ***on Blackboard (Talis Aspire)***



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CMP9780M
Applied Signal & Image Processing - Lecture One

Introduction to Signal Processing

What is a Signal?

- In Signal Processing:
  - A function that conveys information
  - Any physical quantity that can vary over time or space to share messages between observers

- Mathematically, we describe a signal as a function of **one or more** independent variables**:**

1. $S_1(t) = 5t$
2. $S_2(t) = 20t^2$
3. $S(x, y) = 3x + 2xy + 10y^2$

Signal - Graphical Representation

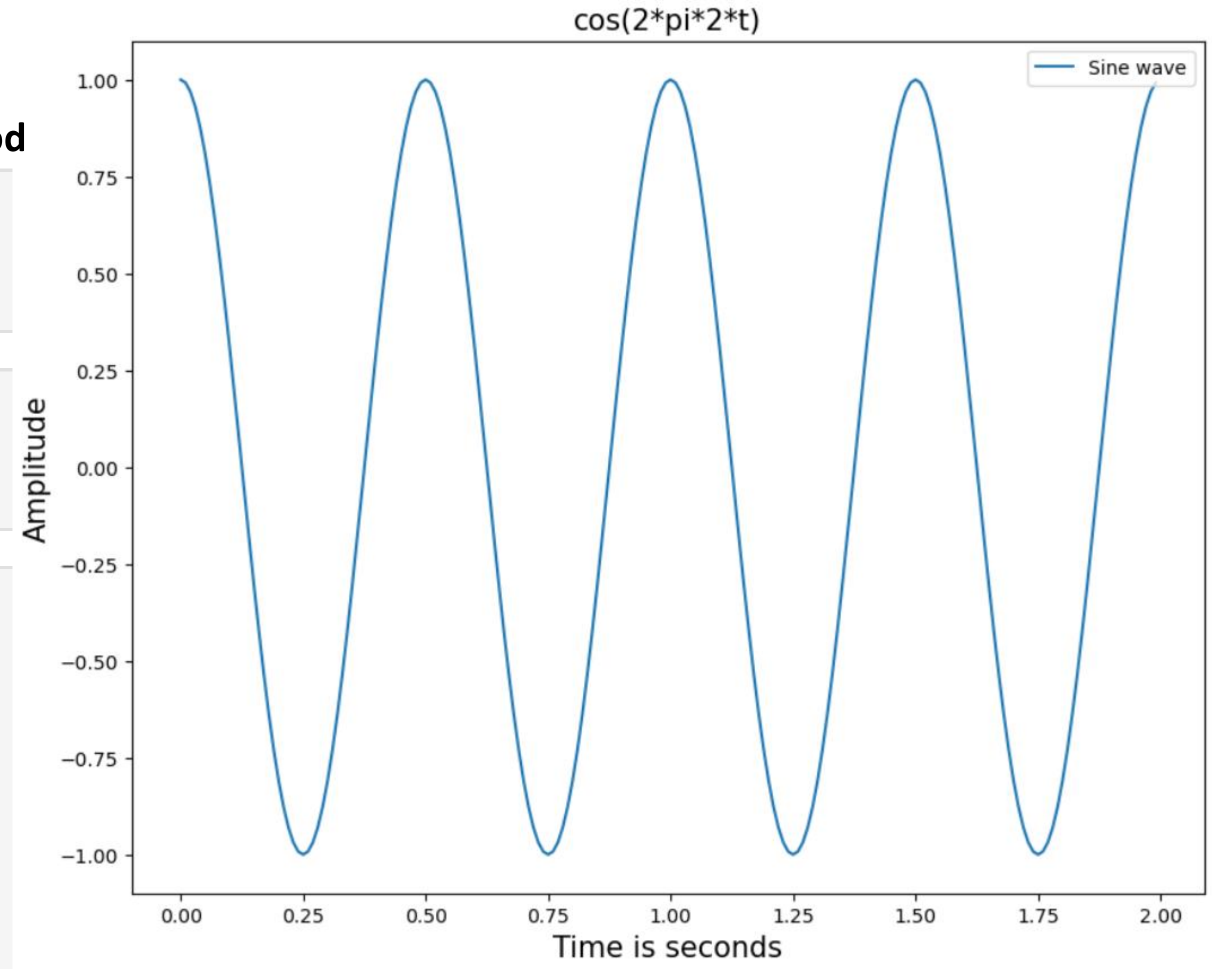
Example #1

Sampling Period

```
import numpy as np
import matplotlib.pyplot as plt
```

```
t = np.arange(0, 2, 0.01)
x = np.cos(2*np.pi*2*t)
```

```
plt.figure(figsize=(10, 8))
plt.plot(t, x, label='Sine wave')
plt.xlabel('Time is seconds', fontsize=15)
plt.ylabel('Amplitude', fontsize=15)
plt.title('cos(2*pi*2*t)', fontsize=15)
plt.legend(fontsize=10, loc='upper right')
```



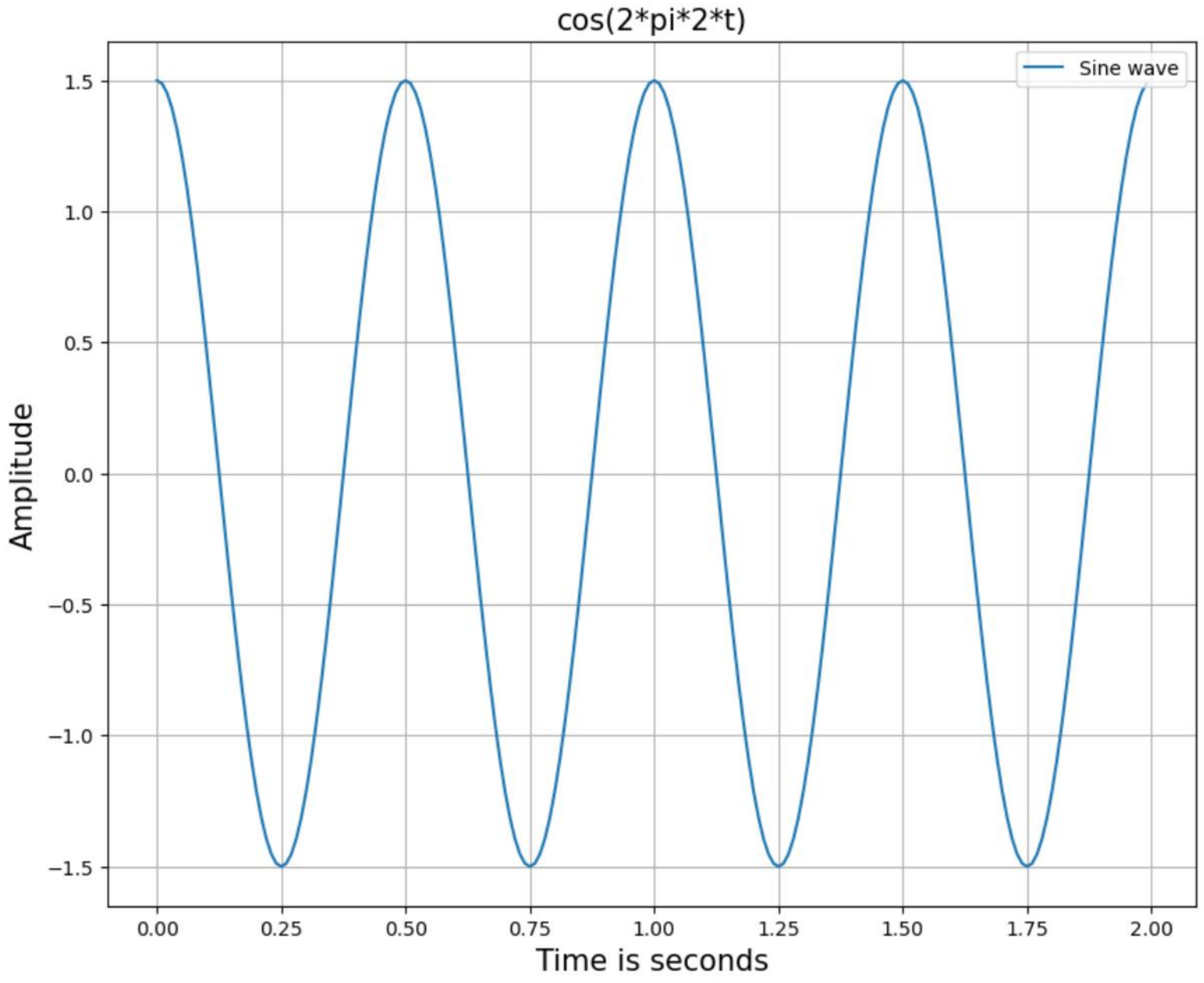
Signal - Graphical Representation

Example #2

```
[8]: import numpy as np
     import matplotlib.pyplot as plt
```

```
[11]: t = np.arange(0, 2, 0.01)
      x = 1.5 * np.cos(2*np.pi*2*t)
```

```
[13]: plt.figure(figsize=(10, 8))
      plt.plot(t, x, label='Sine wave')
      plt.xlabel('Time is seconds', fontsize=15)
      plt.ylabel('Amplitude', fontsize=15)
      plt.title('cos(2*pi*2*t)', fontsize=15)
      plt.legend(fontsize=10, loc='upper right')
      plt.grid()
```

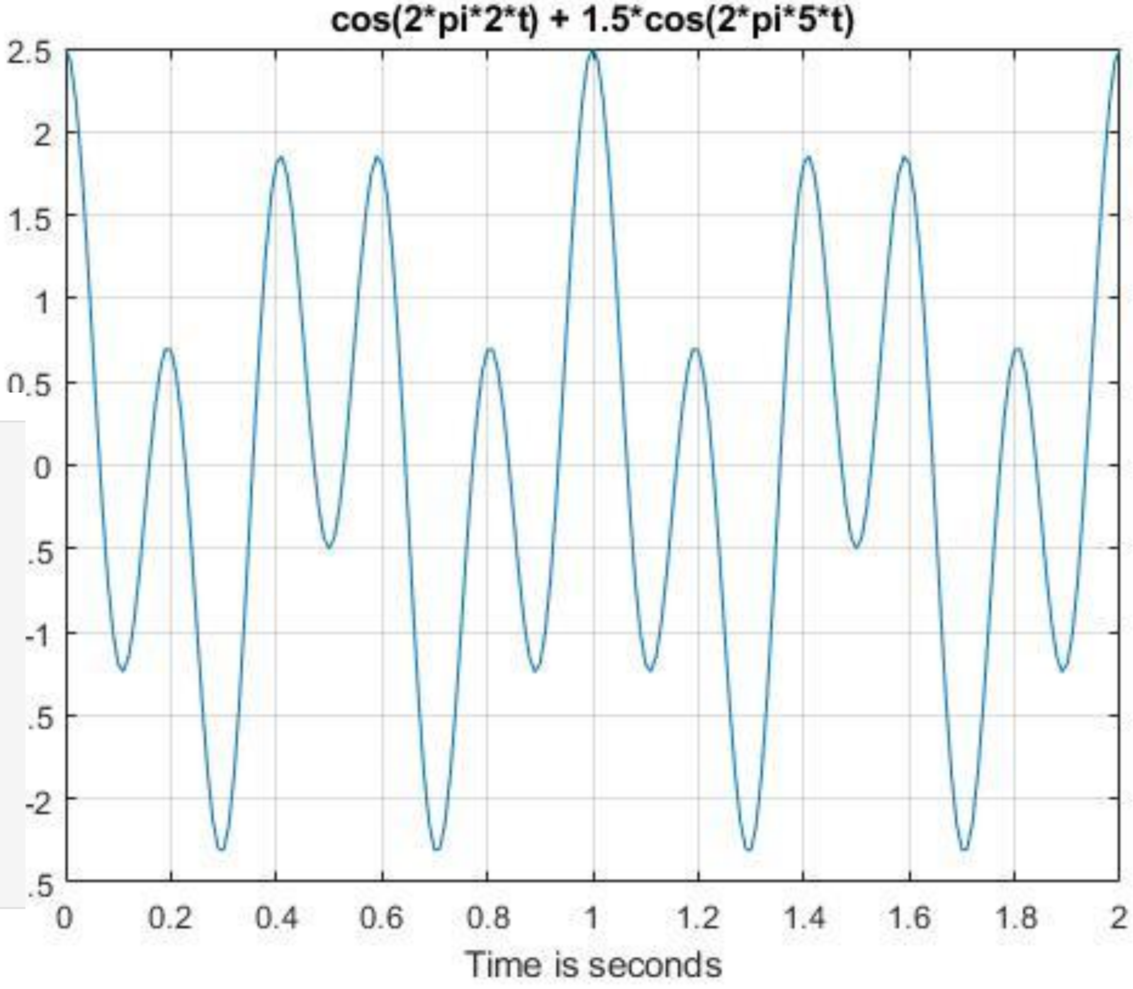


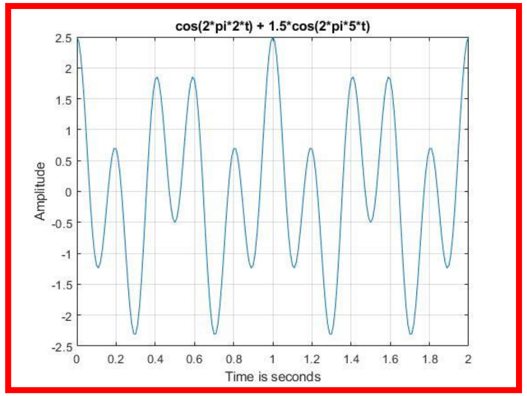
Signal - Graphical Representation

Example #3

```
t = np.arange(0, 2, 0.01)
x =  np.cos(2*np.pi*2*t) +
(1.5 * np.cos(2*np.pi*5*t))
```

```
plt.figure(figsize=(10, 8))
plt.plot(t, x, label='Sine wave')
plt.xlabel('Time is seconds', fontsize=15)
plt.ylabel('Amplitude', fontsize=15)
plt.title('cos(2*pi*2*t) + 1.5 * cos(2*pi*5*t)'
          , fontsize=15)
plt.legend(fontsize=10, loc='upper right')
plt.grid()
```

