Financial Frictions, Intangible Capital and Productivity: A Model of Skill-Biased Stagnation

Alejandro Vicente[1]

[1]University of Alicante

PRELIMINARY AND INCOMPLETE, DO NOT CIRCULATE
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Abstract

This paper studies how financial frictions prevent economies from fully exploiting productivity gains from rising human capital. Using Portuguese matched employer-employee and firm balance sheet data for 2011-2022, I document three empirical facts: (1) intangible capital and skilled labor are complements in production, (2) financially constrained firms underinvest in intangibles and R&D labor, and (3) constrained firms underexploit the complementarity: during Portugal's skill supply expansion, wage premia declined more sharply at constrained firms despite hiring skilled workers at similar rates. I develop a quantitative heterogeneous-firm model with nested CES production, endogenous intangible accumulation through R&D, and collateral constraints with differential pledgeability ($\alpha_K > \alpha_S$). The model rationalizes the empirical patterns and reveals a *skill-biased stagnation* mechanism: when skilled labor becomes more abundant, unconstrained firms exploit the complementarity by investing in both intangibles and skills, while constrained firms hire skilled workers but cannot finance complementary intangible investments, leading to underexploitation and muted aggregate productivity gains. Counterfactual experiments quantify the role of financial frictions in limiting returns to human capital accumulation.

I. Introduction

Over the past decades, advanced economies have experienced substantial increases in educational attainment alongside rising importance of intangible capital in production. Yet aggregate productivity growth has been disappointing. This paper proposes a mechanism through which financial frictions can prevent economies from fully exploiting the potential gains from higher human capital: *skill-biased stagnation.*

The mechanism operates through three key channels. First, intangible capital and skilled labor are complements in production, implying that firms with high intangible intensity benefit more from skilled workers. Second, intangibles are harder to pledge as collateral than tangible assets, creating a pecking-order distortion where financially constrained firms underinvest in intangibles. Third, when skilled labor becomes more abundant, unconstrained firms exploit the complementarity by investing in both intangibles and skilled workers, while constrained firms cannot finance the necessary intangible investments and therefore underexploit their skilled labor.

I develop this argument in two stages. Using Portuguese matched employer-employee and balance sheet data for 2011-2022, I establish three empirical facts: (1) intangible intensity and skilled labor share are complements in production, (2) financially constrained firms invest less in intangibles and R&D labor, and (3) the complementarity between intangibles and skills is significantly weaker for constrained firms, with dynamic evidence showing that wage premia declined more sharply at constrained firms during the skill supply expansion. I then build a quantitative heterogeneous-firm model with nested CES production technology, endogenous intangible capital accumulation through R&D, and collateral constraints with differential pledgeability. The model rationalizes the empirical patterns and enables counterfactual analysis of skill supply shocks under different financial friction scenarios.

The paper contributes to three literatures. First, it connects to work on intangible capital and firm dynamics (Peters and Taylor, 2017; Atkeson and Burstein, 2010), emphasizing the role of complementarities with skilled labor. Second, it relates to the literature on financial frictions and capital misallocation (Buera et al., 2013; Moll, 2014), highlighting the specific distortions created by differential pledgeability of capital types. Third, it speaks to the productivity slowdown debate by proposing financial frictions as a mechanism limiting the returns to human capital accumulation.

The remainder of the paper is organized as follows. Section II. presents the data and empirical evidence. Section III. develops intuition with a stylized two-period model. Section IV. presents the full quantitative model. Section V. outlines the quantitative experiments. The Appendix contains details on data construction and cleaning procedures.

II. Data and Empirical Evidence

II.A. Data Sources and Measurement

The empirical analysis uses Portuguese firm-level data from two administrative sources for 2011-2022. The **SCIE (Sistema de Contas Integradas das Empresas)** provides balance sheet and income statement data for all Portuguese firms required to file annual accounts, including assets, liabilities, revenue, costs, and R&D expenditures. The **QP (Quadros de Pessoal)** is a matched employer-employee dataset containing worker-level information that I aggregate to construct firm-level skill composition measures.

I define skilled workers as those with tertiary education (ISCED 5-8) and construct the share of skilled workers as the key skill measure. Intangible capital stocks are built using the perpetual inventory method following a simplified version of the approach by Peters and Taylor (2017), combining knowledge capital from R&D expenditures and balance sheet intangibles. All monetary variables are deflated to constant 2020 prices. After cleaning and sample restrictions (positive employment, non-negative balance sheet items, at least two consecutive observations per firm), the final analytical sample contains 1,759,093 firm-year observations. Details on data construction and cleaning are provided in Appendix A.

Financial constraints are proxied by leverage (total debt relative to total assets). Following standard practice in the corporate finance literature, I classify firms as constrained if their leverage exceeds the sector-year median, allowing for industry heterogeneity in optimal capital structure.

II.B. Empirical Facts

Fact 1: Complementarity Between Intangibles and Skilled Labor

To test for complementarity between intangibles and skilled labor, I estimate the following production function specification:

$$\begin{aligned}\ln Y_{jt} = {} & \beta_1 \,\text{IntangIntensity}_{jt} + \beta_2 \,\text{ShareSkilled}_{jt} + \beta_3 \,(\text{IntangIntensity} \times \text{ShareSkilled})_{jt} \\ & + \boldsymbol{X}'_{jt}\boldsymbol{\gamma} + \alpha_j + \delta_t + \eta_i + \varepsilon_{jt},\end{aligned} \tag{1}$$

where Y_{jt} is gross value added for firm j in year t, $\text{IntangIntensity}_{jt} \equiv K_{jt}^{intang}/K_{jt}^{total}$ is the ratio of intangible to total capital, ShareSkilled_{jt} is the share of workers with tertiary education, $\boldsymbol{X}_{jt}$ includes log total capital, log employment, and log firm age, and α_j, δ_t, η_i denote firm, year, and industry fixed effects respectively. The coefficient of interest is β_3: a positive estimate indicates that the marginal product of skilled labor is increasing in intangible intensity, i.e., complementarity.

Table 1 reports estimates of equation (1). The table presents a nested specification: column (1) includes no controls or fixed effects, column (2) adds firm, year, and industry fixed effects, and column (3) further includes the control variables $\boldsymbol{X}_{jt}$.

Table 1: Complementarity Between Intangibles and Skilled Labor

	Log Gross Value Added		
	(1)	(2)	(3)
Intangible Intensity	0.40***	-0.07***	-0.13***
	(0.01)	(0.01)	(0.01)
Share Skilled Workers	0.46***	-0.06***	-0.02***
	(0.00)	(0.01)	(0.00)
Intangible Intensity × Share Skilled	1.33***	0.27***	0.04**
	(0.03)	(0.02)	(0.02)
Observations	1,759,086	1,759,085	1,759,076
Adjusted R-squared	0.020	0.868	0.895
Firm FE	No	Yes	Yes
Year FE	No	Yes	Yes
Industry FE	No	Yes	Yes
Controls	No	No	Yes

Dependent variable: Log gross value added (GVA).
Intangible intensity = $K_{intangible}$ / K_{total}.
Controls include log total capital, log employment, and log firm age.
Robust standard errors in parentheses.

The interaction coefficient is positive and statistically significant across all specifications, demonstrating robust complementarity between intangibles and skilled labor. The coefficient attenuates from 0.111 without controls to 0.040 with the full set of fixed effects and controls, indicating that the raw correlation partly reflects selection (intangible-intensive firms hiring more skilled workers) but a substantial within-firm complementarity remains. This pattern indicates that the marginal product of skilled labor is increasing in intangible intensity: firms with higher intangible capital benefit more from employing skilled workers. Conversely, the marginal product of intangibles is increasing in skill share. This complementarity is precisely the technological structure embedded in the nested CES production function of the quantitative model. Table 3 in Appendix B shows that results are robust to using revenue or production value as alternative outcome measures.

Figure 1 shows the raw correlation between intangible intensity and skill share using a binscatter with 20 quantiles. The positive relationship is evident even without controlling for firm characteristics, though the regression estimates exploit within-firm variation and control for confounding factors.

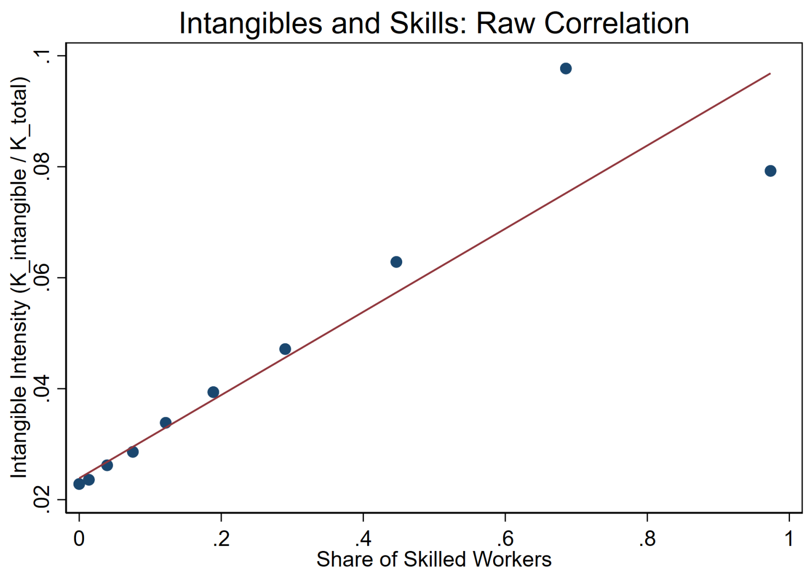
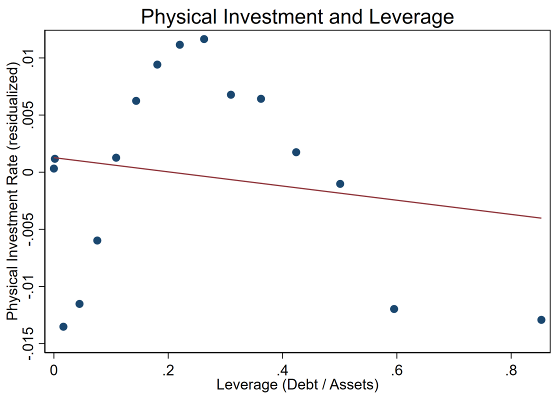


Figure 1: Raw Correlation Between Intangible Intensity and Skilled Labor Share

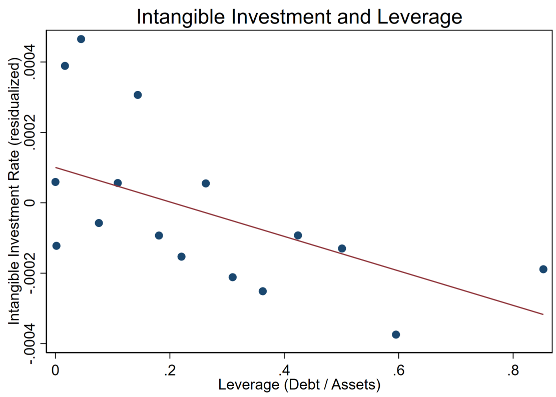
Fact 2: Financial Constraints and the Pecking Order

The model predicts that financially constrained firms underinvest in intangibles relative to tangibles because intangibles have lower pledgeability. Figure 2 presents residualized binscatters showing the relationship between leverage (the constraint proxy) and various investment decisions, after partialling out firm, year, and industry fixed effects along with log capital, employment, and age.

