KAUNAS UNIVERSITY OF TECHNOLOGY

Faculty of Mathematics
and Natural Sciences

Optimization Methods

Laboratory work report
Lab 3

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KAUNAS

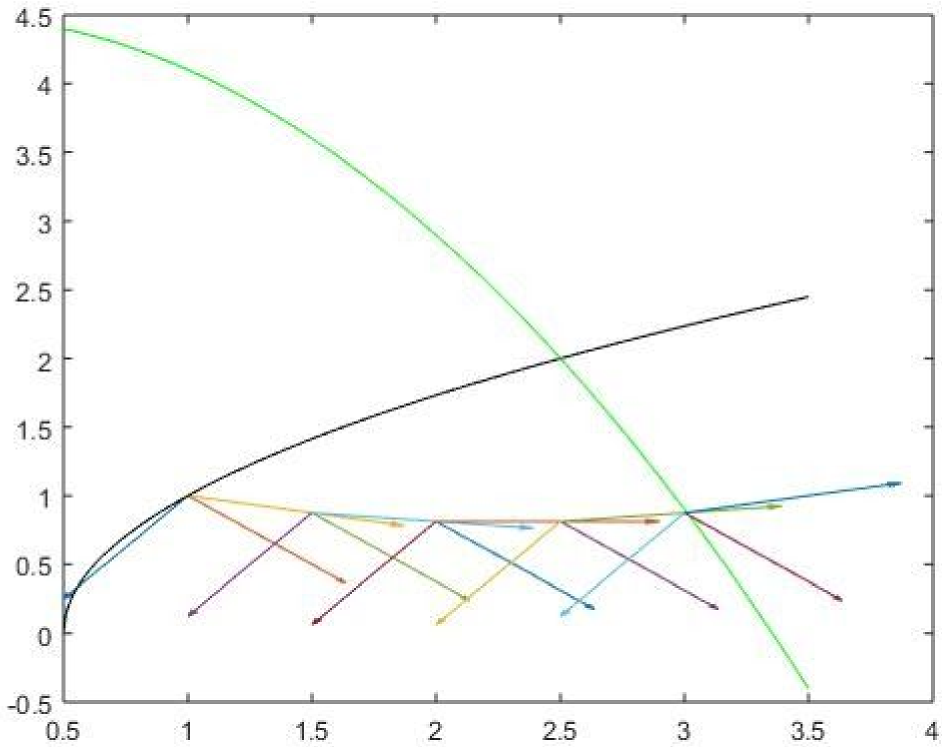
REPORT FOR LABORATORY 3:

Firstly, my list number is 26 ⇒ (26%20+1)=7, so I have Target function #2 ,
First Constraint #1 and Second Constraint #1.
Target Function #2: f(x) = (x(1) – 3)^2 + (x(2) - 4)^2
First Constraint #1: g1(x) = 2*x(1) – (x(2))^2 - 1 >= 0
Second Constraint #1: g2(x) = 9 – 0.8*(x(1))^2 – 2*x(2) >= 0

