The Empirical Distribution of Firm Dynamics and Its Macro Implications*

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Abstract

Idiosyncratic shocks shape firm decisions and the value functions they maximize. Using a comprehensive firm-level dataset, we document significant departures from the widely assumed Gaussian AR(1) stochastic process, including fat-tailed, leptokurtic revenue transitions and lower persistence in the tails. This discrepancy has two key implications. First, these dynamics flatten the revenue-to-value mapping and create a more clustered firm value distribution. Second, solving a canonical general equilibrium heterogeneous firm dynamics model nonparametrically to align with these observed empirical patterns reveals a first-order quantitative impact on the economy's responsiveness to aggregate shifts. Accurately modeling firm-level shocks is imperative for macroeconomics.

Keywords: firm dynamics, nonparametric shocks, selection, subsidy policy

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"The overall effect on aggregate employment seems ambiguous, depending on the stochastic structure of firm-level shocks. This being the case, evidence on the firm-level stochastic environment is necessary." Hopenhayn and Rogerson (1993)

1 Introduction

In dynamic macroeconomic models, the forward-looking decisions of firms are entirely driven by the value function they seek to maximize. The shape of this value function, in turn, is intimately linked to the idiosyncratic shocks firms face. In other words, accurately modeling the optimal decisions of the firm requires an adequate characterization of the shock process. This realization raises three questions for modelers:

1. *Does the commonly assumed Gaussian AR(1) model in the literature offer a good approximation of the dynamics observed in the real world?* Our answer is no, the Gaussian AR(1) model offers a poor approximation of the dynamics faced by firms.

2. *Does the difference matter for firm value functions?* Our answer is yes, the shape of the firm value function is significantly different under realistic modeling of firm shocks.

3. *Are there aggregate implications of accurately modeling firm-level shocks?* Our answer is yes. We show that in the context of a standard heterogeneous-firm model, a realistic shock process generates bunching in the distribution of firm values. This, in turn, has quantitatively large implications for the macroeconomic predictions of this class of models.

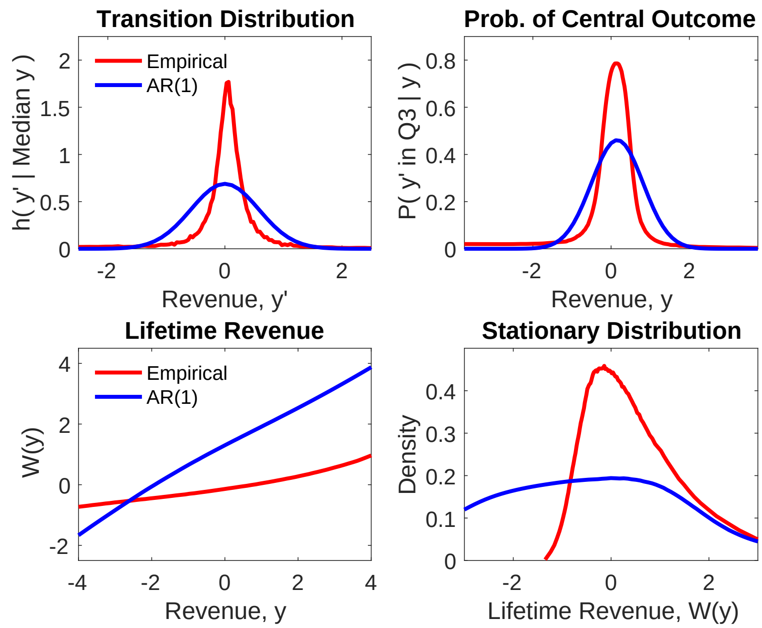
For our empirical analysis, we rely on historical ORBIS firm-level panel data. For many countries, this dataset offers coverage that is representative of the size distribution of the universe of both private and public firms, a feature that is crucial for our purposes. We use Spain as our benchmark, but show that our findings hold across a number of countries and are robust to multiple alternative treatments of the data.

To study our first question, we nonparametrically characterize revenue dynamics from the ORBIS data. We then compare those dynamics to the ones implied by the common parametric AR(1) assumption used in the literature and document significant differences. As we discuss later, these discrepancies persist even when compared to richer parametric models. As an illustration, consider the top left panel in Figure 1 which compares the density of next year's firm revenue conditional on currently being at the median revenue level, both from the data (in red) and a standard Gaussian AR(1) process (in blue). Relative to the commonly assumed parametric specification, a firm with revenue close to the median this year is much more likely in the data to experience a small revenue transition and remain around the same level the following year. Conversely, we plot in the top right panel the probability that a firm of a given size transits to the third quintile of the revenue distribution the following year. One can see that transition from the tails towards the center is more prevalent than under the standard parametric assumption, particularly from low revenue states. To summarize, firm revenue dynamics are leptokurtic or fat-tailed.

To answer our second question, we then investigate whether these differences between the empirical dynamics and those from the commonly assumed shock process have a material impact on the shape of the firm value function. Specifically, we show that the divergence substantially alters the relationship between current firm revenue and its expected future values, leading to a flatter value function than implied by the standard parametric specification. To do so, we rely on the ORBIS dataset to construct a novel model-free proxy for firm value, which we refer to as "lifetime revenue:" the expected present discounted value of future firm revenue. As demonstrated in Section 2, this metric can be computed from any dataset that includes information on firm revenue transitions, entrant size distributions, and exit hazards. We focus on revenue given its role in canonical firm dynamics models as an outcome linked to both shock processes and a firm's production choices. Moreover, we show for the subsample of publicly listed firms that our measure of lifetime revenue is a strong predictor of observed market value.

The bottom left panel in Figure 1 plots lifetime revenue as a function of current revenue for the purely empirical specification (red) and under a parametric AR(1) assumption (blue). The two functions differ markedly, with the empirical function being significantly flatter than its AR(1) counterpart. This difference arises directly from the revenue dynamics described above: the standard AR(1) specification exhibits too little persistence around the center of the revenue distribution and too much persistence in the tails. As a result, the forward-looking value of the firm in the AR(1) model becomes overly sensitive to current revenue. By contrast, the data

Figure 1: Revenue Dynamics and Lifetime Revenue



Notes: The top left panel of the figure plots the distribution of next year's firm revenue y' conditional upon median revenue y in the current year. The top right panel plots the probability that next year's firm revenue y' lies within the central or 3rd quintile, conditional upon revenue y in the current year. The bottom left plots lifetime revenue, a term summarized in the introduction and defined formally in Section 2, as a function of current revenue. The bottom right panel plots the stationary distribution of lifetime revenue. Here, y is log revenue residualized by sector and year from our baseline Spanish ORBIS sample described in Section 2 covering around five million firm years for around one million firms over the 2005-2014 period. In each panel, the red line is computed from empirical nonparametric estimates while the blue line reflects estimates implied by a parametric AR(1) model.

suggest that firms with different current revenue levels have lifetime revenue values that are much closer to each other than the AR(1) model implies.

Building on these results, we address our third question: does accurately modeling the firm-level shock process matter for macroeconomic analysis? To explore this, we rely on a standard heterogeneous firm model in the spirit of Hopenhayn (1992) and Hopenhayn and Rogerson (1993). This framework serves as a natural laboratory to investigate the economic implications of our findings, as revenue dynamics in this class of models directly influence expected firm lifetime outcomes. These outcomes are pivotal for key decisions such as entry, exit, hiring, and investment, making it an ideal setting to assess the macroeconomic relevance of firm-level shock processes. We develop a solution and calibration technique that allows our model to perfectly match the nonparametric revenue dynamics extracted from the data and, as a result, the implied empirical lifetime revenue distribution. We then consider two simple policy experiments in the spirit of Hopenhayn (1992): a fixed subsidy to each operating firm and a subsidy to entrants. We implement these policies in both our baseline model and a version using a calibrated standard AR(1) specification for the idiosyncratic shock process.

In both policy experiments, we find a significantly larger response of exit rates under the nonparametric calibration, resulting in notable differences in the reactions of aggregate variables. This divergence arises from the fact that, in the nonparametric case, the flattening of the value function leads to a highly clustered firm value distribution around the exit thresholds, which amplifies the sensitivity of exit rates. This is evident in the bottom right panel of Figure 1, which contrasts the distribution of firm values, or more specifically lifetime revenues, under the empirical and AR(1) specifications.

With our benchmark results in hand, we undertake validation exercises, robustness checks, and extensions. First, as a validation exercise, we compute a measure of lifetime revenue clustering at the sector level. Consistent with our model's central mechanism, we show that exit rates covary more negatively with sales growth in sectors where firm value clustering is more pronounced. Second, for robustness, we replicate our empirical and model findings across different countries, time periods, sub-sectors, and alternative data treatments. Crucially, we also demonstrate that our results are not driven by firm age and that fat-tailed firm shocks persist. We also evaluate the forecasting performance of our nonparametric shock process against richer parametric specifications, showing that even these approaches fall short. For instance, while a Gaussian AR(1) process with fixed effects and noise performs poorly, a nonlinear AR(1) process with non-Gaussian shocks, though moderately better, still fails to match the predictive accuracy of the nonparametric method. Third, we confirm the robustness of our main results to alternative model specifications: incorporating lumpy investment, we find that fat-tailed investment flows alone cannot account for the empirically observed fat-tailed revenue dynamics.

We view this paper as contributing to two main strands of the literature. First, we add to the work contrasting empirical and "conventional" distributions (Midrigan, 2011; Carvalho and Grassi, 2019; Forneron, 2020; Guvenen et al., 2021; Sterk et al., 2021; Guvenen et al., 2023; Boar et al., 2023; Barro and Ursúa, 2012; Parham, 2024a,b), demonstrating that common parametric assumptions in heterogeneous agent models are poor approximations of reality.

Second, our paper relates to theoretical and empirical research on firm dynamics (Dunne et al., 1989; Hopenhayn, 1992; Davis and Haltiwanger, 1992; Hopenhayn and Rogerson, 1993; Luttmer, 2011; Kehrig, 2015; Clementi and Palazzo, 2016; Karahan et al., 2022), which uses firm heterogeneity to explain stylized facts and draw macroeconomic conclusions. Additionally, it connects to extensive policy analysis work within this framework (Hopenhayn and Rogerson, 1993; Guner et al., 2008; Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Davies and Eckel, 2010; Gourio and Miao, 2010; Asker et al., 2014; Garicano et al., 2016; Catherine et al., 2018; Kehrig and Vincent, 2020; Ottonello and Winberry, 2020; Bils et al., 2021; Sraer and Thesmar, 2021; Celik and Tian, 2023).[1]

Hence, to summarize, our main contributions relative to these bodies of work are: (i) documenting new nonparametric facts about firm dynamics, (ii) demonstrating how these facts significantly influence and reshape firms' value functions, and (iii) illustrating how our empirical findings have a substantial quantitative impact on aggregate outcomes. Unlike previous work, such as Sterk et al. (2021), we do not focus on the reasons behind the discrepancy between the data and the standard parametric assumptions in the literature. Instead, we take a step back to broadly characterize this discrepancy and explore its implications.

The rest of the paper is organized as follows. Section 2 introduces our data and facts. To build intuition, Section 3 examines the predictions of a simple model. Section 4 builds then a canonical quantitative general equilibrium firm dynamics model, while Section 5 outlines the nonparametric and parametric approaches used to solve and calibrate the model. Section 6 analyzes our two quantitative policy experiments

[1]This framework has been used to study, among many other topics, the contributions of labor frictions to aggregate outcomes (Hopenhayn and Rogerson, 1993); the cyclical implications of firm entry and exit (Bilbiie et al., 2012; Clementi and Palazzo, 2016; Lee and Mukoyama, 2018); the decline in business dynamism (Decker et al., 2016, 2020; Karahan et al., 2022); the role of firm heterogeneity in shaping aggregate investment dynamics (Khan and Thomas, 2008, 2013; Winberry, 2021); the propagation of financial frictions (Moll, 2014; Midrigan and Xu, 2014; Ottonello and Winberry, 2020); uncertainty shocks (Bloom et al., 2018); and the drivers and consequences of resource misallocation (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Bento and Restuccia, 2017; Kehrig and Vincent, 2020). This broad family of models has also been highly influential in the trade literature (Melitz, 2003).

within each version of the model and highlights the aggregate implications of the nonparametric calibration. Section 7 discusses our empirical and quantitative robustness checks. Section 8 concludes. Online appendices provide further details on our empirical analysis (Appendix A) and quantitative analysis (Appendix B).

2 Data

In this section, we introduce our representative firm microdata and present our nonparametric approach to extracting several key empirical objects. With this framework in hand, we then describe empirical firm revenue dynamics.

2.1 ORBIS Data

We rely on Moody's, formerly Bureau van Dijk's, historical ORBIS dataset for our empirical analysis. ORBIS is drawn mostly from government business registers and contains many firm-level outcomes for both private and publicly listed companies at yearly frequency. Crucially, this dataset allows us to conduct the analysis for multiple countries, ensuring that our findings are not specific to a given jurisdiction. Coverage and representativeness, however, vary greatly across countries, and researchers working with ORBIS data must also be mindful of differentiating the commercial from the historical ORBIS datasets, with varying sample selection criteria. Despite these subtleties, Kalemli-Ozcan et al. (2022) and Bajgar et al. (2020) demonstrate in detail that for multiple European countries, the historical ORBIS data is of high quality: it yields a sample that covers 80% to 90% of total economic activity and displays a size distribution that is in line with that from the official Eurostat Structural Business Statistics database, considered to represent the most comprehensive portrait of business activity for EU countries.

We also note that relative to alternative US datasets, ORBIS is advantageous in providing, for multiple countries, a representative firm size distribution through the inclusion of both private and publicly listed firms. ORBIS also includes information on a broad range of firm-level outcomes.[2]

[2] Getting access to firm-level datasets that are representative of the universe of firms is notoriously difficult in the US. In addition, they are not well suited for this study, for the following reasons. The Longitudinal Business Database, for instance, includes a very limited set of economic and financial outcome variables. The U.S. Economic Census includes more information but is conducted only

Our baseline sample consists of just over one million private and public Spanish firms that are active at some point over the years 2005-2014, for a total of around five million firm-year observations. Appendix Table A.1 presents some summary statistics on this sample. Although our Spanish ORBIS data is a useful benchmark, we show in Section 7 that our results are robust to instead using data from Italy, Portugal, France, and Norway, nations for which ORBIS data is also representative.

2.2 Measuring Three Key Empirical Objects

ORBIS includes many economic and financial variables, but we focus on revenue given its role in canonical firm dynamics models as an outcome linked to both shock processes and a firm's production choices. Revenue is also, helpfully, one of the most populated outcomes in ORBIS across countries. Given our focus on idiosyncratic patterns, we analyze log firm revenue residualized with respect to both sector and year effects, sometimes succinctly referring to this measure as "revenue" below. Omitting subscripts, we denote log revenue for a given firm year by y and let y' indicate the following year's outcome at the same firm. We also construct indicators for firm entry and exit, a task enabled by ORBIS' firm panel structure. We separate firms into the potentially overlapping categories of "incumbents" including all those operating in a given year; "entrants" including only first-year incumbents; and "continuing" firms which operate in future year(s). With this dataset in hand, we nonparametrically measure three objects.

1. The transition distribution, i.e., the distribution of next-year's revenue conditional upon current revenue for continuing incumbent firms, denoted $H(y'|y)$
2. The distribution of revenue for entrant firms, denoted $H_E(y)$
3. The exit hazard for incumbent firms, denoted $\mathbb{P}(\text{Exit}|y)$

Our extensive sample allows us to perform straightforward nonparametric estimation. First, we discretize firm revenue into $N_y = 101$ equally weighted intervals.[3] Next, we

every five years, a limitation incompatible with a study of firm dynamics. Other surveys conducted at an annual frequency, such as the Annual Survey of Manufactures, are too limited in coverage. Finally, more accessible sources such as Compustat tend to limit their scope to publicly listed – large, nonrepresentative – firms.

[3]We note that the exact value of N_y is quantitatively unimportant for our results.

estimate three objects: the matrix $H(y'|y)$ describing incumbent dynamics, obtained using transitions of firm revenue across intervals for continuing incumbents; the vector $H_E(y)$, defined as the distribution of entrants across revenue intervals; and the vector $\mathbb{P}(\text{Exit}|y)$, capturing the exit rates of incumbent firms across revenue intervals. Below, we will interchangeably refer to these objects as "nonparametric" or "empirical," and all of the facts we lay out below are functions of these three items.

For comparison, we also consider a Gaussian AR(1) parametric model for log revenue $y' = \rho_y y + \sigma_y \varepsilon$, where $\varepsilon \sim N(0, 1)$. We estimate ρ and σ to match the autocorrelation and unconditional variance of revenue in our data.[4] This parametric model implies a transition distribution $H_{AR(1)}(y'|y)$, different from the empirical distribution $H(y'|y)$. Our analysis below contrasts nonparametric empirical facts and those implied by the parametric Gaussian AR(1) case.

We make the three measured nonparametric objects $H(y'|y)$, $H_E(y)$, and $\mathbb{P}(\text{Exit}|y)$ available online, together with code for processing and using them in firm dynamics models in continuous or discretized form.[5]

Appendix A presents more detailed information on our sample construction, data treatment, and estimation approaches. Section 7 demonstrates our results' robustness to a range of alternative data treatment choices and sample splits. Notably, our robustness exercises also confirm that our results are not driven by firm age, a variable not directly incorporated in our baseline analysis.

2.3 Facts

This section lays out some key stylized facts. Where relevant, red lines indicate our empirical nonparametric estimates while blue lines indicate outcomes implied by a parametric Gaussian AR(1).

