Capital-Skill Complementarity in Firms and in the Aggregate Economy*

September 7, 2024

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Abstract

We study capital-skill complementarity in a multi-sector framework featuring firm-specific, multi-factor production functions and allowing for firm-specific factor-price wedges. We characterize the elasticity of the skill premium to the price of capital equipment in terms of firm-level elasticities of substitution across factors, elasticities of substitution across firms and sectors, and factor intensities. Using French data, we provide credible identification of these firm-level elasticities. Combining these elements we offer the first identification of aggregate capital-skill complementarity that allows for arbitrary trends in the unobservable skill-bias of productivity at the firm, industry, and aggregate levels. We find an economically and statistically significant degree of aggregate capital-skill complementarity, but this force alone is insufficient to generate the full increase in the relative demand for high-skilled workers observed in the data. There is a substantial role for skill-augmenting technical change not embodied in capital equipment.

*We thank Julieta Caunedo, Jonathan Dingel, Erik Hurst, Ezra Oberfield, Elena Pastorino, Pascual Restrepo, Felix Tintelnot, Giovanni Violante, and numerous seminar participants for helpful comments. The project has benefitted from funding from the TOPAZE project (ANR-18-CE26-0009). Access to French condential data was made possible thanks to the secure environment provided by CASD - Centre d'acces securise aux donnees (Ref. 10.34724/CASD).

1 Introduction

Modern technological advances are often embodied in the development of more efficient and cost-effective capital equipment and machinery. Following Griliches (1969), an extensive literature has proposed that such capital-embodied technical advances have been more complementary with high-skilled than low-skilled labor (*capital-skill complementarity*). According to this hypothesis, declining equipment prices have increased the relative demand for skilled labor and, therefore, the skill premium. This mechanism is conceptually attractive: the measurable stock of capital equipment proxies for latent skill-biased technical change. It has also proven successful in accounting for the evolution of the skill premium in the United States given particular aggregate elasticities between equipment and high- and low-skilled labor (Krusell et al., 2000).

Despite its virtues, the capital-skill complementarity hypothesis faces a central critique: the aggregate elasticities of substitution that determine the strength of this mechanism are difficult to identify in macro-level time series. In such data, it is challenging to differentiate capital-skill complementarity from other mechanisms that generate skill-biased technical change but are not embodied in capital equipment (Acemoglu, 2002). Hence, whether capital is complementary with skilled labor at the aggregate level and, if it is, the strength of this complementarity remain open questions.

In this paper, we tackle this issue using theory-guided aggregation, obtaining macro-level elasticities of substitution from credibly identified micro-level elasticities. We first develop a theory that characterizes the relevant aggregate elasticities of substitution that determine the degree of capital-skill complementarity in terms of firm-level elasticities of production and demand. Using data from the universe of French firms, we estimate these micro-level elasticities by leveraging exogenous variation that addresses standard identification concerns, both at the aggregate and at the firm level. We then apply our theoretical results to the French economy. We find a statistically and economically significant degree of capital-skill complementarity; but this force alone is insufficient to generate the full increase in the relative demand for high-skilled workers observed in the data.

In Section 2, we begin by constructing a multi-sector, general equilibrium model in which firms produce with firm-specific, differentiable, constant returns-to-scale production technologies using three factors (high-skilled labor, low-skilled labor, and capital equipment) in fixed aggregate supply. We assume monopolistic competition and a nested CES demand structure across firms and sectors. We show how the response of the real wage of each skill group to a small change in equipment prices depends on aggregate elasticities of substitution between different production factors. These elasticities account for the effect of changes in the price of a given factor on the aggregate demand for another factor. We characterize these aggregate elasticities in terms of firm-level elasticities of substitution across factors, factor intensities across firms, and firm-level demand elasticities. Aggregate capital-skill complementarity may emerge if capital and skill are complementary in firm-level production functions or if capital and skill intensities positively (negatively) covary across firms when the elasticity of demand is greater (smaller) than the elasticity of substitution between equipment and labor.

We generalize our baseline theoretical results along a number of dimensions. First, we extend our results to allow for exogenous *changes in the supplies of each skill group*, and also to endogenous changes caused by changes in equipment prices when labor supplies are elastic. Second, we extend our environment to allow for firm-specific wedges that create heterogeneity in factor prices across firms. In particular, we show how our baseline results generalize to a case with heterogeneous shocks to the price of equipment, introducing *heterogeneity-adjusted* aggregate elasticities of substitution that incorporate firm-specific exposures to the the aggregate equipment-price shock. Moreover, we show how our results generalize to cases in which we consider *composite factors,* e.g., a composite of labor that aggregates both skill types, and to cases involving *arbitrarily many factors* where the shock to equipment capital simultaneously shifts the price of multiple other factors.

To use our framework to assess the impact of falling equipment prices on the skill premium, we exploit a number of French administrative firm-level data sources described in Section 3, including firm-level balance sheet data, matched employer-employee data, and transaction-level import and export data. These data sources allow us to directly measure some of the key micro-level variables required for our theory-guided aggregation (including factor intensities across firms) and to estimate the rest (including the the elasticities of substitution in production and demand).

We estimate the key micro-level elasticities needed for our aggregation in Section 4. First, we estimate the two demand elasticities that characterize substitutability across sectors and across firms within sectors. We construct instruments for firm and sector-level price changes using shift-share cost shocks based on variation across firms in their initial allocation of imports across origin countries and movements in the real exchange rate between these countries and France.

Second, we estimate the elasticities of substitution across different production factors imposing two distinct functional form assumptions on firm-level technology: (1) the CRESH family of production functions (Hanoch, 1971), and (2) a nested CES production function proposed by Krusell et al. (2000). Like nested CES specifications, the CRESH family generalizes the CES production function by allowing for varying degrees of substitutability among factor inputs. Unlike nested CES specifications, the CRESH family neither requires ex ante assumptions about the nesting of factors nor imposes a strong upper limit on the degree of capital-skill complementarity.[1]

Under CRESH firm-level production functions, we estimate the three factor-specific elasticity parameters that characterize factor demand using changes in observed firm-level factor demand, revenue, and factor prices. We construct instruments for firm-level equipment prices using shift-share shocks based on variation across firms in the initial allocation of their equipment investments across products and origin countries and the movements in transport costs between these countries and France, following Hummels et al. (2014). We predict the latter movements based on observed changes in the price of oil and jet fuel, the mode of transport for each product, and the distance between origin countries and France. We add to these instruments additional shift-share instruments for firm-level revenue (based on global demand shocks to firm suppliers) and local wages for each skill group (based on shift-share national demand shocks to local industries).[2] We also rely on the latter set of instruments in the identification of the nested CES specification. Under both specifications, we identify a modest, but statistically significant degree of capital-skill complementarity at the firm level.

Armed with the estimated elasticities and the observed distribution of factor intensities, we perform our main aggregation exercise in Section 5. Over the 1997-2007 period, we compute a measure of the aggregate degree of capital-skill complementarity—defined as the difference between the aggregate elasticity of substitution between low-skilled labor and equipment and between high-skilled labor and equipment—as well as the aggregate elasticity of the skill premium with respect to the equipment price. These elasticities are substantially smaller than previous estimates obtained from variation in the aggregate time series. For example, our measure of the aggregate degree of capital-skill complementarity is an order of magnitude smaller than in Krusell et al. (2000). This is because our strategy does not attribute the aggregate increase in the relative demand for skill *entirely* to capital-skill complementarity.[3] We find that a 1% decline in the price of equipment

[1]The latter property implies that a decline in equipment prices can reduce the real wage of low-skilled labor when the aggregate production function is in the CRESH family and the degree of capital-skill complementarity is sufficiently strong (an outcome that cannot occur when the aggregate production function is nested CES).

[2]Our theoretical extensions incorporate firm-level heterogeneity in factor prices, both for equipment and for the skill groups.

[3]This difference in the strength of capital-skill complementarity is not driven by our focus on France: we obtain similar elasticities of the skill premium with respect to the price of capital equipment in France and the US using the approach in Krusell et al. (2000). Nor is it driven by our use of the CRESH family of production functions: we obtain similar results estimating our model using nested CES firm-level production functions.

generates a 0.06% increase in the observed skill premium in France.

Finally, we feed into our full model the observed annual changes in equipment prices and the observed changes in labor supplies over the period 1997-2007. The observed increase in the relative supply of skilled labor decreases the skill premium over this time period. The decline in equipment prices counteracts part of this, but not enough to match the observed evolution of the skill premium. Skill-biased technical change not embodied in capital equipment—which we explicitly incorporate in the model—is then responsible for the gap between the observed small rise in the skill premium and the reduction in the skill premium caused by the evolutions of labor supply and the equipment price.[4]

We view our central contributions in three parts. First, we provide a general theory—with many factors, firm-specific production functions, firm-specific factor-price wedges, etc.—to characterize the aggregate elasticity of the skill premium with respect to the price of capital equipment in terms of firm-level factor intensities and elasticities of production and demand. Second, we provide credible identification of these firm-level elasticities under various functional form assumptions. Third, combining these elements we offer the first identification of aggregate capital-skill complementarity that allows for arbitrary trends in the unobservable skill-bias of productivity at the firm, industry, and aggregate levels.

Literature. Our paper contributes to a large literature in labor, international trade, and macroeconomics that studies the implications of technological change on inequality across skill groups. Following Katz and Murphy (1992), many earlier studies measure latent skill-biased technical change as a residual to match the observed evolution of the skill premium given the observed evolution of the aggregate relative supply of skill (see Acemoglu, 2002; Card and DiNardo, 2002, for reviews).[5]

Our approach builds more directly on studies of capital-embodied technical change, which focus on changes in the quality-adjusted price of capital equipment as a well-defined empirical proxy for the technological shock affecting the labor market (Greenwood and Yorukoglu, 1997; Hornstein et al., 2005). This strand of the literature in turn builds on earlier work by Griliches (1969, 1970), who first hypothesized capital-skill complementarity. In a seminal paper, Krusell et al. (2000) use this idea to provide an account

[4]We show that our quantitative results are largely robust to assuming and estimating nested CES firm-level production functions, incorporating heterogeneous equipment price changes across firms, and generalizing our demand system.

[5]A more recent approach in the literature emphasizes the importance of studying the endogenous assignment of production tasks to different skill groups for understanding the effect of technology on inequality (Costinot and Vogel, 2010; Acemoglu and Autor, 2011).

of the rise of the skill premium in the U.S. by estimating a four-factor aggregate production function representation of the U.S. economy using aggregate time-series variation.[6]

A large empirical literature studies the impact of capital equipment on skill intensity at the level of sectors, occupations, or firms, finding strong evidence for the capital-skill complementarity hypothesis at these levels of disaggregation; see, e.g., Autor et al. (1998), Caroli and Van Reenen (2001), Bresnahan et al. (2002), Autor et al. (2003), Bartel et al. (2007), Beaudry et al. (2010), Lewis (2011), Akerman et al. (2015), and Gaggl and Wright (2017).[7] Our contribution relative to this literature is aggregating from micro/firm-level estimates to quantify macro implications.

By providing a framework for explicit aggregation of micro-level firm responses, we build on recent work by Oberfield and Raval (2021), Lashkari et al. (2022), and Baqaee and Farhi (2019). The first two consider two-factor models—thereby abstracting from capital-skill complementarity—with firm heterogeneity under constant returns-to-scale and variable returns-to-scale technologies, respectively. The latter provides a multi-factor model abstracting from firm heterogeneity.[8] In this spirit, our paper is also closely related to Burstein et al. (2019) and Acemoglu and Restrepo (2022), which quantify the impact of computerization and automation, respectively, on between-group inequality. Relative to these papers, we incorporate firm heterogeneity in our theory and firm-level data in our empirics, and use these to estimate elasticities of substitution at the micro level rather than assuming their values.[9]

2 Theory

We begin this section by setting up the baseline environment in Section 2.1. Next, we characterize the response of the skill premium to changes in the price of capital equipment in Section 2.2. Finally, we discuss a number of extensions of these baseline results in Section 2.3.

[6]Duffy et al. (2004) instead uses a cross-country panel estimation approach. Unlike these papers, Taniguchi and Yamada (2022) and Caunedo et al. (2023) allow for potential trends in aggregate skill-biased technical change not embodied in capital equipment, with the first leveraging an indusry-and-country panel and the second leveraging an occupation panel.

[7]On the other hand, Aghion et al. (2022) find that equipment is neutral and Curtis et al. (2021) find that it is complementary with low-skilled labor.

[8]Unlike these papers, we additionally allow for deviations from perfect factor markets.

[9]The elasticity of substitution at the occupation/task level between labor and computers and between labor and machines is assumed to be one in Burstein et al. (2019) and infinity in Acemoglu and Restrepo (2022), respectively. We note that the relationship between micro-level and macro-level substitution is also widely studied in the context of the effects of trade liberalization on the skill premium (e.g., Parro, 2013; Burstein et al., 2013; Burstein and Vogel, 2017).

2.1 Environment

Firm Production and Factor Demand. There is a large set $\mathcal{I}$ of firms indexed by $i \in \mathcal{I}$. Firm i produces with a firm-specific CRS production function $Y_i = G_i(\boldsymbol{X}_i)$, where $\boldsymbol{X}_i \equiv (X_{fi})_{f\in\mathcal{F}}$ indicates the vector of factors of production $f \in \mathcal{F}$ deployed by this firm. In our baseline analysis, we focus on three factors: low-skilled labor ($f = \ell$), high-skilled labor ($f = h$), and capital equipment ($f = e$).

In our benchmark setting, we assume that factor markets are perfectly competitive and all firms face uniform factor prices. We let $\boldsymbol{W} \equiv (W_f)_{f\in\mathcal{F}}$ denote the vector of factor prices, normalizing the wage of low-skilled labor to unity, $W_\ell \equiv 1$. Given factor prices, firm-level cost minimization leads to the vector of factor demand $\boldsymbol{X}_i$ as a function of the vector of factor prices $\boldsymbol{W}$ and output Y_i. We characterize firm-level production technologies in terms of a set of elasticities of factor demand. First, let θ_{fi} denote the factor f intensity of firm i (i.e., the factor's expenditure share in firm costs), defined as

$$\theta_{fi} \equiv \frac{W_f X_{fi}}{\sum_{f'} W_{f'} X_{f'i}}. \tag{1}$$

Next, we define the (Allen-Uzawa) elasticity of substitution between factor f and f' (with respect to the price of factor f') for firm i as[10]

$$\sigma_{ff',i} \equiv \frac{1}{\theta_{f'i}} \frac{\partial \log X_{fi}}{\partial \log W_{f'}}, \qquad f \neq f', \tag{2}$$

at fixed firm-level output, Y_i. These cross-factor elasticities of substitution are symmetric ($\sigma_{ff',i} = \sigma_{f'f,i}$) and are, along with factor intensities, sufficient to fully characterize firm-level responses to small changes in factor prices. For instance, note that we can write the own price elasticity of factor f as a weighted sum of the Allen-Uzawa elasticities of substitution as

$$\frac{\partial \log X_{fi}}{\partial \log W_f} = -\sum_{f'\neq f} \theta_{f'i}\sigma_{ff',i}. \tag{3}$$

Product Demand. Firms compete across a number of sectors, and we denote by $\mathcal{I}_s \subseteq \mathcal{I}$ the (large) set of firms in sector s. The product market is monopolistically competitive within each sector, and we assume nested CES preferences across sectors. Demand for

[10]The Allen-Uzawa elasticities $\sigma_{ff'}$ can be contrasted from the alternative Morishima elasticities of substitution $\sigma^M_{ff'}$ (Blackorby and Russell, 1989). Appendix A.1 presents the definition of the Morishima elasticities and contrasts them with the Allen-Uzawa definition used throughout the paper.

firm $i \in \mathcal{I}_s$ is given by

$$Y_i = \Phi_i \left(\frac{P_i}{P_s}\right)^{-\varepsilon} \left(\frac{P_s}{P}\right)^{-\eta} Y \tag{4}$$

where ε is the elasticity of substitution across firms within each sector, η is the elasticity of substitution across sectors, Φ_i is a firm-specific demand shifter, P_s is the price index within sector s, and P is the aggregate price index. With slight abuse of notation, we also use P_i to denote the price that firm i charges for its output Y_i. We use Y to denote aggregate production.

Aggregate Factor Supply. We let $\overline{\boldsymbol{X}} \equiv \left(\overline{X}_f\right)_{f\in\mathcal{F}}$ denote the vector of aggregate factor supply, which in general is a function of the vector of factor prices $\boldsymbol{W}$.[11] In our baseline analysis, we consider a case where factor supplies are inelastic and exogenously given. However, we can more generally characterize these functions in terms of the elasticities of substitution between different factors in the aggregate factor supply.

Aggregate Production and Factor Demand. Given our assumptions about product and factor markets, the allocation of inputs across firms is efficient and we can aggregate the production side of our economy.[12] Let $Y = G\left(\boldsymbol{X}\right)$ denote the resulting aggregate production function, where $\boldsymbol{X} \equiv \sum_{i\in\mathcal{I}} \boldsymbol{X}_i$ denotes the vector of aggregate factor demand across all firms. This definition allows us to define aggregate factor demand $\boldsymbol{X}$ as a function of factor prices $\boldsymbol{W}$, and characterize it using production-function elasticities that parallel those in the case of firm-level factor demand. Correspondingly, we let θ_f denote the aggregate cost share of factor f (factor intensity), that is,

$$\theta_f \equiv \frac{W_f X_f}{\sum_{f'} W_{f'} X_{f'}}, \tag{5}$$

Next, we define the aggregate elasticity of substitution between factor f and f' (with respect to the price of factor f') as

$$\sigma_{ff'} \equiv \frac{1}{\theta_{f'}} \frac{\partial \log X_f}{\partial \log W_{f'}}, \qquad f \neq f', \tag{6}$$

[11] We assume this function is homogenous of degree zero. Note, however, that the assumption that this function only depends on factor prices imposes some restriction on the structure of factor supplies, ruling out, e.g., potential nonhomotheticies in factor supplies.

[12] Note that the assumption of monopolistic competition and the demand system in Equation (4) together imply that firms charge constant markups $\frac{\varepsilon}{\varepsilon-1}$ over their marginal costs. As a result, marginal products are equalized across firms.

at fixed aggregate output Y, allowing firm-level outputs to adjust subject to this constraint. We can find the own price elasticity of each factor following an expression similar to that in Equation (3).

2.2 The Skill Premium and Falling Equipment Prices

We now use the model set up in Section 2.1 to examine the implications of a shock to the price of one factor of production (equipment) on the relative price of two other factors (the skill premium) in general equilibrium. More specifically, we consider a rise in the aggregate supply of equipment $\overline{X}_e$, keeping all other factor supplies fixed. This shock leads to a corresponding fall in the price of equipment. Henceforth, we study the implications of this change in equipment prices, when measured relative to the low-skilled wage (numeraire), for other factor prices.

2.2.1 Response of the Skill Premium

According to the definition in Equation 6, the direct response of the demand for high-skilled relative to low-skilled labor (henceforth, the relative demand for skill) to a change in equipment prices is given by

$$\frac{\partial \log \left(X_h / X_\ell\right)}{\partial \log W_e} = -\theta_e \left(\sigma_{\ell e} - \sigma_{he}\right). \tag{7}$$

A decline in the price of equipment increases the relative demand for skill (capital-skill complementarity) if and only if the elasticity of substitution between low-skilled labor and equipment is higher than the elasticity of substitution between high-skilled labor and equipment, that is, $\sigma_{\ell e} - \sigma_{he} > 0$.

Next, we study the general equilibrium response of the skill premium, defined as the relative wage of high-skilled workers $W_h / W_\ell \equiv W_h$, to this shock in the price of equipment. Assuming inelastic labor supplies, wages must respond to equate labor demands with fixed labor supplies. The following proposition characterizes this response.

Proposition 1. *Consider a setting with three-factors, $\mathcal{F} \equiv \{\ell, h, e\}$, and with exogenous factor supplies. Assume a small shock to the equipment supply $\overline{X}_e$, holding the supplies of other factors fixed, that leads to a change $d \log W_e$ in the price of equipment. The effect on the skill premium, i.e., the relative prices of high-skilled and low-skilled labor, satisfies*[13]

[13]As shown in Appendix A.1, when expressed in terms of the aggregate Morishima elasticicities of substi-

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e} = -\frac{\theta_e\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}, \tag{8}$$

where aggregate factor intensities θ_f and aggregate elasticities of substitution $\sigma_{ff'}$ are defined by Equations (5) and (6).

Proof. Since factor supplies and factor demands equalize in equilibrium, $\overline{\boldsymbol{X}} = \boldsymbol{X}$, and since we have assumed labor supplies remain unchanged, we have

$$0 = \frac{d\log\left(\overline{X}_h/\overline{X}_\ell\right)}{d\log W_e} = \frac{d\log\left(X_h/X_\ell\right)}{d\log W_e} = \frac{\partial\log X_h/X_\ell}{\partial\log W_e} + \left(\frac{\partial\log X_h/X_\ell}{\partial\log W_h}\right)\frac{d\log W_h}{d\log W_e}. \tag{9}$$

Substituting Equation (7) in the first term on the right-hand side, and using Equations (3) and (6) to write $\frac{\partial\log X_\ell/X_h}{\partial\log W_h} = \theta_\ell\sigma_{\ell h}+\theta_e\sigma_{eh}+\theta_h\sigma_{\ell h}$ leads to the desired result. □

When labor is supplied inelastically, Lemma 1 shows that the response of the skill premium depends on two key objects. First, the response depends on the degree of capital-skill complementarity, $\sigma_{\ell e}-\sigma_{he}$. The higher is the gap between these two aggregate substitution elasticities, the more responsive is the relative demand for skill (in the aggregate) to changes in equipment prices.

Second, the response also depends on a weighted average of the aggregate elasticity of substitution between high- and low-skilled labor, $\sigma_{h\ell}$, and between high-skilled labor and equipment, σ_{he}. When falling equipment prices encourage a rise in skill demand due to capital-skill complementarity, the skill premium has to increase to re-establish labor market clearing. When the two labor types are more substitutable (high $\sigma_{h\ell}$), equilibrating the labor market requires a smaller increase in the skill premium.

More generally, we can also derive the implications of falling equipment prices for the *real* wages of the two skill groups. Since markups are constant and the low-skilled wage is the numeraire, we find[14]

$$\frac{d\log\left(W_h/P\right)}{d\log W_e} = -\theta_e\left(1+\frac{\left(1-\theta_h\right)\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}\right),$$

tution, the response can be expressed equivalently as

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e} = -\frac{\sigma^M_{\ell e}-\sigma^M_{he}}{\sigma^M_{\ell h}}.$$

[14] We have $\frac{d\log P}{d\log W_e} = \frac{d\log C}{d\log W_e} = \frac{\partial\log C}{\partial\log W_e} + \frac{\partial\log C}{\partial\log W_h}\frac{d\log W_h}{d\log W_e}$, where C stands for the unit cost of production in the aggregate. Shephard's lemma implies that $\frac{\partial\log C}{\partial\log W_f} = \theta_f$.

$$\frac{d\log\left(W_\ell/P\right)}{d\log W_e}=-\theta_e\left(1-\frac{\theta_h\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}\right) \tag{10}$$

If equipment is equally complementary with high- and low-skilled labor, $\sigma_{\ell e}=\sigma_{he}$, then a reduction in the price of equipment raises the real wage of high- and low-skilled labor proportionately. On the other hand, in the presence of capital-skill complementarity, $\sigma_{\ell e}>\sigma_{he}$, a reduction in the equipment price necessarily raises the real wage of high-skilled labor, but may either raise or lower the real wage of low-skilled labor. Specifically, low-skilled labor experiences a loss in real terms if the degree of capital-skill complementarity is sufficiently large, in the sense that[15]

$$\sigma_{\ell e}-\sigma_{he}>\frac{1}{\theta_h}\left[\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}\right]. \tag{11}$$

Thus, in our general setup, technological progress embodied in capital equipment, in the form of technologies reducing the price of equipment, may reduce the real wage of low-skilled labor, a point to which we return below. This result stands in contrast to those derived in the two-factor CES aggregate production function (referred to as the canonical model), critiqued by Acemoglu and Autor (2011), and the three-factor, nested-CES model of capital-skill complementarity, critiqued by Acemoglu and Restrepo (2022).[16]

In Section 2.3 we generalize Proposition 1 along multiple dimensions by accounting for elastic labor supplies, additional factors of production, and potential heterogeneity in the price of equipment. However, the core insight of Proposition 1 remains in all these extensions: the degree of capital-skill complementarity and the degree of substitutability of the two skill groups play key roles in shaping the response of the skill premium to falling equipment prices.

Examples of Aggregate Technology without Firm Heterogeneity. To further unpack the results of Proposition 1, we consider two specific functional forms for the aggregate production function. For now, we assume that the economy comprises a single sector with a unit mass of identical firms, implying that the aggregate production function coincides with the firm production function $G\left(\boldsymbol{X}\right)\equiv G_i\left(\boldsymbol{X}\right)$.

Example 1 (Nested CES Production Function á la Krusell et al., 2000)**.** Consider a produc-

[15] See the derivation in Appendix A.4.1 on page A15.

[16] Symmetric results apply if equipment is relatively more complementary with low-skilled than high-skilled labor, in which case a reduction in the price of equipment raises the real wage of low-skilled labor but may either increase or decrease the real wage of high-skilled labor.

tion function $Y = G(\boldsymbol{X})$ characterized by the following nested structure

$$Y = \left((Z_\ell X_\ell)^{\frac{\varsigma-1}{\varsigma}} + X_c^{\frac{\varsigma-1}{\varsigma}} \right)^{\frac{\varsigma}{\varsigma-1}}, \qquad X_c \equiv \left((Z_h X_h)^{\frac{\rho-1}{\rho}} + (Z_e X_e)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{12}$$

where X_c is a composite factor that combines high-skilled labor and equipment capital.

As shown in Appendix A.2.1, the functional form in Example 1, which we will henceforth refer to interchangeably as KORV, implies the following elasticities of substitution

$$\sigma_{he} = \varsigma - (\varsigma - \rho) \frac{1}{\theta_e + \theta_h}, \qquad \sigma_{\ell e} = \sigma_{\ell h} = \varsigma. \tag{13}$$

Choosing $\varsigma = \rho$, we recover the standard CES production function, which implies $\sigma_{\ell e} = \sigma_{he}$ and does not allow for capital-skill complementarity. More generally, the degree of capital-skill complementarity is given by $\sigma_{\ell e} - \sigma_{he} = (\varsigma - \rho) / (\theta_e + \theta_h)$, and the production function exhibits capital-skill complementarity if and only if $\varsigma > \rho$.

By construction, the nested CES specification of Example 1 imposes an important restriction on the degree of capital-skill complementarity. In particular, since the KORV specification imposes the restriction $\sigma_{\ell e} = \sigma_{\ell h}$, we can express the response of the real wage of low–skill workers in Equation (10) as[17]

$$\frac{d \log (W_\ell / P)}{d \log W_e} = -\theta_e \frac{(\theta_e + \theta_h) \rho}{\theta_h \varsigma + \theta_e \rho} < 0. \tag{14}$$

The nested CES specification implies that a falling equipment price necessarily increases the real wage of low-skilled workers. Since there is evidence that real wages of low-education workers in the US fell throughout the 1980s—as documented in, e.g., Acemoglu and Autor (2011)—it may be desirable to choose a specification that does not impose such a restriction on the degree of capital-skill complementarity.[18]

Example 2 (CRESH Production Function á la Hanoch, 1971)**.** Consider a production function $Y = G(\boldsymbol{X})$ implicitly defined through the following constraint on unit factor require-

[17] See the derivation in Appendix A.4.1 on page A15.

[18] Another critique of the nested CES specification is the arbitrary choice of the nesting structure, which affects estimation and can affect counterfactual implications; see, e.g., Baum-Snow et al. (2018). Our preferred specification does not suffer from this issue.

ments:[19]

$$\sum_{f\in\{\ell,h,e\}}\left(\frac{Z_f X_f}{Y}\right)^{\frac{\sigma_f-1}{\sigma_f}}=1, \tag{15}$$

where $Z_f \geq 0$ and $\sigma_f > 0$ are factor-specific parameters shaping productivity and elasticities of substitution.

The production function defined in Equation (15) is a homothetic generalization of the standard CES production function allowing for different degrees of substitutability among different factor inputs. The CES production function is nested in this specification for the case of constant parameters $\sigma_f \equiv \sigma$ across all factors f. As shown in Appendix A.2.2, the functional form in Example 2 implies that elasticities of substitution are given by

$$\sigma_{ff'}=\frac{\sigma_f\sigma_{f'}}{\overline{\sigma}}, \tag{16}$$

where we have defined $\overline{\sigma} \equiv \sum_f \theta_f \sigma_f$ as the cost-share weighted average of σ_f across factors. Thus, we find that the degree of capital-skill complementarity is given by $\sigma_{\ell e} - \sigma_{he} = \sigma_e\left(\sigma_\ell - \sigma_h\right)/\overline{\sigma}$ and production exhibits capital-skill complementarity if and only if $\sigma_\ell > \sigma_h$. Parameters σ_h and σ_ℓ characterize the relative substitutability of high- and low-skilled workers with equipment.

Unlike the nested CES specification, the degree of capital-skill complementarity here can be sufficiently large as to allow falling equipment prices to reduce the real wage of low-skilled labor. Under the specification in Example 2, Equation (11) simplifies to[20]

$$\frac{\sigma_\ell}{\sigma_h} > 1+\frac{\theta_e}{\theta_h}+\left(\frac{1-\theta_e}{\theta_h}\right)\frac{\sigma_\ell}{\sigma_e}. \tag{17}$$

In principle, we could impose a functional form such as those in Example 1 or 2 on the aggregate production technology and estimate the resulting elasticity parameters to infer the degree of capital-skill complementarity.[21] However, the presence of factor-augmenting productivity change, and its potential correlation with relative factor prices, makes it challenging to estimate the elasticity parameters credibly using aggregate time-series data. Moreover, imposing simple functional forms on aggregate production abstracts away from well-known evidence on firm-level heterogeneity in technology, e.g., in terms of factor intensities, that may shape the properties of aggregate technology. This

[19] Hanoch (1971) introduced this class of production functions and referred to them as CRESH (constant ratios of elasticities of substitution with homotheticity).

[20] See Appendix A.2.2 on page A4 for the derivation.

[21] In Appendix A.2, we derive the demand systems characterizing factor demand for the nested CES and the CRESH production functions.

abstraction is not without a cost, as the extent of capital-skill complementarity may vary over time and space with these firm-level details.

Our strategy is to focus on firm heterogeneity and characterize (and estimate) production technologies at the firm level. To this end, we use the demand system in Equation (4) to aggregate firm-level production technologies to derive the properties of production at the macro level. More specifically, in what follows we solve for the aggregate elasticities of substitution between factors defined and used above as functions of firm-level variables.

2.2.2 From Firm to Aggregate Production

Let Λ_i denote firm i's share of aggregate factor payments (costs), defined as

$$\Lambda_i \equiv \frac{Y_i C_i}{YC} = \frac{\sum_f W_f X_{fi}}{\sum_f W_f X_f}, \tag{18}$$

and, with a slight abuse of notation, let $\Lambda_s \equiv \sum_{i \in \mathcal{I}_s} \Lambda_i$ denote the corresponding share across all firms in sector s. The following proposition characterizes the elasticities of aggregate technology using these definitions.

Proposition 2. *The aggregate elasticity of substitution $\sigma_{ff'}$ of aggregate factor demand defined by Equation (6) is given by*

$$\sigma_{ff'} = \mathbb{E}_i \left[\frac{\theta_{fi}}{\theta_f} \frac{\theta_{f'i}}{\theta_{f'}} \sigma_{ff',i} \right] - \varepsilon \, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\frac{\theta_{fi}}{\theta_f}, \frac{\theta_{f'i}}{\theta_{f'}} \right) \right] - \eta \, \mathbb{C}_s \left(\frac{\theta_{fs}}{\theta_f}, \frac{\theta_{f's}}{\theta_{f'}} \right) \tag{19}$$

where θ_{fi} and $\sigma_{ff',i}$ are the firm-level factor intensities and production elasticities defined in Equations (1) and (2), the firm-level expectation operator $\mathbb{E}_i[\cdot]$ is defined with respect to the share of firms in aggregate costs (factor payments) Λ_i defined by Equation (18), the within-sector firm-level covariance operator $\mathbb{C}_{i|s}(\cdot,\cdot)$ is defined with respect to the share of firms in their respective sectors $\Lambda_{i|s} \equiv \Lambda_i / \Lambda_s$, and the sector-level expectation and covariance operators $\mathbb{E}_s[\cdot]$ and $\mathbb{C}_s(\cdot,\cdot)$ are defined with respect to the share of sectors in aggregate costs Λ_s.