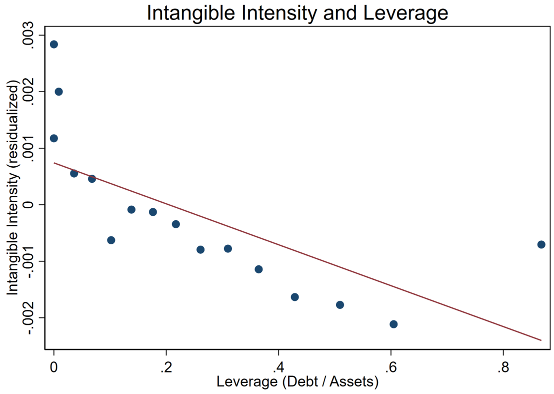
The evidence reveals a clear pecking-order pattern. While physical investment shows little systematic relationship with leverage (Panel a), intangible investment, intangible intensity, and R&D worker allocation all decline sharply with leverage (Panels b-d). Constrained firms shift their investment composition toward tangible assets and allocate fewer skilled workers to R&D activities, precisely as the model predicts when intangibles have lower collateral value than tangibles.



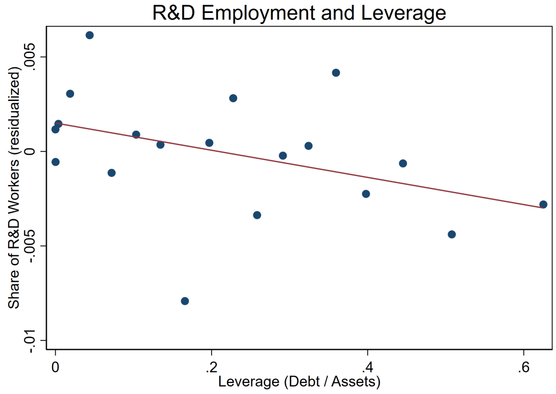
(a) Physical Investment Rate



(b) Intangible Investment Rate



(c) Intangible Intensity



(d) Share of R&D Workers

Figure 2: Financial Constraints and the Pecking Order Distortion

Note: All variables residualized by regressing on firm, year, and industry fixed effects, log total capital, log employment, and log firm age. Binscatters use 20 quantiles. Panel (d) restricts to firms with positive R&D employment.

Fact 3: Underexploitation of Complementarity by Constrained Firms

If financial constraints prevent firms from building intangible capital, they should also reduce the extent to which firms can exploit the technological complementarity between intangibles and skills. Table 2 tests this prediction by estimating equation (1) separately for the pooled sample (column 1), low-leverage firms (column 2), and high-leverage firms (column 3), where leverage groups are defined relative to the sector-year median.

Table 2: Underexploitation of Intangibles-Skills Complementarity

	Log Gross Value Added		
	All Firms	Low Leverage	High Leverage
Main effects:			
Intangible Intensity	-0.13***	-0.13***	-0.12***
	(0.01)	(0.01)	(0.01)
Share Skilled Workers	-0.02***	-0.02***	-0.01
	(0.00)	(0.01)	(0.01)
Complementarity:			
Intangible Intensity × Share Skilled	0.04**	0.06**	-0.01
	(0.02)	(0.03)	(0.03)
Observations	1,759,076	852,524	856,239
Adjusted R-squared	0.895	0.904	0.906

Dependent variable: Log gross value added (GVA).
Low- and high-leverage defined relative to sector-year median leverage.
All specifications include firm, year, and industry fixed effects.
Controls include log total capital, log employment, and log firm age.
Robust standard errors in parentheses.

The interaction coefficient is 0.055 and significant for low-leverage firms (column 2), but statistically indistinguishable from zero (-0.005, $p > 0.10$) for high-leverage firms (column 3). This stark difference indicates that constrained firms cannot exploit the complementarity: even when they employ skilled workers, the lack of complementary intangible capital limits productivity gains. The pooled specification (column 1) masks this heterogeneity, showing an intermediate coefficient of 0.040. Table 4 in Appendix B confirms that this pattern holds when using revenue or production value as alternative outcome measures.

Dynamic Evidence: Wage Premium Decline. The cross-sectional evidence establishes underexploitation at a point in time. I now examine the dynamic implications over the 2011-2022 period, during which Portugal experienced a substantial skill supply expansion. Figure 3 plots the evolution of the skill share and wage premium. The skill share increased substantially, yet the wage premium declined, a puzzle for standard capital-skill complementarity models where skill supply increases induce capital deepening that sustains premia.

Figure 4 decomposes this pattern by financial constraints. Panel (a) shows that both low- and high-leverage firms increased their skilled labor shares over the period at remark-

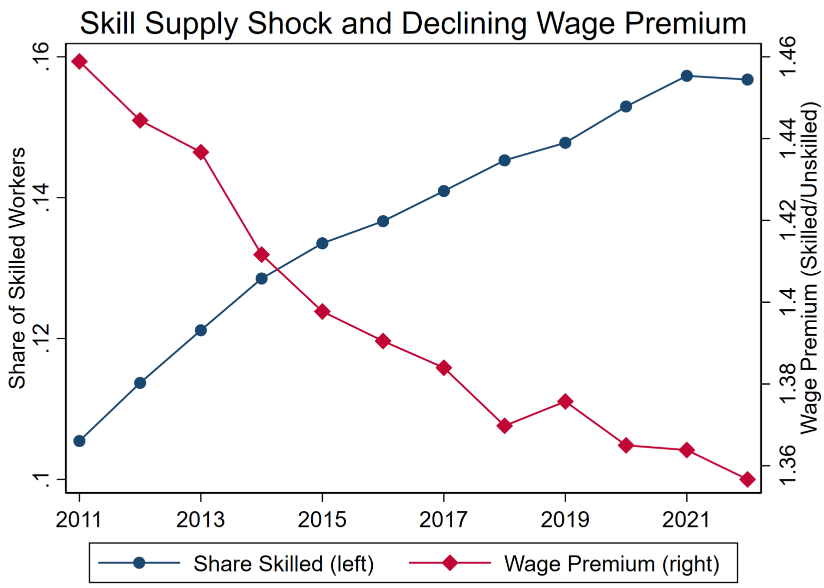


Figure 3: Skill Supply Expansion and Declining Wage Premium

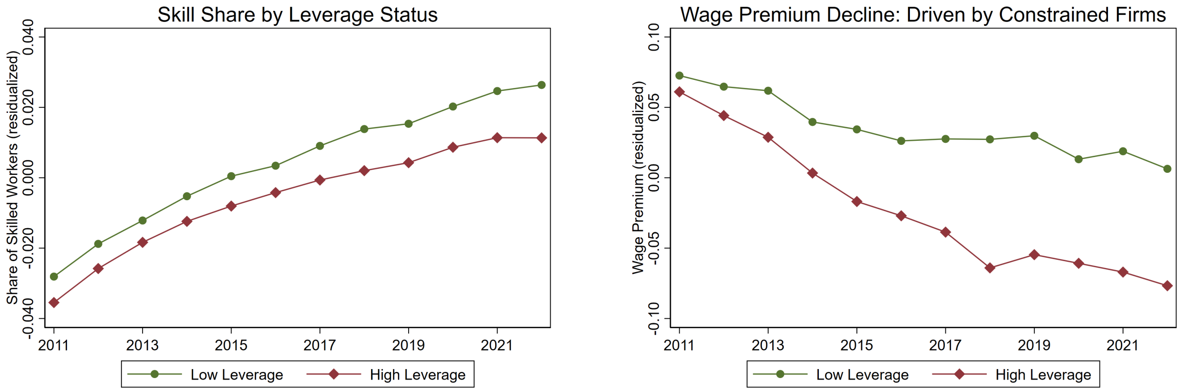
Note: Averages across all firms. Skill share is the fraction of workers with tertiary education. Wage premium is the ratio of average skilled to unskilled wages within firms.

ably similar rates. However, Panel (b) reveals that the wage premium decline was much sharper at high-leverage (constrained) firms. This differential trend reveals the mechanism: constrained firms hire skilled workers at similar rates to unconstrained firms, but cannot finance the complementary intangible investments needed to maintain skilled workers' productivity, causing the wage premium to compress more sharply.

This dynamic evidence completes the empirical case for skill-biased stagnation. Constrained firms respond to the skill supply shock by hiring skilled workers, but their inability to build complementary intangible capital means these workers are underexploited. The wage premium falls more sharply at constrained firms, revealing that financial frictions prevent the economy from fully exploiting productivity gains from rising human capital.

II.C. Summary of Empirical Evidence

The three facts establish the key building blocks of the theoretical mechanism. First, technology features complementarity between intangibles and skills. Second, financial frictions distort investment composition toward tangibles and away from intangibles. Third, the combination of these forces prevents constrained firms from exploiting complementarity in both the cross-section and over time: the interaction coefficient is weak for constrained firms, and wage premia declined more sharply at constrained firms during the skill supply expansion of 2011-2022. The following sections develop a model that rationalizes these patterns and enables quantitative counterfactual analysis.