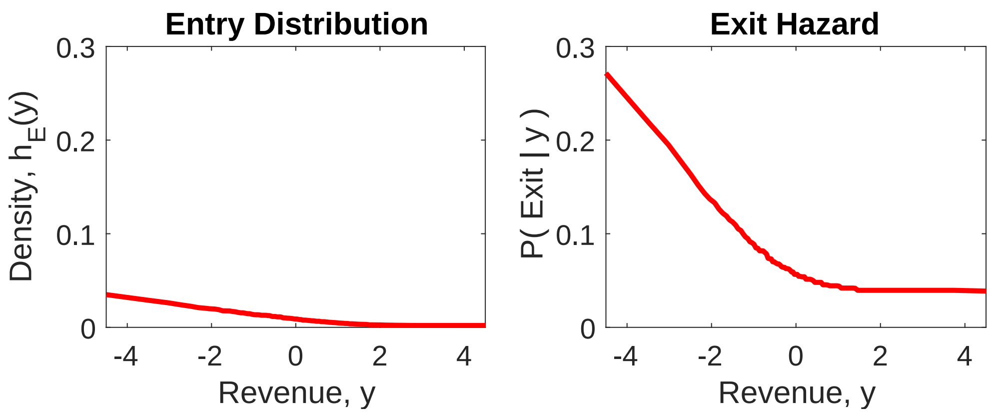
Entry and Exit Patterns Figure 2 plots the entry density $h_E(y)$ (left panel) and exit hazard $\mathbb{P}(\text{Exit}|y)$ (right panel). Both objects are convex and downward sloping in revenue y, although the entry distribution is somewhat flatter. The downward slope of the exit hazard aligns with the predictions of canonical firm dynamics models.[6]

[4]For our baseline Spanish (log) revenue dataset, we find $\hat{\rho}_y = 0.94$ and $\hat{\sigma}_y = 0.57$ for continuing incumbent firms.

[5]See Stephen Terry's website `https://public.websites.umich.edu/~sjterry/`.

[6]In Appendix Table A.7 we also show that firm revenue contains more explanatory power for firm exit than firm profits.

Figure 2: Firm Entry and Exit Patterns in the Data



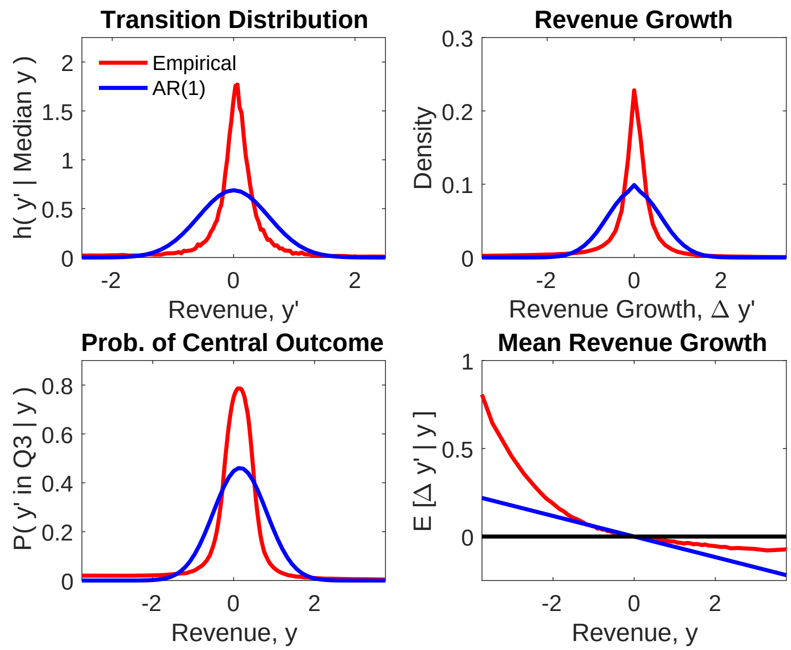
Notes: The figure plots the estimated entry density $h_E(y)$ (left panel) as well as the estimated exit hazard $\mathbb{P}(\text{Exit}|y)$ (right panel) as a function of y, i.e., log revenue residualized by sector and year. Both objects are estimated nonparametrically using our baseline Spanish ORBIS sample covering around five million firm years for around one million firms over the 2005-2014 period.

Revenue Dynamics The top row of Figure 3 plots two measures of revenue dynamics for continuing incumbents. The top left panel plots the densities from our empirical transition distribution $H(y'|y)$ and the parametric AR(1) transition distribution $H_{AR(1)}(y'|y)$, conditional upon a starting level of revenue y equal to the median. Specifically, conditional on being initially at its median value, the probability of log revenue remaining around the median is much higher empirically than under the AR(1) specification. Yet, the empirical distribution also features a higher (though low) likelihood of moving to the tails from the median.

The top right panel plots unconditional distributions of revenue growth. The empirical (parametric) revenue growth distribution features a standard deviation, skewness, and kurtosis of 0.65 (0.59), -0.31 (0), and 29.21 (3) respectively. Hence, although the dispersion and skewness are roughly similar, the empirical revenue dynamics are distinctly leptokurtic or fat-tailed. Intuitively, firms are overwhelmingly more likely to experience very small, but also sometimes very large, yearly revenue growth rates relative to those implied by the standard Gaussian AR(1).

The bottom row of Figure 3 provides insight into the revenue mobility of incumbents, especially mobility from the tails. Specifically, the bottom left panel plots the probability that a firm's revenue next year lies in the 3rd quintile, i.e., the center of

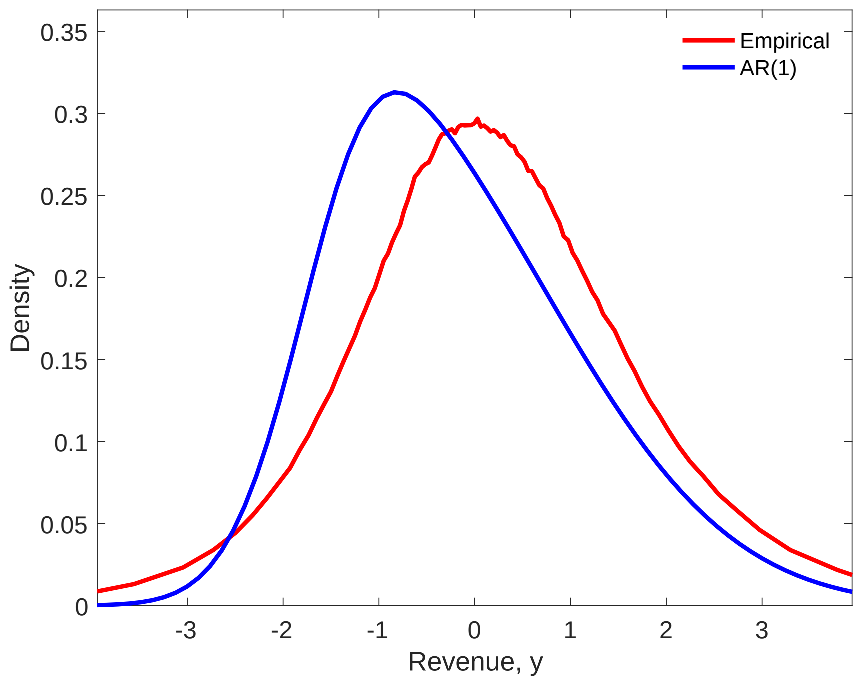
Figure 3: Leptokurtic Revenue Dynamics



Notes: The top left panel of the figure plots the distribution of next year's firm revenue y' conditional upon median revenue y in the current year. The top right panel plots the stationary distribution of yearly revenue growth $\Delta y'$. The bottom left panel plots the probability that next year's firm revenue y' lies within the central or 3rd quintile, conditional upon revenue y in the current year. The bottom right panel plots mean revenue growth $\Delta y'$ over the next year, conditional upon revenue y in the current year. Here, y is log revenue residualized by sector and year from our baseline Spanish ORBIS sample covering around five million firm years for around one million firms over the 2005-2014 period. In each panel, the red line is computed from the empirical nonparametric estimates $H(y'|y)$ while the blue line reflects the transition distribution $H_{AR(1)}(y'|y)$ implied by the parametric AR(1) case.

the distribution, while the bottom right panel plots mean revenue growth over the next year. For the empirical and parametric Gaussian AR(1) versions, outcomes are plotted conditional upon firm revenue today. We can see in the bottom left panel that smaller firms are empirically more likely to grow towards the center of the distribution than an AR(1) would imply. This pattern is echoed in the high conditional mean of revenue growth rates for such firms in the bottom right panel. Despite the fact that AR(1) implied transitions at the right tail are more aligned with those extracted from the data, significant differences remain. In particular, the conditional mean of revenue growth is linear in revenue in the AR(1) case while clearly nonlinear in the data. In summary, these patterns reveal that firms are more empirically likely to grow quickly towards, and then remain within, the center of the distribution than implied by the parametric model.

Figure 4: Stationary Revenue Distributions in the Data



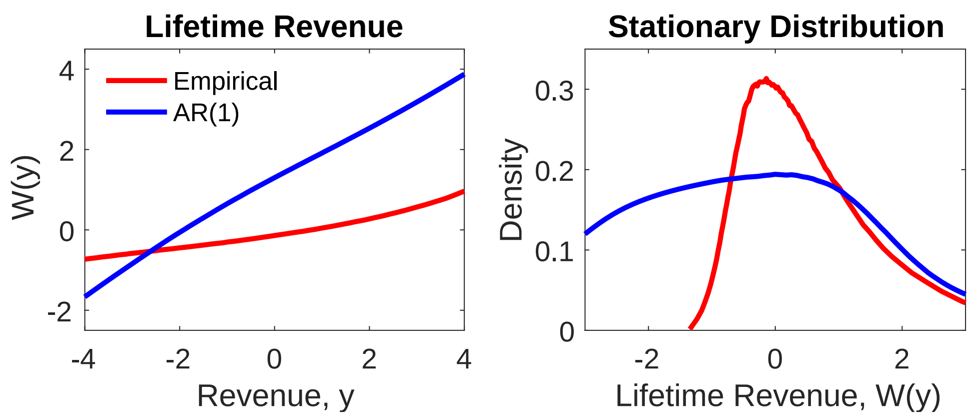
Notes: The figure plots the stationary distribution of firm revenue y. Here, y is log revenue residualized by sector and year from our baseline Spanish ORBIS sample covering around five million firm years for around one million firms over the 2005-2014 period. The red line is computed from our empirical nonparametric estimates, while the blue line reflects the transition distribution implied by the parametric AR(1) case.

Stationary Revenue Distribution Next, we compute the empirical nonparametric stationary distribution $H(y)$ of firm revenue satisfying

$$H(y') = \int H(y'|y)\,(1 - \mathbb{P}(\text{Exit}|y))\,dH(y) + \mathbb{P}(\text{Exit})H_E(y'),$$

where $H(y'|y)$, $\mathbb{P}(\text{Exit}|y)$, and $H_E(y')$ match their data counterparts.[7] For comparison, we also compute an otherwise identical parametric stationary distribution of revenue $H_{AR(1)}(y)$ by simply replacing the empirical transition distribution $H(y'|y)$ with its counterpart $H_{AR(1)}(y'|y)$. Figure 4 plots the densities associated with these two unconditional distributions, which do not appear dramatically different to the naked eye. Of course the two distributions do differ in meaningful ways.[8] In particular, outside the plotted range the empirical distribution $H(y)$ exhibits the well known fat tail consistent with a power law which no Gaussian model can match, a fact we document in Appendix Figure A.3. But, overall, the figure draws a contrast between the reasonably high similarity of the empirical and parametric cross-sectional distributions of firm revenue *levels* versus their markedly divergent implications for firm revenue *dynamics* documented above.

Figure 5: Lifetime Revenue



Notes: The figure plots lifetime revenue as a function of current revenue (left panel) as well the stationary distribution of lifetime revenue (right panel). Here, y is log revenue residualized by sector and year, and for ease of reference we present lifetime revenue $W(y)$ in logs and subsequently demeaned. Our empirical estimates come from our baseline Spanish ORBIS sample covering around five million firm years for around one million firms over the 2005-2014 period. In each panel, the red line is computed from our empirical nonparametric estimates while the blue line reflects the transition distribution implied by the parametric AR(1) case.

[7]Note that for the last term we rely on the fact that in a stationary steady state $\mathbb{P}(\text{Exit}) = \mathbb{P}(\text{Entry}) = \int \mathbb{P}(\text{Exit}|y)dH(y)$.

[8]The standard deviation, skewness, and kurtosis of the empirical (parametric) stationary distributions for revenue levels y are roughly comparable at 1.77 (1.36), 0.01 (0.60), and 7.54 (3.28).

Firm "Lifetime Revenue" If the documented differences in revenue dynamics between the empirical and parametric models do not generate large apparent differences in the stationary distribution of revenue *levels*, do they still matter? The answer is yes. Revenue dynamics impact expected firm *lifetime* outcomes, such as firm value. In canonical firm dynamics models, firm value – the expected present discounted value of payouts – is not only the key decision-relevant measure for firm entry and exit, it also shapes the firm's choices along many dimensions, such as hiring and investment.

Directly obtaining information on firm value for a representative set of firms is, unfortunately, impossible. Since the vast majority of firms are not publicly listed, their market value is not observable. Instead, our approach is to construct a novel proxy of firm value, which we refer to as lifetime revenue: the expected present discounted value of future firm revenue. As we show below, this object can be computed from any dataset that includes information on firm revenue and exit patterns for both listed and unlisted firms, suchs as ORBIS. Note that the measure relies on *revenue* instead of *payouts*, because information on payouts is often missing or of dubious quality in representative firm-level datasets. Yet, in most widely used quantitative firm dynamics models, payouts in firm value equations are highly correlated with and dominated quantitatively by firm revenue.[9] Moreover, we later show in Appendix A.2 that for the small subsample of publicly listed firms, our measure of lifetime revenue is a good predictor of observed market value.

Lifetime revenue $W(y)$ can be easily computed as a function of current log revenue y, using only the estimated objects already in hand, via the Bellman equation

$$W(y) = e^y + \left(\frac{1 - \mathbb{P}(\text{Exit}|y)}{R}\right) \int W(y')dH(y'|y),$$

where we remind the reader that y denotes the log of firm revenues, hence the contemporaneous revenue is given in its level form as e^y. We choose $R > 1$ to deliver a conventional constant yearly real interest rate of 4%. Besides this assumption, lifetime revenue $W(y)$ is otherwise purely a function of our estimated empirical tran-

[9] In particular, note that in most quantitative applications of models in the Hopenhayn (1992) tradition, firm payouts can be divided into two terms. The first term is proportional to firm revenue, and the second term reflects transitory adjustments based on flow factors such as investment, financial frictions, or adjustment costs. The first term tends to be meaningfully larger in magnitude and more persistent than the second, driving a high correlation between revenue outcomes over a firm's lifetime and its underlying difficult-to-measure expected firm payouts.

sition distribution $H(y'|y)$ and exit hazard $\mathbb{P}(\text{Exit}|y)$. We also compute an analogous lifetime revenue object $W_{AR(1)}(y)$ using the same Bellman equation but replacing the transition distribution $H(y'|y)$ with its parametric AR(1) counterpart $H_{AR(1)}(y'|y)$. Finally, relying on the stationary revenue distributions $H(y)$ and $H_{AR(1)}(y)$ computed above, we immediately obtain stationary distributions of lifetime revenue $H(W)$ and $H_{AR(1)}(W)$ for the empirical and nonparametric cases.

In the left panel of Figure 5, we plot the lifetime revenue constructs $W(y)$ and $W_{AR(1)}(y)$ as functions of current revenue y. The two functions are strikingly different, with the empirical version being much flatter than its AR(1) counterpart. This finding is a key insight from our paper with important implications for the predictions of dynamic heterogeneous firm models, as we show in later sections.

To understand the intuition behind this result, recall two facts that we documented earlier. First, revenue dynamics in the data are fat-tailed or leptokurtic, a property highlighted in Figure 3. In particular, for the median firm, small revenue transitions are by far the most common outcomes. Second, firms face a higher likelihood of transitioning out of the tails of the size distribution than what is implied by the AR(1) process (see the bottom row of Figure 3). That is, movement towards the center is more prevalent than generally assumed, particularly from low-y states. Together, these two facts about *dynamics* generate an implied lifetime revenue function that is much less sensitive to current revenue under the nonparametric empirical case than under the parametric AR(1). In other words, firms with different current revenue levels of y have lifetime revenue values that are not as different from one another as the AR(1) model suggests. As a result, the stationary distribution of lifetime revenue, shown in the right panel of Figure 5, is therefore much less dispersed and exhibits more "clustering" or higher densities at low levels in the data than under the parametric AR(1).[10]

In the remainder of the paper, we draw out the implications of this key insight for firm dynamics models. Specifically, we show that the degree of clustering for firm values links directly to the sensitivity of overall firm exit to changes in the economic environment, i.e., that these empirical facts directly discipline and change the quantitative aggregate implications of workhorse firm dynamics models.

[10] The standard deviation, skewness, and kurtosis of the empirical (parametric) log lifetime revenue distributions in the right panel of Figure 5 are 1.17 (1.85), 1.67 (0.20), and 6.66 (2.53), respectively.

3 Simple Model

In the previous section, we showed that the cross-sectional firm size distribution in the data is roughly similar to the one implied by the parametric Gaussian AR(1) case. Yet, firm revenue dynamics differ markedly, generating important differences in expected lifetime outcomes. In this section, we first analyze the implications for firm dynamics of this divergence in a simple and transparent analytical framework. We then proceed in Section 4 to a quantitative general equilibrium heterogeneous firms model.

Time is discrete. Each of a unit mass of existing firms chooses at the start of period $t = 0$ whether to exit or to continue operating. Continuing commits a firm to operate forever, from $t = 0$ onwards, while a firm exiting immediately receives an outside option of 0. Firms are risk neutral and discount the future at the constant exogenous rate $R > 1$. Each firm observes its own current exogenous profitability state z in period 0 before choosing to continue or exit. A firm's net payoff in any period equals its profitability plus an exogenous constant $\mu\left(\frac{R-1}{R}\right)$. At the start of period 0, the cross-sectional firm profitability distribution is exogenously given by $z \sim N(0, \sigma_z^2)$. That is, for all cases discussed below, we assume an identical cross-sectional distribution of z at $t = 0$. However, we examine three distinct cases for the *dynamics* of z and their implications for the time-0 distribution of the firm values V. This exercise allows us to clearly isolate the role of different firm value, a dynamic object, vs. different revenue, a static measure, in the cross section.