Proof. See Appendix A.4 on page A13. □

Equation (19) expresses the aggregate elasticity of substitution as a function of firm-level elasticities of substitution and factor intensities, elasticities of demand, and the distribution of costs across firms. The first term on the right-hand side of Equation (19) is a weighted average across firms of firm-level elasticities of substitution across factors scaled by factor intensities. The second term accounts for the effect of cross-firm reallocation within sectors on the aggregate elasticity of substitution. If factor intensities for the two factors f and f' positively covary across firms within each sector, a rise in the price of either of the two factors lowers the aggregate demand for the other factor by shifting production away from firms intensive in both factors. The magnitude of this effect is governed by the demand-side, within-sector elasticity of substitution ε. The last term on the right-hand side of Equation (19) captures a similar force that operates across sectors, governed in magnitude by the demand-side, cross-sector elasticity of substitution η.

From Micro to Macro Capital-Skill Complementarity. We can use Equation (19) to compute the three aggregate elasticities $\sigma_{\ell e}$, σ_{he}, and $\sigma_{\ell h}$, featured in the results of Proposition 1. In particular, for the degree of capital-skill complementarity we find[22]

$$\begin{aligned}\sigma_{\ell e}-\sigma_{he} = {} & \overbrace{\mathbb{E}_i\left[\left(\frac{1}{2}\frac{\theta_{\ell i}}{\theta_\ell}+\frac{1}{2}\frac{\theta_{hi}}{\theta_h}\right)\frac{\theta_{ei}}{\theta_e}\left(\sigma_{\ell e,i}-\sigma_{he,i}\right)\right]}^{\text{within-firm capital-skill complementarity}} - \overbrace{\mathbb{E}_i\left[\left(\frac{\theta_{hi}}{\theta_h}-\frac{\theta_{\ell i}}{\theta_\ell}\right)\frac{\theta_{ei}}{\theta_e}\left(\frac{1}{2}\sigma_{\ell e,i}+\frac{1}{2}\sigma_{he,i}\right)\right]}^{\text{within-firm equipment-labor substitution}} \\ & + \underbrace{\varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\frac{\theta_{hi}}{\theta_h}-\frac{\theta_{\ell i}}{\theta_\ell},\frac{\theta_{ei}}{\theta_e}\right)\right]}_{\text{cross-firm (within-sector) reallocation}} + \underbrace{\eta\,\mathbb{C}_s\left(\frac{\theta_{hs}}{\theta_h}-\frac{\theta_{\ell s}}{\theta_\ell},\frac{\theta_{es}}{\theta_e}\right)}_{\text{cross-sector reallocation}}. \end{aligned} \tag{20}$$

As with the decomposition of each aggregate elasticity of substitution, the first term on the right-hand side of Equation (20) accounts for the contribution of within-firm capital-skill complementarity on the aggregate. The second term on the right-hand side accounts for the contribution of reallocation between high- and low-skilled labor *at fixed output for each firm*. When equipment prices fall, those firms that are more equipment-intensive more strongly substitute labor for equipment capital. If these firms are more intensive in skilled compared to unskilled labor, the aggregate demand for skill falls due to the contribution of this term. In contrast, the third and fourth terms account for the contributions of *output reallocation across firms* within sectors and across sectors, respectively. When more equipment-intensive firms are also more intensive in skilled compared to unskilled labor, the aggregate demand for skill rises in response to falling equipment prices due to these latter components.

Below, we study the expression in Equation (20) under a different potential choices for firm-level technology.

Example 3 (No Firm-Level Capital-Skill Complementarity)**.** Consider a firm-level CES production function for all three inputs with elasticity of substitution σ. The elasticities

[22] See the derivation in Appendix A.4.1 on page A15.

of firm-level technology are given by $\sigma_{ff',i} \equiv \sigma$. In this case, there is no capital-skill complementarity at the firm level: the first term in Equation (20) is zero. Assuming a single-sector model (equivalently, $\eta = \varepsilon$), Equation (20) simplifies further to

$$\sigma_{\ell e} - \sigma_{he} = (\varepsilon - \sigma)\,\mathbb{C}_i\left(\frac{\theta_{hi}}{\theta_h} - \frac{\theta_{\ell i}}{\theta_\ell}, \frac{\theta_{ei}}{\theta_e}\right).$$

When the elasticity of substitution across products exceeds that across inputs, $\varepsilon > \sigma$, and when equipment intensity covaries positively with the relative demand for skill across firms, we find capital-skill complementarity in the aggregate even though firm-level production does not feature such complementarity. This result generalizes the logic of Houthakker (1955) and Oberfield and Raval (2021) from a two-factor to a multi-factor setting.

Example 4 (Nested CES/CRESH Firm-Level Production Function)**.** Consider firm-level production functions characterized by $Y_i = G_i(\boldsymbol{X}_i)$ with the same nested CES and CRESH production functions defined in Examples 1 and 2, where firm heterogeneity is expressed in terms of the vector of firm-specific factor augmenting productivities $\boldsymbol{Z}_i \equiv (Z_{fi})_{f\in\{\ell,h,e\}}$. In the case of the nested CES specification, the elasticities of production technology are given by $\sigma_{he,i} = \varsigma - (\varsigma - \rho)/(\theta_{ei} + \theta_{hi}) + \varsigma\theta_{ei}/(\theta_{ei} + \theta_{hi})$, and $\sigma_{\ell e,i} = \sigma_{h\ell,i} = \varsigma$. In the case of the CRESH specification, the elasticities are given $\sigma_{ff',i} = \sigma_{f'}\sigma_f/\overline{\sigma}_i$, where we have defined $\overline{\sigma}_i \equiv \sum_f \theta_{fi}\sigma_f$. In each case, we substitute the corresponding elasticities in Equation (20) to find the degree of aggregate capital-skill complementarity.

2.3 Theoretical Extensions

In this section, we discuss a number of theoretical extensions to our benchmark setup in Section 2.1. We also provide an additional result in our benchmark setup.

2.3.1 Shifts in the Relative Labor Supply

So far, we studied only the impact of changes in equipment prices on the skill premium. As is well-known (Katz and Murphy, 1992; Goldin and Katz, 2007), the relative supply of skill in the U.S. and across most other economies has been increasing for several decades. Here, we additionally account for the contribution of such changes in the supply of skill on the skill premium. Following the proof of Proposition 1, it is straightforward to derive the response of the skill premium to small changes both in the price of equipment and the relative supply of skill as

$$d\log\left(\frac{W_h}{W_\ell}\right)=-\frac{\theta_e\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}d\log W_e-\frac{1}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}d\log\left(\frac{\overline{X}_h}{\overline{X}_\ell}\right). \quad (21)$$

The first term on the right-hand side captures the elasticity of the skill premium with respect to the price of equipment at fixed supplies of labor, given already in Equation (8), and the second term captures the reciprocal of the elasticity of relative skill demand with respect to the skill premium.

The previous expression characterizes the impact on the skill premium of arbitrary changes in the price of equipment and the relative supply of skill. As another extension, in Appendix A.3.1, we also consider an environment in which the relative supply of skill responds endogenously to changes in the skill premium, with an elasticity $\xi > 0$. In this case, changes in the price of equipment impact the skill premium both directly, through the first term in Equation (21)—as in our baseline—but also indirectly through the second term in Equation (21). Our key result holds in the sense that Equation (7) still characterizes the extent of capital-skill complementarity and is the key determinant of the response of the skill premium to changes in the price of equipment capital, which is given by

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e}=-\frac{\theta_e\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}+\xi}, \quad (22)$$

as shown in Proposition A.1 in Appendix A.3.1. The previous expression approaches Equation (8) as the elasticity of skill supply approaches zero, $\xi \to 0$, and approaches zero when this elasticity approaches the perfectly elastic limit, $\xi \to \infty$.

2.3.2 Factor-Price Heterogeneity

In our benchmark setting, we assume perfectly competitive factor markets in which all firms face common factor prices. However, our empirical application in the next section relies on variation in observed prices of equipment capital across firms. In Appendix A.3.2 we present a generalization of our results to cases involving exogenous wedges in equipment prices across firms, in which firm i faces equipment price $W_{ei} = T_{ei}W_e$ with T_{ei} denoting an exogenous firm-specific wedge and W_e a shadow price of equipment. In Appendix A.6, we provide a micro-foundation for firm-level heterogeneity in the price of equipment, motivating the exogenous equipment wedges T_{ei} across firms.

Let $d\log W_{ei}$ denote a small change in the log price of equipment faced by firm i. Define the change in the aggregate equipment price index as $d\log\overline{W}_e \equiv \sum_i \Lambda_i \frac{\theta_{ei}}{\theta_e} d\log W_{ei}$, where Λ_i is defined in Equation (18), and define (potentially negative) exposure weights ω_{ei} satisfying $d\log W_{ei} = \omega_{ei}\, d\log \overline{W}_e$. We fix the exposure weights ω_{ei} across firms and consider comparative statics with respect to the average magnitude $d\log \overline{W}_e$ of the shock to equipment prices. In parallel to our definition of the aggregate elasticity of substitution (6), we can define the *heterogeneity-adjusted* aggregate elasticity of substitution capturing the effect of the shock to equipment prices on the relative demand and payments for factor f as

$$\sigma^{\omega}_{fe} \equiv \frac{1}{\theta_{f'}} \frac{\partial \log X_f}{\partial \log \overline{W}_e}\bigg|_{(\omega_{ei})}, \tag{23}$$

where the superscript ω indicates the fact that the elasticity is defined with respect to a given set of firm-specific exposure weights (ω_{ei}). Note that the aggregate elasticity of substitution defined in Equation (6) is a special case of the aggregate wedge elasticity defined in Equation (23) corresponding to the case of $\omega_{ei} \equiv 1$.

Using the above definition, Proposition A.2 in Appendix A.3.2 shows that the response of the skill premium to the heterogeneous price shocks satisfies

$$d\log\left(\frac{W_h}{W_\ell}\right) = -\frac{\theta_e\left(\sigma^{\omega}_{\ell e} - \sigma^{\omega}_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h} + \theta_e \sigma_{eh}} d\log \overline{W}_e, \tag{24}$$

where the heterogeneity-adjusted elasticity of substitution can be related to the micro-level elasticities of production and demand based on the following generalization of Equation (19):

$$\sigma^{\omega}_{fe} \equiv \mathbb{E}_i\left[\frac{\theta_{fi}}{\theta_f}\frac{\theta_e}{\theta_e}\sigma_{fe,i}\omega_{ei}\right] - \varepsilon\, \mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\frac{\theta_{fi}}{\theta_f}, \frac{\theta_{ei}}{\theta_e}\omega_{ei}\right)\right] - \eta\, \mathbb{C}_s\left(\frac{\theta_{fs}}{\theta_f}, \frac{\theta_{es}}{\theta_e}\omega_{es}\right). \tag{25}$$

Comparing Equations (25) and (19), we find that the expressions for heterogeneity-adjusted elasticities of substitution parallel those for aggregate elasticities of substitution with one modification: they replace the firm-specific equipment intensity θ_{ei} with $\theta_{ei}\omega_{ei}$. This result is intuitive. The heterogeneity in equipment price shocks implies different exposures of firms to the overall average equipment price shock $d\overline{W}_e$. In so far as we are interested in the aggregate effects of the shock, the heterogeneous exposures translate into effective factor-intensities that are smaller or larger than the underlying firm-level factor intensities depending on the size of the exposure weight.

Appendix A.3.2 further generalizes all our results to cases in which firms experience heterogeneity in factor prices across all factors, including skilled and unskilled labor. As we show, in this case a distinction emerges between the *observed skill premium*, which accounts for observed heterogeneity in wages across firms, and the theoretical notion of the *shadow skill premium* that ensures the clearance of skill markets in general equilibrium. Our results characterize the equilibrium responses of both these variables to a fall in equipment prices.

2.3.3 Composite Factors of Production

In Appendix A.3.3, we show how our results can be used to characterize the impact of shocks to the price of equipment on labor demand, aggregating across high- and low-skilled labor. For this purpose, we consider defining composite factors, e.g., labor n that aggregates high- and low-skilled labor, such that $X_n \equiv X_\ell + X_h$. The appendix derives the aggregate- and firm-level elasticities of substitution for this composite factor in terms of the substitution elasticities defined for the underlying, more disaggregated factors of production.

2.3.4 Additional Factors of Production

We can generalize our framework to accommodate additional factors of production beyond the three considered in Section 2.1. Our definitions for firm- and aggregate-level elasticities of substitution remain intact in this case. However, now we must account for the response in the prices of the other factors of production to the change in the price of equipment in computing the response of the skill premium. Proposition A.6 in Appendix A.3.4 generalizes Proposition 1 and characterizes the responses of all factor prices, including the skill premium, to a change in the price of equipment in terms of aggregate factor intensities and aggregate elasticities of substitution.

3 Data

We exploit a number of French administrative data sources on firms, workers, and firm-level export and import transactions. See Section B in the Online Appendix for a full description of these sources and additional details on the variables used.

3.1 Data sources

Balance sheet and administrative information are retrieved from BRN (*Bénéfice Réel Normal*), a dataset jointly administered by the *Direction Générale des Finances Publiques* (DGFiP) and the French *Institute National de la Statistique et des Études Économiques* (INSEE) that covers the universe of French enterprises that fill normal tax returns over the period 1993-2009.[23] The dataset includes 24% of French firms, which account for 94.3% of total gross output and for over 90% of the aggregate value of trade flows in customs records.[24] This data source has been widely used in the literature (e.g., Eaton et al., 2011) and, crucially for us, it contains the detailed breakdown of firm capital by asset type.

Firm-level international transaction data are provided by the French *Directorate-General of Customs and Indirect Taxes* (DGDDI), which provides information on the annul value of imports and exports by country of origin/destination at the level of 8-digit CN product codes for all firms involved in international transactions (with simplified declarations below certain thresholds). We make use of the data over the 1994–2007 period and our final dataset describes the international trade flows of French firms concerning over 5,000 products from/to all origins/destinations.

As we will discuss in detail in Section 3.2 below, firm-level employment and wage bill data are computed by aggregating worker-level data on hours worked and wages. These data, together with worker characteristics, are retrieved from the DADS (*Déclarations Annuelles de Données Sociales*) Poste, a matched employer-employee dataset provided by INSEE that covers the whole population of French workers.

We construct two different samples based on the datasets mentioned above. We use the "full sample" of all firm/year observations for the aggregation exercise of Section 5 and a smaller "estimation sample" for the estimation of firm-level production elasticities in Section 4 (see the details therein). Table B.1 in Appendix B displays the summary statistics of the data for the two samples.

We use sectoral data on employment, investment, and capital by asset type from the Annual National Accounts compiled by Eurostat. Industry-level data on depreciation rates by asset type come from the EU KLEMS Database.

3.2 Variable definitions

In this section we provide a brief overview of how we compute the main variables of interest used in the analysis. See Appendix C for further details.

Firm value added. In our model, firm revenue equals value added. Annual value added for each firm i is obtained from BRN as gross output less intermediate expenditures,

[23]Firms with revenues above a certain threshold must be affiliated with the BRN regime, while firms below the threshold are not required but always have the option to opt for it (instead of the simplified regime RSI). In 2007, the thresholds were 763,000 euros if a firm operates in trade or real estate sectors and 230,000 euros otherwise.

[24]See INSEE (2004) for further information about the representativeness of BRN.

where gross output is defined as the sum of sales and changes in inventories while intermediates include all the purchases of the firm.

Skill groups. The DADS data does not provide information on the education of workers, but includes their detailed occupational codes. Following prior work (e.g., Caliendo et al., 2015; Carluccio et al., 2015), we assign workers to two skill groups based on their occupation, using occupational classifications in French social security data (CS - catégories socioprofessionnelles). High-skill workers are those employed as Managers, Middle managers and professionals, and Qualified workers. Low-skill workers are employed as Clerks and Blue-collar.

Changes in firm-and-skill-specific wages and employment. In the model, all workers in a skill group are identical; but in practice, workers within the same group vary in observable and unobservable characteristics. To measure five-year, firm-and-skill specific wage changes in the data (Δw_{hit} and $\Delta w_{\ell it}$), we correct changes in observed wages both for changes in returns to worker observable characteristics and for changes in worker composition (along observable and unobservable characteristics). To do so, we estimate a national Mincerian regression of log wage changes between each consecutive time period $t-1$ and t. In this regression, we control for worker observables (age, sex, 2-digit CS category), including additionally a firm-skill-time effect, in the sample of *stayers*, defined as workers in a given skill group who are employed by the same firm between $t-1$ and t.[25] We define the year-on-year log change in the firm-and-skill-specific wage to be equal to the firm-skill-year fixed effect, that is, the average log change in wages controlling for observables in the estimation sample. We chain these year-on-year changes together to obtain five-year changes. We measure five-year changes in employment (Δx_{hit} and $\Delta x_{\ell it}$), by deflating changes in firm-and-skill-specific wage bills by the corresponding wage change (Δw_{hit} and $\Delta w_{\ell it}$).

The level and change in equipment stock. To construct measures of the firm-level equipment stock, we rely on data on firm investments in equipment products, obtained from BRN, and on transaction-level data on the imports of equipment products, obtained from the customs dataset.[26] We first deflate the investment values by a firm-level price investment index. To this end, we first aggregate the transaction-level import data to compute quantities and unit values for the imports in each equipment product classification and

[25]We exclude from the data atypical workers (e.g., cross-border workers, interns and trainees) and those who did not meet a minimum hours-worked threshold.

[26]In the customs data, we define equipment products as the ones belonging to category 4 of the BEC (Rev. 4) classification, *Capital goods (except transport equipment), and parts and accessories thereof*. For more details, see Section C.1.1 in the Appendix.

from each origin country for all firms. We then average year-on-year log changes in unit values for each firm across equipment product codes and origins, weighed by Sato-Vartia weights. Since we do not observe the product-level composition of the domestic investment, we use the series of production price indices for equipment goods from the French national accounts for all firms' domestic equipment purchases. Finally, we chain the firm-level equipment price changes to construct a firm-specific equipment price level in each year (P^e_{it}). We obtain equipment investment quantities by dividing firm-level equipment investment values by the firm-level equipment price. Finally, we construct our measures of equipment stocks by accumulating the investment quantities over time using the perpetual inventory method. We compute five-year log changes in the stocks of equipment from the resulting series of firm-and-year-specific equipment stocks.

User cost of Equipment. We define the firm-level user cost of equipment W_{eit} (the effective rental price of equipment capital) as follows:

$$W_{eit} = P^e_{it} \left(R^e_t + \delta^e_{st} - \frac{p^e_{s,t+1} - p^e_{s,t-2}}{3} \right),$$

where P^e_{it} is the firm-specific price of equipment products purchased by firm i in time t defined above, R^e_t is the required rate of return on investment, and p^e_{st} and δ^e_{st} are the log price and the depreciation rate of equipment products at the sector level from the national accounts, respectively.

Finally, we obtain payments to capital equipment by multiplying the firm-level user cost of equipment with the measure of equipment stock. Since the coverage of the variables required to construct equipment prices and stocks is lower compared to employment variables, we impute capital equipment expenditures to extend the sample of firms used in the aggregation exercise of Section 5 (see Appendix C.1.6 for the details).

4 The Estimation of Elasticities

Section 2 characterizes the elasticity of the skill premium with respect to equipment prices. To compute this elasticity, in addition to the firm-level distribution of factor intensities we also need the demand elasticity across sectors η, the demand elasticity across firms within each sector ε, and the elasticities of firm-level production functions, $\sigma_{ff',i}$. In this section, we estimate these demand and production function elasticities. Whereas our general theoretical results apply for any constant returns to scale firm-level production function, in our baseline analysis we assume that the firm-level production function belongs to the

CRESH family presented in Example 2 with three factors (low- and high-skilled labor and equipment capital). We then repeat the same exercise using a production function characterized by a nested CES structure, as presented in Example 1; we use this alternative production structure in robustness exercises.

4.1 Estimating Equations

Under the CRESH specification for firm-level production, the three parameters σ_ℓ, σ_h, and σ_e together determine the firm-specific production elasticities of substitution $\sigma_{ff',i}$. Alternatively, under the nested-CES specification for firm-level production, the two parameters ρ and ε serve this purpose. In both cases, these parameters—together with data on firm-specific factor shares θ_{fi}—are sufficient to calculate $\sigma_{ff',i}$ for all factors and firms.

Below, we begin with the parameters determining demand elasticities and then turn our attention to the parameters above.

Demand Elasticities ε and η. Given our assumptions on the nested CES structure of demand, we can jointly estimate the elasticity of substitution across firms within each sector, ε, and the elasticity of substitution across sectors, η. From Equation (4), we can write the change in log firm revenue Δr_{it} as

$$\Delta r_{it} = (1-\varepsilon)\ \Delta p_{it} + (\varepsilon - \eta)\ \Delta p_{st} + \alpha_t + \Delta\phi_{it}, \tag{26}$$

where α_t is a time-t fixed effect, Δp_{it} is the change in firm i's log price, Δp_{st} is the change in the log price index within sector s, and $\Delta\phi_{it}$ is the log change in firm i's demand shifter.[27] In our baseline, we estimate Equation (26) using five-year changes at the firm level, where $\Delta x_t \equiv x_t - x_{t-5}$ for any variable x. Since markups are constant under our model, we can approximate changes in firm-level prices by aggregating changes in firm-level input costs using their respective costs shares and subtracting from this changes in firm-level productivity, and then approximate changes in sector-level prices by further aggregation across firms. Denoting by $\Delta\widetilde{p}_{it}$ and $\Delta\widetilde{p}_{st}$ the log change in prices at fixed firm-level productivity, we have

$$\Delta\widetilde{p}_{it} = \Delta\widetilde{c}_{it} \approx \sum_{f\in\{\ell,h,e\}} \tfrac{1}{2}\left(\theta_{fi,t} + \theta_{fi,t-1}\right)\Delta w_{fi,t}, \tag{27}$$

$$\Delta\widetilde{p}_{st} = \Delta\widetilde{c}_{st} \approx \sum_{i\in\mathcal{I}_s} \tfrac{1}{2}\left(\Lambda_{i|s,t} + \Lambda_{i|s,t-1}\right)\Delta\widetilde{c}_{it}, \tag{28}$$

[27] In an alternative specification we impose sector × time fixed effects and estimate ε alone.

where, as before, θ_{fit} stands for the share of factor-f in firm-i costs at time t, and where $\Lambda_{i|s,t}$ stands for the share of firm-i in total sector-s costs at time t. In estimating Equation (26) we proxy for firm and sector-level price changes using these changes in prices at fixed productivities. This introduces an additional component into the error term corresponding to firm-level productivity changes and their aggregation within sector.

CRESH Production Function Elasticities σ_ℓ, σ_h, and σ_e. The production function specified in Equation (15) implies that firm i's factor demand satisfies[28]

$$\Delta x_{eit} - \Delta x_{\ell it} = \beta_{\ell t} - \sigma_\ell \left(\Delta w_{eit} - \Delta w_{\ell it}\right) + \left(\frac{\sigma_\ell}{\sigma_e} - 1\right)\left(\tfrac{\varepsilon}{\varepsilon-1}\Delta r_{it} - \Delta x_{eit}\right) + \nu_{\ell it} \tag{29}$$

$$\Delta x_{eit} - \Delta x_{hit} = \beta_{ht} - \sigma_h \left(\Delta w_{eit} - \Delta w_{hit}\right) + \left(\frac{\sigma_h}{\sigma_e} - 1\right)\left(\tfrac{\varepsilon}{\varepsilon-1}\Delta r_{it} - \Delta x_{eit}\right) + \nu_{hit}, \tag{30}$$

where x_{fit} is the logarithm of employment of factor f in firm i at time t and where we have replaced the change in firm i's log output with the change in log revenue (value added) r_{it} using the structure imposed by the CES demand for firm products. The structural residuals $\nu_{\ell it}$ and ν_{hit}, therefore, depend not only on changes in firm-specific factor-augmenting productivities Δz_{fi}, but also on the change in firm i's demand shifter and the change in the relevant sectoral price index.

In our baseline, we estimate Equations (29) and (30) using five-year changes in factor employment, prices, and revenues. We measure firm i's employment of and payment to each factor as described in Section 3.2, computing five-year changes by chaining the year-on-year log changes for each firm. Our approach ensures that the measurement of firm-level wage changes for each skill group is not affected by changes in the composition of the firm's workforce.[29]

Nested CES Production Function Elasticities ς and ρ. The production function specified in Equation (12) implies that firm i's factor demand satisfies[30]

$$\Delta x_{hit} - \Delta x_{\ell it} = -\varsigma \left(\Delta w_{hit} - \Delta w_{\ell it}\right) + \frac{\varsigma - \rho}{1-\rho}\Delta \log\left(\frac{\theta_{hit}}{\theta_{eit} + \theta_{hit}}\right) + u_{\ell it}, \tag{31}$$

$$\Delta x_{hit} - \Delta x_{eit} = -\rho \left(\Delta w_{hit} - \Delta w_{eit}\right) + u_{eit}, \tag{32}$$

[28]See the proof of Equations (29) and (30) in Appendix A.4.1 (on page A16).

[29]There is an alternative approach to estimating σ_f that does not require the inclusion of output (or revenue) as an independent variable. There are two benefits of using Equations (29) and (30). First, the inclusion of additional factors of production leave these estimating equations unchanged. Hence, our estimates of these parameters is robust to additional factors. Second, these equations can be estimated using 2SLS.

[30]See the derivation based on the demand system in Equations (A.3) and (A.4) in Appendix A.2.1.

where, as before, x_{fit} is the logarithm of employment of factor f in firm i at time t. The structural residuals $u_{\ell it}$ and u_{eit} depend on changes in firm-specific factor-augmenting productivities Δz_{fi}. All variables are measured as in the case of the CRESH specification discussed above.

4.2 Identification Strategy and Instruments

Next, we describe our identification strategy and describe the instruments used in this strategy. Appendix C.3 contains further details about the construction of these instruments.

Estimation Strategy. We first estimate ε and η using Equation (26) via GMM. We then use the implied value ε to estimate σ_ℓ, σ_h, and σ_e using equations (29) and (30) via GMM, since the three structural parameters we aim to identify depend on parameter estimates from both equations. We jointly bootstrap both estimation steps so that our confidence intervals for σ_ℓ, σ_h, and σ_e incorporate the dispersion of our estimated values for ε and η. Finally, we estimate ς and ρ using equations (31) and (32) via GMM.

Instruments for the Estimation of Demand Elasticities. In general, changes in firm-level log prices Δp_{it} may covary with firm-specific changes in demand shifters $\Delta\phi_{it}$ due, e.g., to shocks to the quality of firm output. This creates a standard endogeneity problem, since the residual on the right-hand side of Equation (26) is a function of these demand shifters. Moreover, since we proxy for price changes using the component induced by factor-price changes alone, $\Delta\widetilde{p}_{it}$, this creates an additional endogeneity problem, since the residuals are also functions of productivity shifters. To address these problems, we construct an instrument that shifts firm i's marginal cost and is plausibly independent of changes in its demand and productivity.

We leverage differences in firm-specific import exposure to different source countries and movements in real exchange rates between France and these countries. If firm i obtains a large share of its imports from a given source country c and the real exchange rate of country c depreciates relative to France, then firm i's import cost falls, thereby reducing firm i's marginal cost. We refer to this as the real-exchange-rate instrument and define it formally as

$$\Delta RER_{it} = \sum_c \frac{M_{ci,t-5}}{M_{i,t-5}} \Delta \log \left(NER_{ct} \cdot \frac{Defl_{FR,t}}{Defl_{ct}} \right). \tag{33}$$

Here, NER_{ct} is the nominal exchange rate (defined as country c currency per euros), $Defl_{ct}$ is the GDP deflator in the origin country c and $Defl_{FR,t}$ is the French GDP deflator in year t. Exposure shares are constructed as firm i's total import value in year $t-5$

from country c, $M_{ci,t-5}$, relative to total imports by firm i in $t-5$, $M_{i,t-5}$. A real depreciation of country c's currency relative to the euro is reflected in an increase in ΔRER_{it}. We use this instrument for changes in firm-level costs in Equation (26) and then define an analogous instrument for the sectoral prices as follows

$$\Delta RER_{st} = \sum_{c} \frac{M_{cs,t-5}}{M_{s,t-5}} \Delta \log \left(NER_{ct} \cdot \frac{Defl_{FR,t}}{Defl_{ct}} \right). \tag{34}$$

Instruments for the Estimation of Firm-Level Production Elasticities. The structural residuals $\nu_{\ell it}$ and ν_{hit} in Equations (29) and (30) depend on changes in firm-specific factor-augmenting productivities Δz_{fi}, and on changes in firm i's demand shifter and changes in the relevant sectoral price index. To address these endogeneity problems, we construct instruments that shift for each firm the following variables: 1) cost of equipment relative to low-skilled labor, 2) cost of equipment relative to high-skilled labor, and 3) revenue relative to its equipment stock. These instruments are plausibly uncorrelated with changes in the firm's factor-augmenting productivities and its demand shifter.

Our first instrument is intended to shift the price of equipment across and within firms. The logic is similar to our real-exchange-rate instrument. Following Hummels et al. (2014), we leverage differences in firm-specific import exposure to different equipment types and source countries, along with movements in equipment-type-specific predicted changes in transport costs between France and these source countries. These movements are induced by changes in oil and jet fuel prices. A decrease in the transport cost of shipping type-k equipment from source country c to France will reduce the equipment cost of a firm i for which this country-equipment pair constitutes a large share of equipment imports. Using French micro-data, we obtain mode-of-transport frequencies and weight-value ratios by HS6 product. We use data on oil and jet fuel prices, weighted distances between France and all other countries, and transportation charge elasticities. For each origin country c and equipment product $k \in \mathcal{K}_e$, we compute predicted five-year changes in transport costs and construct a firm-specific average weighting by initial equipment import shares. We refer to this as the equipment transit-cost instrument and define it formally as

$$\Delta ETC_{it} = \sum_{c} \sum_{k \in \mathcal{K}_e} \frac{M^e_{cki,t-5}}{M^e_{i,t-5}} \Delta \log TC_{ckt}. \tag{35}$$

Here, $\Delta \log TC_{ckt}$ is the predicted change in the transport cost of equipment product k from country c to France (given oil and fuel prices, the distance between France and country c, and the predicted shipping mode). Exposure shares are constructed as firm

i's import value of equipment products in year $t-5$ from country c, denoted by $M^e_{cki,t-5}$, relative to the value of total imports of equipment products in year $t-5$ by firm i, denoted by $M^e_{i,t-5}$.[31]

Our second instrument is intended to shift firm revenues. Its logic is also similar to our real-exchange-rate instrument. Again following Hummels et al. (2014), we create an instrument that leverages differences in firm-specific intermediate import exposure to different products and source countries, along with the movements in the supplies of those products from the corresponding source countries to the rest of the world. An increase in productivity for source country c in producing intermediate product k will reduce the marginal cost of a firm i for which this country-product pair constitutes a large share of intermediate imports. For each exporting source country c and HS6 product k in the set of intermediate products $\mathcal{K}_I$, we measure the value of total exports towards all countries excluding France in year t, denoted by Exp_{ckt}. We use this variable to construct a firm-i-specific average of changes in export supplies weighting by initial intermediate product and source country import shares. We refer to this as the world-export-supply instrument and define it formally as

$$\Delta WES_{it} = \sum_c \sum_{k \in \mathcal{K}_I} \frac{M^I_{cki,t-5}}{M^I_{i,t-5}} \Delta \log Exp_{ckt}. \tag{36}$$

Here, $\Delta \log Exp_{ckt}$ is the change in the exports of intermediate product $k \in \mathcal{K}_I$ from source country c to the rest of the world (excluding France). Exposure shares are constructed as firm i's import value of intermediate good k in year $t-5$ from country c, $M^I_{cki,t-5}$, relative to the value of total imports of intermediate goods in year $t-5$ by firm i, $M^I_{i,t-5}$.[32]

Our third and fourth instruments are intended to shift the wages of skilled and unskilled workers facing each firm. A rise in the nationwide level of output of a sector s that is particularly skilled (or unskilled) intensive will raise the skilled (unskilled) wage in a region with a large share of employment in that sector. This will raise the effective skilled (unskilled) wage facing firm i if it has a large share of its employment in that region. Following this intuition, our last two instruments leverage differences in the spatial compositions of firm production, the industrial mix of different French regions, and the variations in skill intensities and nationwide growth across industries. We refer to these

[31] See Appendix C.3 for further details.

