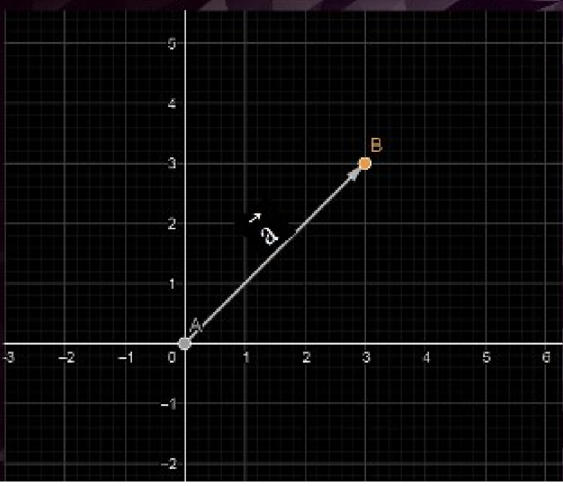
SCALAR

Real number, and represents a single value.

Eg: 24, 998 , 42, etc.

x, y, k , etc.

What is Vector ?



A - Initial Point

B - Terminal Point

AB - Magnitude

TYPES OF VECTORS

UnitVector

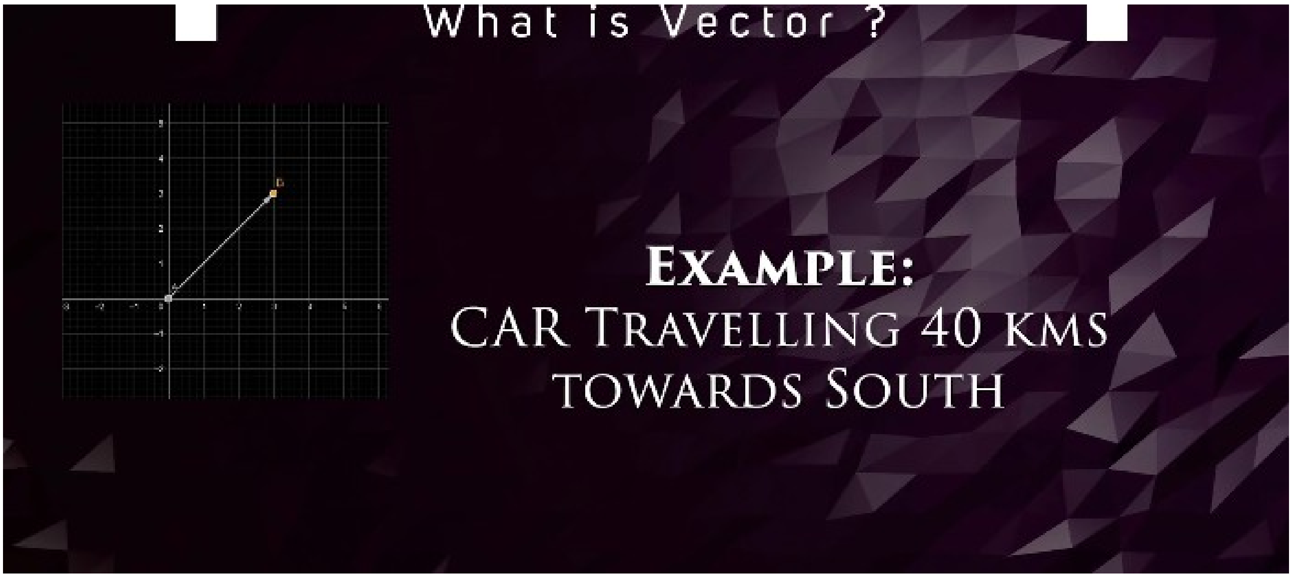
magnitude is equal to one

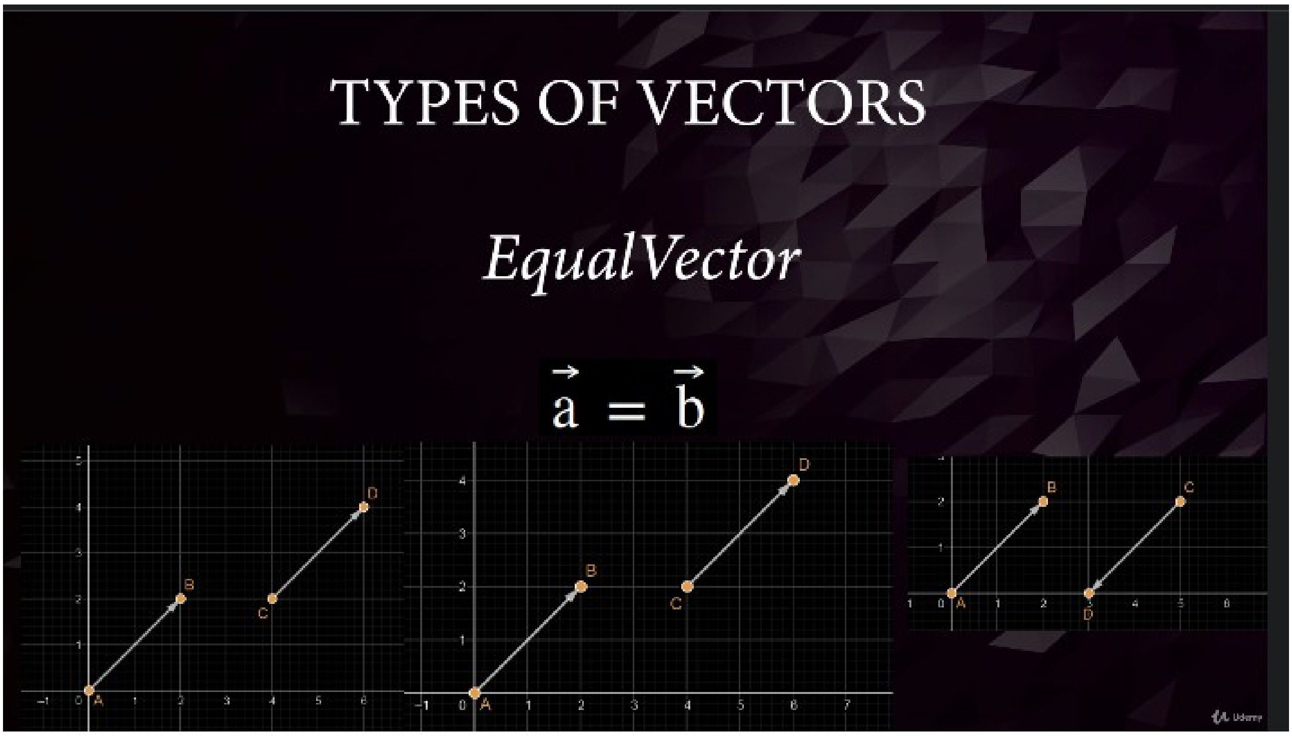
$\hat{a}$ $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

TYPES OF VECTORS

Zero Vector

initial and terminal points coincide.


What is Vector ?
EXAMPLE:
CAR TRAVELLING 40 KMS
TOWARDS SOUTH


TYPES OF VECTORS
EqualVector
$\vec{a} = \vec{b}$

TYPES OF VECTORS

Coinitial Vectors

Two or more vectors having same initial Points

Collinear Vectors

Two or more vectors paralell to same line irrespective of magnitude and direction

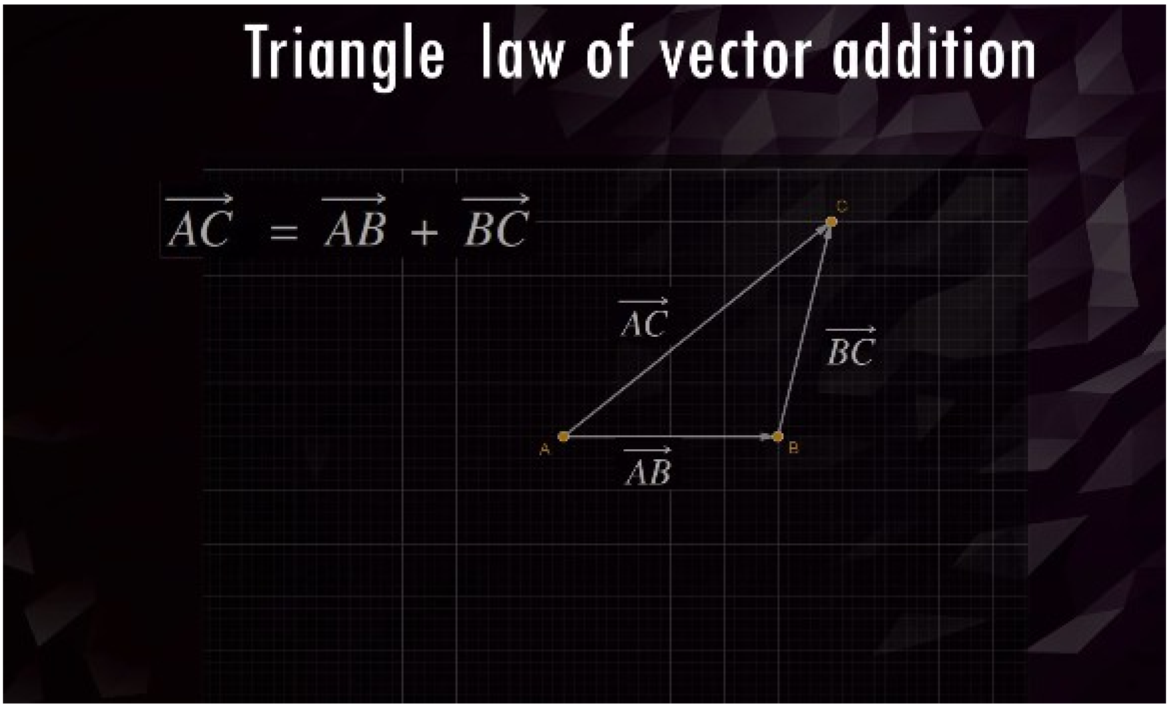
What is Vector ?

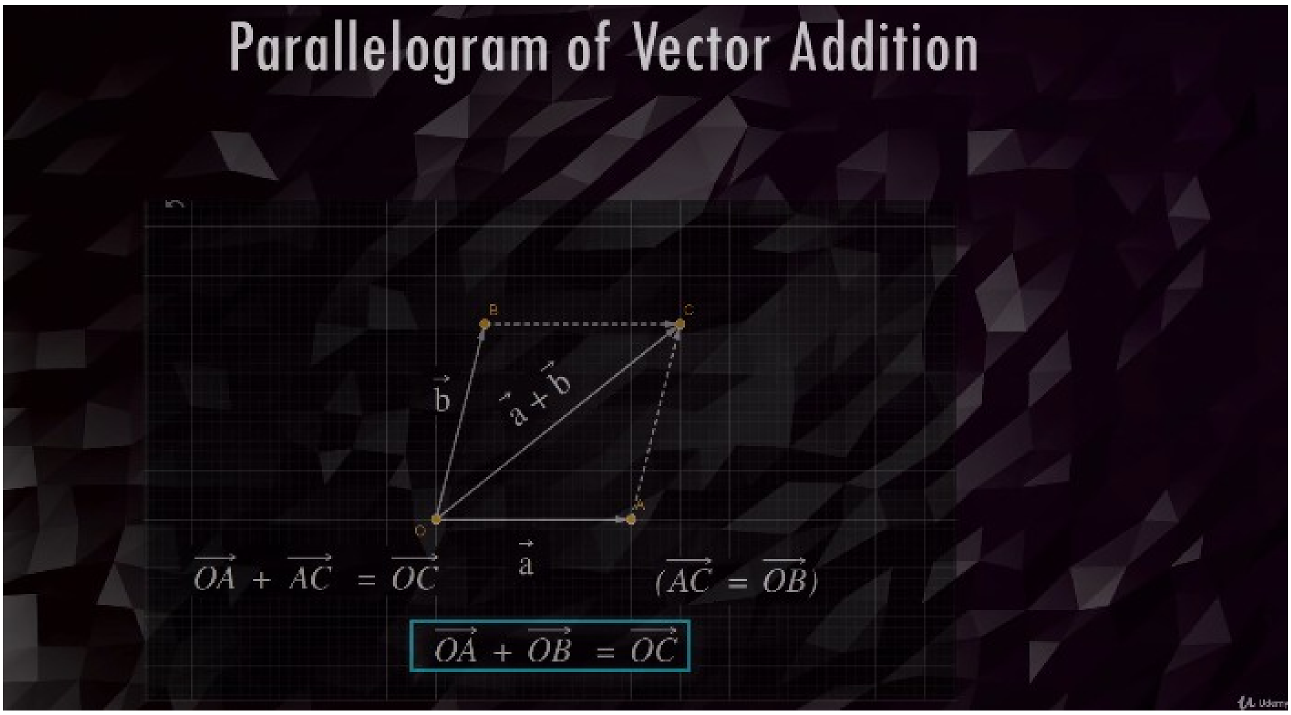
Objects that move around the Space

Car Travelling 50 kms towards south

LIST OF ATTRIBUTES OF AN OBJECT

$$\text{Iphone Xs} = \begin{bmatrix} 5.8 \\ 4 \\ 2915 \\ 13 \\ 1200 \end{bmatrix}$$