(a) Skill Share by Leverage

(b) Wage Premium by Leverage

Figure 4: Skill Adoption vs. Wage Premium by Financial Constraints

Note: Averages by leverage group. Low/high leverage defined relative to sector-year median. Both panels show residualized values, controlling for firm size (log capital, log employment), age, and industry fixed effects. Constrained firms hire skilled workers at similar rates as unconstrained firms (Panel a), yet experience sharper wage premium declines (Panel b), consistent with underexploitation of the intangible-skill complementarity.

III. Building Intuition: A Toy Model

III.A. Environment

Consider a two-period economy with a representative firm and no aggregate uncertainty. The firm is endowed with initial tangible capital $K_0 > 0$, initial wealth $W_0 \geq 0$, and a lifetime allocation of skilled labor $\bar{H} > 0$. Productivity $z > 0$ is constant and known.

Period 1 (Investment Phase): The firm chooses tangible investment $I^K \geq 0$ and allocates skilled labor between R&D ($H^R \geq 0$) and production for period 2 ($H^P = \bar{H} - H^R$). Intangible capital is produced linearly:

$$S = \Gamma H^R, \quad \Gamma > 0. \tag{2}$$

Period 2 (Production Phase): The firm produces output using accumulated capital and the skilled labor allocated in period 1. No further investment decisions are made.

III.B. Technology

Capital Stocks. Period 2 tangible capital is:

$$K = K_0 + I^K. \tag{3}$$

Intangible-Skill Composite. The intangible capital S from period 1 combines with the allocated production labor $H^P = \bar{H} - H^R$ to form:

$$Q(S, H^P) = \left[\omega S^\rho + (1-\omega)(H^P)^\rho\right]^{1/\rho}, \quad \omega \in (0,1),\ \rho < 0. \tag{4}$$

The elasticity of substitution $\sigma_Q = 1/(1-\rho) < 1$ captures complementarity.

Production. Period 2 output is:

$$Y = zK^\alpha Q(S, H^P)^{1-\alpha}, \quad \alpha \in (0,1). \tag{5}$$

For tractability, I abstract from unskilled labor and normalize away depreciation.

III.C. Financial Constraint

The firm may borrow only against tangible assets. At the end of period 1, the pledgeable capital stock equals $K_0 + I^K$. Thus:

$$D \le \alpha_K(K_0 + I^K), \quad \alpha_K \in (0,1), \tag{6}$$

where D denotes borrowing in period 1. Intangible capital is non-pledgeable: $\alpha_S = 0$.

Period 1 Budget Constraint:

$$I^K + w_H H^R \le W_0 + D. \tag{7}$$

If the constraint binds, $D = \alpha_K(K_0 + I^K)$, so:

$$I^K + w_H H^R \le W_0 + \alpha_K(K_0 + I^K), \tag{8}$$

$$I^K(1-\alpha_K) + w_H H^R \le W_0 + \alpha_K K_0. \tag{9}$$

Define *effective wealth*:

$$W_0' \equiv W_0 + \alpha_K K_0,$$

so the constraint becomes:

$$I^K(1-\alpha_K) + w_H H^R \le W_0'. \tag{10}$$

Interpretation. Each additional unit of tangible investment increases borrowing capacity by α_K, effectively reducing its cost to $(1-\alpha_K)$. R&D yields no such benefit. The firm's effective resources include its financial wealth W_0 plus the borrowing capacity from its initial capital stock ($\alpha_K K_0$).

III.D. Firm Problem

The firm chooses (I^K, H^R) to maximize net output:

$$\max_{I^K \ge 0,\, H^R \in [0,\bar{H}]} \quad Y - I^K - w_H \bar{H} \tag{11}$$

subject to the budget constraint (9), where:

$$Y = z(K_0 + I^K)^\alpha Q(\Gamma H^R, \bar{H} - H^R)^{1-\alpha}, \tag{12}$$

$$Q(\Gamma H^R, \bar{H} - H^R) = \left[\omega(\Gamma H^R)^\rho + (1-\omega)(\bar{H} - H^R)^\rho\right]^{1/\rho}. \tag{13}$$

Note that total skilled labor cost $w_H \bar{H}$ is fixed, so the firm's choice concerns only the allocation of $\bar{H}$ between R&D and production.

1. Unconstrained Solution

When W_0' is sufficiently large that constraint (9) does not bind, the first-order conditions are:

$$\frac{\partial Y}{\partial I^K} = \alpha z K^{\alpha-1} Q^{1-\alpha} = 1, \tag{14}$$

$$\frac{\partial Y}{\partial H^R} = (1-\alpha) z K^\alpha Q^{-\alpha}\left[\frac{\partial Q}{\partial S}\Gamma - \frac{\partial Q}{\partial H^P}\right] = 0. \tag{15}$$

From (15), the optimal allocation balances the marginal product of intangibles (via R&D) against the marginal product of direct production labor:

$$\frac{\partial Q}{\partial S}\Gamma = \frac{\partial Q}{\partial H^P}. \tag{16}$$

This is an interior solution exploiting the complementarity structure (4).

2. Constrained Solution

When constraint (9) binds, form the Lagrangian:

$$\mathcal{L} = Y - I^K - w_H \bar{H} - \mu[I^K(1-\alpha_K) + w_H H^R - W_0'], \tag{17}$$

where $\mu > 0$ is the shadow value of resources.

The first-order conditions become:

$$\frac{\partial Y}{\partial I^K} = 1 + \mu(1-\alpha_K), \tag{18}$$

$$\frac{\partial Y}{\partial H^R} = \mu w_H. \tag{19}$$

Key Observation. Comparing (14) with (18): the constrained firm requires a higher marginal product of capital, implying lower I^K.

Comparing (15) with (19): the constrained firm requires a positive marginal product of R&D labor (instead of zero), implying lower H^R (and correspondingly higher H^P) than the unconstrained firm.

III.E. Pecking-Order Distortion

Before stating the main proposition, I establish that the constrained firm produces a lower intangible-skill composite.

Lemma 1 (Suboptimal Allocation)**.** *The constrained firm produces a lower intangible-skill composite than the unconstrained firm:* $Q_c < Q_u$*.*

Proof. Define $\tilde{Q}(H^R; \bar{H}) \equiv Q(\Gamma H^R, \bar{H} - H^R)$ as the intangible-skill composite viewed as a function of the R&D allocation for given $\bar{H}$. The unconstrained firm chooses H_u^R to satisfy (16), which is equivalent to maximizing $\tilde{Q}$ over $H^R \in [0, \bar{H}]$:

$$H_u^R = \arg\max_{H^R \in [0,\bar{H}]} \tilde{Q}(H^R; \bar{H}).$$

The constrained firm chooses $H_c^R < H_u^R$ (as established below in Proposition 1). Since $\tilde{Q}$ is strictly concave in H^R and $H_c^R \neq H_u^R$, we have $\tilde{Q}(H_c^R; \bar{H}) < \tilde{Q}(H_u^R; \bar{H})$, i.e., $Q_c < Q_u$. □

Remark (Parameter Dependence and Limiting Cases)**.** *The result* $Q_c < Q_u$ *and its magnitude depend critically on the elasticity parameter* ρ*. The proof requires strict concavity of* $\tilde{Q}(H^R) = [\omega(\Gamma H^R)^\rho + (1-\omega)(\bar{H} - H^R)^\rho]^{1/\rho}$*, which holds for* $\rho < 1$ *with an interior optimum. We examine the limiting cases:*

Case 1: $\rho \to -\infty$ ***(Leontief,*** $\sigma_Q \to 0$***).*** *In this limit,*

$$Q \to \min\{S, H^P\}$$

after appropriate normalization of ω*. The composite becomes* $\tilde{Q}(H^R) = \min\{\Gamma H^R, \bar{H} - H^R\}$*, which is maximized at the balanced allocation* $H_u^R = \bar{H}/(1+\Gamma)$*, yielding* $Q_u = \Gamma\bar{H}/(1+\Gamma)$*. Any deviation creates complete bottlenecks: the constrained firm's underinvestment in* S *implies* $Q_c = S_c = \Gamma H_c^R < \Gamma H_u^R \leq Q_u$*, with* H_c^P *entirely wasted beyond the binding constraint. The distortion is maximally severe.*

Case 2: $\rho \to 0$ ***(Cobb-Douglas,*** $\sigma_Q \to 1$***).*** *Taking the limit via L'Hôpital's rule,*

$$Q \to S^\omega (H^P)^{1-\omega}.$$

The composite $\tilde{Q}(H^R) = \Gamma^\omega (H^R)^\omega (\bar{H} - H^R)^{1-\omega}$ is strictly concave with interior maximum at $H_u^R = \omega \bar{H}$. The result $Q_c < Q_u$ holds, with the gap determined by the curvature $\tilde{Q}''(H_u^R) = -\omega(1-\omega)\Gamma^\omega \bar{H}^{-1}(H_u^R)^{\omega-2}(\bar{H} - H_u^R)^{-1-\omega} < 0$.

Case 3: $\rho \in (0,1)$ ***(Gross Substitutes,*** $\sigma_Q > 1$***).*** *The CES function remains strictly concave for $\rho < 1$, so the analysis proceeds unchanged. The composite $\tilde{Q}$ has a unique interior maximum, and $Q_c < Q_u$ holds. However, because S and H^P are substitutes, the constrained firm can partially compensate for low S by allocating more H^P, reducing the magnitude of $Q_u - Q_c$ relative to the complementary case.*

Case 4: $\rho \to 1$ ***(Perfect Substitutes,*** $\sigma_Q \to \infty$***).*** *In this limit,*

$$Q \to \omega S + (1-\omega) H^P,$$

and the composite becomes $\tilde{Q}(H^R) = \omega \Gamma H^R + (1-\omega)(\bar{H} - H^R) = [\omega\Gamma - (1-\omega)]H^R + (1-\omega)\bar{H}$. This is linear *in H^R, so $\tilde{Q}$ is no longer strictly concave. Three subcases arise:*

- *If $\omega\Gamma > 1 - \omega$: corner solution $H_u^R = \bar{H}$ (all labor to R&D).*
- *If $\omega\Gamma < 1 - \omega$: corner solution $H_u^R = 0$ (all labor to production).*
- *If $\omega\Gamma = 1 - \omega$: the firm is indifferent over all allocations, and $Q = (1-\omega)\bar{H}$ regardless of H^R.*

In the knife-edge case $\omega\Gamma = 1 - \omega$, the financial constraint creates no distortion in Q (though it still distorts K). Otherwise, the constraint may or may not bind depending on which corner is optimal.