[32] For both the equipment transit-cost (ETC) and the world-export-supply (WES) instruments, we drop the bottom and top 1% of the distribution of 5-year firm-level changes.

Table 1: Between-Firm Demand Elasticity Estimates

	(1)	(2)	(3)	(4)
ϵ	0.750	**4.481**	5.398	3.412
	(0.010)	**(0.773)**	(1.306)	(0.508)
η	0.796	**3.124**	3.548	2.500
	(0.023)	**(0.426)**	(0.721)	(0.279)
Observations	162,303	**162,303**	78,912	162,132
Eta level	4 dig	**4 dig**	4 dig	4 dig
Year FE	Yes	**Yes**	Yes	Yes
Excl Exp Mkt	No	**No**	Yes	No
IV	-	**RER**	RER	RER+SSW
SW Min F-Stat		**45.69**	20.71	24.93
KP F-Stat		**22.85**	10.35	18.70

Notes: This table reports estimation results of Equation (26). The dependent variable is the 5-year change in firm i's value added and the independent variables are corresponding changes in total input costs at the firm and 4-digit sector level. All instrumented columns use the firm-level RER instrument defined in Equation (33) and the sectoral RER instrument defined in Equation (34). Additionally, column (4) uses the skill-specific-wage instruments defined in Equations (37) and (38). All variables are in deviations from year means in all columns. In column (3), we exclude firms that import from any country in which they also export more than 10% of their total export value over the period. The final two rows report the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics and the Kleibergen and Paap (2006) rk Wald (KP F stat) obtained by running the equivalent 2SLS estimation. Analytical standard errors are reported in parenthesis.

as the skill-specific-wage instruments and define them formally as

$$\Delta SSW_{it}^{h,-s} = \sum_{r} \sum_{s \neq s_i^r} \left(\frac{L_{ni,t-5}^r}{L_{ni,t-5}} \right) \cdot \left(\frac{L_{hs,t-5}^r}{L_{h,t-5,-s_i^r}^r} \right) \cdot \Delta \log GO_{st}, \tag{37}$$

$$\Delta SSW_{it}^{\ell,-s} = \sum_{r} \sum_{s \neq s_i^r} \left(\frac{L_{ni,t-5}^r}{L_{ni,t-5}} \right) \cdot \left(\frac{L_{\ell s,t-5}^r}{L_{\ell,t-5,-s_i^r}^r} \right) \cdot \Delta \log GO_{st}. \tag{38}$$

Here, $\Delta \log GO_{st}$ is the change in gross output for sector s at the national level in France. Exposure shares are constructed as the product of two components: 1) firm i's employment in region r, $L_{ni,t-5}^r$, relative to its total employment, $L_{ni,t-5}$ across all domestic regions; 2) region r employment of skill-type f in sector s, $L_{fs,t-5}^r$, relative to region r total employment of skill-type f excluding the sector s of firm i in region r, $L_{f,t-5,-s_i^r}^r$.

4.3 Estimation Results

Demand Elasticity Estimates. Table 1 displays the estimates of ε and η obtained by estimating Equation (26) via GMM. Column (1) shows the estimates obtained using each covariate as its own instrument whereas columns (2)–(4) display the estimates obtained using the instruments indicated in the bottom panel of the table. In our baseline, displayed in column (2), industries are defined at the four-digit level and we use the two RER instruments defined in Equations (33) and (34). Since the instruments require import values for each firm, in our baseline we restrict the sample of firms to those with import values that are at least 1% of their gross output, when averaged over the sample period.

Whereas estimates in column (1) are close to one, the instrumented estimates for ε range from 3.4 to 5.4 and those for η from 2.5 to 3.5, consistent with a positive correlation between demand shocks and prices. Column (2) represents our baseline estimate, $\varepsilon = 4.48$, which is in the range of values reported by Broda and Weinstein (2006) for products at the five-digit level. In our baseline specification, the minimum of the two SW first-stage F statistics obtained by running the corresponding 2SLS estimation of Equation (26) is above 40 and remains sufficiently high even in more demanding specifications.

One potential concern with the moment condition in our baseline specification is that firms may import from and export to the same market. In this case, the RER instruments may be correlated with export demand shocks. To address this concern, in column (3) we exclude firms that import from any country to which they also export more than 10% of their total export value over the period. This halves the size of the sample, but the estimate of ε and η remain relatively stable at levels not significantly different from our baseline. Finally, in column (4) we additionally incorporate the two skill-specific-wage instruments defined in Equations (37) and (38). Our estimates are largely robust, although the first-stage in the 2SLS specification is slightly weaker and the resulting estimates are slightly lower.[33]

Firm-Level Production Elasticity Estimates (CRESH). Table 2 displays the estimates of the structural parameters of interest, σ_ℓ, σ_h, and σ_e, estimated using GMM for different values of ε and using different fixed effects. In all columns, we use the instruments described in Section 4.2: the equipment transport-cost in Equation (35), the world-export-supply instrument of Equation (36), and the two skill-specific-wage instruments defined in Equations (37) and (38). Column (1) displays our baseline estimates of $\sigma_\ell = 1.28$, $\sigma_h = 1.11$, and $\sigma_e = 1.2$, using our baseline estimate of $\varepsilon = 4.48$ and estimated including year $\times$ 2-digit sector fixed effects. We find that $\sigma_\ell > \sigma_h$, which implies that equipment is more substitutable with low- than high-skilled labor: *at the firm level, production exhibits equipment-skill complementarity*. This difference in parameters is statistically significant

[33]Table D.1 in the Appendix replicates Table 1 varying the level of sectoral aggregation and using the equipment transport cost instrument defined in Equation instead of the RER instrument of Equations 33. Table D.2 estimates ε alone by 2SLS, and controls for η using sector-year fixed effects.

Table 2: Firm-Level Production Elasticities (CRESH)

	(1)	(2)	(3)	(4)
σ_ℓ	**1.275**	1.081	1.283	1.261
	(0.164)	(0.122)	(0.164)	(0.158)
σ_h	**1.107**	0.932	1.100	1.122
	(0.151)	(0.107)	(0.152)	(0.157)
σ_e	**1.200**	1.080	1.202	1.198
	(0.214)	(0.152)	(0.220)	(0.193)
Observations	**67,863**	67,904	67,863	67,863
Year-Sector FE	**2 dig**	1 dig	2 dig	2 dig
$\Pr[\sigma_\ell < \sigma_h]$	**0.06**	0.04	0.06	0.07
ϵ	**4.48**	4.48	5.40	3.41

Notes: This table reports regression coefficients for the system of Equations (29) and (30) estimated using GMM for a given value of ε and with variables computed in deviations from year-sector means with sectors defined at the 2 digit or at the 1 digit level (column 2). Corresponding point estimates of σ_ℓ, σ_h, and σ_e are displayed. Bootstrap standard errors are reported in parenthesis, obtained by bootstrapping the estimation of ε and of this system 200 times and $\Pr[\sigma_\ell < \sigma_h]$ reports the share of bootstrapped estimates in which σ_ℓ is lower than σ_h.

based on our bootstrapped samples, as shown in the bottom panel of Table 2.

In column (2), we present estimates of the same model, but when including year $\times$ 1-digit sector (instead of 2-digit sector) fixed effects.[34] Whereas point estimates of our structural parameters fall, the extent of firm-level equipment-skill complementarity remains very similar in magnitude. In columns (3) and (4), we replicate the specification in column (1), but we use the higher and lower estimates of ε reported in columns (3) and (4) of Table 1, respectively. Across all columns, we find a consistent pattern of elasticity estimates: $\sigma_\ell > \sigma_e > \sigma_h$. Moreover, across all columns we find that equipment-skill complementarity is statistically significant.

Firm-Level Production Elasticity Estimates (nested CES). Table 3 displays the estimates of the structural parameters of interest, ς and ρ, estimated using GMM and different fixed effects. This exercise allows us to investigate the extent of capital-skill complementarity using a different functional form and extending the sample to all firms, not only importers. In the baseline estimates of column (1) we use the two SSW instruments defined in Section 4.2, and we find supporting evidence for capital-skill complementarity at the firm level ($\varsigma > \rho$). We show in the next section that these point estimates imply an estimated aggregate capital-skill complementarity virtually identical to CRESH. However, the table shows that for nested CES the difference in the parameters is not statistically

[34] In Table D.4 in the Appendix, we estimate each individual equation separately via 2SLS and display coefficient estimates of Equations (29) in the odd columns and (30) in the even columns. This exercise allows us to gauge the strength of the first stage.

Table 3: Firm-Level Production Elasticities (KORV)

	(1)	(2)	(3)
ς	**1.196**	1.422	1.092
	(0.335)	(0.386)	(0.236)
ρ	**1.084**	1.165	1.037
	(0.154)	(0.184)	(0.098)
Observations	**400,303**	67,734	67,863
Year-Sector FE	**4 dig**	4 dig	2 dig
IV	**SSW**	SSW+Imp	SSW+Imp
KP F 2SLS (ς)	**35.70**	1.54	1.31
KP F 2SLS (ρ)	**31.57**	5.46	10.91
Pr[$\varsigma < \rho$]	**0.27**	0.11	0.34

Notes: This table reports regression coefficients for the system of Equations (31) and (32) estimated estimated using GMM and with variables computed in deviations from year-sector means with sectors defined at the 4 digit (columns 1-2) or at the 2 digit level (column 3). Column 1 uses the two SSW IVs defined in Section 4.2; columns (2) and (3) additionally use the import IVs (ETC and WES) and hence are conditioned on firms firms importing both intermediate and equipment products. Corresponding point estimates of ς and ρ are displayed. Analytical standard errors are reported in parenthesis. Pr[$\varsigma < \rho$] report the result of a Wald test with a null of no capital-skill complementarity within the firm.

significant. In columns (2) and (3), we replicate the same setting as that in the estimation of the CRESH specification, conditioning on importing firms by adding the two import instrumental variables, namely ETC from Equation (35) and WES from Equation (36). We again find that ς is larger than ρ, with a degree of capital-skill complementarity of a comparable magnitude. However, in the corresponding 2SLS specifications the first stage is relatively weak and the difference in the parameters is again not statistically significant.

5 Aggregation

We now use our framework to study the implications of observed shocks to equipment prices (and factor supplies) on the skill premium. In our baseline, we consider a setting featuring CRESH firm-level production functions in which the economy faces a uniform fall in the relative price of equipment. We compute the response of the skill premium using the results developed in Section 2.2. In robustness exercises we consider nested-CES firm-level production functions and use the extensions developed in Section 2.3 to study the impacts of the heterogeneity in price shocks observed in our firm-level data and of an alternative model of nested consumer demand featuring three (instead of two) levels of aggregation.

Table 4: Aggregate Elasticities

	(1)	(2)	(3)	(4)
$\sigma_{\ell e} - \sigma_{he}$	0.204	0.200	0.248	0.151
	[0.107, 0.294]	[0.124, 0.276]	[0.146, 0.350]	[0.056, 0.238]
$\frac{d\log(W_h/W_\ell)}{d\log W_e}$	-0.058	-0.062	-0.065	-0.048
	[-0.083, -0.028]	[-0.086, -0.037]	[-0.092, -0.036]	[-0.077, -0.018]
ϵ	4.481	4.481	5.398	3.412
η	3.124	3.124	3.548	2.500
σ_ℓ	1.275	1.081	1.283	1.261
σ_h	1.107	0.932	1.100	1.122
σ_e	1.200	1.080	1.202	1.198

Notes: $\sigma_{\ell e} - \sigma_{he}$ is the aggregate degree of capital-skill complementarity and $d\log(W_h/W_\ell)/d\log W_e$ is the elasticity of the skill premium with respect to the equipment price. These elasticities are constructed using firm factor shares in each year and then averaging across all sample years. Column 1 is our baseline, Column 2 uses our estimates of σ_ℓ, σ_h, and σ_e estimated within 1-digit (and year) sectors, columns 2 and 3 use our lowest and highest estimates of ε. The table displays 90% confidence intervals obtained by bootstrapping the estimation of ε and each σ_f 200 times.

5.1 Baseline Results

We first compute the aggregate degree of capital-skill complementarity and the elasticity of the skill premium with respect to the equipment price, each calculated as averages across sample years. We then solve for the implications of the observed decline in the equipment price.

Capital-Skill Complementarity. Table 4 displays the aggregate degree of capital-skill complementarity $\sigma_{\ell e} - \sigma_{he}$ defined in Equation (20) and the elasticity of the skill premium with respect to equipment prices $d\log(W_h/W_\ell)/d\log W_e$ constructed using Equation (8), each calculated using shares in each year and then averaging across all years.

Column (1) is our baseline specification, corresponding to the micro elasticities presented in Column (1) of Table 2. The results imply that our measure of capital-skill complementarity is approximately 0.2 and that a decrease in the equipment price of 1% increases the skill premium by approximately 0.06%. Column (2) uses values of σ_ℓ, σ_h, and σ_e estimated within 1-digit sector (and year) from Column 2 of Table 2. Finally, Columns (3) and (4) replace ε with the lower and upper bounds of estimates from Table 1 and the corresponding estimates of σ_ℓ, σ_h, and σ_e from Columns (3) and (4) of Table 2. Qualitative results—a decline in the price of equipment raises the skill premium—are robust across specifications. Quantitative elasticities are largely robust as well, with the elasticity of the skill premium ranging from -0.048 to -0.065 and the aggregate degree of capital-skill

Table 5: Decomposition of Main Aggregate Elasticities

	Capital-Skill Compl	Skill Premium
	$\sigma_{\ell e} - \sigma_{he}$	$\frac{d\log(W_h/W_\ell)}{d\log W_e}$
Aggregate Elasticity	0.204	-0.058
	[0.107, 0.294]	[-0.083, -0.028]
Within firm complementarity	0.169	-0.048
	[-0.012, 0.338]	[-0.095, 0.003]
Within firm substitution	-0.101	0.028
	[-0.185, -0.016]	[0.005, 0.051]
Cross firm	0.136	-0.038
	[0.136, 0.136]	[-0.042, -0.034]

Notes: $\sigma_{\ell e} - \sigma_{he}$ and $d\log(W_h/W_\ell)/d\log W_e$ are the elasticities of aggregate capital-skill complementarity and the skill premium with respect to the equipment price. The table decomposes the baseline estimates of Table 4 into within-firm and cross-firm components according to Equation (20). These elasticities are constructed using firm factor shares in each year and then averaging across all sample years. The table displays 90% confidence intervals obtained by bootstrapping the estimation of ε and each σ_f 200 times.

complementarity from 0.151 to 0.248.

Table 5 decomposes these elasticities into within-firm and cross-firm components using Equation (20), focusing on our baseline specification of Column (1) in Table 4.[35] We find that within-firm capital-skill complementarity accounts for a large share of the aggregate elasticities, with within-firm equipment-labor substitution and cross-firm reallocation roughly offsetting each other.

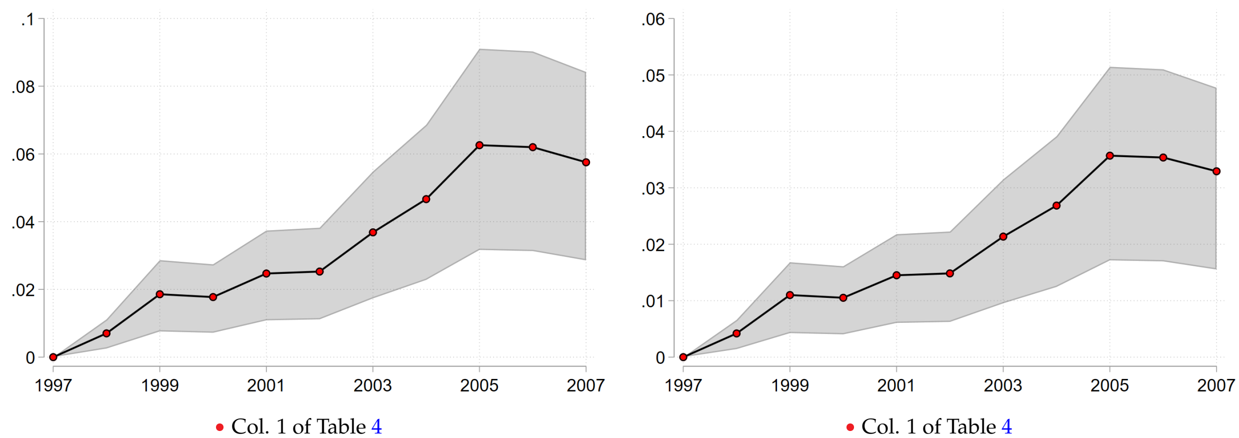
Capital-Skill Complementarity and the Evolution of the Skill Premium. Next, we use our framework to quantify the contribution of the observed fall in the relative price of equipment in France to the evolution of the skill premium in our sample period. In addition to computing the elasticity of the skill premium with respect to the equipment price separately in each year of the data, we also need to measure the change in the equipment price between each year. Since we take the wage of low-skilled labor as the numeraire in our theory in Section 2, we measure the change in equipment prices relative to the low-skilled wage.

We measure year-on-year changes in the equipment price by aggregating industry-specific equipment investment prices taken from French national accounts and using industry-specific depreciation rates from EU-KLEMS. We measure the low-skilled wage as the nominal hourly minimum wage, sourced from the French Ministry of Labor (see

[35]The decomposition for the skill premium is obtained by combining the decomposition of the degree of capital-skill complementarity in Equation (20) with the solution for the skill premium in Equation (8).

Figure 1: Predicted Skill Demand and Skill Premium, Uniform Shock

(a) Predicted Skill Demand (b) Predicted Skill Premium



Notes: The figure plots the evolution of the predicted skill demand (Panel a) and skill premium (Panel b), in response to the observed fall in the equipment price relative to the low skill shadow wage. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low-skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour. See Appendix C.1.7 for further details. The figure displays a 90% confidence interval obtained by bootstrapping the estimation of ε and each σ_f 200 times.

Appendix C.1.7 for further details).[36] Based on this data, the cumulative decline in the relative equipment price is approximately 58 log points over the period 1997-2007, with the fall in equipment prices and the rise in low-skilled wages each accounting for roughly half of this decline; see details in Table D.6 in the appendix.

Panel (a) of Figure 1 shows the model-predicted impact of the fall in the relative equipment price on skill demand between 1997-2007, computed by accumulating the predicted year-on-year response according to Equation (7), along with the associated 90% confidence interval. Panel (b) shows the model-predicted impact of the fall in the relative equipment price on the skill premium over the same time period. To compute the predicted change in the skill premium between years $t-1$ and t, we compute aggregate elasticities using the distribution of factor intensities in year $t-1$ and t, and use the relative equipment price change between $t-1$ and t as the corresponding shock.[37] We then compute the implied cumulative change in the skill premium starting from 1997. The confidence interval is constructed by re-estimating the micro-level elasticities of substitution across bootstrapped samples of the data (200 times). Between 1997 and 2007, the decline in the price of equipment relative to the wage of low-skilled labor generates a 3.3

[36] We obtain very similar results by using the changes in the hourly wage of low-skilled workers from the Socio-Economic Accounts of the WIOD database (2013 Release).

[37] To compute the cumulative predicted change, we rely on second-order approximations results presented in Appendix A.5.

Table 6: Decomposition of the Predicted Change in the Skill Premium, 1998-2007

Total	W/in firm compl.	W/in firm subst.	Cross firm
0.033	0.028	-0.017	0.021
[0.016, 0.048]	[-0.002, 0.056]	[-0.030, -0.003]	[0.019, 0.023]

Notes: This table reports the decomposition of the predicted log change in the skill premium in response to the observed fall in the equipment price relative to the low skill shadow wage. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour. See Appendix C.1.7 for further details. The table displays 90% confidence intervals obtained by bootstrapping the estimation of ε and each σ_f 200 times.

log point increase in the skill premium, which is statistically different from zero.

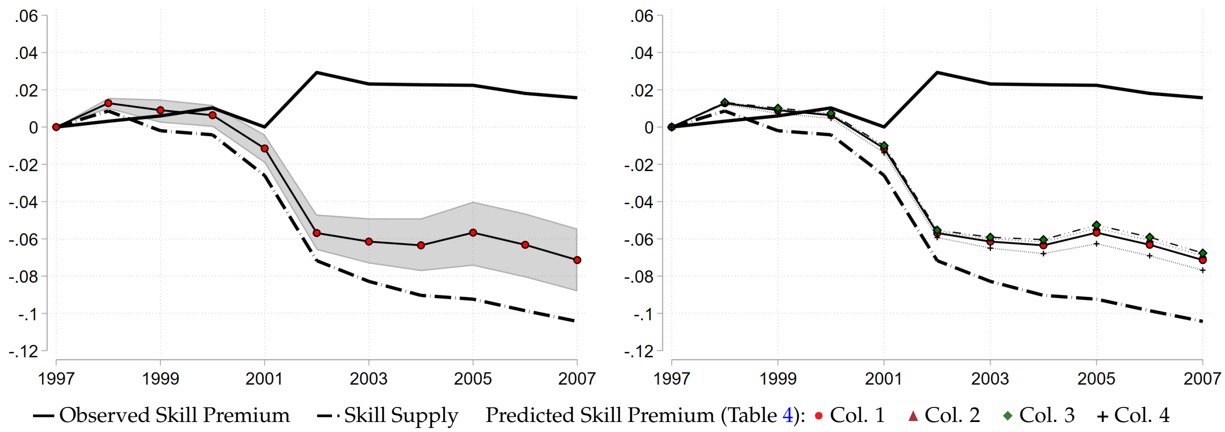
We obtain similar results when we perform a simpler exercise multiplying the total cumulative change in the relative equipment (rental) price by the average aggregate elasticity over the entire period from Column (1) of Table 4. The results are displayed in Table D.6 and they deliver similar conclusions with a change of 3.4 log points over the analyzed period. At the same time, this approach allows us to extend the time frame of the analysis to periods in which our micro data is not available, to obtain the predicted changes over almost three decades. According to our model, the decline in equipment prices generate an approximately 7% rise in the skill premium in France between 1990 and 2015.

Table 6 decomposes the predicted change in the skill premium, confirming the results previously shown for the decomposition of the aggregate elasticities. The within-firm complementarity component accounts for almost 90% of the predicted change, with within-firm equipment-labor substitution and cross-firm reallocation roughly offsetting each other.

Panel (a) of Figure 2 displays our model's predicted change in the skill premium taking into account not only changes in equipment prices—as in Panel (a) of Figure 1—but also changes in the relative supply of skill using Equation (21). Figure 2 additionally displays the change in the skill premium observed in our data. The substantial rise in the relative supply of skilled labor leads to a predicted decline in the skill premium. The size of this decline is close to the size of the change in the relative skill supply because $\sigma_{\ell h}$ is close to 1. Capital-skill complementarity is not sufficiently strong to counteract the impact of the rise in the supply of skill on the skill premium. The remaining gap between the model's predicted change in the skill premium in response to changing equipment prices and skill supply and the observed evolution of the skill premium can be attributed

Figure 2: Predicted Skill Premium vs Observed Skill Premium and Skill Supply

(a) Equipment and Skill Supply Shocks (b) Alternative Estimates



Notes: Panel (a) plots the evolution of the predicted skill premium ($d\log(W_h/W_\ell)$) in response to the observed fall in the relative equipment price and the observed change in the skill supply. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour. See Appendix C.1.7 for further details. The figure displays a 90% confidence interval obtained by bootstrapping the estimation of ε and each σ_f 200 times. Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 4.

to all remaining shocks, including skill-biased technical change that is not embodied in capital equipment and changes in wedges between different inputs. Panel (b) displays the robustness of our quantitative results across the estimation specifications displayed in Table 2. Consistent with the elasticities in Table 4, we obtain very similar results across specifications.

5.2 Extensions

5.2.1 Nested CES (KORV) Specification

We repeat the same exercise with the nested CES production function estimated in Table 3. Table 7 replicates Table 4 and shows that the aggregate elasticities estimated with a nested CES production function are virtually identical to the CRESH case. The baseline estimates of the micro-elasticities ς and ρ used in Column 1 result in an aggregate degree of capital-skill complementarity of 0.2 and an elasticitiy of the skill premium with respect to equipment prices of -0.058. The alternative estimates used in columns 2 and 3 produce aggregate elasticities that are also in the range of estimates found with a CRESH production function.

Under the nested CES specification, Figure D.1 displays the predicted change in the

Table 7: Aggregate Elasticities - Nested CES firm production functions

	(1)	(2)	(3)
$\sigma_{\ell e} - \sigma_{he}$	0.203	0.316	0.159
	[-0.036, 0.377]	[-0.063, 0.551]	[-0.037, 0.292]
$\frac{d\log(W_h/W_\ell)}{d\log W_e}$	-0.058	-0.085	-0.047
	[-0.091, 0.014]	[-0.128, 0.023]	[-0.078, 0.014]
ϵ	4.481	4.481	4.481
η	3.124	3.124	3.124
ς	1.196	1.422	1.092
ρ	1.084	1.165	1.037

Notes: $\sigma_{\ell e} - \sigma_{he}$ is the aggregate degree of capital-skill complementarity and $d\log(W_h/W_\ell)/d\log W_e$ is the elasticity of the skill premium with respect to the equipment price. These elasticities are constructed using firm factor shares averaged across all sample years. Column 1 is our baseline, columns 2 and 3 use our lowest and highest estimates of ε, and column 4 uses our estimates of σ_ℓ, σ_h, and σ_e including year-sector FE. The table displays 95% confidence intervals obtained by bootstrapping the estimation of ε and each σ_f 200 times.

skill premium accounting for changes in the relative skill supply and the equipment price and plots it against the change in skill premium observed in our data, paralleling Figure 2. While the observed fall in the relative equipment price results in a change in the skill premium that counteracts the observed increase in the relative skill supply between high- and low-skilled workers, we again find that capital-skill complementarity is not enough to match the observed change in the skill premium. The result holds across all specifications of Table 7.

While the CRESH specification has several advantages as discussed in Section 2.2.1, we conclude that our findings are not driven by specific functional form assumptions and they carry over to the full sample of firms, beyond those heavily active in import markets.

5.2.2 Heterogenous Shocks to Equipment Prices

We next use the theory presented in Section 2.3 to account for the heterogeneity in shocks experienced by different firms. In particular, we measure firm-specific equipment prices as outlined in Appendix A.3.2. We then construct model-consistent year-on-year changes in the skill premium following the methodology presented in Section 2.3.2.

The results of the exercise are shown in Figure D.2 and in Table D.7 in the Online Appendix. Compared to the case of the uniform shock studied before, the decline in the firm-specific relative price of equipment now generates a larger 4.1 log point increase in the skill premium. While the within-firm complementarity component accounts for a large share of the predicted increase in the skill premium as in the case of uniform shocks, the increase in the prediction is mostly driven by the cross-firm component. This result is driven by the positive covariance between price changes and skill intensity, which is particularly strong after 2002 when skill and equipment intensive firms experienced large declines in the relative price of imported equipment (see Figures D.3 and D.4).[38]

5.2.3 Aggregation with Three-Level Nested CES Preferences

We perform an additional exercise in which the CES preferences are defined over three nests, with a top nest defined at the 1-digit sector level. We set the elasticity of substitution across 1-digit sectors to 0.5, as in Buera and Kaboski (2009). Figure D.5 shows that results are robust both for our baseline CRESH specification as well as the alternative estimates of Table 4. In the exercise we have kept the same estimates for the firm-level elasticities, and in the Online Appendix we show that the estimates remain very similar when the demand elasticities across firms and sectors (ϵ and η) are re-estimated controlling for 1-digit sector by year fixed effects (see Tables D.3 and D.5).

5.3 Comparison with the Prior Literature

In our baseline exercise above, we obtain a measure for the aggregate degree of capital-skill complementarity, defined as $\sigma_{\ell e} - \sigma_{he}$, equal to 0.2 (Table 4); and we find an elasticity of the skill premium with respect to the equipment price equal to -0.06. How do these results compare to the prior work in the literature?

In a seminal contribution to the literature, Krusell et al. (2000) (KORV) used aggregate data on skill supply, the skill premium, and aggregate factor intensities from the U.S. to estimate a nested CES aggregate production function in line with Example 1. The core identification assumption behind their approach rules out the presence of trends in skill-augmenting productivity.[39] They use the resulting estimates to derive predictions for the magnitude of capital-skill complementarity in the US economy. In particular, as the results presented in Appendix A.2.1 show, the nested CES aggregate production function implies

$$\sigma_{\ell e} - \sigma_{he} \qquad = \tfrac{\varsigma-\rho}{\theta_e+\theta_h}, \tag{39}$$

[38] For a better comparison with the heterogeneous price shocks, Table D.7 also displays the same decomposition using a uniform shock obtained by aggregating the firm-level heterogeneous shocks to the user cost of equipment (see Section C.1.7 for more details on the construction of the two shocks), but the conclusions from the decomposition are virtually identical when using the equipment price changes from French national accounts as in Table 6. See Figure C.3 for a comparison between the two aggregate uniform shocks.

[39] For more recent work revisiting their estimates and extending the data to more recent time periods, see, e.g., Castex et al. (2022), Maliar et al. (2022), and Ohanian et al. (2023).

Table 8: Comparison with the Nested CES Aggregate Production Function (KORV)

	Baseline France 1997-2007	KORV US 1963-1992	Our GMM approach US 1963-1992	 France 1997-2007
ς	–	1.67	1.50	0.91
ρ	–	0.67	0.71	0.39
$\sigma_{\ell e} - \sigma_{he}$	0.20	2.39	1.90	0.77
$\frac{d\log(W_h/W_\ell)}{d\log W_e}$	-0.06	-0.39	-0.33	-0.37

Notes: $\sigma_{\ell e} - \sigma_{he}$ is the aggregate degree of capital-skill complementarity and $d\log(W_h/W_\ell)/d\log W_e$ is the elasticity of the skill premium with respect to the equipment price. Column 1 is our baseline, column 2 displays the value for the elasticity estimated by Krusell et al. (2000) using U.S. data between 1963-92, column 3 replicates column 2 but using the estimation approach we provide in Appendix C.4. Finally column 4 displays the value for the elasticities we estimate using French data over our sample years of 1997-2007. We use aggregate factor payments together with a quality-adjusted aggregate equipment price shock $\widetilde{W}_{et}$ and compositionally-adjusted aggregate average wages. We further re-scale factor shares to match the labor share for the entire economy in 1997. Details of the data construction for France are provided in Appendix C.1.7 and C.2.3.

$$\frac{d\log\left(W_h/W_\ell\right)}{d\log W_e} = -\frac{\theta_e(\varsigma-\rho)}{\theta_h\,\varsigma+\theta_e\,\rho}. \tag{40}$$

The first and second columns in Table 8 compare our baseline results on the strength of capital-skill complementarity in the French economy from Section 5.1 with those of Krusell et al. (2000). While both results are consistent with capital-skill complementarity at the aggregate level, the magnitude of our estimates are substantially lower.

