Financial Frictions, Intangible Capital and Productivity: A Model of Skill-Biased Stagnation

Alejandro Vicente[1]

[1]University of Alicante

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Abstract

I develop a dynamic general-equilibrium model of heterogeneous firms where intangible capital and skilled labor are complements in production, and intangibles are produced with skilled labor in R&D. Firms face collateal frictions, where tangible capital is more pledgeable than intangibles, creating a pecking-order distortion: constrained firms over-invest in tangibles. When the skilled-labor supply increases, unconstrained firms are able to exploit the complementarity investing in intangibles, while constrained firms do it suboptimally. The result is a *skill-biased stagnation* mechanism: higher human capital delivers muted TFP gains because of misallocation induced by financial frictions.

I. Environment

I.A. Time and Agents

Time is discrete. A continuum of firms $j \in [0,1]$ faces idiosyncratic productivity $z_{j,t}$. Each holds tangible capital $K_{j,t}$, intangible capital $S_{j,t}$, and debt $D_{j,t-1}$. Firms exit with probability $\zeta \in (0,1)$; entrants pay entry cost $c_e > 0$ (financed by the household via lump-sum transfers), draw $z_0 \sim F_z$ on $[\underline{z}, \overline{z}]$, and start with $K_0 = S_0 = D_{-1} = 0$.

I.B. Household

A representative household owns all firms, supplies skilled and unskilled labor inelastically, and has standard preferences over consumption:

$$\mathop{\mathbb{E}}_{0} \sum_{t=0}^{\infty} \beta^t u(C_t), \qquad 0 < \beta < 1.$$

The household supplies $\bar{L}_t$ units of unskilled labor and $\bar{H}_t$ units of skilled labor, where $\bar{L}_t + \bar{H}_t = 1$.

The household's budget constraint is:

$$C_t + D_t^H = w_L \bar{L}_t + w_H \bar{H}_t + RD_{t-1}^H + \Pi_t - \zeta c_e N_t, \tag{1}$$

where D_t^H denotes household deposits at banks, Π_t aggregate firm profits (dividends), and N_t the mass of active firms. The term $\zeta c_e N_t$ represents the consumption goods transferred to new entrants to cover entry costs, replacing exiting firms.

The household's Euler equation with linear utility pins down:

$$R = \frac{1}{\beta}. \tag{2}$$

I.C. Financial Intermediation

Competitive banks accept deposits from households at rate R and lend to firms. Banks perfectly enforce repayment up to collateral value and make zero profits. No default occurs in equilibrium—the collateral constraint binds ex ante, preventing default, following Kiyotaki and Moore (1997).

II. Technology

II.A. Idiosyncratic Productivity

$$\log z_{j,t+1} = \rho_z \log z_{j,t} + \sigma_z \varepsilon_{j,t+1}, \quad \varepsilon_{j,t+1} \sim N(0,1),$$

with $0 < \rho_z < 1$, $\sigma_z > 0$.

II.B. Production Technology

Production involves a nested CES structure. First, define the capital composite:

$$X_{j,t} = \left[\theta_K K_{j,t}^{\rho_K} + \theta_Q Q_{j,t}^{\rho_K}\right]^{1/\rho_K}, \tag{3}$$

where $Q_{j,t}$ is an intangible-skill bundle defined as:

$$Q_{j,t} = \left[\omega S_{j,t}^{\rho_Q} + (1-\omega)(H_{j,t}^P)^{\rho_Q}\right]^{1/\rho_Q}. \tag{4}$$

The final production function is:

$$Y_{j,t} = z_{j,t}\left[X_{j,t}^{\alpha} L_{j,t}^{\gamma}\right]^{\nu}, \tag{5}$$

where $\alpha, \gamma, \nu \in (0,1)$, $\theta_K, \theta_Q, \omega \in (0,1)$, and $\alpha + \gamma = 1$ (constant returns in the inner nest).

Assumption 1 (Complementarity Structure)**.** *Define elasticities $\sigma_K \equiv 1/(1-\rho_K)$ and $\sigma_Q \equiv 1/(1-\rho_Q)$. We assume that $\sigma_Q < \sigma_K$ (i.e., $\rho_Q < \rho_K < 0$), so that intangibles and skilled labor are stronger complements than tangible capital and the intangible-skill bundle.*

Remark. *Assumption 1 captures the idea that intangible capital and skilled labor work together in a more integrated way than tangible capital does with the intangible-skill composite Q. Empirical evidence from Gozen and Ozkara (2024) supports the existence of sinergies between intangibles and skilled labor.*

II.C. Capital Accumulation

Tangible capital stock evolves via standard time-to-build accumulation:

$$K_{j,t+1} = (1-\delta_K)K_{j,t} + I^K_{j,t}, \tag{6}$$

where $I^K_{j,t}$ is tangible investment and $0 < \delta_K < 1$.

