Intangible Capital, Heterogeneous Borrowing Types, and Firm Dynamics

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Abstract

The rising importance of intangible capital presents a paradox: despite its critical role in innovation and productivity, aggregate U.S. productivity growth has slowed. This paper develops a firm-dynamics framework in which firms, depending on their reliance on intangible capital, face heterogeneous borrowing constraints—earnings-based and asset-based—that influence their ability to expand firm scope and productivity. We highlight a novel channel: firms more reliant on intangible capital are subject to earnings-based borrowing constraints, which tighten particularly during downturns, limiting their scope expansion and affecting firm cycle dynamics. This, in turn, contributes to the decline in aggregate productivity growth. We develop a model that incorporates firm-cycle behavior, including firm entry, where constraints shape investment and scope. This framework helps us understand how heterogeneous financial frictions influence firm scope and productivity through the lens of intangible capital.

Keywords: Intangible Capital, Financial Frictions, Earnings-Based and Asset-Based Borrowing, Firm Dynamics

JEL Codes: E22, E23 E24, E32, G01, G32, O31

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1 Introduction

The rising importance of intangible assets in firm investment decisions presents a paradox: despite their well-documented role in driving innovation and firm-level productivity, aggregate productivity growth in the U.S. has slowed markedly. A key factor in this divergence is the role of financial frictions, which differentially affect firms, particularly during periods of financial distress. The extent to which these constraints shape firms' investment decisions, borrowing behavior, and long-run productivity remains a central question in the literature. While existing research has explored these trends, the underlying mechanisms linking intangible capital accumulation, borrowing constraints, and productivity remain insufficiently understood.

This paper develops a firm-dynamics framework in which firms operate under heterogeneous borrowing constraints that depend on their reliance on intangible versus tangible capital. Our approach is motivated by two key characteristics of intangible capital. First, financing constraints for intangible investment differ fundamentally from those for tangible investment. While tangible assets can be pledged as collateral, intangible assets provide limited borrowing capacity, forcing firms to rely more heavily on internal funds or earnings-based borrowing. Second, intangible capital plays a crucial role in shaping firm scope and productivity growth, with firms investing heavily in intangibles often experiencing significant expansion in both dimensions. However, these gains depend on access to financing, and financial frictions can substantially dampen the benefits of intangible investment.

This interplay creates a trade-off: firms with greater intangible investment have higher productivity potential but face more severe financial constraints, particularly during economic downturns. Building on this insight, we propose a mechanism through which financial conditions influence the extent to which intangible capital contributes to firm and aggregate productivity. Under favorable financial conditions, firms investing in intangible capital can expand and enhance productivity growth through earnings-based borrowing. However, tightening financial constraints—particularly after the global financial crisis—have disproportionately affected intangible-intensive firms, limiting their ability to expand and sustain growth. We argue that the resulting decline in intangible capital investment, driven by tighter earnings-based borrowing constraints, has been a key factor in the post-crisis slowdown in aggregate productivity growth in the U.S. economy.

Our paper's novelty lies in three key contributions. First, we examine the role of the non-rivalry property of intangible capital in shaping firm-level productivity dynamics, offering an alternative explanation for observed productivity patterns. Second, to the best of our knowledge, this study is among the first to systematically investigate the relationship between intangible capital and heterogeneous borrowing types, distinguishing between earnings-based and asset-based financing. Third, we analyze the interaction between non-rivalry in intangible capital and borrowing constraints to provide insights into broader macroeconomic trends, including the observed productivity slowdown.

We empirically and theoretically examine the interplay between intangible capital, financial constraints, and firm dynamics. For the empirical part, using firm-level data from Compustat, intangible capital measures from Peters and Taylor (2017), and loan-level data from DealScan, we document four main findings. First, there is a positive and significant association between intangible capital and labor productivity, suggesting that intangible capital dynamics are a key factor behind productivity patterns. Second, firms with greater intangible capital investment exhibit higher sales growth and expanded firm scope, consistent with the nonrivalrous nature of intangible capital. Third, while intangible capital has historically been a key driver of productivity growth, its contribution has weakened since the early 2000s, coinciding with the broader slowdown in aggregate productivity in the U.S. economy. Fourth, firms more reliant on intangible capital face tighter borrowing constraints, increasingly depending on earnings-based borrowing and experiencing greater financial frictions, particularly in the aftermath of the global financial crisis. These findings highlight the interaction between financial frictions and intangible capital accumulation as a key mechanism underlying recent macroeconomic trends.

Based on the evidence and insights from our empirical framework, we develop a general equilibrium model with heterogeneous firms that differ in their borrowing constraints and production capabilities. The model economy features three key elements that drive firm dynamics and aggregate outcomes. First, firms can produce multiple varieties using a combination of tangible and intangible capital, where intangible capital exhibits non-rivalry in production - it can be deployed across different varieties without diminishing returns. Second, firms face two distinct types of borrowing constraints that are assigned at entry: earnings-based constraints, where borrowing capacity is tied to current profits, and asset-based constraints, where borrowing is limited by the collateral value of physical capital. Third, firms experience both aggregate and idiosyncratic productivity shocks that interact with these financial frictions to generate rich patterns in investment and production decisions.

Our theoretical framework yields several important implications for firm behavior and market structure. When firms face positive productivity shocks, their response depends crucially on their borrowing regime and capital composition. Firms with earnings-based constraints can leverage their improved productivity into greater borrowing capacity, allowing them to scale up both tangible and intangible investments. The model features an endogenous entry mechanism where potential entrants receive a signal about their productivity and face an ex-ante probability of being assigned either an earnings-based or asset-based borrowing constraint. Entry occurs only when the expected value of operation exceeds the fixed entry cost, with the entry decision incorporating the forward-looking value of the firm under its assigned borrowing regime. This endogenous entry condition, combined with the non-rivalry of intangible capital and heterogeneous financial constraints, generates rich firm dynamics that influence both the extensive and intensive margins of production. Through this structure, the model provides a unified framework for analyzing how financial frictions and intangible capital jointly shape firm dynamics, entry decisions, and aggregate outcomes.

Related Literature Our paper is related to several strands of the literature. The first strand of the literature focuses on the productivity slowdown in the U.S. economy. Some potential explanations behind the decline in the related literature are the decline in knowledge diffusion (Akcigit and Ates, 2023), market dominance by superstar firms (Autor et al., 2020), growing productivity dispersion between global frontiers and laggards (Andrews et al., 2016), implementation lags of technology (Brynjolfsson et al., 2018), changing cost structure through intangible capital (De Ridder, 2024), higher market power with rising intangible capital (Crouzet and Eberly, 2019), and many others. Moreover, some studies investigate underlying sources of endogenous productivity (see Howitt et al., 2004; Doraszelski and Jaumandreu, 2013; Anzoategui et al., 2019). Our empirical findings are at the firm level and forthcoming Jordà (2005) local projections in our model framework highlight the relevance of financial frictions.

The second branch of related literature focuses on the steady increase in intangible assets within firms over recent decades (see Corrado et al., 2009; Corrado and Hulten, 2010; McGrattan and Prescott, 2010; Eisfeldt and Papanikolaou, 2014; Corrado et al., 2016; Peters and Taylor, 2017; Ewens et al., 2019; McGrattan, 2020). This body of work demonstrates that the accumulation of intangible assets impacts various aspects of firm behavior, including productivity growth (Corrado et al., 2017; McGrattan, 2020), competition (Ayyagari et al., 2019), market dominance (Crouzet and Eberly, 2019; De Ridder, 2024), markups (Altomonte et al., 2021), profits (Crouzet and Eberly, 2020), and input factors (Chiavari and Goraya, 2020). In our context, Haskel and Westlake (2018) and Crouzet et al. (2022a) discuss the non-rivalrous properties of intangible capital. In our paper non-rivalry of intangible capital generates non-linearity in production factor demand amplifying short-run dynamics to financial shocks.

Lastly, our paper is related to the macro-finance literature concerned with the role of financial shocks and their transmission on the macro-level and micro-level. Canonical papers such as Kiyotaki and Moore (1997) and Bernanke and Gertler (1986) among others rely on a financial accelerator where a sudden decline in collateral value creates an intertemporal feedback loop that amplifies a financial shock throughout the economy. Our forthcoming calibration/estimation approaches resemble Jermann and Quadrini (2012) in estimating the series of financial shocks. Our firm-level estimations and forthcoming implications, when a financial shock occurs, resemble the micro-level analysis conducted by Greenwald (2019); Lian and Ma (2021). The aggregate considerations when the covenants for borrowers can vary as in Drechsel (2023) is a central feature adopted in our framework. The inability of firms to collateralize their intangible capital due to the limited commitment problem of the lenders' inability to seize such assets has been examined by Perez-Orive (2016); Lopez and Olivella (2018); Sun and Xiaolan (2019); Bianchi et al. (2019); Queralto (2020) among others. Our model overcomes this problem by the inclusion of earnings-based borrowing constraints which allow firms to finance their investment and innovation decisions tied to their earnings which endogenously depend on intangible capital (for a summary on financing innovation see Hall and Lerner, 2010; Kerr and Nanda, 2015). Our paper's insights are more closely aligned with Ottonello and Winberry (2024), which examines the importance of financial frictions in the allocation of investment and innovation. In contrast, our focus is on the impact of the non-rivalry channel of intangible capital on productivity and the incorporation of heterogeneous borrowing constraints, including earnings-based and asset-based borrowing. Overall, our contribution to the literature is a rigorous examination of how aggregate shocks are transmitted both at the firm level, considering flexible borrowing constraints.

Layout Our paper is organized as follows: Section 2 describes the databases and discusses the measurement of main variables such as intangible capital, earnings-based, and asset-based borrowing. Section 3 provides stylized facts and reduced-form empirical analysis. Section 4 presents the model setup and framework. Finally, Section 5 concludes by discussing key takeaways and suggesting future extensions.

2 Data

This section provides details on the data sources used in the paper.

Compustat We collect data on the income statement and balance sheet for U.S. publicly held firms from S&P's Compustat database at both yearly and quartely frequency. Our Compustat yearly sample data covers the period from 1980 to 2020. Following the typical sampling procedures found in related literature, we exclude financial firms (SIC codes 4900 - 4999), utilities (SIC codes 6000 - 6999), and firms categorized as public service, international affairs, or nonoperating establishments (SIC code 9000 and above). Additionally, we remove observations with missing or negative assets or sales, as well as small firms with total assets under $5 million. We also trim the selected variables based on industry (NAICS) and year. We measure firm-level labor productivity as the ratio of sales revenue per employee, as commonly used in the standard macroeconomics literature (for some examples see Comin and Philippon, 2005; Gutiérrez and Philippon, 2016; Autor et al., 2020). Table A.1 presents the firm-level constructed data variables and their descriptions in the Compustat sample. Table A.2 reports the summary statistics for selected variables in the Compustat yearly data.

Compustat Segment We use the Compustat Historical Segments File and follow the approach outlined by De Ridder (2024) to proxy the number of business lines operated by firms. Specifically, we count the number of products for each firm by examining the 6-digit NAICS codes associated with their business segments as reported by the firm. Consistent with the methodology described in De Ridder (2024), we assign a count of 1 for firms that are not present in the segments file. We utilize the number of business lines as a measure to quantify the non-rivalry channel of intangible capital. We also use this database to measure the net sales of different business segments (6-digit NAICS codes) for firms. Table A.3 provides the summary statistics for the number of business lines. Figure B.1 depicts the histogram of the number of business lines, revealing that over 90% of firm observations consist of only one business line per year.

Measuring Intangible Capital We do not directly use the $intan$ variable from the firm balance sheet (via Compustat) to measure intangible capital. We follow Peters and Taylor (2017) to measure firm-level intangible capital. The advantage of their method is the consideration of intangible capital's origin: internal and external as the accounting guidelines describe. External intangible capital can be observed in the balance sheet as intangible asset when a firm purchases an intangible asset externally. For example, after acquisitions, the assets of the acquired firm are recorded as part of Intangible Assets in the acquirer firm's balance sheet. On the other hand, they argue that the internally created intangible capital usually appears on the income statement as expenses and not in the balance sheet as intangible asset. For example, a firm's R&D spending to develop knowledge, patents, or software cannot be observed in the balance sheet as intangible asset. Or similarly, marketing expenses to strengthen a firm's brand is included in selling, general, and administrative (SG&A) expenditures. To capture these off-balance-sheet internal intangible expenses, Peters and Taylor (2017) implement the perpetual inventory method to capitalize them, and they call the capitalized R&D expenditures as *Knowledge Capital* and some capitalized portion of SG&A expenditures as *Organizational Capital*. We adopt their approach to measure intangible capital. Given our focus on the non-rivalry channel of intangible capital, we exclusively consider the organizational capital and exclude the knowledge capital. This is because the knowledge capital includes patents, which represent monopoly power and market influence, aspects that fall outside the scope of our study. We compute the intangible capital ratio as the ratio of organizational capital stock to the sum of organizational capital stock and net property, plant, and equipment (*ppent* variable). Table A.4 provides the summary statistics for the components of intangible capital and the intangible capital ratio, while Table A.5 documents the summary statistics for selected Compustat variables by intangible capital ratio. Figure B.2 displays the histogram of the intangible capital ratio in the entire sample, indicating significant heterogeneity across firms.