We consider three alternative explanations for the gap in the two sets of estimates. First, the weaker capital-skill complementarity that we estimate could be driven by our assumption that firm-level production functions are in the CRESH family. However, as we show in Table 7, we obtain very similar results using nested CES production functions at the firm level. Second, the weaker capital-skill complementarity that we estimate could be driven by our focus on France. To check if this is the case, we estimate the aggregate nested CES production function using French data. In Appendix C.4, we provide a strategy to estimate these aggregate parameters using simple log-linear estimation equations, based on the same data and the same identification assumptions as those of Krusell et al. (2000). As the second and third columns of Table 8 show, our strategy leads to similar estimates to those of Krusell et al. (2000) when using U.S. data. The fourth column of the table shows the results of applying this estimation strategy on French data over the 1997-2007 period used in our baseline estimation. We find estimates that are similar in magnitude to those of Krusell et al. (2000). We conclude that the differences in results are neither driven by our assumption on firm-level production functions nor are they driven by differences between the two economies or the time periods under consideration.[40]

What then explains the gap between the two sets of estimates? Appendix A.5 shows how we can use our baseline estimates to infer the trend in skill-augmenting technical change not embodied in capital equipment. This trend corresponds to the gap between the observed path of the skill premium in data and our predicted path accounting for changes in the equipment price and skill supply in Figure 2a. The figure clearly shows that this gap grows substantially over the 1997-2007 period, indicating the presence of skill-augmenting technical change. The identification assumption of Krusell et al. (2000) rules out this trend. Hence, it overestimates the degree of capital-skill complementarity, attributing to this channel the entirety of the gap between the observed path of the skill premium and the predicted path accounting only for changes in skill supply (the dashed line in Figure 2a). Since our identification strategy relies on exogenous instruments at the firm level, we can avoid this bias by accommodating the presence of such potential trends in factor-augmenting productivity at both the firm, industry, and aggregate levels.

6 Conclusion

Does the aggregate production technology feature capital-skill complementarity? If so, to what extent? And is this aggregate capital-skill complementarity driven primarily by capital-skill complementarity at the firm level, or by a positive correlation between firm-level capital intensity and skill intensity?

To make progress on these questions, we characterized the response of the observed skill premium to changes in the price of equipment capital in a multi-factor, multi-sector framework featuring general, firm-specific, constant-returns-to-scale production functions. We showed how the aggregate elasticities of substitution that determine the degree of capital-skill complementarity depend on firm-level elasticities of production and demand and firm-level factor intensities. We then exploited a number of French administrative data sources to measure firm-level factor intensities and wages for each skill group and to estimate the micro-level elasticities of substitution. We identified statistically significant capital-skill complementarity at the firm level: a 1% decline in the price of equipment generates an approximately 0.06% increase in the observed skill premium in France. And we found that this quantitative result is broadly robust to an alternative

[40] To better match aggregate data and the approach in Krusell et al. (2000), we implement two adjustments to our aggregated data. We rescale factor shares to match the aggregate labor share in 1997 and we apply a quality adjustment to our aggregate capital series to match the average price difference between the series used in Krusell et al. (2000) and the official NIPA data. These adjustments do not drive our results: in their absence we find an even larger aggregate elasticity of the skill premium.

firm-level production function, heterogeneous shocks to the price of equipment across firms, and a more flexible demand system across firms.

Although we found a significant degree of capital-skill complementarity, our identification assumptions and, therefore, results differ markedly from previous estimates obtained from the aggregate time series. In particular, our empirical and theoretical strategies do not necessarily attribute the aggregate increase in the relative demand for skill *entirely* to capital-skill complementarity. And our results imply that capital-skill complementarity is not sufficiently strong to generate the full increase in the relative demand for high-skilled workers observed in the data. Explaining the evolution of the skill premium in France requires a sizable degree of skill-augmenting technical change not embodied in capital equipment.

Appendix to "Capital-Skill Complementarity in Firms and in the Aggregate Economy"

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August 2024

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A Theoretical Appendix

A.1 Allen-Uzawa vs. Morishima Elasticities of Substitution

Define the Morishima elasticity of substitution between factors f and $f' \neq f$ as

$$\sigma^M_{ff'} \equiv \frac{\partial \log \left(X_f / X_{f'} \right)}{\partial \log W_{f'}}, \qquad f \neq f'.$$

Note that, unlike the Allen-Uzawa elasticities, the Morishima elasticities are not symmetric: $\sigma^M_{ff'} \neq \sigma^M_{f'f}$.

The gap between two Morishima elasticities—between factors f and $f'' \neq f$ and between factors f' and $f'' \neq f'$—are given by

$$\sigma^M_{ff''} - \sigma^M_{f'f''} = \frac{\partial \log \left(X_f / X_{f'} \right)}{\partial \log W_{f''}} = \theta_{f''} \left(\sigma_{ff''} - \sigma_{f'f''} \right), \qquad f \neq f' \neq f''.$$

This implies that the numerator of Equation (8) satisfies:

$$\sigma^M_{\ell e} - \sigma^M_{he} = \theta_e \left(\sigma_{\ell e} - \sigma_{he} \right).$$

More generally, we can relate Morishima elasticities to the Allen-Uzawa elasticities defined in Equation (6). Using Equation (6) again, we can write the Morishima elasticity as

$$\begin{aligned}
\sigma^M_{ff'} &= \frac{\partial \log X_f}{\partial \log W_{f'}} - \frac{\partial \log X_{f'}}{\partial \log W_{f'}}, \\
&= \theta_{f'} \sigma_{ff'} + \sum_{f'' \neq f'} \theta_{f''} \sigma_{f'f''}, \\
&= \left(1 - \sum_{f'' \notin \{f, f'\}} \theta_{f''} \right) \sigma_{ff'} + \left(\sum_{f'' \notin \{f, f'\}} \theta_{f''} \right) \sum_{f'' \notin \{f, f'\}} \frac{\theta_{f''}}{\sum_{f'' \notin \{f, f'\}} \theta_{f''}} \sigma_{f'f''}.
\end{aligned}$$

The Morishima elasticity $\sigma^M_{ff'}$ is given by the convex combination of the Allen-Uzawa elasticity $\sigma_{ff'}$ and the factor-intensity weighted mean of $\sigma_{f'f''}$ within the set of factors

$f'' \in \mathcal{F} \setminus \{f, f'\}$, where the latter average is weighted in the linear combination by the cost share of all factors $\mathcal{F} \setminus \{f, f'\}$ in total costs. For instance, for the elasticity $\sigma^M_{\ell h}$, the set $\mathcal{F} \setminus \{\ell, h\} = \{e\}$ and we find the denominator of the expression of Equation (8):

$$\sigma^M_{\ell h} = (1 - \theta_e)\, \sigma_{\ell h} + \theta_e \sigma_{he}.$$

Just like the Allen-Uzawa elasticities, in the case of the CES(σ) production function we find $\sigma^M_{ff'} = \sigma$.

A.2 Functional Forms for Production Functions

A.2.1 Nested CES

Elasticities of Substitution. Solving for factor demand given the production function defined in Equation (12), we find $X_h/X_c = (Z_h W_h / W_c)^{-\rho}$, $X_\ell / Y = (W_\ell / Z_\ell P)^{-\varsigma}$, and $X_c / Y = (W_c / P)^{-\varsigma}$, where we have defined the price of the bundle of equipment and high-skill labor as $W_c = \left((W_h/Z_h)^{1-\rho} + (W_e/Z_e)^{1-\rho} \right)^{\frac{1}{1-\rho}}$ and the price of output as $P = \left((W_\ell/Z_\ell)^{1-\varsigma} + W_c^{1-\varsigma} \right)^{\frac{1}{1-\varsigma}}$. We can now compute the elasticities of substitution as

$$\begin{aligned}
\sigma_{\ell e} &= \frac{1}{\theta_e} \frac{\partial \log X_\ell}{\partial \log W_e} = \frac{1}{\theta_e} \left(\varsigma \frac{\partial \log P}{\partial \log W_e} \right) = \varsigma, \\
\sigma_{\ell h} &= \frac{1}{\theta_h} \frac{\partial \log X_\ell}{\partial \log W_h} = \frac{1}{\theta_h} \left(\varsigma \frac{\partial \log P}{\partial \log W_h} \right) = \varsigma, \\
\sigma_{he} &= \frac{1}{\theta_e} \left(\frac{\partial \log (X_h/X_c)}{\partial \log W_c} \frac{\partial \log W_c}{\partial \log W_e} + \frac{\partial \log X_c}{\partial \log W_e} \right), \\
&= \frac{1}{\theta_e} \left(\rho \frac{\partial \log W_c}{\partial \log W_e} - \varsigma \left(\frac{\partial \log W_c}{\partial \log W_e} - \frac{\partial \log P}{\partial \log W_e} \right) \right), \\
&= \frac{1}{\theta_e} \left((\rho - \varsigma) \frac{\theta_e}{\theta_e + \theta_h} + \varsigma\, \theta_e \right) = \varsigma - \frac{\varsigma - \rho}{\theta_e + \theta_h},
\end{aligned}$$

where we have substituted from the definitions of W_c and X_c above, and have used the equality $\frac{\partial \log W_c}{\partial \log W_e} = \left(\frac{W_e/Z_e}{W_c} \right)^{1-\rho} = \frac{W_e X_e}{W_c X_c} = \frac{\theta_e}{\theta_e + \theta_h}$.

Demand System. Let us consider a sequence of factor prices $\boldsymbol{W}_t$ and input choices $\boldsymbol{X}_t$ characterized by the nested CES factor demand system defined in Equation (12). Relative factor demands satisfy

$$\frac{X_{\ell t}}{X_{ct}} = \left(\frac{W_{\ell t}/Z_{\ell t}}{W_{ct}}\right)^{-\varsigma}, \qquad \frac{X_{ht}}{X_{ct}} = \left(\frac{W_{ht}/Z_{ht}}{W_{ct}}\right)^{-\rho}, \tag{A.1}$$

Using these expressions, we can write the skill premium as

$$\frac{W_{ht}}{W_{\ell t}} = \frac{Z_{ht}}{Z_{\ell t}} \left(\frac{X_{\ell t}}{X_{ht}}\right)^{\frac{1}{\varsigma}} \left(\frac{X_{ht}}{X_{ct}}\right)^{\frac{1}{\varsigma}-\frac{1}{\rho}}. \tag{A.2}$$

Next, we use Equation (A.1) again to write

$$\frac{\theta_{ht}}{\theta_{et}+\theta_{ht}} = \frac{W_{ht}X_{ht}}{W_{ct}X_{ct}} = Z_{ht}^{\rho} \left(\frac{W_{ht}}{W_{ct}}\right)^{1-\rho},$$

which we use to substitute for X_{ht}/X_{ct} in Equation (A.2) to find

$$\frac{W_{ht}}{W_{\ell t}} = Z_{ht}^{\frac{\rho}{1-\rho}\left(\frac{1}{\varsigma}-\frac{1}{\rho}\right)} \frac{Z_{ht}}{Z_{\ell t}} \left(\frac{X_{\ell t}}{X_{ht}}\right)^{\frac{1}{\varsigma}} \left(\frac{\theta_{ht}}{\theta_{et}+\theta_{ht}}\right)^{\frac{\rho}{1-\rho}\left(\frac{1}{\rho}-\frac{1}{\varsigma}\right)}.$$

Combining this equation with the remaning relative factor demand equation,

$$\frac{X_{ht}}{X_{et}} = \left(\frac{Z_{et}W_{ht}}{Z_{ht}W_{t}}\right)^{-\rho}$$

we obtain the following demand system

$$x_{ht} - x_{\ell t} = -\varsigma\left(w_{ht} - w_{\ell t}\right) + \frac{\varsigma-\rho}{1-\rho}\log\left(\frac{\theta_{ht}}{\theta_{et}+\theta_{ht}}\right) + z_{ht} - z_{\ell t} + \frac{\rho-\varsigma}{1-\rho} z_{ht}, \tag{A.3}$$

$$x_{ht} - x_{et} = -\rho\left(w_{ht} - w_{et}\right) + \rho\left(z_{ht} - z_{et}\right), \tag{A.4}$$

where $v \equiv \log V$ for any variable V.

A.2.2 CRESH

Elasticities of Substitution. We compute factor cost shares by minimizing unit costs:

$$\min_{(X_f)} \sum_f W_f X_f + \Xi\left(1 - \sum_{f\in\{\ell,h,e\}} \left(\frac{Z_f X_f}{Y}\right)^{\frac{\sigma_f - 1}{\sigma_f}}\right),$$

where Ξ is the Lagrange multiplier on the constraint defining the production function in Equation (15). The first order condition with respect to factor f gives us:

$$W_f = \Xi \frac{\sigma_f - 1}{\sigma_f} \frac{1}{X_f} \left(\frac{Z_f X_f}{Y}\right)^{\frac{\sigma_f - 1}{\sigma_f}},$$

implying $\theta_f \propto W_f X_f \propto \frac{\sigma_f - 1}{\sigma_f} \left(\frac{Z_f X_f}{Y}\right)^{\frac{\sigma_f - 1}{\sigma_f}}$.

It follows that the ratio of payments to factors f and f' satisfy:

$$\frac{W_f}{W_{f'}} = \frac{\frac{\sigma_f - 1}{\sigma_f} \left(\frac{Z_f}{Y}\right)^{\frac{\sigma_f - 1}{\sigma_f}} X_f^{-\frac{1}{\sigma_f}}}{\frac{\sigma_{f'} - 1}{\sigma_{f'}} \left(\frac{Z_{f'}}{Y}\right)^{\frac{\sigma_{f'} - 1}{\sigma_{f'}}} X_{f'}^{-\frac{1}{\sigma_{f'}}}}. \tag{A.5}$$

It then follows that for any $f \neq f'$, we find the following condition on the price elasticities of factor demand

$$1 = \frac{1}{\sigma_f} \frac{\partial \log X_f}{\partial \log W_{f'}} - \frac{1}{\sigma_{f'}} \frac{\partial \log X_{f'}}{\partial \log W_{f'}}. \tag{A.6}$$

Using the constraint (15) again, we can find another constraint on the price elasticities of factor demand

$$0 = \frac{\partial}{\partial \log W_{f'}} \left[\sum_{f \in \mathcal{F}} \left(\frac{Z_f X_f}{Y}\right)^{\frac{\sigma_f - 1}{\sigma_f}} \right] = \sum_f \left(\frac{\sigma_f - 1}{\sigma_f}\right) \left(\frac{Z_f X_f}{Y}\right)^{\frac{\sigma_f - 1}{\sigma_f}} \frac{\partial \log X_f}{\partial \log W_{f'}} = \sum_f \theta_f \frac{\partial \log X_f}{\partial \log W_{f'}}, \tag{A.7}$$

where in the last equality we have used the result that $\theta_f \propto \frac{\sigma_f - 1}{\sigma_f} \left(\frac{Z_f X_f}{Y}\right)^{\frac{\sigma_f - 1}{\sigma_f}}$ proven above.

From Equation (A.7), we find

$$\begin{aligned}
0 &= \theta_{f'} \frac{\partial \log X_{f'}}{\partial \log W_{f'}} + \sum_{f \in \mathcal{F}/\{f'\}} \theta_f \left(\sigma_f + \frac{\sigma_f}{\sigma_{f'}} \frac{\partial \log X_{f'}}{\partial \log W_{f'}} \right), \\
&= \left(\sum_{f \in \mathcal{F}} \theta_f \sigma_f \right) \frac{\partial \log X_{f'}}{\partial \log W_{f'}} + \sigma_{f'} \left(\sum_{f \in \mathcal{F}/\{f'\}} \theta_f \sigma_f \right), \\
&= \bar{\sigma} \left(\frac{\partial \log X_{f'}}{\partial \log W_{f'}} + \sigma_{f'} \left(1 - \frac{\theta_{f'} \sigma_{f'}}{\bar{\sigma}} \right) \right),
\end{aligned}$$

where in the first equality we have substituted for $\frac{\partial \log X_f}{\partial \log W_{f'}}$ from Equation (A.6), where we have multiplied the expression by $\sigma_{f'}$ in the second equality, and where we have defined

$\overline{\sigma} \equiv \sum_{f \in \mathcal{F}} \theta_f \sigma_f$ in the third equality. It then follows that:

$$\frac{\partial \log X_{f'}}{\partial \log W_{f'}} = \frac{\sigma_{f'}^2}{\overline{\sigma}} \theta_{f'} - \sigma_{f'}.$$

Substituting this expression in Equation (A.6), we find

$$\begin{aligned}\frac{\partial \log X_f}{\partial \log W_{f'}} &= \sigma_f + \frac{\sigma_f}{\sigma_{f'}} \frac{\partial \log X_{f'}}{\partial \log W_{f'}}, \\ &= \frac{\sigma_f \sigma_{f'}}{\overline{\sigma}} \theta_{f'}.\end{aligned}$$

Equation (16) then follows from the definition of the elasticities of substitution.

Demand System. Let us consider a sequence of factor prices $\boldsymbol{W}_t$ and input choices $\boldsymbol{X}_t$ characterized by the CRESH factor demand system defined in Equation (15). Rewriting Equation (A.5) in logs, we find for $f \in \{\ell, h\}$:

$$\begin{aligned}w_{et} - w_{ft} = \frac{1}{\sigma_f} x_{ft} - \frac{1}{\sigma_e} x_{et} + \left(\frac{1}{\sigma_e} - \frac{1}{\sigma_f}\right) y_t + \left(1 - \frac{1}{\sigma_e}\right) z_{et} - \left(1 - \frac{1}{\sigma_\ell}\right) z_{ft} \\ + \log\left(\frac{\sigma_e - 1}{\sigma_e}\right) - \log\left(\frac{\sigma_f - 1}{\sigma_f}\right).\end{aligned}$$

From this expression, the following demand system follows for $f \in \{\ell, h\}$

$$x_{et} - x_{ft} = -\sigma_f \left(w_{et} - w_{ft}\right) + \left(\frac{\sigma_f}{\sigma_e} - 1\right)(y_t - x_{et}) + (1 - \sigma_f) z_{ft} + \left(\frac{\sigma_f}{\sigma_e} - 1\right) z_{et} + \omega_f, \tag{A.8}$$

where we have defined $\omega_f \equiv \sigma_f \log\left(\frac{\sigma_f - 1}{\sigma_f}\right) - \sigma_f \log\left(\frac{\sigma_e - 1}{\sigma_e}\right)$.

A.3 Theoretical Extensions of the Baseline Setting

A.3.1 Elastic Relative Labor Supply

The following proposition generalizes our main result in Proposition 1 to the case with a relative supply of skilled workers that responds to the relative wages.

Proposition A.1. *Consider the setting in Proposition 1 but assume that the skill supplies $\overline{X}_\ell\left(W_\ell, W_h\right)$ and $\overline{X}_h\left(W_\ell, W_h\right)$ are homogeneous of degree-zero functions of wages, such that the elasticity of the* *relative skill supply with respect to the skill premium is ξ:*

$$\frac{\partial \log \overline{X}_h / \overline{X}_\ell}{\partial \log W_h / W_\ell} = \xi \geq 0.$$

Assume a small shock to the equipment supply $\overline{X}_e$. The effect on the skill premium, i.e., the relative prices of high-skilled and low-skilled labor, satisfies Equation (22) where the aggregate elasticities of substitution $\sigma_{ff'}$ for $f, f' \in \mathcal{F}$ are defined by Equation (6).

Proof. We use the fact that factor supply and factor demand equalize in equilibrium, $\overline{\boldsymbol{X}} = \boldsymbol{X}$. In this case, with the low-skilled wage as the numeraire $W_\ell \equiv 1$, the left hand side of Equation (9) is given by $\frac{d \log(\overline{X}_h / \overline{X}_\ell)}{d \log W_e} = \frac{\partial \log(\overline{X}_h / \overline{X}_\ell)}{\partial \log W_h} \frac{d \log W_h}{d \log W_e} = \xi \frac{d \log W_h}{d \log W_e}$. Using the same additional steps as those in the proof of Lemma 1 leads to the desired result. □

A.3.2 Factor Price Heterogeneity

We begin this section with a proposition that generalizes our results in Section 2.2 to the case involving heterogeneous equipment prices, as discussed in the extension presented in Section 2.3.2. We then generalize the environment to one that features factor price heterogeneity across all factors, including unskilled and skilled labor.

Heterogeneity in Equipment Prices and the Response of the Skill Premium The first proposition below is the formal statement of the results presented in Section 2.3.2.

Proposition A.2. *Consider a setting with three-factors, $\mathcal{F} \equiv \{\ell, h, e\}$, with exogenous factor supplies and with firm-specific factor prices. Assume small shocks to equipment factor prices characterized by $d\overline{W}_e$ for a given vector of exposure weights $\boldsymbol{\omega}_e \equiv (\omega_{ei})$. Then, the response of the skill premium to this collection of heterogeneous shocks in the price of equipment is given by Equation (24), where the heterogeneity-adjusted elasticity of substitution satisfies Equation (25).*

Proof. This result is a special case of the results of Propositions A.3 and A.4 below for the case in which labor wages are uniform across firms. □

General Heterogeneity-Adjusted Elasticities of Substitution Let us now assume that firms face potential wedges in *all* factor markets and pay firm-and-factor-specific prices. We assume a fixed and exogenous wedge T_{fi} for each firm i and factor f. The firm-level factor price is $W_{fi} = W_f T_{fi}$, where we refer to W_f as the shadow price (or wage) of factor f, since it is unobservable in the data. The shadow factor price is common across firms and equates supply and demand, $X_f = \sum_i X_{fi}$. We take the shadow wage of low-skilled workers as the numeraire and normalize $W_\ell = 1$. In Appendix A.6, we present a micro-foundation for the labor-market wedges based on firm-and-labor-type specific compensating differentials in competitive factor markets (the appendix also offers a micro-foundation for firm-level heterogeneity in the price of equipment).

At the micro-level, we can express the elasticity of the factor intensity of factor f with respect to the price of factor f' in terms of the elasticity of substitution from Equation (2) as

$$\frac{\partial \log \theta_{fi}}{\partial \log W_{f'}} = \frac{\partial \log \left(W_{fi} X_{fi} / C_i\right)}{\partial \log W_{f'}} = \theta_{f'i} \left(\sigma_{ff',i} - 1\right).$$

Moving to the aggregate case, in the presence of factor price heterogeneity across firms, in general we have $\theta_f \neq W_f X_f / C$, thus preventing us from writing a direct macro counterpart to the above equation. Instead, we may define a distinct measure of substitutability across factors in terms of factor payments. Let $\sigma^*_{ff'}$ for $f' \neq f$ denote the aggregate elasticity of factor payment substitution as

$$\sigma^*_{ff'} \equiv 1 + \frac{1}{\theta_{f'}} \frac{\partial \log \theta_f}{\partial \log W_{f'}}, \tag{A.9}$$

where $\theta_f \equiv \left(\sum_i W_{fi} X_{fi}\right) / \left(\sum_{f',i'} W_{f'i'} X_{f'i'}\right)$ denotes the aggregate factor-f intensity of all factor payments in the economy. When there are no wedges ($T_{fi} \equiv T_f$ for all i), the two aggregate elasticities of substitution defined in Equations (6) and (A.9) coincide, $\sigma_{ff'} \equiv \sigma^*_{ff'}$. More generally, however, the two elasticities may diverge due to the fact that reallocations across firms have distinct effects on aggregate factor demand and factor payments.

In parallel to the discussion in Section 2.3.2, we can also consider heterogeneous shocks to factor prices. Let $d \log W_{fi}$ denote a small change in the log price of factor f faced by firm i, and define $d \log \overline{W}_f \equiv \sum_i \Lambda_i \frac{\theta_{ei}}{\theta_e} d \log W_{ei}$ where Λ_i is defined in Equation (18), and exposure weights ω_{fi} satisfying $d \log W_{fi} = \omega_{fi} d \log \overline{W}_f$. As in the case of equipment price changes, we fix the shares ω_{fi} across firms and consider the comparative statics with respect to the average magnitude $d \log \overline{W}_f$ of the shock in the price of equipment. In parallel to our definition of the aggregate elasticities of substitution (6) and (A.9), we can define the heterogeneity-adjusted aggregate elasticities capturing the effect of the firm-specific shocks to factor-f' prices on the relative demand and payments for factor f as

$$\sigma^{\omega}_{ff'} \equiv \frac{1}{\theta_{f'}} \frac{\partial \log X_f}{\partial \log \overline{W}_{f'}} \Bigg|_{\left(\omega_{f'i}\right)}, \qquad \sigma^{*,\omega}_{ff'} \equiv 1 + \frac{1}{\theta_f} \frac{\partial \log \left(\theta_f / \theta_{f'}\right)}{\partial \log \overline{W}_{f'}} \Bigg|_{\left(\omega_{f'i}\right)}, \tag{A.10}$$

where the superscript ω indicates the fact that the elasticity is defined with respect to a given set of firm-specific weights $(\omega_{f'i})$. Note that the aggregate elasticities of substitution defined in Equations (6) and (A.9) are special cases of the aggregate wedge elasticity defined in Equation (A.10) corresponding to the case of $\omega_{fi} \equiv 1$. The next proposition characterizes the aggregate wedge elasticities in their general form.

Proposition A.3. *The aggregate heterogeneity-adjusted elasticities of substitution $\sigma^{\omega}_{ff'}$ and $\sigma^{*,\omega}_{ff'}$ between factors f and f' defined by Equations (A.10) are given by*

$$\sigma^{\omega}_{ff'} = \mathbb{E}_i\left[\frac{\overline{W}_f}{W_{fi}}\frac{\theta_{fi}}{\theta_f}\frac{\theta_{f'}}{\theta_{f'}}\omega_{f'i}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\frac{\overline{W}_f}{W_{fi}}\frac{\theta_{fi}}{\theta_f}, \frac{\theta_{f'i}}{\theta_{f'}}\omega_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\frac{\overline{W}_f}{W_{fs}}\frac{\theta_{fs}}{\theta_f}, \frac{\theta_{f's}}{\theta_{f'}}\omega_{f's}\right), \tag{A.11}$$

$$\sigma^{*,\omega}_{ff'} = \mathbb{E}_i\left[\frac{\theta_{fi}}{\theta_f}\frac{\theta_{f'}}{\theta_{f'}}\omega_{f'i}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\frac{\theta_{fi}}{\theta_f}, \frac{\theta_{f'i}}{\theta_{f'}}\omega_{f'i}\right)\right] - \eta\,\mathbb{C}_s\left(\frac{\theta_{fs}}{\theta_f}, \frac{\theta_{f's}}{\theta_{f'}}\omega_{f's}\right). \tag{A.12}$$

where we have defined the mean sector-specific wedge shock ω_{fs} as

$$\omega_{fs} \equiv \frac{\sum_{i\in\mathcal{I}_s}\Lambda_{i|s}\theta_{fi}\omega_{fi}}{\sum_{i\in\mathcal{I}_s}\Lambda_{i|s}\theta_{fi}}, \tag{A.13}$$

as the mean of the exposure weights based on the distribution of the share of sector-s payments to factor f paid by firm i.

Proof. See Appendix A.4 on page A18. □

It is straightforward to see that in the cases where factor price shocks are uniform across firms, the above results simplify to

$$\sigma_{ff'} = \mathbb{E}_i\left[\frac{\overline{W}_f}{W_{fi}}\frac{\theta_{fi}}{\theta_f}\frac{\theta_{f'}}{\theta_{f'}}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\frac{\overline{W}_f}{W_{fi}}\frac{\theta_{fi}}{\theta_f}, \frac{\theta_{f'i}}{\theta_{f'}}\right)\right] - \eta\,\mathbb{C}_s\left(\frac{\overline{W}_f}{W_{fs}}\frac{\theta_{fs}}{\theta_f}, \frac{\theta_{f's}}{\theta_{f'}}\right), \tag{A.14}$$

$$\sigma^{*}_{ff'} = \mathbb{E}_i\left[\frac{\theta_{fi}}{\theta_f}\frac{\theta_{f'}}{\theta_{f'}}\sigma_{ff',i}\right] - \varepsilon\,\mathbb{E}_s\left[\mathbb{C}_{i|s}\left(\frac{\theta_{fi}}{\theta_f}, \frac{\theta_{f'i}}{\theta_{f'}}\right)\right] - \eta\,\mathbb{C}_s\left(\frac{\theta_{fs}}{\theta_f}, \frac{\theta_{f's}}{\theta_{f'}}\right). \tag{A.15}$$

In the presence of exogenous distortions, Equations (A.14) and (A.15) decompose the two aggregate elasticities of substitution into three components: the first term accounts for the appropriate weighted average of the within-firm elasticities of substitution across firms, the second term accounts for the effect of within-sector reallocations across firms, and the last term accounts for the effect of cross-sectoral reallocations. In the case of the aggregate elasticity of substitution for demand (payments), the averages across firms are weighted by their shares in aggregate demand for (payment to) factors.

The Response of the Skill Premium with Heterogeneous Wages In the presence of wage heterogeneity, we define the *observed skill premium* Ψ as the average wage of high-skilled workers relative to the average wage of low-skilled workers,

$$\Psi \equiv \frac{\overline{W}_h}{\overline{W}_\ell} = \frac{\sum_i W_{hi} X_{hi} / X_h}{\sum_{i'} W_{\ell i'} X_{\ell i'} / X_\ell}, \tag{A.16}$$

where $X_\ell \equiv \sum_i X_{\ell i}$ and $X_h \equiv \sum_i X_{hi}$ are the aggregate demands for high- and low-skilled labor, respectively. We aim to characterize the response of the skill premium to a shock to the prices of different factors, particularly equipment prices. In our model, such changes in equipment prices can stem from changes in the aggregate supply of equipment goods X_e or from changes in firm-specific wedges T_{ei}.

Proposition A.4. *Consider a setting with three-factors, $\mathcal{F} \equiv \{\ell, h, e\}$, with exogenous factor supplies and with firm-specific factor prices. Assume a small shocks to the equipment factor prices characterized by $d\overline{W}_e$ and the vector of exposure weights $\boldsymbol{\omega}_e \equiv (\omega_{ei})$. Then, the responses of the shadow and observed skill premium to this collection of heterogeneous shocks in the price of equipment are given by*

$$d \log W_h = -\frac{\theta_e \left(\sigma^{\omega}_{\ell e} - \sigma^{\omega}_{he}\right)}{(1-\theta_e)\,\sigma_{\ell h} + \theta_e \sigma_{eh}} d \log \overline{W}_e, \tag{A.17}$$

$$\begin{aligned} d \log \Psi = &-\frac{\theta_e \left(\sigma^{\omega}_{\ell e} - \sigma^{\omega}_{he}\right)}{(1-\theta_e)\,\sigma_{\ell h} + \theta_e \sigma_{he}} d \log \overline{W}_e \\ &+ \left[(1-\theta_e)\,\sigma^*_{\ell h} + \theta_e \sigma^*_{eh}\right] \left(\frac{\theta_e \left(\sigma^{\omega}_{\ell e} - \sigma^{\omega}_{he}\right)}{(1-\theta_e)\,\sigma_{\ell h} + \theta_e \sigma_{he}} - \frac{\theta_e \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he}\right)}{(1-\theta_e)\,\sigma^*_{\ell h} + \theta_e \sigma^*_{he}} \right) d \log \overline{W}_e. \end{aligned} \tag{A.18}$$

Proof. See Appendix A.4 on page A20. □

In the special case where the equipment price changes are uniform, e..g, driven by a small shocks to the equipment factor prices characterized by supply $\overline{X}_e$ that leaves the supplies of other factors intact, we can specialize the above results as above. The effect on the shadow skill premium $\frac{d \log W_h}{d \log W_e}$, i.e., the relative shadow prices of high-skilled and low-skilled labor, satisfies Equation (8), and the effect on the observed skill premium is given by

$$\frac{d\log\Psi}{d\log W_e} = \frac{d\log W_h}{d\log W_e} - \left[(1-\theta_e)\,\sigma^*_{\ell h} + \theta_e\sigma^*_{eh}\right]\left(\frac{d\log W_h}{d\log W_e} + \frac{\theta_e\left(\sigma^*_{\ell e} - \sigma^*_{he}\right)}{(1-\theta_e)\,\sigma^*_{\ell h} + \theta_e\sigma^*_{he}}\right), \quad \text{(A.19)}$$

where the aggregate elasticities of substitution $\sigma_{ff'}$ and $\sigma^*_{ff'}$ for $f \neq f' \in \mathcal{F}$ are defined by Equations (6) and (A.9), respectively. Comparing Equations (A.19) and (8), we see that the gap between the observed and the shadow skill premia exists to the extent that the aggregate elasticities in terms of factor demand and factor payments deviate from one another, that is, $\frac{\theta_e\left(\sigma^*_{\ell e}-\sigma^*_{he}\right)}{(1-\theta_e)\sigma^*_{\ell h}+\theta_e\sigma^*_{he}} \neq \frac{\theta_e(\sigma_{\ell e}-\sigma_{he})}{(1-\theta_e)\sigma_{\ell h}+\theta_e\sigma_{he}}$.