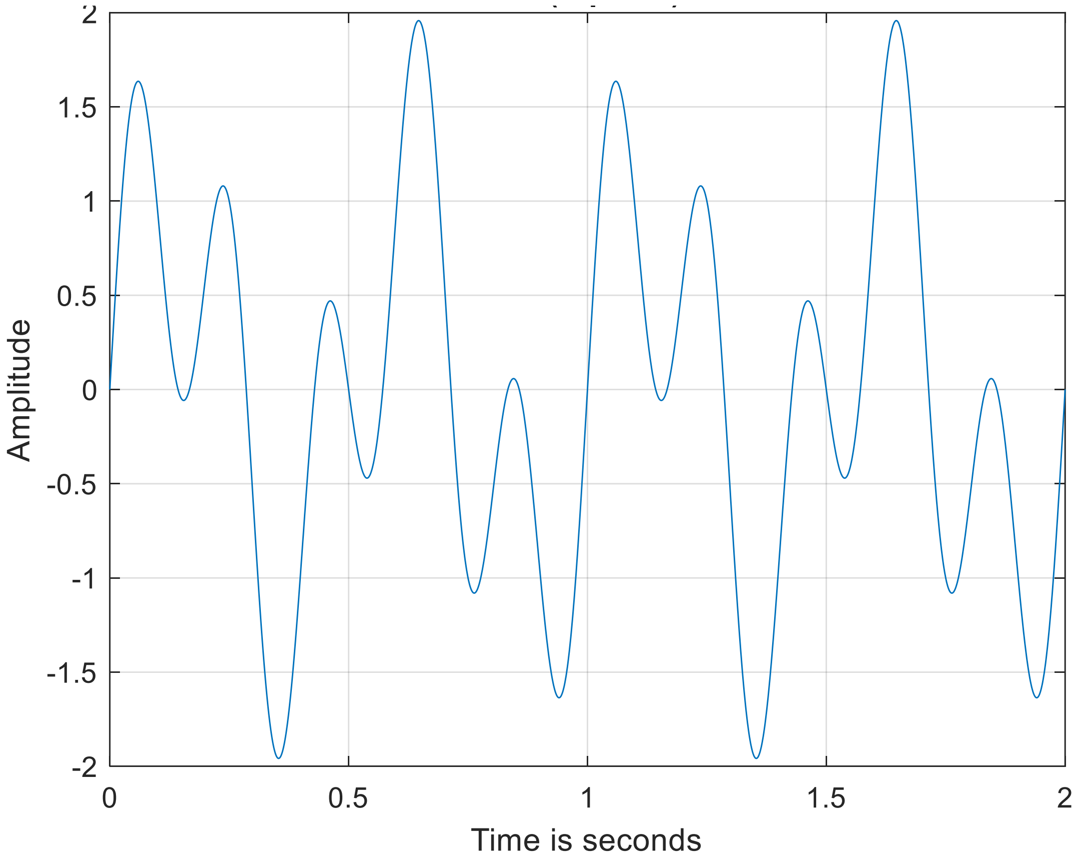




Question

- The signal in the figure is

A. x = cos(2*pi*2*t) + cos(2*pi*5*t);
B. x = sin(2*pi*2*t) + 1.5*sin(2*pi*5*t);
C. x = 1.5*sin(2*pi*2*t) + sin(2*pi*5*t);
D. x = cos(2*pi*2*t) + 1.5cos(2*pi*5*t);



Signals - Examples

- So far, the **signals** we have seen can be **precisely defined** by specifying independent variables.

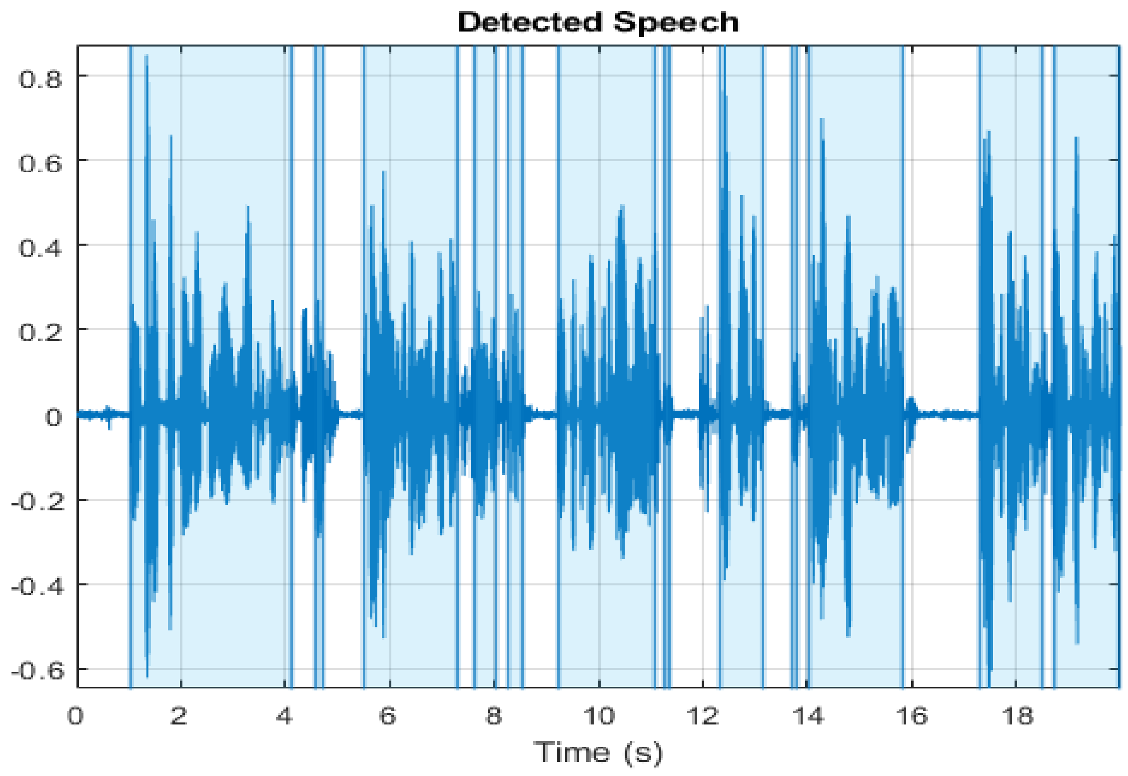
- There are cases where such **functional relationship are unknown or complicated** to be of practical use.

- Can you give some examples?
  - Speech signal
  - Heart rate

Signals – Examples

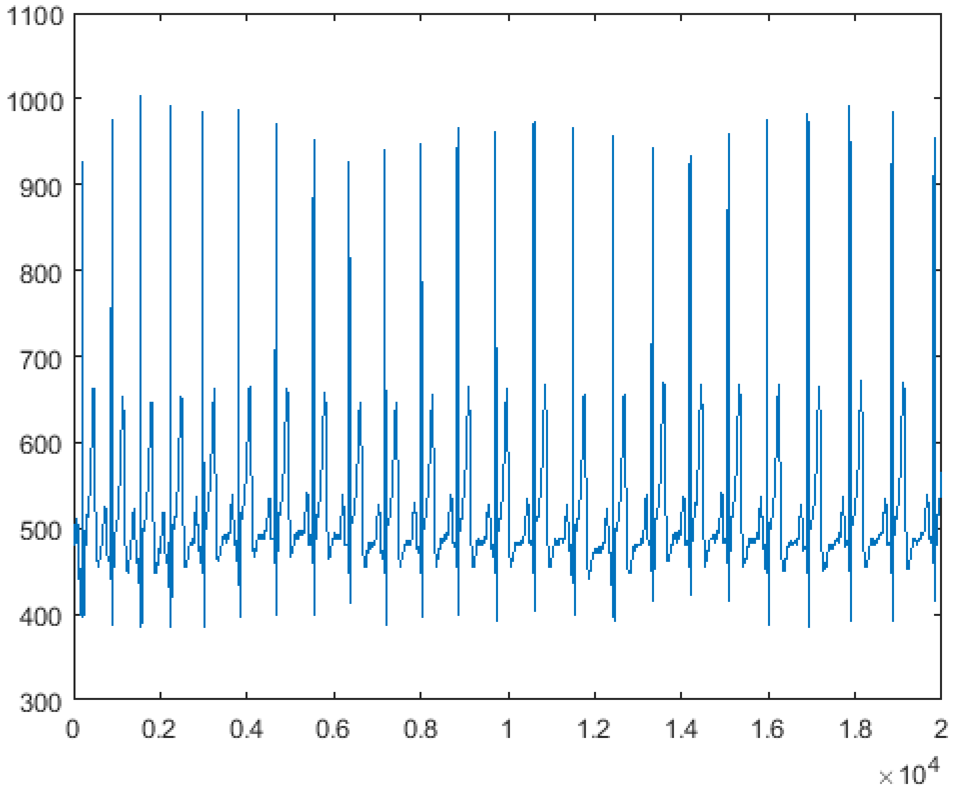
- Functional Relationship Unknown or Complicated
  - **Speech signal** can be represented to a high degree by **sum of several sinusoids** of different amplitudes and frequencies.

$$\sum_{i=1}^{N} \underbrace{\boxed{A_i(t)}}_{\text{Amplitude}} \sin[2\pi \underbrace{\boxed{F_i(t)t}}_{\text{Frequency}} + \underbrace{\boxed{\theta_i(t)}}_{\text{Phase}}]$$



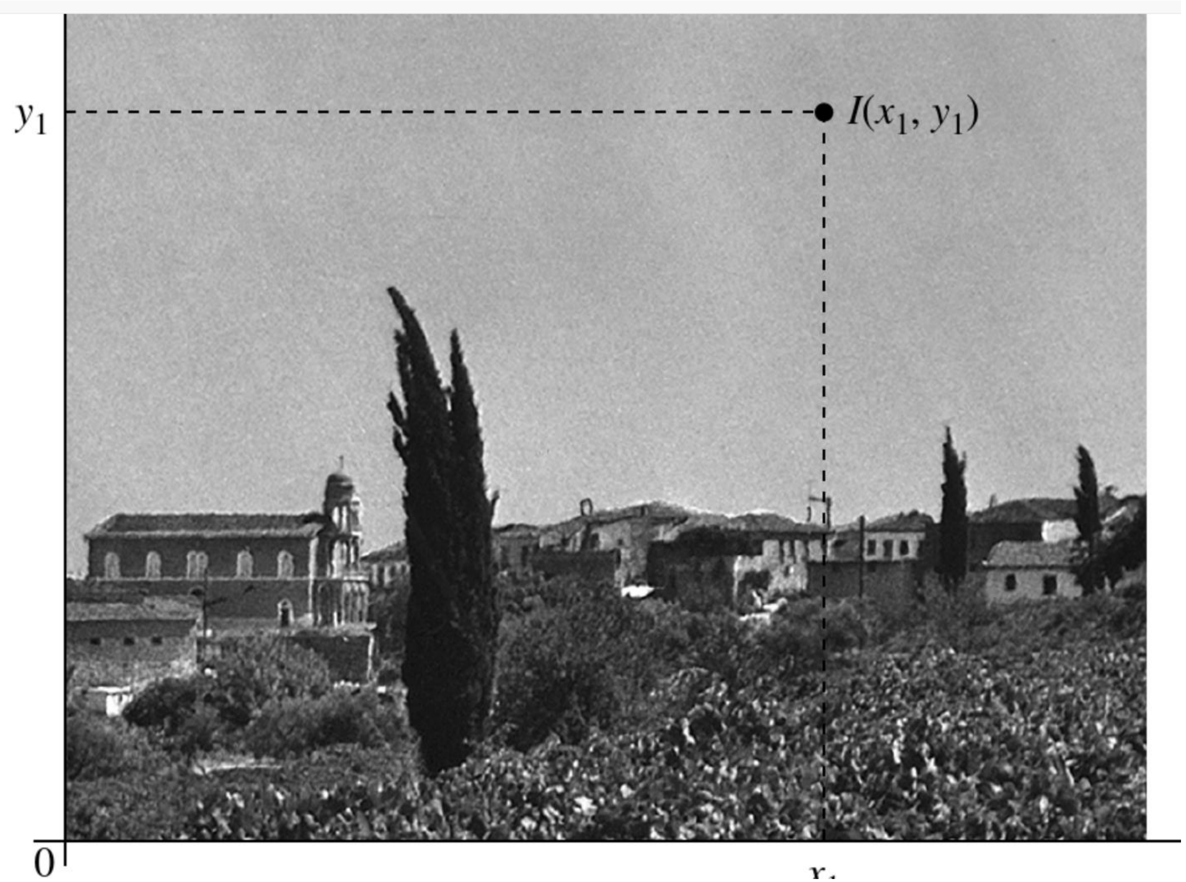
Signals – Example

- Functional Relationship Unknown or Complicated
  - **ECG Signal** provides the doctor with information about a patient's heart.
  - Like Speech signal, ECG is an examples of information-bearing signals that evolve as functions of a single independent variable, namely, time.
  - Visualisation of Bitalino ECG (raw ADC data) acquired data of heart rate measurement (Voltage). (source MathWorks)



