Laboratory work 2

CONTROLLER DESIGN USING THE GUILLEMIN-TRUXAL METHOD

1. OBJECTIVES

- Study the method of controller design using the Guillemin-Truxal approach
- Understand the output variation y(t) for different performance sets
- Interpretation of the effects of imposed performances using simple control structures
- Understand the method of controller design using graphical representation

2. THEORETICAL APPROACH

For fast systems – where the Guileman-Truxal method for computing control structures is convenient – the performance set provides:

$$\begin{cases} \varepsilon_{stp} = 0 \\ \varepsilon_{stv} < \varepsilon_{stv}^{*} \\ t_r < t_r^{*} \\ \sigma < \sigma^{*} \\ \Delta\omega_B < \Delta\omega_B^{*} \end{cases}$$

The fundamental hypothesis on which the method is based, is to consider the equivalent second order transfer function as the closed system:

$$H_o(s) = H_{02} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (3.1)$$

We know the fixed part $H_f(s)$, usually in the form:

$$H_f(s) = \frac{K_f}{s(T_f s + 1)} \tag{3.2}$$

The controller can thus be determined:

$$H_R(s) = \frac{1}{H_f(s)} \cdot \frac{H_{02}(s)}{1 - H_{02}(s)} \tag{3.3}$$

Problem: to determine ξ and ω_n according to the performance set.

A recuring problem which consists of the approximation of the computed control structure – complicated, hard to implement – with a more simple and easy to implement controller, but which could lead to the fulfillement of the performance set. If the controller is computed with the above mentioned formulae, then the following results:

$$H_R(s) = \frac{\omega_n / 2\xi \cdot (T_f s + 1)}{K_f \cdot \left(1/2\xi\omega_n \, s + 1 \right)} \tag{3.4}$$

If $T_f \cong \dfrac{1}{2\xi\omega_n}$, a P controller, would meet the performance. The fallback is the necessity of validating the approximated controller.

In some cases, with processes having more than two poles (complex systems), different approximations of the transfer function that describes the fixed part can be done in order to obtain the form as given in equation (3.1). The most used method for the approximation of the dynamics of superior order is by simplificationof the transfer function, by determining the dominant poles.

3. COMPUTATIONAL STEPS

From the imposed value of the overshoot, $\sigma < \sigma^*$, the value of the damping ratio is computed, ξ, corresponding:

$$\boxed{\xi = \frac{|\ln(\sigma)|}{\sqrt{\ln^2(\sigma) + \pi^2}}} \tag{3.5}$$

Taking into account the value for the settling time, $t_r < t_r^*$, we compute the natural frequency, ω_n :

$$\boxed{\omega_n = \frac{4}{t_r \cdot \xi}} \tag{3.6}$$

The value of the velocity coefficient is verified considering the values computed at (3.5), and (3.6):

$$\boxed{c_v = \frac{\omega_n}{2 \cdot \xi}} \tag{3.7}$$

Using the velocity coefficient the steady state error can be computed :

$$\boxed{\varepsilon_{stv} = \frac{1}{c_v}} \tag{3.8}$$

If the value computed at (3.8) does not meet the performance criteria $\varepsilon_{stv} < \varepsilon_{stv}^*$, other values for the settling time and/or overshoot are chosen (smaller than the ones imposed in the performance specification), which should ensure the fulfillement of this requirement.

The fulfillement of the bandwidth requirement is verified using the equation:

$$\boxed{\Delta\omega_B = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}} \tag{3.9}$$

The final computations of the controller is done based on (3.3).

4. PROBLEMS

1. Consider the fixed parts: $K_f = 2$, $T_f = 2$ sec, the performance sets are:

$$a)\ \begin{cases} \varepsilon_{stp} = 0 \\ t_r^* \le 40\ \text{sec} \\ \sigma^* \le 15\% \\ c_v \ge 0{,}2 \\ \Delta\omega_B^* \le 2\ \text{rad/sec} \end{cases} ;\ b)\ \begin{cases} \varepsilon_{stp} = 0 \\ t_r^* \le 8\ \text{sec} \\ \sigma^* \le 15\% \\ c_v \ge 1 \\ \Delta\omega_B^* \le 2\ \text{rad/sec} \end{cases} ;\ c)\ \begin{cases} \varepsilon_{stp} = 0 \\ t_r^* \le 8\ \text{sec} \\ \sigma^* \le 10\% \\ c_v \ge 1{,}5 \\ \Delta\omega_B^* \le 1{,}2\ \text{rad/sec} \end{cases}$$

Compute the controllers using the Guilemin-Truxal method, if necessary simplify the controller structure and check the performance of the closed loop system using Matlab.