A.3.3 Composite Factors of Production: the Case of Labor Share

We can use our results to derive the elasticities of substitution for composite factors that are bundles of other factors. Let factor c be defined as a composite of a set $\mathcal{F}_c$ of other factors. For instance, we typically define labor (which we denote with label n) as the sum of all workers irrespective of their skill type such that $X_n \equiv X_\ell + X_h$ where we have $\mathcal{F}_n \equiv \{\ell, h\}$. We can show that the aggregate elasticity of factor payments substitution in Equation (A.9), when defined between a composite factor c and any other factor f' that is not within the composite factor ($f' \notin \mathcal{F}_c$), is simply the convex combination of all the factors constituting the composite, that is,

$$\sigma^*_{cf'} \equiv 1 + \frac{\partial\log\theta_c}{\partial\log W_{f'}} = \sum_{f\in\mathcal{F}_c}\frac{\theta_f}{\theta_c}\,\sigma^*_{ff'} = \sum_{f\in\mathcal{F}_c}\frac{\theta_f}{\theta_c}\,\sigma_{ff'}, \qquad f' \notin \mathcal{F}_c, \quad \text{(A.20)}$$

where the intensity of the composite factor is sum of the intensities of all the factors included in it, $\theta_c \equiv \sum_{f\in\mathcal{F}_c}\theta_f$, and where $\sigma^*_{ff'}$ is the aggregate elasticity of factor payments between f and f', given by Equation (A.15). In the example of labor as the composite factor, the aggregate elasticity of substitution between labor and equipment is a convex combination of the aggregate elasticities of the factor payments for the two skill groups, with each group's weight given by the share of that skill group in labor payments, that is, $\sigma^*_{ne} = \frac{\theta_\ell}{\theta_n}\sigma^*_{\ell e} + \frac{\theta_h}{\theta_n}\sigma^*_{he}$.

The following result characterizes the elasticity of the aggregate share of a composite factor c with respect to the price of factor f'.

Proposition A.5. *The elasticity of the aggregate intensity of a composite factor c relative to the* *price of a given factor f' is given by*

$$\frac{\partial \log \theta_c}{\partial \log W_{f'}} = \begin{cases} \theta_{f'} \left(\sigma^*_{cf'} - 1 \right), & f' \notin \mathcal{F}_c, \\ \theta_{f'} \left[\frac{\theta_{\bar{c}_{f'}}}{\theta_c} \left(\sigma^*_{\bar{c}_{f'} f'} - 1 \right) - \frac{\theta_{\bar{f}'}}{\theta_c} \left(\sigma_{\bar{f}' f'} - 1 \right) \right], & f' \in \mathcal{F}_c, \end{cases} \tag{A.21}$$

where we have defined two composite factors: a composite factor $\overline{f}$ as the set of all factors other than f (with $\mathcal{F}_{\overline{f}} \equiv \mathcal{F} / \{f\}$) and composite factor $\bar{c}_f$ as the set of all factors in the composite factor c other than f (with $\mathcal{F}_{\bar{c}_f} \equiv \mathcal{F}_c / \{f\}$).

Proof. See Appendix A.4 on page A21. □

In the example of the composite labor factor n, and in a three-factor model with $\mathcal{F} = \{\ell, h, e\}$, Equation (A.21) implies that the elasticity of the labor share with respect to the price of equipment and the wage of high skilled workers are given by $\frac{\partial \log \theta_n}{\partial \log W_e} = \theta_e \left(\sigma^*_{ne} - 1 \right)$, and $\frac{\partial \log \theta_n}{\partial \log W_h} = \theta_e \left(\frac{\theta_\ell}{\theta_n} \sigma^*_{\ell h} - \sigma_{eh} + \frac{\theta_h}{\theta_n} \right)$ since in this case $\mathcal{F}_{\bar{n}_h} = \{\ell\}$ and $\mathcal{F}_{\bar{e}} = \mathcal{F}_n$.

A.3.4 Additional Factors of Production

To state our main result, we express the aggregate elasticities of substitution in vector and matrix form. Since low-skilled labor is the numeraire and equipment is the factor experiencing the factor price shock, we do not include them in the vector and matrix form, defining the matrix $\mathbf{\Sigma}$ and vectors $\boldsymbol{\sigma}_{\cdot f'}$ as

$$\mathbf{\Sigma} \equiv \left[\sigma_{ff'} \right]_{f \in \mathcal{F}}, \qquad \boldsymbol{\theta} \equiv \left(\theta_\ell, \theta_h, \cdots, \theta_e \right)', \tag{A.22}$$

and where the aggregate substitution elasticity of factor demand $\sigma_{ff'}$ is given by Equation (6), with the additional extension $\sigma_{ff} \equiv 0$ for all f. Using these definitions, the following proposition characterizes the response.

Proposition A.6. *Consider a small shock to the supply of equipment that shifts the price of equipment, holding all other factor supplies and wedges constant. Let $\widetilde{\boldsymbol{W}} \equiv \left(W_f \right)_{f \in \mathcal{F} / \{\ell, e\}}$ be the vector of the prices of all factors other than equipment and unskilled labor. The elasticity of this vector with respect to the price of capital equipment is given by*

$$\frac{d \log \widetilde{\boldsymbol{W}}}{d \log W_e} = -\theta_e \left(\widetilde{\boldsymbol{P}}'_R \, \boldsymbol{D} \, \widetilde{\boldsymbol{P}}_L \right)^{-1} \left(\sigma_{\ell e} - \widetilde{\boldsymbol{\sigma}}_e \right), \tag{A.23}$$

where we have defined $\widetilde{\boldsymbol{\sigma}} \equiv \left(\sigma_{fe} \right)_{f \in \mathcal{F} / \{\ell, e\}}$, the matrix $\boldsymbol{D}$ is the demand elasticity matrix defined for the matrix of elasticities of substitution and the vector of factor intensities in Equation (A.22) *as*

$$\boldsymbol{D} = diag\left(\boldsymbol{\Sigma}\,\boldsymbol{\theta}\right) - \boldsymbol{\Sigma}\,diag\left(\boldsymbol{\theta}\right), \tag{A.24}$$

and where the projection matrices $\widetilde{\boldsymbol{P}}_R$ and $\widetilde{\boldsymbol{P}}_L$ are $N \times (N-2)$ dimensional matrices defined as

$$\widetilde{\boldsymbol{P}}_R \equiv \begin{bmatrix} -1 & -1 & \cdots & -1 \\ 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \qquad \widetilde{\boldsymbol{P}}_L \equiv \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{A.25}$$

Proof. See page A17 in Appendix A.4. □

It is straightforward to show that the above results leads to Proposition 1 in the special case of $N = 3$. In this case, we have

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0 & \sigma_{\ell h} & \sigma_{\ell e} \\ \sigma_{\ell h} & 0 & \sigma_{he} \\ \sigma_{\ell e} & \sigma_{he} & 0 \end{bmatrix}, \qquad \boldsymbol{D} = \begin{bmatrix} \theta_h\sigma_{\ell h} + \theta_e\sigma_{\ell e} & -\theta_h\sigma_{\ell h} & -\theta_e\sigma_{\ell e} \\ -\theta_\ell\sigma_{\ell h} & \theta_\ell\sigma_{\ell h} + \theta_e\sigma_{he} & -\theta_e\sigma_{he} \\ -\theta_\ell\sigma_{\ell e} & -\theta_h\sigma_{he} & \theta_\ell\sigma_{\ell e} + \theta_h\sigma_{he} \end{bmatrix},$$

and we also have

$$\widetilde{\boldsymbol{P}}_R \equiv \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \qquad \widetilde{\boldsymbol{P}}_L \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

which leads to

$$\widetilde{\boldsymbol{P}}_R'\,\boldsymbol{D}\,\widetilde{\boldsymbol{P}}_L = \theta_h\sigma_{\ell h} + \theta_\ell\sigma_{\ell h} + \theta_e\sigma_{he} = \left(1-\theta_e\right)\sigma_{\ell h} + \theta_e\sigma_{he}.$$

Noting $\widetilde{\boldsymbol{W}} = W_h$ and $\widetilde{\boldsymbol{\sigma}}_e = \sigma_{he}$, we find the desired result in Equation (8).

A.4 Proofs and Derivations

A.4.1 Proofs

Proof for Proposition 2. Let C_i, C_s, and C stand for the firm-level, sector-level, and aggregate unit cost of production, respectively. To compute the aggregate elasticities of substitution defined in Equations (6), first note that

$$\frac{\partial \log X_f}{\partial \log W_{f'}} = \frac{1}{X_f} \sum_i X_{fi} \frac{\partial \log X_{fi}}{\partial \log W_{f'}},$$

$$= \sum_i \frac{X_{fi}}{X_f} \left[\frac{\partial \log \left(X_{fi}/Y_i \right)}{\partial \log W_{f'}} + \frac{\partial \log Y_i}{\partial \log W_{f'}} \right], \tag{A.26}$$

$$= \sum_i \Lambda_i \frac{\theta_{fi}}{\theta_f} \left(\theta_{f'i} \sigma_{ff',i} + \frac{\partial \log Y_i}{\partial \log W_{f'}} \right), \tag{A.27}$$

where in the last equality, we have used the fact that the production function is CRS and that the elasticity of substitution is defined for a constant scale of output, and the fact that $X_{fi}/X_f = C_i/C \times \left(W_f X_{fi}/C_i \right) / \left(W_f X_f / C \right) = \Lambda_i \theta_{fi}/\theta_f$. Crucially, note that the term $\frac{\partial \log Y_i}{\partial \log W_{f'}}$ captures the reallocations of production Y_i due to the response of firm-level demand in the product markets.

With monopolistic competition and CES demand in Equation (4) and cost minimization, we obtain

$$\frac{\partial \log Y_i}{\partial \log W_{f'}} = -\varepsilon \left(\frac{\partial \log C_i}{\partial \log W_{f'}} - \frac{\partial \log C_s}{\partial \log W_{f'}} \right) - \eta \left(\frac{\partial \log C_s}{\partial \log W_{f'}} - \frac{\partial \log C}{\partial \log W_{f'}} \right),$$

$$= -\varepsilon \left(\theta_{f'i} - \theta_{f's} \right) - \eta \left(\theta_{f's} - \theta_{f'} \right), \tag{A.28}$$

where, we have kept the scale of aggregate output Y as fixed, and where in the second equality, we have used Shephard's lemma, given the fact that the allocation of factors across firms within sectors is efficient. Substituting Equation (A.28) in Equation (A.27), we find

$$\sigma_{ff'} = \frac{1}{\theta_f \theta_{f'}} \left\{ \sum_i \Lambda_i \theta_{fi} \theta_{f'i} \sigma_{ff',i} - \varepsilon \sum_i \Lambda_i \theta_{fi} \left(\theta_{f'i} - \theta_{f's} \right) - \eta \sum_i \Lambda_i \theta_{fi} \left(\theta_{f's} - \theta_{f'} \right) \right\},$$

$$= \frac{1}{\theta_f \theta_{f'}} \left\{ \sum_i \Lambda_i \theta_{fi} \theta_{f'i} \sigma_{ff',i} - \varepsilon \sum_s \Lambda_s \sum_{i \in \mathcal{I}_s} \Lambda_{i|s} \theta_{fi} \left(\theta_{f'i} - \theta_{f's} \right) - \eta \sum_s \Lambda_s \sum_{i \in \mathcal{I}_s} \Lambda_{i|s} \theta_{fi} \left(\theta_{f's} - \theta_{f'} \right) \right\},$$

$$= \frac{1}{\theta_f \theta_{f'}} \left\{ \mathbb{E}_i \left[\theta_{fi} \theta_{f'i} \sigma_{ff',i} \right] - \varepsilon \, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\theta_{fi}, \theta_{f'i} \right) \right] - \eta \, \mathbb{C}_s \left(\theta_{fs}, \theta_{f's} \right) \right\},$$

$$= \mathbb{E}_i \left[\frac{\theta_{fi}}{\theta_f} \frac{\theta_{f'i}}{\theta_{f'}} \sigma_{ff',i} \right] - \varepsilon \, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\frac{\theta_{fi}}{\theta_f}, \frac{\theta_{f'i}}{\theta_{f'}} \right) \right] - \eta \, \mathbb{C}_s \left(\frac{\theta_{fs}}{\theta_f}, \frac{\theta_{f's}}{\theta_{f'}} \right),$$

where we have defined the expected value and covariance operators $\mathbb{E}_i \left(\cdot \right)$ and $\mathbb{C}^i \left(\cdot, \cdot \right)$, respectively, across firms i with a distribution implied by the shares of firms in aggregate payments Λ_i. Similarly, the same operators are defined across sectors $\mathbb{E}_s(\cdot)$ and $\mathbb{C}_s(\cdot,\cdot)$, with the corresponding shares across sectors Λ_s, and across firms within a given sector s as $\mathbb{E}_{i|s}(\cdot)$ and $\mathbb{C}_{i|s}(\cdot,\cdot)$, with the corresponding shares across sectors $\Lambda_{i|s} \equiv \Lambda_i/\Lambda_s$. □

Proof of Equation (11). From Equation (10), we have the following condition for a fall in the real wages of low-skilled workers:

$$\begin{aligned}
1 &< \frac{\theta_h\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}, \\
&= \frac{\theta_h\left(\sigma_{\ell}-\sigma_{h}\right)}{\left(1-\theta_e\right)\sigma_{\ell}+\theta_e\sigma_{e}}\frac{\sigma_e}{\sigma_h},
\end{aligned}$$

where we have substituted for $\sigma_{ff'}$ from Equation (16). This simplifies to

$$\begin{aligned}
\frac{\sigma_\ell}{\sigma_h}-1 &> \frac{\left(1-\theta_e\right)\sigma_\ell+\theta_e\sigma_e}{\theta_h\sigma_e}, \\
&= \frac{1}{\theta_h\sigma_e}\left(\sigma_e+\left(1-\theta_e\right)\left(\sigma_\ell-\sigma_e\right)\right), \\
&= \frac{1}{\theta_h}\left(\theta_e+\left(1-\theta_e\right)\frac{\sigma_\ell}{\sigma_e}\right),
\end{aligned}$$

leading to the desired result. □

Proof of Equation (14). Substituting the expressions in Equation (13) in Equation (10), we find

$$\begin{aligned}
\frac{d\log\left(W_\ell/P\right)}{d\log W_e} &= -\theta_e\left(1-\frac{\theta_h\left(\sigma_{\ell e}-\sigma_{he}\right)}{\left(1-\theta_e\right)\sigma_{\ell h}+\theta_e\sigma_{he}}\right), \\
&= -\theta_e\left(1-\frac{\frac{\theta_h}{\theta_e+\theta_h}\left(\varsigma-\rho\right)}{\varsigma-\frac{\theta_e}{\theta_e+\theta_h}\left(\varsigma-\rho\right)}\right), \\
&= -\theta_e\left(\frac{\varsigma-\frac{\theta_e}{\theta_e+\theta_h}\left(\varsigma-\rho\right)-\frac{\theta_h}{\theta_e+\theta_h}\left(\varsigma-\rho\right)}{\varsigma-\frac{\theta_e}{\theta_e+\theta_h}\left(\varsigma-\rho\right)}\right), \\
&= -\theta_e\frac{\rho}{\frac{\theta_h}{\theta_e+\theta_h}\varsigma+\frac{\theta_e}{\theta_e+\theta_h}\rho},
\end{aligned}$$

reaching the desired result. □

Proof for Equation (20). Using Equation (19), we have

$$\sigma_{\ell e}-\sigma_{he} = \mathbb{E}_i\left[\left(\frac{\theta_{\ell i}}{\theta_\ell}\sigma_{\ell e,i}-\frac{\theta_{hi}}{\theta_h}\sigma_{he,i}\right)\frac{\theta_{ei}}{\theta_e}\right]$$

$$
\begin{aligned}
&- \varepsilon\, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\frac{\theta_{\ell i}}{\theta_\ell} - \frac{\theta_{hi}}{\theta_h}, \frac{\theta_{ei}}{\theta_e} \right) \right] - \eta\, \mathbb{C}_s \left(\frac{\theta_{\ell s}}{\theta_\ell} - \frac{\theta_{hs}}{\theta_h}, \frac{\theta_{es}}{\theta_e} \right), \\
= {} & \mathbb{E}_i \left[\left(\frac{1}{2} \frac{\theta_{\ell i}}{\theta_\ell} + \frac{1}{2} \frac{\theta_{hi}}{\theta_h} \right) (\sigma_{\ell e,i} - \sigma_{he,i}) \frac{\theta_{ei}}{\theta_e} \right] + \mathbb{E}_i \left[\left(\frac{\theta_{hi}}{\theta_h} - \frac{\theta_{\ell i}}{\theta_\ell} \right) \left(\frac{1}{2} \sigma_{\ell e,i} + \frac{1}{2} \sigma_{he,i} \right) \frac{\theta_{ei}}{\theta_e} \right] \\
&- \varepsilon\, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\frac{\theta_{\ell i}}{\theta_\ell} - \frac{\theta_{hi}}{\theta_h}, \frac{\theta_{ei}}{\theta_e} \right) \right] - \eta\, \mathbb{C}_s \left(\frac{\theta_{\ell s}}{\theta_\ell} - \frac{\theta_{hs}}{\theta_h}, \frac{\theta_{es}}{\theta_e} \right),
\end{aligned}
$$

where in the second equality we have expanded the term $\frac{\theta_{\ell i}}{\theta_\ell}\sigma_{\ell e,i} - \frac{\theta_{hi}}{\theta_h}\sigma_{he,i}$. □

Proof for Equations (29) and (30). Writing Equation (A.8) from Appendix A.2.2 in first differences for each firm i, we find for $f \in \{\ell, h\}$:

$$
\Delta x_{ei,t} - \Delta x_{fi,t} = -\sigma_f \left(\Delta w_{et} - \Delta w_{ft} \right) + \left(\frac{\sigma_f}{\sigma_e} - 1 \right) \left(\Delta y_{it} - \Delta x_{ei,t} \right) + \left(1 - \sigma_f \right) \Delta z_{fi,t}. \tag{A.29}
$$

Using the demand equation (26), we find the following relationship between the change in output and the change in revenues:

$$
\begin{aligned}
\Delta y_{it} &= -\varepsilon\, \Delta p_{it} + (\varepsilon - \eta)\ \Delta p_{st} + \alpha_t + \Delta \phi_{it} \\
&= \frac{\varepsilon}{\varepsilon - 1} \left(\Delta r_{it} - (\varepsilon - \eta)\ \Delta p_{st} - \alpha_t - \Delta \phi_{it} \right) + (\varepsilon - \eta)\ \Delta p_{st} + \alpha_t + \Delta \phi_{it}, \\
&= \frac{\varepsilon}{\varepsilon - 1} \Delta r_{it} - \frac{\varepsilon - \eta}{\varepsilon - 1} \Delta p_{st} - \frac{1}{\varepsilon - 1} \left(\Delta \alpha_t + \Delta \phi_{it} \right).
\end{aligned}
$$

Substituting the above result in Equation (A.29) leads to the desired result. □

Proof for Equations (31) and (32). Writing Equation (A.8) from Appendix A.2.2 in first differences for each firm i, we find for $f \in \{\ell, h\}$:

$$
\Delta x_{ei,t} - \Delta x_{fi,t} = -\sigma_f \left(\Delta w_{et} - \Delta w_{ft} \right) + \left(\frac{\sigma_f}{\sigma_e} - 1 \right) \left(\Delta y_{it} - \Delta x_{ei,t} \right) + \left(1 - \sigma_f \right) \Delta z_{fi,t}. \tag{A.30}
$$

Using the demand equation (26), we find the following relationship between the change in output and the change in revenues:

$$
\begin{aligned}
\Delta y_{it} &= -\varepsilon\, \Delta p_{it} + (\varepsilon - \eta)\ \Delta p_{st} + \alpha_t + \Delta \phi_{it} \\
&= \frac{\varepsilon}{\varepsilon - 1} \left(\Delta r_{it} - (\varepsilon - \eta)\ \Delta p_{st} - \alpha_t - \Delta \phi_{it} \right) + (\varepsilon - \eta)\ \Delta p_{st} + \alpha_t + \Delta \phi_{it}, \\
&= \frac{\varepsilon}{\varepsilon - 1} \Delta r_{it} - \frac{\varepsilon - \eta}{\varepsilon - 1} \Delta p_{st} - \frac{1}{\varepsilon - 1} \left(\Delta \alpha_t + \Delta \phi_{it} \right).
\end{aligned}
$$

Substituting the above result in Equation (A.30) leads to the desired result. □

Proof for Proposition A.6. The changes in aggregate factor demand and factor prices satisfy

$$\begin{aligned} d\log X_f &= \sum_{f'} \frac{\partial \log X_f}{\partial \log W_{f'}} d\log W_{f'}, \\ &= \frac{\partial \log X_f}{\partial \log W_f} d\log W_f + \sum_{f' \neq f} \frac{\partial \log X_f}{\partial \log W_{f'}} d\log W_{f'}, \\ &= -\left(\sum_{f' \neq f} \theta_{f'} \sigma_{ff'} \right) d\log W_f + \sum_{f' \neq f} \theta_{f'} \sigma_{ff'} d\log W_{f'}, \end{aligned}$$

where in the last equality we have used the expression for the own price elasticity from Equation (3). Accordingly, we can write the above equation in vector form as

$$d\log \boldsymbol{X} = -\boldsymbol{M}\, d\log \boldsymbol{W},$$

where we have organized the vectors as $\boldsymbol{X} \equiv (X_\ell, X_h, \cdots, X_e)'$ and $\boldsymbol{W} \equiv (W_\ell, W_h, \cdots, W_e)'$, and where we have have defined the matrix $\boldsymbol{M}$ according to Equation (A.24). Note that matrix $\boldsymbol{M}$ is not invertible since factor demand is homogeneous of degree zero and we have $\boldsymbol{M}\boldsymbol{1} = \boldsymbol{0}$. Now, define the vectors $\widetilde{\boldsymbol{X}} \equiv (X_f/X_\ell, \cdots)'$ and $\widetilde{\boldsymbol{W}} \equiv (W_f/W_\ell, \cdots)'$ as the vectors of the demand and prices for factors $f \in \mathcal{F} \setminus \{\ell, e\}$. Using the definitions in Equation (A.25) and the additional vector $\widetilde{\boldsymbol{1}}_e \equiv (0, \cdots, 0, 1)'$, we can now write

$$\begin{aligned} d\log \widetilde{\boldsymbol{X}} &= \widetilde{\boldsymbol{P}}_R'\, d\log \boldsymbol{X}, \\ &= -\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, d\log \boldsymbol{W}, \\ &= -\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, \left(d\log \boldsymbol{W} - \boldsymbol{1} d\log W_\ell\right), \\ &= -\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, \widetilde{\boldsymbol{P}}_L\, d\log \widetilde{\boldsymbol{W}} - \widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, \widetilde{\boldsymbol{1}}_e\, d\log \left(\frac{W_e}{W_\ell}\right). \end{aligned} \tag{A.31}$$

Since factor supplies are not impacted by the change in the price of equipment $\frac{d\ln(\overline{X}_f/\overline{X}_\ell)}{d\ln W_e} = 0$ for each $f \notin \{\ell, e\}$, the left hand side of Equation (A.31) is zero. We thus obtain the following equation

$$\begin{aligned} d\log \widetilde{\boldsymbol{W}} &= -\left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, \widetilde{\boldsymbol{P}}_L\right)^{-1} \left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, \widetilde{\boldsymbol{P}}_e\right) d\log W_e, \\ &= -\left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, \widetilde{\boldsymbol{P}}_L\right)^{-1} \left(M_{fe} - M_{\ell e}\right)_{f \in \mathcal{F} \setminus \{\ell, e\}} d\log W_e, \\ &= -\left(\widetilde{\boldsymbol{P}}_R'\, \boldsymbol{M}\, \widetilde{\boldsymbol{P}}_L\right)^{-1} \theta_e \left(\sigma_{\ell e} - \widetilde{\boldsymbol{\sigma}}_e\right) d\log W_e, \end{aligned}$$

Since $0 = \frac{d\log\left(\overline{X}_f/\overline{X}_\ell\right)}{d\log W_e}$, we find for each $f \notin \{\ell, e\}$

$$
\begin{aligned}
0 &= \frac{d\log\left(X_f/X_\ell\right)}{d\log W_e}, \\
&= \frac{\partial \log X_f/X_\ell}{\partial \log W_e} + \frac{\partial \log X_f/X_\ell}{\partial \log W_f}\frac{d\log W_f}{d\log W_e} + \sum_{f' \notin \{e,f,\ell\}} \frac{\partial \log X_f/X_\ell}{\partial \log W_{f'}}\frac{d\log W_{f'}}{d\log W_e}, \\
&= -\theta_e\left(\sigma_{\ell e} - \sigma_{fe}\right) - \left(\sum_{f' \neq f} \theta_{f'}\sigma_{ff'} + \theta_f \sigma_{\ell f}\right)\frac{d\log W_f}{d\log W_e} \\
&\quad + \sum_{f' \notin \{e,f,\ell\}} \theta_{f'}\left(\sigma_{ff'} - \sigma_{\ell f'}\right)\frac{d\log W_{f'}}{d\log W_e}.
\end{aligned}
$$

Rewriting the above expression in matrix form, we find

$$
\begin{aligned}
-\theta_e\left(\sigma_{\ell e} - \boldsymbol{\sigma}_{\cdot e}\right) &= \left[\operatorname{diag}\left(\boldsymbol{\Sigma}\boldsymbol{\theta}\right) + \operatorname{diag}\left(\boldsymbol{\sigma}_{\cdot \ell}\right)\operatorname{diag}\left(\boldsymbol{\theta}\right)\right]\frac{d\log \boldsymbol{W}}{d\log W_e} \\
&\quad + \left[\mathbf{1}\boldsymbol{\sigma}_{\cdot\ell}'\operatorname{diag}\left(\boldsymbol{\theta}\right) - \boldsymbol{\Sigma}\operatorname{diag}\left(\boldsymbol{\theta}\right)\right]\frac{d\log \boldsymbol{W}}{d\log W_e},
\end{aligned} \tag{A.32}
$$

which leads to the desired result presented in Equation (6). □

Proof for Proposition A.3. We follow the same steps as in the proof of Proposition 2 above. First, we have the following relationship between factor demand and firm-specific factor price shocks

$$
\begin{aligned}
d\log X_f &= \frac{1}{X_f}\sum_i \frac{X_{fi}}{X_f}\left[\frac{\partial \log\left(X_{fi}/Y_i\right)}{\partial \log W_{f'i}} d\log W_{f'i} + d\log Y_i\right], \\
&= \frac{1}{X_f}\sum_i \frac{X_{fi}}{X_f}\left[\frac{\partial \log\left(X_{fi}/Y_i\right)}{\partial \log W_{f'i}} \omega_{f'i}\, d\log \overline{W}_{f'} + d\log Y_i\right].
\end{aligned} \tag{A.33}
$$

Moreover, the firm's share of aggregate factor demand now satisfies

$$
\frac{X_{fi}}{X_f} = \frac{C_i}{C}\frac{W_{fi}X_{fi}/C_i}{\overline{W}_f X_f/C}\frac{\overline{W}_f}{W_{fi}} = \Lambda_i \frac{\theta_{fi}}{\theta_f}\frac{\overline{W}_f}{W_{fi}}. \tag{A.34}
$$

Finally, we can rewrite Equation (A.28) as

$$
d\log Y_i = -\varepsilon\left(\frac{\partial \log C_i}{\partial \log W_{f'i}} d\log W_{f'i} - d\log C_s\right) - \eta\left(d\log C_s - d\log C\right),
$$

$$
\begin{aligned}
&= -\varepsilon \left(\frac{\partial \log C_i}{\partial \log W_{f'i}} d\log W_{f'i} - \sum_i \Lambda_{i|s}\theta_{f'i} d\log W_{f'i} \right) \\
&\qquad - \eta \left(\sum_i \Lambda_{i|s}\theta_{f'i} d\log W_{f'i} - \sum_i \Lambda_i \theta_{f'i} d\log W_{f'i} \right), \\
&= -\varepsilon \left(\theta_{f'i}\omega_{f'i} - \sum_i \Lambda_{i|s}\theta_{f'i}\omega_{f'i} \right) d\overline{W}_{f'} - \eta \left(\sum_i \Lambda_{i|s}\theta_{f'i}\omega_{f'i} - \sum_i \Lambda_i \theta_{f'i}\omega_{f'i} \right) d\overline{W}_{f'}, \\
&= -\varepsilon \left(\theta_{f'i}\omega_{f'i} - \mathbb{E}_{i|s}\left[\theta_{f'i}\omega_{f'i} \right] \right) d\overline{W}_{f'} - \eta \left(\theta_{f's}\omega_{f's} - \mathbb{E}_s \left[\theta_{f's}\omega_{f's} \right] \right) d\overline{W}_{f'}, \quad \text{(A.35)}
\end{aligned}
$$

where in the second equality we have used Shephard's lemma for the unit cost of sectoral production, and in the fourth equality we have let $\theta_{f's} \equiv \sum_i \Lambda_{i''s}\theta_{f'i}$ and defined $\omega_{f's}$ as in Equation (A.13).