Signals – Example

- Function of two independent variables
  - An example of a signal that is a function of **two independent variables** is an **image signal**.
  - The independent variables in this case are the spatial coordinates.
  - More on this during image processing



Signals Converters

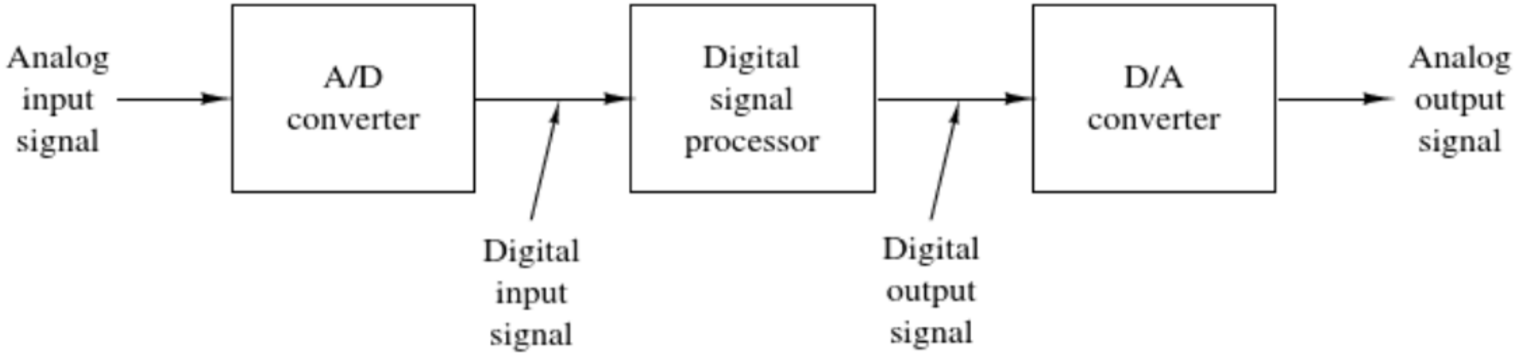
- Most signals in science and engineering are **analogue in nature**

  – It is a **function of a continuous variable** (takes values in a continuous range)

  – Processed directly by appropriate analogue systems (Filters, Frequency multipliers etc.) to change their characteristics or extract some desired information.

Signals Converters

- Digital signal processing is an alternative to processing these signals
  - Requires an interface between the analog device and digital to perform the conversion
    - A/D converters
    - Output is a digital signal suitable for a digital processor (large programmable digital computer or small microprocessor programmed)
    - In speech communications usually the digital signals needs to be converted into analog using D/A converters



Signal

- Signals **represent** or **encode** information

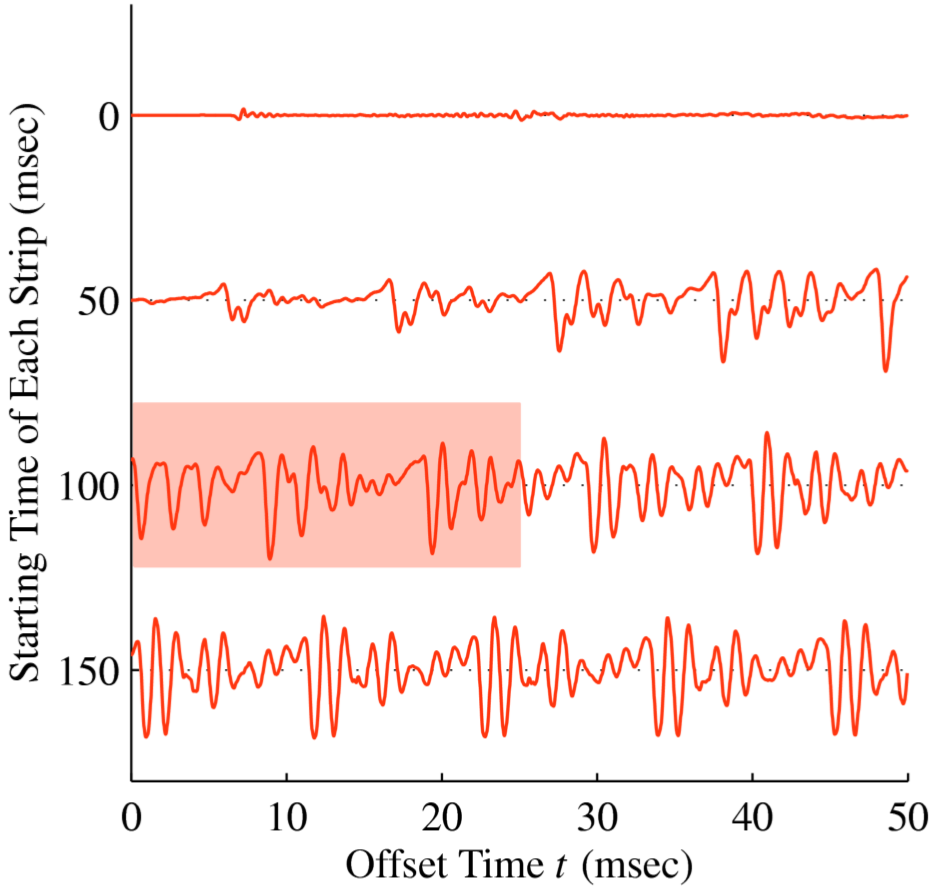
  – In communications applications the information is almost always encoded

  – In the probing of medical and other physical systems, where signals occur naturally, the information is not purposefully encoded

Signal – Speech

- In human speech we create a waveform as a function of time when we force air across our vocal cords and through our vocal tract

- A microphone has converted the sound pressure from the vocal tract into an electrical signal that varies over time, *t*



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

Signal - Continuous

- Signals, such as the above speech signal, are continuous functions of time, and denoted as a continuous-time signal
  - The **independent variable** in this case is **time, t**, but could be another variable of interest, e.g., **position**, **depth**, **temperature**, **pressure**
  - The **mathematical notation** for the speech signal recorded by the microphone might be **s(t)**

Signal - Discrete

- **To process this signal by computer**, we may sample this signal at **regular interval $\boldsymbol{T_s}$**, resulting in

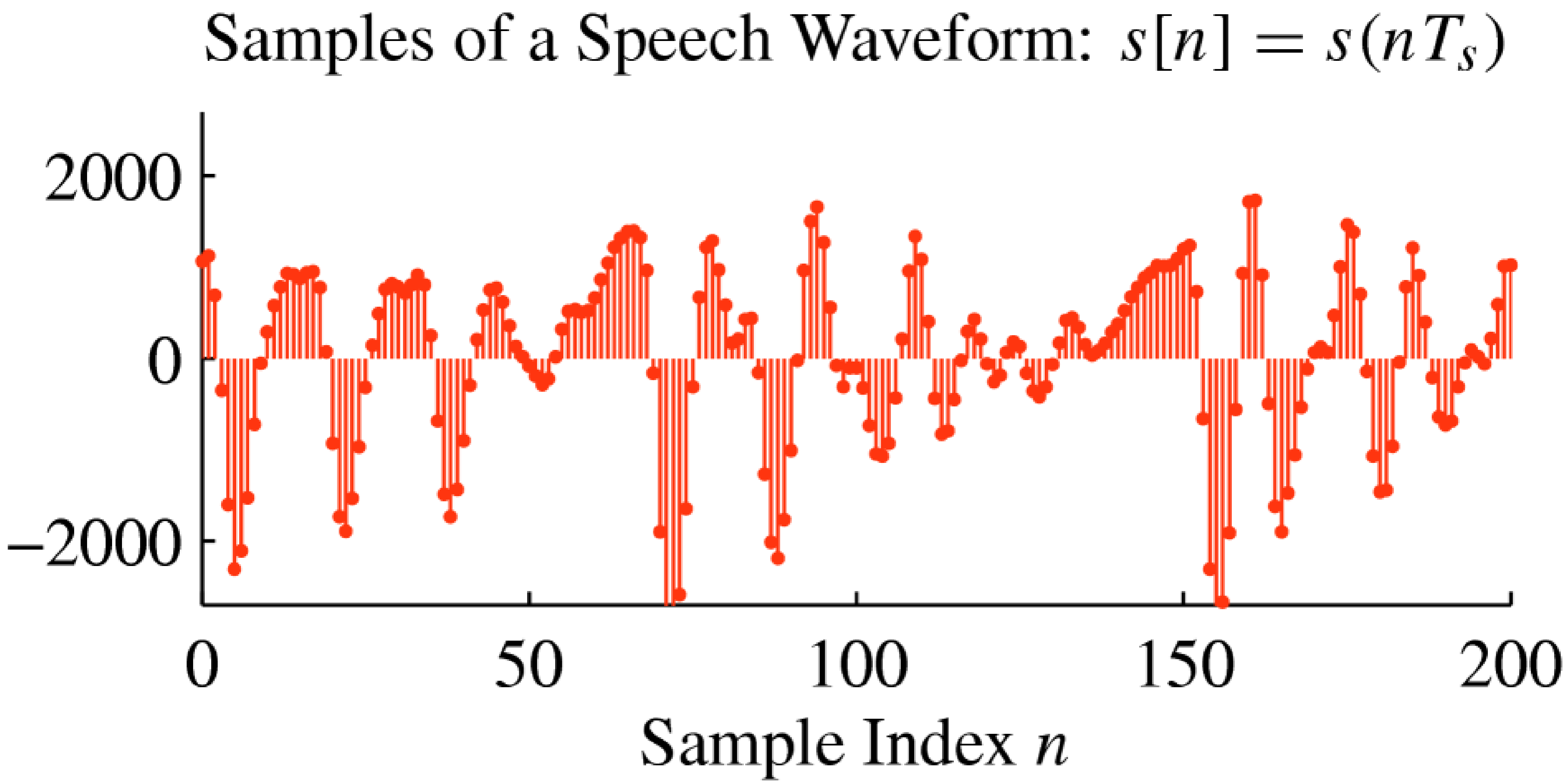
$$s[n] = s(nT_s)$$

Where n is the number of times, we sample the signal.

Signal - Discrete

- The signal $s[n]$ is known as a discrete-time signal, and T_s is the sampling period
  - Note that the independent variable of the sampled signal is the integer sequence
  - $n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$
  - Discrete-time signals can only be evaluated at integer values

Signal - Discrete



Continuous or Discrete Signal?