Intangible capital is produced via R&D labor:

$$S_{j,t+1} = (1-\delta_S)S_{j,t} + \Gamma(H^R_{j,t})^\xi, \qquad 0 < \xi \leq 1, \tag{7}$$

where $H^R_{j,t}$ is skilled labor allocated to R&D, $\Gamma > 0$ and $\xi \leq 1$ captures (weakly) decreasing returns to R&D labor in knowledge creation.[1]

Total skilled labor satisfies:

$$H_{j,t} = H^P_{j,t} + H^R_{j,t}. \tag{8}$$

III. Financial Frictions

III.A. Collateral Constraint Microfoundation

Following the costly-state-verification literature (Townsend, 1979; Bernanke et al., 1999), we assume lenders can recover only a fraction of firm assets in case of default. Specifically, define recovery rates:

$$\alpha_K \in (0,1), \qquad \alpha_S \in [0, \alpha_K), \tag{9}$$

where α_K is the recovery rate on tangible capital and α_S is the recovery rate on intangible capital, with $\alpha_S < \alpha_K$ reflecting the lower pledgeability of intangibles (Holttinen et al., 2025).

To ensure no default in equilibrium, banks lend only up to the recoverable collateral value at the risk-free rate. Given the timing structure described below, the collateral constraint is:

$$D_{j,t} \leq \alpha_K K_{j,t} + \alpha_S S_{j,t}. \tag{10}$$

Assumption 2 (Low Intangible Pledgeability)**.** *Intangible assets are less pledgeable than tangible assets: $0 \leq \alpha_S < \alpha_K < 1$. The baseline case considers $\alpha_S = 0$ (intangibles fully non-pledgeable).*

III.B. Within-Period Timing

To clarify the sequence of decisions within each period t, I specify the following timing:

1. **Beginning of period:** Firm enters with state $(z_{t-1}, K_t, S_t, D_{t-1})$, where z_{t-1} denotes the lagged productivity realization and D_{t-1} is the debt obligation carried forward from the previous period.

2. **Productivity shock:** Firm draws current productivity z_t from the AR(1) process conditional on z_{t-1}.

1 The case $\xi < 1$ captures diminishing returns in R&D production. The limiting case $\xi = 1$ corresponds to linear intangible production, as in Atkeson and Burstein (2010) in the context of innovation. Extensions could incorporate knowledge spillovers by adding $S_{j,t}$ to the R&D production function, creating path dependence in innovation capabilities.

3. **Financial decisions:** Firm simultaneously repays old debt RD_{t-1} and obtains new debt D_t from banks, subject to the collateral constraint (10) based on current capital stocks (K_t, S_t). The net financial flow to the firm is $D_t - RD_{t-1}$.

4. **Static production decisions:** Firm chooses unskilled labor L and skilled production labor H^P to maximize current gross profits. These are static choices determined by first-order conditions (16), (17).

5. **Production:** Output Y is produced using current capital stocks (K_t, S_t) and chosen labor (L, H^P).

6. **Exit shock:** With probability ζ, firm exits the market. Exiting firms:

   - Collect production revenue net of wages: $Y - w_L L - w_H H^P$
   - Liquidate undepreciated capital: $(1-\delta_K)K_t + (1-\delta_S)S_t$
   - Repay current debt: D_t
   - Distribute residual to shareholders: $\text{Div}^{exit} = Y - w_L L - w_H H^P + (1-\delta_K)K_t + (1-\delta_S)S_t - D_t$

   With probability $(1-\zeta)$, firm continues to Stage 7.

7. **Investment decisions (for survivors only):** Surviving firms simultaneously choose:

   - Tangible investment $I^K_{j,t}$, determining $K_{j,t+1}$ via (6)
   - R&D labor $H^R_{j,t}$, determining $S_{j,t+1}$ via (7)

   These choices are subject to the budget constraint (11), which requires that investment expenditures not exceed available resources from gross profits and net borrowing.

8. **End of period:** Surviving firms pay dividends (if any) and enter next period with state $(z_t, K_{j,t+1}, S_{j,t+1}, D_{j,t})$.