Substituting Equations (A.35) and (A.34) in Equation (A.33), and using the definition (A.10), we find

$$
\begin{aligned}
\sigma^{\omega}_{ff'} &= \frac{\overline{W}_f}{\theta_f \theta_{f'}} \Bigg\{ \sum_i \Lambda_i \frac{\theta_{fi}}{W_{fi}} \theta_{f'i}\omega_{f'i}\sigma_{ff',i} - \varepsilon \sum_i \Lambda_i \frac{\theta_{fi}}{W_{fi}} \left(\theta_{f'i}\omega_{f'i} - \mathbb{E}_{i|s}\left[\theta_{f'i}\omega_{f'i} \right] \right) \\
&\qquad - \eta \sum_i \Lambda_i \frac{\theta_{fi}}{W_{fi}} \left(\theta_{f's}\omega_{f's} - \mathbb{E}_s \left[\theta_{f's}\omega_{f's} \right] \right) \Bigg\}, \\
&= \frac{\overline{W}_f}{\theta_f \theta_{f'}} \Bigg\{ \sum_i \Lambda_i \frac{\theta_{fi}}{W_{fi}} \theta_{f'i}\sigma_{ff',i} - \varepsilon \sum_s \Lambda_s \sum_{i\in\mathcal{I}_s} \Lambda_{i|s} \frac{\theta_{fi}}{W_{fi}} \left(\theta_{f'i} - \mathbb{E}_{i|s}\left[\theta_{f'i}\omega_{f'i} \right] \right) \\
&\qquad - \eta \sum_s \Lambda_s \sum_{i\in\mathcal{I}_s} \Lambda_{i|s} \frac{\theta_{fi}}{W_{fi}} \left(\theta_{f's}\omega_{f's} - \mathbb{E}_s \left[\theta_{f's}\omega_{f's} \right] \right) \Bigg\}, \\
&= \frac{1}{\theta_f \theta_{f'}} \left\{ \mathbb{E}_i \left[\frac{\overline{W}_f}{W_{fi}} \theta_{fi}\theta_{f'i}\sigma_{ff',i} \right] - \varepsilon\, \mathbb{E}_s \left[\mathbb{C}_{i|s} \left(\frac{\overline{W}_f}{W_{fi}} \theta_{fi}, \theta_{f'i}\omega_{f'i} \right) \right] - \eta\, \mathbb{C}_s \left(\frac{\overline{W}_f}{W_{fs}} \theta_{fs}, \theta_{f's}\omega_{f's} \right) \right\}.
\end{aligned}
$$

To compute the elasticity for factor payments $\sigma^{*,\omega}_{ff'}$, we follow similar steps

$$
\begin{aligned}
\sigma^*_{ff'} &\equiv 1 + \frac{1}{\theta_{f'}} \frac{\partial \log \theta_f}{\partial \log \overline{W}_{f'}}, \\
&= \frac{1}{\theta_{f'}} \frac{\partial \log \left(\sum_i W_{fi} X_{fi} \right)}{\partial \log \overline{W}_{f'}}, \\
&= \frac{1}{\theta_{f'}} \sum_i \frac{W_{fi} X_{fi}}{\sum_{i'} W_{fi'} X_{fi'}} \left[\frac{\partial \log \left(X_{fi}/Y_i \right)}{\partial \log \overline{W}_{f'}} + \frac{\partial \log Y_i}{\partial \log \overline{W}_{f'}} \right],
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\theta_{f'}} \sum_i \frac{C_i \theta_{fi}}{C \theta_f} \left[\frac{\partial \log \left(X_{fi}/Y_i \right)}{\partial \log \overline{W}_{f'}} + \frac{\partial \log Y_i}{\partial \log \overline{W}_{f'}} \right], \\
&= \frac{1}{\theta_{f'} \theta_f} \sum_i \Lambda_i \theta_{fi} \left[\theta_{f'i} \sigma_{ff',i} + \frac{\partial \log Y_i}{\partial \log \overline{W}_{f'}} \right].
\end{aligned}
$$

Once again, substituting Equations (A.35) and (A.34) in the above equation, and following the same steps as before leads to the desired result. □

Proof for Proposition A.4. Applying the same logic as the previous proofs, since $\frac{d \ln \overline{X}_h / \overline{X}_\ell}{d \ln \overline{W}_e} = 0$, we have

$$
\begin{aligned}
0 &= \frac{\partial \ln X_h / X_\ell}{\partial \ln \overline{W}_e} + \left(\frac{\partial \ln X_h / X_\ell}{\partial \ln W_h} \right) \frac{\partial \ln W_h}{\partial \ln \overline{W}_e}, \\
&= -\theta_e \left(\sigma^{\omega}_{\ell e} - \sigma^{\omega}_{he} \right) - \left(\theta_\ell \sigma_{h\ell} + \theta_e \sigma_{he} + \theta_h \sigma_{h\ell} \right) \frac{\partial \ln W_h}{\partial \ln \overline{W}_e},
\end{aligned}
$$

leading to Equation (A.17).

The same argument as that in the proof of Proposition 1 still applies here. Since relative skill demand X_h / X_ℓ does not change, we have

$$
\begin{aligned}
\frac{d \log \Psi}{d \log \overline{W}_e} &= \frac{d}{d \log \overline{W}_e} \log \left(\frac{\overline{W}_h}{\overline{W}_\ell} \right), \\
&= \frac{d}{d \log \overline{W}_e} \log \left(\frac{\overline{W}_h X_h}{\overline{W}_\ell X_\ell} \right), \\
&= \frac{d}{d \log \overline{W}_e} \log \left(\frac{\theta_h}{\theta_\ell} \right), \\
&= \frac{\partial \log \theta_h / \theta_\ell}{\partial \log \overline{W}_e} + \left(\frac{\partial \log \theta_h / \theta_\ell}{\partial \log W_h} \right) \frac{d \log W_h}{d \log \overline{W}_e}, \\
&= -\theta_e \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he} \right) + \left[\frac{\partial \log \theta_h}{\partial \log W_h} - \theta_h \left(\sigma^{*}_{\ell h} - 1 \right) \right] \frac{d \log W_h}{d \log \overline{W}_e}. \qquad \text{(A.36)}
\end{aligned}
$$

Note that we have

$$
\begin{aligned}
\frac{\partial \log \theta_f}{\partial \log W_f} &= \frac{1}{\theta_f} \frac{\partial}{\partial \log W_f} \left(1 - \sum_{f' \neq f} \theta_{f'} \right), \\
&= -\frac{1}{\theta_f} \sum_{f' \neq f} \theta_{f'} \frac{\partial \log \theta_{f'}}{\partial \log W_f}, \\
&= -\frac{1}{\theta_f} \sum_{f' \neq f} \theta_{f'} \theta_f \left(\sigma^{*}_{f'f} - 1 \right),
\end{aligned}
$$

$$= -\sum_{f' \neq f} \theta_{f'} \sigma^*_{f'f} + 1 - \theta_f.$$

Substituting this expression in Equation (A.36), we find

$$\begin{aligned}
\frac{d \log \Psi}{d \log \overline{W}_e} &= -\theta_e \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he}\right) + \left[-\theta_\ell \sigma^*_{\ell h} - \theta_e \sigma^*_{eh} + 1 - \theta_h - \theta_h \left(\sigma^*_{\ell h} - 1\right)\right] \frac{d \log W_h}{d \log \overline{W}_e}, \\
&= \frac{d \log W_h}{d \log \overline{W}_e} - \theta_e \left(\sigma^{*,\omega}_{\ell e} - \sigma^{*,\omega}_{he}\right) - \left[(1-\theta_e)\, \sigma^*_{\ell h} + \theta_e \sigma^*_{eh}\right] \frac{d \log W_h}{d \log \overline{W}_e}, \\
&= \frac{d \log W_h}{d \log W_e} - \left[(1-\theta_e)\, \sigma^*_{\ell h} + \theta_e \sigma^*_{eh}\right] \left(\frac{d \log W_h}{d \log \overline{W}_e} + \frac{\theta_e \left(\sigma^*_{\ell e} - \sigma^*_{he}\right)}{(1-\theta_e)\, \sigma^*_{\ell h} + \theta_e \sigma^*_{he}} \right),
\end{aligned}$$

leading to Equation (A.18). □

Proof for Proposition A.5. The of case of $f' \notin \mathcal{F}_c$ follows from the definition (A.20). In the case of $f' \in \mathcal{F}_c$, we have

$$\begin{aligned}
\frac{\partial \log \theta_c}{\partial \log W_{f'}} &= \sum_{f \in \mathcal{F}_c} \frac{\theta_f}{\theta_c} \frac{\partial \log \theta_f}{\partial \log W_{f'}}, \\
&= \sum_{f \in \mathcal{F}_c \setminus \{f'\}} \frac{\theta_f}{\theta_c} \theta_{f'} \left(\sigma^*_{ff'} - 1\right) - \frac{\theta_{f'}}{\theta_c} \left(1 - \theta_{f'}\right) \left(\sigma_{\overline{f} f'} - 1\right), \\
&= \theta_{f'} \frac{\theta_{\overline{c}_{f'}}}{\theta_c} \left(\sigma^*_{\overline{c}_{f'} f'} - 1\right) - \frac{\theta_{f'}}{\theta_c} \left(1 - \theta_{f'}\right) \left(\sigma_{\overline{f}' f'} - 1\right), \\
&= \theta_{f'} \left(\frac{\theta_{\overline{c}_{f'}}}{\theta_c} \sigma^*_{\overline{c}_{f'} f'} - \frac{\theta_{\overline{f}'}}{\theta_c} \sigma_{\overline{f}' f'} - \frac{\theta_{\overline{c}_{f'}} - \left(1 - \theta_{f'}\right)}{\theta_c} \right),
\end{aligned}$$

where in the second line we have used the following result for the own price elasticity:

$$\frac{\partial \log \theta_f}{\partial \log W_f} = 1 - \theta_f + \frac{\partial \log X_f}{\partial \log W_f} = 1 - \theta_f - \sum_{f' \in \mathcal{F} \setminus \{f\}} \theta_{f'} \sigma_{ff'} = -\left(1 - \theta_f\right) \left(\sigma_{\overline{f} f} - 1\right).$$

□

A.5 Aggregation in the Data

In this section, we present the second-order approximations that we use in our empirical exercise to perform the aggregation results presented in Section 5.

We first define the following notation. Consider a function $v(\cdot)$ of time-varying factor intensities $\boldsymbol{\theta}_t \equiv \left(\theta_{fi,t}, \theta_{f,t}\right)_{i,f'}$, elasticities $\boldsymbol{\sigma}_t \equiv \left(\sigma_{ff'i,t}, \sigma\right)$, and shock exposure weights $\boldsymbol{\omega}_t \equiv \left(\omega_{f'i,t}\right)$ defined following $\omega_{f'i,t} \equiv \Delta \log W_{f'i,t} / \Delta \log \overline{W}_{f't}$ with $\Delta \log \overline{W}_{f't} \equiv \sum_i \Lambda_i \frac{\theta_{ei}}{\theta_e} \Delta \log W_{f'i}$ at time t. Define the time averaging operator $\langle \cdot \rangle$ for the function between two consecutive periods $t-1$ and t as

$$\langle v(\boldsymbol{\theta}_t, \boldsymbol{\sigma}_t, \boldsymbol{\omega}_t) \rangle \equiv \frac{1}{2} \left(v(\boldsymbol{\theta}_t, \boldsymbol{\sigma}_t, \boldsymbol{\omega}_t) + v(\boldsymbol{\theta}_{t-1}, \boldsymbol{\sigma}_{t-1}, \boldsymbol{\omega}_t) \right),$$

where we keep the same fixed exposure weights $\boldsymbol{\omega}_t$ in both terms.

We now apply a second order approximation of the relative changes in the skill suply and demand in the general case with heterogenous changes in equipment prices, following the proof of Proposition A.4 in Appendix A.4.1 above, to find

$$\begin{aligned} \Delta \log \left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}} \right) &= \Delta \log \left(\frac{X_{h,t}}{X_{\ell,t}} \right), \\ &\approx - \left\langle \theta_{e,t} \left(\sigma^{\omega}_{\ell e,t} - \sigma^{\omega}_{he,t} \right) \right\rangle \Delta \log \overline{W}_{e,t} - \left\langle (1-\theta_{e,t}) \sigma_{h\ell,t} + \theta_{e,t} \sigma_{he,t} \right\rangle \Delta \log W_{h,t}. \end{aligned} \tag{A.37}$$

This leads to the following second-order approximation of the predicted change in the skill premium:

$$\Delta \log W_{h,t} \approx - \frac{\left\langle \theta_{e,t} \left(\sigma^{\omega}_{\ell e,t} - \sigma^{\omega}_{he,t} \right) \right\rangle}{\left\langle (1-\theta_{e,t}) \sigma_{h\ell,t} + \theta_{e,t} \sigma_{he,t} \right\rangle} \Delta \log \overline{W}_{e,t} - \frac{1}{\left\langle (1-\theta_{e,t}) \sigma_{h\ell,t} + \theta_{e,t} \sigma_{he,t} \right\rangle} \Delta \log \left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}} \right). \tag{A.38}$$

In practice, we find that the observed shifts in the skill premium $\Delta \log \widehat{W}_{h,t}$ do not equalize with the left hand side of Equation (A.38). To close this gap, consider uniform skill-biased shifts in technology of the form

$$\Delta a_t \equiv \log \left(\frac{Z_{hi,t} / Z_{\ell i,t}}{Z_{hi,t-1} / Z_{\ell i,t-1}} \right).$$

Then, we can further generalize Equation (A.37) to write

$$\begin{aligned} \Delta \log \left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}} \right) &= \Delta \log \left(\frac{X_{h,t}}{X_{\ell,t}} \right), \\ &\approx - \left\langle \theta_{e,t} \left(\sigma^{\omega}_{\ell e,t} - \sigma^{\omega}_{he,t} \right) \right\rangle \Delta \log \overline{W}_{e,t} - \left\langle (1-\theta_{e,t}) \sigma_{h\ell,t} + \theta_{e,t} \sigma_{he,t} \right\rangle \left(\Delta \log W_{h,t} - \Delta a_t \right). \end{aligned}$$

This allows us to infer equivalent changes

$$\Delta a_t \approx \Delta \log \widehat{W}_{h,t} + \frac{\left\langle \theta_{e,t} \left(\sigma^{\omega}_{\ell e,t} - \sigma^{\omega}_{he,t} \right) \right\rangle}{\left\langle \left(1-\theta_{e,t}\right) \sigma_{h\ell,t} + \theta_{e,t}\sigma_{he,t} \right\rangle} \Delta \log \overline{W}_{e,t} + \frac{1}{\left\langle \left(1-\theta_{e,t}\right) \sigma_{h\ell,t} + \theta_{e,t}\sigma_{he,t} \right\rangle} \Delta \log \left(\frac{\overline{X}_{h,t}}{\overline{X}_{\ell,t}} \right), \tag{A.39}$$

Note that in general we can interpret the inferred shifts in Equation (A.39) either as changes in production technology or equivalent changes in distortions that effective act as taxes on the hiring unskilled workers for firms.

A.6 Microfoundations for the Factor Price Wedges

A.6.1 Heterogeneous Equipment Equipment

We provide a microfoundation for firm-specific equipment prices stemming from variation in the composition of equipment capital across firms. We make two key assumptions. First, we assume that the stock of equipment for firm i is a CRS aggregate across J varieties of equipment capital given by

$$X_{ei} = \mathcal{G}_i \left(X_{ei1}, \cdots, X_{eij}, \cdots, X_{eiJ} \right),$$

where each variety j is a combination of equipment type and origin country (from which the variety is purchased). Second, we assume that the aggregator $\mathcal{G}_i$ is such that the marginal product of all varieties is bounded above by a constant

$$\frac{\partial G_i}{\partial X_{eij}} \leq B < \infty, \qquad \forall i, j,$$

ensuring that equipment variety demand of all firms has a finite choke price above which demand falls to zero.

We introduce firm×variety-specific price distortions $W_{eij} = T_{eij} W_{ej}$, which may be driven, for instance, by firm-specific iceberg costs of sourcing a given variety from the corresponding origin country. Given these assumptions, the price of equipment capital for firm i is given by

$$W_{ei} = \mathcal{P}_i \left(\frac{W_{eit}}{T_{ei1}}, \cdots, \frac{W_{eij}}{T_{eij}}, \cdots, \frac{W_{eiJ}}{T_{eiJ}} \right),$$

where $\mathcal{P}_i$ is the unit cost function corresponding to the CRS aggregator $\mathcal{G}_i$.

The assumption of the choke price implies that for a given collection of firm-specific wedges $\left(T_{eij}\right)_{j=1}^{J}$, the firm may find its firm specific price W_{eij} to be above the variety-specific choke price for some varieties, and therefore the extensive margin of the composition of equipment varieties is heterogeneous across firms. When we map our framework to the data on firm-level imports, this is indeed what we find, since the composition and the sources of equipment goods imported varies across firms. However, since marginal equipment varieties have a zero marginal product, changes in the extensive margin of sourcing decisions due to small firm-specific price changes do not affect the firm-level equipment price W_{ei} to the first order of approximation.

A.6.2 Compensating Differentials

Here, we provide a microfoundation for firm- and skill-type-specific wages stemming from variations in compensating differentials across firms. In this microfoundation, the labor market is perfectly competitive.

Suppose that firms are characterized by variations in amenities that are specific to worker skill. More specifically, if a high-skilled (or low-skilled) worker is employed by firm i, then her utility is her real income divided by T_{hi} (or $T_{\ell i}$). The market equilibrium then leads to firm-specific compensating differentials that are equivalent to amenity differences.

For two firms i and i' to have positive employment of skilled (or unskilled) labor in a competitive labor market, their wages must satisfy $W_{hi}/T_{hi} = W_{hi'}/T_{hi'}$ (or $W_{\ell i}/T_{\ell i} = W_{\ell i'}/T_{\ell i'}$). Letting W_h and W_ℓ denote the shadow wages of skilled and unkilled labor, which we set equal to the wage of a (potentially counterfactual) firm i' with $T_{hi'} = 1$ or $T_{\ell i'} = 1$, we then have $W_{hi} = W_h T_{hi}$ and $W_{\ell i} = W_\ell T_{\ell i}$ for all i, exactly as in our model of Section 2.

B Data Appendix

In this appendix, we present a comprehensive description of the data sources and the main variables utilised in the analysis. Additionally, we discuss how we construct the sample used in the estimation of the micro-elasticities and the sample used in the aggregation exercise.

B.1 List of All Data Sources Used

BRN: The *Bénéfices Réels Normaux* dataset provides information on income and balance sheets for companies with sales exceeding certain thresholds as well as some smaller firms that opt in.[A1] The information are sourced from the official document CERFA 2050-9, which serves as the French counterpart to the US IRS Form 1120 (Corporate Income Tax Return). In 2003, the dataset covered 24% of French firms, accounting for over 94.3% of total gross output (INSEE, 2004) and over 90% of the aggregate value of trade flows in customs records (authors' computation). The available data span from 1994 to 2007 and cover employment, sales, value added, wage bill, payroll taxes, and a breakdown of investment. As we discuss in detail in Section B.2, we use variables retrieved from this dataset to compute firm-level measures of gross output, intermediates and equipment investments.

DADS: The *Déclarations Annuelles de Données Sociales* (INSEE, 2010) is a matched employer-employee dataset that covers the entire population of French private sector workers. Among the different versions of the dataset made available by INSEE, we exploit DADS Poste (*Fichiers Régionaux des Postes*), which provides information at the level of each job spell. The information includes the worker's establishment, wage, hours worked, and 4-digit occupational classification. As we will discuss in detail in Section C.2.2 below, firm-level employment and wage bill data are computed aggregating worker-level data on hours worked and wages.

Customs dataset: Firm-level international transaction data are provided by the *French Directorate-General of Customs and Indirect Taxes (DGDDI)*, offering information on the annual value of imports and exports by country of origin/destination and NC8 product for all firms involved in international transactions. This dataset combines information gathered under two separate legal frameworks: the *Déclaration d'Echange de Biens (DEB)*, which addresses intra-EU trade flows, and the *Document Administratif Unique (DAU)*, which pertains to transactions with non-EU countries. The number of variables collected within each statistical framework depends on thresholds specific to the firm and the transactions (Bergounhon et al., 2018). For intra-EU DEB transactions, the information available depends on the firm's size, measured by the total value of intra-EU trade it conducts during the relative calendar year. During the study period, the minimum threshold requiring firms to provide the information used in this study increased from €38,000 (1993-2000) to €100,000 (2002-2006), eventually reaching €150,000 in 2007. For the purpose of this study, we apply the latter threshold throughout the entire sample to ensure consistency. On the other hand, the extra-EU DAU data include all transactions above €1,000

[A1] In 2007, the threshold was €763,000 for trade and real estate sector firms and €230,000 for firms operating in all other sectors INSEE (2004).

or 1,000 kilos. Data available from 1994 to 2007 include the value, weight, and number of units exported by each firm. These data are used to compute firm-level equipment variables (see Section C.1.1), as well as the trade instruments presented in Section 4.2.

Répertoire Sirene: The *Répertoire Sirene* is a national directory of companies and their establishments (business register), managed by the National Institute of Statistics and Economic Studies (INSEE, 2021).

BACI Dataset: The *Base pour l'Analyse du Commerce International* dataset (Gaulier and Zignago, 2010) contains information on bilateral trade flows by year and product. We use this dataset to measure the world export supply of intermediates, which is a key component of the world-export-supply instrument presented in Section 4.2.

KLEMS: Data on depreciation rates by industry and asset category, as well as industry-level gross output, are retrieved from the *Growth and Productivity Accounts* published by EU KLEMS (*EU level analysis of capital (K), labour (L), energy (E), materials (M), and service (S) inputs*). The dataset includes industry-level measures of economic growth, productivity, employment and capital formation for all European Union member states from 1970 onwards (Stehrer et al., 2019). We use these data to compute equipment stock and user cost of capital, as discussed in Section C.1.4 and C.1.5, and the skill-specific wage instrument presented in Section 4.2.

Eurostat: We retrieve industry-level aggregate data from the *Eurostat National Accounts – Main Aggregates* series (Eurostat, 2019). This dataset, compiled in accordance with the European System of Accounts (ESA, 2013), includes a coherent and consistent set of macroeconomic indicators, which provide an overall picture of the economic situation of each country. For the purpose of this study, we exploit information on aggregate levels and price indices of capital and investment by asset type and by sector, as well as industry-level data on value added and employment. These data are used in the construction of industry-specific prices and depreciation indices (see Section B.2).

IMF WEO: Data on exchange rates are retrieved from the *IMF World Economic Outlook Database*. These series are used to construct the exchange rate instrument presented in Section 4.2.

EIA: The *US Energy Information Administration* provides data on the average annual price (dollars per gallon) of the U.S. Gulf Coast kerosene-type jet fuel (EIA, 2020). This information is used to construct the equipment transit-cost instrument presented in Section 4.2.

BP: A second input for the transit-cost instrument is oil prices, which are retrieved from the 2019 edition of the *British Petroleum Statistical Review of World Energy* (Dudley et al., 2019).

CEPII: The third input for the transit-cost instrument is the bilateral distances between France and 190 countries, which are retrieved from the dataset produced by the *Centre for Prospective Studies and International Information (CEPII)* (Mayer and Zignago, 2011).

WIOD: We use the Socio-Economic Accounts of the *World Input-Output Database* (2013 Release) to construct an alternative proxy for low-skilled wages used as the numeraire in the construction of the aggregate shock to equipment prices (see Section C.1.7).

FRED: Data on historical government bond yields are retrieved from the *Federal Reserve Economic Data (FRED)*.

INSEE: Data on the monthly rate of the nominal hourly minimum wage – *Salaire Minimum Interprofessionnel de Croissance (Smic)* – are provided by the *Institut National de la Statistique et des Études Économiques (INSEE)*.

B.2 Variable Definition

Gross Output: The gross output variable is based on data retrieved from the BRN dataset. It is defined as the sum of three components: sales (*Chiffre d'affaires nets, r310*), capitalised production (*Production immobilisée, r312*), and change in inventories (*Production stockée, r311*). The variable is reported as missing when its main component, sales, is missing or equal to zero.

Intermediates: The intermediates variable is also based on data retrieved from the BRN dataset. It is defined as the sum of three components: purchases of goods and raw materials (*Achats de marchandises, matiéres premiéres et approvisionnements, r210+r212*), change in inventory of goods and raw materials (*Variation de stock des marchandises, matiéres premiéres et approvisionnements, r211 + r213*), and other supplies and external expenses

(*Autres achats et charges externes, r214*). The variable is reported as missing when its main component, purchase of raw materials, is missing or equal to zero.

Value Added: We define value added as the difference between gross output and intermediates. This variable largely coincides with the definition of *valeur ajoutée hors taxe* (value added before taxes) provided by *Systéme Unifié des Statistiques d'Entreprises* (SUSE). The only difference is that we exclude two components: "other products" (*Autres produits, r315*) and "other charges" (*Autres charges, r222*). Despite this minor difference, the two variables have a correlation of 0.992.

Number of Employees: This variable is defined as the full-time equivalent (FTE) employment. We obtain this variable in two steps. First, we extract data on the number of hours worked reported for each employee of a firm in the DADS Poste and divide it by 1,820, which is the expected number of hours worked in a year by a full-time worker with a 35-hour week contract. To avoid discontinuities caused by the reform of the workweek system that took place during the study period, we cap our proxy at 1.[A2] Subsequently, we aggregate this variable at the firm level.

Observed wages: Hourly wages are obtained using data on gross wages and paid hours retrieved from DADS. Gross wages correspond to all the sums received by the employee under their employment contract,[A3] while paid hours represent the cumulative total of all periods during which the employee remained connected to a firm, including overtime, periods of illness, and work-related accidents, except for unpaid leave periods.

Equipment stock (book value): Firm-level data available in BRN include a breakdown of tangible capital by asset type. Our measure of domestic equipment is the sum of two components: Machinery, equipment and tools (*AR - Installations techniques, matériel et outillage industriels*) and Other tangible fixed assets (*AT - Autres immobilisations corporelles*).

Equipment depreciation account: Similar to the book value variable, the depreciation account variable is the sum of two components: Machinery, equipment and tools (*AS - Installations techniques, matériel et outillage industriels*) and Other tangible fixed assets (*AU - Autres immobilisations corporelles*).

[A2] Between 1998 and 2002, France gradually introduced a 35-hour workweek system.

[A3] This salary is understood before any mandatory deductions and is calculated based on the CSG base.

Disposed assets: The variable (*Charges exceptionnelles sur opérations en capital, HF*) records the net book value of the disposed assets.

Total imports/exports (values): Firm-level data on imports and exports by partner country and HS6 product are retrieved from the Customs dataset. These data are are used to select the sample used in Equation (26), Col 3.[A4] Moreover, imports data are used to compute real-exchange-rate instrument presented in Section 4.2.

Intermediate imports (values): Firm-level data on intermediate imports by partner country and HS6 product are retrieved from the Customs dataset and are used as weights to compute the world-export-supply instrument presented in Section 4.2.

Imported equipment (values and quantities): Data on imported equipment are retrieved from the Customs dataset. We identify imported equipment as all transactions belonging to three BEC Rev. 4 groups (*4 - Capital goods (except transport equipment), and parts and accessories thereof*; *521 - Transport equipment, other, industrial*; and *51 - Transport equipment, passenger motor cars*). These data are used to construct the equipment variables used in the estimation of production function elasticities (see Section C.1). Moreover, firm-level data on imported equipment by partner country and HS6 product are used as weights to compute the equipment transit-cost instrument presented in Section 4.2.

Depreciation Rates: We retrieve depreciation rates from KLEMS. Each industry is assigned an equipment depreciation rate by aggregating four ESA 2010 asset categories (*N11321G - Computer hardware*; *N11322G - Telecom equipment*; *N1131G - Transports*; and *N110G - Other machinery equipment and weapons*) using the time-varying industry-level investments in each asset group as weights.

Domestic Equipment Prices: Domestic prices are retrieved from Eurostat. The aggregation procedure implemented to obtain time-varying industry-specific equipment prices is symmetrical to the one used to compute depreciation rates.

Interest Rates: The interest rate used to compute our proxy for the user cost of capital (see Section C.1.5) is the interest associated with the 10-year French bond, as recorded by the Federal Reserve Bank of St. Louis.

[A4] We exclude firms that import from any country in which they also export more than 10% of their total export value over the period.

Table B.1: Summary Statistics

		All firms						Estimation Sample (CRESH)					
	Source	Obs.(Nb)	Mean	Sd	p25	p50	p75	Obs.(Nb)	Mean	Sd	p25	p50	p75
Output	BRN	3,316,889	6,888,053	144,908,457	497,922	1,060,867	2,567,384	358,718	36,685,093	422,912,451	2,067,085	5,122,782	14,794,086
Value Added	BRN	3,316,889	1,869,950	36,710,491	194,813	381,259	859,239	358,718	8,752,715	101,010,070	525,127	1,339,758	3,671,408
Equipment stock	BRN	3,316,889	810,271	29,690,945	25,153	57,434	153,115	358,718	5,429,262	84,657,546	114,288	370,523	1,348,482
High-Skill Wage Bill	DADS	3,316,889	442,433	6,781,880	35,204	79,821	196,319	358,718	2,189,704	19,295,690	135,221	341,465	954,852
Low-Skill Wage Bill	DADS	3,316,889	387,182	8,318,383	32,198	85,200	221,247	358,718	1,552,821	20,984,595	82,451	262,296	773,062
High-Skill Working Hours	DADS	3,316,889	20,784	335,064	2,028	4,375	10,179	358,718	96,621	967,560	6,723	16,069	43,857
Low-Skill Working Hours	DADS	3,316,889	35,554	739,271	3,223	8,314	21,402	358,718	134,165	1,814,480	7,552	24,299	71,075
High-Skill Hourly Wage	DADS	3,316,889	19	9	13	17	22	358,718	22	8	17	20	25
Low-Skill Hourly Wage	DADS	3,316,889	10	3	9	10	11	358,718	11	3	10	11	12
Exports - Total	Customs	3,316,889	773,407	34,970,828	0	0	0	358,718	6,036,394	98,668,055	0	59,808	940,799
Imports - Total	Customs	3,316,889	845,054	32,181,043	0	0	0	358,718	6,545,377	95,579,766	25,484	468,196	1,988,704
Imports - Equipment Products	Customs	3,316,889	222,708	14,597,314	0	0	0	358,718	1,892,044	43,482,627	0	10,421	230,563
Imports - Intermediate Products	Customs	3,316,889	479,670	22,964,181	0	0	0	358,718	3,801,673	68,031,178	561	171,056	969,316

Notes: Observations is the number of firm-year pairs in manufacturing and non-financial services from BRN over the 1998-2007 period. The units for all variables are euros except for those involving working hours. The full sample includes all firms used in the aggregation exercise of Section 5, which is conditional on firms with positive equipment capital stock and employing both low- and high-skill workers. The estimation sample is restricted to the set of firms used in the estimation of the firm-level CRESH production elasticities (Table 2), which further conditions on firms importing both intermediate and equipment products (and having both high-skill and low-skill stayers).

Table B.2: List of CS occupational categories

Classification	Catégories socioprofessionnelles (CS)
High-skilled	**2 - Managers** 21 - Managers of craft enterprises 22 - Managers of industrial or commercial enterprises with $<$ 10 employees 23 - Managers of industrial or commercial enterprises with $\geq$ 10 employees **3 - Executives and Higher Intellectual Professions** 31 - Health professionals and lawyers 33 - Public service executives 34 - Professors, scientific professions 35 - Information, arts, and entertainment professions 37 - Administrative and commercial executives of enterprises 38 - Engineers and technical executives of enterprises **4 - Intermediate Professions** 42 - Primary school teachers and equivalents 43 - Intermediate professions in health and social work 44 - Clergy, religious workers 45 - Intermediate administrative professions in the Public Service 46 - Intermediate administrative and commercial professions in enterprises 47 - Technicians 48 - Foremen, supervisors
Low-skilled	**5 - Employees** 52 - Civil employees and service agents of the Public Service 53 - Security agents 54 - Administrative employees of enterprises 55 - Sales employees 56 - Personal services personnel **6 - Blue-collar workers** 62 - Skilled industrial workers 63 - Skilled craft workers 64 - Drivers 65 - Skilled workers in handling, warehousing, and transport 67 - Unskilled industrial workers 68 - Unskilled craft workers 69 - Agricultural workers

B.3 Industry and Occupational Classifications

B.3.1 Constant Industry

The available data report the NACE Rev. 1.1 industry classification between 1994 and 2007. However, certain macroeconomic statistics (such as aggregate prices and depreciation rates) are available exclusively at the NACE Rev. 2 level. We address this discontinuity by assigning each firm in our dataset a constant NACE Rev. 2 industry code.

To achieve the above assignment, we first construct a correspondence table. We retrieve information on the industry classification recorded by all firms registered in the Répertoire Sirene (see Section B.1). Each firm is assigned a 5-digit NAF code, with the first 4 digits corresponding to the NACE classification. We first select all firms that are

Table B.3: Observed Wages by Occupational Category

	Low-skilled					High-skilled					
	Mean	10p	median	90p	sd	Mean	10p	median	90p	sd	N fims
Hourly Wages											
Manufacturing [C]	13.2	10	12.6	17.2	3.1	23.3	17	22.9	29.7	5.8	54,664
Electricity, gas, water, and waste [D-E]	13.2	10.7	13	15.9	2.3	23.2	17.3	22	30	6.2	2,067
Construction [F]	12.9	10.6	12.7	15.2	2	23.4	15.9	22.2	31.8	8.1	40,906
Non-Financial Market Services [G-N]	11.7	9.1	11.2	14.9	2.6	21	14.1	19.5	29.2	7.5	212,076
Non Market Services [O-U]	11.4	9.5	11.2	13.2	1.8	19.5	14	18.4	24.9	8.5	21,095
Tot	12.2	9.4	11.6	15.7	2.8	21.7	14.5	20.6	29.4	7.3	330,807
Paid Hours											
Manufacturing [C]	1,573	1,303	1,618	1,781	215	1,658	1,425	1,694	1,852	214	54,664
Electricity, gas, water, and waste [D-E]	1,512	1,296	1,545	1,690	179	1,601	1,370	1,648	1,777	195	2,067
Construction [F]	1,465	1,183	1,486	1,719	232	1,575	1,232	1,610	1,890	297	40,906
Non-Financial Market Services [G-N]	1,274	669	1,301	1,760	400	1,501	829	1,588	1,856	371	212,076
Non Market Services [O-U]	1,316	979	1,355	1,601	270	1,354	946	1,408	1,685	308	21,095
Tot	1,377	806	1,445	1,756	363	1,542	1,042	1,620	1,848	334	330,807

present both in 2007 and 2008 and identify the industry codes reported in those two years. Subsequently, we compute the relative frequency of each NACE 1.1-NACE 2 match and assign each NACE 1.1 code to the NACE 2 code that provides the best match. This process results in a many-to-one crosswalk table between the two classifications, which we use to assign a NACE Rev. 2 code to all observations.