DealScan We complement our benchmark analysis with detailed loan-level data from the LPC DealScan database provided by Thomson Reuters. Our DealScan sample data covers the period from 1994 to 2017. The database covers approximately 75% of the overall volume of the U.S. commercial loan market. LPC DealScan database includes rich information on loan deals, which is the unit of observation, and depending on the deal, it can consist of several loan facilities. Following Lian and Ma (2021) and Drechsel (2023), we focus on loan covenants to extract information on earning-based borrowing. Loan covenants are contractual obligations that a borrowing firm must adhere to throughout the duration of a loan. Typically, covenants are associated with specific indicators and a numerical maximum or minimum value for this indicator is set for the loan deal. Since our focus is on earnings-based borrowing, we look for covenants related to earnings of a firm. For example, Drechsel (2023) shows that the most frequently used covenant, present at over 60% of the covenants, stipulates that the borrower's total debt cannot surpass earnings by a multiple of 4.58 at any point in time. Additionally, the top four most commonly mentioned covenants are linked to earnings before interest, taxes, depreciation and amortization (EBITDA) in the loan database.

We merge Compustat and LPC DealScan databases using the identifier links following Chava and Roberts (2008).[1] Table A.6 provides the summary statistics for number of deals and total deal amounts by distinguishing the deals with and without covenants. Figure B.3 illustrates the histogram of the number of loan deals, indicating that more than 80%

[1]January 2024 version of the link file is obtained from Michael Roberts' website.

of firm observations involve only one loan deal per year.

3 Empirical Analysis

In this section, we present empirical evidence that informs our theoretical model. First, we provide evidence on the dynamics of intangible capital and its relationship with productivity. Second, we present several facts that support the non-rivalry channel of intangible capital. Third, we analyze the relationship between firm-level non-rivalry and productivity through both cross-sectional and time-series analyses. Finally, we offer a set of stylized facts and reduced-form firm-level regression analyses to explore the empirical relationship between intangible capital and firms' heterogeneous borrowing constraints (earnings-based vs. asset-based borrowing).

3.1 Intangible Capital and Productivity

This section provides empirical evidence on the relationship between intangible capital and labor productivity at both the aggregate and firm levels.

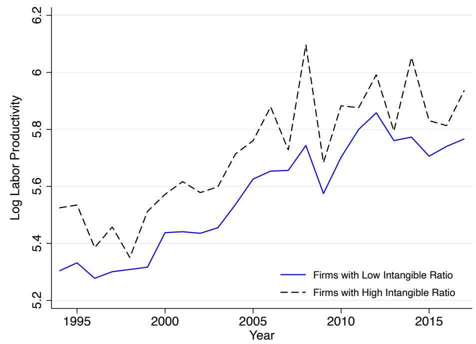
Aggregate Level We begin by examining the time trends of the intangible capital ratio and labor productivity at the aggregate level. Figure 1 presents the annual average growth rates of labor productivity (left axis) and intangible capital (right axis) using the Compustat sample. First, we observe a sharp decline in labor productivity growth after 2000, with no subsequent recovery to pre-2000 levels. A similar pattern emerges for intangible capital growth. Second, the figure indicates that labor productivity growth and intangible capital growth tend to co-move over time, suggesting a potential link between their dynamics. This provides preliminary evidence that changes in intangible capital may be associated with variations in labor productivity.

Figure 1: Intangible Capital Growth and Labor Productivity Growth

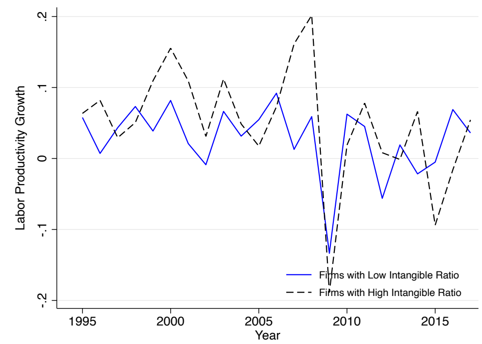
Note: This figure shows the annual average of labor productivity growth (left axis) and intangible capital growth (right axis) using the Compustat sample. Intangible capital is defined as the organizational capital stock. Labor productivity is measured as firm-level sales per worker.

Firm Level We also examine the relationship between intangible capital and productivity at the firm level. Figure 2a shows that firms with a higher intangible capital ratio (above the industry-year median) consistently exhibit higher labor productivity than those with a lower intangible capital ratio (below the industry-year median). However, productivity levels stagnate following the financial crisis. Figure 2b further illustrates that while intangible-intensive firms generally experience higher labor productivity growth, they suffer a sharp decline during crisis periods, such as the 2000–2001 dot-com crash and the 2008–2009 financial crisis. In later sections, we will explore how these productivity declines, particularly for intangible-intensive firms, relate to firm capital structure.

Figure 2: Labor Productivity - By Intangible Ratio



(a) Labor Productivity Level



(b) Labor Productivity Growth

Note: Panel (a) displays the annual average of log labor productivity, while Panel (b) illustrates the annual average of log labor productivity for firms with low and high intangible capital ratios using the Compustat sample. The division for high and low intangible ratios is determined by whether they are above or below the median of the intangible capital ratio within each industry and year. Labor productivity is measured as the firm-level sales per worker.

We further investigate the relationship between firm-level labor productivity and the intangible capital ratio using a regression model. Table 1 presents the results across different model specifications, all of which include control variables (total assets, markup, and Tobin's Q). Standard errors are clustered at the firm level. Across all specifications, the coefficient on the intangible capital ratio is positive and statistically significant, indicating a robust association between higher intangible capital intensity and increased labor productivity. These results hold even after accounting for firm-specific, industry-specific, and year-specific factors.

Table 1: Labor Productivity and Intangible Capital Ratio

	(1)	(2)	(3)	(4)
	Log Labor Productivity	Log Labor Productivity	Log Labor Productivity	Log Labor Productivity
L.Intangible Ratio	0.771***	0.320***	0.595***	0.374***
	(0.00900)	(0.0333)	(0.0333)	(0.0397)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
R^2	0.153	0.401	0.609	0.841
N	162936	162936	151374	150124

Note: This table displays the results of the regression specification where the dependent variable is the firm-level logarithm of labor productivity, and the main explanatory variable is the one-year lag of the intangible capital ratio. Control variables include the one-year lag of firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

In summary, we find that intangible capital and labor productivity significantly co-move both cross-sectionally and over time. Building on this insight, the following two sections explore an alternative explanation based on the non-rivalry aspect of intangible capital and its impact on firm productivity.

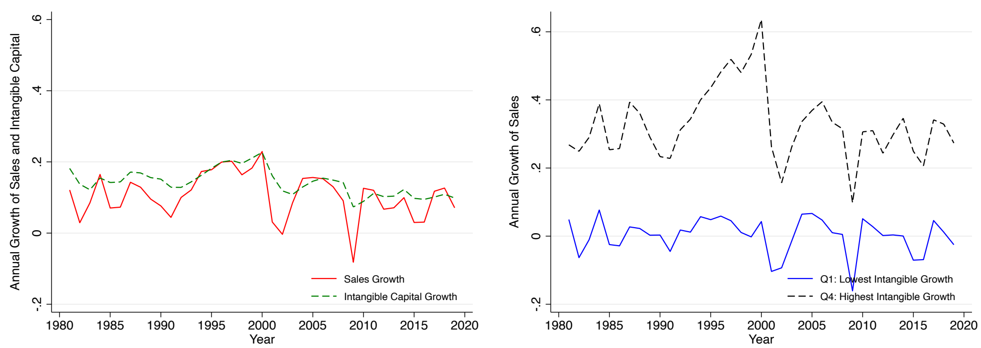
3.2 Intangible Capital and Non-Rivalry

In this section, we document several pieces of empirical evidence that quantify the non-rivalry channel of intangible capital, using different firm-level measures such as sales growth, the number of business lines, and firm scope measures from Hoberg and Phillips (2022).

3.2.1 Non-rivalry through Sales Growth

One key characteristic of intangible capital is its ability to be deployed across multiple business lines, which we argue can drive higher sales growth. Figure 3a provides suggestive evidence for this argument, showing that sales growth and intangible capital growth strongly co-move over time. Figure 3b illustrates that firms in the highest intangible capital growth quartile (Quartile 4) experience greater sales growth over time compared to those in the lowest intangible capital growth quartile (Quartile 1). Moreover, we observe a sharp increase in sales growth for firms with higher intangible capital during the IT boom period, followed by a dramatic decline during the financial crisis. We will link these findings to the various borrowing constraints faced by intangible capital-intensive firms.

Figure 3: Sale Growth and Intangible Capital Growth



(a) All Sample (b) By Intangible Growth Quartiles

Note: Panel (a) shows the annual growth rates of sales and intangible capital, while Panel (b) shows the annual growth rate of sales by Quartile 1 and Quartile 4 of intangible capital growth using the Compustat sample. Quartiles are constructed based on the intangible capital growth within each industry (NAICS) and year.

Given the various components involved in measuring intangible capital, we also explore which components exhibit stronger correlations with sales growth. Table A.7 reveals that the growth of internally acquired intangible capital (knowledge and organizational capital) is significantly and strongly correlated with sales growth. In contrast, externally acquired intangible capital (such as goodwill) shows a low and statistically insignificant correlation with sales growth. This evidence suggests that internally acquired intangible capital is likely more non-rivalrous than externally acquired intangible capital, supporting the notion that firms can more easily leverage their knowledge and organizational capital across multiple units simultaneously.

To systematically investigate the relationship between sales growth and intangible capital growth, we employ the following regression framework:

$$y_{it} = \beta_0 + \beta_1 \textit{intangible growth}_{it} + \beta_2' X_{it} + u_i + u_t + u_s + \epsilon_{it} \tag{1}$$

where the dependent variable, y_{it}, represents the firm-level annual sales growth for firm i at time t, and $\textit{intangible growth}_{it}$ is the firm-level annual growth of intangible capital. The vector X_{it} denotes firm-level control variables, including firm size (total assets), markup, and Tobin's Q. Firm size is measured as the logarithm of total assets. To control for the market power channel, we include firm-level markup, calculated using the method outlined by De Loecker et al. (2020).[2] To account for unobserved heterogeneity, we include firm (u_i), industry (u_s), and year (u_t) fixed effects. We also incorporate one-period lagged explanatory variables to address potential endogeneity concerns.

Table 2 demonstrates a positive and statistically significant association between intangible capital growth and sales growth, providing empirical support for the non-rivalrous channel of intangible capital on firm sales. Specifically, we observe that a one-unit increase in intangible capital growth is associated with an increase in sales growth ranging from 0.05 to 0.13, depending on the fixed effects considered in the regression specification. These results suggest that intangible capital growth plays a significant role in enhancing firm sales, supporting the hypothesis that intangible assets can be leveraged across multiple business lines, thereby contributing to sales performance.

[2]The details for the markup measure can be found in Table A.1.

Table 2: Sales Growth and Intangible Capital Growth

	(1)	(2)	(3)	(4)
	Sales Growth	Sales Growth	Sales Growth	Sales Growth
L.Intangible Growth	0.135***	0.136***	0.123***	0.0510***
	(0.00380)	(0.0106)	(0.0108)	(0.00876)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
R^2	0.0336	0.0534	0.0667	0.189
N	155245	155245	143730	142678

Note: This table displays the results of the regression specification where the dependent variable is the firm-level annual sales growth, and the main explanatory variable is the one-year lag of firm-level intangible capital growth. Control variables include the one-year lag of firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Using the Compustat Historical Segments File, we are able to observe the net sales of different business segments (6-digit NAICS codes) of firms. Utilizing this firm-level industry sales data, we investigate its relationship with firm-level annual intangible capital growth. Table 3 demonstrates a positive and statistically significant relationship between intangible capital growth and firm-level industry sales. Specifically, we observe that higher levels of intangible capital, measured by lagged log intangible assets, are associated with higher sales across different industry segments. This relationship holds across all model specifications, indicating its robustness.

Notably, the coefficient on the lagged log intangible capital variable is consistently positive and significant, with values ranging from 0.0961 to 0.331 across different specifications. This suggests that a 1% increase in intangible capital is associated with a

0.096% to 0.331% increase in sales in the firm's various business segments, depending on the model specification. These results remain significant even after controlling for firm-specific, industry-specific, and year-specific factors, further strengthening the evidence for the non-rivalrous nature of intangible capital. This finding implies that firms with greater intangible capital are able to leverage these assets across multiple industries, driving sales growth in various business lines simultaneously. These results provide additional support for the hypothesis that intangible capital can be effectively deployed across business segments, reinforcing the non-rivalry feature of intangible assets.

Table 3: Firm-level Industry Sales and Intangible Capital

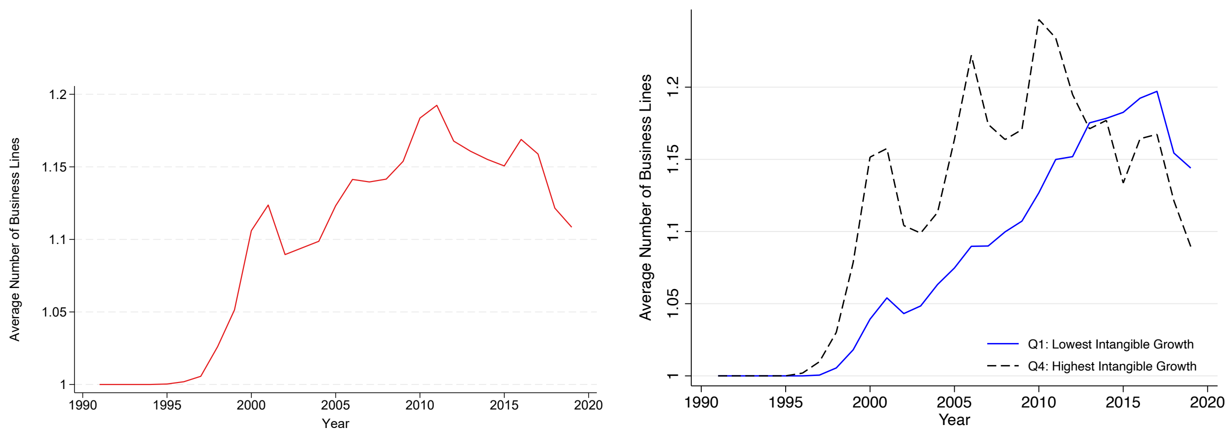
	(1)	(2)	(3)	(4)
	Firm Log Industry Sales	Firm Log Industry Sales	Firm Log Industry Sales	Firm Log Industry Sales
L.Log Intangible	0.317***	0.331***	0.232***	0.0961***
	(0.00216)	(0.0153)	(0.0124)	(0.0103)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
R^2	0.786	0.810	0.850	0.935
N	159500	159500	159468	158347

Note: This table displays the results of the regression specification where the dependent variable is the logarithm of net sales of different business segments (6-digit NAICS codes) of firms, and the main explanatory variable is the one-year lag of firm-level logarithm of intangible capital. Control variables include the one-year lag of firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

3.2.2 Non-rivalry through Firm Business Lines and Scope

We also examine the pattern in the number of business lines to assess the potential impacts of non-rivalrous intangible capital on firm-level product diversification. Figure 4a presents the annual average number of business lines using Compustat segment data, revealing an upward trend until the 2008 financial crisis, followed by a subsequent decline. To further investigate, we analyze the time-series pattern of the average number of business lines across quartiles of intangible capital growth. Figure 4b illustrates that firms in the highest quartile of intangible capital growth (Quartile 4) exhibit a higher and increasing average number of business lines until the financial crisis. However, these firms also experience a more pronounced decline in the number of business lines following the crisis compared to those in the lowest quartile of intangible capital growth (Quartile 1).

