Laboratory work 4

CORRECTION IN GUILLEMIN-TRUXAL (ROOT LOCUS) METHOD

1. THE GOAL OF THE WORK

- To highlight the advantages and disadvantages of the correction
- Controller design using the Guillemin-Truxal method

2. THEORETICAL BACKGROUND

The second order structure $H_{02}(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ is simple, easy to use, but with limited possibilities. In the case of a demanding performance specification set, not all of the performance criteria can be met using just two tuning parameters, ξ and ω_n. In order to meet more performance specifications, additional tuning parameters are required. In this case, the closed loop transfer function is modified to include two extra parameters, a zero z_c and a pole p_c:

$$H_{0C}(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot \frac{s + z_c}{s + p_c} \cdot \frac{p_c}{z_c} \tag{4.1}$$

The closed loop performance specifications are given as:

$$\begin{cases} \varepsilon_{stp} = 0 \\ \varepsilon_{stv} < \varepsilon_{stv}^* \\ t_r < t_r^* \\ \sigma < \sigma^* \\ \Delta\omega_B < \Delta\omega_B^* \end{cases}$$

The steady state error to a step reference signal requirement is already met, based on the specific structure of the closed loop system in (4.1), where $H_{0C}(0)=1$. Then, the four parameters in (4.1) are used to meet the subsequent performance criteria.

The choice of the z_c and p_c parameters is performed in such a way as to have little effect upon the settling time and closed loop bandwidth. Nevertheless, the overshoot and the velocity error coefficient c_v (or steady state error to a ramp input) will be affected by the additional dipole (p_c, z_c).

3. DESIGN STEPS

The design of the controller is performed indirectly, based on the design of the closed loop system $H_{0C}(s)$.

Step 1. A proper ratio $\frac{p_c}{z_c} \cong (1{,}0 \div 1{,}10)$ is selected. The overshoot that corresponds to this dipole is then computed according to $\Delta\sigma_c \cong \frac{p_c}{z_c} - 1\,[\%]$.

Step 2. Determine the overshoot that corresponds to the second order system in the closed loop transfer function $H_{0C}(s)$, according to

$$\boxed{\sigma_2 = \sigma^* - \Delta\sigma}$$

where σ^* is the imposed overshoot given in the performance set.

Determine the damping ratio ξ based on the overshoot σ_2 and then the natural frequency ω_n of the second order system according to the imposed settling time.

Step 3. Verify that the closed loop bandwidth meets the performance criteria. The closed loop bandwidth is computed based on the previously computed ξ and ω_n. If this is not met, go back to Step 2 (or even Step 1) to recompute ξ and ω_n until the closed loop bandwidth is met (Iterative procedure similar to Guillemin Truxal method).

Step 4. Determine the velocity coefficient that corresponds to the second order system in the closed loop transfer function: $c_{v2} = \frac{\omega_n}{2\xi}$. The additional dipole will be used to correct this value.

The following system of two equations is used, where the second equation is obtained according the computation of the steady state error to a ramp imput for the $H_{0C}(s)$ closed loop transfer function (see Lecture 4 for explanations):

$$\begin{cases} \dfrac{p_c}{z_c} = 1 + \Delta\sigma_c \\[2ex] \dfrac{1}{c_v^*} = \dfrac{1}{c_{v2}} - \dfrac{1}{z_c} + \dfrac{1}{p_c} \end{cases} \tag{4.2}$$

Overall steady state error for $H_{0C}(s)$

Steady state error for dipole

According to (4.2), an adequate selection of z_c and p_c will correct the c_{v2} value and increase it up to the imposed value c_v^*. Based on (4.2), the pole and zero are determined as:

$$\boxed{\begin{cases} p_c = \dfrac{\Delta\sigma_c}{2\dfrac{\xi}{\omega_n} - \dfrac{1}{c_v^*}} \\[3ex] z_c = \dfrac{p_c}{1 + \Delta\sigma_c} \end{cases}} \tag{4.3}$$

The controller can be determined from the equation: $\boxed{H_R(s) = \frac{1}{H_f(s)} \cdot \frac{H_{0C}(s)}{1 - H_{0C}(s)}}$. The resulting controller is complex, but it can be simplified according to:

a) $\frac{Ts+1}{s+\beta} = \frac{Ts+1}{\beta\left(\frac{1}{\beta}s+1\right)} \cong \frac{1}{\beta}$ if $\frac{T}{1/\beta} \in \left(\frac{1}{5} \div 5\right)$

b) a small time constant is attached to a large time constant using the equation: $(Ts+1)(T_1s+1) \cong (T+T_1)s+1$, if $T >> T_1$.

c) $\frac{Ts+1}{\beta\left(1/\beta\, s+1\right)} \cong \frac{1}{\beta}\left[\left(T - \frac{1}{\beta}\right)s+1\right]$ if $T > 5 \cdot \frac{1}{\beta}$, respectively $\frac{Ts+1}{\beta\left(1/\beta\, s+1\right)} \cong \frac{1}{\beta\left[\left(\frac{1}{\beta} - T\right)s+1\right]}$

if $T < \frac{1}{5} \cdot \frac{1}{\beta}$.

Since the simplifications lead to a different structure of the controller ($H_{Rsim}(s)$) the overall performance specifications are verified based on the modified closed loop system:

$$\boxed{H_{0C}(s) = \frac{H_f(s)H_{Rsim}(s)}{1 + H_f(s)H_{Rsim}(s)}}$$

3. PROBLEMS

For the process described by $H_f(s) = \frac{K_f}{s(T_f s+1)}$, with $K_f = 2$, $T_f = 5\sec$ and the performance indicators set

$$\begin{cases} \varepsilon_{stp} = 0 \\ t_r < 8\sec \\ \sigma \le 10\% \\ c_v \ge 1{,}5 \\ \Delta\omega_n \le 1{,}2 \mathrm{rad/sec} \end{cases}$$

design the simplest controller possible using the dipole correction method. Demonstrate by plots that the imposed performance indicator set is met. What is happening when the process parameters (K_f and T_f) are changing?

$$K_f = K_{fN} \pm 20\% K_{fN}$$
$$T_f = T_{fN} \pm 20\% T_{fN}$$