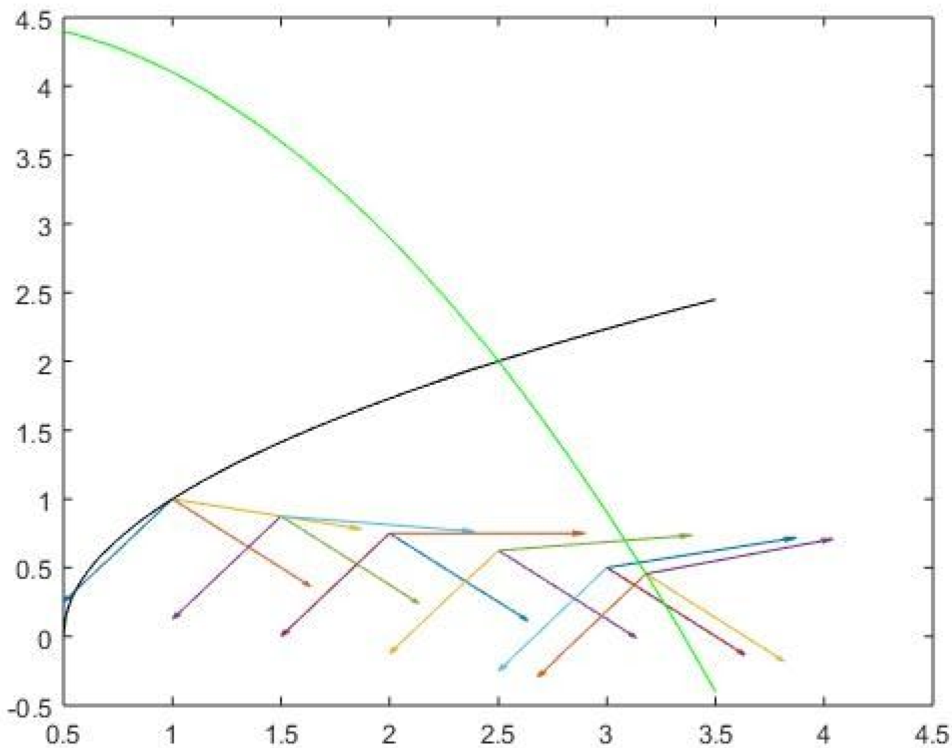
<u>Task 1</u>

Construct the program and visualize the results for the individual optimization problem given in the Instructions file.

In every iteration , we are making sure that the constraints are not violated.
We tried it with 2 ways:
1st way: X1 being affected always by the new d
2nd way: X1 being affected only by the first d.
g1 is plotted with black and g2 is with green.
Output:



Plotting the 1st way.



Plotting the 2nd way

Task 2

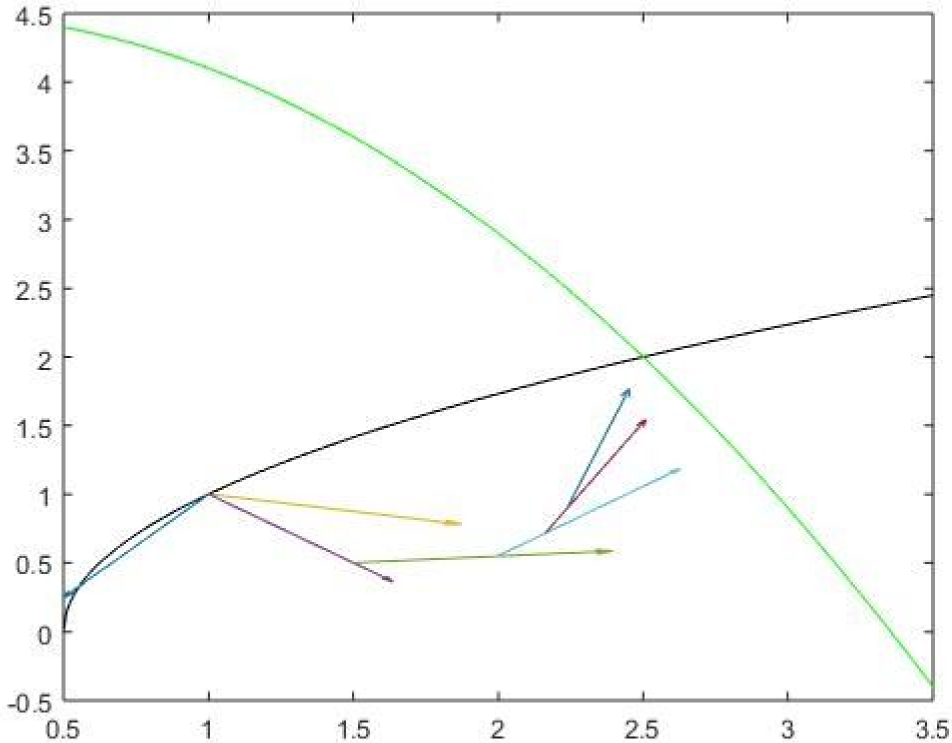
Implement the Veinott modification. Comment the results.

We observe that θ, d change value. Theta increases to 4 and d changes its secondary value to -1 from -0.25. Therefore, we can conclude that theta and d value are less affected now by the gradient since the constraint value will counterbalance the effect of the gradient term. This ensures that no sudden changes are introduced in the search direction.