Figure 4: Average Number of Business Lines



(a) All Sample

(b) By Intangible Growth Quartiles

Note: Panel (a) shows the annual average number of business lines using the Compustat segment data. Panel (b) shows the annual average number of business lines by Quartile 1 and Quartile 4 of intangible capital growth using the Compustat segment data. Quartiles are constructed based on the intangible capital growth.

We also investigate the empirical relationship between the number of business lines and intangible capital. Since the dependent variable represents a count of business lines (an integer value), we use a Poisson regression model to estimate the relationship. The model includes the one-period lagged intangible capital growth, control variables (firm size and Tobin's Q), and fixed effects. Table 4 illustrates that higher levels of intangible capital are positively associated with an increase in the number of business lines. Specifically, an increase in intangible capital growth is significantly linked to the number of business lines, with a one-unit increase in intangible capital growth resulting in an increase of 0.01 to 0.06 in the number of business lines, depending on the fixed effects included in the model. These results remain significant even when controlling for industry-specific (NAICS) and year-specific factors, indicating that the relationship between intangible capital and the number of business lines is robust across different model specifications. This suggests that intangible capital growth plays a key role in driving firm diversification, reflected in an expanded range of business lines.

Table 4: Number of Business Lines and Intangible Capital Growth

	(1)	(2)	(3)	(4)
	Number of Business Lines	Number of Business Lines	Number of Business Lines	Number of Business Lines
L.Intangible Growth	0.0429***	0.0630***	0.0519***	0.0125***
	(0.00459)	(0.00661)	(0.00596)	(0.00355)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
N	144191	144191	144160	143108

Note: This table displays the results of the regression specification where the dependent variable is the firm-level number of business lines, and the main explanatory variable is the one-year lag of firm-level annual intangible capital growth. The table employs a Poisson regression model, as the dependent variable is a count variable. Control variables include the one-year lag of the firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

We also examine the relationship between intangible capital growth and a firm's scope, as measured using the data provided by Hoberg and Phillips (2022). Table 5 shows that intangible capital growth is positively and significantly associated with firm scope. Specifically, the results suggest that as firms increase their intangible capital, they are able to expand their scope over time. This relationship remains robust even after accounting for firm-specific, industry-specific, and time-specific effects, highlighting the importance of intangible capital in shaping the boundaries of firms. In particular, Column (4) of Table 5 reveals that a one-unit increase in intangible capital growth is associated with a significant increase in firm scope. This suggests that firms with higher intangible capital growth tend to diversify their activities and broaden their scope more effectively. These findings provide strong empirical support for the hypothesis that intangible assets enable firms to expand their business scope, which could involve entering new markets or introducing new product lines. This reinforces the role of intangible capital in facilitating firm growth and boundary expansion.

Table 5: Firm Scope and Intangible Capital Growth

	(1)	(2)	(3)	(4)
	Firm Scope	Firm Scope	Firm Scope	Firm Scope
L.Intangible Growth	-0.00201	0.0747**	0.0532*	0.0541***
	(0.0171)	(0.0245)	(0.0210)	(0.0156)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
N	15892	15891	15855	15590

Note: This table displays the results of the regression specification where the dependent variable is the firm scope, and the main explanatory variable is the one-year lag of firm-level annual intangible capital growth. The table employs a Poisson regression model, as the dependent variable is a count variable. Control variables include the one-year lag of the firm-level logarithm of total assets, markup, and Tobin's Q. Firm scope measure is based on the data provided by Hoberg and Phillips (2022). Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

All of our various measures of the non-rivalry aspect of intangible capital suggest that, after controlling for various firm-level variables, firms with higher levels of intangible capital and growth are better positioned to increase their sales growth, expand their number of business lines, and broaden their firm scope. These findings collectively highlight the non-rivalrous nature of intangible capital, demonstrating that intangible assets can be leveraged across multiple dimensions of firm performance and expansion, benefiting the firm as a whole without diminishing returns.

3.3 Non-rivalry and Productivity

Building on the strong suggestive evidence for the non-rivalry feature of intangible capital presented in the previous section, we first introduce a novel measure of firm-level non-rivalry and examine its relationship with firm-level productivity. We then explore how the evolution of non-rivalry over time is linked to the broader slowdown in productivity growth.

Measurement of Firm-level Non-rivalry As discussed in the previous section, an intuitive way to conceptualize the non-rivalry of intangible capital is to examine the proportional increase in sales across a firm's different business segments when the firm expands its intangible capital. Our empirical analysis shows that sales growth across a firm's various industries is positively and significantly associated with increases in intangible capital, supporting this intuition.

To formalize this relationship, we construct a firm-level measure of non-rivalry using the Compustat Historical Segments File, which reports net sales for different business segments (classified by 6-digit NAICS codes). Specifically, we calculate the annual sales growth of each business segment within a firm, compute the average segment-level sales growth, and normalize it by the firm's annual intangible capital growth. This approach captures the extent to which intangible capital expansion translates into growth across multiple business segments. A higher ratio indicates greater firm-level non-rivalry, as it reflects a stronger capacity to scale intangible capital across different industries.

Our non-rivalry measure is defined as follows:

$$\textit{non-rivalry}_{i,t} = \frac{\frac{1}{N_{i,t}} \sum_{s=1}^{N_{i,t}} \textit{sales growth}_{i,s,t}}{\textit{intangible growth}_{i,t}} \tag{2}$$

where i represents a firm, t denotes a year, and s indexes the firm's business segments. $N_{i,t}$ is the number of business segments firm i operates in during year t. *sales growth*$_{i,s,t}$ measures the annual sales growth of segment s for firm i in year t, while *intangible growth*$_{i,t}$

represents the firm's overall annual intangible capital growth. For firms operating in only a single business segment in a given year ($N_{i,t} = 1$), we use the total sales variable from the benchmark Compustat dataset. This adjustment ensures that our measure remains consistent across firms with varying degrees of diversification.

To validate whether our measure effectively captures the non-rivalry aspect of intangible capital, we analyze its relationship with firm-level intangible capital. Table 6 presents the regression results, where the dependent variable is our firm-level non-rivalry measure, and the key explanatory variable is the lagged logarithm of intangible capital. Across all model specifications, we find a strong and statistically significant positive relationship between intangible capital and non-rivalry.

In column (1), without fixed effects, we observe a significant coefficient of 0.425, suggesting that a 1% increase in intangible capital is associated with a 0.425 percentage point increase in non-rivalry. As we introduce year fixed effects in column (2), the coefficient remains positive and significant, though slightly attenuated to 0.219, indicating that the relationship holds even after accounting for time trends. Column (3) further includes industry fixed effects, strengthening the model's explanatory power, as reflected by the increase in R^2 to 0.118. Finally, column (4) incorporates firm fixed effects, controlling for time-invariant firm-specific characteristics. The coefficient increases to 1.466, suggesting that within-firm variations in intangible capital are strongly linked to changes in non-rivalry over time.

These findings confirm that firms with higher levels of intangible capital exhibit greater non-rivalry, reinforcing the idea that intangible capital is more easily scalable across different business segments. Moreover, the persistence of this relationship across different model specifications highlights the robustness of our measure in capturing the core feature of non-rivalrous intangible capital.

Table 6: Non-rivalry and Intangible Capital

	(1)	(2)	(3)	(4)
	Non-rivalry	Non-rivalry	Non-rivalry	Non-rivalry
L.Log Intangible	0.425***	0.219***	0.987***	1.466***
	(0.0154)	(0.0256)	(0.0332)	(0.0557)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
R^2	0.0101	0.0484	0.118	0.294
N	106248	106248	106204	104592

Note: This table displays the results of the regression specification where the dependent variable is the measure of firm-level non-rivalry derived from equation (2), and the main explanatory variable is the one-year lag of the firm-level logarithm of intangible capital. Control variables include the one-year lag of the firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Since our main objective is to explore whether the dynamics of non-rivalrous intangible capital are associated with firm-level productivity and productivity slowdown, we begin by investigating the relationship between firm-level non-rivalry and labor productivity. In Table 7, we find that, on average, firms with higher non-rivalry also exhibit higher labor productivity (except in regression specification (3)), even after controlling for various firm-level variables and fixed effects. This suggests that non-rivalrous intangible capital has the potential to enhance firm productivity.

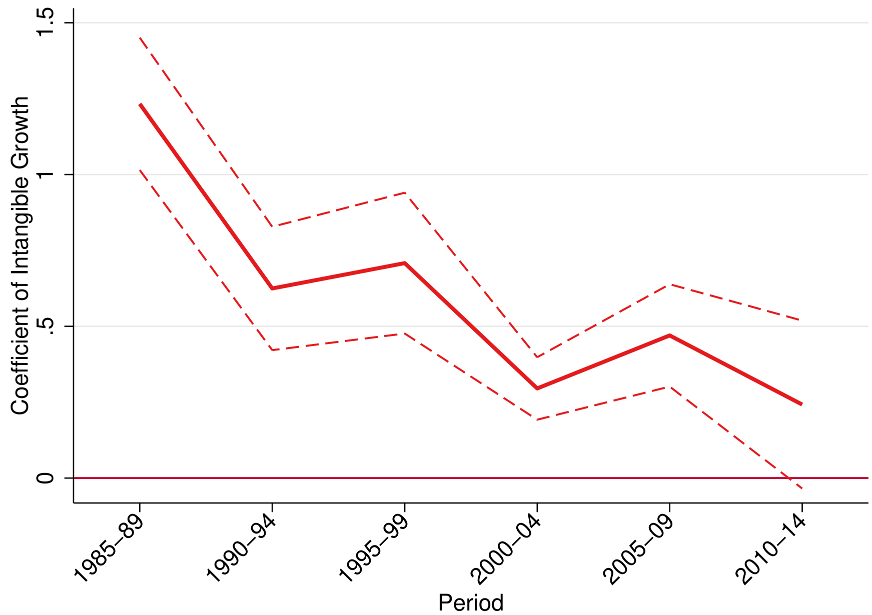
Table 7: Labor Productivity and Non-rivalry

	(1)	(2)	(3)	(4)
	Log Labor Productivity	Log Labor Productivity	Log Labor Productivity	Log Labor Productivity
L.Non-rivalry	0.0959***	0.0272***	-0.00650**	0.00473**
	(0.00246)	(0.00337)	(0.00238)	(0.00167)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
R^2	0.117	0.411	0.664	0.885
N	101682	101682	101639	100210

Note: This table displays the results of the regression specification where the dependent variable is the firm-level logarithm of labor productivity, and the main explanatory variable is the measure of firm-level non-rivalry derived from equation (2). Control variables include the one-year lag of the firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Given the strong association between firm-level non-rivalry and productivity, we next examine whether time-varying firm-level non-rivalry could be a contributing factor to the observed productivity slowdown. To do so, we analyze how the non-rivalry channel of intangible capital evolves over time. Figure 5 presents the coefficient of intangible capital growth from a variation of regression (1) estimated over rolling 5-year periods, which we interpret as the effect of the non-rivalry channel on sales growth. We find that the non-rivalry channel remains positive and significant in nearly all periods but follows a declining trend over time. Notably, its stagnation during the financial crisis period coincides with broader productivity slowdown.

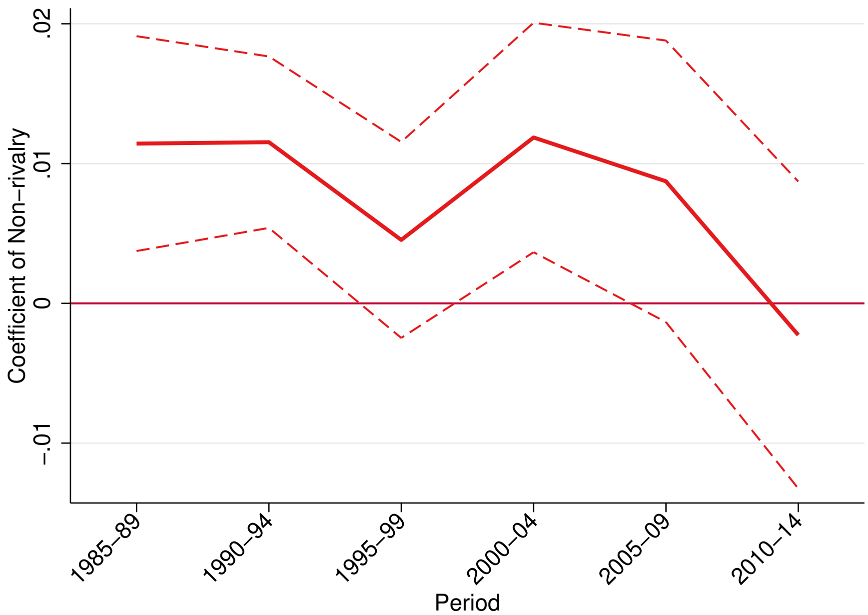
Figure 5: Non-Rivalry Channel of Intangible Capital Across Periods



Note: This figure documents the coefficient of contemporaneous firm-level annual intangible capital growth in a similar regression specification (1) for each 5-year period window, where the dependent variable is firm-level annual sales growth. Control variables include firm size (total assets) and firm-level Tobin's Q. Confidence intervals are at the 95% significance level.