Permanent z In this case, firm profitability z is permanent and fixed, so that a firm's value is

$$V_{perm}(z) = \mu + z + \frac{1}{R}z + \frac{1}{R^2} + ... = \mu + z\left(\frac{R}{R-1}\right).$$

Hence, the distribution of firm values at the start of period 0 is

$$V_{perm} \sim N\left(\mu, \sigma_z^2\left(\frac{R}{R-1}\right)^2\right) = N(\mu, \sigma_{perm}^2).$$

Persistent z In this case, each firm's profitability follows an independent Gaussian AR(1) with persistence satisfying $0 < \rho < 1$ and shock variance $\sigma^2 = (1-\rho^2)\sigma_z^2$. A firm's expected value is

$$V_{pers}(z) = \mu + z + \frac{1}{R}\rho z + \frac{1}{R^2}\rho^2 z + ... = \mu + z\left(\frac{R}{R-\rho}\right).$$

The distribution of firm values at the start of period 0 is therefore

$$V_{pers} \sim N\left(\mu, \sigma_z^2\left(\frac{R}{R-\rho}\right)^2\right) = N(\mu, \sigma_{pers}^2).$$

Transitory z In this case, firm profitability $z \sim N(0, \sigma_z^2)$ is iid across time and firms. A firm's expected value is

$$V_{iid}(z) = \mu + z + \frac{1}{R}\mathbb{E}(z) + \frac{1}{R^2}\mathbb{E}(z) + ... = \mu + z.$$

As a result, the firm value distribution at the start of period 0 is

$$V_{iid} \sim N\left(\mu, \sigma_z^2\right) = N(\mu, \sigma_{iid}^2).$$

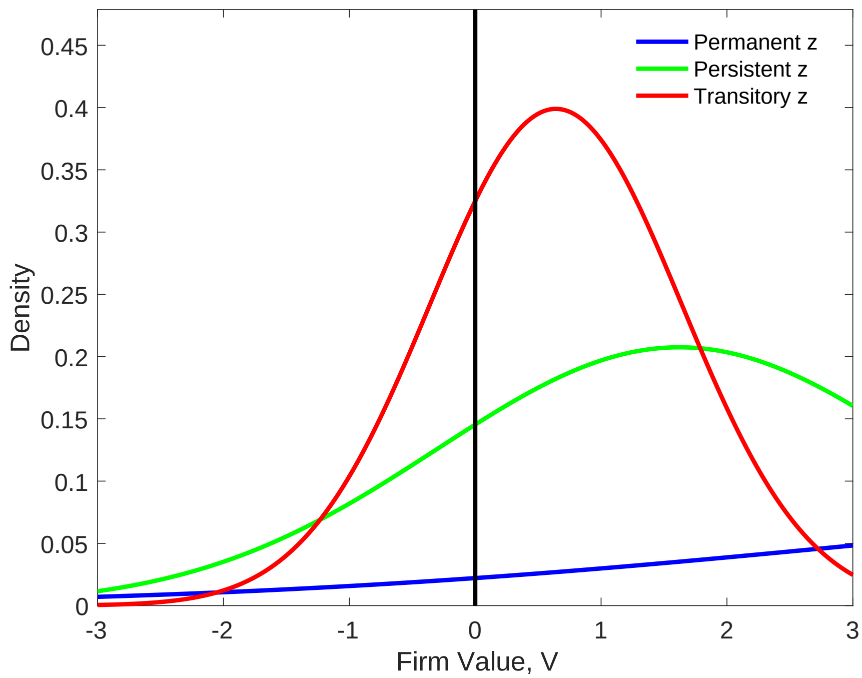
Note that since $0 < \rho < 1 < R$, we can rank the variances of the underlying firm value distributions across cases as $\sigma_{iid}^2 < \sigma_{pers}^2 < \sigma_{perm}^2$. Intuitively, faster mean reversion at the firm level generates a more compressed distribution of firm value. Hence, although all cases exhibit an identical cross-sectional distribution of profitability or size z at $t = 0$, the divergent dynamics of profitability imply different distributions of firms' decision-relevant object: firm value.

For an illustrative parameterization, Figure 6 plots the firm value distribution under each scenario. Firms with negative value below the plotted threshold level at 0 choose to exit in period 0. To allow comparison across cases, we choose the mean payoff parameter μ in each case to guarantee an identical exit rate.[11] Firm value dispersion varies widely across the three cases, despite identical cross-sectional size distributions, with higher mean reversion generating more compressed firm value distributions in the transitory and persistent cases relative to the permanent case.

But do these distributional differences matter for firm dynamics, and in particular

[11] We target 10%, a round value, in Figure 6. We pick μ_i such that $\Phi\left(\frac{z_i^* - \mu_i}{\sigma_i}\right) = p_{exit}$, where z_i^* is such that $V_i(z_i^*) = 0$, σ_i is the standard deviation of the potential firm value distribution as defined in the text for each case i, and Φ is the standard normal CDF.

Figure 6: Firm Value Distributions in the Simple Model



Notes: The figure plots the distribution of firm value at the start of period 0 in the simple model. For this illustrative parameterization we choose $\rho = 0.4$, $\sigma_z^2 = 1$, and $R = 1/1.2$. The figure plots the firm value distributions for the permanent z case (blue line), the persistent z case (green line), and the transitory or iid z case (red line) together with the exit threshold of 0 (black vertical line). We also normalize μ separately for each case to generate an exit rate of 10%.

exit rates? To answer this question, first note that the exit rate is simply $\mathbb{P}(\text{Exit}) = F(0)$, where $F(V)$ is the firm value CDF. Next, consider the implementation of a one-time subsidy $s > 0$ paid to all firms. This policy naturally shifts the firm value distribution to the right and implies a new, lower exit rate $\mathbb{P}(\text{Exit}|s) = F(0 - s)$. Specifically, the local sensitivity of the exit rate to the subsidy is $\frac{\partial \mathbb{P}(\text{Exit})}{\partial s}|_{s=0} = -f(0)$, where $f(V) = F'(V)$. Consequently, when the distribution of firm value is more clustered, with a higher density of firms at the exit threshold $V = 0$, the exit rate is more responsive to the subsidy. Such clustering varies widely across the cases in Figure 6, with more persistent shocks leading to larger dispersion in firm value, i.e., less clustering.

As we showed in Section 2, the rich firm revenue dynamics observed in the data generate an empirical distribution of lifetime revenue outcomes that is more clustered, with higher densities at low levels, than the one implied by the standard Gaussian AR(1) process ubiquitous in the literature. The basic intuition from our simple model suggests that such clustering should cause the aggregate exit rate to be more sensitive to changes in the economy. To formalize and quantify this intuition, we now turn our attention to a quantitative general equilibrium model of firm dynamics.

4 Quantitative Model

Our quantitative general equilibrium firm dynamics model is in the spirit of Hopenhayn (1992), Hopenhayn and Rogerson (1993) and the literature spawned by their seminal work. Before proceeding with the description of the model, we note that our focus is solely on the idiosyncratic shock process faced by firms: we replace the standard parametric specification used in this literature by a flexible, nonparametric, and empirically disciplined process. As such, we intentionally keep the other aspects of the model closely aligned with the established benchmarks in the field. This approach allows us to isolate and underscore the impact on the model predictions of replacing the standard parametric specification used in this literature. Among the maintained assumptions are: (i) the stationarity of driving processes, (ii) the use of a first-order Markov process, and (iii) a steady-state analysis that excludes aggregate shocks or stochastic discount factor movements.

4.1 Operating Firms and Exit

Two types of firms in the economy, incumbents and entrants, form a mass M_O of operating firms in any given period. Each operating firm produces a homogenous numeraire good in the amount $y = zn^{\alpha}$. Firms hire undifferentiated labor n at a competitive wage W. Production exhibits decreasing returns to scale with $0 < \alpha < 1$ and is shifted by an exogenous idiosyncratic profitability shock $z > 0$ following a first-order Markov chain with transition distribution $F(z'|z)$.[12] An operating firm's

[12]Importantly, we note that the decreasing returns to scale production function specification is isomorphic to a monopolistic competition framework with love of variety; our results are not specific to the formulation chosen. So we refer to z as "profitability" instead of the narrower term "productivity." In order to match the revenue dynamics we do *not* need to take a stand on whether the

dynamic problem is summarized by its value function

$$V(z) = \max_n \left[zn^\alpha - Wn + \mathbb{E}_{\phi_F} \max \left\{ 0, -\phi_F + \frac{1}{R}(1-\delta) \int V(z')dF(z'|z) \right\} \right]. \quad (1)$$

Time is discrete, and at the start of a period, each operating firm solves a static profit maximization problem for labor input n, which is the sole input used in production. Then, each operating firm receives an iid fixed production cost draw ϕ_F which is denominated in output units and drawn from an exogenous distribution $G(\phi_F)$. Operating firms must pay the fixed cost ϕ_F in order to continue to produce in the future, although a firm can alternatively choose to exercise an option to endogenously exit with limited liability and outside option 0. If an operating firm chooses not to exit, and avoids an iid exogenous death shock with probability satisfying $0 < \delta < 1$, then the firm transits as an incumbent to the next period. An operating firm therefore trivially chooses to endogenously exit if and only if its fixed cost ϕ_F exceeds a threshold level, or equivalently a continuation value, given by

$$\phi_F^*(z) = \frac{1}{R}(1-\delta) \int V(z')dF(z'|z). \quad (2)$$

4.2 Entry

A mass of potential entrants in the economy considers whether to enter at the start of each period. Entry requires that a firm pay an exogenous constant sunk cost $\phi_E > 0$, denominated in output units, in order to obtain an initial profitability shock draw z from the exogenous distribution $F_E(z)$. After receiving an initial profitability draw z, each entrant firm joins the mass of currently operating firms in the current period. Free entry implies that the sunk cost ϕ_E must weakly exceed the value to entry

$$\phi_E \geq \int V(z)dF_E(z), \quad (3)$$

with equality whenever the mass M_E of entry is greater than zero.

driving shocks are supply or demand shocks; the revenue function in such a model is given by

$$Revenue = z^{\nu-1} \daleth^{\nu} \text{Aggregates},$$

where z and $\daleth$ respectively denote idiosyncratic productivity shocks and demand shocks.

4.3 Households

The economy is populated by a measure one of identical households. Households consume the numeraire good and supply labor inelastically in the exogenous amount $\bar{N} > 0$. In addition to labor income, households also receive dividends from operating firms. The household problem reflects an optimal choice of the level of consumption, C, to maximize welfare given by the discounted sum of log utility payoffs. The simple dynamic problem is represented by

$$S = \max_{C} \{\log(C) + \beta S'\} \tag{4}$$

where the time discount rate satisfies $0 < \beta < 1$ and a standard budget constraint holds. As usual, household intertemporal optimization in a stationary steady state implies that the real interest rate is proportional to the household time discount rate

$$\beta = \frac{1}{R}. \tag{5}$$

4.4 Timing

To summarize, the timing of the model within each period is as follows:

1. New entrant firms pay entry costs.
2. Incumbent firms and new entrants receive their idiosyncratic profitability draws z, drawn from $F_E(z)$ for entrants and according to the transition distribution $F(z|z_{-1})$ for incumbents with previous profitability z_{-1}.
3. Operating firms, i.e., both incumbents and entrants, produce output $y = zn^{\alpha}$ by combining z and labor n hired at the prevailing wage W.
4. Operating firms draw an iid fixed cost $\phi_F \sim G(\phi_F)$.
5. Operating firms form expectations of their continuation value $\phi_F^*(z)$, choosing whether to exit endogenously or remain in operation for next period. Operating firms that choose to remain pay the fixed cost ϕ_F.
6. Households receive firm profits and labor income and then consume.
7. A fraction δ of operating firms exogenously exits.

8. Surviving operating firms transition to the next period as incumbents.

4.5 Stationarity and Aggregates

Stationarity requires that the distribution $F_O(z)$ of operating firms is stable across periods according to the mapping

$$M_O F_O(z) = (1-\delta) M_O \int G(\phi_F^*(z_{-1})) F(z|z_{-1}) dF_O(z_{-1}) + M_E F_E(z), \tag{6}$$

which implicitly defines the distribution $F_O(z)$ but also implies proportionality of the operating and entrant masses according to

$$M_O \mathbb{P}(\text{Exit}) = M_E.\text{[13]} \tag{7}$$

Aggregates in the economy can be written as a function of the stationary distribution. Output Y, total fixed costs Φ_F, and total sunk costs Φ_E satisfy the equations

$$Y = M_O \int y(z, W) dF_O(z) \tag{8}$$

$$\Phi_F = M_O \int \int_{\{\phi_F \leq \phi_F^*(z)\}} \phi_F dG(\phi_F) dF_O(z) \tag{9}$$

$$\Phi_E = M_E \phi_E \tag{10}$$

which together imply the level of consumption C via the aggregate resource constraint

$$Y = C + \Phi_F + \Phi_E. \tag{11}$$

Total labor demand N is given by

$$N = M_O \int n(z, W) dF_O(z), \tag{12}$$

[13] Take $z \to \infty$ in Equation (6) to obtain $M_O = M_O(1-\delta)\int G(\phi_F^*(z_{-1}))dF_O(z_{-1}) + M_E$. Since $\int (1-\delta) G(\phi_F^*(z_{-1}))dF_O(z_{-1}) = \int \left[1 - \mathbb{P}(\text{Exit}|z_{-1})\right] dF_O(z_{-1}) = 1 - \mathbb{P}(\text{Exit})$, we immediately obtain $M_O \mathbb{P}(\text{Exit}) = M_E$.

which, of course, must equal exogenous labor supply if markets clear

$$N = \bar{N}. \tag{13}$$

4.6 General Equilibrium

A stationary general equilibrium in this economy is a value function $V(z)$, exit thresholds $\phi_F^*(z)$, a stationary distribution $F_O(z)$ of operating firms, an operating mass M_O, an entrant mass M_E, aggregate output Y, aggregate fixed operating costs Φ_F, aggregate sunk entry costs Φ_E, aggregate consumption C, aggregate labor demand N, a wage W, and an interest rate R such that operating firms' optimal value satisfies (1), exit thresholds are optimal according to (2), the stationary distribution replicates itself according to (6), the operating mass is proportional to entry via (7), free entry holds in (3), the aggregate production and resource constraints in (8), (9), (10), and (11) hold, the labor market clears with demand in (12) equal to supply via (13), and household intertemporal optimality holds in (5).

5 Calibration and Solution

In this section, we lay out our approach to calibrating and solving the quantitative general equilibrium framework described in Section 4. We consider two versions of the same model. The first case is based on our empirical nonparametric estimates from Section 2, while the second case employs the standard Gaussian AR(1) parametric assumptions adopted in the literature.

5.1 Calibration

We calibrate the model at annual frequency. As an initial step, we first externally calibrate four parameters shared by both versions of our model. In particular, we choose $\alpha = 0.67$ to match a labor share of two thirds. The value $\beta = 1/1.04$ is picked to generate a yearly net real interest rate $R - 1$ of 4%. We also normalize $\bar{N}$ to the mean employment rate of 59.7% in Spain during our sample period. And, finally, we set the exogenous exit rate δ equal to the 3.9% exit rate observed among the largest firms in our empirical sample, another normalization.

Table 1: Model Calibration

	Value	Empirical Target
Panel A: Nonparametric Case		
Profitability transition, $F(z'\|z)$	-	$H(z'\|z)$
Entrant distribution, $F_E(z)$	-	$H_E(z)$
Fixed cost distribution, $G(\phi_F)$	-	$\mathbb{P}(\text{Exit}\|z)$
Sunk entry cost, ϕ_E	22.9	Employees per firm, 12.3
Panel B: Parametric AR(1) Case		
Profitability persistence, ρ	0.94	Profitability autocorr., 0.94
Profitability volatility, σ	0.19	Profitability st. dev., 0.56
Entrant profitability mean, μ_E	-0.43	Mean entrant vs operating $\log z$, -0.36
Fixed cost support, $\bar{\phi}_F$	2.30	Exit rate $\mathbb{P}(\text{Exit})$, 6.9%
Sunk entry cost, ϕ_E	5.18	Employees per firm, 12.3

Notes: Panel A of the table lists internally calibrated model objects, their calibrated values where relevant, and the associated empirical targets for the nonparametric version of the model, while Panel B reports the same information for the parametric version of the model. All empirical targets come from our baseline Spanish ORBIS sample covering around five million firm years for around one million firms over the 2005-2014 period. In Panel A, dash placeholders are used to denote three distributions pinned down nonparametrically, as discussed in the main text, to exactly match the indicated empirical targets.

In both versions of our model, we discipline our calibration of profitability shocks z using empirical evidence on firm revenue.[14] In fact, by inverting a firm's static labor demand from the optimization problem in (1), we obtain and employ the simple formula

$$\log z = (1-\alpha)y + \text{Constant}, \tag{14}$$

which allows us to obtain z directly, up to a normalizing aggregate constant, from observed log firm revenue y.

Nonparametric Empirical Calibration In Section 2, we nonparametrically estimated three key objects: the empirical revenue transition distribution for continuing

[14]In our model, 'y' represents value added, whereas in the data we primarily measure revenue or sales. However, given our production structure, which is widely used in this research area, value added and revenue are proportional to each other. This proportionality implies that our measurement of log value added is essentially equivalent to measuring revenue, adjusted by a constant factor. Consequently, this relationship enables us to apply the observed dynamics of firm revenue from the data directly to the concept of value added in the model.

incumbents $H(y'|y)$, the entrant revenue distribution $H_E(y)$, and the revenue exit hazard $\mathbb{P}(\text{Exit}|y)$. Inverting firm profitability z from revenue y via equation (14) directly yields equivalent empirical estimates as functions of z, which we label $H(z'|z)$, $H_E(z)$, and $\mathbb{P}(\text{Exit}|z)$.

Helpfully, the profitability transition and entrant profitability distributions are primitive exogenous objects in our model. Hence, to empirically calibrate the nonparametric version of our model we simply set $F(z'|z) = H(z'|z)$ and $F_E(z) = H_E(z)$, i.e., we can directly choose the model distributions to exactly replicate their empirical equivalents. Calibrating the exogenous fixed cost distribution, by contrast, requires more care, since exit is endogenous in the model. To do so, we exploit the theoretical identity

$$\mathbb{P}(\text{Exit}|z) = 1 - (1-\delta)G(\phi_F^*(z)) \tag{15}$$

linking the endogenous, but observable, exit hazard $\mathbb{P}(\text{Exit}|z)$ to the exogenous, but unknown, fixed cost distribution $G(\phi_F)$. We observe that the exit thresholds $\phi_F^*(z)$ can be determined straightforwardly as a function of the firm value function $V(z)$ through equation (2). Therefore, taking the value function $V(z)$ and the exit thresholds $\phi_F^*(z)$ as given, the identity in equation (15) directly and nonparametrically implies a unique fixed cost distribution $G(\phi_F)$ which is precisely consistent with the observed exit hazard $\mathbb{P}(\text{Exit}|z)$.[15]

Of course, we do not observe $V(z)$ ex ante. However, our solution algorithm for the nonparametric version of the model, summarized below, employs conventional dynamic programming or value function iteration to solve the Bellman equation (1). Within each step of this iteration, we employ our ongoing updated guesses for the value function $V(z)$, and hence continuation values $\phi_F^*(z)$, to compute ongoing updated guesses for the fixed cost distribution $G(\phi_F)$. Convergence of $V(z)$ then delivers convergence of the fixed cost distribution $G(\phi_F)$.