Triangle law of vector addition
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
C
$\overrightarrow{AC}$
$\overrightarrow{BC}$
A
B
$\overrightarrow{AB}$


Parallelogram of Vector Addition
B
C
$\vec{b}$
$\vec{a}+\vec{b}$
O
A
$\vec{a}$
$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$
$(\overrightarrow{AC} = \overrightarrow{OB})$
$\boxed{\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}}$

1. $\vec{a} = \begin{bmatrix} 8 \\ 13 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 26 \\ 7 \end{bmatrix}$

$$\vec{a} + \vec{b} = \begin{bmatrix} 8 \\ 13 \end{bmatrix} + \begin{bmatrix} 26 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8+26 \\ 13+7 \end{bmatrix}$$

$$= \underline{\begin{bmatrix} 34 \\ 20 \end{bmatrix}}$$

$$\begin{bmatrix} \\ 4+11 \end{bmatrix}$$

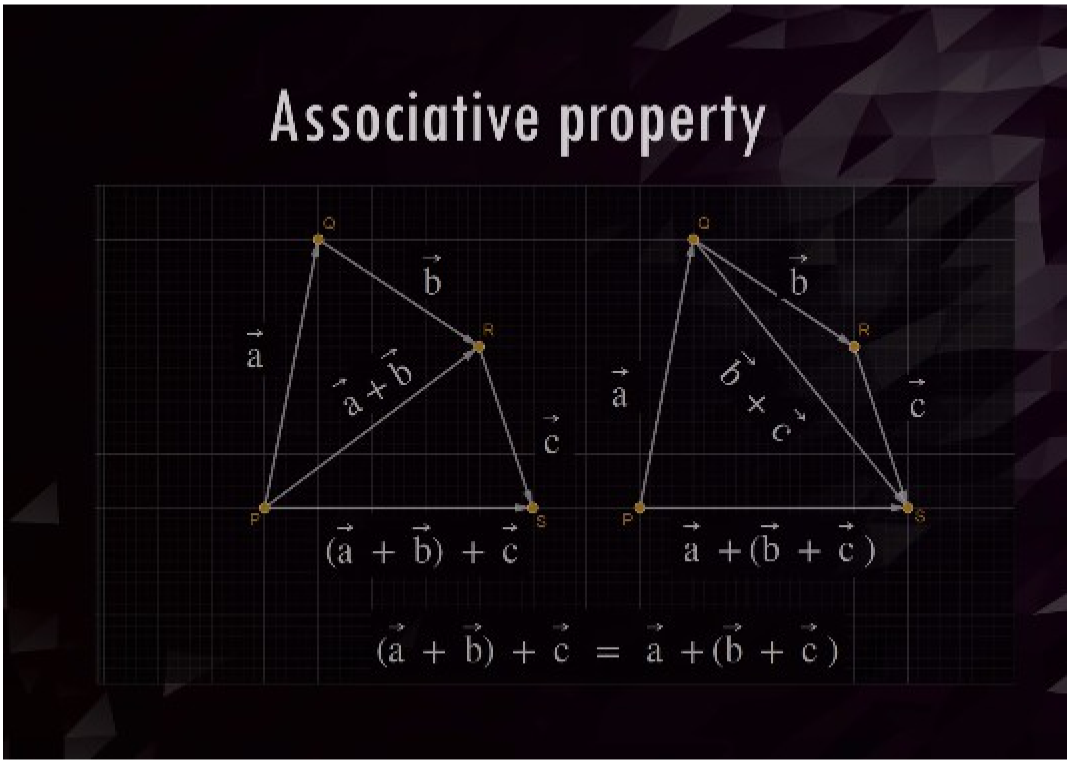
$$= \begin{bmatrix} 5 \\ 16 \\ 15 \end{bmatrix}$$

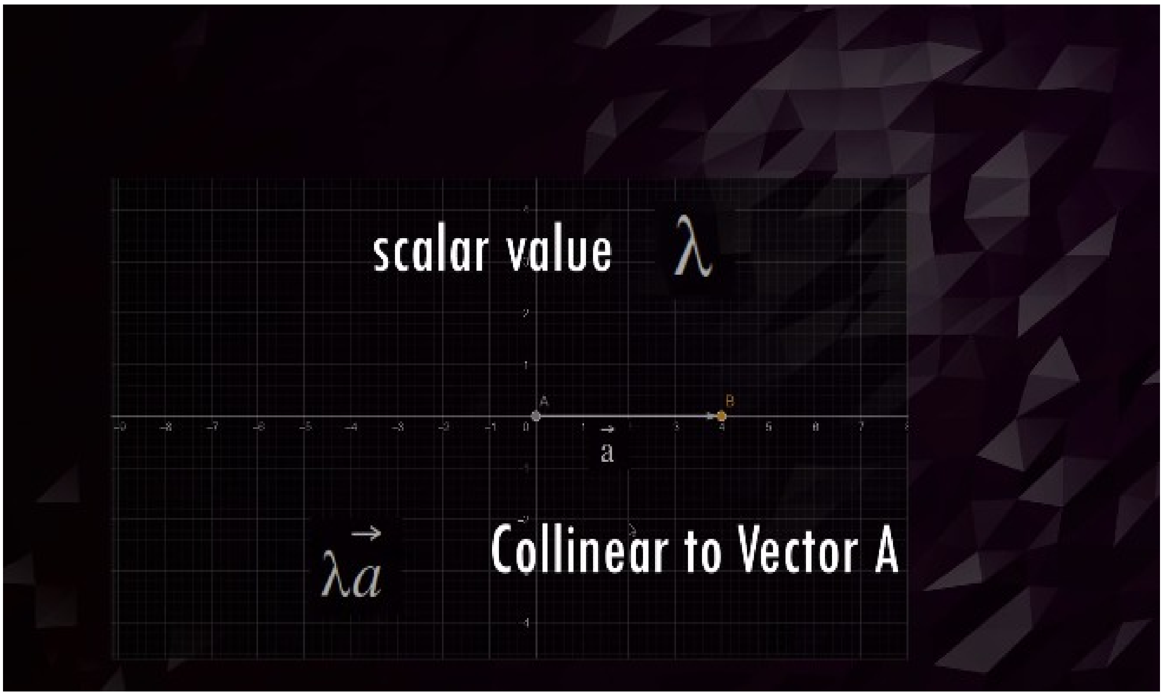
3. $\vec{a} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 54 \\ 34 \\ 20 \end{bmatrix}$ find $\vec{a} + \vec{b}$

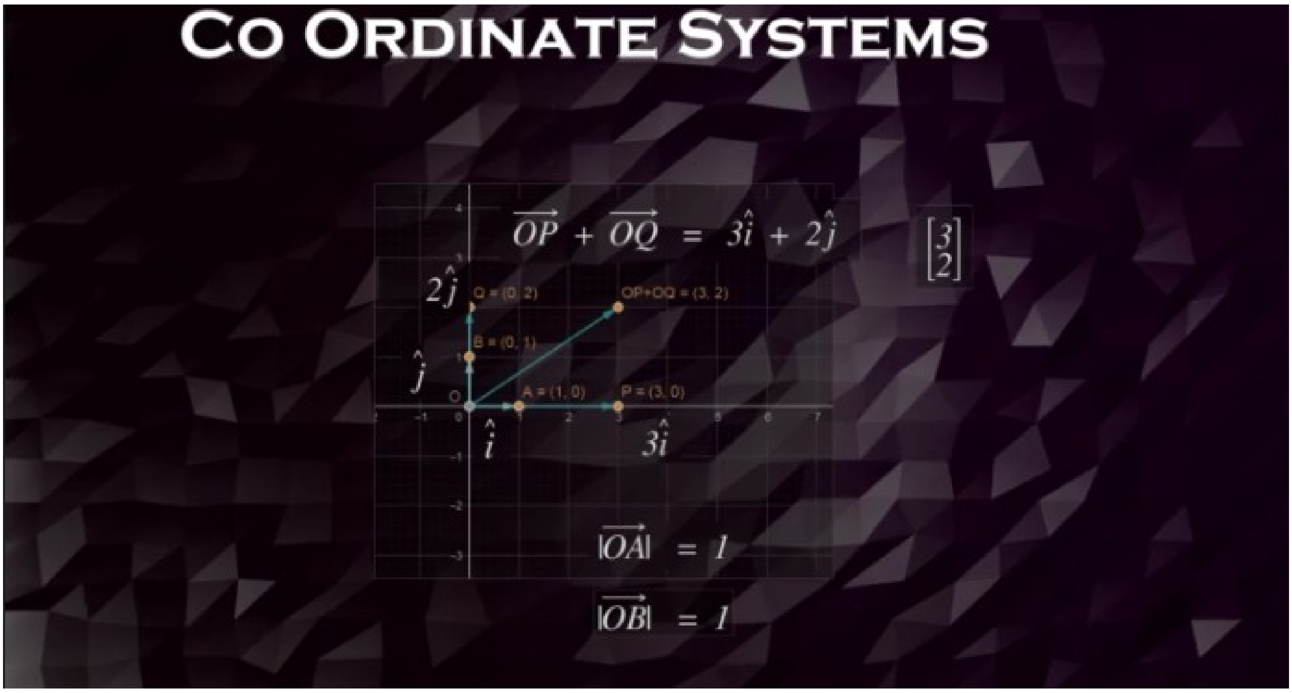
$\underline{2 \times 1} \qquad \underline{3 \times 1}$