Having demonstrated that the non-rivalry channel of intangible capital has weakened over time, we now examine how the productivity gains from firm-level non-rivalry have evolved over the same period. Figure 6 shows that the association between firm-level non-rivalry and labor productivity begins to decline sharply after the 2000s, coinciding with the broader slowdown in productivity. This suggests that while firm-level non-rivalry remains an important driver of labor productivity, its impact has diminished over time. This evidence supports our argument that the productivity slowdown is endogenously linked to changes in the degree of non-rivalry associated with intangible capital.

Figure 6: Labor Productivity Channel of Non-rivalry Across Periods



Note: This figure documents the coefficient of the one-year lagged firm-level measure of non-rivalry, derived in equation (2), in a similar regression specification shown in Table 7 for each 5-year period window, where the dependent variable is the firm-level logarithm of labor productivity. Control variables include the one-year lag of the firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. Confidence intervals are at the 95% significance level.

As the next step, we conduct further analysis to investigate the potential factors driving the declining importance of firm-level non-rivalry in productivity. In this context, we examine the role of heterogeneous borrowing constraints, which may influence this evolving relationship over time.

3.4 Intangible Capital and Earnings-Based vs. Asset-Based Borrowing

Since intangible capital is a key factor driving firm-level non-rivalry and is known to involve distinct financing structures, as discussed in the related literature, we investigate how firm-level heterogeneous borrowing constraints relate to intangible capital. As a novel approach, we distinguish between earnings-based and asset-based borrowing constraints and present empirical findings on the relationship between intangible capital and both forms of borrowing. We use the DealScan database to retrieve loan covenants and follow the methodologies of Lian and Ma (2021) and Drechsel (2023) to categorize borrowing as earnings-based or asset-based. Then, we match firms in the DealScan database with Compustat using identifier links, following Chava and Roberts (2008).[3]

3.4.1 Cross-sectional Analysis

This section analyzes the cross-sectional relationship between firm-level financial characteristics and intangible capital.

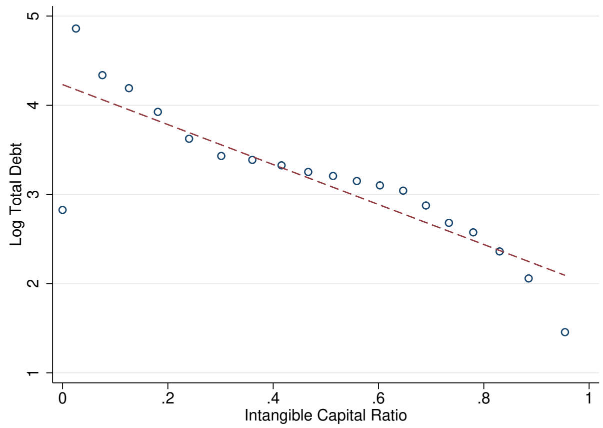
Descriptive Analysis We first provide a descriptive analysis of the association between selected firm-level financial characteristics and the intangible capital ratio. Notably, we use the secured debt ratio, defined as secured loans (backed by mortgages or other properties) divided by total assets, as a proxy for the degree of asset-based borrowing.

Figure 7 constructs various bins for each selected firm-level financial variable and its intangible capital ratio. It shows that as the firm-level intangible capital ratio increases, (a) total debt, (b) secured debt ratio, and (c) leverage ratio (total debt to total assets) decline, while (d) the cash ratio (total cash to total assets) increases. This set of descriptive evidence suggests that firms with a higher intangible capital ratio are likely to face tighter borrowing constraints and have lower asset-based borrowings.

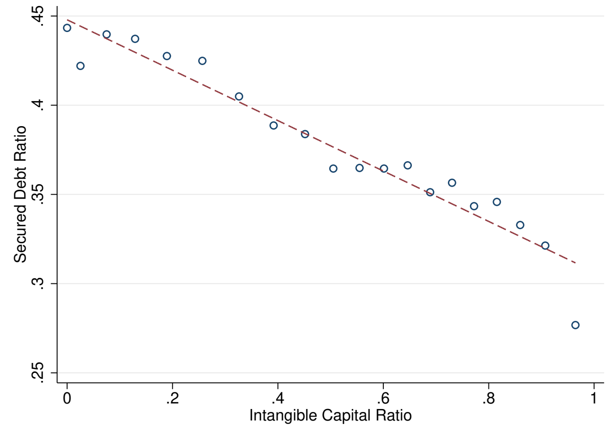
Analysis using the Compustat Data Building on the insights from the descriptive analysis, we conduct a regression analysis using our Compustat sample to investigate the systematic association between firm-level debt characteristics and the intangible capital ratio.

[3]The version of the linked file we retrieved is the most up-to-date as of January 2024.

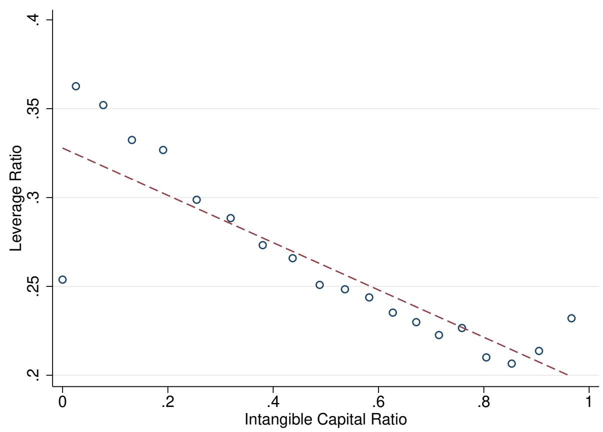
Figure 7: Financial Characteristics vs. Intangible Capital Ratio



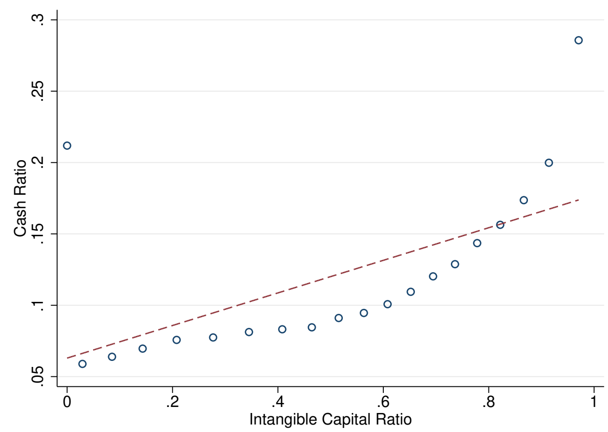
(a) Total Debt vs. Intangible Ratio



(b) Secured Debt Ratio vs. Intangible Ratio



(c) Leverage Ratio vs. Intangible Ratio



(d) Cash Ratio vs. Intangible Ratio

Note: This figure shows bin scatter plots between selected firm-level financial variables and the intangible capital ratio for the entire sample. The financial variables are (a) the logarithm of total debt, (b) the secured debt ratio, (c) the leverage ratio (total debt per total assets), and (d) the cash ratio (total cash per total assets).

Table 8 presents regression results examining the relationship between total debt and the intangible capital ratio at the firm level. The dependent variable is the logarithm of firm-level total debt, regressed on the one-year lagged intangible capital ratio. Control variables (assets, markup, and Tobin's Q) and various fixed effects (firm, year, and industry) are included in all specifications to account for potential confounding effects. The coefficients for the intangible capital ratio across different specifications (columns (1) to (4)) consistently show negative and statistically significant associations. This indicates that higher levels of intangible capital are associated with lower levels of total debt. Overall, the results suggest a robust negative relationship between intangible capital and total debt, implying that firms with higher intangible capital ratios tend to have lower total debt levels, even after accounting for other factors included in the regression models.

Table 8: Firm-level Total Debt and Intangible Capital Ratio

	(1)	(2)	(3)	(4)
	Log Total Debt	Log Total Debt	Log Total Debt	Log Total Debt
L.Intangible Ratio	-0.924***	-0.827***	-0.535***	-0.557***
	(0.0126)	(0.0329)	(0.0466)	(0.0684)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
R^2	0.721	0.726	0.752	0.858
N	149426	149426	137680	136261

Note: This table displays the results of the regression specification where the dependent variable is the firm-level logarithm of total debt, and the main explanatory variable is the one-year lag of the firm-level intangible capital ratio. Control variables include the one-year lag of the firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 9 presents the results of regressions analyzing the relationship between the secured debt ratio and the intangible capital ratio at the firm level. The dependent variable is the secured debt ratio, defined as the ratio of debt secured by mortgages or other assets to total debt, including current liabilities. The control variables are the same as those in Table 8. Overall, the results reveal a robust negative relationship between intangible capital intensity and the secured debt ratio, suggesting that firms with higher intangible capital are likely to rely less on asset-based borrowings.

Table 9: Firm-level Secured Debt Ratio and Intangible Capital Ratio

	(1)	(2)	(3)	(4)
	Secured Debt Ratio	Secured Debt Ratio	Secured Debt Ratio	Secured Debt Ratio
L.Intangible Ratio	-0.186***	-0.220***	-0.197***	-0.148***
	(0.00404)	(0.00971)	(0.0119)	(0.0190)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	No	No	No	Yes
Industry FE	No	No	Yes	Yes
Year FE	No	Yes	Yes	Yes
R^2	0.0627	0.0862	0.171	0.508
N	104547	104547	103468	101986

Note: This table displays the results of the regression specification where the dependent variable is the firm-level secured debt ratio, and the main explanatory variable is the one-year lag of the firm-level intangible capital ratio. The secured debt ratio is calculated as the share of secured debt (backed by mortgages or other secured assets) in total debt ($\frac{\textit{Debt - Mortgages \& Other Secured (dm)}}{\textit{Total Debt (dlc + dltt)}}$). Control variables include the one-year lag of the firm-level logarithm of total assets, markup, and Tobin's Q. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Analysis using the Dealscan-Compustat Merged Data Even though using Compustat data provides crucial insights into the relationship between firm-level financial characteristics and intangible capital, we supplement this with the DealScan database, which offers rich information on loan deals and covenants to distinguish between earnings-based and asset-based borrowings. We merge this data with our Compustat sample. In this context, to provide more systematic evidence and quantify the role of intangible capital in heterogeneous borrowing constraints, such as earnings-based and asset-based borrowings, we specify the following regression model:

$$y_{it} = \beta_0 + \beta_1 \textit{intangible capital}_{it} + \beta_2' X_{it-1} + u_i + u_t + u_s + \epsilon_{it} \tag{3}$$

where the dependent variable takes the following values: i) the firm-level total number of earnings-based loan deals per year, ii) the firm-level total number of asset-based loan deals per year, iii) the firm-level total volume of earnings-based loan amounts per year, and iv) the firm-level total volume of asset-based loan amounts per year. The variable $\textit{intangible ratio}_{it}$ represents the the firm-level logarithm of intangible capital. The vector of firm-level control variables, denoted as X_{it-1}, includes earnings before interest, taxes, depreciation, and amortization (*EBITDA*), one-year lagged *EBITDA*, Tobin's Q, and net property, plant, and equipment (*ppent*), with all control variables normalized by one-year lagged book assets. To account for unobserved heterogeneity, we incorporate firm (u_i), year (u_t), and industry (u_s) fixed effects. One-year lagged explanatory variables are included to mitigate potential endogeneity issues. Standard errors are clustered at the firm level.

Following the specifications of earnings-based and asset-based borrowings in Lian and Ma (2021) and Drechsel (2023), Tables 10 and 11 show that firms with a higher intangible capital ratio are more likely to have a greater share of earnings-based borrowings. Our empirical estimates suggest that a one-unit increase in the intangible capital ratio is associated with an approximate 0.005 increase in the share of earnings-based borrowing. Together with the evidence in Table 9, we find that firms with a higher intangible capital ratio are more likely to rely on earnings-based borrowing and less on asset-based borrowing.

Table 10: Borrowing Types (based on Lian and Ma (2021) specification) and Intangible Capital

	(1) #EBS Deals	(2) #ABS Deals	(3) EBS Log Deal Amounts	(4) ABS Log Deal Amounts
Intangible Capital	0.00540*	-0.0596**	0.00706***	-0.0578**
	(0.00263)	(0.0185)	(0.00212)	(0.0191)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
N	15174	8986	14058	8343

Note: The table employs a Poisson regression model, where the borrowing types are specified based on Lian and Ma (2021). EBS (ABS) stands for earnings-based (asset-based) borrowing type. Control variables include earnings before interest, taxes, depreciation, and amortisation (*EBITDA*), one-year lagged *EBITDA*, Tobin's Q, net property, plant and equipment (*ppent*), where all control variables are normalized by one-year lagged book assets. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 11: Borrowing Types (based on Drechsel (2023) specification) and Intangible Capital

	(1) #EBS Deals	(2) #ABS Deals	(3) EBS Log Deal Amounts	(4) ABS Log Deal Amounts
Intangible Capital	0.00582*	-0.0763***	0.00715**	-0.0766***
	(0.00284)	(0.0188)	(0.00247)	(0.0189)
Control Variables	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
N	14226	6796	13108	6204

Note: The table employs a Poisson regression model, where the borrowing types are specified based on Drechsel (2023). EBS (ABS) stands for earnings-based (asset-based) borrowing type. Control variables include earnings before interest, taxes, depreciation, and amortisation (*EBITDA*), one-year lagged *EBITDA*, Tobin's Q, net property, plant and equipment (*ppent*), where all control variables are normalized by one-year lagged book assets. Standard errors (in parentheses) are clustered at the firm level. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Overall, our findings in this section suggest that firms with a higher intangible capital ratio tend to face tighter borrowing constraints and rely more on earnings-based borrowings.