Taken as a whole, our approach to calibration of this version of the model delivers nonparametric distributions $F(z'|z)$, $F_E(z)$, and $G(\phi_F)$ that allow us to perfectly replicate both our empirical estimates of $H(z'|z)$, $H_E(z)$, and $\mathbb{P}(\text{Exit}|z)$, as well as their revenue-indexed versions $H(y'|y)$, $H_E(y)$, and $\mathbb{P}(\text{Exit}|y)$. We emphasize that since the empirical results in Section 2 were computed as functions of these empirical

[15] The implied fixed cost distributions, for both our nonparametric and parametric model calibrations, are plotted in Appendix Figure B.3.

targets, our calibrated model also, *by construction*, matches all of the nonparametric results we presented in that section, including observed revenue dynamics, revenue mobility, the stationary distributions of current and lifetime revenue, exit hazards, and exit rates.

One parameter, the sunk entry cost ϕ_E, remains to be calibrated. Note that higher levels of the sunk cost ϕ_E cause, via the free entry condition (3), an increase in mean entry values, requiring lower equilibrium wages W and driving up mean employment per firm. We therefore calibrate ϕ_E, jointly with the distributions above, to exactly match the mean ratio of employees per firm in our baseline Spanish dataset. Panel A of Table 1 summarizes the results of our internal calibration of the nonparametric version of the model.

Parametric AR(1) Calibration The calibration of our parametric model is more conventional and relies on various distributional assumptions. We assume that the transition distribution for profitability z, $F(z'|z)$, is governed by an exogenous Gaussian AR(1) process

$$\log z' = \rho \log z + \sigma\varepsilon', \quad \varepsilon \sim N(0,1).$$

where persistence and volatility satisfy $0 < \rho < 1$ and $\sigma > 0$. For the entrant distribution $F_E(z)$, we also assume that entrants' log profitability $\log z \sim N(\mu_E, \sigma^2)$ is drawn from a Gaussian distribution with mean μ_E. We further assume that the distribution of iid fixed costs $G(\phi_F)$ is uniformly distributed between 0 and an upper bound $\bar{\phi}_F > 0$, so $G(\phi_F) = U(0, \bar{\phi}_F)$.

Our distributional assumptions, together with the sunk entry cost parameter ϕ_E, imply a total of five internally calibrated parameters. Panel B of Table 1 lists the parameters, the resulting calibrated values, and the empirical targets which we exactly match in our calibrated parametric model through a joint procedure. Following standard practice, ρ and σ are disciplined by the observed autocorrelation and variance of profitability z in our sample. The mean difference between the log profitability of entrants versus operating firms varies directly with the entrant profitability mean μ_E. Higher values of the upper bound $\bar{\phi}_F$ for the distribution of the fixed cost generate higher mean fixed costs and, therefore, higher mean exit rates in the model. Finally, as in the nonparametric case, we target the mean number of employees per firm in order to help identify the sunk cost of entry ϕ_E. The results of our calibration in

Table 1 are unsurprising, with high persistence of profitability of $\rho = 0.94$, moderately high conditional volatility of $\sigma \approx 20\%$ annually, and a meaningful reduction of $\mu_E = -43\%$ in entrant profitability relative to all operating firms.

Note that both the nonparametric and parametric AR(1) versions of the model offer exact fits to their empirical targets. But the parametric model, as is conventional in the firm dynamics literature, only matches a narrower, selected set of moments implied by the empirical profitability distributions and exit hazards. By contrast, the nonparametrically calibrated model offers an exact fit to all of these distributions at all points of the support. Consequently, the nonparametric version of our model also matches by construction each moment targeted by the parametric version, while simultaneously exploiting far more information from our empirical dataset.

5.2 Solution

To numerically solve both versions of the model, we employ conventional dynamic programming methods, i.e., value function iteration. We approximately solve the key operating firm Bellman equation (1) over a continuous state space for profitability z in an "inner loop." We embed this firm-level solution inside a general equilibrium "outer loop" over the wage W and entry mass M_E in order to satisfy the free entry and labor market clearing conditions (3) and (13). At a high level, this approach is quite standard within the quantitative firm dynamics literature.

Successfully solving the nonparametric version of our model in a manner fully consistent with our empirical targets requires a few novel ingredients beyond the standard approach, however. First, we lightly regularize the empirical transition distribution $H(z'|z)$, imposing that the z process exhibits persistence in a first-order stochastic dominance sense. Second, we also regularize the exit hazard $\mathbb{P}(\text{Exit}|z)$, imposing that the hazard is nonincreasing in profitability z. The first condition improves the stability of our value function iteration algorithm, while the second condition ensures that our recovered fixed cost distribution $G(\phi_F)$ is in fact nondecreasing. Fortunately, as we document in Appendix Section B.1, the raw empirical objects quite nearly satisfy both conditions, resulting in only extremely light adjustments in practice. Finally, as mentioned in our calibration discussion above, we must nonparametrically recover updated guesses for the fixed cost distribution $G(\phi_F)$ which are consistent with observed exit hazards within each step of our value function iteration algorithm. We defer further technical information on our solution techniques for both versions of the model to a detailed discussion in Appendix B.

6 Inspecting the Mechanism: Empirical vs AR(1)

Our goal in this section is to assess the quantitative relevance of our empirical findings for the predictions of macroeconomic models featuring heterogeneous firms. To do so, we consider the impact of two simple experiments: (i) a subsidy s_F for all operating firms, and (ii) a subsidy s_E to entrant firms. These experiments conveniently mirror the changes in the fixed operating cost and the sunk entry cost theoretically studied in Hopenhayn (1992). In each experiment, we compute and analyze the quantitative response of macroeconomic aggregates, taking into account general equilibrium. Note that in this model with perfectly competitive output and labor markets, our focus is on descriptively analyzing the impacts of each experiment rather than on normative questions.

6.1 The Model-Implied Distribution of Firm Value

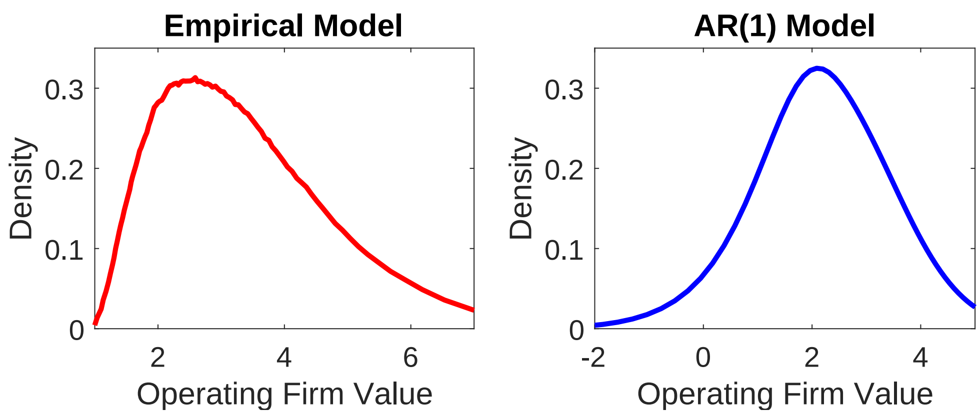
Using our simple analytical model, we argued earlier that clustering of the firm value distribution at low values, where exit mostly occurs, causes a higher sensitivity of the exit rate to economic changes.

Figure 7 plots the stationary distribution of operating firm value in our quantitative model in both the calibrated empirical (left panel) and parametric AR(1) (right panel) cases. Recall that the empirical model matches, by construction, the more clustered distribution of lifetime revenue in the data in Figure 5. We see from Figure 7 that our earlier intuition about lifetime revenue carries over to the underlying firm value functions.[16] The distribution of firm value is indeed more clustered at the low end, with higher densities towards the left of the distribution where exit is most likely.

To demonstrate the quantitative relevance of the firm value distribution, we proceed with our two simple experiments.

[16] In addition to the fact that Figure 5 and Figure 7 plot different objects conceptually, i.e., lifetime revenue vs firm value distributions, one additional technical detail differentiates the two figures. The empirical vs AR(1) lifetime revenue distributions in Figure 5 both rely on the empirical exit hazard and entry distributions, while varying only the incumbent revenue transition distributions. In Figure 7, with the parametric model's structural exit hazards and entry distributions already defined and in hand, we also vary the entry and exit patterns.

Figure 7: Empirical vs AR(1) Firm Value Distributions



Notes: The figure plots the stationary distribution of operating firm continuation values ϕ_F^*, in logs, in the calibrated nonparametric (left panel) and parametric AR(1) (right panel) models.

6.2 Subsidy to Operating Firms

In the first experiment, the subsidy s_F to operating firms effectively lowers their net fixed operating costs. We distribute the subsidy, denominated in numeraire output, to all operating firms each period, financing the subsidy through lump sum taxes on households. Figure 8 plots the response of various aggregates to the subsidy, for both the empirical nonparametric (red lines) and parametric AR(1) (blue lines) models. We find that while the response of the exit rate is significantly more pronounced under the nonparametric case, the response of output is smaller. In what follows, we provide an overview of the economic forces behind these responses.

Exit and Selection The subsidy mechanically raises the value of every operating firm, which leads to a decline in the exit rate in both models (top left panel). Our intuition from Figure 7 suggests that the exit rate should be more sensitive to the subsidy under the empirical model, since firm values are more clustered where exit is more likely. Figure 8 confirms that this intuition holds in our full quantitative model. Specifically, with a subsidy of 5% of output, the exit rate decline in the empirical model is three times as large as in the parametric AR(1) model. The fall in exit in turn triggers a negative selection effect, as lower-z firms now survive with higher probability. Given the higher decline in the exit rate, this negative selection effect is

Figure 8: Impact of the Fixed Operating Subsidy s_F

Notes: Each panel in the figure plots an aggregate outcome as the subsidy is increased from zero for the calibrated nonparametric empirical (red lines) and parametric AR(1) (blue lines) models. For comparability, the horizontal axis in each panel is the aggregate size of the subsidy as a percentage of zero-subsidy aggregate output in each economy. The vertical axes plot either the levels of the outcomes or, where natural, percent changes from the zero-subsidy level.

more pronounced in the empirical version, partly counteracting the direct increase in mean firm value due to the subsidy.

Wages The equilibrium wage W in the top right panel of Figure 8 rises to offset the increase in the expected value of entry due to the subsidy, ensuring that the free entry condition (3) continues to hold.[17] While this logic holds in both models, the size of the response is different. Because a stronger decline in the exit rate in the empirical model generates a stronger negative selection effect, average firm value after

[17]Recall that firms enter based on an expected continuation value: only after entry do they learn their profitability level, produce and then choose whether to exit. For this reason, there is no selection through entry, and thus the sunk entry costs must always equal the average firm value across the entrant profitability distribution in the free entry condition (3).

entry rises less as a direct result of the subsidy. As a result, the equilibrium wage increase required by the free entry condition is smaller in the empirical model.

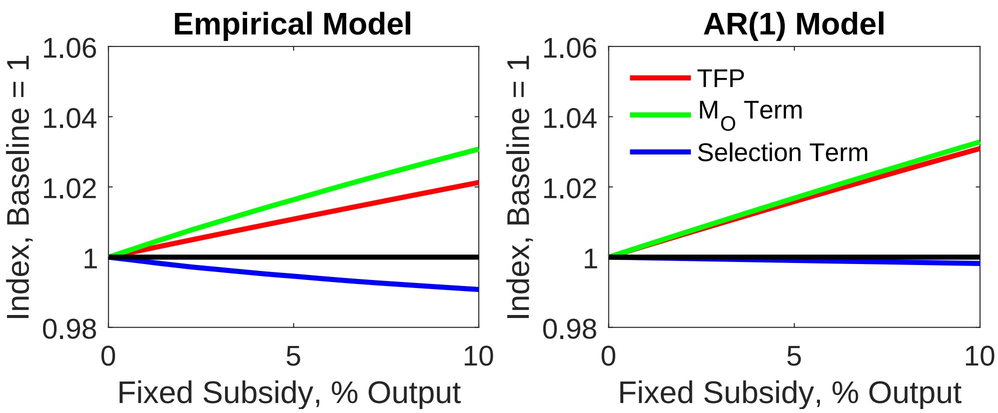
Labor and the Mass of Firms Recall that total labor in the economy is in fixed supply $\bar{N}$. The labor market clearing condition

$$N = M_O \int n(z, W) dF_O(z) = \bar{N} \tag{16}$$

equalizes total demand N to this fixed supply. But, as a result, the mass of operating firms, M_O, must move inversely to the average labor per firm, i.e., $\int n(z, W) dF_O(z)$. Next, note that average labor per firm is pinned down by two margins: the size of operating firms $n(z, W)$, which is governed by the wage; and the selection of operating firms through the distribution $F_O(z)$. The operating subsidy acts on both. Higher wages reduce firm size $n(z, W)$ for a given z (a stronger force in the AR(1) model), while negative selection in $F_O(z)$ lowers the mean of z across firms (a stronger force in the empirical model). Because of the offsetting strengths of these two channels, the overall decline in average labor per firm in the bottom left panel of Figure 8 turns out to be very similar across in the empirical and AR(1) models. Now, since labor per firm declines, the mass of operating firms M_O must rise to restore labor market clearing. But because the decline in average labor per firm is comparable across models, the rise in the mass of operating firms in the bottom right panel of Figure 8 is also similar.

TFP and Output In this economy with fixed labor supply, the aggregate levels of the wage W, output Y, and measured $TFP = Y/N^{\alpha}$ are proportional to one another. As a result, the percentage changes in output and TFP under the subsidy exactly match those plotted in the top right panel of Figure 8 for the wage. We conclude that output and TFP in this economy respond more in the AR(1) model than in the empirical version. Specifically, we note that with a subsidy of 5% of output, the percent change in these outcomes is only two thirds as strong in the empirical model as in the parametric AR(1). Our discussion above suggested that this difference is the result of a strong negative selection force generated by the sharp decline in exit in the empirical economy. To highlight this point, we decompose measured aggregate

Figure 9: Decomposed TFP under the Fixed Operating Subsidy s_F



Notes: The figure plots each component of the TFP decomposition in (17) as the subsidy is increased from zero for the calibrated nonparametric empirical (left panel) and parametric AR(1) (right panel) models. For comparability, the horizontal axis in each panel is the aggregate size of the subsidy as a percentage of zero-subsidy aggregate output in each economy. The vertical axes index each component to one in the zero-subsidy baseline. The black line depicts the no-change line.

TFP in this economy into two margins

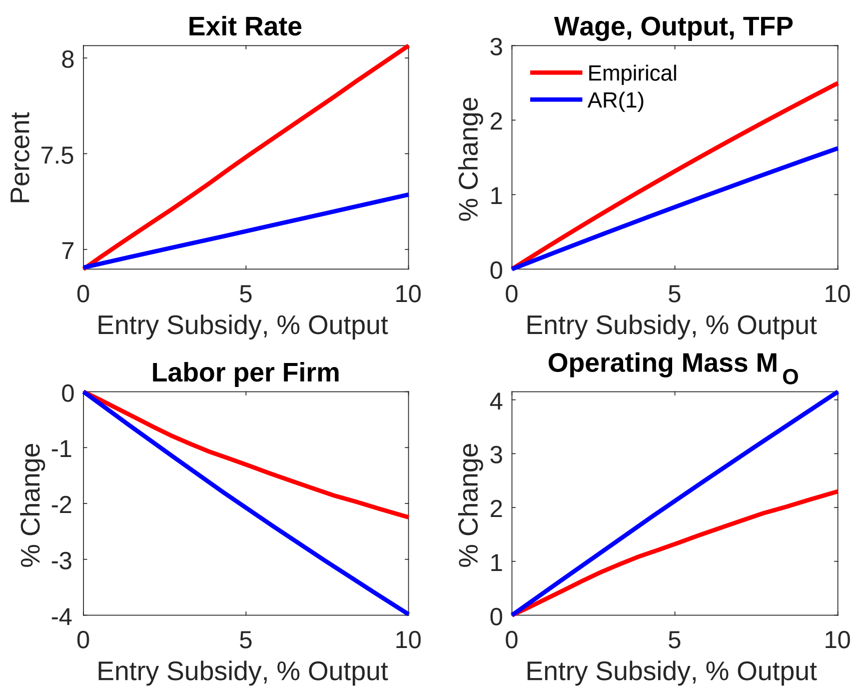
$$TFP = \underbrace{M_O^{1-\alpha}}_{\text{Operating Mass of Firms}} \underbrace{\left(\int z^{\frac{1}{1-\alpha}} dF_O(z)\right)^{1-\alpha}}_{\text{Selection}}. \tag{17}$$

The first term increases with the operating mass of firms through a standard extensive margin effect under decreasing returns. The second term is a geometric mean of operating profitability, i.e., a measure of firm selection. Figure 9 plots the respective contributions of these two components from equation (17) to the change in TFP in the empirical (left panel) and AR(1) (right panel) models. Our decomposition confirms that the negative contribution from selection is indeed more pronounced in the empirical model, due to the larger fall in the exit rate. This selection margin entirely explains the more muted response of TFP (and output) relative to the AR(1) case.

6.3 Subsidy to Entrants

In our next experiment, the subsidy s_E is given only to entrants, lowering their net entry costs. We again finance this output-denominated subsidy with a simple lump sum tax on households. As in the case of the operating subsidy, we find that the exit rate is more sensitive in the empirical model. This time, however, we note that the response of output is significantly larger than under the AR(1) specification. As in our first experiment, we conclude that a selection effect driven by shifts in the exit rate is key to understanding differences in the response across our two model versions.