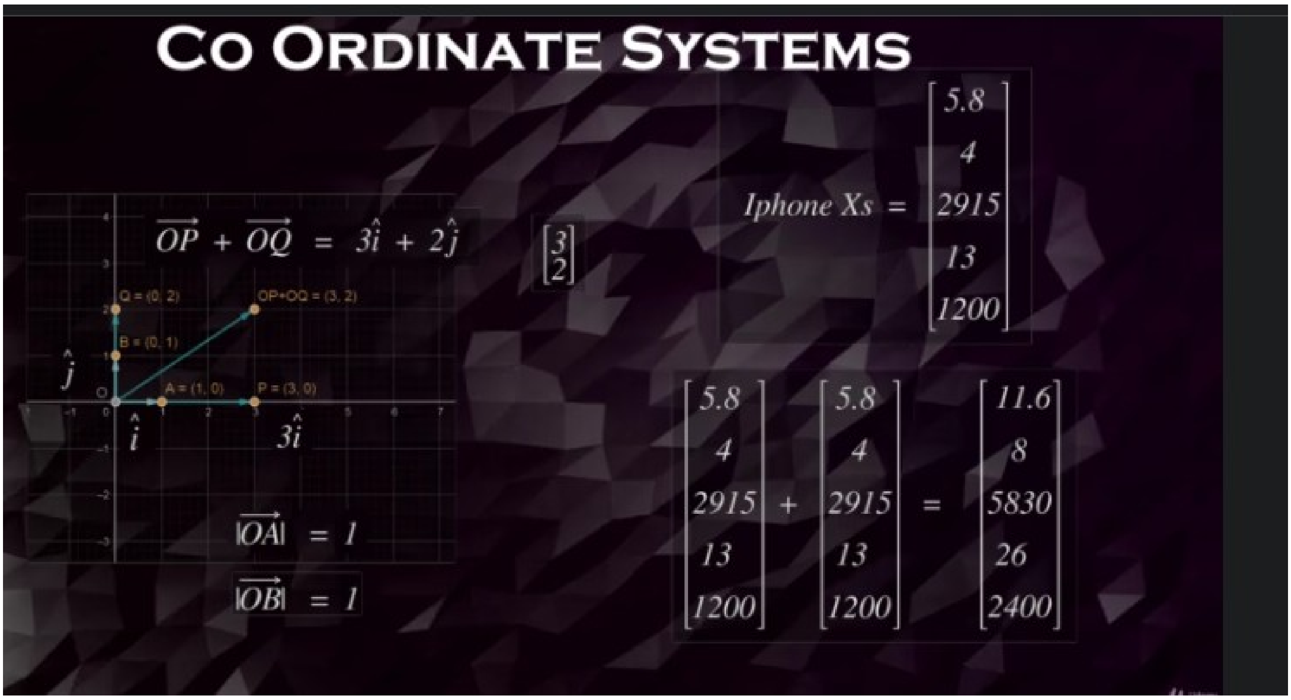
Not possible to add when

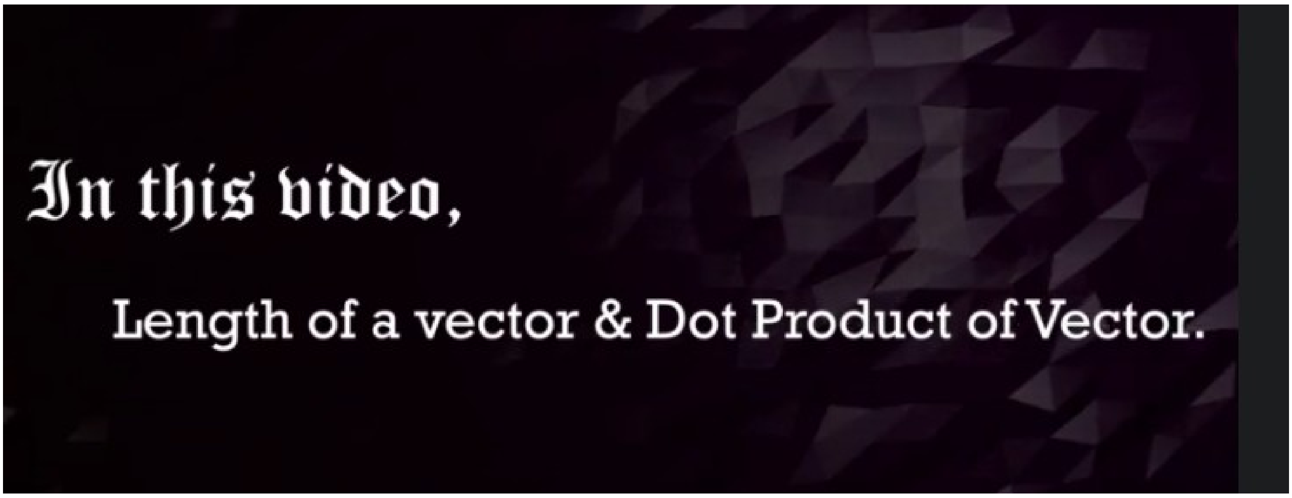
→ vectors has <u>different</u> <u>dimensions</u>

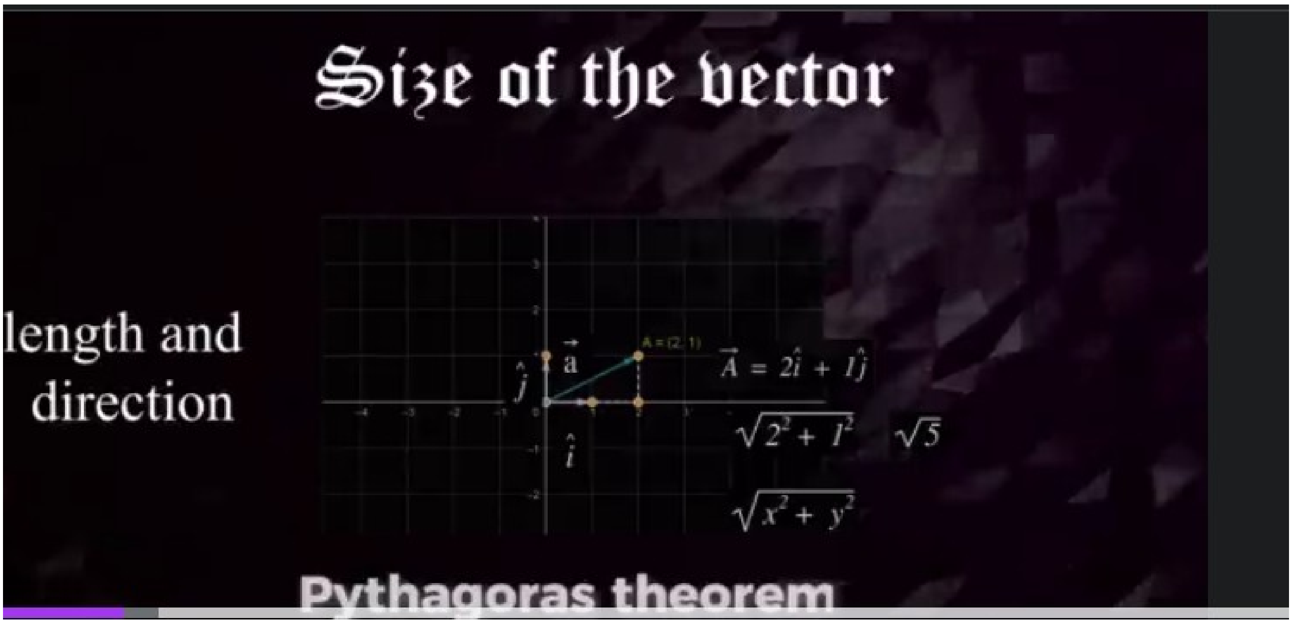

Associative property
Q
R
P
S
$\vec{a}$
$\vec{b}$
$\vec{c}$
$\vec{a}+\vec{b}$
$\vec{b}+\vec{c}$
$(\vec{a}+\vec{b})+\vec{c}$
$\vec{a}+(\vec{b}+\vec{c})$
$(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

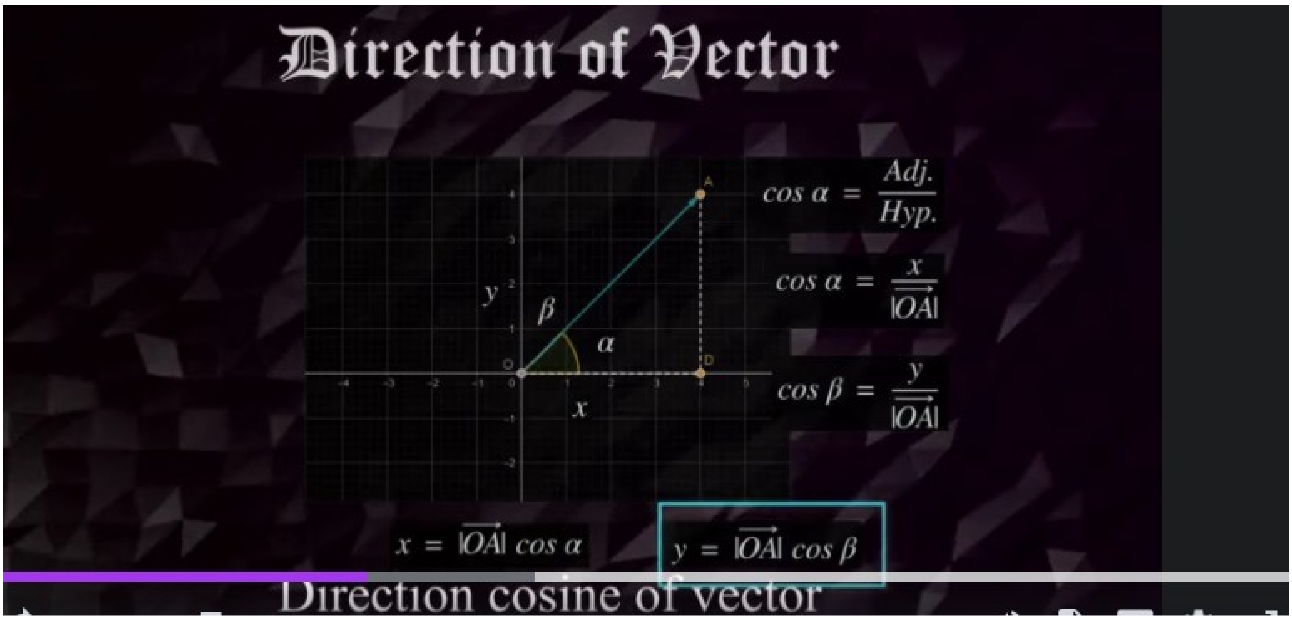

scalar value λ
A
B
$\vec{a}$
$\lambda\vec{a}$
Collinear to Vector A


Co Ordinate Systems
$\overrightarrow{OP} + \overrightarrow{OQ} = 3\hat{i} + 2\hat{j}$
$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$
$2\hat{j}$
Q = (0, 2)
OP+OQ = (3, 2)
B = (0, 1)
$\hat{j}$
A = (1, 0)
P = (3, 0)
$\hat{i}$
$3\hat{i}$
$|\overrightarrow{OA}| = 1$
$|\overrightarrow{OB}| = 1$


Co Ordinate Systems
Iphone Xs = $\begin{bmatrix} 5.8 \\ 4 \\ 2915 \\ 13 \\ 1200 \end{bmatrix}$
$\overrightarrow{OP} + \overrightarrow{OQ} = 3\hat{i} + 2\hat{j}$
$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$
Q = (0, 2)
OP+OQ = (3, 2)
B = (0, 1)
$\hat{j}$
A = (1, 0)
P = (3, 0)
$\hat{i}$
$3\hat{i}$
$|\overrightarrow{OA}| = 1$
$|\overrightarrow{OB}| = 1$
$\begin{bmatrix} 5.8 \\ 4 \\ 2915 \\ 13 \\ 1200 \end{bmatrix} + \begin{bmatrix} 5.8 \\ 4 \\ 2915 \\ 13 \\ 1200 \end{bmatrix} = \begin{bmatrix} 11.6 \\ 8 \\ 5830 \\ 26 \\ 2400 \end{bmatrix}$


In this video,
Length of a vector & Dot Product of Vector.


Size of the vector
length and
direction
$\vec{a}$
$\hat{j}$
$\hat{i}$
A = (2, 1)
$\vec{A} = 2\hat{i} + 1\hat{j}$
$\sqrt{2^2 + 1^2}$
$\sqrt{5}$
$\sqrt{x^2 + y^2}$
Pythagoras theorem


Direction of Vector
A
D
O
y
β
α
x
$\cos \alpha = \frac{Adj.}{Hyp.}$
$\cos \alpha = \frac{x}{|\overrightarrow{OA}|}$
$\cos \beta = \frac{y}{|\overrightarrow{OA}|}$
$x = |\overrightarrow{OA}| \cos \alpha$
$y = |\overrightarrow{OA}| \cos \beta$
Direction cosine of vector

Dot Product of Vector

When dot product is performed - resultant is scalar

$\vec{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$$\vec{x} \cdot \vec{y} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \overset{a_1 b_1}{(2 \times 3)} + (4 \times 5)$$