- Classify these as continuous of discrete signals:

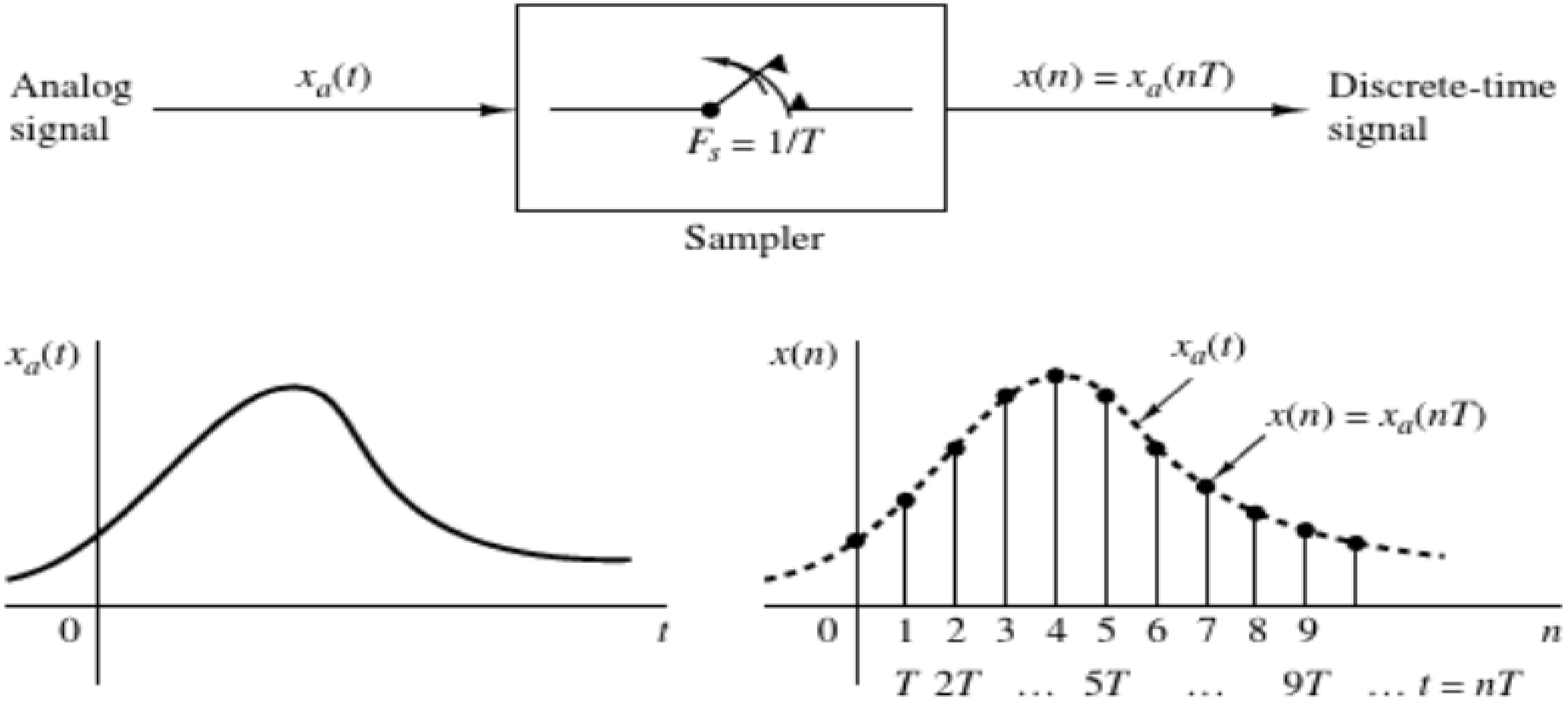
A. The temperature recorded every minute throughout the day.
B. The voltage output of a microphone capturing live sound.
C. The number of cars passing through a toll booth every hour.
D. A music file stored on a computer in MP3 format.

Analog or Digital Signal?

- Classify these signals as analog or digital:

A. The height of a plant measured every day for a month.
B. The brightness of sunlight changing throughout the day.
C. The number of students attending class each day.
D. A photograph stored as a JPEG file on a smartphone.

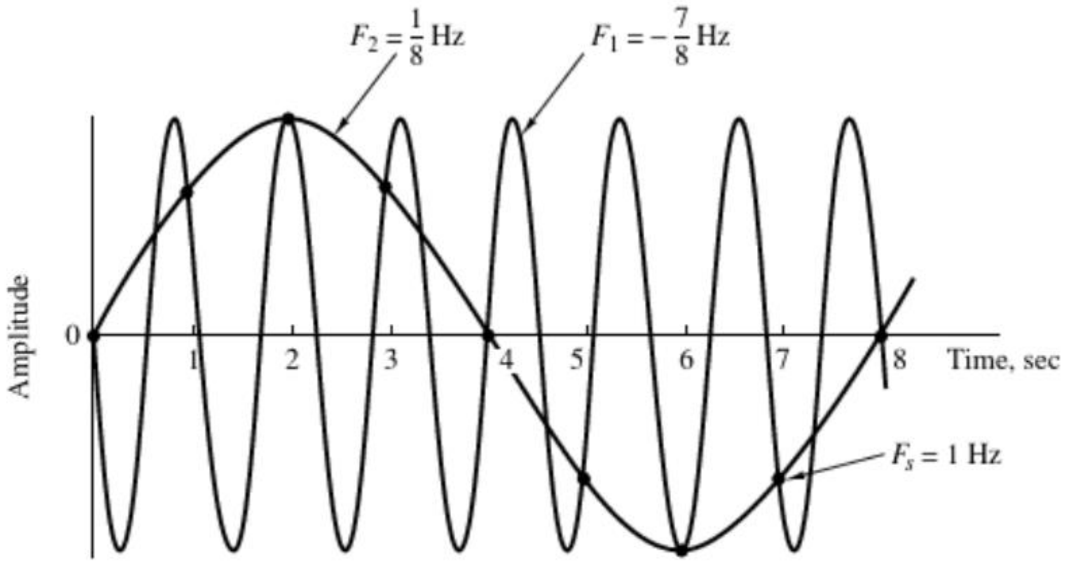
Periodic Sampling of an analog signal



Proakis, John, and Dimitris Manolakis 2013

Aliasing

- Sampling signal could lead to aliasing, if the Nyquist criterion is not followed
  - Fs >= 2Fmax
- An example of aliasing is illustrated in Fig. 4.5, where two sinusoids with frequencies F0 = 1/8Hz and F1 = − 7/8Hz yield identical samples when a sampling rate of Fs = 1 Hz is used.



Proakis, John, and Dimitris Manolakis 2013

Question

- Consider the signal
  - X(t) = 3*cos(100*pi*t)
- Determine the minimum sampling rate required to avoid aliasing.

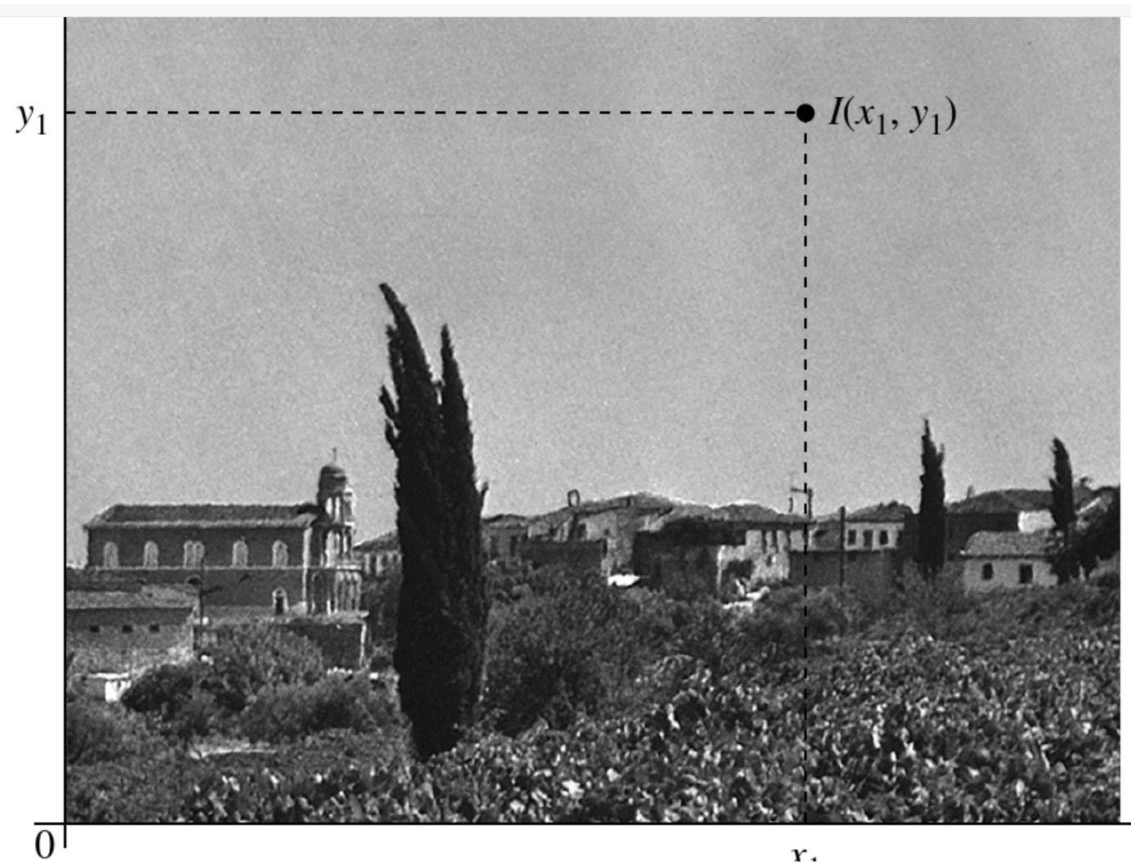
A. 100 Hz
B. 50 Hz
C. 200 Hz
D. 300 Hz

Signal Classification

- The speech waveform is an example of a **one-dimensional signal**, but we may have more than one dimension

- An image, say a photograph, is an example of a **two-dimensional signal**, being a function of two spatial variables, e.g. $s(x, y)$.

- You will revisit this in the image processing section



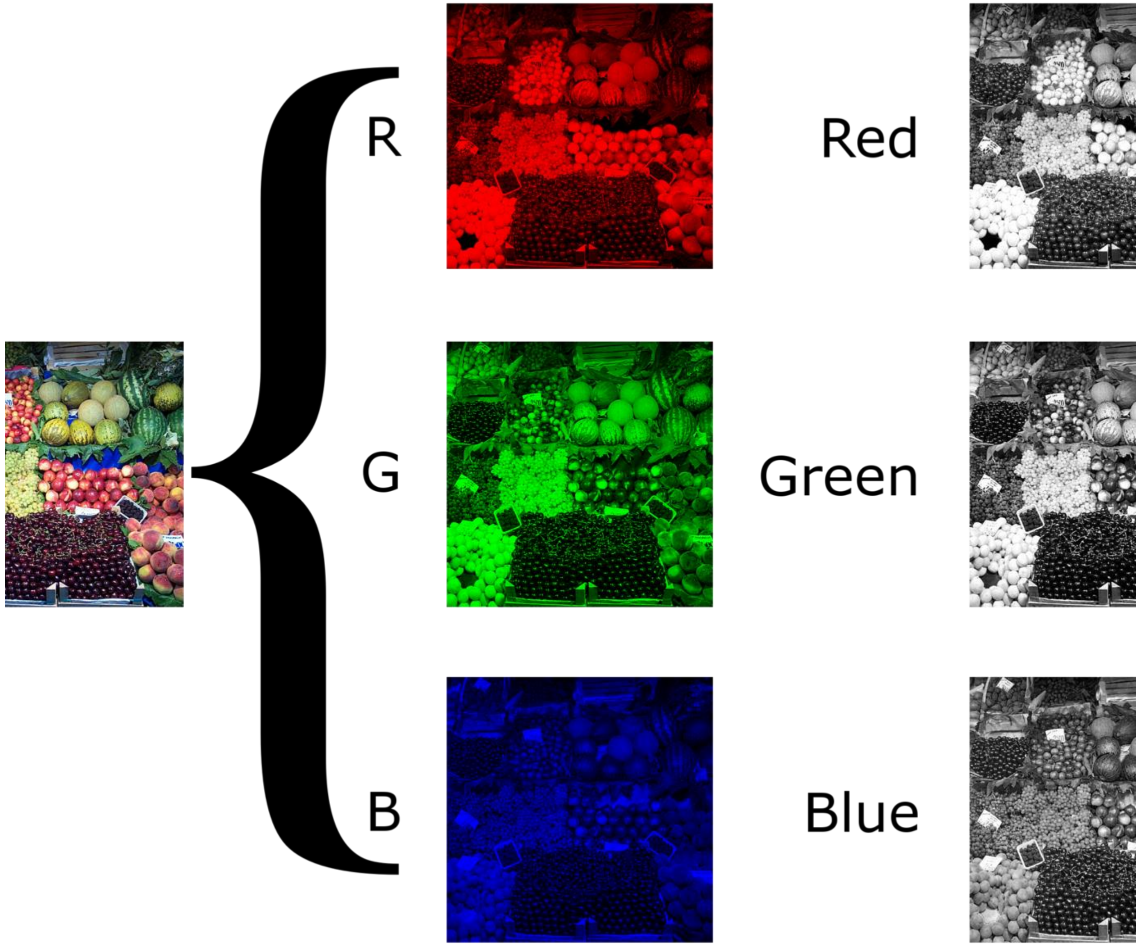
Signal Classification

- In some application, the signal is generated by multiple sources or sensors.
  - They may be represented as a vector
  - An example is an electrical signal from 3 sensors represented as the vector below:

$$\mathbf{S}_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

  - We refer to this signal as a **Multichannel signal**

Signal Classification



- If a signal is a function of **one independent variable,** then it is one-dimensional
- With **n independent variables**, the signal is called an n-dimensional signal.
- The image on the left shows a two-dimensional signal (2D)
  - The intensity at each pixel is a function of two independent variables
- We can have an **n-channel, m-dimensional** signal.
  - e.g., RGB

https://en.wikipedia.org/wiki/Grayscale

Digital vs Analogue Pros

- In Analog signal transmission
  - the main issue is degradation due to noise.
    - Caused by interferences like electrical interference
    - if the transmission is through wires also impact quality.
    - The transmission rate is also slow.
- Digital Signals
  - Advances in Semiconductor Technology makes them powerful in some cases.
  - Easily store on disk without losing signal quality apart from those introduced by the A/D converters.
  - Affected by the speed of A/D converters

Question

- Is Wi-Fi digital or analog?