Figure 10: Impact of the Entry Subsidy s_E



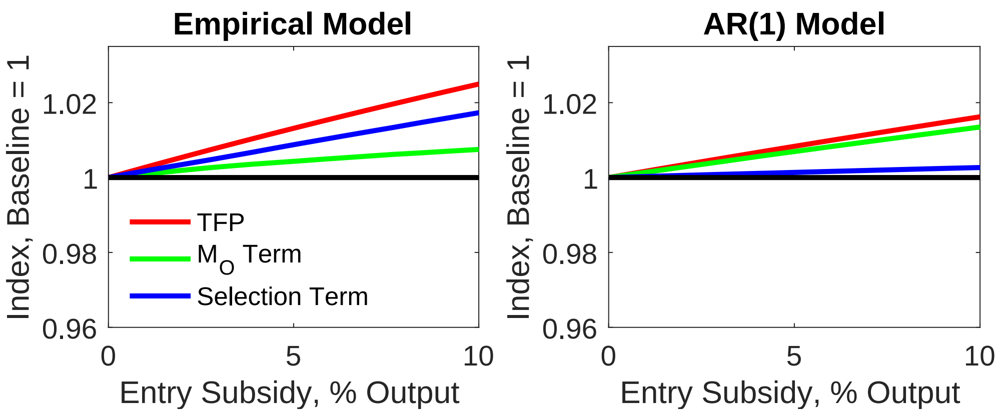
Notes: Each panel in the figure plots an aggregate outcome as the subsidy is increased from zero for the calibrated nonparametric empirical (red lines) and parametric AR(1) (blue lines) models. For comparability, the horizontal axis in each panel is the aggregate size of the subsidy as a percentage of zero-subsidy aggregate output in each economy. The vertical axes plot either the levels of the outcomes or, where natural, percent changes from the zero-subsidy level.

Wages, Exit, and Selection Our entry subsidy lowers the cost of entry to $\phi_E - s_E$ in the free entry condition (3). In order to restore equilibrium, the wage must increase in both versions of our model to reduce the post-entry expected value of operating. Wage increases trigger exit among low-profitability firms and hence generate a positive selection effect. However, due to the shape of the value distributions in Figure 7 with more clustering of firm value at low levels, we see in the top left panel of Figure 10 that the exit rate increases more sharply in the empirical versus the AR(1) model. Specifically, with a subsidy of 5% of output, the exit rate increase in the empirical model is three times as large as in the parametric AR(1) model. To offset the resulting stronger selection effect and maintain free entry, the wage rises by more in the empirical model in the top right panel of Figure 10.

Labor and the Mass of Firms A higher wage following the subsidy directly drives down labor demand $n(z, W)$ at individual firms, conditional upon profitability z. Yet the selection channel, driving more low-z firms to exit following the subsidy, has the opposite effect through indirect changes in the distribution of operating firms. Ultimately, the net effect on labor per firm is ambiguous. In the bottom left of Figure 10 we see that the direct effect dominates, since average labor demand per operating firm falls in both models. But the nonparametric model features a stonger indirect selection effect, generating a smaller overall fall in labor per firm in this case. Finally, recall from the labor market clearing condition in equation (16) that labor per firm and the mass of operating firms must move inversely due to the fixed total labor supply. As a result, we see in the bottom right panel of Figure 10 that the shift in the mass of operating firms M_O is also smaller in the empirical than in the AR(1) model.

TFP Recall that in this economy, measured TFP and aggregate output are both proportional to the wage and therefore rise following the subsidy (top right panel of Figure 10). In Figure 11, we rely on equation (17) to decompose the rise in TFP into separate contributions from the operating mass and selection effects for the empirical (left panel) and AR(1) (right panel) models. The sharper response of exit in the empirical model generates a stronger positive selection effect. The operating mass of firms, however, rises more in the parametric case. On net, the selection channel induced by exit is stronger, underlying the larger impact of the subsidy on TFP, and

Figure 11: Decomposed TFP under the Entry Subsidy s_E



Notes: The figure plots each component of the TFP decomposition in (17) as the subsidy is increased from zero for the calibrated nonparametric empirical (left panel) and parametric AR(1) (right panel) models. For comparability, the horizontal axis in each panel is the aggregate size of the subsidy as a percentage of zero-subsidy aggregate output in each economy. The vertical axes index each component to one in the zero-subsidy baseline.

output, in our empirical nonparametric model. Specifically, with a subsidy of 5% of output, the increase in TFP and output in the empirical model is one and a half times as large as in the parametric AR(1) model.

6.4 Taking Stock

Our analysis across both subsidy experiments highlights the quantitative importance of the shape of the firm value distribution in driving aggregate responses in a canonical general equilibrium model of firm dynamics. Specifically, our nonparametric model matching the more pronounced empirical clustering of the distribution of lifetime revenue at lower levels where exit is more likely to occur in Figure 5 also features a more clustered distribution of underlying firm value in Figure 7 relative to the standard AR(1) case. As a result, exit rates and hence selection shift more strongly in our nonparametric model, causing a large difference in the quantitative predictions of the two models. We conclude that embedding a shock process that adequately matches the rich distributional dynamics found in the data proves to be crucial for quantitative work with this class of models.

7 Discussion, Extensions, and Robustness

In this section, we discuss a range of additional robustness checks and extensions to our empirical and quantitative analysis. We frame our discussion around a number of natural and sensible questions. In each case, our empirical approach or quantitative conclusions prove to be robust.

Do country, data treatment, time period, or industry composition drive our results? Our analysis naturally involves many decisions regarding data treatment and sample construction. We investigate whether these choices drive our conclusions using a combination of empirical but also quantitative model robustness checks.

Focusing first on the empirical moments that are key to our findings, Appendix Table A.1 reports a number of statistics on revenue and revenue growth moments in our baseline ORBIS dataset as well as in a number of robustness checks. We first consider the role of the sample period by splitting our dataset into pre- and post-2009 samples. Second, we divide our broad representative dataset into manufacturing and non-manufacturing subsamples. Third, we use unconsolidated firm-level accounts instead of the consolidated statements from our baseline. Fourth, we exclude firm-years with reported M&A activity. Fifth, we demean log revenue by year only, rather than our baseline year and sector demeaning. Sixth, we consider different treatments of outliers relative to our baseline baseline trimming of 0.1% of revenue outliers. Seventh, we consider data from Italy, Portugal, France, and Norway instead of our Spain baseline. To judge the results in Appendix Table A.1, recall a key fact from Section 2: revenue growth in Figure 3 is leptokurtic or fat-tailed. Appendix Table A.1 reveals extremely high baseline revenue growth kurtosis of about 30, compared to exactly 3 in any Gaussian case. Uniformly, we find fat-tailed revenue growth in all of our robustness checks.

Next, for each of these alternative data samples, we perform a full recalibration of our empirical nonparametric and parametric AR(1) models. Appendix Table B.2 lists the recalibrated parameters for all the quantitative model robustness checks. We recompute the changes in the aggregate exit rate and output induced by an operating subsidy s_F totaling 5% of pre-subsidy output. Appendix Table B.3 reports the ratio of these responses in the empirical vs parametric AR(1) models. Our baseline non-parametric model's exit rate response is 3 times as strong as the one in the parametric model, driving a negative selection effect which dampens the output response to only around two-thirds that of the parametric model. The same overall pattern is evident in all of our robustness checks.

Does firm age drive our results? Our baseline analysis, like much work following Hopenhayn (1992), features no separate role for firm age conditional upon size in predicting growth or exit. Yet, many papers rationalize related evidence, recently documented authoritatively by Sterk et al. (2021), with mechanisms such as learning (Jovanovic, 1982; Arkolakis et al., 2018), demand accumulation (Foster et al., 2008; Gourio and Rudanko, 2014; Moreira, 2018), or financial frictions (Moll, 2014). This rich firm age literature is complementary to, but quite distinct from, our analysis contrasting nonparametric versus parametric approaches. Nevertheless, we observe firm demographic data in ORBIS and can calculate firm age. We therefore conduct another robustness check by residualizing revenue against firm age – with a full set of age indicators denominated in years – in addition to our baseline demeaning by sector and fiscal year. The moments for this alternative dataset, shown in Appendix Table A.1, reveal that revenue growth remains strongly fat-tailed or leptokurtic, i.e., features that are incompatible with a Gaussian AR(1). Finally, Appendix Table B.3 reports that once the model is calibrated, solved and simulated based on the dataset controlling for age, the relative impact of the operating subsidy across the two models is in line with that in our baseline for firm exit and in fact stronger in the case of firm output.

Do firm heterogeneity and noise drive our results? Following the standard specification adopted in the literature, we naturally contrast our nonparametric empirics to the predictions from a Gaussian AR(1) parametric benchmark. One might however wonder whether a richer extension of our AR(1) that includes permanent firm fixed effects and transitory shocks, ingredients ubiquitous in household incomplete markets analyses, might allow us to match the predictions of our nonparametric model. In Appendix Section A.1.1, we therefore specify and estimate an extended parametric Gaussian AR(1) model augmented with a Pareto distribution of firm fixed effects as well as Gaussian transitory shocks. We then subject all our models to a battery of tests gauging their predictive accuracy for both the mean and full distribution of observed revenue dynamics. Appendix Table A.2 reports our extended model's estimates and shows that the extended model, while still failing to capture the fat-tailed nature of revenue growth observed in the data, does predict firm revenue somewhat better than our benchmark Gaussian AR(1). Ultimately, however, even this richer parametric model remains less accurate for prediction than our nonparametric structure, confirming that our comparison of parametric versus nonparametric approaches is not unduly driven by our choice of parametric benchmark.

Do any alternative parametric models perform well? A reader might accept that nonparametric models such as the one we employ more easily capture apparent nonlinearities or non-Gaussian revenue dynamics than the conventional Gaussian AR(1) model but still prefer to work with parametric models for the sake of tractability. So, in Appendix Section A.1.2 we present a parametric but nonlinear, non-Gaussian AR(1) model in which the autocorrelation of the process is allowed to vary with the level of profitability and the shocks are allowed to follow a potentially fat-tailed Gaussian mixture distribution. We estimate this nonlinear AR(1) model on our benchmark ORBIS sample from Spain, reporting the results in Appendix Table A.3 and Appendix Figure A.1. We recover substantial nonlinearities, with smaller firms growing more quickly, and fat-tailed innovations. Although a gap in predictive accuracy remains relative to our fully nonparametric model, the nonlinear AR(1) does improve substantially upon the forecasting performance of the benchmark Gaussian AR(1), closing more than half the gap in each metric we consider in Appendix Table A.3 while, arguably, allowing for more tractability than our fully nonparametric process.[18]

Can we empirically link lifetime revenue and market value? Outcomes summarizing a firm's lifetime prospects such as firm value are not typically available for unlisted private firms. For this reason, in Section 2 we proposed a new measure, lifetime revenue, defined as the expected presented discounted value of firm revenue. This proxy for firm value can be constructed simply using information on revenue and exit alone. We found earlier that the distributions of data-driven lifetime revenues (Figure 5) and model-implied firm values (Figure 7) both display similar clustering, providing support for our proxy. But for the small subset of publicly listed Spanish

[18]See two interesting contributions in this area, Parham (2024a) and Parham (2024b), for another example of a tractable parametric model which may capture fat-tailed revenue dynamics.

firms in our sample, we can proceed further empirically without directly relying on our structural model. Regressions in Appendix Table A.4 reveal that, while both current revenue and lifetime revenue are highly correlated with observed market value, current revenue loses its predictive power for firm value once we account for lifetime revenue. This result confirms that our lifetime revenue measure does in fact capture useful variation in a firm's long-term prospects, as captured by realized market value.

Can we empirically link clustered distributions and exit rate sensitivity? A reader might accept our empirical evidence of fat-tailed revenue dynamics and clustered lifetime revenue outcomes for firms but still harbor two natural objections. First, we do not provide a formal definition of distributional clustering in our analysis above. Second, we use our quantitative model, inevitably laden with assumptions, rather than a more direct empirical approach to link our intuitive notion of clustering to higher exit rate sensitivity.

In Appendix Section A.3, we further expand and develop the analysis in both directions. To begin, we develop a reduced-form, purely statistical model that allows us to predict the aggregate exit rate based on the distribution of firm lifetime revenue. Within this framework, we analytically derive the predicted local response of the exit rate to a hypothetical one-time revenue windfall for all firms. This derivative, which is directly computable in our data, has a natural interpretation as a clustering statistic. This clustering statistic is higher when the lifetime revenue distribution has, on average, higher density in regions with steeper exit hazards. Empirically, we compute and report the value of the statistic in Appendix Table A.5 for each two-digit sector within our sample. Clustering varies widely, with particularly high values in sectors including construction and retail trade and particularly low values in sectors including finance and health care. We then exploit this cross-sectoral heterogeneity by running a set of panel regressions, whose results are presented in Appendix Table A.6, demonstrating that exit rates covary more negatively with sales growth at the industry level in the presence of higher clustering. Our quantitative model, of course, does not incorporate sectoral shocks or clustering heterogeneity. Yet we view these empirical results as consistent with our quantitative model's central prediction, which links the high sensitivity of the exit rate in the nonparametric specification to the high level of clustering of the firm value distribution.

Do our exact model assumptions drive our results? Our quantitative model is purposefully conventional within the Hopenhayn (1992) tradition, but we explore our results' robustness to multiple alternative assumptions. First, while we fix aggregate labor in our baseline, as a robustness check we instead consider the case of endogenous labor supply.[19] Second, the parameter α, which plays an important role in the TFP decomposition (17), has multiple interpretations. A literal view links α to the labor share, rationalizing our baseline external calibration $\alpha = 2/3$. But a revenue function view of our production technology under imperfect competition links α to production and demand elasticities. We therefore entertain values of α of 0.6 and 0.75. After performing model recalibrations and counterfactual analyses for each scenario, Appendix Table B.3 reports the relative impacts of an operating subsidy in the nonparametric vs parametric models. Our conclusions are little changed from baseline.

Can a lumpy capital adjustment model explain our results? One natural question is whether our findings of fat-tailed revenue dynamics can be explained by an otherwise standard model with Gaussian AR(1) shocks but non-convex capital adjustment costs leading to lumpy, fat-tailed investment rate distributions. In Appendix Section B.5 and Table B.1 we present and solve a canonical firm-level lumpy investment model in the Cooper and Haltiwanger (2006) tradition with Gaussian AR(1) firm-level shocks. With fixed capital adjustment costs, this model is well known to generate a fat-tailed investment rate distribution in simulated data, since firms alternate between frequent periods with zero investment and more infrequent investment spikes even with Gaussian AR(1) shocks. However, revenue in lumpy investment models depends upon input *stocks* such as employment or capital, so fat tails for investment rates – a measure of input *flows* – do not immediately imply that revenue growth will be fat-tailed. By contrast, we find in simulated data that even with a relatively large fixed capital adjustment cost equal to around 2.5% of the capital stock per adjustment, a cost which generates quite extreme investment spikes, delivers revenue growth kurtosis of only around 4.5 in simulated data. This value is

[19]The extension is straightforward. We replace household log consumption preferences $\log C$ with log-linear utility $\log C - \omega N$ for some $\omega > 0$. Then, we replace the labor market clearing condition with the household intratemporal optimality condition $W = \omega C$. Otherwise, the equilibrium structure remains unchanged. We calibrate $\omega = 1.51$ (empirical case) and $\omega = 1.39$ to match labor supply N to the Spanish employment rate.

very close to the Gaussian benchmark of 3, and far below the high degree of revenue growth kurtosis observed in our baseline empirical sample (around 29). We conclude that lumpy input adjustment costs are not enough by themselves, without changes to the underlying firm-shock process, to rationalize our evidence of fat-tailed revenue dynamics.

8 Conclusion

In this paper, we argue that the standard parametric assumption for firm-level shocks—a Gaussian AR(1) process—commonly used in the heterogeneous firms literature, is not well-suited to accurately capture real-world dynamics. In particular, we find that nonparametrically solving a model consistent with the firm-level revenue dynamics we observe in the data, i.e., fat-tailed growth and high mobility from the tails, has a large impact on the behavior of a canonical firm dynamics model at the macro level. The standard parametric model implies a firm value distribution which is far too dispersed relative to the firm value distribution consistent with empirical firm dynamics. As a result, the empirical, nonparametric model's more clustered value distribution generates substantially higher sensitivity of the exit rate to a set of standard policy experiments. The stronger extensive margin reaction in our nonparametric model drives strong selection effects serving to amplify or dampen the response of aggregate output, depending upon the exact details of the underlying policy. As a result, we conclude that the standard parametric assumptions adopted in the quantitative firm dynamics literature are far from innocuous but instead directly change the macro implications of firm-level mechanisms. Given the central role of these models in modern macroeconomics, we see our paper as a call to reevaluate how these models are calibrated and employed for both normative and positive analyses.

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Appendices for Online Publication Only

A Data

We begin with annual ORBIS data on firm financials in Spain from 2005-2019. Since our interest is in firms as a legal concept rather than on, say, physical locations or lines of work, we restrict our sample to consolidated financial statements.

Our revenue measure is the ORBIS variable `opre`, i.e., operating revenue or turnover measured at the firm-year level. This variable, in logs, residualized with respect to 4-digit NAICS industry and year effects, is our baseline revenue measure referred to as y in the text.

We trim our panel of residualized revenue y data at the 0.1% and 99.9% thresholds. We also lightly clean our sample in some other ways to guard against the possibility that observed exit or entry might be driven by missing information in a specific year. We first form candidate indicators for firm entry and exit events based on that firm's data availability in our historical ORBIS panel dataset. If for a given year the firm has data for at least one of the most populated variables (e.g. employment and payroll) but not revenue, then the firm is dropped altogether. Second, we ensure that data "holes" do not generate spurious entry or exit by verifying that the firm is not ever present in the dataset before (after) the candidate entry (exit) year, with the "after" window extending for a buffer of four years.[20]

The benchmark ORBIS sample we construct following the guidelines above results in a panel dataset in Spain with a total of 5,157,769 firm-years for 1,032,098 firms in the 2005-2014 period. In the analysis that follows in this appendix, we provide statistics from various alternative datasets we consider as part of robustness checks to our baseline empirical approach. We also introduce various ancillary empirical results referenced throughout the main text.