$$= 6 + 20$$

$$= \underline{\underline{26}}$$

Consider, $a = (4, 8, 10)$

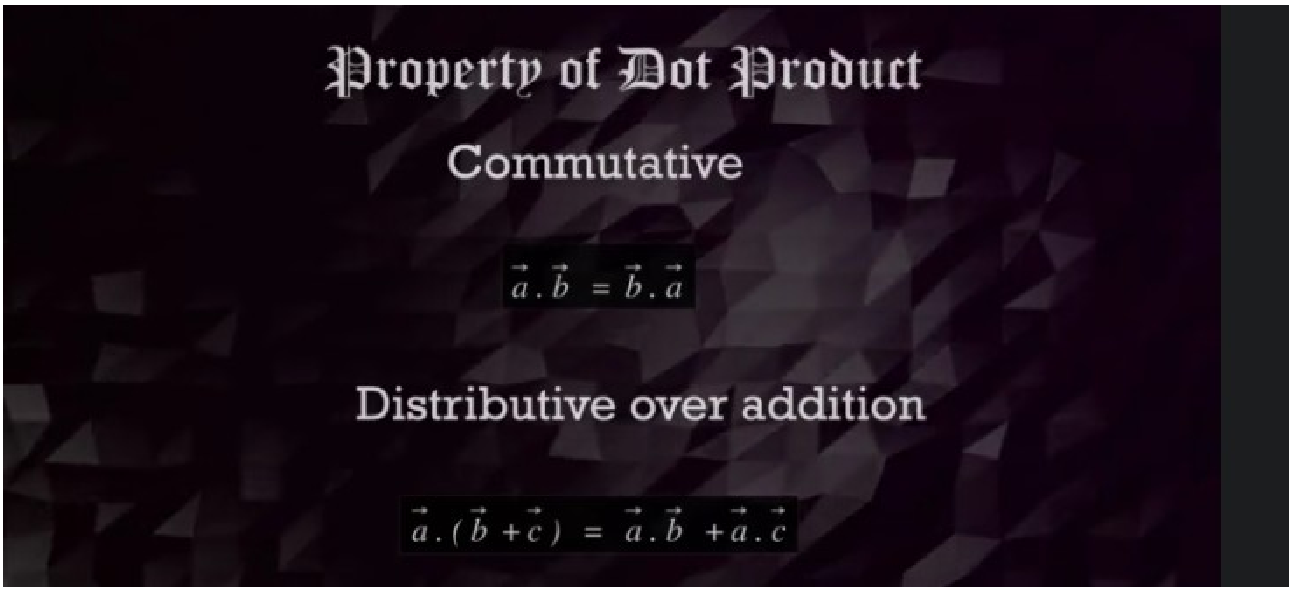
$b = (9, 2, 7)$

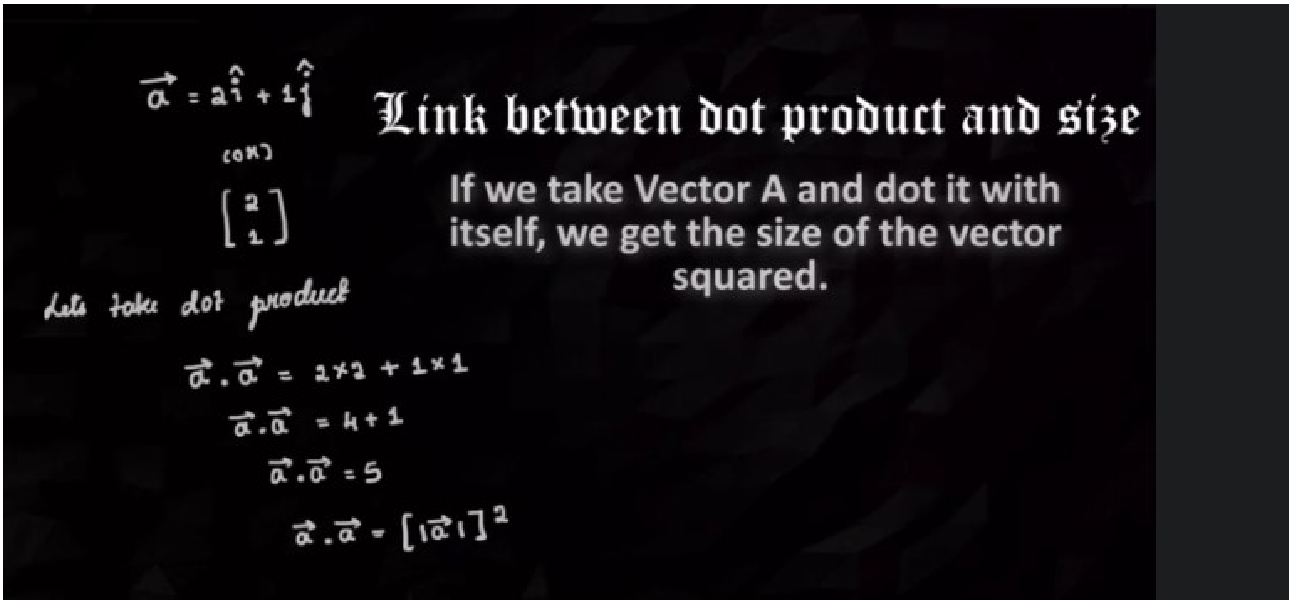
$a_1 b_1 + a_2 b_2 + a_3 b_3$

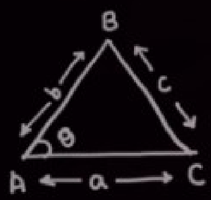
$a.b = 9 \times 4 + 2 \times 8 + 7 \times 10$

$a.b = 36 + 16 + 70$

$\Rightarrow a.b = \underline{\underline{122}}$

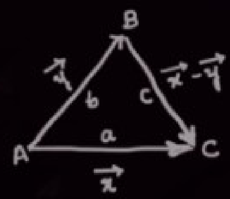

Property of Dot Product
Commutative
$\vec{a}.\vec{b} = \vec{b}.\vec{a}$
Distributive over addition
$\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$


$\vec{a} = 2\hat{i} + 1\hat{j}$
(OR)
$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
Lets take dot product
$\vec{a}.\vec{a} = 2\times2 + 1\times1$
$\vec{a}.\vec{a} = 4+1$
$\vec{a}.\vec{a} = 5$
$\vec{a}.\vec{a} = [|\vec{a}|]^2$
Link between dot product and size
If we take Vector A and dot it with itself, we get the size of the vector squared.

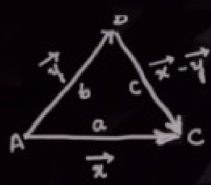


$\Rightarrow \quad c^2 = a^2 + b^2 - 2ab\cos\theta$

$\Longrightarrow$ Now lets translate to vector notation



$\Rightarrow \boxed{|x-y|^2} = |\vec{x}|^2 + |\vec{y}|^2 - 2|x||y|\cos\theta$



vector

$\Rightarrow |x-y|^2 = |\vec{x}|^2 + |\vec{y}|^2 = 2|x||y|\cos\theta$

consider LHS i.e $|x-y|^2$

$(\vec{x} - \vec{y}) \cdot (\vec{x} - \vec{y})$

$= \vec{x}\cdot\vec{x} - \vec{x}\cdot\vec{y} - \vec{y}\cdot\vec{x} + \vec{y}\cdot\vec{y}$

$= |\vec{x}|^2 - 2\vec{x}\cdot\vec{y} + |\vec{y}|^2$

$= \cancel{|\vec{x}|^2} + |\vec{y}|^2 - 2\vec{x}\cdot\vec{y} = \cancel{|\vec{x}|^2} + |\vec{y}|^2 - 2|x||y|\cos\theta$

$= |x| - 2x\cdot y + |y|$

$= \cancel{|\vec{x}|^2} + \cancel{|\vec{y}|^2} - \cancel{2}\vec{x}\cdot\vec{y} = \cancel{|\vec{x}|^2} + \cancel{|\vec{y}|^2} - \cancel{2}|x||y|\cos\theta$

$\vec{x}\cdot\vec{y} = |x||y|\cos\theta$

$\theta = 0, \quad \cos 0 = 1$

$\vec{x}\cdot\vec{y} = |x||y|$

If $\theta = 90^\circ$,

$\cos 90 = 0$

$\vec{x}\cdot\vec{y} = |\vec{x}||\vec{y}|\cdot 0$

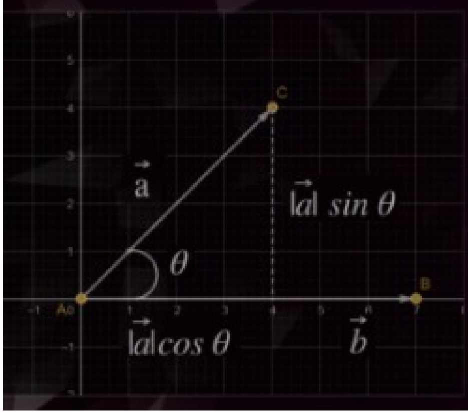
$= \underline{\underline{0}}$

If $\theta = 180^\circ$

$\cos 180 = -1$

Two types of projection

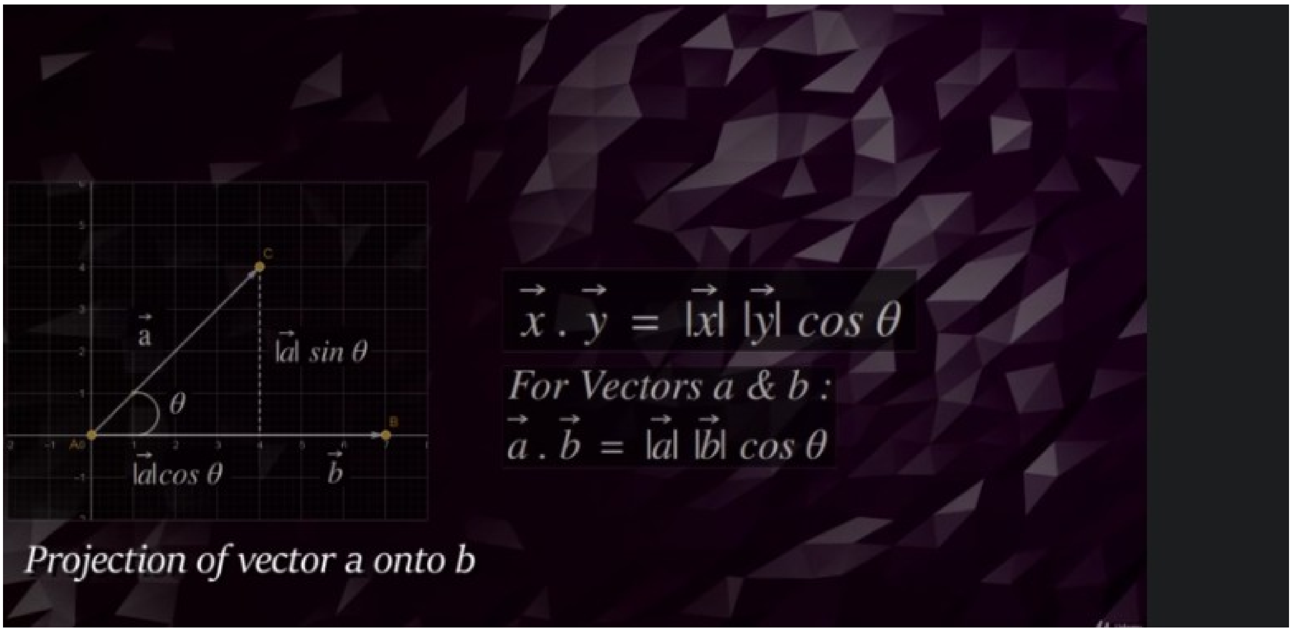
1. Scalar projection
2. Vector Projection



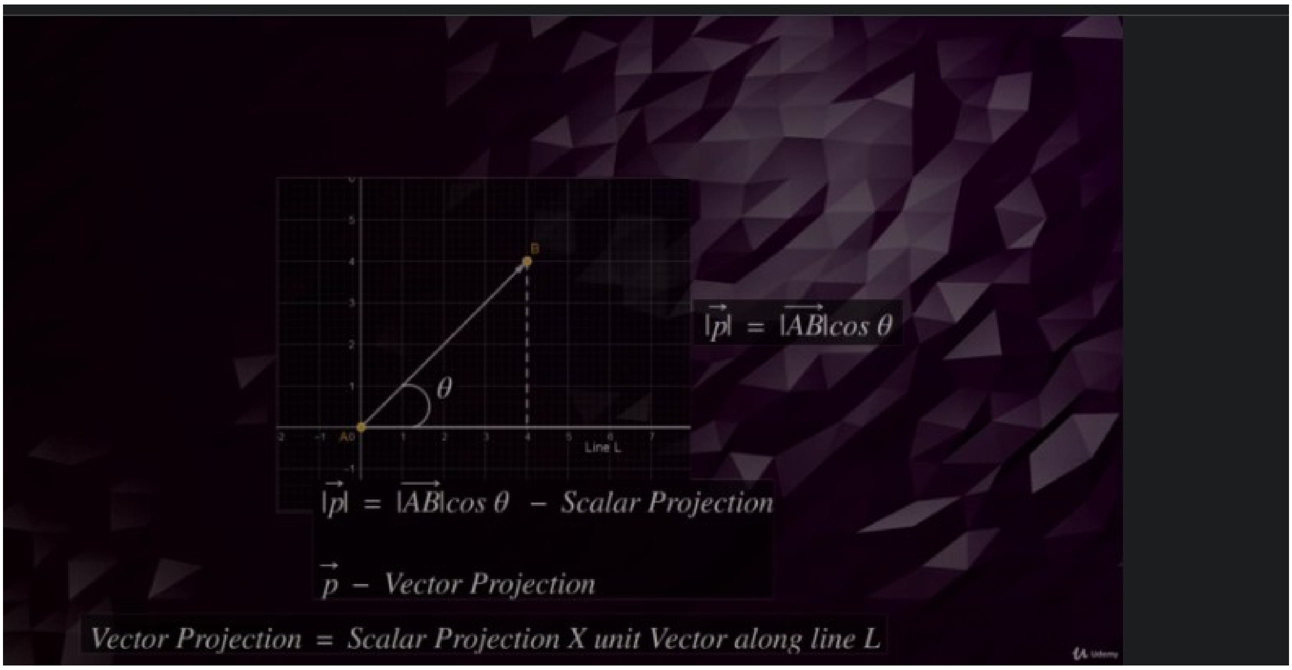
$$\vec{x} \cdot \vec{y} = |\vec{x}|\,|\vec{y}| \cos\theta$$

For Vectors a & b :

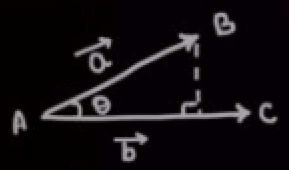
$$\boxed{\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta}$$