3.4.2 Time-series Analysis

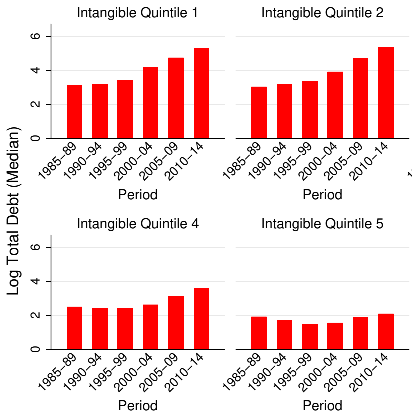
In this section, we analyze the time-series dynamics of firm-level debt characteristics and their association with intangible capital to investigate how this relationship evolves over time and relates to periods of productivity slowdown.

Analysis using the Compustat Data We begin by analyzing firm-level financial variables from the Compustat data, examining various measures of financial characteristics to assess the relationship between borrowing constraints and intangible capital. To capture how debt dynamics evolve over time in relation to intangible capital, we divide the sample into five-year periods and analyze trends in firm-level debt characteristics across intangible capital quintiles.

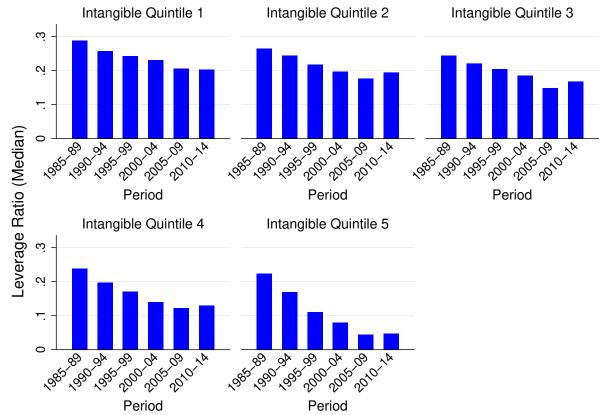
Figure 8 provides several key insights. First, total debt, leverage ratio, and secured debt ratio are consistently lower for the most intangible-intensive firms (Quintile 5) compared to other firms across all periods. Second, as shown in Figure 8a, total debt exhibits an increasing trend for less intangible-intensive firms after the 2000s, whereas it remains nearly stagnant for the highest intangible-intensive firms (Quintile 5) during the same period. Third, Figure 8b shows that the decline in the leverage ratio over time is more pronounced for the most intangible-intensive firms (Quintile 5) compared to other quintiles. Lastly, Figure 8c indicates that the secured debt ratio drops sharply for the most intangible-intensive firms (Quintile 5) after the 2000s, a trend not observed in other quintiles.

Collectively, these findings suggest that borrowing constraints have tightened disproportionately for the most intangible-intensive firms since the 2000s—a period also associated with the productivity slowdown.

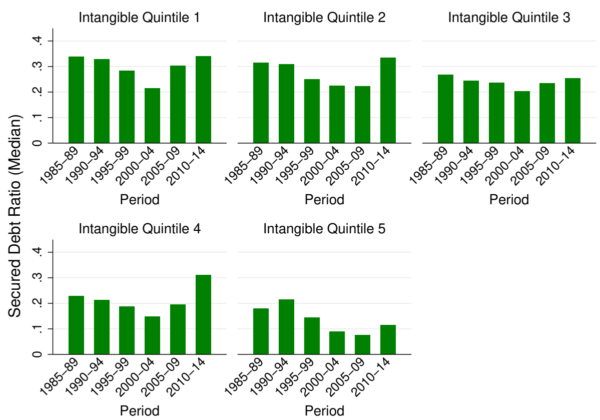
Figure 8: Total Debt, Leverage Ratio, and Secured Debt Ratio by Intangible Capital Ratio Over Time



(a) Total Debt



(b) Leverage Ratio

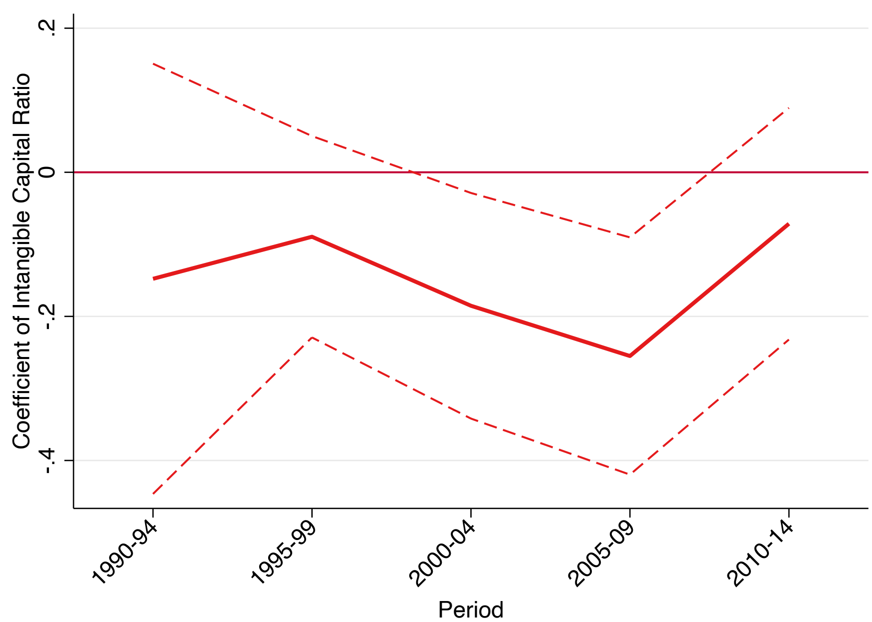


(c) Secured Debt Ratio

Note: This figure shows the median of selected firm-level debt characteristics by 5-year time periods and quintiles of the intangible capital ratio. The firm-level debt characteristics are: (a) the logarithm of total debt, (b) the leverage ratio (total debt to total assets), and (c) the secured debt ratio (the share of secured debt backed by mortgages or other secured assets relative to total debt). The quintiles are constructed based on the firm-level intangible capital ratio within each industry and year.

Analysis using the Dealscan-Compustat Merged Data We extend our analysis by utilizing the merged DealScan-Compustat dataset to incorporate more detailed information on loan deals. To examine how borrowing constraints evolve with the intangible capital ratio over time, Figure 9 plots the coefficient of the intangible capital ratio from a regression framework similar to that in (3) across five-year periods, where the dependent variable is the firm-level logarithm of total deal amounts. Our findings indicate that firms with a higher intangible capital ratio face the tightest borrowing constraints, particularly after the 2000s, including during the financial crisis. This result supports our argument that borrowing constraints become relatively tighter for intangible-intensive firms, especially during periods of adverse financial shocks and productivity slowdowns.

Figure 9: Association between Intangible Capital and Loan Deal Amounts Over Time



Note: This figure presents the coefficient of the contemporaneous intangible capital ratio from a regression specification similar to (3), including year fixed effects, estimated separately for each five-year period. The dependent variable is the firm-level logarithm of loan deal amounts. Control variables include firm size (measured as total assets) and firm-level Tobin's Q. Confidence intervals are shown at the 95% significance level.

Overall, our key empirical findings can be summarized as follows: i) There is a positive and significant association between intangible capital and labor productivity, suggesting that intangible capital dynamics are a key factor behind productivity patterns, ii) Firms with a higher intangible capital ratio experience greater sales growth and an expansion in the number of business lines, consistent with the non-rivalry properties of intangible capital, iii) The positive and significant relationship between firm-level non-rivalry and labor productivity weakens substantially after the 2000s, aligning with the observed productivity slowdown, and iv) Firms with a higher intangible capital ratio tend to have lower debt levels and a greater reliance on earnings-based borrowing, with borrowing constraints becoming increasingly restrictive after the 2000s. A striking insight is that the last two findings coincide with periods of productivity slowdown. We argue that the dependence of intangible-intensive firms on earnings-based borrowing, which is particularly vulnerable to financial distress during and after the global financial crisis, may be a key driver of the productivity slowdown. Motivated by this empirical evidence, we now develop a model framework that integrates the non-rivalry properties of intangible capital with heterogeneous borrowing constraints to analyze their joint impact on firm dynamics.

4 Model

The economy is set as a discrete-time infinite horizon economy with time denoted t indexed starting from 1. Firms, capital goods producers, and representative households populate the economy. The firm sector is comprised of final goods producers and intermediate producers. Final consumption goods producers purchase intermediate goods and are perfectly competitive. Intermediate producers retain monopolistic rights and produce varieties that can produce more than one variety. They also face borrowing constraints where some firms are subject to earnings-based conditions while others are subject to asset-based conditions. These intermediate producers utilize tangible and intangible capital in their production process, with intangible capital being nonrivalrous in producing varieties.

Final Good Producers There is a continuum of final good producers where the final good priced as the numeraire is a composite given as a CES aggregator of intermediate goods

$$Y_t = \left(\int_0^{A_t} y_{i,t}^{\frac{1}{\eta}} di \right)^{\eta} \tag{4}$$

where $y_{i,t}$ is the intermediate variety i purchased from an associated producer and $\eta > 1$ is the elasticity of substitution between different varieties. The variable A_t represents the stock of intermediate varieties, which intermediate firms endogenously determine. We arrive at the standard demand function for intermediate goods

$$p_{i,t} = P_t Y_t^{\frac{\eta-1}{\eta}} y_{i,t}^{\frac{1-\eta}{\eta}}. \tag{5}$$

Intermediate prices aggregate to the economy's price level as $P_t = \left(\int_0^{A_t} p_{i,t}^{\frac{1}{1-\eta}} di \right)^{1-\eta} = 1.$

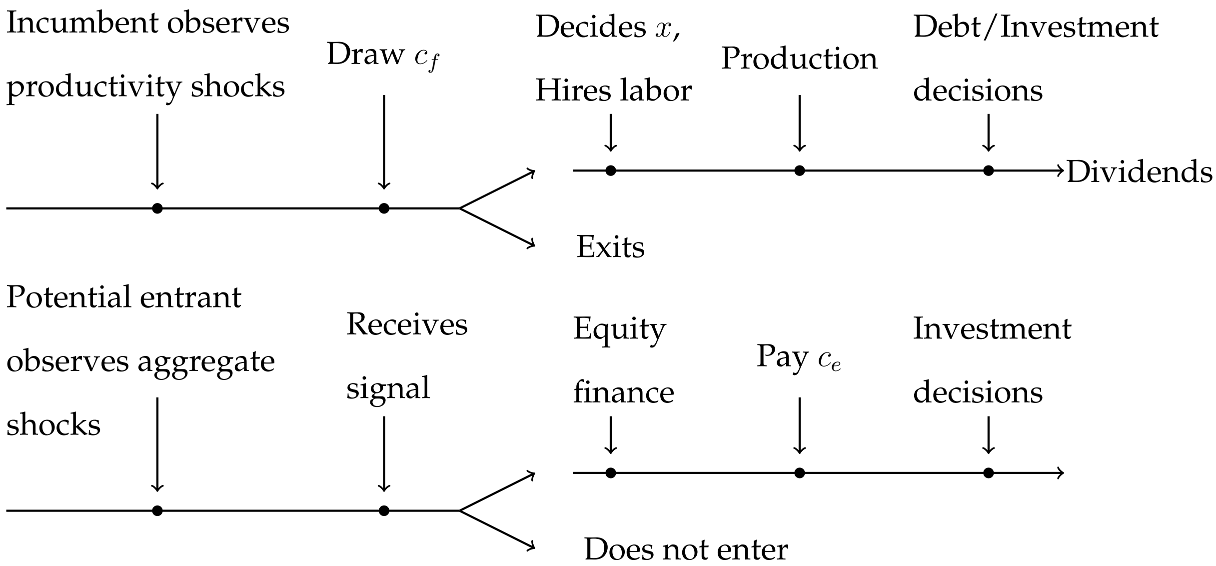
Intermediate Producers Intermediate firms incur a common and idiosyncratic productivity shock denoted by z_t and s_t, respectively, given by

$$\log z_{t+1} = \rho_z \log z_t + \sigma_z \varepsilon_{z,t+1},$$
$$\log s_{t+1} = \rho_s \log s_t + \sigma_s \varepsilon_{s,t+1},$$

where $\varepsilon_{z,t}, \varepsilon_{s,t} \sim N(0,1)$. Let us denote the conditional distribution of s_{t+1} as $\mathcal{F}(s_{t+1}|s_t)$. At the beginning of each period before production, incumbent firms realize z_t and s_t and an operating cost c_f distributed over a time-invariant common distribution G. If an incumbent decides to exit, they recoup the market price Q_t of their current capital stock k_t, less any debts due b_t adjusted for dividend frictions. Firms that decide to produce decide how many varieties produce x_t and allocate the current stock of physical and firm-specific intangible capital k_t, n_t, respectively, and hiring labor l_t at wage w_t. In the same period, firms that choose to remain in the market also decide on investments in physical and intangible capital, i_t^k, i_t^n, constrained by current debts due b_t less new loans received b_{t+1}/R_t where R_t denotes the gross interest rate. Households own all incumbent producers that aim to maximize dividend payouts and can access equity financing subject to an adjustment cost. Each firm faces an earnings-based or asset-based borrowing constraint, which remains fixed over the firm's life cycle.