Summary. *The baseline assumption $\rho < 0$ ensures: (i) strict concavity of $\tilde{Q}$ with interior optimum, (ii) the result $Q_c < Q_u$ for any constrained allocation, and (iii) economically significant complementarity that amplifies the distortion. As $\rho \to -\infty$, the distortion becomes arbitrarily severe; as $\rho \to 1$, the distortion vanishes in the knife-edge case or the problem degenerates to corner solutions.*

Proposition 1 (Pecking Order)**.** *Under the collateral constraint (6) with $\alpha_K \in (0,1)$ and $\alpha_S = 0$, constrained firms ($\mu > 0$) exhibit:*

(i) Lower R&D investment: $H_c^R < H_u^R$

(ii) Lower intangible capital: $S_c < S_u$

(iii) Higher production labor: $H_c^P > H_u^P$

(iv) Lower tangible investment: $I_c^K < I_u^K$

where subscripts c and u denote constrained and unconstrained firms respectively.

Proof. **Parts (i)-(iii):** From (19), the constrained firm requires $\partial Y/\partial H^R = \mu w_H > 0$, while the unconstrained firm has $\partial Y/\partial H^R = 0$ at optimum. Since

$$\frac{\partial Y}{\partial H^R} = (1-\alpha) z K^\alpha Q^{-\alpha} \left[\frac{\partial Q}{\partial S}\Gamma - \frac{\partial Q}{\partial H^P} \right],$$

and the term $(1-\alpha) z K^\alpha Q^{-\alpha} > 0$, the constrained firm has

$$\frac{\partial Q}{\partial S}\Gamma > \frac{\partial Q}{\partial H^P}$$

at its optimum, whereas the unconstrained firm has equality. Computing the partial derivatives of (4):

$$\frac{\partial Q}{\partial S} = \omega S^{\rho-1} Q^{1-\rho}, \qquad \frac{\partial Q}{\partial H^P} = (1-\omega)(H^P)^{\rho-1} Q^{1-\rho}.$$

The condition $\frac{\partial Q}{\partial S}\Gamma > \frac{\partial Q}{\partial H^P}$ implies a lower S/H^P ratio than the unconstrained optimum. Given the constraint $H^P = \bar{H} - H^R$ and $S = \Gamma H^R$, this requires $H_c^R < H_u^R$. Parts (ii) and (iii) follow immediately.

Part (iv): Define $f(I^K; Q) \equiv \alpha z (K_0 + I^K)^{\alpha-1} Q^{1-\alpha}$, which is decreasing in I^K (since $\alpha - 1 < 0$) and increasing in Q.

The unconstrained firm satisfies $f(I_u^K; Q_u) = 1$.

For the constrained firm, evaluate f at I_u^K with Q_c:

$$f(I_u^K; Q_c) = \alpha z (K_0 + I_u^K)^{\alpha-1} Q_c^{1-\alpha} < \alpha z (K_0 + I_u^K)^{\alpha-1} Q_u^{1-\alpha} = 1,$$

where the inequality uses $Q_c < Q_u$ (Lemma 1) and $(1-\alpha) > 0$.

The constrained firm's FOC (18) requires $f(I_c^K; Q_c) = 1 + \mu(1-\alpha_K) > 1$. Since $f(\cdot; Q_c)$ is strictly decreasing in I^K and $f(I_u^K; Q_c) < 1 < f(I_c^K; Q_c)$, we must have $I_c^K < I_u^K$. □

Remark. *The asymmetry arises from differential collateral value. Each unit of I^K reduces the effective resource constraint by $(1-\alpha_K)$ because it generates α_K in borrowing capacity. In contrast, H^R provides no collateral benefit since $\alpha_S = 0$. This creates an implicit subsidy for tangible investment relative to R&D.*

III.F. Underexploitation of Complementarity

Lemma 2 (Complementarity). *The intangible-skill composite (4) with $\rho < 0$ satisfies:*

$$\frac{\partial^2 Q}{\partial S\, \partial H^P} = \omega(1-\omega)(1-\rho) S^{\rho-1} (H^P)^{\rho-1} Q^{1-2\rho} > 0. \tag{20}$$

Proof. Direct differentiation of (4). All terms are positive when $\rho < 0$ since $(1-\rho) > 0$ and $(1-2\rho) > 0$. □

Corollary 1 (Underexploitation of Skilled Labor). *Despite having higher production labor ($H_c^P > H_u^P$), the constrained firm has lower marginal product of that labor:*

$$\frac{\partial Q}{\partial H^P}(S_c, H_c^P) < \frac{\partial Q}{\partial H^P}(S_u, H_u^P). \tag{21}$$

Proof. The marginal product of production labor can be rewritten as:

$$\frac{\partial Q}{\partial H^P} = (1-\omega)(H^P)^{\rho-1} Q^{1-\rho} = (1-\omega)\left(\frac{Q}{H^P}\right)^{1-\rho}. \tag{22}$$

Since $\rho < 0$, we have $1 - \rho > 1$, so (22) is strictly increasing in the ratio Q/H^P.

From Lemma 1, $Q_c < Q_u$. From Proposition 1(iii), $H_c^P > H_u^P$. Therefore:

$$\frac{Q_c}{H_c^P} < \frac{Q_u}{H_u^P},$$

which implies $\frac{\partial Q}{\partial H^P}(S_c, H_c^P) < \frac{\partial Q}{\partial H^P}(S_u, H_u^P)$. □

Economic Interpretation. The constrained firm overallocates skilled labor to production (H^P) relative to intangibles (S) because of the financial distortion. However, due to complementarity, the productivity of that labor is limited by the low stock of intangibles. This creates a *double inefficiency*: wrong allocation *and* underexploitation of the resources allocated to production.

III.G. Skill-Biased Stagnation

Consider a comparative static exercise: an increase in the skilled labor endowment $\bar{H}$.

1. Unconstrained Firm Response

For the unconstrained firm, the optimality condition (16) determines the allocation of $\bar{H}$ between H^R and H^P. Using the expressions for marginal products:

$$\omega\Gamma(\Gamma H^R)^{\rho-1} = (1-\omega)(\bar{H} - H^R)^{\rho-1}.$$

This condition pins down the *ratio* $H^R/(\bar{H} - H^R)$ independently of $\bar{H}$. Therefore, the R&D share $\phi^R \equiv H^R/\bar{H}$ is constant:

$$\frac{dH_u^R}{d\bar{H}} = \phi_u^R, \qquad \frac{dH_u^P}{d\bar{H}} = 1 - \phi_u^R. \tag{23}$$

Both margins expand proportionally, fully exploiting the complementarity.

2. Constrained Firm Response

The binding constraint implies:

$$I_c^K(1-\alpha_K) + w_H H_c^R = W_0'.$$

Differentiating:

$$(1-\alpha_K)\, dI_c^K + w_H\, dH_c^R = 0.$$

Because both constrained FOCs depend on

$$Q_c = Q(\Gamma H_c^R, \bar{H} - H_c^R),$$

which varies with $\bar{H}$, the pair (I_c^K, H_c^R) generally adjusts with $\bar{H}$.

However, the constraint imposes a strong wedge: additional skilled labor primarily enters production. Formally, the constrained R&D response satisfies:

$$0 \le \frac{dH_c^R}{d\bar{H}} < \phi_u^R, \qquad \frac{dH_c^P}{d\bar{H}} = 1 - \frac{dH_c^R}{d\bar{H}}. \tag{24}$$

Proposition 2 (Skill-Biased Stagnation)**.** *When the skilled labor endowment increases ($\bar{H}\uparrow$):*

(i) The unconstrained firm increases both H^R and H^P proportionally, building higher S and exploiting complementarity.

(ii) The constrained firm expands H^P strictly more than H^R:

$$0 \le \frac{dH_c^R}{d\bar{H}} < \phi_u^R.$$

(iii) The intangible-skill composite grows faster for the unconstrained firm:

$$\frac{dQ_u}{d\bar{H}} > \frac{dQ_c}{d\bar{H}} > 0. \tag{25}$$

(iv) The output gap between firms widens:

$$\frac{d(Y_u - Y_c)}{d\bar{H}} > 0. \tag{26}$$

Proof. Parts (i)-(ii) follow from the derived responses.

Part (iii): By the envelope theorem, since the unconstrained firm maximizes Q over the allocation of $\bar{H}$:

$$\frac{dQ_u}{d\bar{H}} = \left.\frac{\partial Q}{\partial H^P}\right|_{(S_u, H_u^P)}.$$

For the constrained firm:

$$\frac{dQ_c}{d\bar{H}} = \frac{\partial Q}{\partial S}\Gamma\frac{dH_c^R}{d\bar{H}} + \frac{\partial Q}{\partial H^P}\left(1 - \frac{dH_c^R}{d\bar{H}}\right).$$

Since the unconstrained firm satisfies $\frac{\partial Q}{\partial S}\Gamma = \frac{\partial Q}{\partial H^P}$, we can write:

$$\frac{dQ_u}{d\bar{H}} = \left.\frac{\partial Q}{\partial H^P}\right|_{(S_u, H_u^P)} = \left.\frac{\partial Q}{\partial S}\right|_{(S_u, H_u^P)}\Gamma = \left[\frac{\partial Q}{\partial S}\Gamma\phi_u^R + \frac{\partial Q}{\partial H^P}(1 - \phi_u^R)\right]_{(S_u, H_u^P)}.$$

From Corollary 1, $\frac{\partial Q}{\partial H^P}(S_u, H_u^P) > \frac{\partial Q}{\partial H^P}(S_c, H_c^P)$. Moreover, from the constrained firm's FOC, $\frac{\partial Q}{\partial S}(S_c, H_c^P)\Gamma > \frac{\partial Q}{\partial H^P}(S_c, H_c^P)$. Since $\frac{dH_c^R}{d\bar{H}} < \phi_u^R$ (part ii), the constrained firm places less weight on the higher-valued margin (R&D) and more weight on the lower-valued margin (production), establishing (25).