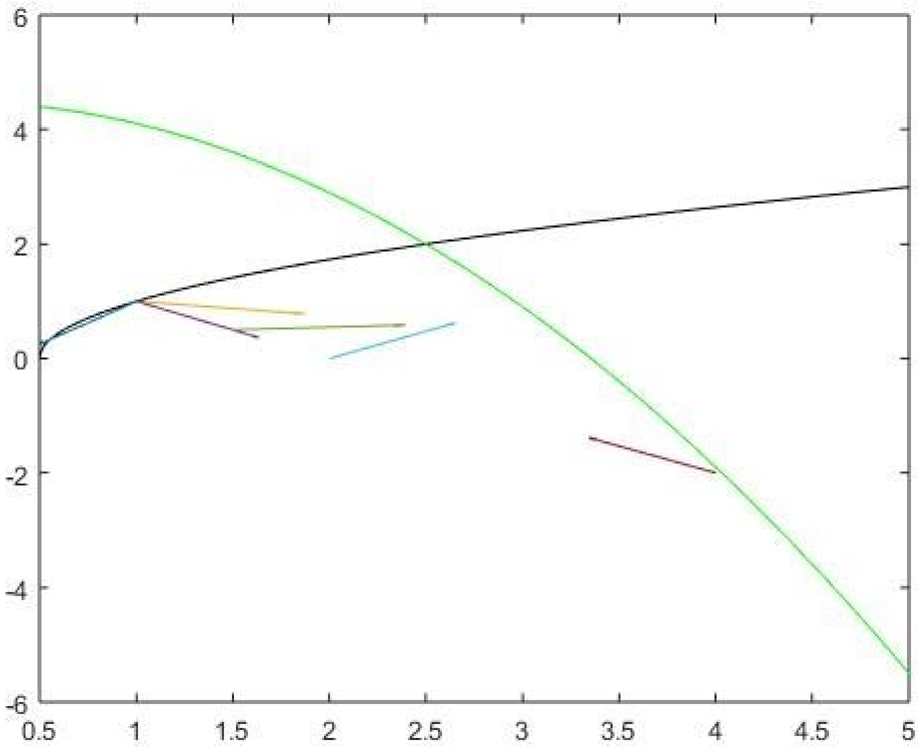
But here , trying again the same 2 ways, we observe that moving along the decent using the 1st way, it will never reach the boundaries because now d always change direction way more intensively than before.

And with the second way we see again that d changes direction more intensively than before but at least now it reaches the boundary of the feasible region.

Output:



plotting the 1st way



plotting the 2nd way

Program Code :
task1.m -1st way:

```
close all;
clear all;

%TASK 1
%target function: f(x) =  (x(1) – 3)^2 + (x(2) - 4)^2
%constraint 1: g1(x) =  2*x(1) – (x(2))^2 - 1 >= 0
%constraint 2: g2(x) =  9 – 0.8*(x(1))^2 – 2*x(2) >= 0

X0=[1;1];
%because The constraint g1(X) is active at the point X0= [1; 1].
%Note that the vector grad g1(X0) is directed into the interior of the feasible region.
%grad g1=[2 -2*x(2)]
%grad g2=[-1.6*x(1) -2]
%grad f=[2*(x(1)-3) 2(x(2)-4)]
% Find w that minimizes f = -1t + 0d1 + 0d2
%
% subject to
% 1t - 2d1 +2d2 <= 0
% 1t -4d1 -6d2 <= 0
% -1 <= d1 <= 1; -1 <= d2 <= 1.
f = [-1; 0; 0];
A =  [1 -2  2; 1 -4 -6];
b = [0; 0];
```

```
lb = [-1; -1; -1];
ub = [10000; 1; 1];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag


X = 0.5:0.01:3.5;
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;

% grad(f) at X0 = [-4; -6]
% grad(g1) at X0 = [2; -2]

gf = [-4; -6];
gg1 = [2; -2];
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(1,1,-4/norm(gf),-6/norm(gf)); hold on;
quiver(1,1,2/norm(gg1),-2/norm(gg1)); hold on;
quiver(1,1,x(2)/norm(d),x(3)/norm(d)); hold on
varfordisplay=['visualization of d, d1 = ',num2str(x(2))];
display(varfordisplay);
varfordisplay=['visualization of d, d2 = ',num2str(x(3))];
display(varfordisplay);
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));

%step 4
X1= X0 + 0.5*d;
%X1=[1.5 0.875];
%gfH=gf([1.5 ;0.75]);
%gg1H=gg1(X1);
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
```

```
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));

X1= X1 + 0.5*d;
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));

X1= X1 + 0.5*d;
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));
```

```
X1= X1 + 0.5*d;
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));
```