  A. Analog
  B. Digital
  C. Both analog and digital

- The electromagnetic waves traversing, carrying the data from one point to another, is analog.
- During the data transfer, its digital signal

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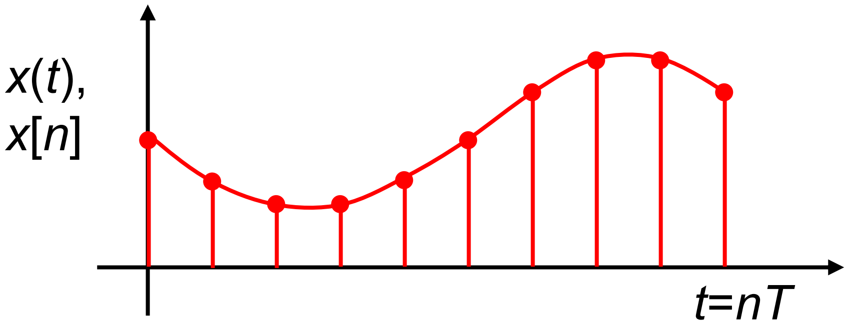
Sampling

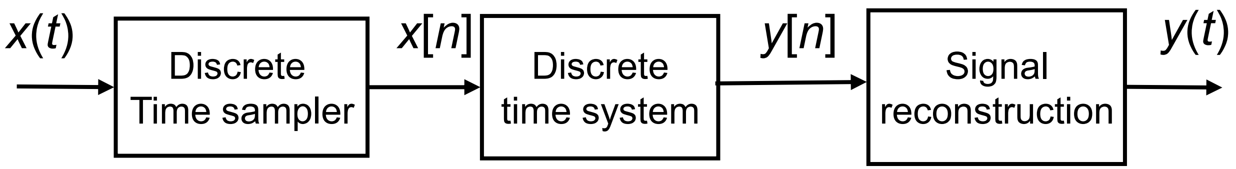
What is Discrete Time Sampling?



T is the sampling period

- Discrete Time Sampling is the transformation of a continuous signal into a discrete signal

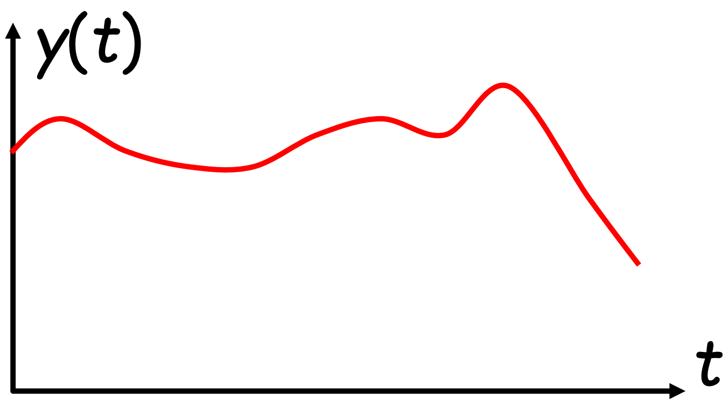
- Why?
  - computational convenience and mathematical tractability
  - Done to compress signal
  - Done to remove noise or certain artifacts from the signal

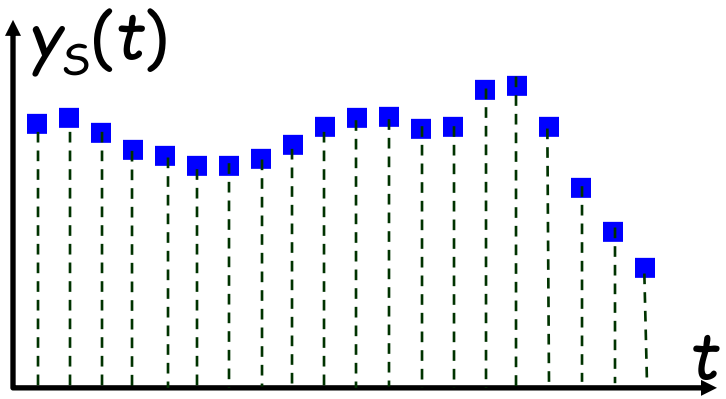


Sampling and Recording

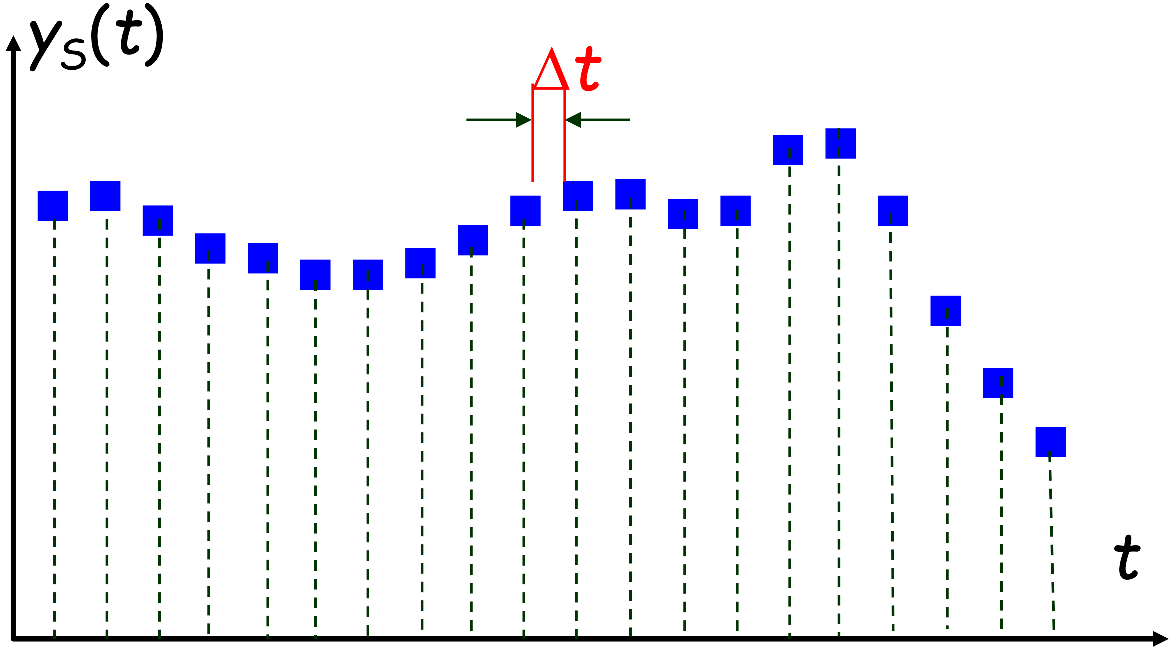
When perform a measurement … a *transducer* converts the *measurand* into an electrical signal … and this signal is *"sampled"* using a digital computer

We normally record a continuous signal $y(t)$ by a set of samples $y_s(t)$ at discrete intervals of time Δt.





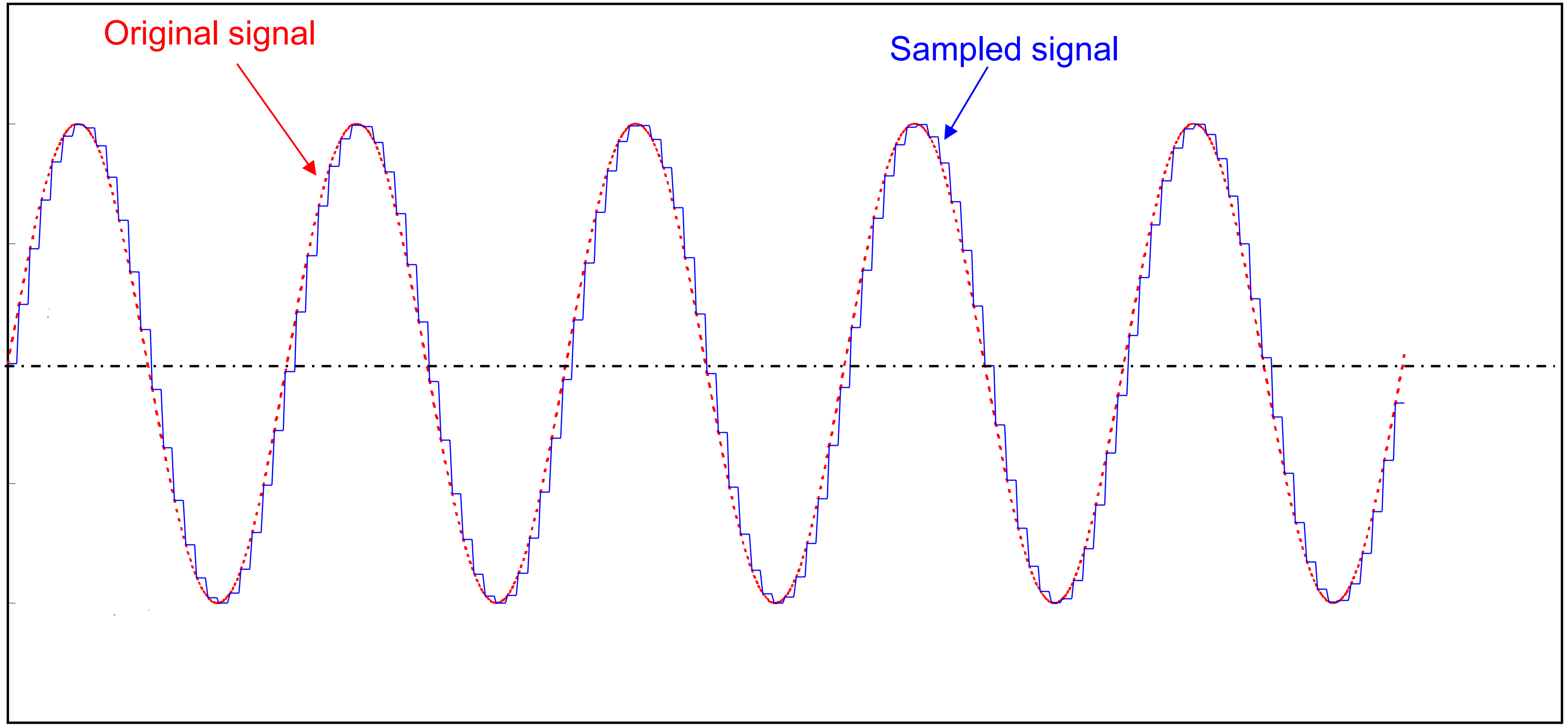
Sampling Frequency



$$f_S = \frac{1}{\Delta t}$$

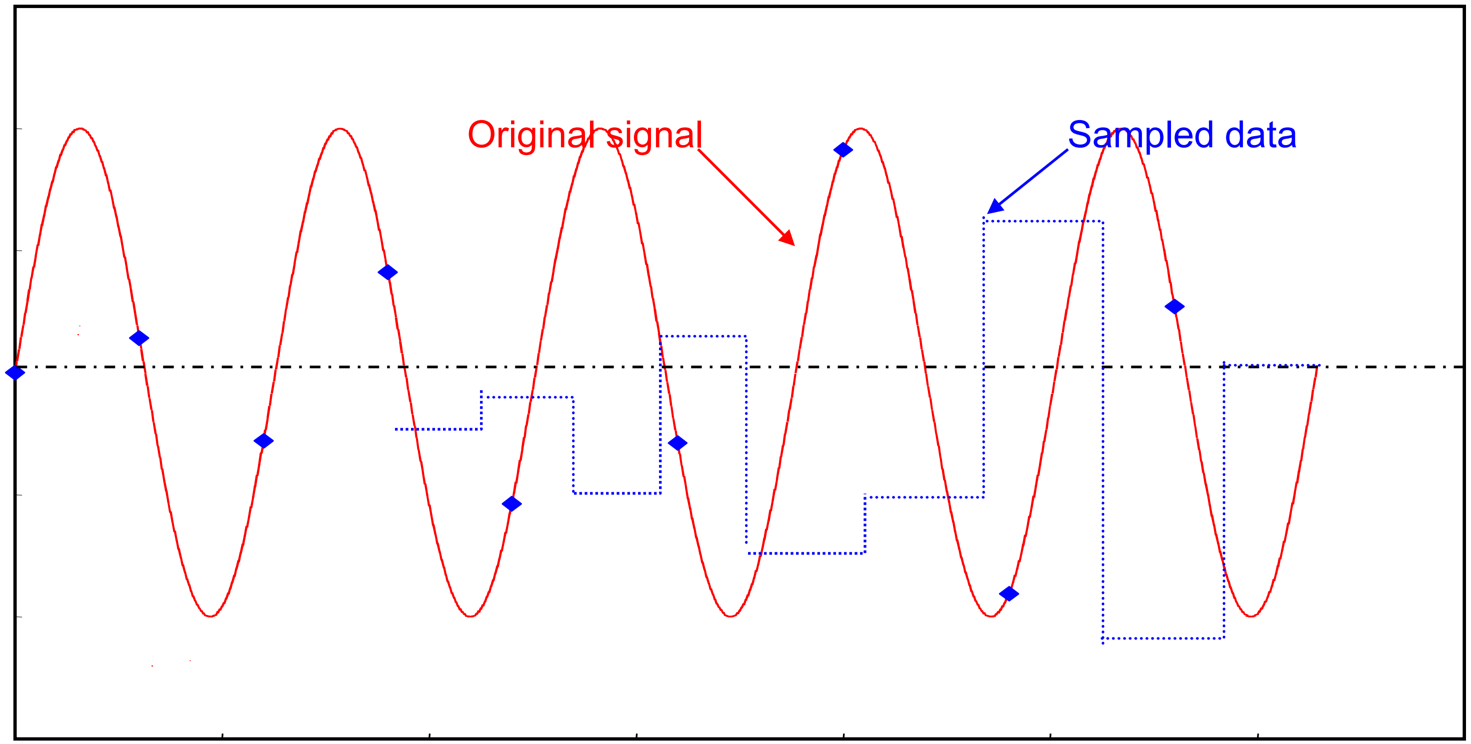
The number of samples recorded each second is defined as the sampling frequency, f_S.