In each period, there is a constant mass $M > 0$ of prospective entrants where each receives a signal j drawn from a common time-invariant distribution J and a realization with probability χ of facing an earnings-based borrowing constraint and probability $1-\chi$ of facing an asset-based borrowing constraint. Entrants that decide to enter pay an entry cost $c_e \geq 0$. When an entrant decides to enter the market, they rely only on equity financing for their initial investments and begin operations in the following period. Let $\Gamma_t(s, k, n, b)$ denote the distribution of operating intermediate producers over the modeled heterogeneity. However, Γ_t does not capture the conditional distribution by borrowing type. Let Γ_t^x, where $x = \text{eb}, \text{ab}$ denote the conditional distribution by earnings-based borrowers and asset-based borrows, respectively. Additionally, $\lambda_t \in \Lambda$ denotes the vector of aggregate state variables with transition operator $T(\lambda_{t+1}|\lambda_t)$. Figure 10 provides a diagram of the timing of events that abstractly holds for each time of borrowing-constraint firm.

Figure 10: Timing in Period t



Note: This figure provides a diagram of the timing of events that abstractly holds for each time of borrowing-constraint firm.

Variety Problem In the spirit of Crouzet et al. (2022b), the production of a variety $i \in A_t$ follows a three-input Cobb-Douglass function of the following:

$$y_{i,t} = z_t s_t n_t(i)^{\alpha} k_t(i)^{\beta} l_t(i)^{1-\alpha-\beta} \tag{6}$$

where $0 \leq \alpha, \beta$ and $\alpha + \beta < 1$ are the elasticities of production for intangible and physical capital, respectively. For a stock of varieties $[0, x_t] \subset A_t$, we can describe the total firms' revenue problem as allocating the appropriate factors of production for the production of each variety:

$$\Pi_t \equiv \max_{\{n_t(i),k_t(i),l_t(i)\}_{i\in[0,x_t]}} Y_t^{\frac{\eta-1}{\eta}} \int_0^{x_t} y_{i,t}^{1/\eta} di \tag{7}$$

subject to

$$\int_0^{x_t} k_t(i) di \leq k_t \tag{8}$$

$$\int_0^{x_t} l_t(i) di \leq l_t \tag{9}$$

$$\left(\int_0^{x_t} n_t(i)^{\frac{1}{1-\rho}} d_i\right)^{1-\rho} \leq n_t \tag{10}$$

The parameter $0 \leq \rho \leq 1$ captures the firm's non-rivalry aspect of intangible capital. The limits of ρ offers insight:

$$\begin{cases} \int_0^{x_t} n_t(i)di \leq n_t & \text{if } \rho = 0; \\ \max\limits_{i\in[0,x_t]} n_t(i) \leq n_t & \text{if } \rho = 1. \end{cases}$$

When $\rho = 1$, intangible capital can be used in the production stream of every variety exhibiting complete non-rivalry.

The firm demand for production factors satisfies

$$\begin{aligned} k_t(i) &= x_t^{-1} k_t, \\ l_t(i) &= x_t^{-1} l_t, \\ n_t(i) &= x_t^{-(1-\rho)} n_t, \qquad \forall\, i \in [0, x_t]. \end{aligned} \tag{11}$$

We can reduce equation (7) to express the firm's revenue as:

$$\pi_t^x = x_t^{\frac{\eta+\alpha\rho-1}{\eta}} \left[z_t s_t n_t^\alpha k_t^\beta l_t^{1-\alpha-\beta}\right]^{1/\eta} Y_t^{\frac{\eta-1}{\eta}}. \tag{12}$$

Incumbent's Optimization Problem Given the aggregate state λ_t, idiosyncratic productivity s_t, operating cost c_f, and stock of capital, k_t, n_t, incumbent firms choose the number of varieties and labor decisions as the following static problem:

$$\pi_t = \max_{x_t, l_t} \pi_t^x - w_t l_t.$$

Incumbents can access debt markets through one-period risk-free bonds, b_t. The gross interest rate incumbents face is R_t denoted in final consumption units.

We describe the law of motion of capital and the intertemporal budget constraint of incumbents necessary for the continuation value. The law of motion of capital is given by:

$$k_{t+1} = i_t^k + (1-\delta_k)k_t$$
$$n_{t+1} = i_t^n + (1-\delta_n)n_t,$$

where δ_k, i_t^k, δ_n, i_t^n denote depreciation rates and investment for physical and intangible capital, respectively. We arrive at the following intertemporal budget constraint

$$\Psi(d_t) = \pi_t - w_t l_t - c_f - Q_t i_t^k - i_t^n - b_t + \frac{b_{t+1}}{r_t} \tag{13}$$

where d_t denotes the firm's free cash flow in period t distributed as a dividend to investors subject to costly change in equity payout $\Psi(\cdot)$ modeled similarly as Jermann and Quadrini (2012):

$$\Psi(d_t) = d_t + \psi(d_t - \overline{d})^2 \tag{14}$$

where $\psi \geq 0$ and $\overline{d}$ are the long-run payout target.

Motivated by the limited commitment problem, incumbents face heterogeneous borrowing constraints as documented by Lian and Ma (2021). Similar to Drechsel (2023), we characterize these borrowing constraints as asset-based or earnings-based. We assume the same borrowing constraint type applies across the entire life-cycle of a firm determined by an exogenous probability at the entry phase. Incumbents with an earnings-based borrowing constraint are modeled as

$$\frac{b_{t+1}}{r_t} \leq \theta_e \pi_t.$$

The parameter $\theta_e > 1$ represents a multiple of current gross profits. Whereas incumbents with an asset-based borrowing constraint are modeled as

$$\frac{b_{t+1}}{r_t} \leq \theta_a E_t\{Q_{t+1}(1-\delta_k)k_t\}$$

where the parameter $0 < \theta_a < 1$ represents the fraction of physical capital that can be pledged as collateral. A negative value $b_t < 0$ means a firm is a saver. If the firm decides to exit, the exit occurs before production and the value is

$$V^{\text{ex}} = \Psi(Q_t k_t - b_t).$$

We arrive at two value functions dependent on the borrowing constraint a firm faces with time subscripts omitted of the following:

$$V^{\text{eb}}(s, k, n, b, c_f; \lambda) =$$
$$\max_{i^k, i^n, b'} \left\{ d + \tilde{\Lambda} \int_{\Lambda} \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \max \left\{ V^{\text{eb}}(s', k', n', b', c'_f; \lambda'), V^{\text{ex}}(s', k', n', b'; \lambda') \right\} dG(c'_f) d\mathcal{F}(s'|s) dT(\lambda'|\lambda) \right\},$$

subject to

$$\begin{aligned}
\Psi(d) &= \pi^*(x, l, s, k, n; \lambda) - c_f - Qi^k - i^n - b + \frac{b'}{r}, \\
\frac{b'}{R} &\leq \theta_e \pi^*(x, s, k, n; \lambda), \\
k' &= i^k + (1 - \delta_k)k, \\
n' &= i^n + (1 - \delta_n)n. \\
\pi^*(x, l, s, k, n; \lambda) &= \max_{x,l} \pi(x, l, s, k, n; \lambda) - wl,
\end{aligned}$$

and

$$V^{\text{ab}}(s, k, n, b, c_f; \lambda) =$$
$$\max_{i^k, i^n, b'} \left\{ d + \tilde{\Lambda} \int_{\Lambda} \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \max \left\{ V^{\text{ab}}(s', k', n', b', c'_f; \lambda'), V^{\text{ex}}(s', k', n', b'; \lambda') \right\} dG(c'_f) d\mathcal{F}(s'|s) dT(\lambda'|\lambda) \right\},$$

subject to

$$\Psi(d) = \pi^*(x, l, s, k, n; \lambda) - c_f - Qi^k - i^n - b + \frac{b'}{r},$$

$$\frac{b'}{r} \le E_t\{\theta_a Q'(1-\delta_k)k\},$$

$$k' = i^k + (1-\delta_k)k,$$

$$n' = i^n + (1-\delta_n)n,$$

$$\pi^*(x,l,s,k,n;\lambda) = \max_{x,l} \pi(x,l,s,k,n;\lambda) - wl.$$

Where V^{eb} is the cum-dividend value function of incumbents facing an earnings-based borrowing constraint, and similarly V^{ab} is for incumbents facing an asset-based borrowing constraint. $\tilde{\Lambda}$ denotes the stochastic discount factor with $\tilde{\Lambda} \equiv \beta\frac{\partial u(\cdot)/\partial c'}{\partial u(\cdot)/\partial c}$ based on the household instantaneous utility function $u(\cdot)$ evaluated at appropriate temporal consumption values and β being the subjective discount factor for households.

Entry For an aggregate state λ and signal $j \sim J(j)$ the value for a prospective entrant can take either form

$$V_e^{\text{eb}}(j;\lambda) = \max_{k',n'} \Psi(-Qk'-n') + \tilde{\Lambda}\int_{\Lambda}\int_{\mathbb{R}_+}\int_{\mathbb{R}_+} V^{eb}(s',k',n',b',c_f';\lambda')dG(c_f')d\mathcal{F}(s'|q)dT(\lambda'|\lambda),$$
$$V_e^{\text{ab}}(j;\lambda) = \max_{k',n'} \Psi(-Qk'-n') + \tilde{\Lambda}\int_{\Lambda}\int_{\mathbb{R}_+}\int_{\mathbb{R}_+} V^{ab}(s',k',n',b',c_f';\lambda')dG(c_f')d\mathcal{F}(s'|q)dT(\lambda'|\lambda),$$

depending on whether an earnings-based or asset-based borrowing constraint is anticipated. Entry will occur if and only if $V_e^{eb}(j;\lambda) \ge c_e$ or $V_e^{ab}(j;\lambda) \ge c_e$ for earnings-based and asset-based entrants, respectively.

Physical Capital Producers Physical capital is created by a continuum of producers sold to intermediate producers at a uniform price, Q_t. Production of physical capital, I_t^k, entails transforming the final good while incurring the standard flow adjustment cost $\Phi\left(\frac{I_t}{I_{t-1}}\right)$ which is increasing and concave, with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(1) > 0$. We arrive at

Tobin's Q:

$$Q_t = 1 + \Phi\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}\Phi'\left(\frac{I_t}{I_{t-1}}\right) - \tilde{\Lambda}_{t,t+1}\int_\Lambda \left\{\left(\frac{I_{t+1}}{I_t}\right)^2 \Phi'\left(\frac{I_{t+1}}{I_t}\right)\right\} dT(\lambda_{t+1}|\lambda_t)$$

with the law of motion of aggregate capital as

$$K_{t+1} = I_t^k + (1-\delta_k)K_t.$$

Households There is a unit continuum of representative households that provide labor and debt financing for intermediate producers and consume the final good. Households can transfer wealth intertemporally by lending to intermediate firms and trading equity shares of intermediate producers among themselves. During the beginning of the period, households pool their resources and form a financial intermediary that lends directly to intermediate firms. The financial intermediary's diversified asset portfolio guarantees households receive a risk-free bond, B_t, with future rate of return, r_t, (see Diamond, 1984). Additionally, at the beginning of the economy in time $t = 0$, all households pool their initial allocation of equity shares and form an aggregate index to which receive back equity shares on the index, ind_t, with index dividends, D_t, and price, $p_{\text{ind},t}$.

Household consumption and leisure preferences are assumed to be consistent with the following additively separable instantaneous utility function:

$$u(C_t, L_t) = \log(C_t) - \tilde{\psi}\frac{L_t^{1+\frac{1}{\rho_l}}}{1+\frac{1}{\rho_l}},$$

where L_t is labor supplied, ρ_l is the Frisch elasticity of labor. Lastly, let β denote the subjective household discount factor. The household intertemporal budget constraint can be expressed as

$$C_t + \frac{B_{t+1}}{1+r_t} + p_{\text{ind},t}\text{ind}_{t+1} = w_t L_t + B_t + \text{ind}_t(D_t + p_{\text{ind},t}).$$

The representative household value function can be expressed as

$$V^h(B, \text{ind}; \lambda) = \max_{C,L,B',\text{ind}'} u(C, L) + \beta \int_\Lambda V^h(B', \text{ind}'; \lambda') dT(\lambda'|\lambda)$$

subject to

$$C + \frac{B'}{1+r} + p_{\text{ind}}\text{ind}' = wL + B + \text{ind}(D + p_{\text{ind}}).$$