A.1 Empirical Robustness Checks

Table A.1 reports moments of residualized log revenue y and revenue growth Δy from our baseline Spanish ORBIS dataset on over one million firms for over five million

[20]For this reason, our effective sample period never goes beyond 2014, the end date quoted in the text. This restriction allows us to verify that a firm does not show up again between 2015 and 2019, since 2019 is the formal end of the ORBIS historical dataset in the ORBIS vintage we used.

Table A.1: Empirical Moments under Alternative Datasets

	Revenue y			Revenue Growth Δy		
	Std dev	Skewness	Kurtosis	Std dev	Skewness	Kurtosis
Baseline	1.548	0.025	4.196	0.656	-0.312	29.212
Before 2009	1.515	0.007	4.182	0.692	-0.071	29.920
After 2009	1.583	0.025	4.224	0.652	-0.482	26.974
Mfg	1.561	0.056	3.862	0.480	-0.829	41.572
Non-Mfg	1.546	0.021	4.250	0.679	-0.285	27.767
Unconsolidated only	1.527	-0.031	4.125	0.655	-0.321	29.160
No M&A	1.545	0.028	4.214	0.655	-0.320	28.937
Year Effects Only	1.699	0.131	3.756	0.660	-0.415	28.765
No Trimming	1.641	-0.068	5.088	0.774	-0.774	35.153
1% Trimming	1.416	-0.008	3.336	0.586	-0.201	21.646
Remove Firm Age	1.496	-0.063	4.356	0.664	-0.593	25.882
Italy	1.764	-0.660	6.070	0.956	-0.165	33.889
Portugal	1.514	-0.122	5.077	0.736	0.347	28.683
France	1.379	-0.152	6.483	0.604	0.637	72.380
Norway	1.705	-0.233	4.274	0.681	-0.153	26.698

Notes: This table reports moments of firm revenue y and revenue growth Δy under alternative empirical approaches. The Baseline moments in the top row represent our benchmark ORBIS sample of just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain. In this case, revenue levels y represent log firm revenue demeaned by sector and year, while revenue growth Δy is the first difference of revenue levels. In subsequent rows, we report moments from datasets constructed from ORBIS using different nations, subsamples, time periods, or data treatment approaches.

firm years together with analogous moments for a range of robustness checks and alternative samples described in the main text in Section 7.

A.1.1 A Parametric Model with Fixed Effects and Transitory Shocks

In a robustness check we consider an extended parametric AR(1) model. First, we recover observed profitability z_{it} for firm i in year t from residualized revenue y_{it} by inverting the labor optimality condition according to (14). In our extended model, we decompose profitability as

$$z_{it} = \mu_i e^{\log \hat{z}_{it} + \nu_{it}}. \tag{18}$$

Table A.2: Parametric AR(1) with Fixed Effects and Transitory Shocks

Panel A: Extended AR(1) Parameters	Symbol	Value
Autocorrelation	ρ	0.9522
Persistent shock variance	σ_ε^2	0.0189
Transitory shock variance	σ_ν^2	0.0150
Pareto fixed effects lower bound	μ_{min}	0.7941
Pareto fixed effects shape	μ_{shape}	4.4176
Panel B: Extended AR(1) Moments	**Data**	**Model**
Autocorrelation, log z	0.9071	0.9071
Variance , Δ log z	0.0480	0.0480
Variance, log z	0.2692	0.2692
Top 1% share, z	0.0408	0.0408
Mean, z	1.1441	1.1441
Panel C: Model Predictive Accuracy	**RMSE**	**LPS**
Nonparametric	1.000	-3.25
Extended AR(1)	1.021	-3.6
Benchmark AR(1)	1.033	-3.7

Notes: Panels A and B reports simulated method of moment estimates and fit for our extended parametric AR(1) model (18). The estimates were computed using a simulated panel of identical size to our benchmark ORBIS data for just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain. As in our baseline model calibration, we recover profitability z from log firm revenue demeaned by sector and year using the labor optimality condition (14). Panel C reports a battery of predictive accuracy tests for $\log z$ – relative root mean squared errors (RMSE) and log predictive scores (LPS) — for our nonparametric model from Section 2.2, our benchmark AR(1), and the extended AR(1) from Panels A and B.

In the equation above μ_i is a firm fixed effect cross-sectionally distributed Pareto with scale parameter μ_{min} and shape parameter μ_{shape}. Inside the exponent, $\log \hat{z}_{it} = \rho \log z_{it-1} + \varepsilon_{it}$ is a Gaussian AR(1) component with $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$, and $\nu_{it} \sim N(0, \sigma_\nu^2)$ is an iid transitory shock.

We estimate the model (18) with an exactly identified simulated method of moments strategy. While the identification is joint, roughly speaking, the autocorrelation of profitability disciplines ρ, the variance of both profitability growth and levels discipline shock innovations σ_ε^2 and σ_ν^2, and finally the mean and top 1% share of profitability discipline the Pareto scale and shape parameters μ_{min} and μ_{mean}. Panels A and B in Table A.2 report our point estimates and targeted moments, revealing an exact fit as well as high estimated persistence, conditional volatility close to evenly split between persistent and transitory sources, and a nontrivial distribution of firm heterogeneity.

Panel C of A.2 subjects three models – the empirical or nonparametric model defined in Section 2.2, our benchmark calibrated parametric AR(1), and the extended AR(1) model – to a battery of predictive accuracy tests for log profitability. In the second column we report the root mean squared error (RMSE) of the mean one-year predictions implied by each model, normalizing the nonparametric model's RMSE to 1. This statistic measures the point forecast accuracy of each model, with higher values indicating a poorer performance. In the column "LPS" we report the log predictive score, a measure of a model's predictive accuracy over the full distribution of one-year ahead profitability in which higher values indicate more accurate prediction. Under either measure, the performances of both the baseline and extended parametric models are poor relative to the nonparametric model, although the extended model does improve on the benchmark AR(1)'s performance meaningfully. Quantitatively, the extended model closes only around a third of the accuracy gap with the nonparametric model measured using mean forecast RMSE's and around a quarter of the accuracy gap measured using the broader LPS measure.

A.1.2 A Nonlinear, Non-Gaussian Parametric Model

Consider a nonlinear, non-Gaussian AR(1) model for observed profitability z_{it} for firm i in year t that takes the form

$$\log z_{it} = \rho(z_{it-1}) \log z_{it-1} + \varepsilon_{it}, \tag{19}$$

where the autocorrelation parameter $\rho(z_{it-1})$ depends upon profitability according to the function

$$\rho(z_{it-1}) = \rho_0 + \rho_1 \log z_{it-1} \tag{20}$$

for some parameters ρ_0 and ρ_1. The shocks ε_{it} are assumed to be non-Gaussian and drawn from the mixture distribution

$$\varepsilon_{it} \sim_{iid} p_1 N(\mu_1, \sigma_1^2) + (1 - p_1) N(\mu_2, \sigma_2^2) \tag{21}$$

for some $p_1 \in (0, 1)$ and Gaussian mixture component parameters μ_1, μ_2, σ_1^2, and

Table A.3: A Nonlinear, Non-Gaussian AR(1) Model

Panel A: Nonlinear AR(1) Parameters	Symbol	Value
Mixture component 1 weight	p_1	0.9052
Mixture component 1 mean	μ_1	0.0276
Mixture component 2 mean	μ_2	-0.4552
Mixture component 1 std. dev.	σ_1	0.0949
Mixture component 2 std. dev.	σ_2	0.4906
Panel B: Nonlinear AR(1) Moments	**Data**	**Model**
$\mathbb{E} \log \hat{z}$	0.0000	-0.0014
$\mathbb{E}(\log \hat{z})^2$	0.2692	0.2624
$\mathbb{E}(\log \hat{z})^3$	-0.0045	-0.0023
$\mathbb{E}(\log \hat{z})^4$	0.3066	0.3090
$\mathbb{E}(\Delta \log \hat{z})^2$	0.0481	0.0547
$\mathbb{E}(\Delta \log \hat{z})^3$	-0.0009	-0.0352
$\mathbb{E}(\Delta \log \hat{z})^4$	0.0685	0.0473
Panel C: Model Predictive Accuracy	**RMSE**	**LPS**
Nonparametric	1.000	-3.25
Nonlinear AR(1)	1.010	-3.4
Benchmark AR(1)	1.033	-3.7

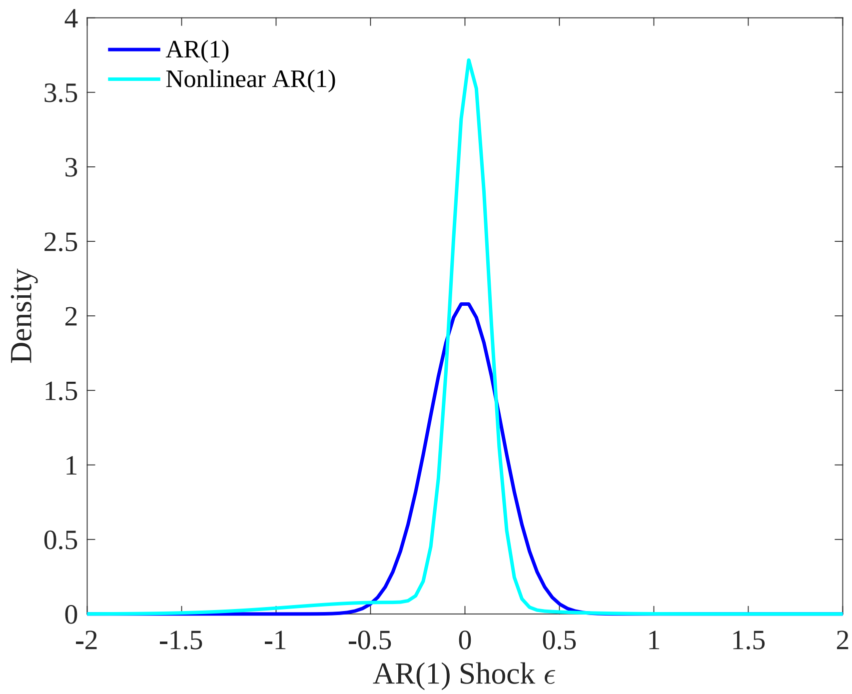
Notes: Panels A and B reports calibrated parameters and model fit for our nonlinear, non-Gaussian AR(1) model. The parameters were computed using a simulated panel of identical size to our benchmark ORBIS data for just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain. As in our baseline model calibration, we recover profitability z from log firm revenue demeaned by sector and year using the labor optimality condition (14). Panel C reports a battery of predictive accuracy tests for $\log z$ – relative root mean squared errors (RMSE) and log predictive scores (LPS) — for our nonparametric model from Section 2.2, our benchmark AR(1), and the nonlinear, non-Gaussian AR(1) from Panels A and B.

σ_2^2. Note that the conditional mean of future profitability in this model, i.e.,

$$\begin{aligned} \mathbb{E}(\log z_{it} | \log z_{it-1}) &= \rho(z_{it-1}) \log z_{it-1} && (22) \\ &= \rho_0 \log z_{it-1} + \rho_1 (\log z_{it-1})^2, && (23) \\ & && (24) \end{aligned}$$

is linear in the parameters ρ_0 and ρ_1. So we can estimate these parameters via conventional OLS. In our baseline sample, using our recovered firm-level profitability data $\hat{z}$ from the Spanish ORBIS data, we estimate $\hat{\rho}_0 = 0.9117$ and $\hat{\rho}_1 = 0.0854$. We then calibrate the remaining mixture parameters for the shock distribution by minimizing the sum of squared deviations between the first four moments of the profitability and profitability growth distributions, effectively computing the point estimates associated with a simple overidentified simulated method of moments estimator. Table A.3 reports the resulting parameters, moment fit, and predictive accuracy of the model. The mixture distribution for the shock process ε_{it}, does in fact exhibit fat tails as shown in Figure A.1.

Figure A.1: Shocks in the Nonlinear, Non-Gaussian AR(1)



Notes: The figure plots the density of the distribution of the profitability shock ε_{it} for our nonlinear, non-Gaussian model calibrated in Table A.3 (in light blue) compared with the baseline AR(1) model with Gaussian innovations (in dark blue). Both models were calibrated using our benchmark ORBIS data for just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain.

We see that as measured by forecasting RMSE or log predictive scores, the nonlinear, non-Gaussian AR(1) outperforms the benchmark AR(1) process by a wide margin, closing more than half the performance gap with our full nonparametric model in each case.

A.2 Predicting Market Value with Lifetime Revenue

Table A.4: Lifetime Revenue, Current Revenue, and Market Value

	Market Value$_{it}$			
	(1)	(2)	(3)	(4)
Revenue$_{it}$	0.284***	0.141***	0.141***	-0.057
	(0.029)	(0.018)	(0.018)	(0.036)
Lifetime Revenue$_{it}$				0.362***
				(0.076)
Fixed Effects	-	Industry	Industry Year	Industry Year
Firm-Years	4273	4273	4273	4273

Notes: The table reports OLS estimates of market value, in logs, for firm i in year t on log revenue and log lifetime revenue. Industry refers to four-digit industry codes. The sample is drawn from the subset of publicly listed firms within our baseline Spanish ORBIS dataset spanning 2005-2014 for both listed and unlisted firms. Unconditionally, the correlation of log revenue and market value is 0.24, and the correlation of log lifetime revenue and log market value is 0.27. Standard errors are clustered at the firm level. Significance is indicated as * = 10% level, ** = 5% level, and *** = 1% level.

To examine the predictive content of our lifetime revenue measure $W(y)$ defined in Section 2, over and above current revenue y, we restrict our baseline Spanish ORBIS dataset to a subset of only publicly listed firms. For this subsample, we observe realized market value. We see in Table A.4's regression results in columns 1-3 that contemporaneous revenue is highly correlated with a firm's market value. Yet, our constructed firm lifetime revenue variable is a better predictor of a firm's market value. In particular, once lifetime revenue is included, contemporaneous revenue ceases to be statistically significant. We view these results as validating the empirical relevance of our lifetime revenue measure.

A.3 Industry Clustering and Exit Rates

We develop a framework linking firm exit to our notion of observed lifetime revenue $W(y)$ developed in Section 2. Recall that the stationary distribution $H(y)$ of current

Table A.5: Clustering across Sectors

Clustering Statistic $\mathcal{C}$	NAICS Sector
0.091	Construction, 23
0.0543	Real Estate, 53
0.0492	Professional Technical Services, 54
0.0484	Retail Trade, 44
0.0453	Retail Trade, 45
0.0447	Information, 51
0.0401	Manufacturing, 33
0.0399	Wholesale Trade, 42
0.0399	Arts & Entertainment, 71
0.0392	Administrative Support Services, 56
0.0389	Accommodation and Food Services, 72
0.0376	Manufacturing, 32
0.0371	Educational Services, 61
0.0369	Other Services, 81
0.0351	Manufacturing, 31
0.0324	Transportation and Warehousing, 48
0.0288	Finance and Insurance, 52
0.0236	Health Care and Social Assistance, 62

Notes: This table reports the value of the clustering statistic $\mathcal{C}$ defined in (25) at the 2-digit NAICS sector level. The underlying data is our benchmark ORBIS data for just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain.

revenue y implies a stationary distribution $H(W)$ of lifetime revenue W. Similarly, the revenue exit hazard $\mathbb{P}(\text{Exit}|y)$ implies a lifetime revenue exit hazard $\mathbb{P}(\text{Exit}|W)$. We rewrite the exit rate as

$$\mathbb{P}(\text{Exit}) = \int \mathbb{P}(\text{Exit}|W)dH(W).$$

In this *purely statistical* model, the new exit rate predicted in partial equilibrium after a windfall increase ϵ in lifetime revenue W is given by

$$\mathbb{P}(\text{Exit}) = \int \mathbb{P}(\text{Exit}|W + \epsilon)dH(W).$$

Table A.6: Clustering and Exit

	Exit Rate$_{jt}$			
	(1)	(2)	(3)	(4)
Δ Revenue$_{jt}$	-0.045***	-0.046***	-0.050***	-0.039***
	(0.008)	(0.012)	(0.011)	(0.011)
Δ Revenue$_{jt}$ $\times$ Clustering$_s$	-0.011*	-0.013*	-0.014*	
	(0.006)	(0.006)	(0.007)	
Clustering$_s$	0.428**	0.413**		
	(0.211)	(0.209)		
Δ Revenue$_{jt}$ $\times$ I(Highly Clustered$_s$)				-0.052**
				(0.022)
Fixed Effects	-	Year	Year, Sector	Year, Sector
Industry-Years	1584	1584	1584	1584
Years	2006-13	2006-13	2006-13	2006-13

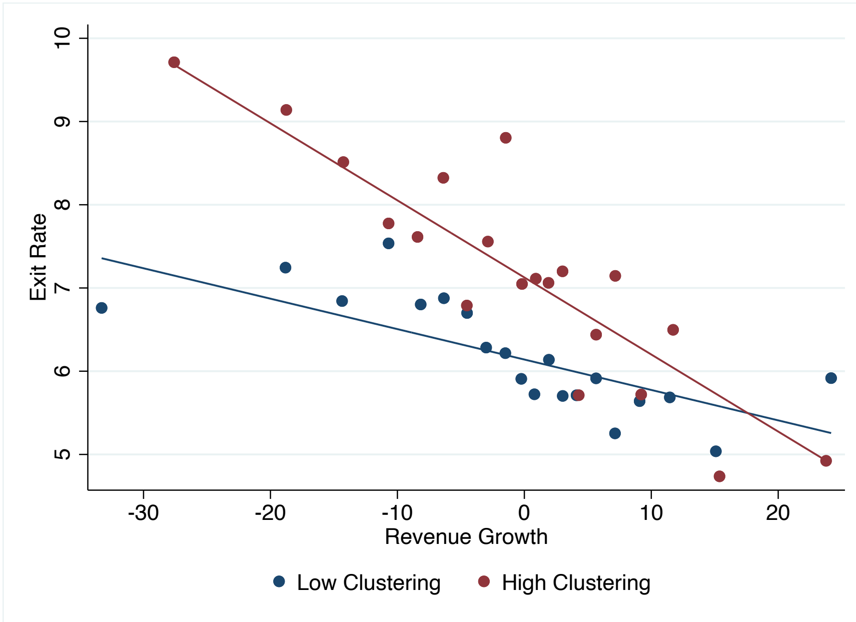
Notes: The table reports OLS estimates from (26) of 4-digit NAICS industry j exit rates in year t on industry j's revenue growth in year t and standardized clustering statistics $\mathcal{C}_s$ for 2-digit NAICS sector s containing j. "Highly Clustered" sectors are those with clustering in the top quartile across sectors. Standard errors are clustered at the 4-digit industry j level. Significance is indicated as * = 10% level, ** = 5% level, and *** = 1% level. The underlying data is our benchmark ORBIS data for just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain.