Resemble Sampling Data



If a signal is sampled and recorded relative rapidly, the sampled data will closely resemble the original signal.

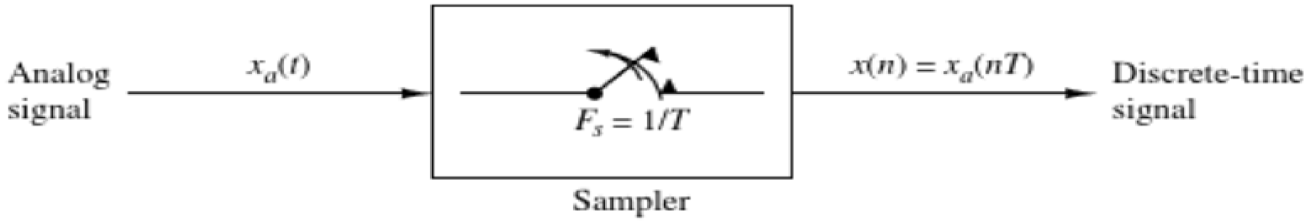
Under Sampling of Test Data

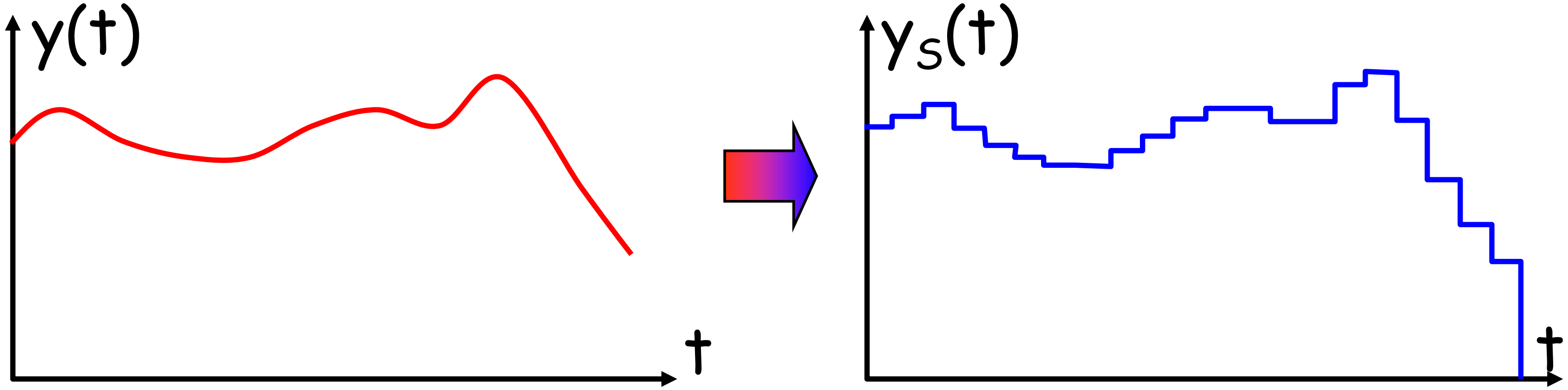


If we sampled too slowly, a recorded data will present a distortion from the original signal. Such distortion will introduce some measurement errors.

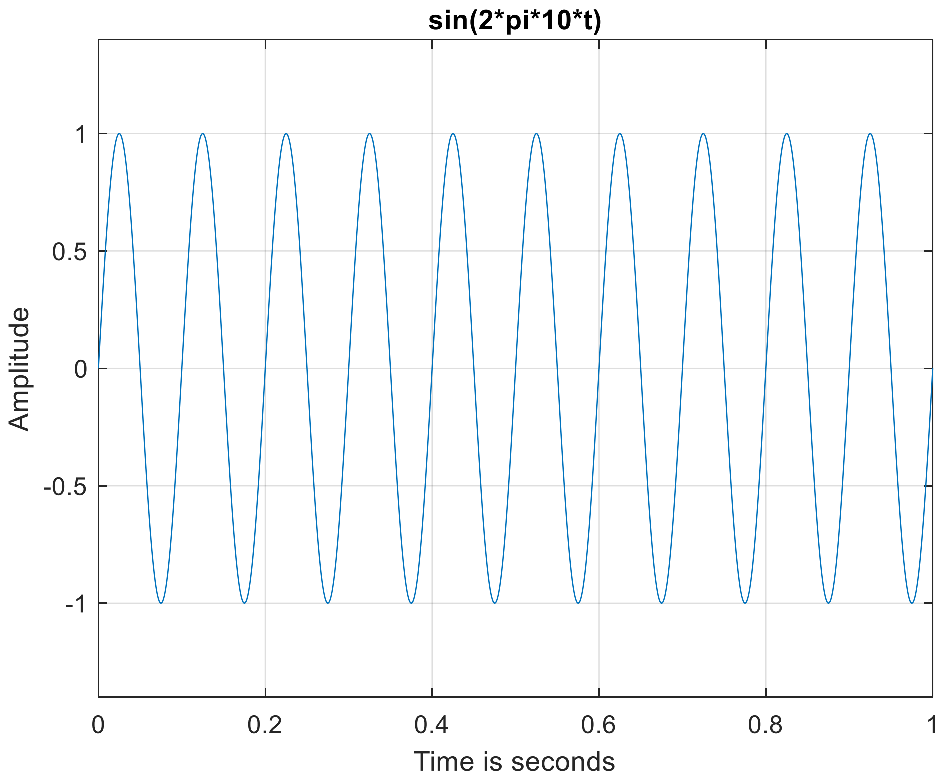
Sampling and Hold

Almost any **analog to digital converter** will have some form of voltage "hold" before sampling. A sampling and hold unit is used to hold each sample value until the next pulse occurs. The sampling and hold unit is necessary because the A/D converter requires a finite amount of time.





A 10 Hz Sine Wave Sampled at 1KHz



Python

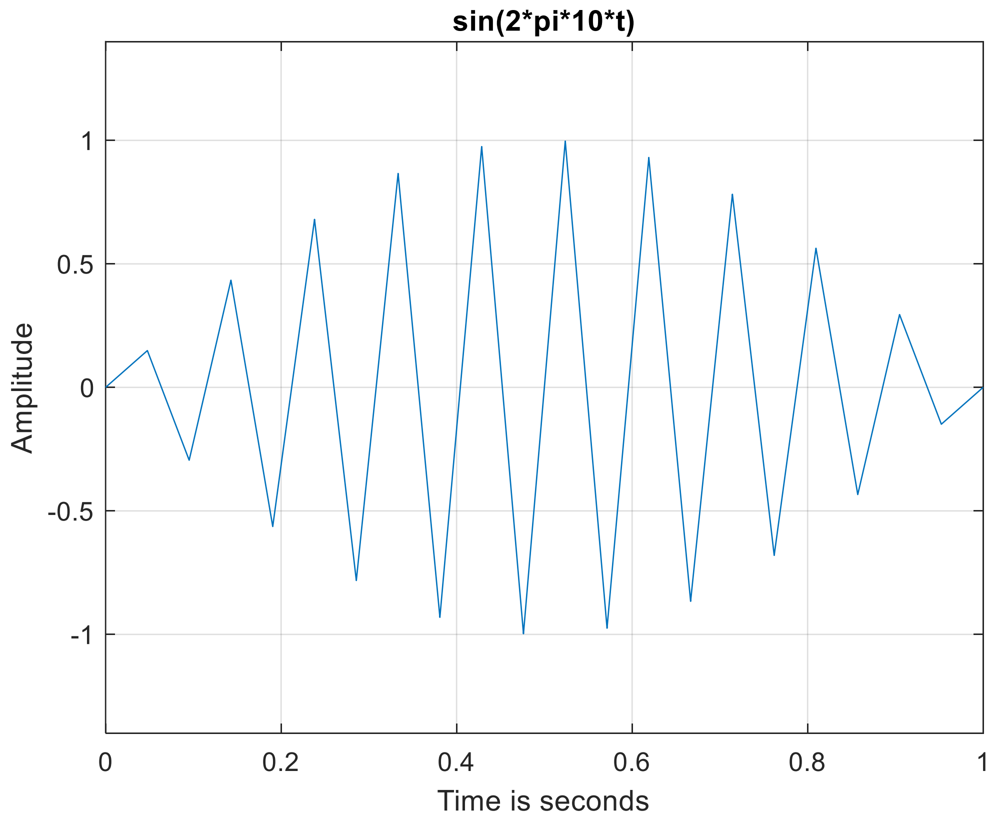
[20]:
```
fs = 1000
dt = 1/fs
t = np.arange(0, 1, dt)
x = np.sin(2*np.pi*10*t)
```

[21]:
```
plt.figure(figsize=(10, 8))
plt.plot(t, x, label='Sine wave')
plt.xlabel('Time is seconds', fontsize=15)
plt.ylabel('Amplitude', fontsize=15)
plt.title('sin(2*pi*10*t)', fontsize=15)
plt.legend(fontsize=10, loc='upper right')
plt.ylim([-1.4, 1.4])
plt.grid()
```

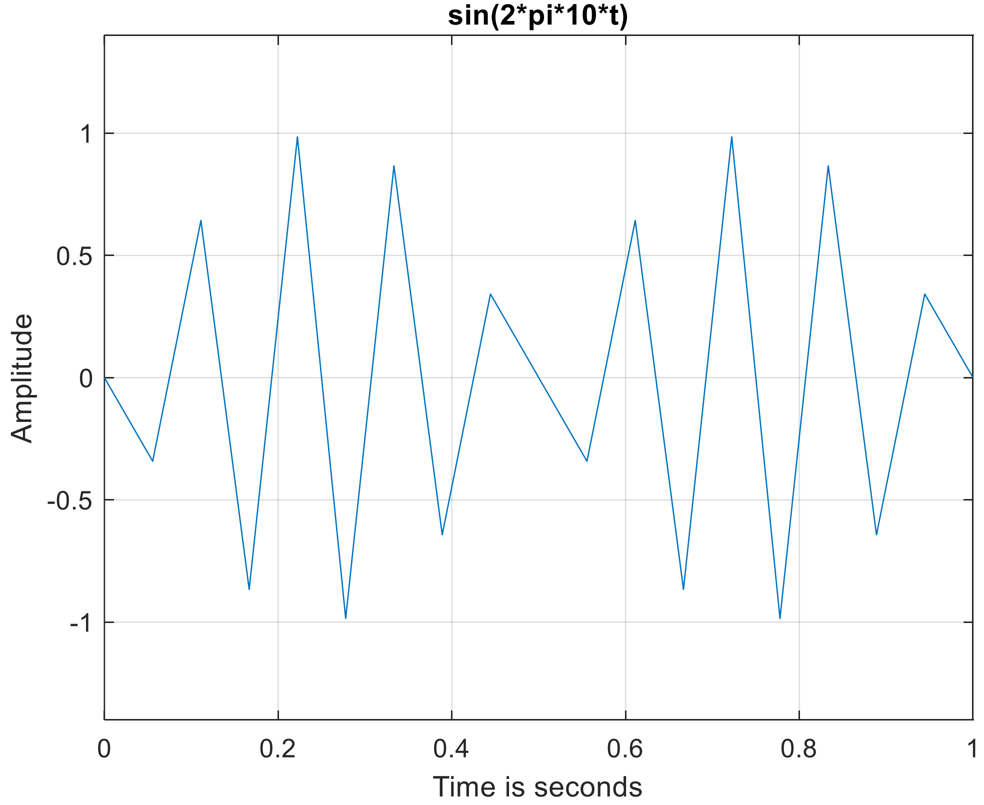
A 10 Hz Sine Wave Sampled at 21Hz

```
fs = 21
dt = 1/fs
t = np.arange(0, 1, dt)
x = np.sin(2*np.pi*10*t)
```

```
plt.figure(figsize=(10, 8))
plt.plot(t, x, label='Sine wave')
plt.xlabel('Time is seconds', fontsize=15)
plt.ylabel('Amplitude', fontsize=15)
plt.title('sin(2*pi*10*t)', fontsize=15)
plt.legend(fontsize=10, loc='upper right')
plt.ylim([-1.4, 1.4])
plt.grid()
```