III.C. Budget Constraint

Given the timing above, the firm's per-period budget constraint is:

$$I^K_{j,t} + w_L L_{j,t} + w_H H^P_{j,t} + w_H H^R_{j,t} + RD_{j,t-1} \leq Y_{j,t} + D_{j,t}. \tag{11}$$

We can rewrite this in terms of gross profits from production:

$$\Pi^{gross}_{j,t} := Y_{j,t} - w_L L_{j,t} - w_H H^P_{j,t},$$

so that:

$$I^K_{j,t} + w_H H^R_{j,t} + RD_{j,t-1} \leq \Pi^{gross}_{j,t} + D_{j,t}. \tag{12}$$

Dividends (profits) distributed to shareholders are:

$$\text{Div}_{j,t} = Y_{j,t} - I^K_{j,t} - w_L L_{j,t} - w_H(H^P_{j,t} + H^R_{j,t}) - RD_{j,t-1} + D_{j,t}. \tag{13}$$

IV. Firm Problem

IV.A. Recursive Formulation

The firm's state is (z, K, S, D_{-1}). Given the timing structure, the firm's problem can be written recursively. Let $\Pi^*(z, K, S)$ denote the maximized gross profits from production:

$$\Pi^*(z, K, S) = \max_{L, H^P} \Big\{ Y(z, K, S, L, H^P) - w_L L - w_H H^P \Big\}. \tag{14}$$

Then the value function satisfies:

$$\begin{aligned} V(z, K, S, D_{-1}) = \max_{D} \Bigg\{ & \Pi^*(z, K, S) - RD_{-1} + D \\ & + \max_{I^K, H^R} \Big\{ -I^K - w_H H^R + \beta(1-\zeta)\, \mathbb{E}[V(z', K', S', D) \mid z] \Big\} \Bigg\}, \end{aligned} \tag{15}$$

subject to:

- Capital accumulation: $K' = (1-\delta_K)K + I^K$, $S' = (1-\delta_S)S + \Gamma(H^R)^\xi$
- Collateral constraint: $D \leq \alpha_K K + \alpha_S S$
- Non-negative dividends: $\text{Div} = \Pi^* - I^K - w_H H^R - RD_{-1} + D \geq 0$

Interpretation: The firm first chooses debt D subject to the collateral constraint, then maximizes static production profits $\Pi^*(z, K, S)$, and finally chooses investment (I^K, H^R) subject to the non-negative dividend constraint to maximize continuation value.

IV.B. First-Order Conditions

Let $\mu_{j,t}$ denote the shadow value of internal funds.[2] Let $\lambda_{j,t}$ denote the multiplier on the collateral constraint (10).

Static Production FOCs. From (14):

$$\frac{\partial Y}{\partial L} = w_L, \tag{16}$$

$$\frac{\partial Y}{\partial H^P} = w_H. \tag{17}$$

These determine $L^*(z, K, S)$ and $H^{P*}(z, K, S)$ as functions of the state.

2 **Notational Convention:** Formally, $\mu_t \equiv 1 + \mu_t^{raw}$ where μ_t^{raw} is the Lagrange multiplier on the non-negative dividend constraint. This normalization ensures $\mu_t = 1$ when the constraint does not bind (dividends are positive), and $\mu_t > 1$ when internal funds are scarce (dividends are zero). This convention eliminates "1+" terms throughout the derivations.

Envelope Conditions. Taking derivatives of $V(z, K, S, D_{-1})$ with respect to state variables yields:

$$V_K(z, K, S, D_{-1}) = \mu \frac{\partial Y}{\partial K} + \lambda \alpha_K + (1 - \delta_K)\beta(1 - \zeta)\,\mathbb{E}[V_{K'}(z', K', S', D) \mid z], \tag{18}$$

$$V_S(z, K, S, D_{-1}) = \mu \frac{\partial Y}{\partial S} + \lambda \alpha_S + (1 - \delta_S)\beta(1 - \zeta)\,\mathbb{E}[V_{S'}(z', K', S', D) \mid z], \tag{19}$$

$$V_{D_{-1}}(z, K, S, D_{-1}) = -\mu R. \tag{20}$$

Derivation note: By the envelope theorem applied to (14), we have $\partial \Pi^*/\partial K = \partial Y/\partial K$ (the static optimization does not involve μ). In the dynamic value function (15), this cash flow is valued at the shadow price μ, yielding the first term in (18). The second term $\lambda \alpha_K$ reflects the value of additional collateral, and the third term is the continuation value of undepreciated capital. Note that $V_{D_{-1}}$ in equation (20) denotes the derivative with respect to the predetermined debt stock D_{-1}, as D_{-1} is a state variable while current debt D is a choice variable.