As a final step, we identify the first industry code associated with each firm over the study period and assign it to the observations recorded in subsequent years. Using this method, we create an industrial classification dataset where each private sector firm recorded between 1994 and 2007 is assigned a unique NACE Rev. 2 industrial code. We then univocally match these 4-digit NACE Rev. 2 codes to the 38 aggregate industries of the Eurostat classification.

B.3.2 Skill groups

The DADS dataset assigns to each job spell (*poste*) a unique occupational code, based on a two-digit classification called *catégories socioprofessionnelles* (see Table B.2). Following prior work (e.g., Caliendo et al., 2015; Carluccio et al., 2015), we assign workers to two skill groups based on this classification. High-skill workers are those employed as Managers, Middle managers and professionals, and Qualified workers. Low-skill workers are employed as Clerks and Blue-collar. To account for a minor variation in the classification over the study period, we aggregate the categories 31 (Health professionals and lawyers), 34 (Professors, scientific professions), and 37 (Administrative and commercial executives of enterprises). In Table B.3, we report summary statistics on hourly wages by skill group

Table B.4: Aggregation Sample (average 1998-2007)

	N. of firms			
	BRN	DADS	Merge	Aggregation
Manufacturing [C]	80,959	137,097	69,132	54,664
Electricity, gas, water, and waste [D-E]	4,185	7,410	2,798	2,067
Construction [F]	94,576	203,937	64,813	40,906
Non-Financial Market Services [G-N]	535,064	869,053	349,165	212,076
Non Market Services [O-U]	43,887	334,788	34,798	21,095
Tot	758,671	1,552,285	520,706	330,808
	Employment			
Manufacturing [C]		3,071,032	2,886,393	2,858,400
Electricity, gas, water, and waste [D-E]		160,429	143,062	140,256
Construction [F]		1,220,655	932,322	813,194
Non-Financial Market Services [G-N]		8,131,740	6,997,741	5,966,905
Non Market Services [O-U]		4,345,177	752,852	660,311
Tot		16,929,033	11,712,370	10,439,066

and sector.

B.4 Sample selection

The sample used in this study represents the intersection of the DADS and BRN datasets. Our analysis focuses exclusively on firms included in the BRN dataset, specifically those subject to the fiscal regime *réel normal*.[A5] In 2003, the dataset covered 24% of French firms, accounting for over 94.3% of total gross output (INSEE, 2004). We further exclude the NACE/ISIC Sections *'A - Agriculture, Forestry and Fishing'*, *'B - Mining and Quarrying'*, and *'K - Financial and insurance activities'*, which are under-represented in BRN. Moreover, by conditioning on DADS, we exclude all firms that do not report employees in a given year, also excluding firms that are still reported as "active" but are de facto out of the market.

After this initial selection, we further reduce the dataset to two different samples for the purposes of the aggregation and the estimation exercises, as discussed below.

B.4.1 Aggregation Sample

In the sample used for the aggregation, we include only firms that report a non-missing number of both high- and low-skilled workers. However, we retain firms with missing values of equipment payments, for which we impute values using the approach presented in Section C.1.6.

[A5] In 2007, the threshold was €763,000 for trade and real estate sector firms and €230,000 for firms operating in all other sectors INSEE (2004).

Table B.5: Estimation Sample (average 2002-2007)

	Tab. 1, Col. (2)	Tab 1, Col. (3)	Tab 2, Col (1)	Table 3, Column (1)
	N. of Firms			
Manufacturing [C]	12,189	4,621	6,560	18,691
Electricity, gas, water, and waste [D-E]	138	44	68	610
Construction [F]	507	370	218	8,067
Non-Financial Market Services [G-N]	14,063	8,034	4,407	35,852
Non Market Services [O-U]	155	83	58	3,498
Tot	27,051	13,152	11,311	66,717
	Employment			
Manufacturing [C]	1,712,132	412,484	1,410,691	1,841,887
Electricity, gas, water, and waste [D-E]	9,480	3,209	20,228	86,318
Construction [F]	54,977	35,339	71,605	379,553
Non-Financial Market Services [G-N]	971,711	436,374	957,113	2,987,962
Non Market Services [O-U]	26,232	8,112	25,939	237,662
Tot	2,774,532	895,518	2,485,576	5,533,382

In Table B.4 we provide details about the coverage of the dataset used for the aggregation. The first two columns of the first panel report the average number of firms, respectively, in BRN and DADS, by sector. The third column reports sector-specific statistics obtained using only firms covered in both datasets. Finally, in the fourth column, we report the coverage of the final dataset, which is obtained by excluding firms that do not report either low- or high-skilled workers and a small number of firms for which it is not possible to impute the value of equipment stock (see Section C.1.6). Similarly, the second panel reports statistics on total employment, apart from the first column since the employment data are retrieved from DADS and not BRN.

B.4.2 Estimation Sample

The sample used to estimate the micro-elasticities is smaller, as we only focus on firms that are continuously active over a 5-year period and impose further restrictions based on the specific features of the model used. In particular, the baseline estimates presented in Equation (26) are based on a sample of importers, excluding firms reporting extreme values in the used import instruments. Instruments based on trade shocks result in outliers for some firms that trade a limited range of specialised products to small destinations. Following Aghion et al. (2024), we address this issue by regressing trade instrument shocks on a firm fixed effect and trimming observations with residuals beyond the 99th and 1st percentiles. Thus, observations with the largest deviations in their trade shock (relative to their firm mean) are excluded from our sample.

In Table B.5, we report the average number of firms and employment by sector for the samples used to estimate the baseline model reported in Tables 1, 2, and 3. In the first column, we report the sector breakdown of the average number of firms and employment of the sample used in our baseline estimates of Equation (26). The sample is limited to firms that import at least 1% of their gross output (on average over the sample period) and do not report anomalous values in the adopted instruments.[A6] In the second column, we impose a further restriction by excluding firms that import from any country to which they also export more than 10% of their total export value over the period. In the third column, we report sector-specific statistics for the sample used to estimate Equations (29) and (30). This sample is limited to firms that import equipment products and survive the same outlier cleaning routine adopted for the other import instruments.[A7] Finally, in the fourth column, we report the sector breakdown for the sample used to estimate Equations (31) and (32). In this case, the sample is significantly larger, as the model relies only on the skill-specific wage instruments.

C Empirical Appendix

In this appendix we provide a detailed account on how we construct the firm-level capital stock for each asset category and our measure of firm- and skill-specific wages. Additionally, we provide additional details on the construction of the instrumental variables used in the estimation of the micro-elasticities, and our strategy to estimate the nested CES aggregate production function.

C.1 Measuring Stock and User Cost of Equipment

The data used in building our measure of the stock of equipment come from the BRN and the customs datasets. In 2003, BRN covered 24% of French firms, accounting for over 94.3% of total gross output (INSEE, 2004) and over 90% of the aggregate value of trade flows in customs records (authors' computation).

C.1.1 Equipment Imports

For the imported part of equipment investment, we rely on customs data, which provide information on partner country, NC8 product, firm, and year-specific transaction values and quantities (in kilograms). The initial dataset includes over 28 million import and 24

[A6] The instrument exploits differences in firm-specific import exposure across source countries and changes in real exchange rates between France and its trading partners.

[A7] As explained in section 4.2, we use an equipment transit-cost instrument, a skill-specific-wage instruments and a world-export-supply instrument. The sample coincides with the one used to estimate Column (3) of Table 3, where we use the same set of instruments.

million export transactions, involving 551,008 French firms, 13,901 different CN8 products, and 190 partner countries.

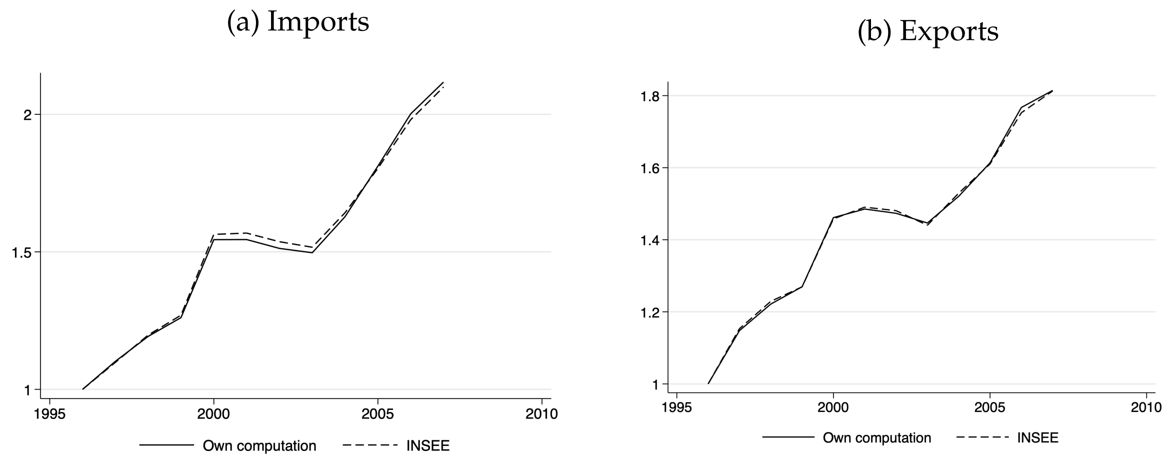
Unit-value prices are defined as the ratio between the value and quantity (typically measured in kilograms) associated with each transaction. Since certain firms had the option to report the number of units transacted instead of the quantity in kilograms, 5% of firms report missing values for the import quantity variable and 3.6% for the export quantity. However, they all report the units sold or purchased. We impute the missing values using CN8-year-level quantity-unit ratios. This exercise reduces the share of missing values to 1.9% for imports and 1.4% for exports.

As explained in Section B.1, the number of variables gathered within each statistical framework depends on thresholds specific to the firm and the transactions. Firms are required to report individual transactions with non-European countries when they exceed €1,000. Moreover, intra-EU transactions are subject to an aggregate annual threshold that grows over the study period from €38,000 to €150,000. This features of the dataset create two potential issues. First, the increase over time of the minimum threshold for reporting intra-EU transactions could bias our results. Second, firms reporting under-threshold transactions might be different from firms that do not. To address these issues, we drop extra-EU transactions below the threshold of €1,000 and all transactions with European partners where the cumulative annual flow is below the threshold of €150,000. After these cleaning steps, we are left with 82% of the original imports, and 93% of the original exports, corresponding, respectively, to 99.4% and 99.7% of the total value.

In Figure C.1, we conduct a simple validation exercise, comparing the aggregate import/export trends produced with our data with those reported by INSEE. The high correlation between the two series provides reassurance regarding the representativeness of our data.

The creation of the import variables follows two steps. First, we compute a constant CN8 product classification for the period 1996-2007 (hereafter "CN8plus") by implementing the method proposed by Van Beveren et al. (2012). Subsequently, we create a constant HS6 product classification (hereafter "HS6plus"), so that each CN8plus corresponds to a single HS6plus code. Finally, we link HS6plus product codes to (overlapping) macro-product categories: equipment, intermediate inputs and capital inputs, using the BEC product classification. In particular, we define equipment products as the ones that be-

Figure C.1: Aggregate Trends



long to the following categories of the BEC Rev. 4 classification: *Capital goods (except transport equipment), and parts and accessories thereof*; *521 - Transport equipment, other, industrial*; and *51 - Transport equipment, passenger motor cars*. The second step consists in excluding from the equipment variable all transactions with product category belonging to the macro-groups 'raw materials' (HS 01-15, 25-27, 31, 41) and 'services' (HS 97-99). Finally, we aggregate all import variables at the firm-country-product-year level.

C.1.2 Domestic Equipment Investments

The proxy for domestic investment in equipment capital is computed using variables retrieved from the BRN dataset, namely the book-value stock (K^e_{it}) and depreciation account (D^e_{it}) for the equipment variable, defined as discussed in Section B.2, as well as a measure of the total disposed assets (U_{it}). These three variables are used to compute a proxy for investment in equipment, using the following formula:

$$I^e_{it} = \frac{T}{T-1} \cdot \left[(K^e_{it} - K^e_{it-1}) - (D^e_{it} - D^e_{it-1}) + \sum_{s=t-T}^{t-1} \frac{I^e_{is}}{T} + \psi_{it} + U^e_{it} - \lambda_{it-1} \right],$$

where we have defined

$$U^e_{it} = U_{it} \frac{K^e_{it-1}}{K_{it-1}},$$

$$\psi_{it} = \begin{cases} \min\left\{\frac{K^e_{i0}}{T}, K^e_{i0} - D^e_{i0} - \frac{K^e_{i0}}{T}(t-1)\right\}, & \text{if } K^e_{i0} - D^e_{i0} - \frac{K^e_{i0}}{T}(t-1) > 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$\lambda_{it} = \begin{cases} 0, & \text{if } t = 1, \\ \min\left\{U^e_{i1}, \psi + \frac{I^e_{i1}}{T}\right\}, & \text{if } t = 2, \\ \min\left\{U^e_{it-1} + \max\left\{U^e_{it-2} - \sum_{s=t-T-1}^{t-2} \frac{K^e_{i0}}{T}, 0\right\}, \sum_{s=t-T}^{t-1} \frac{K^e_{i0}}{T}\right\}, & \text{if } t > 2. \end{cases}$$

We validated our proxy against real investment data, only accessible to INSEE employees, obtaining a correlation above 0.9.[A8]

Finally, we compute the measure of domestic investments as the difference between overall investments and import equipment values: $I^{e^{dom}}_{it} = I^e_{it} - I^{e^{for}}_{it}$.

C.1.3 Firm-Level Price Indices for Equipment Investment

Prices for imported equipment products are defined as unit values at the firm-, year-, product-, and country of origin-level. For domestic investment, we do not have firm-level information on prices. Therefore we exploit sector-, asset-, and year-specific data provided by KLEMS (See section B.2).

At this point, we have annual firm-level data on domestic equipment investments and imported equipment investments at the level of product-origins, together with their prices. We next compute a firm-level price index for equipment to deflate the investment values and obtain the quantity of equipment investment.

Our index is a slight variation on the standard Sato-Vartia price index. We treat the domestic investment as a single equipment product originating from France, and define the share of equipment product k coming from the country of origin c as $S^e_{ckit} = I^e_{ckit}/\tilde{I}^e_{it}$, where $\tilde{I}^e_{it}$ is the total firm-level equipment investment at time t on product-country varieties common between $t-1$ and t. Similarly, we define the share of equipment products k coming from country of origin c at $t-1$ as $S^e_{ckit-1} = I^e_{ckit-1}/\tilde{I}^e_{it-1}$, where $\tilde{I}^e_{it-1}$ is the total firm-level equipment investment at time $t-1$ on common (product-country) varieties between $t-1$ and t. Using these shares, we define product-country weights ω^e_{ckit} as:

$$\omega^e_{ckit} = \begin{cases} \frac{S^e_{ckit} - S^e_{ckit-1}}{s^e_{ckit} - s^e_{ckit-1}} & \text{if } S^e_{ckit} \neq S^e_{ckit-1}, \\ S^e_{ckit} & \text{if } S^e_{ckit} = S^e_{ckit-1}, \end{cases}$$

[A8]We are grateful to Jocelyn Boussard for help with this analysis.

Table C.1: Correlation Table - Alternative Price Indices

ε	∞	**5.40**	**4.48**	**3.41**
∞	1			
5.40	0.977	1		
4.48	0.976	0.993	1	
3.41	0.952	0.979	0.985	1

where, as before, small-cap letters denote the logarithm of the corresponding variable. We then compute the firm-level Sato-Vartia log price indices as:

$$\mathbb{P}^e_{it} = \exp\left(\sum_{ck}(p_{cki,t} - p_{cki,t-1})\frac{\omega^e_{ckit}}{\sum_{ck}\omega^e_{ckit}}\right).$$

Subsequently, we compute equipment investment quantities Q^e_{it} using equipment investment values (I^e_{it},) and the price indices computed in the previous stage: $Q^e_{it} = I^e_{it}/P^e_{it}$ where $P^e_{it} \equiv \prod_{\tau=0}^{t}\mathbb{P}^e_{i\tau}$.

We also construct an alternative price index which takes into account changes in import varieties:

$$\hat{\mathbb{P}}^e_{it} = \mathbb{P}^e_{it}\left(\frac{\Lambda_t}{\Lambda_{t-1}}\right)^{\frac{1}{\varepsilon-1}}$$

where $\left(\frac{\Lambda_t}{\Lambda_{t-1}}\right)^{\frac{1}{\varepsilon-1}}$ is the standard Feenstra (1994) variety correction term, which takes account of the entry and exit of varieties. If entering varieties are more attractive than exiting varieties, the share of common varieties in total expenditure will be smaller in period t than in period $t-1$ $\left(\frac{\Lambda_t}{\Lambda_{t-1}}^{\frac{1}{\varepsilon-1}} < 1\right)$, which reduces the price index (since $\varepsilon \geq 1$). In Table C.1, we report the correlation matrix between our baseline variable (equivalent to a Feenstra-corrected variable with $\varepsilon = \infty$) and alternative measures constructed with estimates of the elasticity of substitution from Table 1.

C.1.4 Construction of the Equipment Stocks

Finally, we compute the equipment stock X_{eit}, applying the perpetual inventory method:

$$X_{eit} = \begin{cases} \frac{1}{\delta^e_{st}}\frac{\sum_{\tau=0}^{T_i-1}Q^e_{i\tau}}{T_i}, & \text{if } t = 1, \\ (1-\delta^e_{st})Q^e_{it-1} + Q^e_{it}, & \text{if } t > 1, \end{cases}$$

Table C.2: Equipment Stock - Summary Statistics

	Value Added (000)					Equipment stock (000)					
	Mean	10p	median	90p	sd	Mean	10p	median	90p	sd	N fims
Manufacturing [C]	3,856	160	630	4,400	67,438	11,348	0	590	8,311	216,699	54,664
Electricity, gas, water, and waste [D-E]	5,347	190	811	7,692	29,154	48,064	0	1,325	24,677	701,905	2,067
Construction [F]	1,078	150	470	1,774	5,229	692	0	191	1,193	4,649	40,906
Non-Financial Market Services [G-N]	1,691	98	358	1,882	34,642	3,838	0	194	1,962	176,765	212,076
Non Market Services [O-U]	1,273	67	356	2,498	7,574	1,417	0	208	1,986	14,930	21,095
Tot	2,024	111	414	2,302	40,339	4,809	0	231	2,546	176,249	330,807

where δ^e_{st} is a sector-specific depreciation rate obtained from the EU-KLEMS dataset. Following Mueller (2008), we set the initial capital stock for each firm i as the average investment over all T_i years in which the firm is present in the sample divided by the depreciation rate. To decrease the sensitivity of the equipment stock to the initialization strategy we set the first two years to missing. For most firms we can observe at least two years of data before our sample period, so this strategy has no impact on the coverage. However, the sample coverage will be reduced for firms that enter during our sample period.[A9]
In Table C.2 we report the summary statistics for our proxy of equipment stock.

For the construction of our baseline equipment stock variable, we rely on an initial value of the capital stock that is computed using the dual of the price index for equipment investment defined in Section C.1.3. This approach ensures maximum consistency with the overall strategy adopted to construct the variable. However, an alternative method to construct the equipment stock would be to initialise the process with the book value contained in the balance sheet reported by the firm. While more imprecise, we construct this alternative equipment stock variable and compare it against our baseline variable. We find that the two equipment stock variables exhibit a correlation of 0.96 and they only present slight deviations at the extremes of the distribution (Figure C.2).

C.1.5 Construction of the User Cost of Equipment

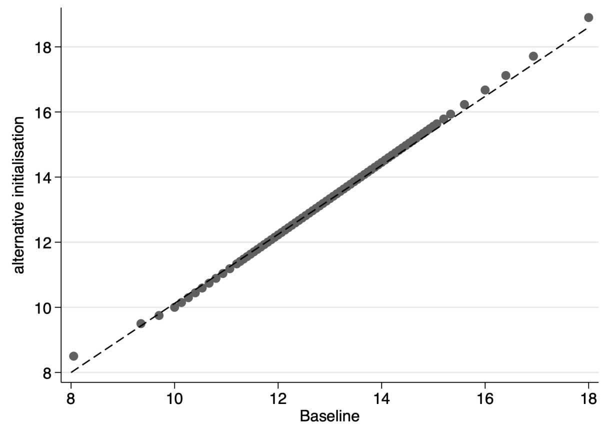
We define the firm-level user cost of equipment W_{eit} (the effective rental price of equipment capital) as follows:

$$W_{eit} = P^e_{it}\left(R^e_t + \delta^e_{st} - \frac{p^e_{s,t+1} - p^e_{s,t-2}}{3}\right)$$

where P^e_{it} is the firm-specific price of equipment products purchased by firm i in time t defined above, R^e_t is the required rate of return on equipment investments, and p^e_{st} and δ^e_{st}

[A9] Moreover, we remove anomalous values in the computed equipment stocks: negative values, permanent falls by a factor of 10, one-off (yearly) increases or decreases by a factor of 50.

Figure C.2: PIM: Alternative Initial Value



are the log price and depreciation rate of equipment products at the sector level retrieved respectively from Eurostat and EU-KLEMS (See Section B.2). Finally, we obtain firm-level payments to capital equipment by multiplying the user cost of equipment W_{eit} by the measure of equipment stock X_{eit}.

C.1.6 Imputation

As a final step, we develop a simple imputation strategy to reduce the number of observations reporting missing values for the equipment payments variable. A large part of these missing values are a direct consequence of the cleaning procedures implemented. In particular, the procedures described in Section C.1.2 and C.1.4 to compute the equipment investment and the equipment stock proxies produce missing values in the first three years in which a firm is observed. In our final sample, we do not observe equipment stock for around 30% of firm-year observations and we cannot construct our variable for equipment payments.[A10]

To address this issue, we impute the expenditures on equipment for these firms based on industry-specific relationships between factor payments and size. Specifically, we run the following regression:

$$\log(W_{eit}X_{eit}) = \delta_s^y y_{it} + \delta_s^n n_{it} + \psi_{st} + v_{eit}$$

where $W_{eit}X_{eit}$ stands for the payments to equipment factors by firm i in industry s in

[A10]Specifically, 15.6% of missing values are due to the initialisation of the investment proxy, 32.4% are due to the initialisation of the perpetual inventory method and 24.6% are caused by other cleaning steps and 27.5% are due to firms not reporting data on BRN in a given year.

year t; y_{it} is the logarithm of gross output, and where n_{it} is the logarithm of employment (number of hours). We implement this imputation procedure on all observations reporting a reliable measure of gross output and this allows us to increase the coverage from 69.3% to 91.4%.[A11]

We use these imputed values to extend the sample of firms used in the aggregation exercise performed in Section 5, which only requires factor payments when studying the case of a uniform shock in the rental price of equipment. To avoid working with different aggregation samples, when studying the case of heterogeneous shocks we simply assign the average aggregate rental price of equipment to the imputed firms. While this approach reduces heterogeneity in our sample, the imputed firms are typically smaller and less likely to import capital equipment. So this assumption is not entirely implausible and these firms are essentially treated as those that purchase capital equipment only domestically, for which we cannot observe a firm-specific price. However, we refrain from using imputed values in the estimation of the micro-elasticities, which requires reliable values of both quantities and rental prices of capital equipment.

C.1.7 Aggregate Equipment Shocks

In this section, we outline the methodology used to construct the uniform equipment shocks employed in the analysis. In the baseline aggregation exercise (Table 6 and Figure 1b), the shock is defined as the log changes in the aggregate rental price based on industry-level data. First, we construct the industry-specific user cost of equipment, W_{est}, following the same approach detailed in Section C.1.5 with firm level data:

$$W_{est} = P^e_{st}\left(R^e_t + \delta^e_{st} - \frac{p^e_{s,t+1} - p^e_{s,t-2}}{3}\right)$$

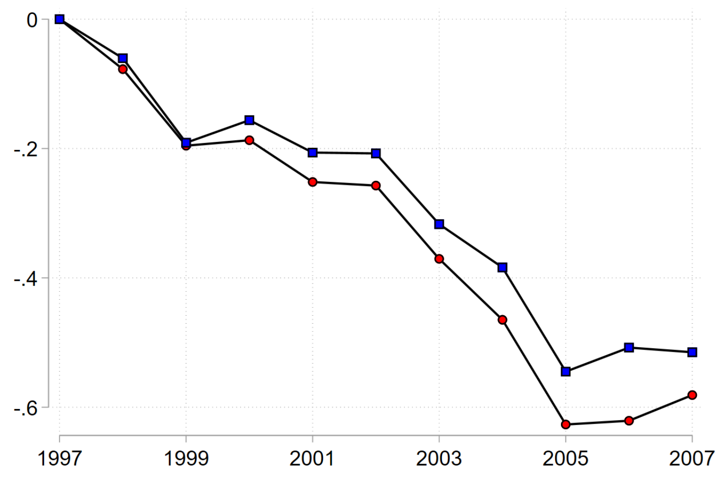
where P^e_{st} is the industry s price index for the ESA 2010 asset category *N110G - Machinery, Equipment and Weapons* retrieved from Eurostat, R^e_t is the required rate of return on equipment investments,[A12] and δ^e_{st} is the industry-specific depreciation rate obtained from EU-KLEMS.[A13] Subsequently, we aggregate across industries using the revised Sato-Vartia

[A11]The remaining firm-year observations do not report neither equipment stock nor gross output. As discussed in Section B.4, our sample is obtained by intersecting the DADS and BRN datasets. However, some firms appear in DADS for more years than in BRN. This discrepancy may be due to the fact that they did not always meet the minimum threshold to be subject to the 'réel normal' fiscal regime.

[A12]We use the annual average of France 10-year Government bond Yield, as retrieved from FED St. Louis.

[A13]See Section B.2 for more details on the two data sources.

Figure C.3: Uniform Equipment Shocks



Notes: The figure plots the cumulated equipment shocks observed over the period, rescaled by the aggregate change in the low skilled wage proxied with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour. The uniform shock represented with red circles is computed as the log change in the rental price using industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. The uniform shock represented with blue squares is computed by aggregating the firm-level heterogeneous changes in the user cost of equipment (W_{eit}).

price aggregation method described in Section C.1.3:

$$W_{et} = \exp\left(\sum_s (\log W_{est} - \log W_{est-1}) \frac{\omega^e_{st}}{\sum_s \omega^e_{st}}\right).$$

Lastly, we define the aggregate uniform equipment shock as the difference between the year-on-year log change in the aggregate user cost of equipment and the log change in the minimum wage (Smic) $\underline{W}_t$ retrieved from INSEE:

$$d \log \widehat{W}_{et} = d \log W_{et} - d \log \underline{W}_t$$

To facilitate a better comparison with the heterogeneous price shocks in Table D.7 and Figure D.2, we also construct an alternative uniform shock obtained by aggregating the firm-level heterogeneous shocks to the user cost of equipment. Let $\widetilde{\Lambda}_{eit}$ denote firm i's share of aggregate equipment payments following:

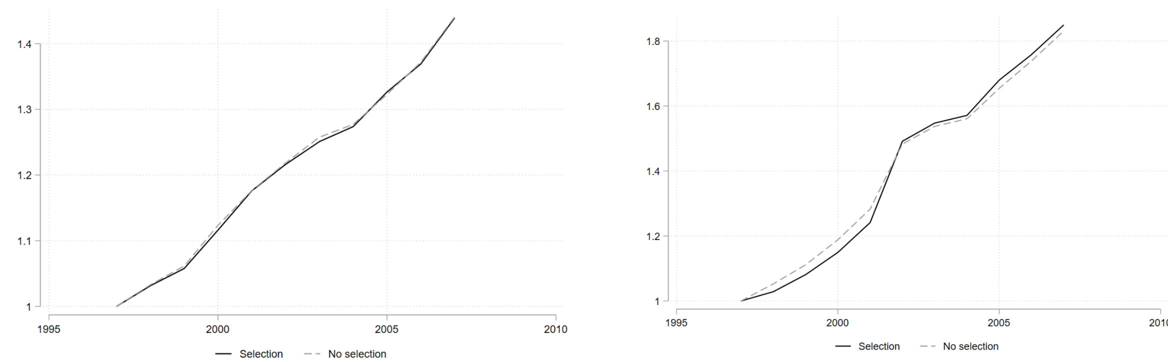
$$\widetilde{\Lambda}_{eit} = \frac{1}{2}\left(\Lambda_{it}\frac{\theta_{eit}}{\theta_{et}} + \Lambda_{it-1}\frac{\theta_{eit-1}}{\theta_{et-1}}\right) = \frac{1}{2}\left(\frac{W_{eit}X_{eit}}{\sum_i W_{eit}X_{eit}} + \frac{W_{eit-1}X_{eit-1}}{\sum_i W_{eit-1}X_{eit-1}}\right)$$

where $W_{eit}X_{eit}$ and $W_{eit-1}X_{eit-1}$ stand for the payments to equipment factors measured in

Figure C.4: Aggregate trends

(a) Low-skilled

(b) High-skilled



t and $t-1$ for firms i that are active in the market in both years.

The alternative uniform shock is defined as:

$$d \log \overline{W}_{et} = \left(\sum_i \widetilde{\Lambda}_{eit} d \log W_{eit} \right) - d \log \underline{W}_t$$

See Figure C.3 for a comparison between the two aggregate uniform shocks.

In the comparison exercise of Section 5.3, we define aggregate equipment payments as the sum of firm-level equipment payments ($\sum_i W_{eit} X_{eit}$). We further apply a quality adjustment to the aggregate price of equipment to match the exercise in Krusell et al. (2000). We take the average annual difference between the official NIPA series and the TORN series computed by Krusell et al. (2000) and redefine our aggregate uniform equipment shock as:

$$d \log \widetilde{W}_{et} = d \log W_{et} - 3.3$$

We then index $\widetilde{W}_{et}$ to one in 1997 and define the aggregate equipment stock as:

$$\widetilde{X}_{et} = \frac{\sum_i W_{eit} X_{eit}}{\widetilde{W}_{et}}.$$

C.2 Measuring firm-and-skill-specific Wages and Employment

In the model, all workers within a skill group are identical. Consequently, we need to correct the observed changes in firm- and skill-specific wages for worker-level unobservables and changes in returns to worker observable characteristics.

Table C.3: Correlation Table - Wage Proxies

	occupation, sex, full-time status	sex, full-time status	no controls
Low-skilled			
occupation, sex, full-time status	1		
sex, full-time status	0.999	1	
no controls	0.997	0.998	1
High-skilled			
	occupation, sex, full-time status	sex, full-time status	no controls
occupation, sex, full-time status	1		
sex, full-time status	0.999	1	
no controls	0.994	0.995	1

C.2.1 Data Cleaning

Our model-consistent measures of firm- and skill-specific wages are computed using data retrieved from DADS Poste. This dataset provides, for each worker, the observed wage in t and $t-1$, as well as the number of hours worked, the duration of the employment contract, and a set of individual characteristics.[A14] To construct reliable proxies for high- and low-skilled wages, we first exclude "atypical workers," namely those younger than 18 or older than 75, cross-border workers, interns and trainees ("apprentis," "stagiaires," and "emploi aidés"). We also exclude workers employed for less than a month or who recorded fewer than 150 paid hours.[A15] Finally, we exclude all workers for whom occupational information is missing.