A 10 Hz Sine Wave Sampled at 18 Hz



Python

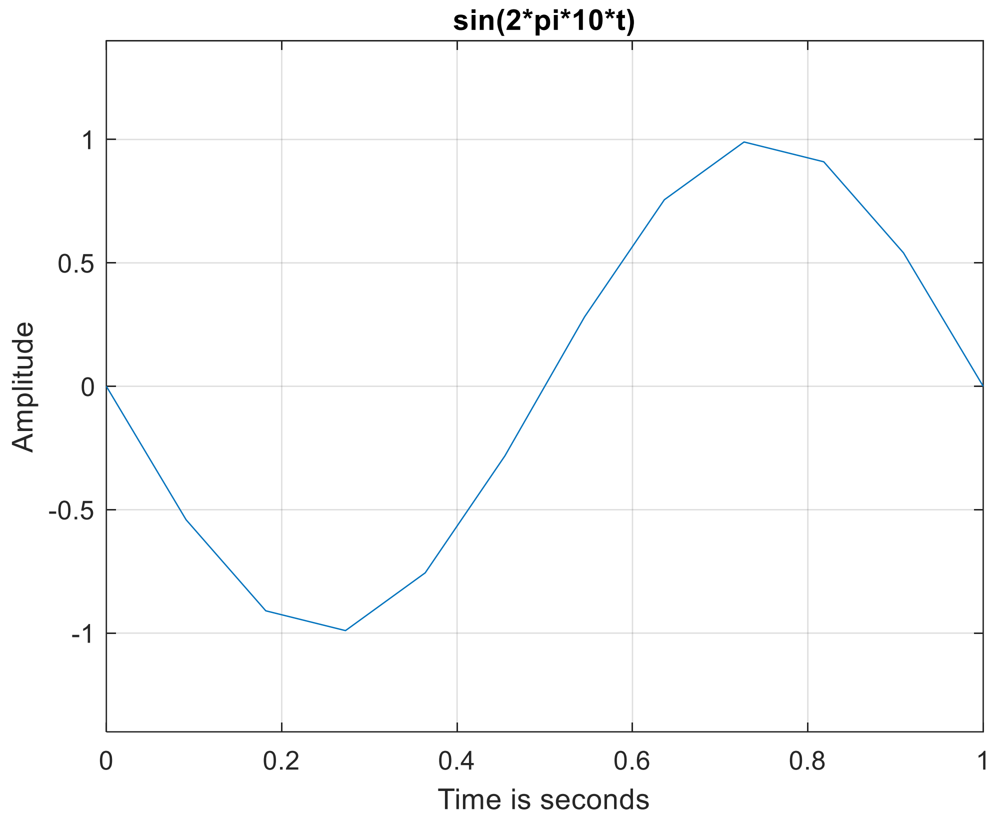
```
[30]: fs = 18
      dt = 1/fs
      t = np.arange(0, 1, dt)
      x = np.sin(2*np.pi*10*t)
```

```
[31]: plt.figure(figsize=(10, 8))
      plt.plot(t, x, label='Sine wave')
      plt.xlabel('Time is seconds', fontsize=15)
      plt.ylabel('Amplitude', fontsize=15)
      plt.title('sin(2*pi*10*t)', fontsize=15)
      plt.legend(fontsize=10, loc='upper right')
      plt.ylim([-1.4, 1.4])
      plt.grid()
```

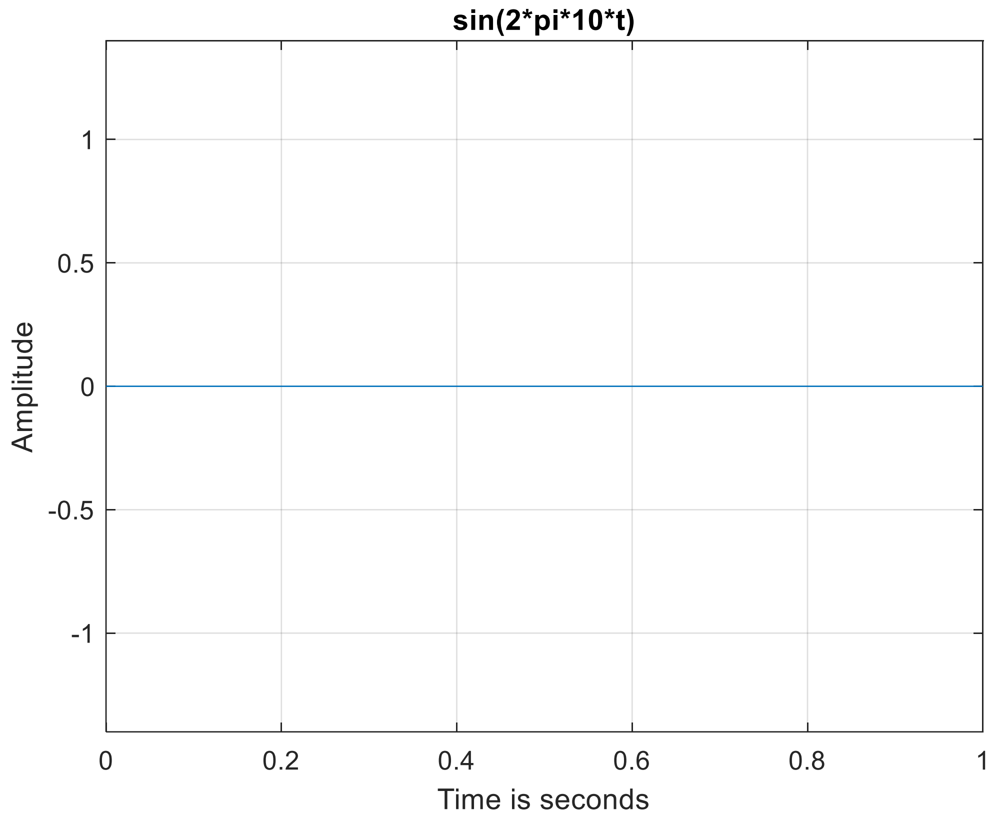
A 10 Hz Sine Wave Sampled at 11Hz

```
fs = 11
dt = 1/fs
t = np.arange(0, 1, dt)
x = np.sin(2*np.pi*10*t)
```

```
plt.figure(figsize=(10, 8))
plt.plot(t, x, label='Sine wave')
plt.xlabel('Time is seconds', fontsize=15)
plt.ylabel('Amplitude', fontsize=15)
plt.title('sin(2*pi*10*t)', fontsize=15)
plt.legend(fontsize=10, loc='upper right')
plt.ylim([-1.4, 1.4])
plt.grid()
```



A 10 Hz Sine Wave Sampled at 5Hz

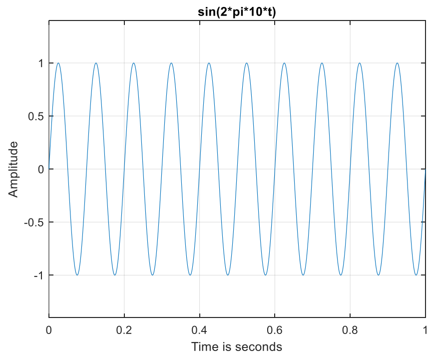



```python
fs = 5
dt = 1/fs
t = np.arange(0, 1, dt)
x = np.sin(2*np.pi*10*t)
```

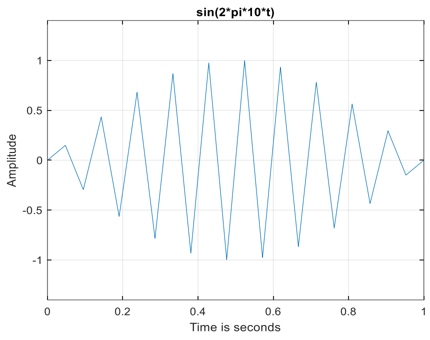
```python
plt.figure(figsize=(10, 8))
plt.plot(t, x, label='Sine wave')
plt.xlabel('Time is seconds', fontsize=15)
plt.ylabel('Amplitude', fontsize=15)
plt.title('sin(2*pi*10*t)', fontsize=15)
plt.legend(fontsize=10, loc='upper right')
plt.ylim([-1.4, 1.4])
plt.grid()
```

The 10 Hz Sine Wave

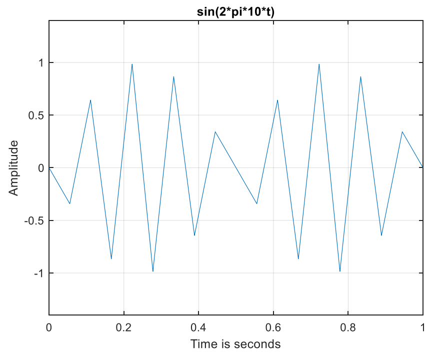
- We examined the characteristics of a 10 Hz sine wave sampled at frequencies of 1 kHz, 21 Hz, 18 Hz, 11 Hz, and 5 Hz.
  - Based on your observations, what conclusions can you draw about the relationship between the sampling frequency (Fs) and the signal frequency (Fmax)?



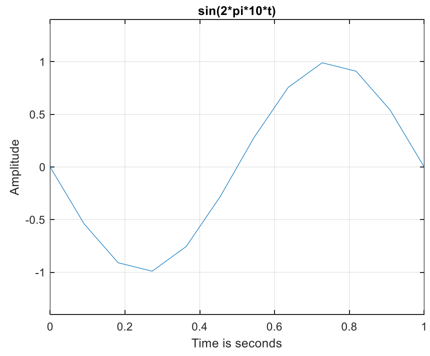
1KHz



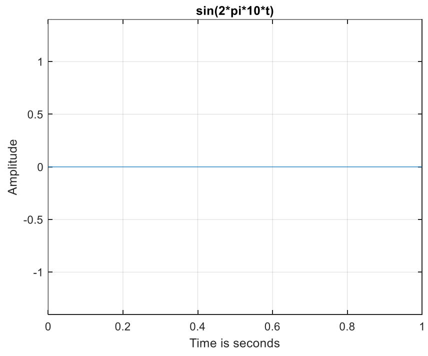
21Hz



18Hz



11Hz



5Hz

Nyquist Sampling Theorem

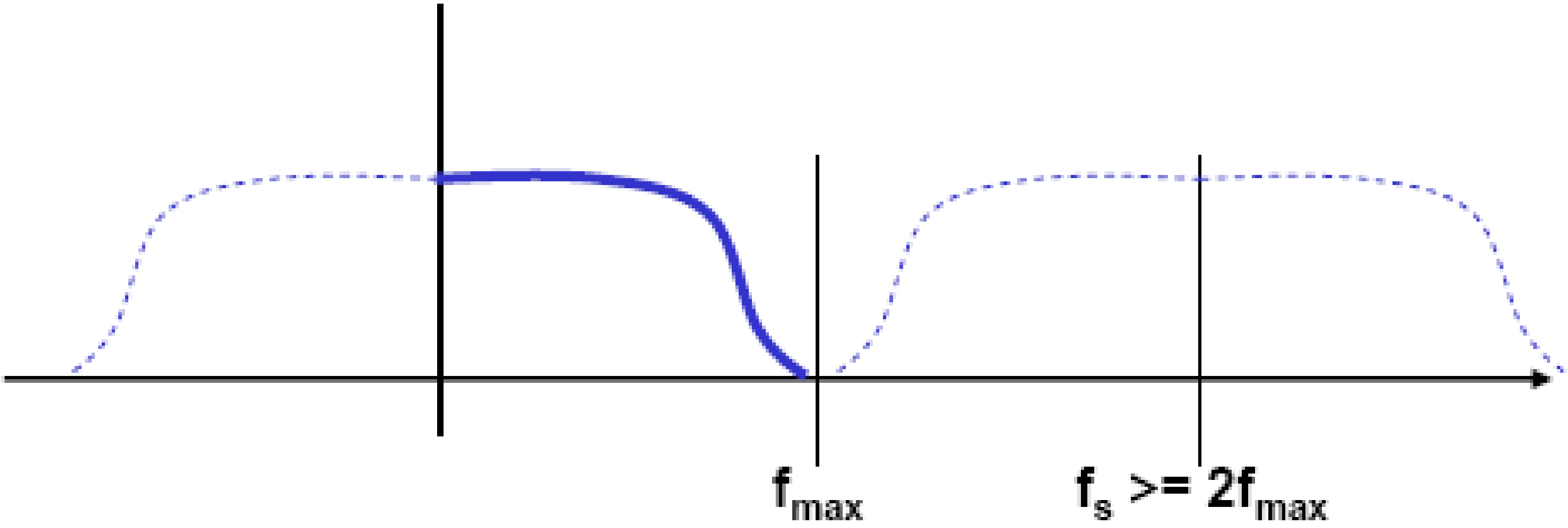
A continuous signal can be represented by, and reconstituted from, a set of sample values providing that the number of samples per second is at least twice the highest frequency presented in the signal.