task1.m -2nd way:

```
close all;
clear all;

%TASK 1
%target function: f(x) =  (x(1) – 3)^2 + (x(2) - 4)^2
%constraint 1: g1(x) =  2*x(1) – (x(2))^2 - 1 >= 0
%constraint 2: g2(x) =  9 – 0.8*(x(1))^2 – 2*x(2) >= 0

X0=[1;1];
%because The constraint g1(X) is active at the point X0= [1; 1].
%Note that the vector grad g1(X0) is directed into the interior of the feasible region.
%grad g1=[2 -2*x(2)]
%grad g2=[-1.6*x(1) -2]
%grad f=[2*(x(1)-3) 2(x(2)-4)]
% Find w that minimizes f = -1t + 0d1 + 0d2
%
% subject to
% 1t - 2d1 +2d2 <= 0
% 1t -4d1 -6d2 <= 0
% -1 <= d1 <= 1; -1 <= d2 <= 1.
f = [-1; 0; 0];
A =  [1 -2  2; 1 -4 -6];
b = [0; 0];
lb = [-1; -1; -1];
ub = [10000; 1; 1];
```

```
[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag


X = 0.5:0.01:3.5;
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;

% grad(f) at X0 = [-4; -6]
% grad(g1) at X0 = [2; -2]

gf = [-4; -6];
gg1 = [2; -2];
dS = [x(2); x(3)];
d=dS;
%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(1,1,-4/norm(gf),-6/norm(gf)); hold on;
quiver(1,1,2/norm(gg1),-2/norm(gg1)); hold on;
quiver(1,1,x(2)/norm(d),x(3)/norm(d)); hold on
varfordisplay=['visualization of d, d1 = ',num2str(x(2))];
display(varfordisplay);
varfordisplay=['visualization of d, d2 = ',num2str(x(3))];
display(varfordisplay);
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));

%step 4
X1= X0 + 0.5*dS;
%X1=[1.5 0.875];
%gfH=gf([1.5 ;0.75]);
%gg1H=gg1(X1);
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];
```

```
%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));

X1= X0 + 1*dS;
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));

X1= X0 + 1.5*dS;
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));

X1= X0 + 2*dS;
```

```
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));
X1= X0 + 2.18*dS;
A =  [1 -2 2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4)];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag

X = 0.5:0.01:3.5; %MAYBE NEEDS UNTIL 3.3
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(X1(1),X1(2),-4/norm(gf),-6/norm(gf)); hold on;
quiver(X1(1),X1(2),2/norm(gg1),-2/norm(gg1)); hold on;
quiver(X1(1),X1(2),x(2)/norm(d),x(3)/norm(d)); hold on
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));
```