Thus, the sensitivity of exit to this windfall revenue increase can then be computed as the distributional "clustering statistic" $\mathcal{C}$ given by

$$\mathcal{C} = -\frac{\partial \mathbb{P}(\text{Exit})}{\partial \epsilon}|_{\epsilon=0} = -\int \frac{\partial \mathbb{P}(\text{Exit}|W)}{\partial W} dH(W). \tag{25}$$

Intuitively, $\mathcal{C}$ is simply a weighted average of the slope of exit hazard. We use $\mathcal{C}$ as a measure of clustering simply because the statistic captures the coincidence of high distributional density with steep exit hazards. In other words, a lifetime revenue distribution with a higher value of $\mathcal{C}$ has higher distributional weight and is "clustered" in regions where exit is more marginal.

Figure A.2: Clustering, Revenue Growth, and Exit Rates



Notes: The binscatter plots exit rates on the vertical axis against revenue growth rates on the horizontal axis. Both variables are measured at the industry (4-digit NAICS) by year level, with a total of 198 industries and 1980 industry-years in total. In red, with associated line of best fit, the plot is based on observations from "high clustering" 2-digit NAICS sectors in which the clustering statistic $\mathcal{C}$ from (25) is in the top quartile across sectors, while the blue observations and line plot data from other "low clustering" sectors with $\mathcal{C}$ below the top quartile. The underlying data is our benchmark ORBIS data for just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain.

We compute the clustering statistic $\mathcal{C}_s$ using the nonparametric lifetime revenue distributions for each 2-digit NAICS sector s in our baseline Spanish ORBIS dataset. Table A.5 reports the bunching statistics by sector, which vary widely. For instance, construction, real estate, professional services and retail trade are characterized by larger clustering statistics than health care, transportation or manufacturing.

We then use more disaggregated industry classifications for 4-digit NAICS industries j within 2-digit sector s for year t in our data to estimate versions of the following specification

$$\mathbb{P}(\text{Exit})_{jt} = \alpha + \beta \Delta \text{Revenue}_{jt} + \gamma \Delta \text{Revenue}_{jt} \times \mathcal{C}_{s(j)} + \delta \mathcal{C}_{s(j)} + \varepsilon_{jt}. \qquad (26)$$

Above, $\mathbb{P}(\text{Exit})_{jt}$ is the exit rate of industry j in year t, $\Delta\text{Revenue}_{jt}$ is the industry j growth rate of revenue in year t, and $\mathcal{C}_{s(j)}$ is the clustering statistic for sector s containing industry j. Note that our model-based intuition predicts $\gamma < 0$ if more clustering is linked to higher exit sensitivity.[21] Table A.6 presents estimates of (26). Column 1 shows that high revenue growth at the 4-digit industry level is associated with lower exit and that – via the estimated interaction term – this negative association is stronger in sectors with a higher degree of clustering, consistent with our model's intuition. Columns 2 and 3 show the robustness of this pattern to the inclusion of year fixed effects as well as fixed effects for 2-digit sector s. In both columns, the interaction term continues to be negative and statistically significant at the 10% level. In column 4, we replace the linear interaction term with a categorical approach. We define a highly clustered sector as a sector with a clustering statistic $\mathcal{C}_s$ in the upper quartile of the distribution of $\mathcal{C}_s$ across sectors. The interaction term is significant at the 5% level, emphasizing again that clustering is indeed statistically linked to the dynamics of industry exit rates. Figure A.2 presents the same fact from column 4, with heterogenous sensitivities in high vs low clustering sectors, using a simple binscatter plot. So, to summarize, Table A.6 shows that industries with more clustered lifetime revenue distributions exhibit higher exit rate sensitivity to changes in revenue growth, intuitively consistent with our key model mechanism.

A.4 Predicting Firm Exit with Profit versus Revenue

Our empirical analysis centers on firm-level revenue. Table A.7 reports the results of a set of predictive regressions demonstrating that our revenue variable is a better predictor of both exit and employment growth at the firm level than a natural alternative measure of firm profits.

[21] Our maintained assumption is that the degree of clustering at the 4-digit level is relatively homogeneous within a given 2-digit sector and stable over our sample period. We rely on this assumption since at the 4-digit level, with too few observations in each cell, the resulting $\mathcal{C}_s$ statistics are too noisy. Also, note that output in our model is stationary while, naturally, output exhibits positive growth in the data. So (26) links the exit rate to the transformed stationary *growth rate* of sectoral revenue rather than its level. This transformation allows the empirical test to be consistent with the interpretation of the model.

Table A.7: Predicting Firm Outcomes with Revenue vs Profits

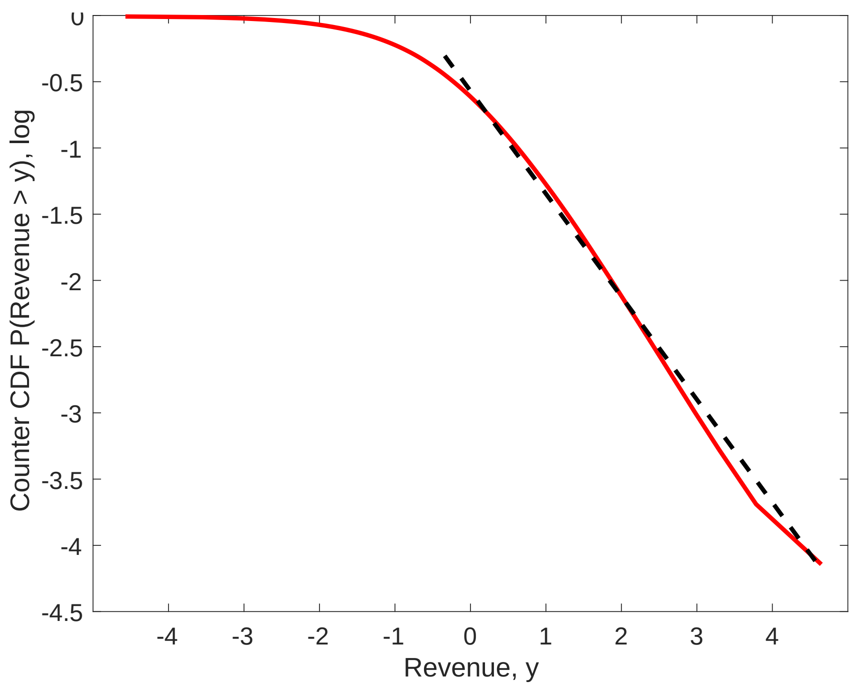
	Regressor	
	Revenue	Profit Margin
Regressand	(1)	(2)
Exit	-0.021***	-0.001***
	[0.001]	[0.002]
R^2	0.037	0.027
Employment Growth	0.021***	0.0011***
	[0.001]	[0.0013]
R^2	0.022	0.018

Notes: The table reports results from a serious of predictive regressions of firm exit (top panel) or firm employment growth (bottom panel) in a given year on the firm's revenue, in logs, or profit margin, the ratio of earnings before interest and taxes to revenue, measured in the previous year. Year and industry fixed effects are included in all specifications. The sample is our benchmark ORBIS data for just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain. p-values, based on clustering at the industry level, are reported in square brackets. Significance is indicated as * = 10% level, ** = 5% level, and *** = 1% level.

A.5 A Power Law Tail in Firm Revenue

Fat-tailed cross-sectional size distributions are ubiquitous in many economic contexts and can in principle be detected by a telltale linear relationship between log size and the log counter-CDF of a distribution (Gabaix, 2016). In Figure A.3, we see that a linear relationship of this sort matches the shape of the right tail of our baseline sample's stationary distribution of revenue $H(y)$.

Figure A.3: A Power Law Tail in Firm Revenue



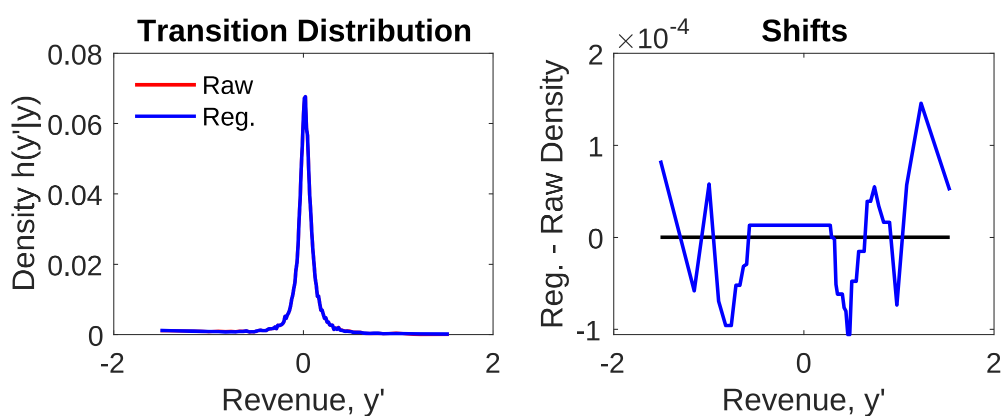
Notes: The solid red line in the figure plots the stationary distribution of revenue $H(y)$ computed from our baseline ORBIS sample of just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain. The horizontal axis is revenue y, in logs, and the vertical axis is the log counter CDF of the revenue distribution. The dotted line is the line of best fit estimated on the upper half of our revenue distribution with a slope coefficient of -0.77.

B Model

In this appendix we provide further information on our solution and calibration of the quantitative model. We start with our approach to (very lightly) regularizing the raw nonparametric empirical objects from Section 2 to satisfy standard assumptions for firm dynamics models. We then discuss the numerical techniques we employ while solving and calibrating both the nonparametric and parametric models. Finally, we present details on our quantitative model robustness checks and recalibrations in a set of summary tables.

B.1 Regularizing the Raw Data

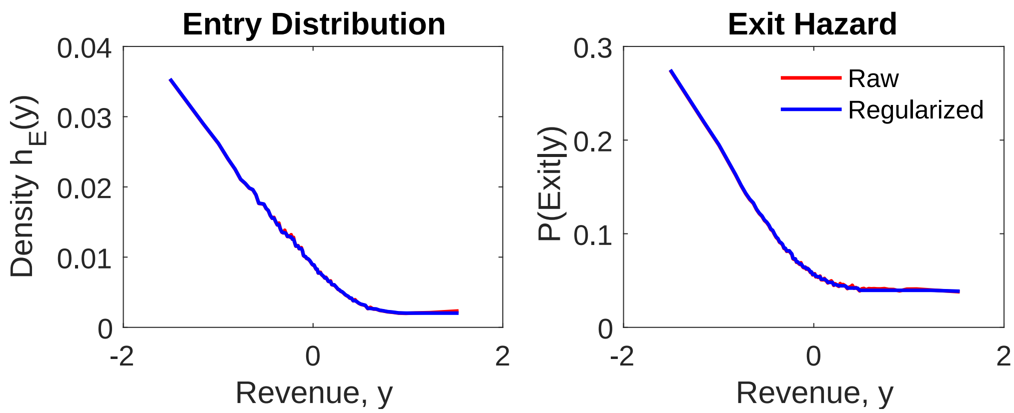
Figure B.1: Regularized vs Raw Transition Distribution



Notes: The left panel of the figure plots raw (in red) and regularized (in blue) transition densities $h(y'|y)$ for next year's revenue y' conditional upon median revenue y in the current year. The right panel plots the regularization shifts or the difference between the regularized and raw densities. The horizontal axis in each figure is next year's revenue y', in logs. The underlying data is drawn from our baseline ORBIS sample of just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain.

In order to embed $H(y'|y)$, $H_E(y)$, and $P(\text{Exit}|y)$ into a canonical heterogeneous firms model, each needs to be adjusted in order to satisfy some standard technical or regularity assumptions. To ensure monotonicity of firm value functions and a well behaved value function iteration algorithm, we first require that the transition distribution H exhibits persistence via first-order stochastic dominance: for two states

Figure B.2: Regularized vs Raw Entry and Exit Patterns



Notes: The left panel of the figure plots the entry density $h_E(y)$, and the right panel plots the exit hazard $\mathbb{P}(\text{Exit}|y)$. The horizontal axis in each figure is the current year's revenue y, in logs. In both panels, the raw object is presented in red, and the regularized object is presented in blue. The underlying data is drawn from our baseline ORBIS sample of just over 5 million firm-years for over one million firms covering the 2005-2014 period in Spain.

such that $y_2 \geq y_1$, we require that $H(y'|y_2) \leq H(y'|y_1)$ for all y'. Second, to ensure that the fixed cost distribution $G(\phi_F)$ has a nondecreasing CDF, i.e., to ensure that the fixed cost distribution is in fact a distribution, we require that the exit hazard $\mathbb{P}(\text{Exit}|y)$ is nonincreasing in y. We do not technically face a need to regularize the entry distribution, but given its overall declining shape in the raw data we also impose that the entry density, not just the exit hazard, is nonincreasing as well.

To impose these regularizations, we design and employ a simple procedure with the intention of making only minimal modifications to the raw data. First, we note that our assumptions of first-order stochastic dominance and a downward sloping hazard can be written as a large set of inequality restrictions which must be satisfied by each value of our extracted distributions and exit hazards. Our procedure then operates as follows. First, we initialize a regularized object, either a transition matrix or an exit hazard, to the raw data equivalent. Second, we compute the existing "gaps" in each of our inequality restrictions. We then distribute weight proportional to the size of this gap to the remaining entries in the corresponding distribution or hazard. Third, we recompute the inequality gaps or errors in our regularized empirical objects, ending the procedure if the gaps are absent but restarting if they are not. An example may help build intuition for our procedure. If an exit hazard is slightly less than downward sloping in the raw data due to apparent noise and a bump upwards in observed exit rates for a given revenue bin, we simply take a small portion of the exit rate in that bin and distribute it elsewhere along the hazard, repeating this approach for all points iteratively until the resulting hazard is downward sloping as a whole.

Helpfully, this regularization procedure turns out to impose only extremely light modifications to the raw data, i.e., the raw data is already very close to satisfying these regularity conditions absent apparent statistical noise. To illustrate this for our baseline ORBIS sample, Figure B.1 compares the raw and regularized transition density for revenue y' conditional upon median revenue y in the current year. The left panel plots the two resulting densities, which are virtually identical to the naked eye. The right panel plots the "shift" or difference between the regularized and raw densities, which remains trivial across revenue levels. Figure B.2 compares the raw and regularized entry distribution and exit hazards which are, again, virtually indistinguishable. In both figures, where applicable, the raw data is presented in red and the regularized objects are plotted in blue.

B.2 Solving the Empirical Nonparametric Model

Note that the static optimality condition for the input n in equation (14) and the residualized log revenue grid y_i (indexing our partition of the revenue space into N_y equally weighted intervals) together imply a quantile-based grid for profitability shocks z_i, $i = 1, ..., N_z$, where $N_z = N_y$ and $\log z_i = (1-\alpha)y_i$ for all i. Similarly, the empirical objects $H(y'|y)$, $H_E(y)$, and $\mathbb{P}(Exit|y)$ imply an incumbent profitability transition $F(z'|z)$, an entry distribution $F_E(z)$, and an exit hazard $\mathbb{P}(\text{Exit}|z)$ on the profitability grid z_i.

We assume that exit occurs for the highest profitability firms in our sample for only exogenous reasons, i.e., that $\delta = \mathbb{P}(\text{Exit}|z_{N_z})$. In our baseline ORBIS sample in Spain, the resulting exogenous exit rate is $\delta = 3.9\%$. The remaining parameters to be calibrated in our nonparametric model include only the labor share α, the household's rate of time preference β, the fixed labor supply $\bar{N}$, and the sunk entry cost ϕ_E. Given a parameterization of the model, i.e., a list of these parameters, we solve the model with an outer loop-inner loop approach as follows.

1. **Outer Loop on GE Objects** Guess values for the wage W and the entry

mass M_E, and fix a GE tolerance $\epsilon^{GE} > 0$.

(a) **Inner Loop on Firm Value Function** Initialize $k = 0$, guess a value function $V^{(k)}(z)$, and fix a value function error tolerance $\epsilon^V > 0$.

i. Compute the implied continuation values $\phi_F^{*(k)}(z)$ via equation (2) and using $V^{(k)}(z)$.

ii. Infer the distribution $G^{(k)}(\phi_F)$ of fixed cost shocks ϕ_F consistent with $\phi_F^{*(k)}(z)$, $V^{(k)}(z)$, and the empirical exit hazard by using the mapping

$$G^{(k)}(\phi_F^{*(k)}(z)) = \frac{1 - \mathbb{P}(\text{Exit}|z)}{1 - \delta}.$$

iii. Compute an updated value function $V^{(k+1)}(z)$ via the Bellman equation

$$V^{(k+1)}(z) = \left\{ \begin{array}{c} \max_n (zn^\alpha - Wz) \\ - \int_0^{\phi_F^{*(k)}(z)} \phi_F dG(\phi_F) \end{array} + \beta(1-\delta) \int V^{(k)}(z') dF(z'|z) \right\}.$$

iv. If the error in the Bellman equation $\max_z |V^{(k+1)}(z) - V^{(k+1)}(z)|$ is smaller than ϵ^V, then the firm value function $V(z) = V^{(k)}(z)$, continuation values $\phi_F^*(z) = \phi_F^{*(k)}(z)$, and the fixed cost distribution $G(\phi_F) = G^{(k)}(\phi_F)$ are computed. Otherwise, set $k = k+1$ and return to step (1(a)i).