$\vec{a}$
$|\vec{a}|\, sin\, \theta$
θ
$|\vec{a}| cos\, \theta$
$\vec{b}$
$\vec{x} \,.\, \vec{y} \,=\, |\vec{x}|\, |\vec{y}|\, cos\, \theta$
For Vectors a & b :
$\vec{a} \,.\, \vec{b} \,=\, |\vec{a}|\, |\vec{b}|\, cos\, \theta$
Projection of vector a onto b



θ
Line L
$|\vec{p}| \,=\, |\overrightarrow{AB}| cos\, \theta$
$|\vec{p}| \,=\, |\overrightarrow{AB}| cos\, \theta$ – Scalar Projection
$\vec{p}$ – Vector Projection
Vector Projection = Scalar Projection X unit Vector along line L



$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$$|\vec{a}|\cos\theta = \vec{a} \cdot \left(\frac{\vec{b}}{|\vec{b}|}\right)$$

$$|\vec{a}|\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}.$$

$\vec{x} = 9\hat{i} + 3\hat{j} + 4\hat{k}$ $\qquad$ $\vec{y} = 2\hat{i} + 5\hat{j} + 7\hat{k}$

$\vec{x} = \begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix}$ $\qquad$ $\vec{y} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$

calculate vector projection of $\underline{\vec{x} \text{ onto } \vec{y}}$

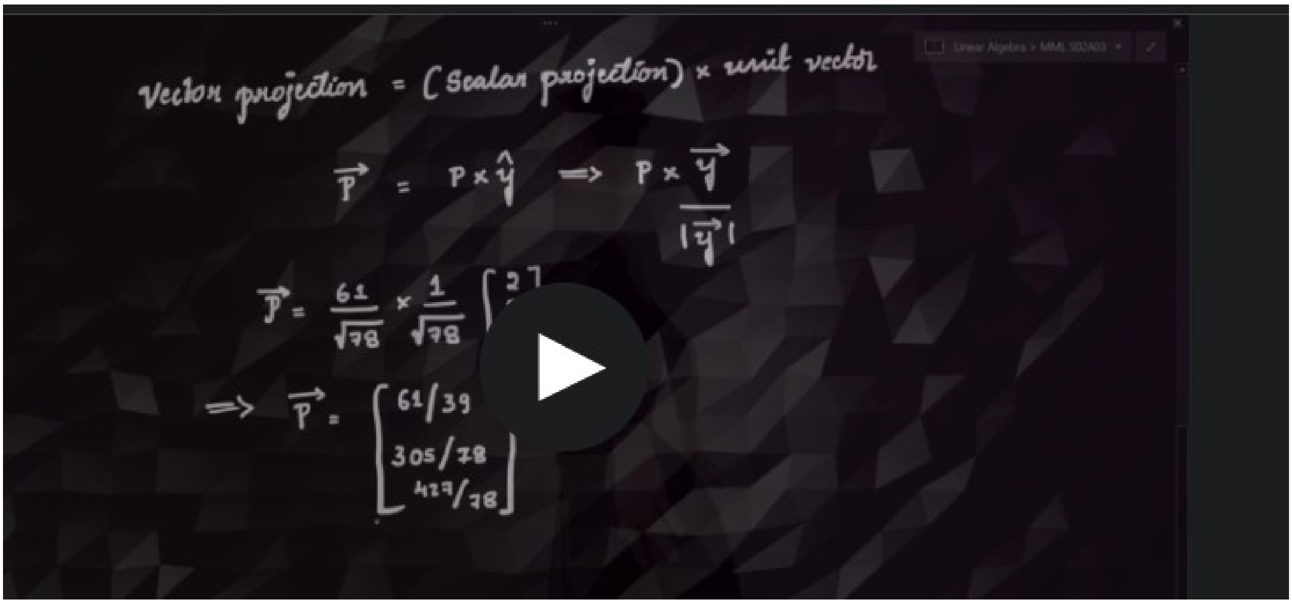
$$\begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix} = P \cdot |\vec{y}| \longrightarrow (9 \times 2) + (3 \times 5) + (4 \times$$

$$18 + 15 + 28 = P \cdot \sqrt{78}$$

$$\Rightarrow 61 = P \cdot \sqrt{78}$$

$$\left[\therefore\ P = \frac{61}{\sqrt{78}}\right]$$

This is magnitude (or)
Scalar projec



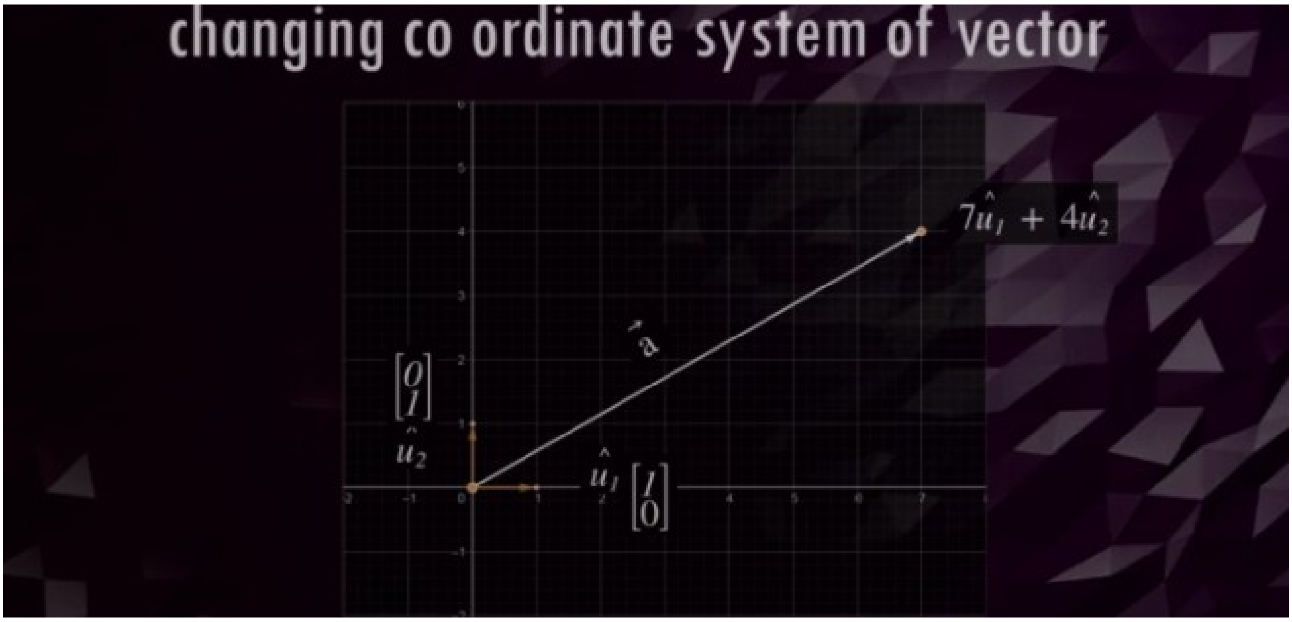
Vector Norm - Finding the Length of the vector

Many ways of finding the length of the vector

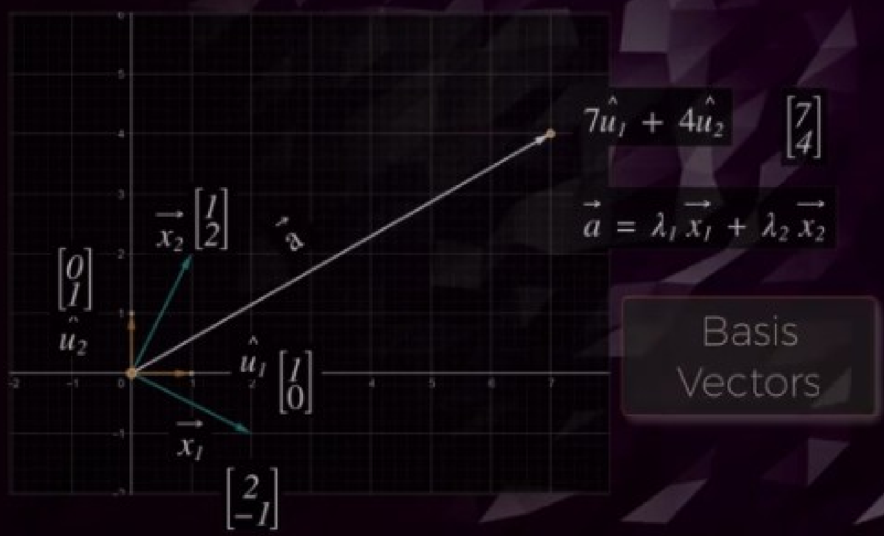
- L1 Norm - Lasso Regression (a.k.a Taxicab Norm or Manhattan Norm)
- L2 Norm - Ridge Regression (A.K.A Eucledian Norm)
- p Norm
- Vector Max Norm

L2 Norm

$$\|\mathbf{x}\|_2 = \left(\sum_{i=1}^{N} |x_i|^2\right)^{1/2} = \sqrt{x_1^2 + x_2^2 + \ldots + x_N^2}$$



changing co ordinate system of vector



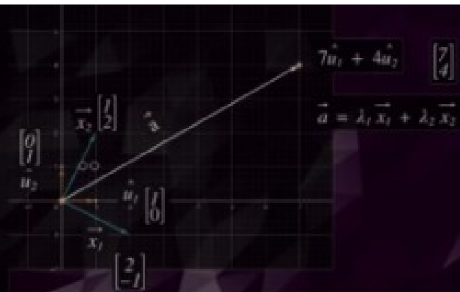
$$x_1 \cdot x_2 = |\vec{x_1}|\,|\vec{x_2}|\cos\theta$$

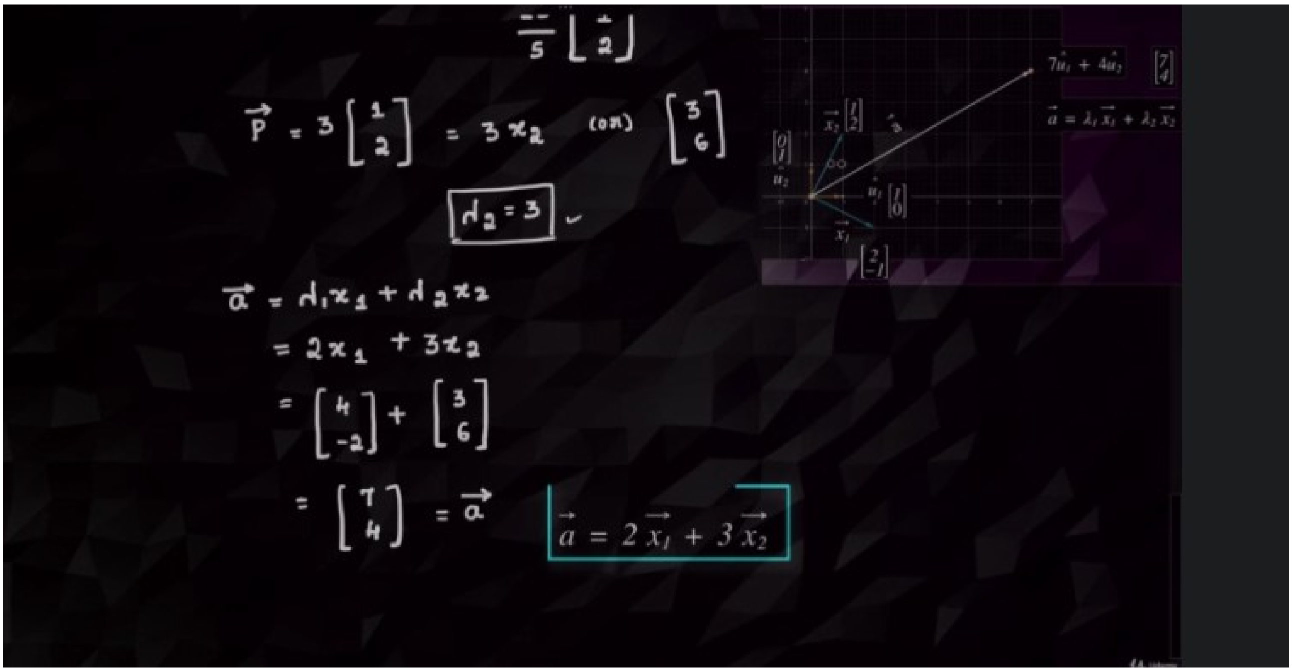
$$\cos\theta = \frac{\vec{x_1}\cdot\vec{x_2}}{|\vec{x_1}|\,|\vec{x_2}|} = \frac{\begin{bmatrix}2\\-1\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix}}{\sqrt{5}\cdot\sqrt{5}} = \frac{(2\times1)+(-1\times2)}{5} = \frac{2-2}{5} = \underline{\underline{0}}$$

$$\cos\theta = 0$$
$$\theta = \cos^{-1} 0$$
$$\boxed{\theta = 90^\circ}$$

$$\vec{P} = P\times\hat{x}_1 = P\times\frac{\vec{x_1}}{|\vec{x_1}|} = \frac{10}{\sqrt{5}}\times\frac{1}{\sqrt{5}}\times\begin{bmatrix}2\\-1\end{bmatrix}$$