task2.m- 1st way

```
close all;
clear all;

%TASK 1
%target function: f(x) =  (x(1) – 3)^2 + (x(2) - 4)^2
%constraint 1: g1(x) =  2*x(1) – (x(2))^2 - 1 >= 0
%constraint 2: g2(x) =  9 – 0.8*(x(1))^2 – 2*x(2) >= 0
```

```
X0=[1;1];
%because The constraint g1(X) is active at the point X0= [1; 1].
%Note that the vector grad g1(X0) is directed into the interior of the feasible region.
%grad g1=[2 -2*x(2)]
%grad g2=[-1.6*x(1) -2]
%grad f=[2*(x(1)-3) 2(x(2)-4)]
% Find w that minimizes f = -1t + 0d1 + 0d2
%
% subject to
% 1t - 2d1 +2d2 <= 0
% 1t -4d1 -6d2 <= 0
% -1 <= d1 <= 1; -1 <= d2 <= 1.
f = [-1; 0; 0];
A =  [1 -2  2; 1 -4 -6];
b = [0; 0];
lb = [-1; -1; -1];
ub = [10000; 1; 1];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag


X = 0.5:0.01:3.5;
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;

% grad(f) at X0 = [-4; -6]
% grad(g1) at X0 = [2; -2]

gf = [-4; -6];
gg1 = [2; -2];
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(1,1,-4/norm(gf),-6/norm(gf)); hold on;
quiver(1,1,2/norm(gg1),-2/norm(gg1)); hold on;
quiver(1,1,x(2)/norm(d),x(3)/norm(d)); hold on
varfordisplay=['visualization of d, d1 = ',num2str(x(2))];
display(varfordisplay);
varfordisplay=['visualization of d, d2 = ',num2str(x(3))];
display(varfordisplay);
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));
% VENOTT Modification
%
% Find w that minimizes f = -1t + 0d1 + 0d2
%
```

```
% subject to
% 1t - 2d1 +2d2 <= 0
% 1t -4d1 -6d2 <= 0
% 1t + 1.6d1 + 2d2 <= 6.2
% -1 <= d1 <= 1; -1 <= d2 <= 1.
%TASK 2
%target function: f(x) =  (x(1) – 3)^2 + (x(2) - 4)^2
%constraint 1: g1(x) =  2*x(1) – (x(2))^2 - 1 >= 0
%constraint 2: g2(x) =  9 – 0.8*(x(1))^2 – 2*x(2) >= 0

X0=[1;1];
%because The constraint g1(X) is active at the point X0= [1; 1].
%Note that the vector grad g1(X0) is directed into the interior of the feasible region.
%grad g1=[2 -2*x(2)]
%grad g2=[-1.6*x(1) -2]
%grad f=[2*(x(1)-3) 2(x(2)-4)]

Amod =  [1 -2  2; 1 -4 6; 1 1.6 2];
bmod = [0; 0; 6.2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag

dmod = [xmod(2); xmod(3)];
quiver(1,1,xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on

X0=X0 +0.5 *dmod;
Amod =  [1 -2  2*X0(2); 1 2*(X0(1)-3) 2*(X0(2)-4); 1 1.6*X0(1) 2];
bmod = [2*X0(1) - (X0(2)).^2 - 1; 0; 9-2*X0(2)-0.8*(X0(1)).^2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag

dmod = [xmod(2); xmod(3)];
quiver(X0(1),X0(2),xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on

X0=X0 +0.5 *dmod;
Amod =  [1 -2  2*X0(2); 1 2*(X0(1)-3) 2*(X0(2)-4); 1 1.6*X0(1) 2];
bmod = [2*X0(1) - (X0(2)).^2 - 1; 0; 9-2*X0(2)-0.8*(X0(1)).^2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag
```

```
dmod = [xmod(2); xmod(3)];
quiver(X0(1),X0(2),xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on
X0=X0 +0.5 *dmod;
Amod =  [1 -2  2*X0(2); 1 2*(X0(1)-3) 2*(X0(2)-4); 1 1.6*X0(1) 2];
bmod = [2*X0(1) - (X0(2)).^2 - 1; 0; 9-2*X0(2)-0.8*(X0(1)).^2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag

dmod = [xmod(2); xmod(3)];
quiver(X0(1),X0(2),xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on
X0=X0 +0.5 *dmod;
Amod =  [1 -2  2*X0(2); 1 2*(X0(1)-3) 2*(X0(2)-4); 1 1.6*X0(1) 2];
bmod = [2*X0(1) - (X0(2)).^2 - 1; 0; 9-2*X0(2)-0.8*(X0(1)).^2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag

dmod = [xmod(2); xmod(3)];
quiver(X0(1),X0(2),xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on
```