(b) **Inner Loop on Firm Distribution** Initialize $k = 0$, guess an operating distribution $F_O^{(k)}(z)$ for firms, guess a mass $M_O^{(k)}$ of operating firms, and fix a tolerance $\epsilon^F > 0$ for distributional convergence.

i. Compute the implied mass of operating firms $M_O^{(k+1)}$ via

$$M_O^{(k+1)} = (1-\delta) M_O^{(k)} \int G(\phi_F^*(z)) dF_O^{(k)}(z) + M_E.$$

ii. Compute the implied distribution of operating firms $F_O^{(k+1)}(z)$ via

$$F_O^{(k+1)}(z') = (1-\delta) \frac{M_O^{(k)}}{M_O^{(k+1)}} \int G(\phi_F^*(z)) F(z'|z) dF_O^{(k)}(z) + \frac{M_E}{M_O^{(k+1)}} F_E(z').$$

iii. If the errors in the operating mass update $|M_O^{(k+1)} - M_O^{(k)}|$ and distributional update $\max_z |F_O^{(k+1)}(z) - F_O^{(k)}(z)|$ are both less than ϵ^F, then the operating mass $M_O = M_O^{(k)}$ and operating distribution $F_O(z) = F_O^{(k)}(z)$ are computed. Otherwise, set $k = k + 1$ and return to step (1(b)i).

2. Compute the implied value to entry V_E via

$$V_E = \int V(z) dF_E(z).$$

3. Compute the implied labor demand N via

$$N = M_O \int n^*(z) dF_O(z),$$

   where $n^*(z)$ is optimal static labor demand for an individual firm with profitability z.

4. If the error in the free entry condition $|V_E - \phi_E|$ and the error in the labor market clearing condition $|N - \bar{N}|$ are both less than the GE tolerance ϵ^{GE}, then the model is solved. Otherwise, update your guesses for the wage and entry mass and return to step (1).

When the algorithm above is complete, the nonparametric version of our model is solved in a manner not only consistent with general equilibrium but also, by construction, with the observed revenue transitions, the entry distribution, and the exit hazard measured nonparametrically.

A few additional technical details are useful. We implement all of the calculations above continuously, linearly interpolating value functions, fixed cost distributions, operating distributions, and continuation values on the grid z_i. Where integration is required, we use Simpson quadrature with densities $f_O(z)$, $f_E(z)$, and $f(z'|z)$ consistent with linear interpolation of the CDFs $F_O(z)$, $F_E(z)$, and $F(z'|z)$ in a manner which preserves the empirical weight on equal-mass intervals containing the revenue quantiles y_i. Because the free entry condition is separable from the entry mass M_E, we first employ bisection on the aggregate wage W to ensure that the free entry condition is satisfied, then we update M_E so that (3) is exactly satisfied. In our baseline, we employ $N_y = N_z = 101$ grid points or quantiles, and on a 2017 iMac

Pro model solution takes around a minute or two in MATLAB without requiring aggressive parallelization.

B.3 Solving the AR(1)/Parametric Model

In our AR(1) or parametric model version, the parameters to be calibrated include the labor share α, the household's rate of time preference β, the fixed labor supply $\bar{N}$, the sunk entry cost ϕ_E, the upper bound $\bar{\phi}_F$ of the fixed cost distribution $G(\phi_F) = U(0, \bar{\phi}_F)$, the persistence of the lognormal AR(1) profitability process ρ, the conditional variance of the lognormal AR(1) profitability process σ^2, and the mean of the lognormal entry distribution μ_E. The exogenous exit hazard δ is carried over identically from our nonparametric model solution as described above. Given a parameterization of the model, i.e., a list of these parameters, we solve the model with an outer loop-inner loop approach as follows.

1. **Outer Loop on GE Objects** Guess values for the wage W and the entry mass M_E, and fix a GE tolerance $\epsilon^{GE} > 0$.
   (a) **Inner Loop on Firm Value Function** Initialize $k = 0$, guess a value function $V^{(k)}(z)$, and fix a value function error tolerance $\epsilon^V > 0$.
      i. Compute an updated value function $V^{(k+1)}(z)$ via the Bellman equation
      $$V^{(k+1)}(z) = \left\{ \begin{array}{c} \max_n (zn^\alpha - Wz) \\ - \int_0^{\phi_F^{*(k)}(z)} \phi_F dG(\phi_F) \end{array} + \beta(1-\delta) \int V^{(k)}(z') dF(z'|z) \right\}.$$
      ii. If the error in the Bellman equation $\max_z |V^{(k+1)}(z) - V^{(k+1)}(z)|$ is smaller than ϵ^V, then the firm value function $V(z) = V^{(k)}(z)$ is computed. Otherwise, set $k = k + 1$ and return to step (1(a)i).
   (b) **Inner Loop on Firm Distribution** Initialize $k = 0$, guess an operating distribution $F_O^{(k)}(z)$ for firms, guess a mass $M_O^{(k)}$ of operating firms, and fix a tolerance $\epsilon^F > 0$ for distributional convergence.
      i. Compute the implied mass of operating firms $M_O^{(k+1)}$ via
      $$M_O^{(k+1)} = (1-\delta) M_O^{(k)} \int G(\phi_F^*(z)) dF_O^{(k)}(z) + M_E.$$

ii. Compute the implied distribution of operating firms $F_O^{(k+1)}(z)$ via

$$F_O^{(k+1)}(z') = (1-\delta)\frac{M_O^{(k)}}{M_O^{(k+1)}}\int G(\phi_F^*(z))F(z'|z)dF_O^{(k)}(z) + \frac{M_E}{M_O^{(k+1)}}F_E(z').$$

iii. If the errors in the operating mass update $|M_O^{(k+1)} - M_O^{(k)}|$ and distributional update $\max_z |F_O^{(k+1)}(z) - F_O^{(k)}(z)|$ are both less than ϵ^F, then the operating mass $M_O = M_O^{(k)}$ and operating distribution $F_O(z) = F_O^{(k)}(z)$ are computed. Otherwise, set $k = k + 1$ and return to step (1(b)i).

2. Compute the implied value to entry V_E via

$$V_E = \int V(z)dF_E(z).$$

3. Compute the implied labor demand N via

$$N = M_O\int n^*(z)dF_O(z),$$

where $n^*(z)$ is optimal static labor demand for an individual firm with profitability z.

4. If the error in the free entry condition $|V_E - \phi_E|$ and the error in the labor market clearing condition $|N - \bar{N}|$ are both less than the GE tolerance ϵ^{GE}, then the model is solved. Otherwise, update your guesses for the wage and entry mass and return to step (1).

Note that unlike in the empirical or nonparametric version of the model, the fixed cost distribution $G(\phi_F) = U(0, \bar{\phi}_F)$ is predetermined. Also note that the entry and transition distributions $F_E(z)$ and $F(z'|z)$ are parametric, following conventional lognormal processes converted to a uniform profitability grid as in Tauchen (1986). Just as in the nonparametric solution of the model, however, we continue to solve the model continuously, storing value functions via linear interpolation, computing integrals via Simpson quadrature, and evaluating entry, operating, and transition distributions using linear interpolation of the CDFs $F_E(z)$, $F_O(z)$, and $F(z'|z)$. In our baseline, we again employ $N_z = N_y = 101$ points for our interpolation procedures, and model solution takes around a minute or two on a 2017 iMac Pro in MATLAB without aggressive parallelization.

B.4 Calibrating the Model

There are multiple model parameters which we fix or calibrate externally before engaging in a moment-matching exercise, as outlined in Section 5.1. We set $\alpha = 2/3$ to generate a conventional labor share of 2/3, we set $\beta = 1/1.04$ to be consistent with a conventional 4% real interest rate and an annual solution of the model, and we set $\bar{N}$ to be equal to the aggregate employment rate (resulting in $\bar{N} = 0.5974$ in our baseline Spanish sample and comparable values for our other samples). We also set the exogenous exit hazard δ based on the observed exit rate of the largest firms in our empirical sample, resulting in $\delta = 3.9\%$ for our baseline Spanish sample and comparable values for our other samples. Each of the versions of our model, nonparametric and parametric, is solved holding these externally calibrated parameters fixed.

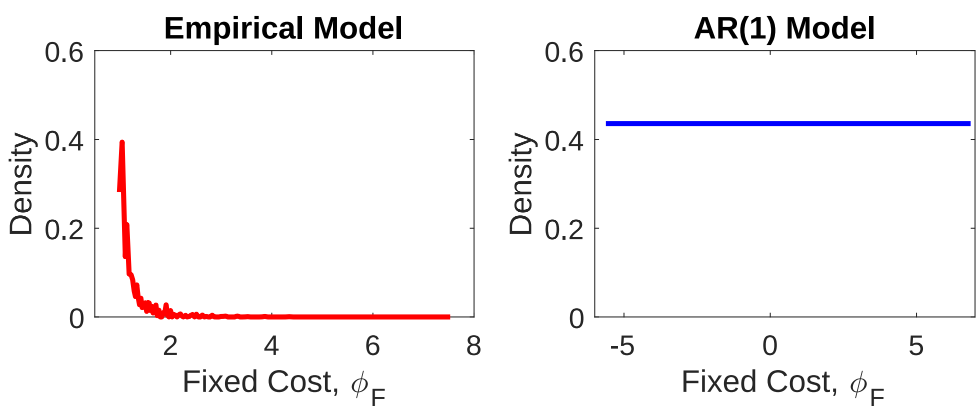
Nonparametric Calibration With the externally calibrated parameters listed above fixed, only the sunk entry cost ϕ_E must be calibrated for the nonparametric model. We choose the value of ϕ_E to match the observed average number of employees per firm. The number of employees per firm declines in the wage W, which adjusts to satisfy the free entry condition as the parameter ϕ_E is shifted.

Parametric Calibration With the externally calibrated parameters above fixed, we must still fix the values of the lognormal AR(1) profitability process (ρ, σ^2), the mean of the lognormal entry distribution μ_E, the upper bound $\bar{\phi}_F$ of the fixed cost distribution $G(\phi_F) = U(0, \bar{\phi}_F)$, as well as the sunk entry cost ϕ_E. Following convention in the parametric firm dynamics literature, we first set ρ to the autocorrelation of the profitability process $\log z$ inferred from our observed revenue series y, and we set σ^2 to match the observed variance of $\log z$.

Then, with ρ and σ^2 fixed, we choose the remaining three parameters $(\mu_E, \bar{\phi}_F, \phi_E)$ to jointly match three moments. As in the nonparametric model, we match (i) the observed average number of employees per firm. We also match (ii) the observed exit rate $\mathbb{P}(\text{Exit})$ which naturally moves with the fixed cost upper bound $\bar{\phi}_F$. Finally, we match (iii) the mean difference between log revenue for entering and operating firms, which naturally moves with the mean of the entry distribution μ_E. One might wonder why we did not target moments (ii) nor (iii) in our nonparametric model solution. But the nonparametric model matches both of these moments by construction, since both moments are implied by the combination of incumbent revenue transitions, exit hazards, and the entry distribution which are fully matched in the nonparametric model.

Fixed Cost Distributions The nonparametric and parametric calibration techniques yield fixed cost distributions $G(\phi_F)$, which we plot in Figure B.3. In the nonparametric case, our procedure yields a distribution with high density at low fixed cost realizations: this is required in order to match the strongly declining exit hazard found empirically and plotted in Figure B.2.

Figure B.3: Calibrated Fixed Cost Distributions



Notes: The left panel plots the density $g(\phi_F)$ of fixed costs recovered in our baseline nonparametric quantitative model analysis, while the right panel plots the same density for our baseline calibrated AR(1) quantitative model. The horizontal axis is the fixed cost shock ϕ_F, in logs, while the vertical axis is the density $g(\phi_F)$.

B.5 A Lumpy Investment Model

Consider a canonical partial equilibrium model of lumpy capital adjustment in the Cooper and Haltiwanger (2006) tradition with a firm-level dynamic problem described by

$$V(z,k) = \max_{k'} \left\{ zk^\alpha - k' + (1-\delta)k - AC(k,k') + \left(\frac{1}{1+r}\right) \mathbb{E}\left[V(z',k')|z\right] \right\}$$

$$AC(k',k) = \phi k \mathbb{I}(k' \neq (1-\delta)k)$$

$$\log z' = \rho \log z + \sigma \epsilon', \quad \epsilon' \sim N(0,1).$$

Above, $\log z$ is a parametric Gaussian AR(1) shock process at the firm level, k is firm-level capital, $y = zk^\alpha$ is a decreasing returns to scale production function with capital elasticity $\alpha \in (0,1)$, $r > 0$ is a constant exogenous real interest rate, and $AC(k',k)$ is an adjustment cost function featured a fixed cost of adjusting capital governed by the parameter $\phi > 0$. In Panel A of Table B.1 we lay out an illustrative calibration of this model, based on round values not unusual for annual calibrations of models of this class. Panel B summarizes the parameters such as grid densities and numerical tolerances for numerical solution and simulation of the model, which we carry out using standard discretization strategies, value function iteration, and Monte Carlo simulation. Panel C reports summary statistics for log revenue and revenue growth from the simulated model, both in our baseline calibration (with fixed adjustment costs and $\phi > 0$) as well as in an alternative no adjustment cost calibration with $\phi = 0$.

B.6 Robustness Checks

Section 7 overviews a large number of quantitative model robustness checks. For each check, we redo the calibration process summarized above for an alternative sample or model assumption, resulting in the recalibrated values list in Appendix Table B.2. The associated counterfactual implications of a subsidy to operating firms, in the nonparametric vs AR(1) cases, are available in Appendix Table B.3.

Table B.1: Lumpy Investment Model

Panel A: Lumpy Investment Model Parameters	Symbol	Value
Autocorrelation	ρ	0.9
Shock variance	σ	0.1
Capital elasticity	α	0.5
Interest rate	r	0.04
Depreciation rate	δ	0.1
Fixed adjustment cost	ϕ	0.025
Panel B: Solution Strategy		
Productivity grid points		61
Productivity grid range		$\pm 3.5\frac{\sigma^2}{1-\rho^2}$
Capital grid points		251
Capital grid range		[0.62,50.0]
Value function tolerance		1×10^{-5}
Distribution tolerance		1×10^{-6}
Simulated periods		100000
Burn-in periods		500
Panel C: Simulation Statistics	**Baseline**	**No AC ($\phi = 0$)**
Revenue standard deviation	0.4230	0.4247
Revenue skewness	-0.0279	-0.0082
Revenue kurtosis	2.8988	2.9475
Revenue growth standard deviation	0.1699	0.1348
Revenue growth skewness	1.1855	-0.0026
Revenue growth kurtosis	4.5588	3.0260

Notes: Panels A reports illustrative calibration parameter values for the baseline lumpy investment model. Panel B reports model solution and simulation parameters for the numerical solution of the model, which was carried out using value function iteration on a grid a productivity discretization procedure following Tauchen (1986). Panel C reports summary statistics based on simulated firm-level data from the baseline model (i.e., the model with calibrated parameters in Panel A) and a No AC model which is otherwise identically parameterized but imposes $\phi = 0$.

Table B.2: Alternative Model Calibrations

	Empirical Case	Parametric AR(1) Case				
	ϕ_E	ρ	σ	μ_E	$\bar{\phi}_F$	ϕ_E
Panel A: Alternative Model Assumptions						
Endogenous Labor Supply	22.9	0.94	0.19	-0.44	2.30	5.18
Higher $\alpha = 0.75$	16.8	0.94	0.14	-0.33	1.96	4.42
Lower $\alpha = 0.60$	28.4	0.94	0.23	-0.53	2.50	5.66
Panel B: Alternative Datasets						
Before 2009	16.7	0.93	0.19	-0.52	2.46	4.02
After 2009	25.1	0.95	0.19	-0.46	1.96	5.48
Manufacturing Only	25.2	0.98	0.13	-0.48	2.11	4.25
Non-Manufacturing Only	21.5	0.93	0.20	-0.44	2.25	4.99
Unconsolidated Accounts	16.9	0.94	0.19	-0.44	2.15	4.79
Excluding M&A	22.7	0.94	0.19	-0.36	2.55	4.97
Year Effects Only	14.3	0.96	0.19	-0.45	2.17	4.84
No Trimming	129.7	0.91	0.24	-0.33	3.12	7.65
Trimming at 1% and 99%	9.39	0.94	0.18	-0.41	1.88	4.00
Remove Firm Age	64.7	0.92	0.21	0.11	2.40	12.1
Italy	28.0	0.91	0.24	-0.65	3.13	5.08
Portugal	13.4	0.95	0.17	-0.84	1.57	2.18
France	22.7	0.96	0.15	-0.65	1.62	3.67
Norway	24.8	0.95	0.19	-0.85	1.23	3.33
Baseline	22.9	0.94	0.19	-0.43	2.30	5.18

Notes: This table reports calibrated parameters for each of our model robustness checks and alternative datasets, for both the empirical and parametric AR(1) model versions. Note that in the extension with endogenous labor supply, we also calibrate $\omega = 1.51$ (empirical case) and $\omega = 1.39$ (parametric case) to match the Spanish employment rate.

Table B.3: Relative Subsidy Impacts in Our Empirical vs AR(1) Models

	Exit Rate	Output
Panel A: Alternative Model Assumptions		
Endogenous Labor Supply	3.0660	0.8125
Higher $\alpha = 0.75$	3.0728	0.6872
Lower $\alpha = 0.60$	3.0765	0.6859
Panel B: Alternative Datasets		
Before 2009	2.5787	0.7095
After 2009	2.4338	0.6176
Manufacturing Only	3.6506	0.4211
Non-Manufacturing Only	2.7832	0.7108
Unconsolidated Accounts	3.1550	0.7109
Excluding M&A	3.0750	0.7161
Year Effects Only	2.8076	0.7015
No Trimming	3.3206	0.7883
Trimming at 1% and 99%	2.6755	0.7461
Remove Firm Age	2.9401	0.3934
Italy	3.9723	0.7040
Portugal	5.8445	0.7905
France	1.8992	0.5814
Norway	2.8302	0.6560
Baseline	3.0701	0.6845

Notes: This table reports relative changes at the aggregate level from a fixed cost subsidy equal to 5% of pre-subsidy output in our calibrated empirical nonparametric versus the parametric AR(1) model. Panel A reports results under various alternative model assumptions, while Panel B considers calibrations based on alternative ORBIS datasets. For each experiment indicated in the first column, we first calculate the change in the aggregate exit rate, in percentage points, and aggregate output, in percent, relative to the no-subsidy values for both the nonparametric and AR(1) models. We then report the ratio of the nonparametric to the AR(1) model's changes. The second column reports this ratio for the exit rate, while the third column reports this ratio for output.