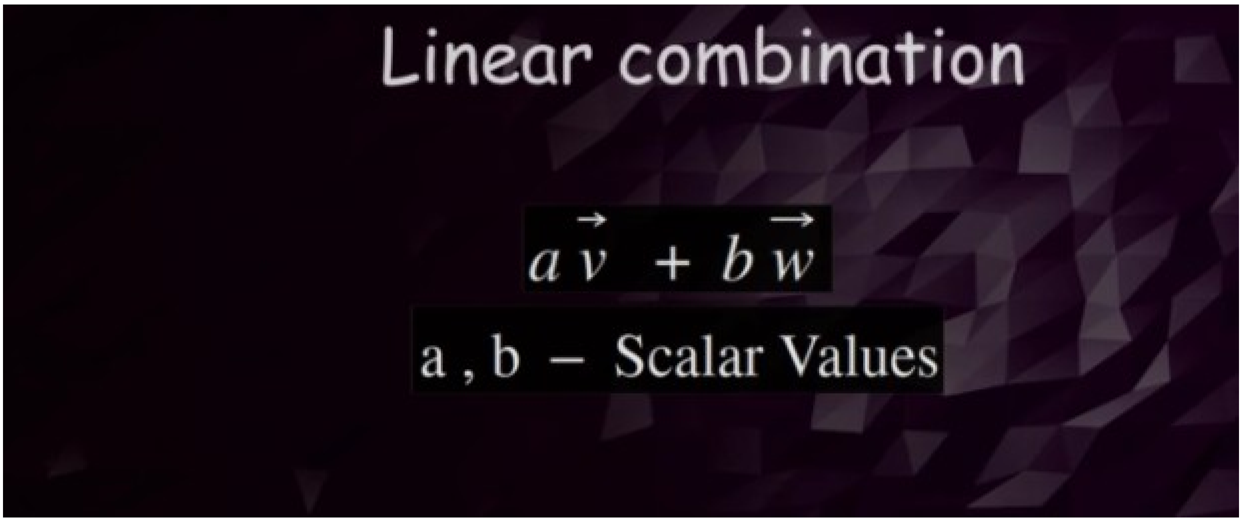
$$= \frac{10}{5}\times\begin{bmatrix}2\\-1\end{bmatrix} = 2\begin{bmatrix}2\\-1\end{bmatrix}$$
$$= 2x_1$$

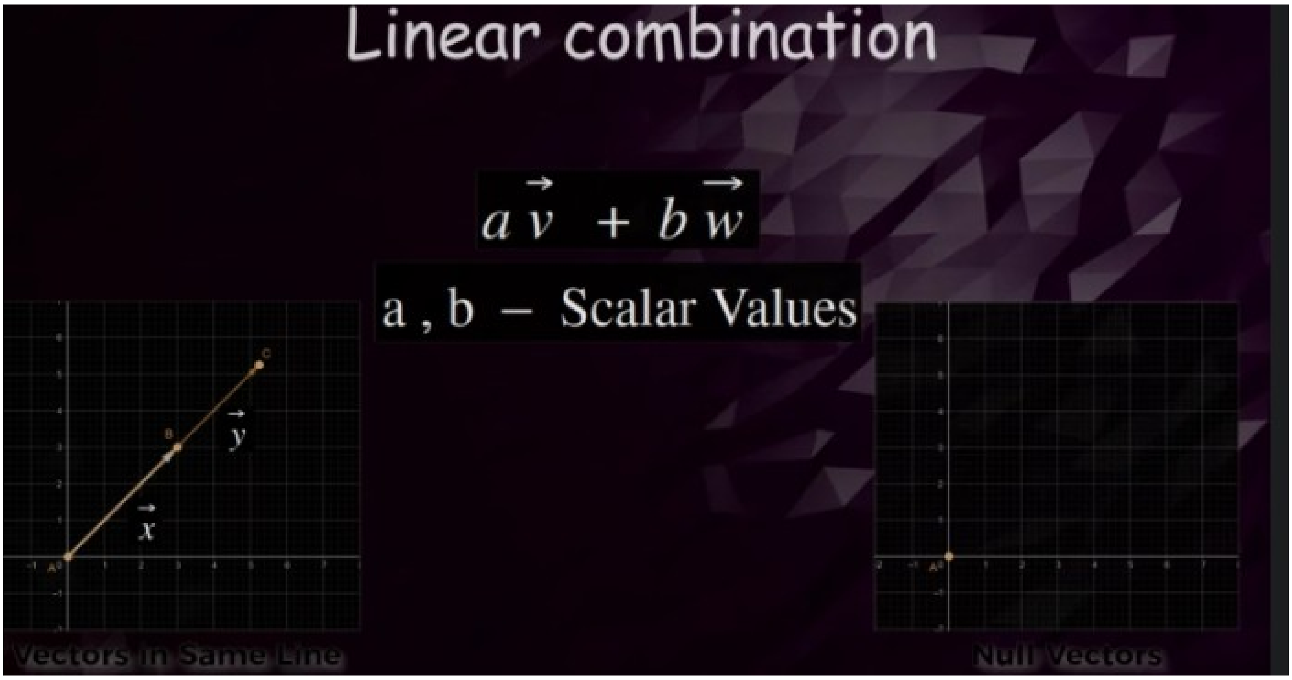




$\vec{P} = 3\begin{bmatrix}1\\2\end{bmatrix} = 3x_2$ (or) $\begin{bmatrix}3\\6\end{bmatrix}$
$\boxed{d_2 = 3}$
$\vec{a} = d_1 x_1 + d_2 x_2$
$= 2x_1 + 3x_2$
$= \begin{bmatrix}4\\-2\end{bmatrix} + \begin{bmatrix}3\\6\end{bmatrix}$
$= \begin{bmatrix}7\\4\end{bmatrix} = \vec{a}$
$\boxed{\vec{a} = 2\vec{x_1} + 3\vec{x_2}}$
$7\vec{u_1} + 4\vec{u_2}$
$\vec{a} = \lambda_1\vec{x_1} + \lambda_2\vec{x_2}$

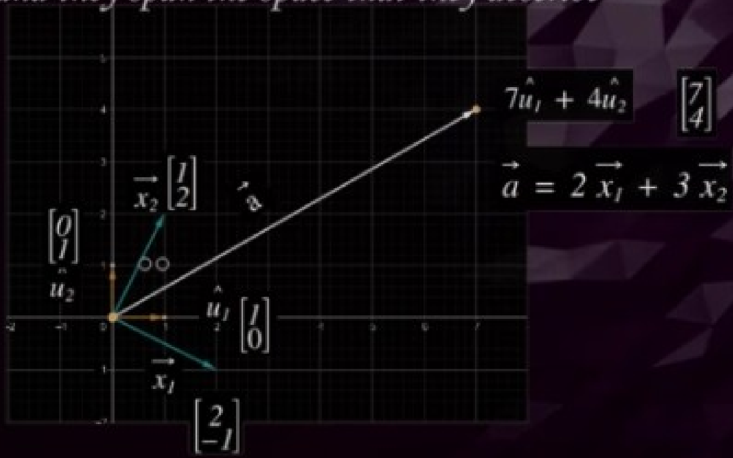
1. Linear Combination
2. Span &
3. Basis Vectors


Linear combination
$a\vec{v} + b\vec{w}$
a , b – Scalar Values


Linear combination
$a\vec{v} + b\vec{w}$
a , b – Scalar Values
$\vec{x}$
$\vec{y}$
Vectors in Same Line
Null Vectors

What is Basis ?

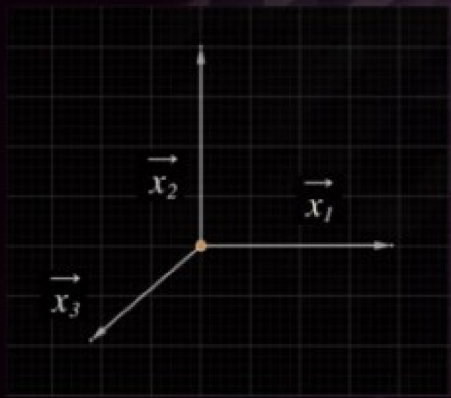
A basis is a set of n vectors that are linearly independent of each other, that means they are not linear combination of the two vectors, and they span the space that they describe



For the pair of vectors are to be basis vectors ,

* **It has to be Linearly Independent to each other**

Vector X3 is linearly independent of other two



Vector X3 is linearly independent of other two

$$\vec{x_3} \neq a_1\vec{x_1} + a_2\vec{x_2}$$

For the two vectors to become basis is:

1. They don't have to be unit vectors, that means vectors of length 1.
2. And they don't have to be orthogonal, that is they don't have to be at 90 degrees to each other.