$$f_S \geq 2 f_{\max}$$

$f_{\max}$ is the signal frequency (or the maximum signal frequency if there is more than one frequency in the signal)

f_S is the sampling rate

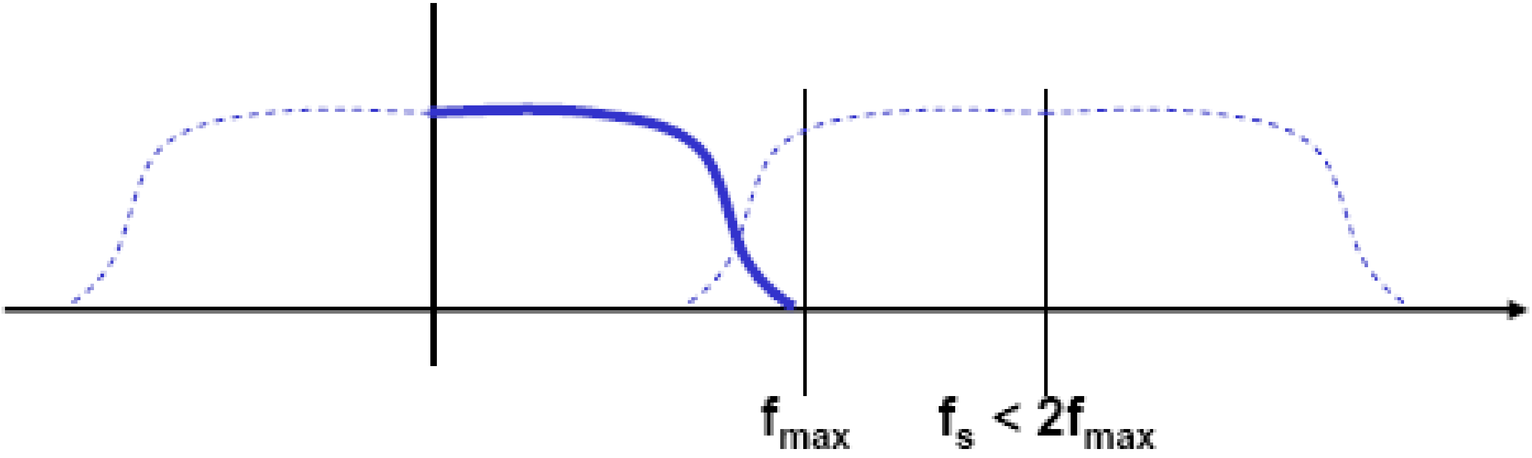
Nyquist Sampling Theorem



A signal that has energy to f_{max} must be sampled at a rate ($2 \times f_{max}$) or greater

Sampling creates an “alias” copy of a signal

Nyquist Sampling Theorem



A signal that has energy to f_{max} must be sampled at a rate ($2 \times f_{max}$) or greater

Sampling creates an "alias" copy of a signal

If the sampling rate is less than twice the highest frequency, the alias overlaps the original, creating distortion

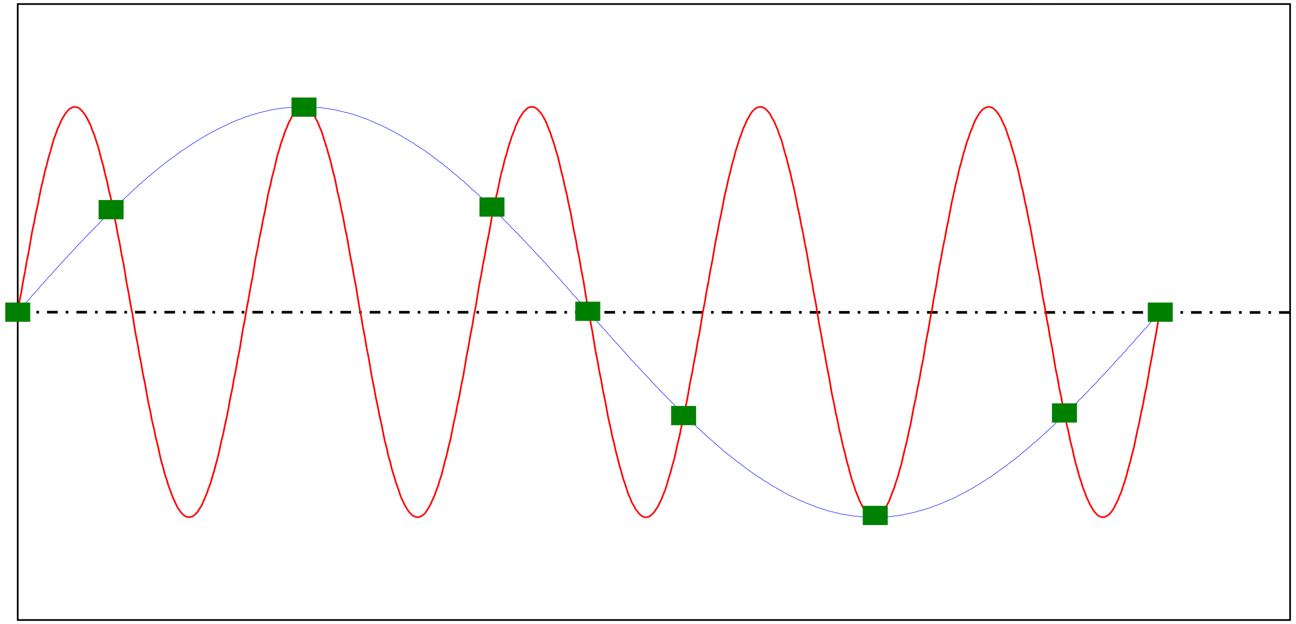
Discussion Question

Nyquist sampling theorem tells us that the sampling frequency should be *at least twice* the highest frequency presented in the signal to be able to resemble the pattern of the original signal.

Use your commonsense knowledge, to determine what is the proper sampling frequency if you are requested to design a vibration monitoring system for a passenger car. Assume that the natural frequency of the car system is 10 Hz, the driving stimulated vibration on driver's seat is 120 Hz, and the **measurement system natural frequency** is 10 kHz.

What is the sampling frequency you will select?

Nyquist Frequency and Aliasing



High frequency signal to be sampled by a low sampling rate may cause to “fold” the sampled data into a false lower frequency signal. This phenomena is known as *aliasing*.

Nyquist Frequency and Aliasing

Definitions:

Sampling Time, T: Total measuring time of a signal $T = N$

Sampling Interval Δt: Time between two samples $\Delta t = N/f_s$ [sec]

Sample Rate : $1/\Delta t$, the number of samples per second

Nyquist Frequency, F_{nyq}: Maximum frequency that can be captured by a sample interval, Δt

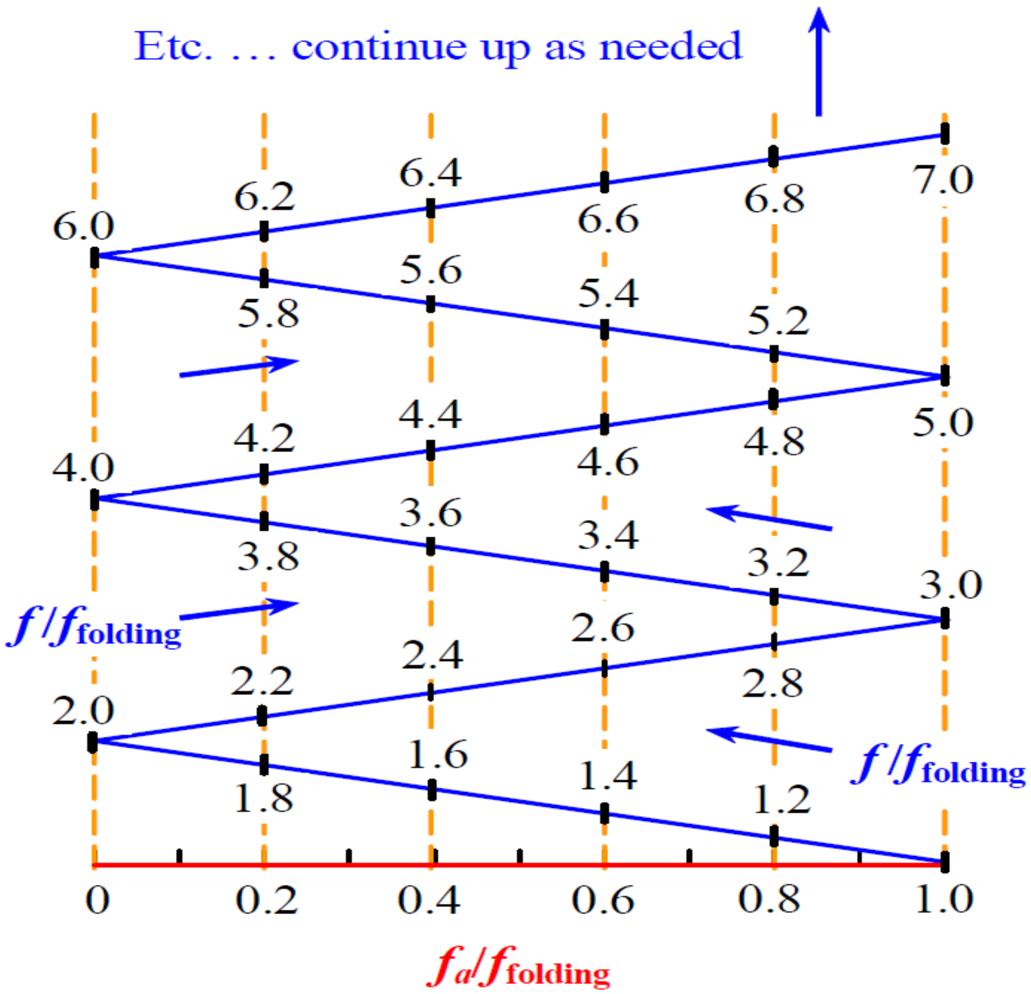
$$\boxed{f_{Nyq} = \frac{f_s}{2} \rightarrow f_s = \frac{1}{\Delta t}}$$

Resolution Bandwidth: Minimum frequency that can be represented by a sample

$$\boxed{\Delta f = \frac{1}{N\Delta t} = \frac{f_s}{N}}$$

Alias Frequency

- **A simple method to estimate alias frequencies** is by using the **folding diagram**.
  - Calculate the folding frequency, $f_{folding} = f_s /2$.
  - Locate $f/f_{folding}$ on the folding diagram. Note: For values of $f/f_{folding}$ greater than 5.0, the folding diagram can easily be extended, following the obvious pattern.
  - Read straight down from the value of $f/f_{folding}$ to obtain the value of $f_a/f_{folding}$ on the bottom (horizontal) axis.



Source: https://www.me.psu.edu/cimbala/me345/Exams/Folding_diagram_for_aliasing.pdf

Aliasing Formulas

- $f_{folding}$ = Folding Frequency (Nyquist Frequency)
- f_s = Sampling frequency
- $f_{folding} = f_s /2$
- f_a = Alias frequency
- $f_a = (f_a / f_{folding})\ f_{folding}$

Question 2

Given that sampling frequency equals 250Hz and the ratio of folding frequency to alias frequency equals 0.5, find the alias frequency.

Solution:

$f_s = 250$

$f_a/f_{folding} = 0.5$

$f_{folding} = f_s/2 = 250/2 = 125$

$f_a = (f_a / f_{folding}) f_{folding} = 0.5 * 125 = 62.5\text{Hz}$

Alias Frequency example

- Compute the lowest alias frequencies for the following cases:

1. $f = 80\ Hz\ and\ f_s = 100Hz$
2. $f = 100\ Hz\ and\ f_s = 60Hz$
3. $f = 100\ Hz\ and\ f_s = 250Hz$

Solution 1

- $f_{folding} = 100/2 = 50Hz$
- $f/f_{folding} = 80/50 = 1.6$

- Find $f/f_{folding}$ on the folding diagram, draw a vertical line down to the intersection with line AB, then read 0.4 on the line AB. The alias frequency can then be determined from

$$f_a = (f_a/f_{folding})f_{folding} = 0.4 * 50 = 20Hz$$

Solution 2

- $f_{folding} = 60/2 = 30Hz$
- $f/f_{folding} = 100/30 = 3.333$

- Find $f/f_{folding}$ on the folding diagram, draw a vertical line down to the intersection with line AB, then read 0.667 on the line AB. The alias frequency can then be determined from

$$f_a = (f_a/f_{folding})f_{folding} = 0.667 * 30 = 20Hz$$

Solution 3

- $f_{folding} = 250/2 = 125Hz$
- $f/f_{folding} = 100/125 = 0.8$

- Find $f/f_{folding}$ on the folding diagram, this falls on the line AB, then read 0.8 on the line AB. The alias frequency can then be determined from

$$f_a = (f_a/f_{folding})f_{folding} = 0.8 * 125 = 100Hz$$

- Which is the same as the sampled frequency

Summary

- Introduction to the Module
- Signals
- Sampling Theorem
- Aliasing
- Examples

Questions

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