Worker-level hourly wages are computed by dividing individual wage bills by the reported number of hours worked. The resulting variable is windsorised between a minimum value equal to 80% of the hourly minimum wage and €1,000.

In Figure C.4, we report the aggregate trend of the observed hourly wage variables, both with and without the selection strategy discussed above.

C.2.2 Estimation of Wage Residuals

In order to compute our residual wage variable, we estimate for both high- and low-skilled workers a Mincerian regression on log wage changes between $t-1$ and t controlling for worker observables (dummy variables for sex, full-time status and 2-digit

[A14]The dataset is at the job-spell level. Starting in 2002, a unique worker identifier is provided so that job spells can be correctly aggregated at the worker level. Before 2002, we cannot implement this aggregation and consider each spell as a separate worker.

[A15]In 2002, 23.5% of job spells fall below one of the two thresholds, but they account for only 1.3% of total hours worked.

occupation categories) and firm-year fixed effect. The specification takes the following form:

$$wage_{ji,t} - wage_{ji,t-1} = \boldsymbol{b}_t' \boldsymbol{X}_{jit} + \gamma_{fit} + e_{jit},$$

where worker j is classified as either $f = h$- or $f = \ell$- skilled depending on its occupation (*catégories socioprofessionnelles*) at time t, as defined earlier, and where $\boldsymbol{X}_{jit}$ denotes the vector of worker characteristics. The year-on-year change in the firm-specific wage is the estimated firm-year fixed effect.

We also produce two alternative proxies, computed by estimating the same model controlling for sex and full-time status and with no controls at all. In Table C.3, we report the correlation tables between our baseline variables and these alternatives.

As a final step, we chain the year-on-year changes together to obtain five-year changes, so that $\Delta w_{fit} = \sum_{\tau=0}^{4} \widehat{\gamma}_{fi,t-\tau}$ with $f \in \{h, \ell\}$. We further measure five-year changes in employment, Δx_{hit} and $\Delta x_{\ell it}$, by deflating changes in firm-and-skill-specific wage bills by the corresponding five-year wage change, Δw_{hit} and $\Delta w_{\ell it}$.

C.2.3 Aggregate Skill Supply and Wages

To construct the aggregate skill premium and skill supply, we construct factor-specific aggregate wages that are not influenced by the compositional adjustment that occurred over the study period. Specifically, we group worker-level observations based on the same set of covariates used to estimate the residuals in the previous section (sex, full-time status, and 2-digit occupation categories). We then measure the (weighted) average wage within each group for both high- and low-skilled workers, and compute the weighted average across all g groups within skilled and unskilled categories, where the weight is defined in terms of number of hours worked and is fixed across time by taking an average over the sample period:

$$\widetilde{W}_{ft} = \sum_{g \in G_f} \frac{W_{fgt} X_{fgt}}{X_{fgt}} \frac{\overline{X_{fg}}}{X_f}$$

where $W_{fgt} X_{fgt}$ is the sum of observed payments (wage bills) to factor $f \in \{h, \ell\}$ within group g and X_{fgt} is the corresponding observed number of hours worked (with slight abuse of notation). The compositionally adjusted hours used to construct the relative skill supply are computed from the observed wage bills by skill and the compositionally adjusted wages defined above.

In the comparison exercise of Section 5.3, we compute aggregate factor shares from the observed firm-level labor payments (wage bills) and the equipment payments computed in Sections C.1.5 and C.1.6, as follows:

$$\theta_{ft} = \frac{\widetilde{W}_{ft}\widetilde{X}_{ft}}{\sum_{f'} \widetilde{W}_{f't}\widetilde{X}_{f't}},$$

where $\widetilde{W}_{et}$ and $\widetilde{X}_{et}$ are defined in Section C.1.7. To better match aggregate data, we further rescale the aggregate factor shares. In particular we target the labor income share for the total economy in 1997 from EU-KLEMS, and we rescale the aggregate factor shares θ_{ft} with the rescaling constants κ_n for $f \in \{\ell, h\}$ and κ_e for $f = e$, defined as:

$$\frac{\theta_{\ell,1997} + \theta_{h,1997}}{\kappa_n} = 0.622,$$
$$\frac{\theta_{e,1997}}{\kappa_e} = 0.378.$$

We then further re-scale the factor shares in the years $t > 1997$ to ensure that they sum to one in every year.

C.3 Additional Details on the Construction of the Instruments

Real-exchange-rate Instrument. In order to address the endogeneity issues that affect the estimation of demand elasticities, in Section 4.2 we introduce a firm- and a sector-specific real-exchange-rate instrument. These instruments leverage variations in firm- and sector-specific import exposure across different source countries, alongside changes in real exchange rates between France and its trading partners. As noted by Borusyak et al. (2018), shift-share instruments are consistent when shocks, in our case to real-exchange rates, are as-good-as-randomly assigned, uncorrelated, large in number, and dispersed in terms of their average exposure. While these conditions are easily met by the firm-specific instrument, they may not always apply when considering industry-level shocks at the 2-digit classification. Therefore, in our baseline model, presented in Table 1, we construct the instrument at the 4-digit industry level. However, we also test a 2-digit industry-level instrument in column (4) of Table D.1.

Equipment Transit-cost Instrument. In Section 4.2, we introduce the transit-cost instrument used in the estimation of firm-level production elasticities. This instrument leverages differences in firm-specific import exposure across source countries and equipment types and product-specific predicted changes in transport costs between France and its trading partners induced by changes in oil and jet fuel prices. The proxy is closely modelled after the one developed by Hummels et al. (2014).

We exploit data on ad-valorem shipping costs to estimate the costs specific to each mode of transport and partner country. These costs are expressed as a function of the average product weight-to-value ratio, fuel prices, and shipping distances. Subsequently, we construct fitted cost measures using French customs data. Finally, we aggregate the country-product-specific changes in transport costs across different transport modes, weighted by the proportion of transactions associated with each mode.

French trade data do not provide data on transportation costs paid by firms, but we have information on the mode of transport which characterised each individual transaction in 2011, the first year in which this information is available. We use these data to estimate the relative share of product- and country-specific trade flows for different transport modes within our study period.[A16] We combine this information with EIA data on the average annual price of the U.S. Gulf Coast kerosene-type jet fuel (EIA, 2020) and with BP data on oil prices (Dudley et al., 2019).

To calculate transportation costs, we follow a three-step procedure:

1. First, we construct fitted cost measures using our variables and the coefficients obtained by Hummels et al. (2014), using the following specification

$$\log\left(\frac{F_{ckmt}}{V_{ckmt}}\right) = \widetilde{\beta}_{m0} + \widetilde{\beta}_{m1}\log\left(\frac{Q_{ckmt}}{V_{ckmt}}\right) + \widetilde{\beta}_{m2}\log P_t^{oil} + \widetilde{\beta}_{m3}\log P_t^{jf} \\ + \widetilde{\beta}_{m4}\log D_c + \widetilde{\beta}_{m5}\log P_t^{oil}\log D_c + \widetilde{\beta}_{m6}\log P_t^{jf}\log D_c,$$

   where F_{ckmt} is the transportation charge for HS6 product k, imported from country c using the transport mode $m = \{air, sea, rail, truck\}$, V_{ckmt} is the value of the shipment, Q_{ckmt} is the weight in kg, P_t^{oil} and P_t^{jf} are respectively oil and jet fuel prices and D_c is the weighted average bilateral distance between cities in the two countries.

2. Next, using the fitted cost measures, we calculate the relative shares of each transport mode by product type to create a proxy for product- and country-level trans-

[A16]While Hummels et al. (2014) use shares from 1994 to proxy for the period 1995-2006, due to data limitations we rely on shares from 2011 for our instrument, which covers the period 1998-2007. It is reasonable to assume that the transport modes for specific products and countries did not change significantly over these time frames.

port costs:

$$\log TC_{ckt} = \sum_m \frac{X_{ckm}}{X_{ck}} \log \left(\frac{\widehat{F_{ckmt}}}{V_{ckmt}} \right)$$

where $\frac{X_{ckm}}{X_{ck}}$ is the share of HS6 products k from country c that is imported using mode of transport m.

3. Lastly, we compute predicted five-year changes in transport costs and construct a firm-specific average weighting by initial equipment import shares:

$$\Delta ETC_{it} = \sum_c \sum_{k \in \mathcal{K}_e} \frac{M^e_{cki,t-5}}{M^e_{i,t-5}} \Delta \log TC_{ckt}. \tag{C.1}$$

Pre-sample weights are constructed using information on pre-sample equipment product k imports from country c, $M^e_{cki,t-5}$, over the total of equipment products imported by firm i, $M^e_{i,t-5}$.

Skill-specific Wage Instruments. The skill-specific-wage instruments presented in Section 4.2 leverage differences in the spatial compositions of firm production, the industrial mix of French regions, and the skill intensities and growth of individual industries. The intuition is that a rise in the nationwide level of output of a sector s that is particularly skill- (or unskill-) intensive will raise the skilled (unskilled) wage in a region with a large share of employment in that sector, affecting firms based on their spatial distribution over the territory. For these instrument, we exploit gross output data retrieved from EU KLEMS. The choice to use this external variable instead of aggregating firm-level gross output is primarily explained with the higher stability of aggregate data. Moreover, while BRN covers only firms subject to the 'réel normal' fiscal regime,[A17] EU KLEMS data cover the whole economy and thus represent a better proxy of the sectorial variations in skill-demand.

C.4 Estimation of the Nested CES Aggregate Production Function

[A17] See Section B.1 for more details

If we assume a nested CES aggregate production function, Equations (A.3) and (A.4) suggest the following estimation equation

$$x_{ht} - x_{et} = -\varsigma \left(w_{ht} - w_{et}\right) + \nu_{1,t}, \tag{C.2}$$

$$x_{ht} - x_{\ell t} = -\varsigma \, \left(w_{ht} - w_{\ell t}\right) + \frac{\varsigma - \rho}{1 - \rho} \log \left(\frac{\theta_{ht}}{\theta_{et} + \theta_{ht}}\right) + \nu_{2,t}, \tag{C.3}$$

where the residuals on the right hand side are related to the factor-augmenting productivity terms

$$\nu_{1,t} \equiv \rho \, \left(z_{ht} - z_{et}\right),$$

$$\nu_{2,t} \equiv \, z_{ht} - z_{\ell t} + \frac{\rho - \varsigma}{1 - \rho} z_{ht}.$$

In contrast to our approach, which is based on a combination of firm-level estimation of elasticities of production and demand together with theory-consistent aggregation, Krusell et al. (2000) rely on the identification assumption that that $\nu_{1,t}$ and $\nu_{2,t}$ are the sum of a constant and a zero-mean residual that is uncorrelated with the regressors on the right hand side. Relying on the same identification assumption, we can jointly estimate the model parameters in Equations (C.2) and (C.3) using a nonlinear GMM approach that uses the right-hand side variables as instruments.

D Additional Tables and Figures

Table D.1: Between-Firm Demand Elasticity Estimates (Alternative Instruments)

	(1)	(2)	(3)	(4)
ϵ	0.750	8.474	5.301	2.923
	(0.010)	(1.750)	(0.985)	(0.303)
η	0.796	6.596	2.061	1.185
	(0.023)	(1.273)	(0.750)	(0.548)
Observations	162,303	92,280	158,779	162,228
Eta level	4 dig	4 dig	2 dig	1 dig
Year FE	Yes	Yes	Yes	Yes
Sel Costs		Yes	Yes	No
IV	-	ETC	RER	RER
SW Min F-Stat		29.13	42.41	184.97
KP F-Stat		14.56	21.15	92.48

Notes: This table reports estimation results of Equation (26). The dependent variable is the 5-year change in firm i's value added and the independent variables are total input costs at the firm and sectoral level, with sectors defined at the 4 digit (columns 1-2), at the 2 digit (column 3), or at the 1 digit level (column 4). All instrumented columns use the sectoral RER instrument defined in Equation (34). Additionally, column (2) uses the transport cost instrument defined in Equation (35), and columns (3)-(4) use the firm-level RER instrument defined in Equation (33). All variables are in deviations from year means in all columns. To avoid weak identification in columns (2) and (3) we drop the bottom and top 1% of 5-year firm-level changes in total input costs (Sel Costs Yes). We report the Kleibergen and Paap (2006) rk Wald (KP F stat) and the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics obtained by running the equivalent 2SLS estimation.

Table D.2: Between-Firm Demand Elasticity Estimates (2SLS - ε only)

	OLS	2SLS					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Δc	0.257	-3.960	-3.948	-2.985	-2.834	-2.133	-1.930
	(0.013)	(1.258)	(1.539)	(0.961)	(1.258)	(0.497)	(0.425)
Observations	162,133	162,133	78,659	161,958	94,904	162,268	162,228
Year-Sector FE	4 dig	4 dig	4 dig	4 dig	4 dig	2 dig	1 dig
Excl Exp Mkt	No	No	Yes	No	No	No	No
IV	-	RER	RER	RER+SSW	ETC	RER	RER
KP F-Stat	.	16.71	11.41	6.75	9.79	57.22	71.95
ϵ	0.74	4.96	4.95	3.98	3.83	3.13	2.93

Notes: This table reports the results of estimating equation (26) by 2SLS when the sector-level price changes are absorbed by sector-year fixed effects. The dependent variable is the 5-year change in firm i's value added and the independent variables are total input costs at the firm level. Columns (2)–(3) and (6)–(7) use the firm-level RER instrument defined in Equation (33), column (4) uses the skill-specific-wage instruments defined in Equations (37) and (38), and column (5) uses the equipment transport cost instrument defined in Equation (35). In column (3), we exclude firms that export on average more than 10% their output to countries from which they also import. To avoid weak identification and implausible estimates in column 3 we drop the bottom and top 1% of 5-year firm-level changes in total input costs (Sel Costs Yes). We report the Kleibergen and Paap (2006) rk Wald (KP F stat) and the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics obtained by running the equivalent 2SLS estimation. Robust standard errors are clustered at the firm level and reported in parenthesis.

Table D.3: Between-Firm Demand Elasticity Estimates (3 Nests)

	(1)	(2)	(3)	(4)
ϵ	0.752	**5.182**	6.313	3.981
	(0.010)	**(0.935)**	(1.666)	(0.592)
η	0.781	**3.143**	3.699	2.505
	(0.023)	**(0.478)**	(0.878)	(0.302)
Observations	162,303	**162,303**	78,912	162,132
Eta level	4 dig	**4 dig**	4 dig	4 dig
Year-1digSect FE	Yes	**Yes**	Yes	Yes
Excl Exp Mkt	No	**No**	Yes	No
IV	-	**RER**	RER	RER+SSW
SW Min F-Stat		**39.28**	16.69	22.74
KP F-Stat		**19.64**	8.35	17.05

Notes: This table reports estimation results of Equation (26) with 1-digit sector by year fixed effects. The dependent variable is the 5-year change in firm i's value added and the independent variables are total input costs at the firm and 4-digit sectoral level. All instrumented columns use the firm-level RER instrument defined in Equation (33) and the sectoral RER instrument defined in Equation (34). Additionally, column (4) uses the skill-specific-wage instruments defined in Equations (37) and (38). All variables are in deviations from year means in all columns. In column (3), we exclude firms firms that import from any country in which they also export more than 10% of their total export value over the period. We report the the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics and the Kleibergen and Paap (2006) rk Wald (KP F stat) obtained by running the equivalent 2SLS estimation. Analytical standard errors are reported in parenthesis.

Table D.4: Firm-Level Production Elasticities (CRESH): First Stage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta w_e - \Delta w_\ell$	-1.339		-1.209		-1.339		-1.339	
	(0.245)		(0.176)		(0.245)		(0.245)	
$\frac{\varepsilon}{\varepsilon-1}\Delta r - \Delta x_e$	-0.033	-0.021	-0.097	-0.074	-0.035	-0.022	-0.028	-0.018
	(0.165)	(0.133)	(0.109)	(0.086)	(0.176)	(0.142)	(0.145)	(0.118)
$\Delta w_e - \Delta w_h$		-0.935		-0.738		-0.935		-0.935
		(0.207)		(0.148)		(0.207)		(0.207)
Observations	67,863	67,863	67,904	67,904	67,863	67,863	67,863	67,863
Year-Sector FE	2 dig	2 dig	1 dig	1 dig	2 dig	2 dig	2 dig	2 dig
ϵ	4.48	4.48	4.48	4.48	5.40	5.40	3.41	3.41
SW Min F-Stat	13.30	12.64	25.59	22.61	12.81	12.20	14.20	13.44
KP F-Stat	9.95	8.71	19.23	17.50	9.60	8.52	10.58	8.98

Notes: This table reports regression coefficients for the system of Equations (29) and (30), with each equation estimated separately by 2SLS using the same set of instruments of Table 2. All columns include year-sector effects with sectors defined at the 2 digit (columns 1-2 and 5-8) or at the 1 digit level (columns 3-4). Robust standard errors are clustered at the firm level and reported in parenthesis. SW Min F-Stat is the minimum value of the Sanderson and Windmeijer (2016) first-stage F statistics. KP F-Stat is the Kleibergen and Paap (2006) first-stage F-statistic.

Table D.5: Firm-Level Production Elasticities (CRESH), 3 Nests

	(1)	(2)	(3)	(4)
σ_ℓ	**1.282**	1.087	1.289	1.270
	(0.164)	(0.123)	(0.165)	(0.160)
σ_h	**1.101**	0.926	1.094	1.113
	(0.151)	(0.107)	(0.152)	(0.156)
σ_e	**1.202**	1.085	1.203	1.199
	(0.222)	(0.156)	(0.228)	(0.201)
Observations	**67,863**	67,904	67,863	67,863
Year-Sector FE	**2 dig**	1 dig	2 dig	2 dig
$\Pr[\sigma_\ell < \sigma_h]$	**0.06**	0.04	0.04	0.07
ϵ	**5.18**	5.18	6.31	3.98

Notes: This table reports regression coefficients for the system of Equations (29) and (30) estimated using GMM for a given value of ε from Table D.3, and with variables computed in deviations from year-sector means with sectors defined at the 2 digit or at the 1 digit level (column 2). Corresponding point estimates of σ_ℓ, σ_h, and σ_e are displayed. Bootstrap standard errors are reported in parenthesis, obtained by bootstrapping the estimation of ε and of this system 200 times and $\Pr[\sigma_\ell < \sigma_h]$ reports the share of bootstrapped estimates in which σ_ℓ is lower than σ_h.

Table D.6: Predicted Change in the Skill Premium, Uniform Shock, Back-of-the-envelope Calculation

	1997-2007	1990-2015
$\Delta \log W_e$	-0.232	-0.421
$\Delta \log W_\ell$	0.349	0.716
$\Delta \log W_e$ - $\Delta \log W_\ell$	-0.581	-1.137
$\Delta \log(W_h/W_\ell)$	0.034	0.066

Notes: This table reports the predicted log change in the skill premium $\Delta \log(W_h/W_\ell)$ in response to the observed fall in the equipment price relative to the low skill shadow wage. The predicted change is obtained by multiplying the total cumulated change in the relative equipment price by the average aggregate elasticities over the entire period from column 1 of Table 4. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour.

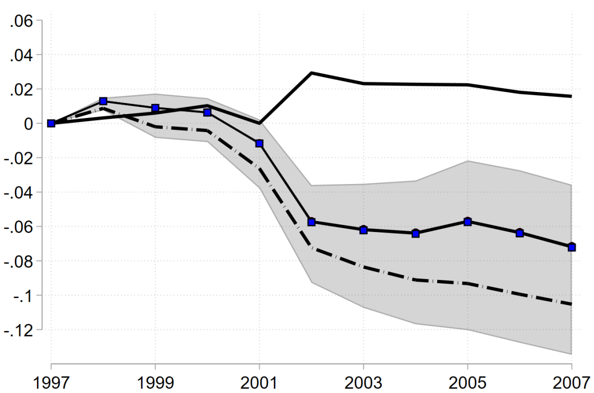
Table D.7: Decomposition of the Predicted Change in the Skill Premium

	Uniform Shock	Heterogeneous Shock
	$\Delta \log(W_h/W_\ell)$	$\Delta \log(W_h/W_\ell)$
Equipment	0.029	0.041
	[0.014, 0.042]	[0.024, 0.055]
Within firm complementarity	0.025	0.021
	[-0.002, 0.049]	[-0.006, 0.046]
Within firm substitution	-0.015	-0.014
	[-0.026, -0.002]	[-0.025, -0.002]
Cross firm	0.019	0.034
	[0.017, 0.021]	[0.031, 0.037]

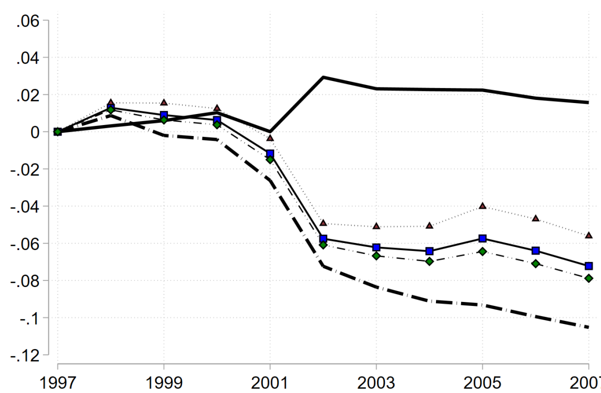
Notes: This table reports the decomposition of the predicted log change in the skill premium in response to the observed fall in the equipment price relative to the low skill shadow wage. The left panel presents the results obtained using a uniform shock computed by aggregating the firm-level heterogeneous price shocks (see Section C.1.7). The right panel reports the estimates obtained using heterogeneous price shocks, analyzed in Section 5.2. The table displays 90% confidence intervals obtained by bootstrapping the estimation of ε and each σ_f 200 times.

Figure D.1: Predicted vs Observed Skill Premium, Uniform Shock, Nested CES

(a) Nested CES vs CRESH



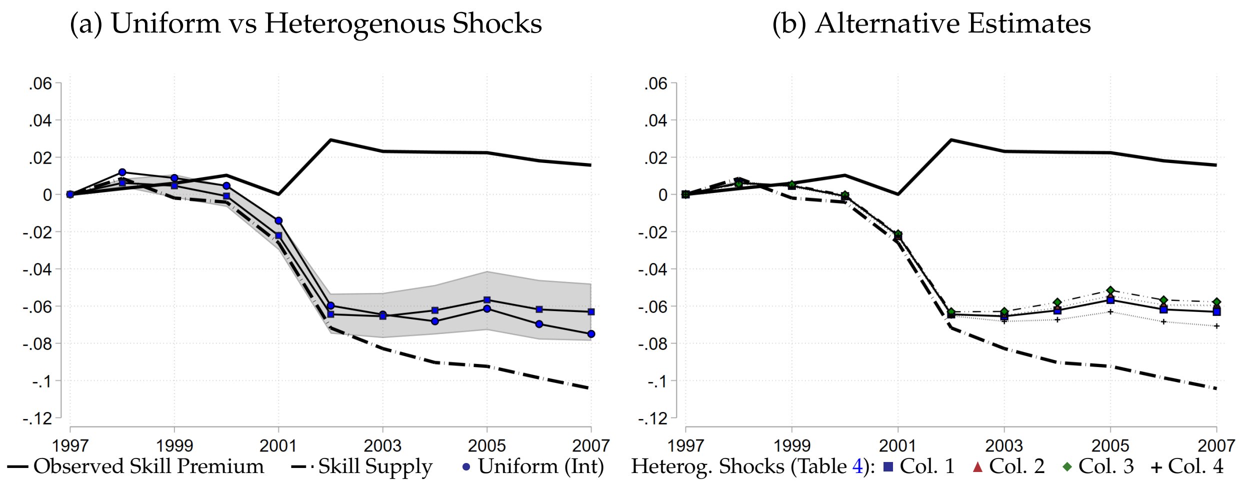
(b) Alternative Estimates



— Observed Skill Premium — · Skill Supply • CRESH (Figure 2a) Nested CES (Table 7): ■ Col. 1 ▲ Col. 2 ◆ Col. 3

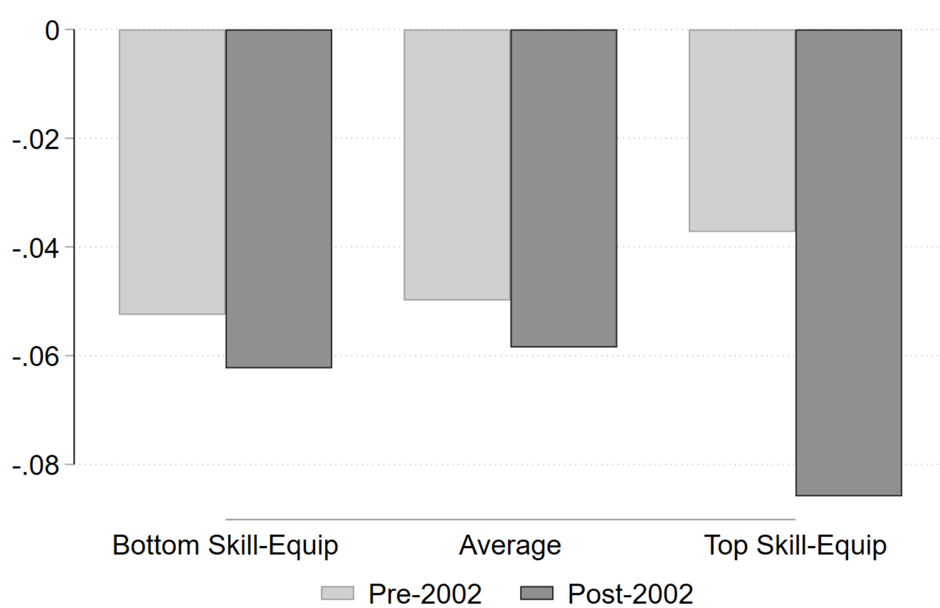
Notes: Panel (a) plots the evolution of the predicted skill premium ($d \log(W_h/W_\ell)$) in response to the observed fall in the relative equipment price and the observed change in the skill supply. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour (see Section C.1.7). The figure displays a 90% confidence interval obtained by bootstrapping the estimation of ε and each σ_f 200 times. Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 7.

Figure D.2: Predicted vs Observed Skill Premium, Heterogenous Shocks, CRESH



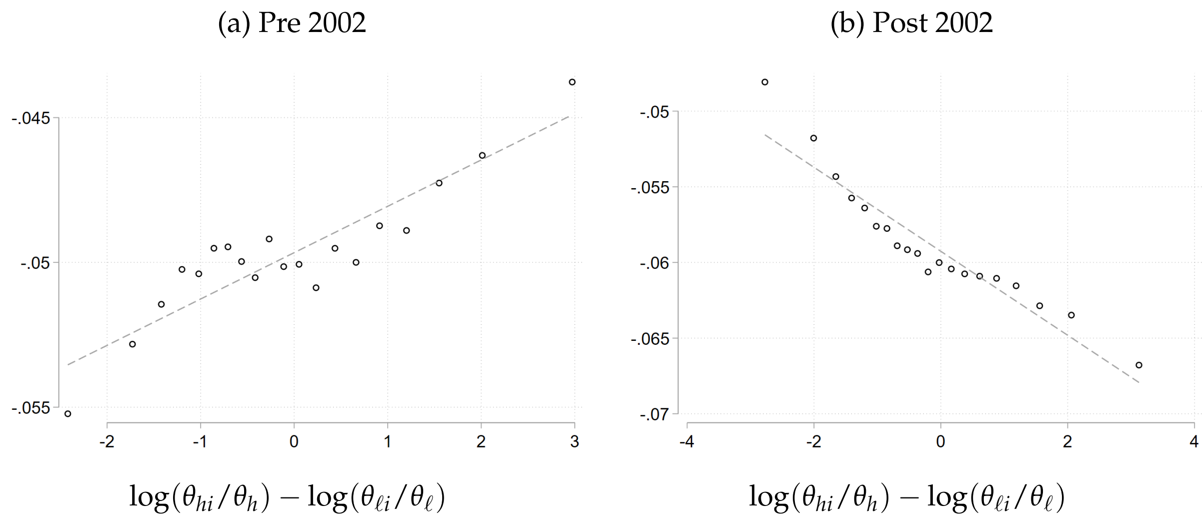
Notes: Panel (a) plots the evolution of the predicted skill premium ($d\log(W_h/W_\ell)$) in response to firm-specific relative equipment prices (see Section A.3.2). We proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour. The figure displays a 90% confidence interval obtained by bootstrapping the estimation of ε and each σ_f 200 times. We compare the results against the evolution of the predicted skill premium in response to a uniform change in relative equipment price computed by aggregating the firm-specific equipment price changes (see Section C.1.7). Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 4.

Figure D.3: Average Annual Equipment Shocks by Skill and Equipment Intensity



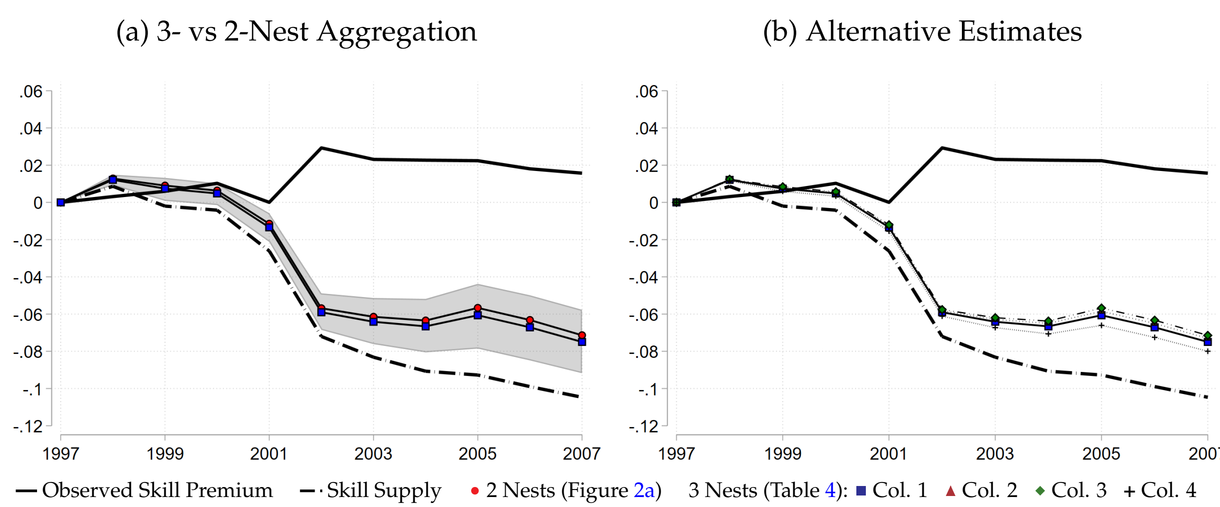
Notes: The chart reports the average firm-level equipment price shock by bins of skill and equipment intensity, $\Lambda_i\left(\frac{\theta_{hi}}{\theta_h}-\frac{\theta_{\ell i}}{\theta_\ell}\right)\theta_{ei}$, and further split into the pre- and post- 2002 periods. The bins are defined on the average skill and equipment intensity over the entire period, and the top and bottom bins represent 1% of firms (around 7000 firms each).

Figure D.4: Equipment Shocks by Skill Intensity



Notes: The figure plots a binscatter of the annual change in firm-specific equipment prices over the log of the skill intensity ratio $(\frac{\theta_{hi}/\theta_h}{\theta_{\ell i}/\theta_\ell})$.

Figure D.5: Predicted Skill Premium, Uniform Shock, 3-nest Aggregation



Notes: Panel (a) plots the evolution of the predicted skill premium ($d\log(W_h/W_\ell)$) in response to the observed fall in the relative equipment price and the observed change in the skill supply. We compute the log change in the rental price from industry-specific equipment investment prices taken from French national accounts and industry-specific depreciation rates from EU-KLEMS. We then proxy the aggregate change in the low skilled wage with the change in the nominal hourly minimum wage sourced from the French Ministry of Labour (see Section C.1.7). The figure displays a 90% confidence interval obtained by bootstrapping the estimation of ε and each σ_f 200 times. Panel (b) plots the variation in the predicted skill premium across the different estimated aggregate elasticities of Table 4.

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