Types of Matrices

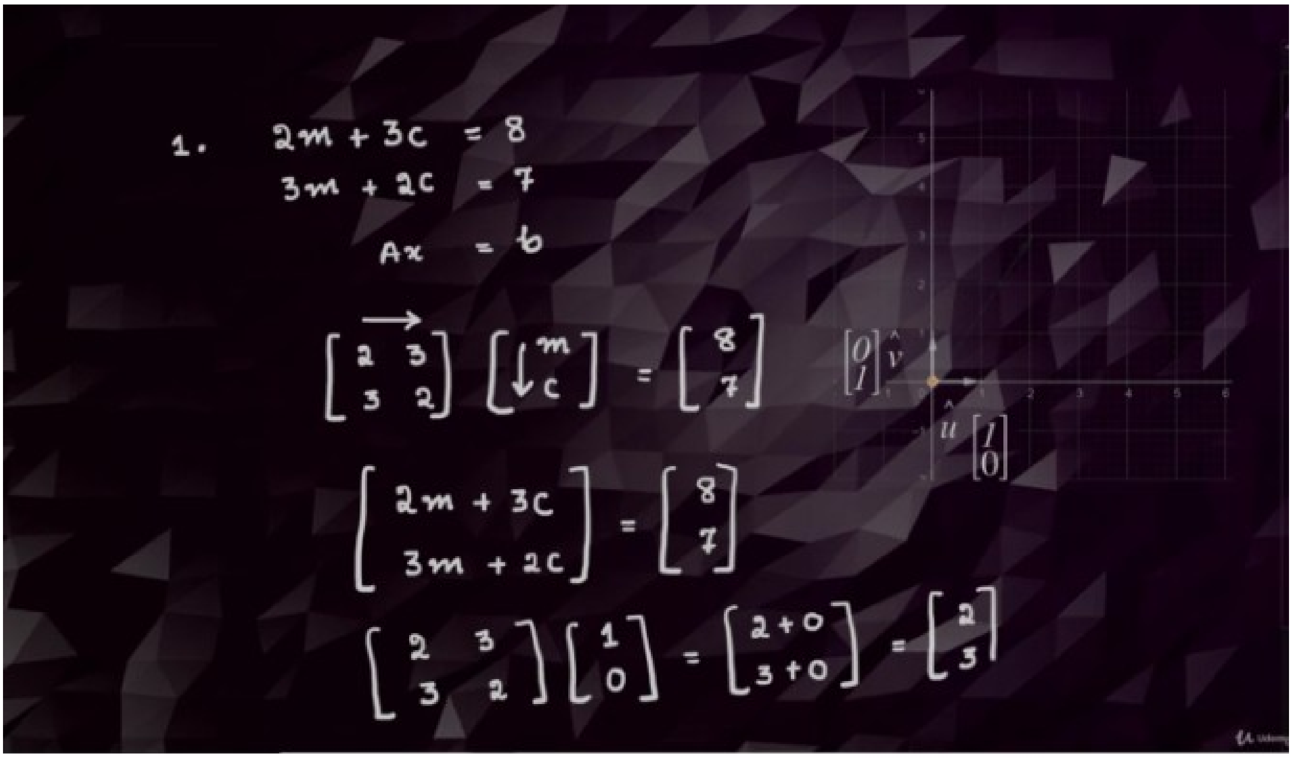
Lets look at the types of Matrices

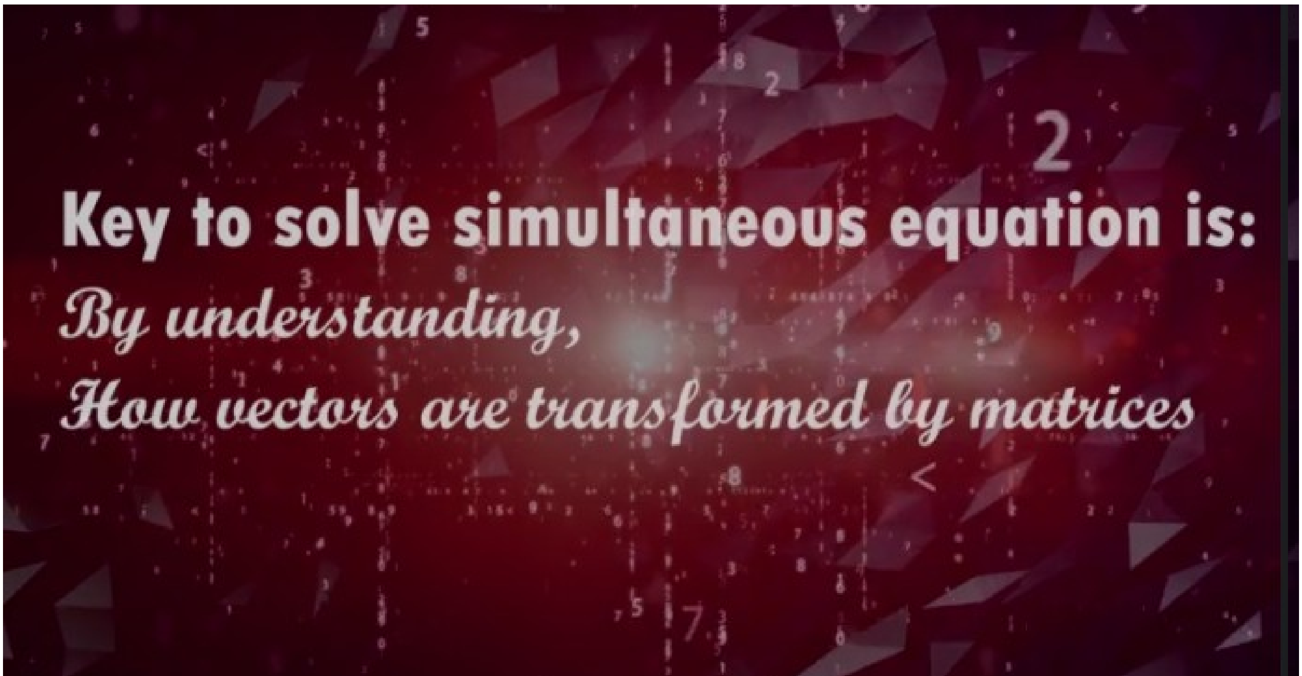
Column matrix , Row Matrix, Square matrix , Diagonal matrix, Scalar matrix,Identity matrix,Zero matrix

Addition of matrices, Subtraction of matrices , Multiplication of a matrix by a scalar:

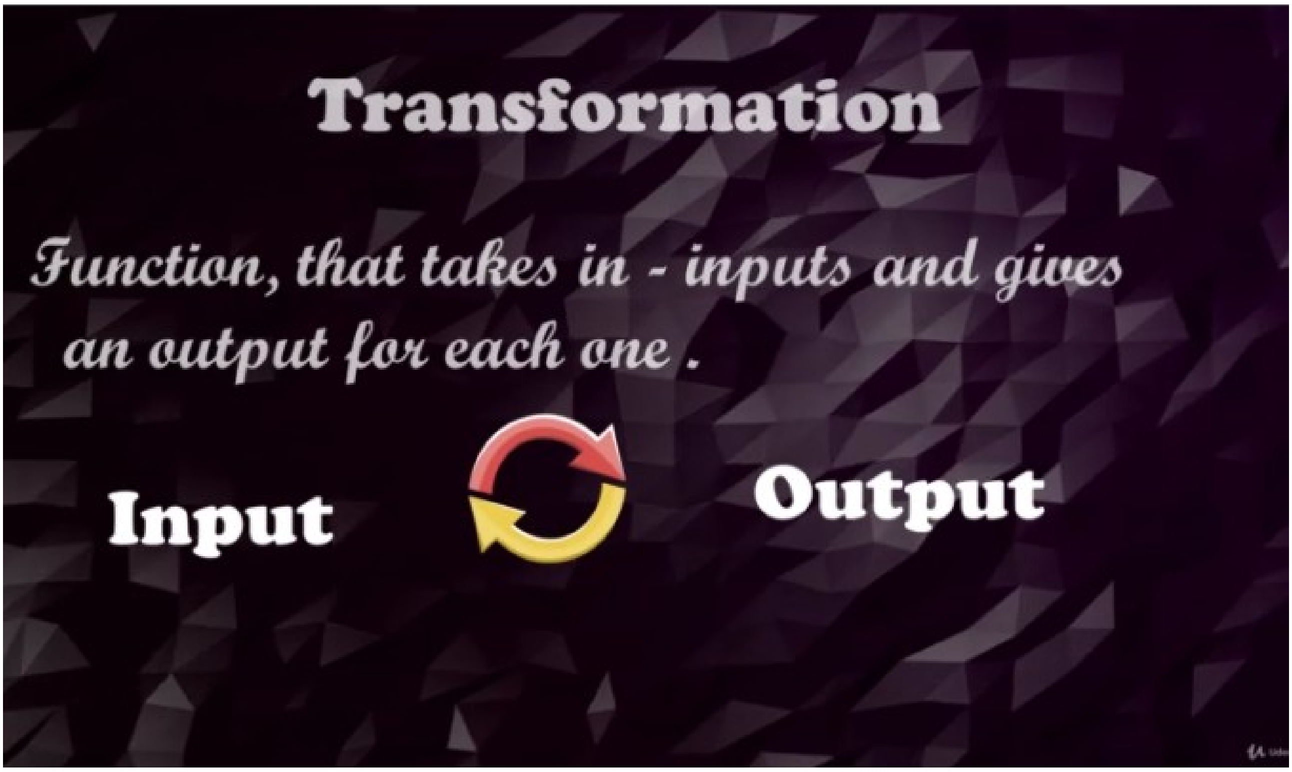
We can multiply the matrices only if the number of columns in the first matrix is the same as the number of rows in the second matrix.

The *transpose* of a matrix is simply a flipped version of the original matrix. We can transpose a matrix by switching its rows with its columns.


1. $2m + 3c = 8$
$3m + 2c = 7$
$Ax = b$
$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$
$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{v}$
$\hat{u} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$\begin{bmatrix} 2m + 3c \\ 3m + 2c \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$
$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 3+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$


Key to solve simultaneous equation is:
By understanding,
How vectors are transformed by matrices


From the last video:
* We have explored as how the matrix transform the unit basis vectors.
Lets understand how matrices transform the vectors.


Transformation
Function, that takes in - inputs and gives an output for each one .
Input
Output

1. $2m + 3c = 8$

$\quad 3m + 2c = 7$

$$Ax = b$$

Transformation Matrix $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \boxed{\begin{bmatrix} 8 \\ 7 \end{bmatrix}}$ Output Vector

Input Vector

$$\left.\begin{aligned} 1x - 1y &= 1 \\ 2x + 1y &= 5 \end{aligned}\right\}$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

lets solve for $\underline{x \,\&\, y}$

$$\begin{aligned} 1x - \cancel{1y} &= 1 \\ 2x + \cancel{1y} &= 5 \\ \hline 3x &= 6 \\ [x &= 2] \end{aligned}$$

$$\begin{aligned} 1x - 1y &= 1 \\ 1(2) - 1y &= 1 \\ 2 - 1y &= 1 \\ -y &= -2 + 1 \end{aligned}$$

-1

$$\vec{a} = m\hat{u} + n\hat{v}$$
$$= 2\hat{u} + 1\hat{v}$$

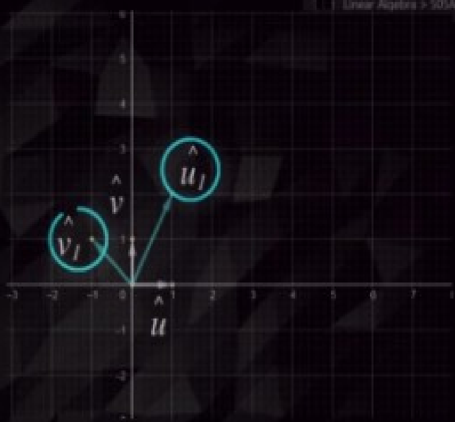
$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \hat{u}_1$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \boxed{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \hat{v}_1$$

$$\vec{a} = m\hat{u} + n\hat{v}$$
$$= 2\hat{u} + 1\hat{v}$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \hat{u}_1$$

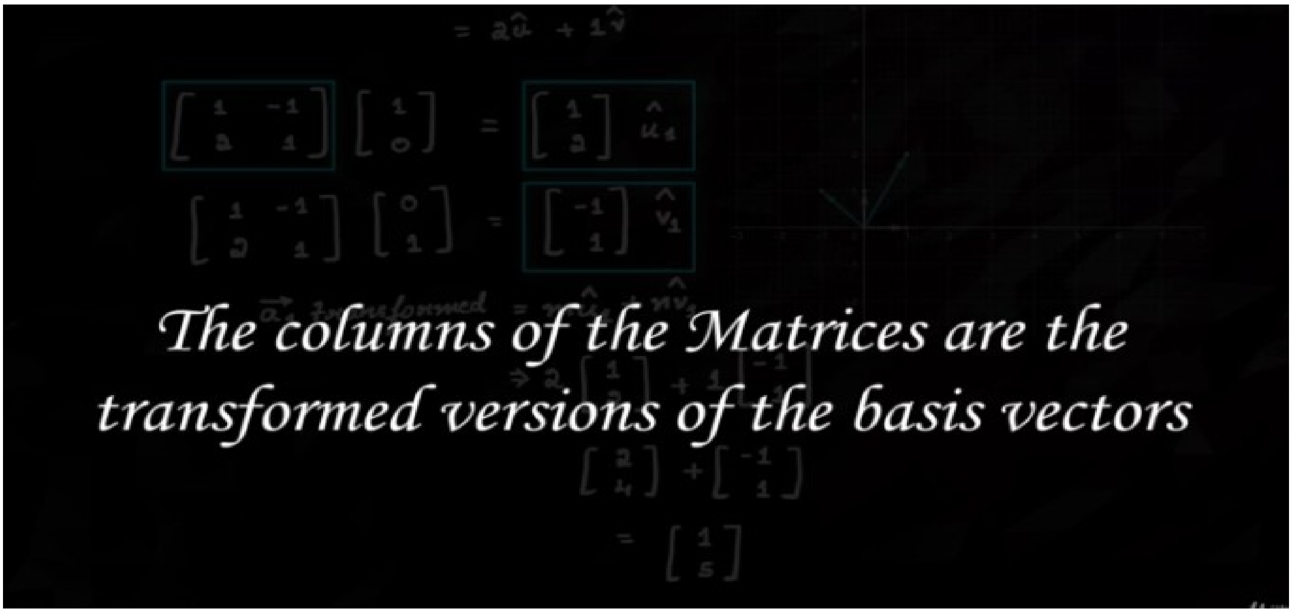
$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \hat{v}_1$$