Part (iv): From $Y = zK^\alpha Q^{1-\alpha}$:

$$\frac{dY}{d\bar{H}} = (1-\alpha)zK^\alpha Q^{-\alpha}\frac{dQ}{d\bar{H}} = (1-\alpha)\frac{Y}{Q}\frac{dQ}{d\bar{H}}.$$

Define the "output-to-composite" ratio $\psi \equiv Y/Q = zK^\alpha Q^{-\alpha}$. Then:

$$\frac{d(Y_u - Y_c)}{d\bar{H}} = (1-\alpha)\left[\psi_u\frac{dQ_u}{d\bar{H}} - \psi_c\frac{dQ_c}{d\bar{H}}\right].$$

From part (iii), $\frac{dQ_u}{d\bar{H}} > \frac{dQ_c}{d\bar{H}}$. Using (22):

$$\frac{dQ}{d\bar{H}} \geq \frac{\partial Q}{\partial H^P} = (1-\omega)\left(\frac{Q}{H^P}\right)^{1-\rho}.$$

Therefore:

$$\begin{aligned}\frac{dY}{d\bar{H}} &\geq (1-\alpha)zK^\alpha Q^{-\alpha}(1-\omega)\left(\frac{Q}{H^P}\right)^{1-\rho} \\ &= (1-\alpha)(1-\omega)zK^\alpha Q^{1-\alpha-\rho}(H^P)^{-(1-\rho)}.\end{aligned} \tag{27}$$

From Proposition 1: $K_u > K_c$, $Q_u > Q_c$, and $H_u^P < H_c^P$. Since $\alpha > 0$, $1 - \alpha - \rho > 0$ (as $\rho < 0$), and $1 - \rho > 0$:

- $K_u^\alpha > K_c^\alpha$
- $Q_u^{1-\alpha-\rho} > Q_c^{1-\alpha-\rho}$

- $(H_u^P)^{-(1-\rho)} > (H_c^P)^{-(1-\rho)}$

All three factors favor the unconstrained firm, so $\frac{dY_u}{d\bar{H}} > \frac{dY_c}{d\bar{H}}$, establishing (26). $\square$

Economic Intuition. The skill-biased stagnation arises because constrained firms cannot finance the R&D needed to build intangibles that would complement the additional skilled labor in production. While they can deploy more H^P, the lack of complementary S means this labor is underexploited. Unconstrained firms, by contrast, expand both margins and realize the full gains from complementarity.

IV. Quantitative Model

IV.A. Environment

1. Time and Agents

Time is discrete. A continuum of firms $j \in [0,1]$ faces idiosyncratic productivity $z_{j,t}$. Each firm holds tangible capital $K_{j,t}$, intangible capital $S_{j,t}$, and debt $D_{j,t}$. Firms exit with probability $\zeta \in (0,1)$; entrants draw $z_0 \sim F_z$ on $[\underline{z}, \overline{z}]$ and start with $K_0 = S_0 = D_0 = 0$, receiving initial equity $a_0 > 0$ financed by household transfers.

2. Household

A representative household owns all firms, supplies skilled and unskilled labor inelastically, and has standard preferences over consumption:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \qquad 0 < \beta < 1.$$

The household supplies $\bar{L}$ units of unskilled labor and $\bar{H}$ units of skilled labor, where $\bar{L} + \bar{H} = 1$.

The household's budget constraint is:

$$C_t + B_{t+1} + T_t = w_L \bar{L} + w_H \bar{H} + (1+r)B_t + \Pi_t, \tag{28}$$

where B_t denotes household deposits at banks, Π_t is aggregate firm dividends, and T_t represents transfers to finance new entrants replacing exiting firms.

The household's Euler equation with linear utility pins down:

$$1 + r = \frac{1}{\beta}. \tag{29}$$

3. Financial Intermediation

Competitive banks accept deposits from households at rate r and lend to firms at the same rate. Banks perfectly enforce repayment up to collateral value and make zero profits. No default occurs in equilibrium—the collateral constraint binds ex ante, preventing default, following Kiyotaki and Moore (1997).

IV.B. Technology

1. Idiosyncratic Productivity

$$\log z_{j,t+1} = \rho_z \log z_{j,t} + \sigma_z \varepsilon_{j,t+1}, \quad \varepsilon_{j,t+1} \sim N(0,1),$$

with $0 < \rho_z < 1$, $\sigma_z > 0$.

2. Production Technology

Production involves a nested CES structure. First, define the capital composite:

$$X_{j,t} = \left[\theta_K K_{j,t}^{\rho_K} + \theta_Q Q_{j,t}^{\rho_K}\right]^{1/\rho_K}, \tag{30}$$

where $Q_{j,t}$ is an intangible-skill bundle defined as:

$$Q_{j,t} = \left[\omega S_{j,t}^{\rho_Q} + (1-\omega)(H_{j,t}^P)^{\rho_Q}\right]^{1/\rho_Q}. \tag{31}$$

The final production function is:

$$Y_{j,t} = z_{j,t}\left[X_{j,t}^{\alpha} L_{j,t}^{\gamma}\right]^{\nu}, \tag{32}$$

where $\alpha, \gamma, \nu \in (0,1)$, $\theta_K, \theta_Q, \omega \in (0,1)$, and $\alpha + \gamma = 1$ (constant returns in the inner nest).

Assumption 1 (Complementarity Structure)**.** *Define elasticities $\sigma_K \equiv 1/(1-\rho_K)$ and $\sigma_Q \equiv 1/(1-\rho_Q)$. We assume $\sigma_Q < \sigma_K$ (i.e., $\rho_Q < \rho_K < 0$), so that intangibles and skilled labor are stronger complements than tangible capital and the intangible-skill bundle.*

Remark. *Assumption 1 captures the idea that intangible capital and skilled labor work together in a more integrated way than tangible capital does with the intangible-skill composite Q. Empirical evidence from Gozen and Ozkara (2024) supports the existence of synergies between intangibles and skilled labor.*

3. Capital Accumulation

Tangible capital evolves via standard accumulation:

$$K_{j,t+1} = (1-\delta_K)K_{j,t} + I_{j,t}^K, \tag{33}$$

where $I_{j,t}^K$ is tangible investment and $0 < \delta_K < 1$.

Intangible capital is produced via R&D labor:

$$S_{j,t+1} = (1-\delta_S)S_{j,t} + \Gamma(H_{j,t}^R)^{\xi}, \qquad 0 < \xi \leq 1, \tag{34}$$

where $H_{j,t}^R$ is skilled labor allocated to R&D, $\Gamma > 0$, and $\xi \leq 1$ captures (weakly) decreasing returns to R&D labor in knowledge creation.[1]

Total skilled labor hired by the firm satisfies:

$$H_{j,t} = H_{j,t}^P + H_{j,t}^R. \tag{35}$$

4. Adjustment Costs

Following the investment literature (?Khan and Thomas, 2008), capital adjustment is subject to convex costs:

$$\Phi_{j,t}^K = \frac{\phi_K}{2}\frac{(I_{j,t}^K)^2}{K_{j,t}}, \qquad \Phi_{j,t}^S = \frac{\phi_S}{2}\frac{(\Delta S_{j,t})^2}{S_{j,t}}, \tag{36}$$

where $\Delta S_{j,t} \equiv S_{j,t+1} - (1-\delta_S)S_{j,t} = \Gamma(H_{j,t}^R)^{\xi}$ is gross intangible investment. Setting $\phi_K = \phi_S = 0$ nests the frictionless case.

1 The case $\xi < 1$ captures diminishing returns in R&D production. The limiting case $\xi = 1$ corresponds to linear intangible production, as in Atkeson and Burstein (2010).

IV.C. Financial Frictions

1. Collateral Constraint

Following the costly-state-verification literature (Townsend, 1979; Bernanke et al., 1999), lenders can recover only a fraction of firm assets in case of default. Define recovery rates:

$$\alpha_K \in (0,1), \qquad \alpha_S \in [0, \alpha_K), \tag{37}$$

where α_K is the recovery rate on tangible capital and α_S is the recovery rate on intangible capital, with $\alpha_S < \alpha_K$ reflecting the lower pledgeability of intangibles (Holttinen et al., 2025).

Assumption 2 (Low Intangible Pledgeability)**.** *Intangible assets are less pledgeable than tangible assets:* $0 \leq \alpha_S < \alpha_K < 1$*. The baseline calibration considers* $\alpha_S = 0$ *(intangibles fully non-pledgeable).*

To ensure no default in equilibrium, banks lend only up to recoverable collateral value. Following the timing convention in Kiyotaki and Moore (1997) and **?**, debt $D_{j,t+1}$ taken in period t (repaid in $t+1$) is collateralized by capital available when repayment is due:

$$D_{j,t+1} \leq \alpha_K K_{j,t+1} + \alpha_S S_{j,t+1}. \tag{38}$$

Since $K_{j,t+1}$ and $S_{j,t+1}$ depend on current investment decisions, this creates a direct link between investment and borrowing capacity within the period.

2. Timing

The stages below describe the sequence of decisions within period t:

1. **Beginning of period:** Firm enters with state (z_{t-1}, K_t, S_t, D_t), where D_t is debt to be repaid this period.

2. **Productivity realization:** Current productivity z_t is drawn from the AR(1) process.

3. **Exit shock:** With probability ζ, the firm exits: it produces, collects revenue net of wages, liquidates capital, repays debt $(1+r)D_t$, and distributes residual value to shareholders. With probability $(1-\zeta)$, the firm survives.

4. **Production decisions:** Surviving firms choose labor (L_t, H_t^P) to maximize static profits.

5. **Investment and financing:** Firms choose tangible investment I_t^K, R&D labor H_t^R, and new debt D_{t+1} subject to the collateral constraint.

6. **Dividend payment:** Firms pay dividends $\text{Div}_t \geq 0$ to shareholders.

7. **End of period:** Firm enters $t+1$ with state $(z_t, K_{t+1}, S_{t+1}, D_{t+1})$.