task2.m- 2nd way

```
close all;
clear all;

%TASK 1
%target function: f(x) =  (x(1) – 3)^2 + (x(2) - 4)^2
%constraint 1: g1(x) =  2*x(1) – (x(2))^2 - 1 >= 0
%constraint 2: g2(x) =  9 – 0.8*(x(1))^2 – 2*x(2) >= 0

X0=[1;1];
%because The constraint g1(X) is active at the point X0= [1; 1].
%Note that the vector grad g1(X0) is directed into the interior of the feasible region.
%grad g1=[2 -2*x(2)]
%grad g2=[-1.6*x(1) -2]
%grad f=[2*(x(1)-3) 2(x(2)-4)]
% Find w that minimizes f = -1t + 0d1 + 0d2
%
% subject to
% 1t - 2d1 +2d2 <= 0
% 1t -4d1 -6d2 <= 0
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% -1 <= d1 <= 1; -1 <= d2 <= 1.
f = [-1; 0; 0];
A =  [1 -2  2; 1 -4 -6];
b = [0; 0];
lb = [-1; -1; -1];
ub = [10000; 1; 1];

[x,fval,exitflag,output,lambda] = linprog(f,A,b,[],[],lb,ub);

x
fval
exitflag


X = 0.5:0.01:5;
%because we have a sqrt of (2*X - 1) ==> X>=1/2
G1 = (2*X - 1).^(1/2) ;
G2 = 4.5 - 0.4*(X).^2;
plot(X,G1,'k'); hold on; plot(X,G2,'g'); hold on;

% grad(f) at X0 = [-4; -6]
% grad(g1) at X0 = [2; -2]

gf = [-4; -6];
gg1 = [2; -2];
d = [x(2); x(3)];

%at the first 2 positions of our quiver we will put X0=[1;1]
quiver(1,1,-4/norm(gf),-6/norm(gf)); hold on;
quiver(1,1,2/norm(gg1),-2/norm(gg1)); hold on;
quiver(1,1,x(2)/norm(d),x(3)/norm(d)); hold on
varfordisplay=['visualization of d, d1 = ',num2str(x(2))];
display(varfordisplay);
varfordisplay=['visualization of d, d2 = ',num2str(x(3))];
display(varfordisplay);
fprintf('visualisation of d, d1=%d and d2=%d .\n',x(2),x(3));
% VENOTT Modification
%
% Find w that minimizes f = -1t + 0d1 + 0d2
%
% subject to
% 1t - 2d1 +2d2 <= 0
% 1t -4d1 -6d2 <= 0
% 1t + 1.6d1 + 2d2 <= 6.2
% -1 <= d1 <= 1; -1 <= d2 <= 1.
%TASK 2
%target function: f(x) =  (x(1) – 3)^2 + (x(2) - 4)^2
%constraint 1: g1(x) =  2*x(1) – (x(2))^2 - 1 >= 0
%constraint 2: g2(x) =  9 – 0.8*(x(1))^2 – 2*x(2) >= 0

X0=[1;1];
```

```
%because The constraint g1(X) is active at the point X0= [1; 1].
%Note that the vector grad g1(X0) is directed into the interior of the feasible region.
%grad g1=[2 -2*x(2)]
%grad g2=[-1.6*x(1) -2]
%grad f=[2*(x(1)-3) 2(x(2)-4)]

Amod =  [1 -2  2; 1 -4 6; 1 1.6 2];
bmod = [0; 0; 6.2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag

dmodS = [xmod(2); xmod(3)];
dmod=dmodS;
quiver(1,1,xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on

X1=X0 +0.5 *dmodS;
Amod =  [1 -2  2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4); 1 1.6*X1(1) 2];
bmod = [2*X1(1) - (X1(2)).^2 - 1; 0; 9-2*X1(2)-0.8*(X1(1)).^2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag

dmod = [xmod(2); xmod(3)];
quiver(X1(1),X1(2),xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on

X1=X0 +1 *dmodS;
Amod =  [1 -2  2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4); 1 1.6*X1(1) 2];
bmod = [2*X1(1) - (X1(2)).^2 - 1; 0; 9-2*X1(2)-0.8*(X1(1)).^2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);

xmod
fvalmod
exitflag

dmod = [xmod(2); xmod(3)];
quiver(X1(1),X1(2),xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on

X1=X0 +3 *dmodS;
Amod =  [1 -2  2*X1(2); 1 2*(X1(1)-3) 2*(X1(2)-4); 1 1.6*X1(1) 2];
bmod = [2*X1(1) - (X1(2)).^2 - 1; 0; 9-2*X1(2)-0.8*(X1(1)).^2];

[xmod,fvalmod,exitflag,output,lambda] = linprog(f,Amod,bmod,[],[],lb,ub);
```

```
xmod
fvalmod
exitflag

dmod = [xmod(2); xmod(3)];
quiver(X1(1),X1(2),xmod(2)/norm(dmod),xmod(3)/norm(dmod)); hold on
```

<u>Control Work-Alexandros Veremis :</u>

<u>Problem 1]</u>. $f(x,y) = (x-4)^2 + (y-5)^2$

$g_1(x,y) = 1 - x^2 - y \geq 0$

$g_2(x,y) = x^2 - y - 1 \geq 0$

$(x_0, y_0) = (-1, 0)$

Construct the LPP:

So $(-1,0)$ makes $g_1(-1,0) = 0$.

$\Rightarrow$ We will calculate the gradients:

$\nabla f(x,y) = [2(x-4), \; 2(y-5)]$

$\Rightarrow \nabla f(-1,0) = [-10, -10]$

$\nabla g_1(x,y) = [-2x, -1]$.