Recursive Competitive Equilibrium For an initial Γ_0, a recursive competitive equilibrium consists of (i) incumbent value functions $V^{\text{eb}}(s, k, n, b, c_f; \lambda)$ and $V^{\text{ab}}(s, k, n, b, c_f; \lambda)$; (ii) entrant value functions $V_e^{\text{eb}}(j; \lambda)$, $V_e^{\text{ab}}(j; \lambda)$; (iii) household value function $V^h(B, \text{ind}; \lambda)$; (iv) incumbent firm policy functions $n'_{\text{eb}}(s, k, n, b, c_f; \lambda)$, $n'_{\text{ab}}(s, k, n, b, c_f; \lambda)$, $k'_{\text{eb}}(s, k, n, b, c_f; \lambda)$, $k'_{\text{ab}}(s, k, n, b, c_f; \lambda)$, $b'_{\text{eb}}(s, k, n, b, c_f; \lambda)$, $b'_{\text{ab}}(s, k, n, b, c_f; \lambda)$, $l_{\text{eb}}(s, k, n; \lambda)$, $l_{\text{ab}}(s, k, n; \lambda)$, $x_{\text{eb}}(s, k, n; \lambda)$, $x_{\text{ab}}(s, k, n; \lambda)$; (v) entrant policy functions $k'_{\text{eb}}(j; \lambda)$, $k'_{\text{ab}}(j; \lambda)$, $n'_{\text{eb}}(j; \lambda)$, $n'_{\text{ab}}(j; \lambda)$; (vi) household policy functions $L(B, \text{ind}; \lambda)$, $B'(B, \text{ind}; \lambda)$, $\text{ind}'(B, \text{ind}; \lambda)$; $\{Q_t\}_{t=0}^\infty$ of prices from physical capital producers and demand schedule $\{p_{i,t}\}_{t=0}^\infty$ from final goods producers; (vii) bounded sequences of wages and interest rate $\{w_t\}_{t=0}^\infty$ and $\{r_t\}_{t=0}^\infty$; (viii) incumbents' measures $\{\Gamma_t\}_{t=0}^\infty$, $\{\Gamma_t^{\text{eb}}\}_{t=0}^\infty$ and $\{\Gamma_t^{\text{ab}}\}_{t=0}^\infty$; (viii) entrants' measures $\{\mathcal{E}_t\}_{t=0}^\infty$, $\{\mathcal{E}_t^{\text{eb}}\}_{t=0}^\infty$, and $\{\mathcal{E}_t^{\text{ab}}\}_{t=0}^\infty$ such that, for all $t \geq 0$:

(i) $V^{\text{eb}}(s, k, n, b, c_f; \lambda)$, $n'_{\text{eb}}(s, k, n, b, c_f; \lambda)$, $k'_{\text{eb}}(s, k, n, b, c_f; \lambda)$ and $b'_{\text{eb}}(s, k, n, b, c_f; \lambda)$ solve the incumbent's problem for earnings-based borrowing constraint;

   (a) $x_{\text{eb}}(s, k, n; \lambda)$, $l_{\text{eb}}(s, k, n; \lambda)$, $\{p_{it}\}_{t=0}^\infty$ solve the myopic incumbent's profit maximization for incumbents with earnings-based borrowing constraint;

(ii) $V^{\text{ab}}(s, k, n, b, c_f; \lambda)$, $n'_{\text{ab}}(s, k, n, b, c_f; \lambda)$, $k'_{\text{ab}}(s, k, n, b, c_f; \lambda)$ and $b'_{\text{ab}}(s, k, n, b, c_f; \lambda)$ solve the incumbent's profit maximization for incumbents with earnings-based borrowing constraint;

   (a) $x_{\text{ab}}(s, k, n; \lambda)$ and $l_{\text{ab}}(s, k, n; \lambda)$ solve the myopic profit maximization problem for incumbents with asset-based borrowing constraint;

(iii) $V_e^{\text{eb}}(j;\lambda)$, $k'_{\text{eb}}(j;\lambda)$ and $n'_{\text{eb}}(j;\lambda)$ solve the entrant's problem with anticipated earnings-based borrowing constraint;

(iv) $V_e^{\text{ab}}(j;\lambda)$, $k'_{\text{ab}}(j;\lambda)$ and $n'_{\text{ab}}(j;\lambda)$ solve the entrant's problem with anticipated asset-based borrowing constraint;

(v) $V^h(B,\text{ind};\lambda)$, $L(B,\text{ind};\lambda)$, $B'(B,\text{ind};\lambda)$ and $\text{ind}'(B,\text{ind};\lambda)$ solve the representative households problem;

(vi) the labor market clears: for all $t \geq 0$ and wage w_t we have

$$\int l_{\text{eb}}(s,k,n;\lambda)d\Gamma_t^{\text{eb}}(s,k,n,b) + \int l_{\text{ab}}(s,k,n;\lambda)d\Gamma_t^{\text{ab}}(s,k,n,b) = L_t;$$

(vii) the debt market clears: for all $t \geq 0$ and interest rate r_t we have

$$\int b'_{\text{eb}}(s,k,n,b,c_f;\lambda)d\Gamma_t^{\text{eb}}(s,k,n,b) + \int b'_{\text{ab}}(s,k,n,b,c_f;\lambda)d\Gamma_t^{\text{ab}}(s,k,n,b) = \frac{B_{t+1}}{1+r_t};$$

(viii) the physical capital market clears: for all $t \geq 0$ and price of physical capital Q_t we have

$$\begin{aligned}&\int k'_{eb}(s,k,n,b,c_f;\lambda)d\Gamma_t^{eb}(s,k,n,b) + \int k'_{ab}(s,k,n,b,c_f;\lambda)d\Gamma_t^{ab}(s,k,n,b)\\&+\theta M\int_{\mathcal{C}^{\text{eb}}} k'_{eb}(j;\lambda)dJ(j) + (1-\theta)M\int_{\mathcal{C}^{\text{ab}}} k'_{ab}(j;\lambda)dJ(j) = K_{t+1},\end{aligned}$$

where

$$\begin{aligned}\mathcal{C}^{eb} &= \{j \in \mathbb{R}_+ : V_e^{eb}(j;\lambda) \geq c_e\},\\ \mathcal{C}^{ab} &= \{j \in \mathbb{R}_+ : V_e^{ab}(j;\lambda) \geq c_e\};\end{aligned}$$

(ix) for all $t \geq 0$ and all Borel sets $\mathcal{S} \times \mathcal{P} \subset \mathbb{R}_+ \times \mathbb{R}_+^3$ we have

$$\mathcal{E}_{t+1}^{\text{eb}}(\mathcal{S} \times \mathcal{P}) = \theta M \int_{\mathcal{S}} \int_{\mathcal{B}_e^{eb}(\mathcal{P},\lambda)} dJ(j)\mathcal{F}(s'|j)$$
$$\mathcal{E}_{t+1}^{\text{ab}}(\mathcal{S} \times \mathcal{P}) = (1-\theta) M \int_{\mathcal{S}} \int_{\mathcal{B}_e^{ab}(\mathcal{P},\lambda)} dJ(j)d\mathcal{F}(s'|j)$$
$$\mathcal{E}_{t+1}(\mathcal{S} \times \mathcal{P}) = \mathcal{E}_{t+1}^{\text{eb}}(\mathcal{S} \times \mathcal{P}) + \mathcal{E}_{t+1}^{\text{ab}}(\mathcal{S} \times \mathcal{P})$$

where

$$\mathcal{B}_e^{eb}(\mathcal{P},\lambda) = \left\{ j \in \mathbb{R}_+ : V_e^{eb}(j;\lambda) \geq c_e \ \wedge (k'_{eb}(j;\lambda), n'_{eb}(j;\lambda), 0) \in \mathcal{P} \right\},$$
$$\mathcal{B}_e^{ab}(\mathcal{P},\lambda) = \left\{ j \in \mathbb{R}_+ : V_e^{ab}(j;\lambda) \geq c_e \ \wedge (k'_{ab}(j;\lambda), n'_{ab}(j;\lambda), 0) \in \mathcal{P} \right\};$$

(x) for all $t \geq 0$ and all Borel sets $\mathcal{S} \times \mathcal{P} \subset \mathbb{R}_+ \times \mathbb{R}_+^3$ we have

$$\Gamma_{t+1}^{\text{eb}}(\mathcal{S} \times \mathcal{P}) = \int_{\mathcal{S}} \int_{\mathcal{B}^{\text{eb}}(\mathcal{P},\lambda')} \int d\Gamma_t^{\text{eb}}(s,k,n,b)dG(c'_f)d\mathcal{F}(s'|s) + \mathcal{E}_{t+1}^{\text{eb}}(\mathcal{S} \times \mathcal{P}),$$
$$\Gamma_{t+1}^{\text{ab}}(\mathcal{S} \times \mathcal{P}) = \int_{\mathcal{S}} \int_{\mathcal{B}^{\text{ab}}(\mathcal{P},\lambda')} \int d\Gamma_t^{\text{ab}}(s,k,n,b)dG(c'_f)d\mathcal{F}(s'|s) + \mathcal{E}_{t+1}^{\text{ab}}(\mathcal{S} \times \mathcal{P}),$$
$$\Gamma_{t+1}(\mathcal{S} \times \mathcal{P}) = \Gamma_{t+1}^{\text{eb}}(\mathcal{S} \times \mathcal{P}) + \Gamma_{t+1}^{\text{ab}}(\mathcal{S} \times \mathcal{P}),$$

where

$$\begin{aligned}\mathcal{B}^{\text{eb}}(\mathcal{P},\lambda') = \big\{ c'_f : & V^{eb}(s',k',b',c'_f;\lambda') \geq V^{\text{ex}}(s',k',n';\lambda') \\ & \wedge\ (k'_{\text{eb}}(s,k,n,b,c_f;\lambda), n'_{\text{eb}}(s,k,n,b,c_f;\lambda), b'_{\text{eb}}(s,k,n,b,c_f;\lambda)) \in \mathcal{P} \big\},\\ \mathcal{B}^{\text{ab}}(\mathcal{P},\lambda') = \big\{ c'_f : & V^{ab}(s',k',b',c'_f;\lambda') \geq V^{\text{ex}}(s',k',n';\lambda') \wedge \\ & (k'_{\text{ab}}(s,k,n,b,c_f;\lambda), n'_{\text{ab}}(s,k,n,b,c_f;\lambda), b'_{\text{ab}}(s,k,n,b,c_f;\lambda)) \in \mathcal{P} \big\}.\end{aligned}$$

5 Conclusion

In this paper, we offer a novel exploration of the relationship between intangible capital, heterogeneous borrowing types, and firm-level productivity dynamics. By developing a firm-dynamics framework that accounts for the heterogeneous borrowing constraints faced by firms depending on their reliance on intangible versus tangible capital, we uncover important insights into the mechanisms driving key macroeconomic patterns, particularly the slowdown in aggregate productivity growth despite rising firm-level investment in intangibles.

Our empirical findings provide strong evidence that intangible capital is positively associated with labor productivity, sales growth, and firm scope. However, we also show that the benefits of intangible investment are significantly constrained by financial frictions, particularly when firms are unable to collateralize intangible assets. These constraints are especially pronounced when firms rely on earnings-based borrowing, which ties their ability to access financing directly to their current earnings rather than the collateral value of their assets. This reliance on earnings-based borrowing creates additional vulnerability, limiting the growth opportunities for firms that invest heavily in intangible capital. In the aftermath of the global financial crisis, the constraints on earnings-based borrowing became even more pronounced, exacerbating the broader productivity slowdown. Firms that were highly reliant on intangible investments faced substantial difficulties in accessing the necessary financing to expand, slowing their growth and, in turn, contributing to the deceleration in aggregate productivity growth.

Our theoretical framework rationalizes these empirical patterns through a general equilibrium model that highlights three key mechanisms. First, the non-rivalry property of intangible capital, combined with earnings-based borrowing constraints, creates an amplification mechanism that explains both the productivity-enhancing effects of intangibles during expansions and their role in exacerbating downturns. Second, heterogeneous borrowing constraints interact with productivity shocks to generate rich firm dynamics, where firms operating under earnings-based constraints exhibit greater volatility in investment and growth rates. Third, our model demonstrates how the endogenous entry condition, influenced by ex-ante probabilities of borrowing types, shapes the distribution of firms and their subsequent growth trajectories.

This paper also suggests avenues for future research, and we intend to expand our analysis in both empirical and theoretical directions. On the empirical side, our aim is to develop a framework to investigate the cyclical properties of non-rivalrous intangible capital, various types of borrowing constraints, and their relationship with productivity. In terms of the theoretical part, our future plans involve documenting additional evidence on the cyclical properties of firms and drawing inferences on the impact of short-run financial shocks within our general equilibrium framework. Furthermore, we intend to conduct local projections (as in Jordà (2005)) to emphasize the relevance of financial frictions.

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Appendix A Tables

Table A.1: Firm-Level Constructed Data Variables and Descriptions - Compustat

Variable	Description
Labor Productivity	$\frac{\text{Sales } (sale)}{\text{Employee } (emp)}$
Intangible Capital Ratio	$\frac{\text{Organizational Capital}}{\text{Organizational Capital + Net Property, Plant and Equipment } (ppent)}$
Tobin's Q	$\frac{\text{Debt in Curr. Liab.}(dlc) + \text{Long-term Debts } (dltt) + (\text{Common Shares } (csho) \text{ x Price Close } (prcc\_f)) - \text{Curr. Assets } (act)}{\text{Total Assets } (at)}$
Markup	$0.85 \frac{\text{Sales } (sale)}{\text{Cost of Goods Sold } (cogs)}$ (De Loecker et al. (2020))
Cash Ratio	$\frac{\text{Cash } (ch)}{\text{Total Assets } (at)}$
Total Debt	Debt in Curr. Liab. (*dlc*) + Long-term Debts (*dltt*)
Leverage Ratio	$\frac{\text{Debt in Curr. Liab.}(dlc) + \text{Long-term Debts } (dltt)}{\text{Total Assets } (at)}$
Secured Debt Ratio	$\frac{\text{Debt - Mortgages \& Other Secured } (dm)}{\text{Total Debt } (dlc + dltt)}$

Note: This table presents the firm-level constructed data variables and their descriptions in the Compustat sample.

Table A.2: Summary Statistiscs - Compustat Yearly Data

	Mean	SD	P50	Observation
Assets - Total (Real, million $)	2616.11	13138.85	240.36	96145
Sales (Real, million $)	2347.96	11783.55	232.85	96145
Employees	8.35	38.4	.97	149149
Property, Plant and Equipment - Total (Net) (Real, million $)	837.03	4994.77	40.83	96010
Research and Development Expense (Real, million $)	92.66	547.8	6.68	58766
Selling, General and Administrative Expense (Real, million $)	433.09	2036.95	56.51	87725
Tobin's Q	1.11	2.07	.67	87282

Note: This table presents the summary statistics for selected variables in the Compustat sample.