Timing and Wage Equality. Both H^P (production) and H^R (R&D) are skilled labor employed within period t at wage w_H. The single wage clears the aggregate skilled labor market: $\int (H_j^P + H_j^R) d\Psi = \bar{H}$.

3. Budget Constraint and Dividends

The firm's budget constraint equates sources and uses of funds:

$$I_{j,t}^{K} + \Phi_{j,t}^{K} + \Phi_{j,t}^{S} + w_L L_{j,t} + w_H (H_{j,t}^{P} + H_{j,t}^{R}) + (1+r) D_{j,t} = Y_{j,t} + D_{j,t+1} + \mathrm{Div}_{j,t}. \tag{39}$$

Define gross profits from production:

$$\Pi_{j,t}^{gross} \equiv Y_{j,t} - w_L L_{j,t} - w_H H_{j,t}^{P}. \tag{40}$$

Dividends distributed to shareholders are:

$$\mathrm{Div}_{j,t} = \Pi_{j,t}^{gross} - I_{j,t}^{K} - \Phi_{j,t}^{K} - \Phi_{j,t}^{S} - w_H H_{j,t}^{R} - (1+r) D_{j,t} + D_{j,t+1}. \tag{41}$$

Firms cannot issue new equity:

$$\mathrm{Div}_{j,t} \geq 0. \tag{42}$$

IV.D. Firm Problem

1. Static Production Problem

Given capital (K, S) and productivity z, the firm chooses labor (L, H^P) to maximize static profits:

$$\pi(z, K, S) = \max_{L, H^P} \Big\{ Y(z, K, S, L, H^P) - w_L L - w_H H^P \Big\}. \tag{43}$$

The first-order conditions are:

$$\frac{\partial Y}{\partial L} = w_L, \qquad \frac{\partial Y}{\partial H^P} = w_H. \tag{44}$$

These yield policy functions $L^*(z, K, S)$ and $H^{P*}(z, K, S)$. Labor decisions are undistorted by financial frictions since labor is paid contemporaneously.

2. Dynamic Problem

The firm maximizes the expected present value of dividends. The state is (z, K, S, D), and the value function satisfies:

$$V(z, K, S, D) = \max_{D', I^K, H^R} \Big\{ \mathrm{Div} + \beta(1-\zeta)\, \mathbb{E}[V(z', K', S', D') \mid z] \Big\}, \tag{45}$$

subject to:

- Dividends: $\mathrm{Div} = \pi(z, K, S) - I^K - \Phi^K - \Phi^S - w_H H^R - (1+r) D + D'$
- Capital accumulation: $K' = (1-\delta_K) K + I^K, \quad S' = (1-\delta_S) S + \Gamma (H^R)^{\xi}$
- Collateral constraint: $D' \leq \alpha_K K' + \alpha_S S'$
- Non-negative dividends: $\mathrm{Div} \geq 0$

3. First-Order Conditions

Let $\mu \geq 0$ denote the Lagrange multiplier on the dividend constraint (42), and $\lambda \geq 0$ the multiplier on the collateral constraint (38). When $\mu = 0$, dividends are positive and internal funds are not scarce; when $\mu > 0$, the dividend constraint binds and internal funds command a premium. When $\lambda = 0$, the collateral constraint is slack; when $\lambda > 0$, it binds.

FOC for Debt (D').

$$1 + \mu = \lambda + \beta(1-\zeta)(1+r)\,\mathbb{E}[(1+\mu')]. \tag{46}$$

For an unconstrained firm ($\mu = \lambda = 0$), this reduces to $1 = \beta(1-\zeta)(1+r)$, which holds when exit risk exactly offsets the discount factor.

FOC for Tangible Investment (I^K).

$$(1+\mu)\left(1 + \phi_K \frac{I^K}{K}\right) = \lambda\alpha_K + \beta(1-\zeta)\,\mathbb{E}[V_{K'}]. \tag{47}$$

The left-hand side is the marginal cost: purchasing one unit plus adjustment costs, valued at $(1+\mu)$ reflecting the shadow cost of internal funds. The right-hand side has two components:

1. $\lambda\alpha_K$: the immediate collateral benefit—investment increases K', relaxing the current-period borrowing constraint.
2. $\beta(1-\zeta)\,\mathbb{E}[V_{K'}]$: the expected continuation value of additional capital.

Envelope Condition for K.

$$V_K = (1+\mu)\frac{\partial \pi}{\partial K} + (1+\mu)\frac{\phi_K}{2}\left(\frac{I^K}{K}\right)^2 + (1-\delta_K)\Big[\lambda\alpha_K + \beta(1-\zeta)\,\mathbb{E}[V_{K'}]\Big]. \tag{48}$$

The term $(1+\mu)\frac{\phi_K}{2}(I^K/K)^2$ reflects that higher capital reduces adjustment costs for given investment. The collateral term $\lambda\alpha_K$ is multiplied by $(1-\delta_K)$ because only undepreciated capital carries forward.

Combining (47) and (48) yields the Euler equation:

$$\begin{aligned}(1+\mu_t)\left(1 + \phi_K \frac{I^K_t}{K_t}\right) = \lambda_t\alpha_K + \beta(1-\zeta)\,\mathbb{E}_t\Bigg[&(1+\mu_{t+1})\frac{\partial \pi_{t+1}}{\partial K_{t+1}} + (1+\mu_{t+1})\frac{\phi_K}{2}\left(\frac{I^K_{t+1}}{K_{t+1}}\right)^2 \\ &+ (1-\delta_K)\Big(\lambda_{t+1}\alpha_K + \beta(1-\zeta)\,\mathbb{E}_{t+1}[V_{K,t+2}]\Big)\Bigg].\end{aligned} \tag{49}$$

FOC for Intangible Investment (via H^R). Since $\partial S'/\partial H^R = \Gamma\xi(H^R)^{\xi-1}$:

$$(1+\mu)\left(w_H + \phi_S \frac{\Delta S}{S}\cdot \Gamma\xi(H^R)^{\xi-1}\right) = \Big[\lambda\alpha_S + \beta(1-\zeta)\,\mathbb{E}[V_{S'}]\Big]\cdot \Gamma\xi(H^R)^{\xi-1}. \tag{50}$$

Define the marginal cost of producing intangibles as $q_S \equiv w_H/[\Gamma\xi(H^R)^{\xi-1}]$. Abstracting from adjustment costs, the FOC simplifies to:

$$(1+\mu)\cdot q_S = \lambda\alpha_S + \beta(1-\zeta)\,\mathbb{E}[V_{S'}]. \tag{51}$$

Envelope Condition for S.

$$V_S = (1+\mu)\frac{\partial \pi}{\partial S} + (1+\mu)\frac{\phi_S}{2}\left(\frac{\Delta S}{S}\right)^2 + (1-\delta_S)\Big[\lambda \alpha_S + \beta(1-\zeta)\,\mathbb{E}[V_{S'}]\Big]. \tag{52}$$

Envelope Condition for D.

$$V_D = -(1+\mu)(1+r). \tag{53}$$

4. The Pecking-Order Distortion

Comparing the FOCs for tangible (47) and intangible (51) investment reveals the core mechanism.

Proposition 3 (Pecking-Order Distortion)**.** *For a financially constrained firm ($\lambda > 0$), the immediate collateral benefit is larger for tangible than intangible investment:*

$$\underbrace{\lambda \alpha_K}_{\textit{collateral benefit of } K'} > \underbrace{\lambda \alpha_S}_{\textit{collateral benefit of } S'} \tag{54}$$

since $\alpha_K > \alpha_S$.

Intuition. Each unit of tangible capital generates borrowing capacity α_K, while intangible capital generates only $\alpha_S < \alpha_K$. When the collateral constraint binds ($\lambda > 0$), this asymmetry creates an implicit subsidy for tangible investment: the effective cost of tangible investment is reduced by $\lambda\alpha_K/(1+\mu)$, while intangible investment is reduced by only $\lambda\alpha_S/(1+\mu)$.

For unconstrained firms ($\lambda = 0$), the collateral terms vanish and investment depends only on marginal products—the first-best allocation.

Capital Composition Distortion. Rearranging the FOCs in steady state:

$$\frac{\mathrm{MB}_K - \lambda\alpha_K}{\mathrm{MB}_S - \lambda\alpha_S} = \frac{(1+\mu)(1+\phi_K I^K/K)}{(1+\mu)q_S}, \tag{55}$$

where MB_K and MB_S denote the continuation value terms. For constrained firms, since $\alpha_K > \alpha_S$, the numerator is reduced more than the denominator, distorting capital composition toward tangibles.

IV.E. Equilibrium

Definition 1 (Stationary Recursive Equilibrium)**.** *A stationary recursive equilibrium consists of:*

(i) Firm value function $V(z,K,S,D)$ and policy functions $\{K'(\cdot), S'(\cdot), D'(\cdot), I^K(\cdot), H^R(\cdot), L(\cdot), H^P(\cdot)$

(ii) Wages (w_L, w_H) and interest rate r,

(iii) A stationary distribution of firms $\Psi^*(z,K,S,D)$,

(iv) Aggregate quantities $\{C, Y_{agg}, K_{agg}, S_{agg}, D_{agg}\}$,

such that:

(a) ***Firms optimize:*** *$V(z, K, S, D)$ solves (45) and policies satisfy the FOCs.*

(b) ***Household optimizes:*** *$1 + r = 1/\beta$ from the Euler equation.*

(c) ***Labor markets clear:***

$$\int L_j \, d\Psi^* = \bar{L}, \qquad \int (H_j^P + H_j^R) \, d\Psi^* = \bar{H}.$$

(d) ***Credit market clears:***

$$B = \int D_j \, d\Psi^*.$$

(e) ***Goods market clears:***

$$C + \int \left(I_j^K + \Phi_j^K + \Phi_j^S\right) d\Psi^* + T = \int Y_j \, d\Psi^*,$$

where $T = \zeta a_0$ finances entrants.

(f) ***Free entry:*** *The expected value of an entrant equals required initial equity:*

$$\int V(z, 0, 0, 0) \, dF_z(z) = a_0.$$

(g) ***Stationarity:*** *Ψ^* is invariant under the transition implied by policy functions, exit, and entry.*

V. Quantitative Experiments

The quantitative model enables several counterfactual experiments to quantify the skill-biased stagnation mechanism.