$\nabla g_1(-1,0) = [2, -1]$.

So LPP will be:

$$\left\{\begin{array}{l} \text{find } w \text{ that minimizes and maximizes } \theta \\ f = -1\theta + 0 \cdot d_1 + 0 d_2 \\ \nabla f(X_0) \cdot d \leq -\theta \Rightarrow \theta - 10 d_1 - 10 d_2 \leq 0 \\ \nabla g_1(X_0) \cdot d \geq \theta \Rightarrow \theta - 2d_1 + d_2 \leq 0 \\ -1 \leq d_1 \leq 1 \text{ and } -1 \leq d_2 \leq 1 \\ -1 \leq f \leq 100.000 \end{array}\right\}.$$

So : $f = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -10 & -10 \\ 1 & -2 & 1 \end{bmatrix}$.

$b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Problem 2 | $\vec{X_0} = (3, 3)$
$\vec{d} = (-1, -1)$

Inactive: $g(x, y) = x^2 - y - 1 \geq 0$

Find the coordinates of the intersection point between the descent line and the constraint.

The coordinates of the intersection point will be: $(x, x^2 - 1)$ since it belongs to $g(x, y)$.

Therefore: $\vec{X_1} = \vec{X_0} + \gamma \cdot \vec{d} \implies$

$$(x, x^2 - 1) = (3, 3) + \gamma(-1, -1)$$

$$\Rightarrow \left.\begin{aligned} x &= 3 - \gamma \\ x^2 - 1 &= 3 - \gamma \end{aligned}\right\} \Rightarrow x = x^2 - 1 \Rightarrow x^2 - x - 1 = 0.$$

$$x_{1,2} = \frac{1 \pm \sqrt{5}}{2} = \begin{cases} (1+\sqrt{5})/2 \\ (1-\sqrt{5})/2 \end{cases}$$

~~So the coordinates will either be~~

$\cancel{\left(\frac{1+\sqrt{5}}{2}, \left(\frac{1+\sqrt{5}}{2}\right)^2\right)}$

γ must be positive:
and $\gamma = 3 - x$

when $x = \frac{1+\sqrt{5}}{2} \Rightarrow \gamma = 3 - \frac{1+\sqrt{5}}{2}$

$\Rightarrow \gamma > 0$

and also $\gamma > 0$ when $x = \frac{1-\sqrt{5}}{2}$.

So, the coordinates will either be:

A$(-0{,}618, -0{,}618)$ or B$(1{,}618, 1{,}618)$

But, we are intersted only at the first time they intersect $\Rightarrow$ therefore $B(1{,}618\ ,\ 1{,}618)$ is our point.

<u>Problem 3</u>| $g_1(x,y) = 1 - x^2 - y \geq 0$

$g_2(x,y) = x^2 - y - 1 \geq 0$

$g_3(x,y) = x + 0{,}5 \geq 0$

$X_0 = (-0{,}5\ ,\ 0)$

Veinott constraints for the inactive constraints.

As we can see:

$$\left.\begin{array}{l} g_1(X_0) \neq 0 \\ g_2(X_0) \neq 0 \end{array}\right\} \text{Inactive}$$

$g_3(X_0) = 0 . \Rightarrow$ Active.

therefore Veinott constraints for g_1, g_2!

$$\left.\begin{array}{l} g_1(X_0) + \nabla g_1(X_0) \cdot \vec{d} \geq 0 \\ g_2(X_0) + \nabla g_2(X_0) \cdot \vec{d} \geq 0 \\ -1 \leq d_1 \leq 1 \quad \text{and} \quad -1 \leq d_2 \leq 1 . \end{array}\right\}$$

$\nabla g_1(X_0) = (-2x, -1) = (1, -1)$

$\nabla g_2(x,y) = (2x, -1) = (-1, -1).$

$$\left.\begin{array}{l} \frac{3}{4} + d_1 - d_2 \geq 0 \\ -\frac{3}{4} - d_1 - d_2 \geq 0 \end{array}\right\} \left\{\begin{array}{ll} 0 - d_1 + d_2 & \leq \frac{3}{4} \\ 0 + d_1 + d_2 & \leq -\frac{3}{4} \\ -1 \leq d_1 \leq 1 & \\ -1 \leq d_2 \leq 1 & \end{array}\right\} .$$