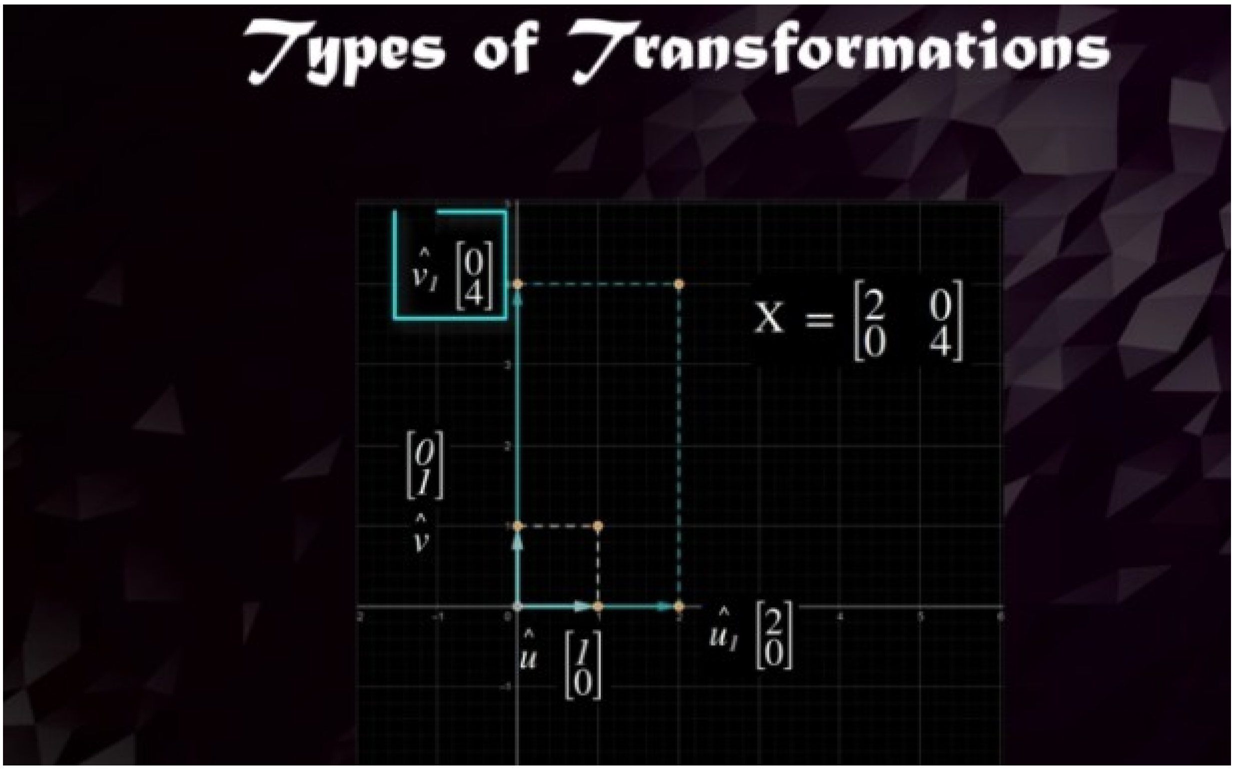
Properties of linear transform

1. All lines must remain lines , without getting curved

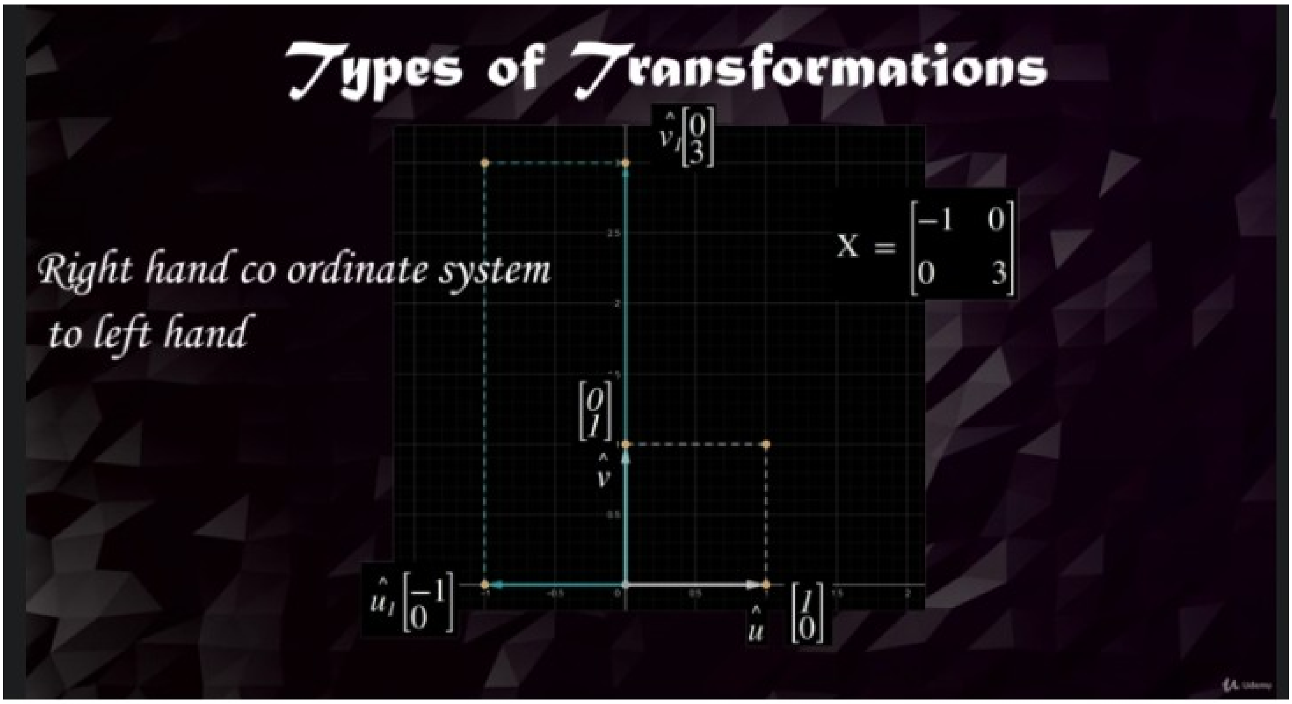
2. Origin must remain fixed in place.


The columns of the Matrices are the
transformed versions of the basis vectors

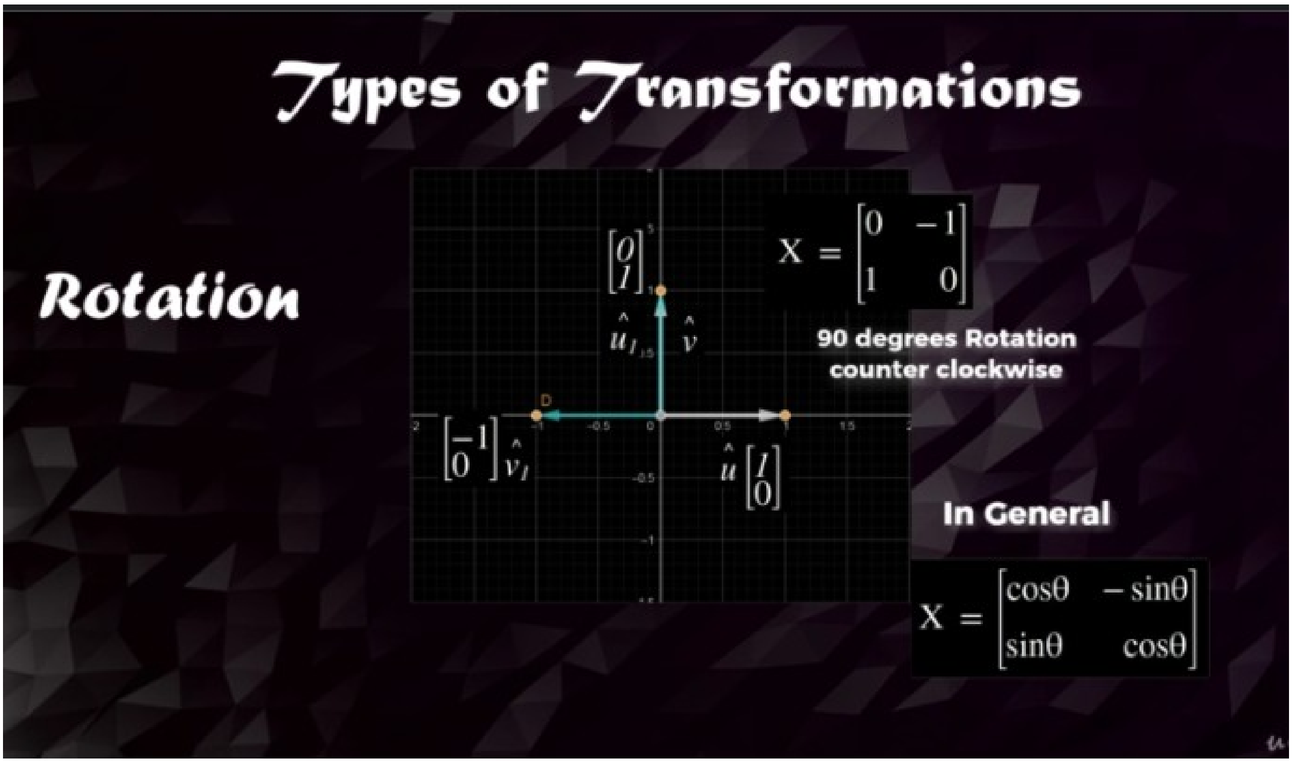

Types of Transformations
To determine how vectors transform, we just need to know
how unit vectors transform.

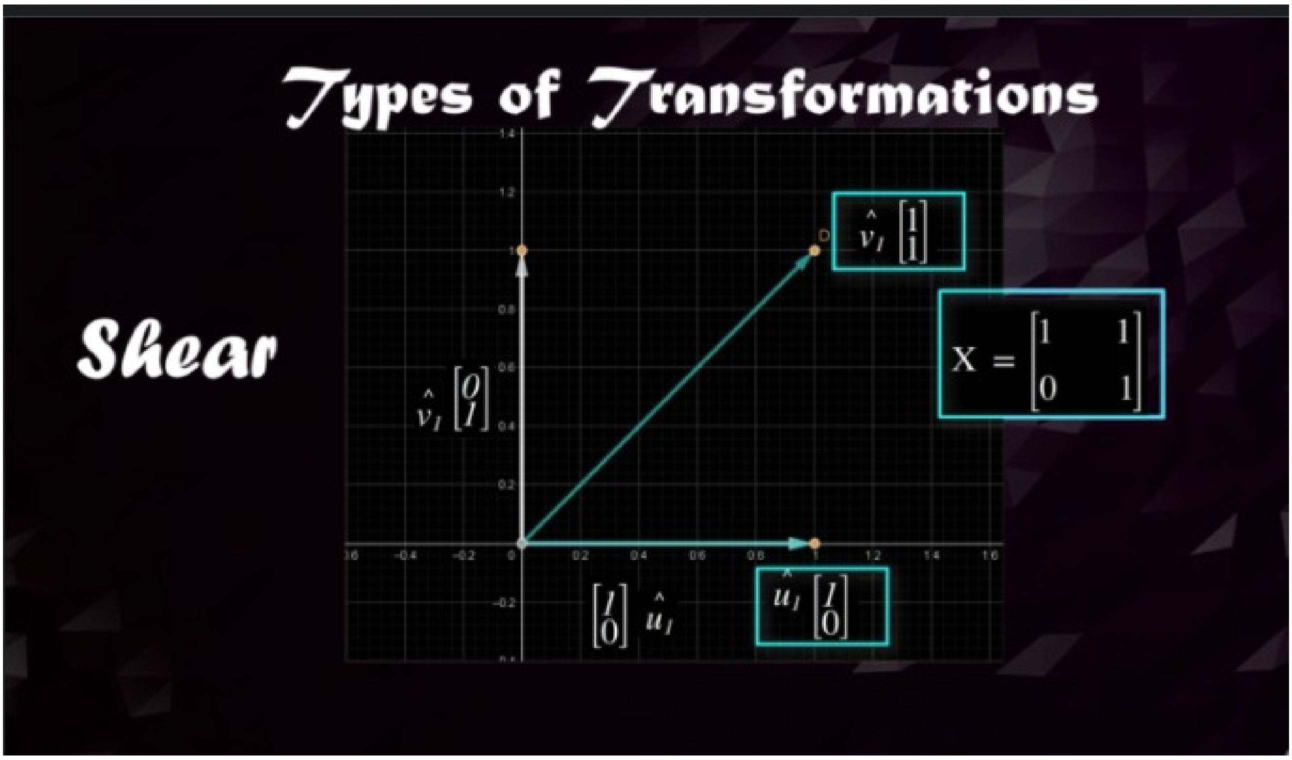


Types of Transformations
$\hat{v}_1 \begin{bmatrix}0\\4\end{bmatrix}$
$X = \begin{bmatrix}2 & 0\\0 & 4\end{bmatrix}$
$\begin{bmatrix}0\\1\end{bmatrix}$
$\hat{v}$
$\hat{u}$ $\begin{bmatrix}1\\0\end{bmatrix}$
$\hat{u}_1 \begin{bmatrix}2\\0\end{bmatrix}$



Types of Transformations
$\hat{v}_1 \begin{bmatrix}0\\3\end{bmatrix}$
$X = \begin{bmatrix}-1 & 0\\0 & 3\end{bmatrix}$
Right hand co ordinate system
to left hand
$\begin{bmatrix}0\\1\end{bmatrix}$
$\hat{v}$
$\hat{u}_1 \begin{bmatrix}-1\\0\end{bmatrix}$
$\hat{u}$ $\begin{bmatrix}1\\0\end{bmatrix}$


Types of Transformations
Rotation
$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$\hat{u}_1$ $\hat{v}$
D
$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $\hat{v}_1$
$\hat{u}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$X = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
90 degrees Rotation
counter clockwise
In General
$X = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$


Types of Transformations
Shear
$\hat{v}_1$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
D
$X = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
$\hat{v}_1$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\hat{u}_1$
$\hat{u}_1$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Types of Transformations

To determine how vectors transform, we just need to know how unit vectors transform.

Concepts of :

Shear, Rotations and Scaling and its combinations has applications in Data Science and Artificial intelligence.