Table A.3: Summary Statistiscs - Number of Business Lines (Compustat Segment)

	Mean	SD	P50	Min	Max	Observation
Number of Business Lines	1.17	0.69	1	1	13	231851

Note: This table presents the summary statistics for the number of business lines in the Compustat Segment sample.

Table A.4: Summary Statistiscs - Intangible Capital

	Mean	SD	P50	Min	Max	Observation
Total Intangible Capital	788.3	5695.17	39.52	0	368905	210459
Knowledge Capital	156.83	1741.73	0.24	0	175725.2	210459
Organizational Capital	276.71	1811.11	20.85	0	166803.9	210459
External Intangible Capital	354.75	3529.27	0.45	0	310197	210459
Intangible Capital Ratio	0.48	0.29	0.5	0	1	209836

Note: This table presents the summary statistics for the components of intangible capital and the intangible capital ratio in the Compustat sample.

Table A.5: Summary Statistics - By Intangible Capital Ratio

	Total Assets	Sales	Number of Employees	Cash Ratio	Leverage Ratio	Log Labor Productivity	Log Labor Productivity Growth
Q1							
Mean	3148.94	2781.78	10.64	0.11	0.26	5.37	0.03
P50	352.86	314.06	1.34	0.06	0.21	5.40	0.02
SD	14877.28	12888.11	49.74	0.15	0.28	1.04	0.45
Q2							
Mean	1375.74	1378.42	4.71	0.14	0.24	5.47	0.04
P50	105.81	112.08	0.42	0.08	0.15	5.47	0.02
SD	8210.17	8970.20	20.18	0.17	0.37	1.02	0.48
All							
Mean	2367.98	2163.26	8.02	0.13	0.25	5.41	0.03
P50	212.58	203.45	0.84	0.07	0.19	5.43	0.02
SD	12422.31	11350.90	39.61	0.16	0.32	1.03	0.46

Note: This table presents the summary statistics for selected variables in the Compustat sample for firms with low (Q1) and high (Q2) intangible capital ratios using the Compustat sample. The division for high and low intangible ratios is determined by whether they are above or below the median of the intangible capital ratio within each industry and year.

Table A.6: Summary Statisticss - Compustat-DealScan Merge

	Mean	SD	P50	Min	Max	Observation
Number of Loan Deals	1.17	0.46	1	1	7	13638
Number of Loan Deals with No Covenants	0.49	0.62	0	0	7	13638
Number of Loan Deals with Covenants	0.68	0.57	1	0	6	13638
Loan Deal Amounts	4.77×10^8	1.04×10^9	1.67×10^8	302593.8	2.66×10^{10}	13638
Loan Deal Amounts - with No Covenants	2.18×10^8	7.88×10^8	0	0	2.66×10^{10}	13638
Loan Deal Amounts - with Covenants	2.59×10^8	6.78×10^8	3.28×10^7	0	1.55×10^{10}	13638

Note: This table presents the summary statistics for selected variables in the Compustat-DealScan merged sample.

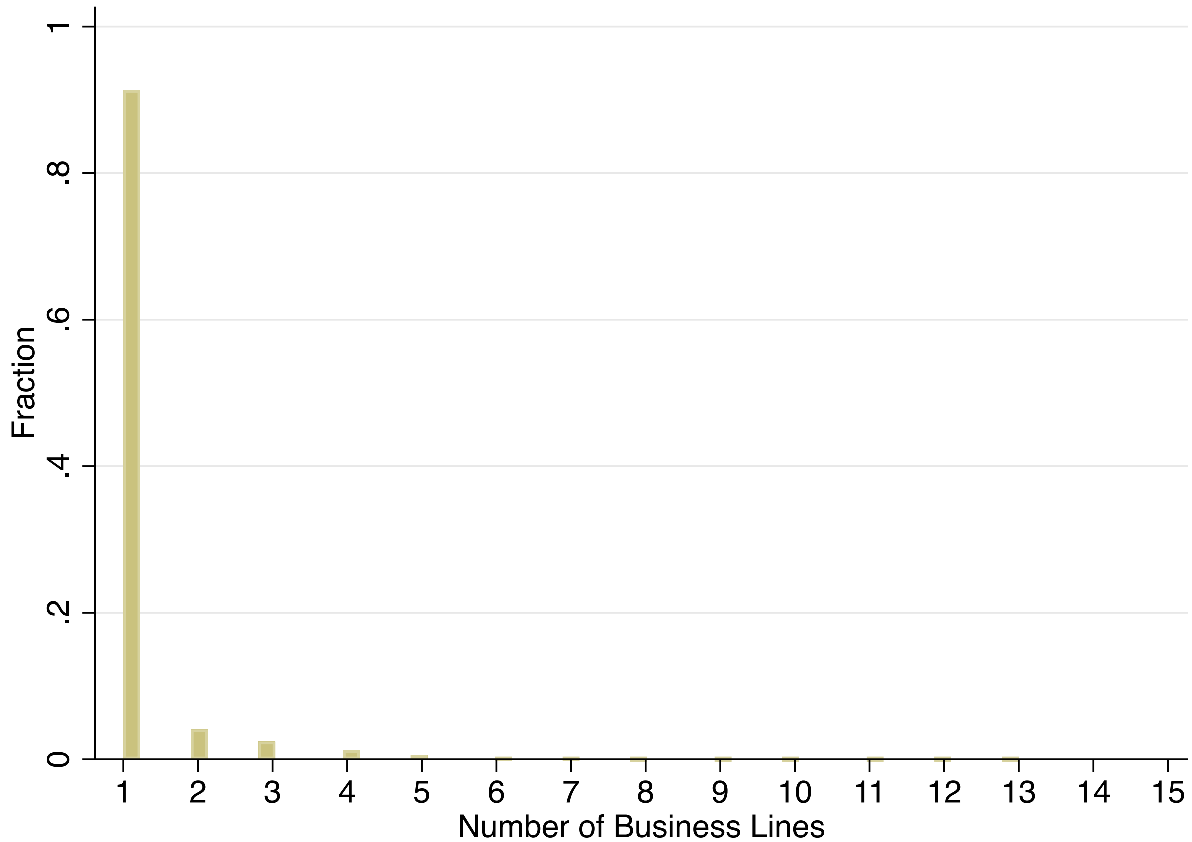
Table A.7: Pairwise Correlation Matrix

	Sales Growth	Total Intangible Growth	Knowledge Capital Growth	Organizational Capital Growth	External Intangible Growth
Sales Growth	1.00				
Total Intangible Growth	0.79*	1.00			
Knowledge Capital Growth	0.81*	0.83*	1.00		
Organizational Capital Growth	0.83*	0.85*	0.83*	1.00	
External Intangible Growth	0.33	0.61*	0.37	0.26	1.00

Note: This table documents pairwise correlation for sales growth and the growth of each component of intangible capital. $^*p < 0.01$

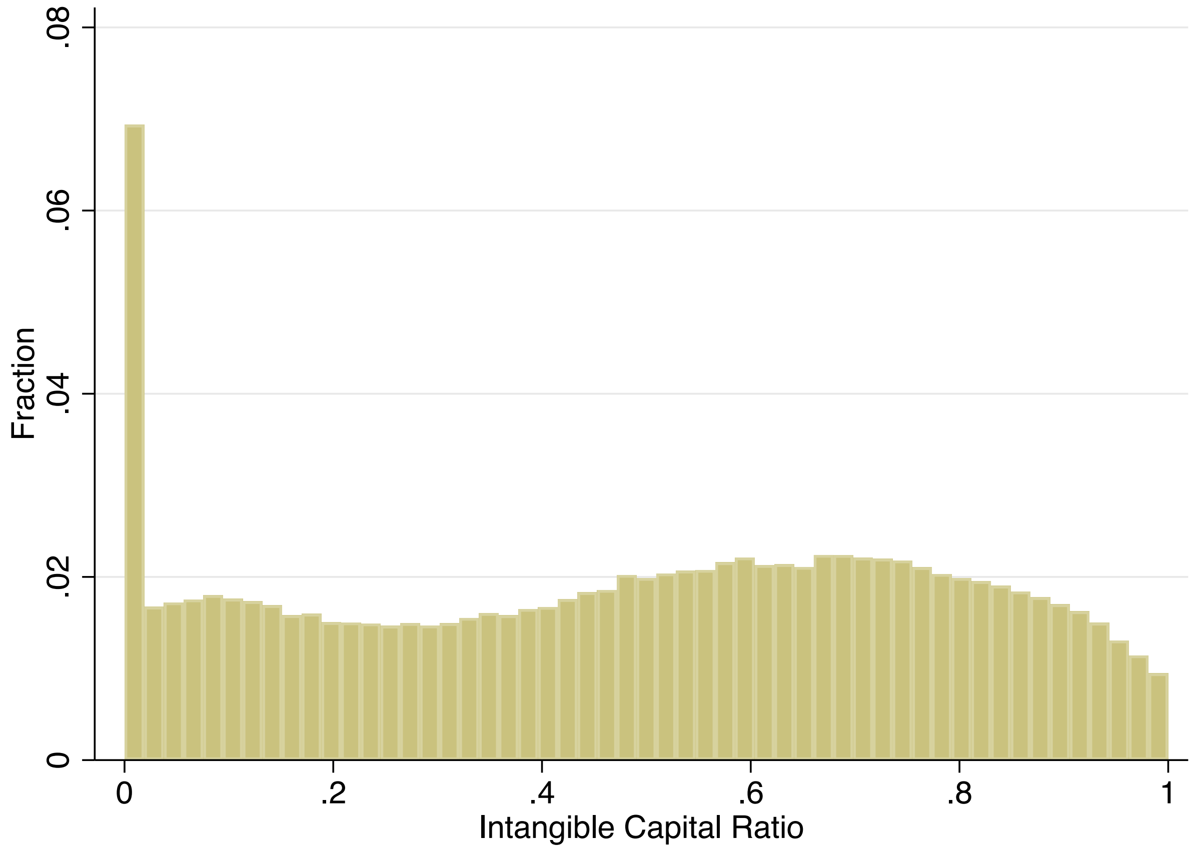
Appendix B Figures

Figure B.1: Histogram of Number of Business Lines



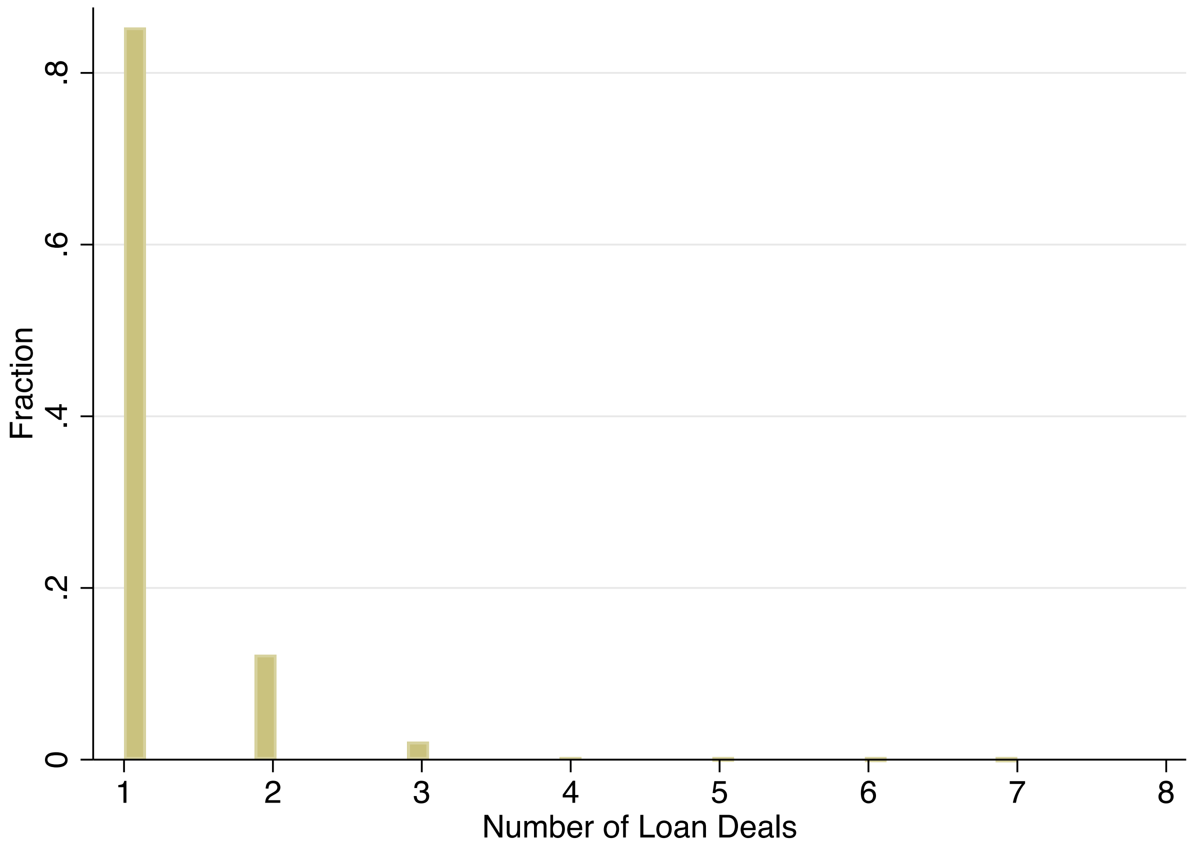
Note: This figure shows the histogram of number of business lines in the Compustat Segment sample.

Figure B.2: Histogram of Intangible Capital Ratio



Note: This figure shows the histogram of intangible capital ratio in the Compustat sample.

Figure B.3: Histogram of Number of Loan Deals



Note: This figure shows the histogram of number of loan deals in the Compustat-DealScan merged sample.