Baseline Experiment: Skill Supply Shock. I compute steady-state economies for different values of the skill ratio $\bar{H}/\bar{L}$, comparing outcomes with financial frictions (baseline calibration) against a frictionless counterfactual (setting $\lambda = 0$ for all firms). This experiment measures how financial frictions mediate the aggregate productivity response to skill accumulation.

Transition Dynamics. An alternative approach computes the full transition path following an unexpected permanent increase in $\bar{H}/\bar{L}$, tracking how constrained and unconstrained firms adjust their capital stocks and employment over time. This captures the dynamic misallocation channel as financially constrained firms slowly build intangible capital.

Financial Liberalization. A third experiment varies the pledgeability of intangibles α_S while holding technology fixed, simulating financial innovations that improve the collateralizability of intangible assets. The baseline calibration uses $\alpha_S = 0.134$ and $\alpha_K = 0.381$ from Holttinen (2025). Increasing α_S toward α_K quantifies the TFP gains from reducing the pledgeability gap.

A Data Construction and Cleaning

AA. Data Sources and Sample Construction

The empirical analysis uses Portuguese firm-level data from two administrative sources for the period 2011-2022:

(i) **SCIE (Sistema de Contas Integradas das Empresas):** Balance sheet and income statement data covering all Portuguese firms required to file annual accounts. Variables include assets, liabilities, equity, revenue, costs, investment flows, and R&D expenditures.

(ii) **QP (Quadros de Pessoal):** Matched employer-employee data containing firm characteristics and worker-level information. I aggregate worker data to construct firm-level skill composition measures, defining skilled workers as those with tertiary education (ISCED 5-8).

Before the cleaning procedure, I restrict the sample to private incorporated businesses (*sociedades*), as balance sheet data is only available for incorporated firms and public firms face different objectives and constraints. The initial merged dataset (SCIE-matched, private incorporated firms) contains 2,329,807 firm-year observations spanning 2011-2022.

AB. Price Deflators

All monetary variables are deflated to constant 2020 prices using three price indices:

- **GDP deflator** (base year 2020=100): Applied to revenue, costs, wages, and R&D expenditures. Source: FRED (Federal Reserve Economic Data), series PRTGDPDEFQISMEI_NBD20200101.

- **GFCF deflator** (Gross Fixed Capital Formation, 2020=100): Applied to investment and disinvestment flows. Source: EU KLEMS-INTAN database, Portuguese data.

- **Capital deflator** (2020=100): Applied to balance sheet stocks. Source: EU KLEMS-INTAN database, Portuguese data.

For 2022, GFCF and capital deflators are extrapolated using the 2021-2022 growth rate of the GDP deflator, as EU KLEMS-INTAN data availability ends in 2021.

AC. Data Cleaning Procedure

The cleaning procedure follows a sequential pipeline to ensure data quality before constructing intangible capital stocks. All monetary variables are first deflated to constant 2020 prices using appropriate deflators: GDP deflator for revenue, costs, and wages; GFCF deflator for investment flows; and capital deflator for balance sheet stocks.

Step 1: Structural Problems. Remove observations with missing firm identifiers or duplicate firm-year observations. This quality control check drops a negligible number of observations.

Step 2: Missing or Negative Values. Drop observations with missing or negative values in key balance sheet and income statement variables. Since deflated monetary values should be non-negative, negative values indicate data errors. Checked variables include:

- Tangible fixed assets (physical capital)
- Balance sheet intangibles (excluding goodwill)
- Revenue, production value, gross value added, wagebill
- Total assets, long-term debt, short-term debt, interest expenses
- Tangible investment flows
- Intangible construction inputs: R&D expenditures, advertising, training

Step 3: Missing Economic Activity. Drop observations indicating absence of genuine economic activity. Specifically, I remove firms with:

- Zero or missing number of workers
- Less than 1,000 euros in tangible fixed assets
- Less than 1,000 euros in revenue
- Less than 1,000 euros in production value
- Less than 500 euros in wagebill

These thresholds eliminate shell companies and data errors while retaining small active firms. Physical capital is defined as tangible fixed assets following standard practice in production function estimation, excluding inventories and working capital.

Step 4: Panel Structure. Require at least two consecutive observations per firm to enable construction of intangible capital stocks via the perpetual inventory method (PIM). Firms with isolated observations cannot be used for dynamic capital stock accumulation.

Final Sample. The sequential cleaning procedure yields a final analytical sample of **1,759,093 firm-year observations** spanning 2011-2022. All observations have complete data for core balance sheet variables, positive economic activity, and sufficient panel length for capital stock construction. Missing values in R&D, advertising, or training expenditures are treated as zero investment in these categories, following standard practice in the intangible capital literature.

AD. Intangible Capital Construction

Following Peters and Taylor (2017), I construct intangible capital stocks using the perpetual inventory method (PIM). The baseline measure combines two components: knowledge capital from R&D and balance sheet intangibles.

Knowledge Capital (from R&D). Accumulated from reported R&D expenditures using sector-specific depreciation rates from Ewens et al. (2025):

$$K_{j,t}^{knowledge} = (1 - \delta_R^{sector})K_{j,t-1}^{knowledge} + RD_{j,t}, \tag{56}$$

where δ_R^{sector} varies by Fama-French 5 industry classification:

- Consumer: $\delta_R = 0.43$
- Manufacturing: $\delta_R = 0.50$
- High Tech: $\delta_R = 0.42$
- Health: $\delta_R = 0.33$
- Other: $\delta_R = 0.35$

Initial stocks are set to zero ($K_{j,2011}^{knowledge} = 0$). The sector-specific rates capture heterogeneity in knowledge obsolescence across industries: manufacturing R&D depreciates faster (50%) than health-related R&D (33%), reflecting differences in product cycles and technological change.

Balance Sheet Intangibles. Externally acquired intangibles recorded on firm balance sheets, measured directly as stock variables (excluding goodwill). These capture purchased patents, software, databases, and other codified intangible assets.

Total Intangible Capital. The baseline measure sums knowledge capital and balance sheet intangibles:

$$K_{j,t}^{intangible} = K_{j,t}^{knowledge} + K_{j,t}^{external}. \tag{57}$$

This *BS+R&D* measure focuses on intangibles with clearer market values and avoids the measurement challenges associated with organization capital (SG&A-based accumulation). Total capital is $K_{j,t}^{total} = K_{j,t}^{physical} + K_{j,t}^{intangible}$, where physical capital equals tangible fixed assets from balance sheets.

AE. Winsorization

To limit the influence of extreme outliers while preserving genuine economic variation, I apply winsorization to key variables by year and sector (3-digit CAE code). Financial variables and investment rates are winsorized at the 5th and 95th percentiles within each year-sector cell, allowing for industry heterogeneity in distributions while mitigating the impact of extreme values. This approach balances outlier treatment with preserving cross-sectional variation that is economically meaningful.

AF. Final Dataset Structure

The final analysis-ready dataset contains 1,759,093 firm-year observations with:

- All monetary variables in constant 2020 prices
- Constructed intangible capital stocks using sector-specific depreciation

- Firm-level skill composition (share of workers with tertiary education, share of R&D workers)
- Multiple intangible capital definitions (BS+R&D baseline, full Peters-Taylor alternative)
- Investment rates and intensity measures (intangible intensity, R&D intensity)
- Financial constraint proxies (leverage, interest rates, credit spreads)
- Winsorized versions of key variables

B Robustness: Alternative Outcome Measures

This appendix presents robustness checks for the main empirical results using alternative outcome measures. The main text focuses on gross value added (GVA) as the preferred output measure because it captures the firm's contribution to aggregate output net of intermediate inputs. Here I show that the key findings are robust to using revenue or production value as alternative measures.

BA. Fact 1: Complementarity

Table 3 replicates the complementarity analysis from Table 1 using log revenue (columns 1–3) and log production value (columns 4–6) as dependent variables. Each panel follows the same nested specification: no controls, fixed effects only, and fixed effects with controls.

Table 3: Complementarity Between Intangibles and Skilled Labor: Alternative Outcomes

	Log Revenue			Log Production		
	(1)	(2)	(3)	(4)	(5)	(6)
Intangible Intensity	0.64***	-0.05***	-0.10***	0.48***	-0.05***	-0.11***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Share Skilled Workers	0.18***	-0.05***	-0.01***	0.36***	-0.05***	-0.01**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Intangible Intensity × Share Skilled	0.98***	0.23***	0.03*	1.18***	0.25***	0.04***
	(0.03)	(0.02)	(0.01)	(0.03)	(0.02)	(0.01)
Observations	1,759,093	1,759,093	1,759,084	1,759,093	1,759,093	1,759,084
Adjusted R-squared	0.012	0.933	0.952	0.017	0.931	0.951
Firm FE	No	Yes	Yes	No	Yes	Yes
Year FE	No	Yes	Yes	No	Yes	Yes
Industry FE	No	Yes	Yes	No	Yes	Yes
Controls	No	No	Yes	No	No	Yes

Intangible intensity = $K_{intangible}$ / K_{total}.
Controls include log total capital, log employment, and log firm age.
Robust standard errors in parentheses.

The interaction between intangible intensity and skilled labor share is positive and statistically significant across all specifications and outcome measures. The magnitude of the complementarity effect is similar across GVA, revenue, and production, confirming that the technological complementarity documented in the main text is not driven by the choice of output measure.

BB. Fact 3: Underexploitation by Leverage

Table 4 replicates the underexploitation analysis from Table 2 using log revenue (columns 1–3) and log production value (columns 4–6) as dependent variables. Each panel reports results for all firms, low-leverage firms, and high-leverage firms.

Table 4: Underexploitation of Complementarity: Alternative Outcomes

	Log Revenue			Log Production		
	All	Low Lev.	High Lev.	All	Low Lev.	High Lev.
Main effects:						
Intangible Intensity	-0.10***	-0.11***	-0.10***	-0.11***	-0.12***	-0.11***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Share Skilled Workers	-0.01***	-0.02***	-0.01	-0.01**	-0.02***	-0.00
	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)
Complementarity:						
Intangible Intensity × Share Skilled	0.03*	0.04**	-0.00	0.04***	0.07***	0.00
	(0.01)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)
Observations	1,759,084	852,528	856,243	1,759,084	852,528	856,243
Adjusted R-squared	0.952	0.957	0.958	0.951	0.955	0.959

Low- and high-leverage defined relative to sector-year median leverage.
All specifications include firm, year, and industry fixed effects.
Controls include log total capital, log employment, and log firm age.
Robust standard errors in parentheses.

The pattern of underexploitation is robust across outcome measures: the complementarity coefficient is positive and significant for low-leverage firms but small and insignificant for high-leverage firms. This confirms that the differential ability to exploit intangible-skill complementarity by financial constraint status is not an artifact of the GVA measure.

C Computational Methods

This appendix provides a detailed description of the numerical methods used to solve and simulate the quantitative model. The computational implementation follows standard practices in the heterogeneous-agent and heterogeneous-firm literature while incorporating several acceleration techniques to handle the large state space.

CA. Overview of the Solution Algorithm

The model is solved by finding a stationary recursive competitive equilibrium, which requires:

(i) Solving the firm's dynamic optimization problem via value function iteration (VFI)

(ii) Computing the stationary distribution of firms over the state space

(iii) Clearing labor markets by iterating on wages until excess demand is zero

The outer loop iterates on wages (w_L, w_H) until labor market clearing conditions are satisfied within tolerance. For each wage guess, the inner loops solve the firm problem and compute the implied stationary distribution and aggregate labor demands.

CB. State and Choice Spaces

State Variables. The firm's state is (z, K, S, D) where z is idiosyncratic productivity, K is tangible capital, S is intangible capital, and D is debt to be repaid this period. Productivity z is discretized using the **?** method with n_z grid points covering ± 3 unconditional standard deviations. The continuous state variables (K, S, D) are discretized on grids:

- Tangible capital K: n_K points, log-spaced from $K_{\min}$ to $K_{\max}$ for better resolution at low capital levels where marginal products are highest
- Intangible capital S: n_S points, linearly spaced from $S_{\min}$ to $S_{\max}$
- Debt D: n_D points, linearly spaced from $D_{\min} = 0$ to $D_{\max}$

The total state space contains $n_z \times n_K \times n_S \times n_D$ points. With the baseline calibration ($n_z = 11$, $n_K = 50$, $n_S = 50$, $n_D = 25$), this yields 687,500 state points.

Choice Variables. The firm chooses tangible investment I^K, R&D labor H^R, and new debt D'. Static labor inputs (L, H^P) are solved analytically from first-order conditions given the state and wages. The investment choices are discretized on grids:

- Tangible investment I^K: n_{I^K} points from $-\delta_K K_{\max}/2$ (disinvestment) to $K_{\max}/5$ (large investment)
- R&D labor H^R: n_{H^R} points from 0 to $\bar{H}$ (total skilled labor supply)

New debt D' is computed analytically rather than searched over a grid. Given investment choices (I^K, H^R), the collateral constraint $D' \leq \alpha_K K' + \alpha_S S'$ and dividend non-negativity $\text{Div} \geq 0$ jointly determine optimal borrowing: $D' = \min(\text{financing gap}, \alpha_K K' + \alpha_S S')$ where the financing gap is total expenditures minus internal resources.

CC. Value Function Iteration with Howard's Policy Improvement

The Bellman equation is solved via value function iteration with Howard's policy improvement algorithm to accelerate convergence. The algorithm alternates between:

Policy Improvement Step. Every n_H iterations, perform full optimization over the choice space to update policy functions:

$$V^{new}(z, K, S, D) = \max_{D', I^K, H^R} \left\{ \text{Div} + \beta(1-\zeta)\mathbb{E}_{z'|z}\left[V(z', K', S', D')\right] \right\} \tag{58}$$

where $\text{Div} = \Pi^*(z, K, S) - I^K - \text{AC}^K(I^K, K) - \text{AC}^S(\Delta S, S) - w_H H^R - RD + D'$ includes adjustment costs $\text{AC}^K = (\phi_K/2)(I^K)^2/K$ and $\text{AC}^S = (\phi_S/2)(\Delta S)^2/S$. The optimization is subject to the collateral constraint $D' \leq \alpha_K K' + \alpha_S S'$ (based on next-period capital), the dividend non-negativity constraint $\text{Div} \geq 0$, and the laws of motion for capital $K' = (1-\delta_K)K + I^K$ and $S' = (1-\delta_S)S + \Gamma(H^R)^\xi$. This step is computationally expensive as it requires searching over $n_{I^K} \times n_{H^R}$ choice combinations at each state point (debt D' is computed analytically given the investment choices).

Policy Evaluation Step. Between policy improvements, update the value function using the *fixed* policy functions without re-optimization:

$$V^{new}(z, K, S, D) = \text{Div}^*(z, K, S, D) + \beta(1-\zeta)\mathbb{E}_{z'|z}\left[V(z', K'^*, S'^*, D'^*)\right] \tag{59}$$

where starred variables denote the stored optimal policies, and Div^* is computed using these policies (including the associated adjustment costs). This step has computational complexity $O(n_z \times n_K \times n_S \times n_D)$ compared to $O(n_z \times n_K \times n_S \times n_D \times n_{I^K} \times n_{H^R})$ for full optimization.

The Howard improvement frequency n_H trades off accuracy (frequent policy updates) against speed (more evaluation steps). With $n_H = 15$ and up to 20 evaluation steps between improvements, convergence is typically achieved in 50–200 VFI iterations.

CD. Computational Optimizations

Four key optimizations reduce computation time by an order of magnitude:

1. OpenMP Parallelization. The state space loops are parallelized using OpenMP directives. Since the Bellman equation at each state point is independent, the outer loops over (z, K) can be executed in parallel across CPU cores. With 8 threads, this yields approximately 6–7× speedup.

2. Local Search. After the first VFI iteration, the optimal policy at each state typically changes only slightly. Instead of searching the entire choice grid, subsequent policy improvement steps search only within ± 3 grid points of the previous optimum for (I^K, H^R). Since debt D' is computed analytically (given the collateral constraint on next-period capital), this reduces the choice space from $n_{I^K} \times n_{H^R} = 400$ combinations to $(2 \times 3 + 1)^2 = 49$ combinations, an 8× reduction.

3. Precomputed Static Labor. The static labor choices (L^*, H^{P*}) depend only on (z, K, S) and wages—not on debt D. Before each policy improvement step, optimal static labor and associated output Y^* and gross profits Π^* are precomputed for all (z, K, S) triplets. This reduces redundant computation by a factor of n_D.

4. Feasibility Screening. Investment choices that violate $K' \in [K_{\min}, K_{\max}]$ or the dividend constraint are skipped immediately, avoiding unnecessary continuation value computation.

CE. Interpolation

Continuation values require evaluating $V(z', K', S', D')$ at points (K', S', D') that may not lie on grid nodes. Trilinear interpolation is used:

$$V(z', K', S', D') = \sum_{i,j,k \in \{0,1\}} \omega_i^K \omega_j^S \omega_k^D \, V(z', K_{i_K+i}, S_{i_S+j}, D_{i_D+k}) \tag{60}$$

where (i_K, i_S, i_D) are the lower grid indices and $(\omega^K, \omega^S, \omega^D)$ are the interpolation weights. Grid location uses binary search for $O(\log n)$ complexity.

CF. Stationary Distribution

The stationary distribution $\Psi(z, K, S, D)$ is computed via forward iteration with entry and exit:

$$\Psi^{new}(z', K', S', D') = (1-\zeta) \sum_{z,K,S,D} \Psi(z, K, S, D) \cdot \pi(z'|z) \cdot \mathbf{1}\{K' = g^K(\cdot), S' = g^S(\cdot), D' = g^D(\cdot)\} \tag{61}$$

plus entry mass at $(z, K_{\min}, S_{\min}, 0)$ drawn from the stationary productivity distribution. Since policy functions map to off-grid points, mass is distributed to neighboring grid nodes using trilinear interpolation weights.

Iteration continues until $\|\Psi^{new} - \Psi\|_\infty < 10^{-4}$, typically achieved in 1,000–3,000 iterations with dampening parameter 0.1.

CG. Equilibrium Computation

The outer wage iteration uses a simple fixed-point updating rule:

$$w_L^{new} = w_L \cdot \left(1 + \lambda \cdot \frac{L^d - \bar{L}}{\bar{L}}\right), \quad w_H^{new} = w_H \cdot \left(1 + \lambda \cdot \frac{H^d - \bar{H}}{\bar{H}}\right) \tag{62}$$

where L^d, H^d are aggregate labor demands from integrating over Ψ, and $\lambda = 0.2$ is the dampening parameter. Convergence is declared when $|L^d - \bar{L}|/\bar{L} < 0.01$ and $|H^d - \bar{H}|/\bar{H} < 0.01$.

CH. Implementation Details

The model is implemented in Fortran 90/95 for computational efficiency, using double precision arithmetic throughout. Key numerical parameters:

Parameter	Symbol	Value
VFI tolerance	ε_{VFI}	10^{-4}
Distribution tolerance	ε_{dist}	10^{-4}
Equilibrium tolerance	ε_{eq}	10^{-2}
VFI dampening	θ_{VFI}	0.70
Distribution dampening	θ_{dist}	0.10
Wage dampening	λ	0.20
Howard frequency	n_H	15
Max Howard evaluations	—	20
Local search radius	—	± 3

Compilation uses Intel Fortran (ifx/ifort) via Microsoft Visual Studio with full optimization and OpenMP enabled. Typical runtime for full equilibrium computation is 30–60 minutes on an 8-core workstation, depending on convergence speed of the wage iteration.

CI. Numerical Validation

Several checks ensure numerical accuracy:

- **Grid refinement:** Key results are verified to be stable when doubling grid density
- **Euler equation errors:** Checked at random state points to ensure errors are below 10^{-3} in consumption-equivalent units
- **Distribution mass:** Verified to sum to unity after each iteration
- **Market clearing:** Final equilibrium satisfies labor market clearing within stated tolerance

The code is available upon request and will be made publicly available upon publication.

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