First-Order Condition for Tangible Investment. From $\partial \mathcal{L}/\partial I^K = 0$:

$$\mu_{j,t} = \beta(1 - \zeta)\,\mathbb{E}[V_{K'}(z_{j,t+1}, K_{j,t+1}, S_{j,t+1}, D_{j,t}) \mid z_{j,t}]. \tag{21}$$

Combining (21) with the envelope condition (18):

$$\mu_{j,t} = \beta(1 - \zeta) \underset{j,t}{\mathbb{E}} \left[\mu_{j,t+1} \frac{\partial Y_{j,t+1}}{\partial K_{j,t+1}} + \lambda_{j,t+1} \alpha_K + (1 - \delta_K)\mu_{j,t+1} \right], \tag{22}$$

where $\mathbb{E}_{j,t}[\cdot] \equiv \mathbb{E}[\cdot \mid z_{j,t}]$ denotes the conditional expectation given the firm's current productivity.

First-Order Condition for Intangible Investment (via R&D). The firm chooses H^R to balance the wage cost against the marginal value of additional intangible capital. Since $S' = (1 - \delta_S)S + \Gamma(H^R)^\xi$, we have $\partial S'/\partial H^R = \Gamma \xi (H^R)^{\xi - 1}$.

The FOC with respect to H^R is:

$$\mu_{j,t} w_H = \beta(1 - \zeta)\,\mathbb{E}[V_{S'}(z_{j,t+1}, K_{j,t+1}, S_{j,t+1}, D_{j,t}) \mid z_{j,t}] \cdot \Gamma \xi (H^R_{j,t})^{\xi - 1}. \tag{23}$$

Using the envelope condition (19) and substituting recursively, we obtain:[3]

$$\mu_{j,t} w_H = \beta(1 - \zeta)\Gamma\xi(H^R_{j,t})^{\xi - 1} \underset{j,t}{\mathbb{E}} \left[\mu_{j,t+1} \frac{\partial Y_{j,t+1}}{\partial S_{j,t+1}} + \lambda_{j,t+1}\alpha_S + (1 - \delta_S)\mu_{j,t+1} \frac{w_{H,t+1}}{\Gamma\xi(H^R_{j,t+1})^{\xi - 1}} \right]. \tag{24}$$

First-Order Condition for Debt. The firm balances the benefit of additional borrowing today against the cost of repayment tomorrow. Given that the firm borrows D_t today and repays $RD_t = D_t/\beta$ tomorrow (since $R = 1/\beta$), and accounting for the survival probability $(1 - \zeta)$, the FOC is:

$$\mu_{j,t} = (1 - \zeta) \underset{j,t}{\mathbb{E}}[\mu_{j,t+1}] + \lambda_{j,t}. \tag{25}$$

3 Undepreciated intangible capital must be valued at its marginal replacement cost $q_{S,t+1} \equiv w_{H,t+1}/[\Gamma\xi(H^R_{j,t+1})^{\xi-1}]$, not at unity. From the period $t+1$ FOC for H^R_{t+1}, we have $\beta(1 - \zeta)\mathbb{E}_{t+1}[V_{S,t+2}] = \mu_{t+1} w_{H,t+1}/[\Gamma\xi(H^R_{j,t+1})^{\xi-1}]$. Substituting this expression into the envelope condition yields the correct continuation value term.

IV.C. Economic Interpretation

The first-order conditions (22), (24) reveal the core distortion created by financial frictions. Consider two firms with identical productivity z but different financial positions.

An unconstrained firm has $\lambda = 0$, so its Euler equations reduce to standard intertemporal optimality conditions equating the marginal cost of investment today to the expected discounted marginal benefit tomorrow. The firm's investment decisions depend only on fundamentals: productivity, depreciation rates, and the interest rate.

A constrained firm has $\lambda > 0$, introducing wedges into both Euler equations. From (22), the presence of $\lambda_{j,t+1}\alpha_K$ in the expectation makes tangible capital investment more attractive because tangible assets relax future borrowing constraints. Similarly, from (24), the term $\lambda_{j,t+1}\alpha_S$ provides an analogous benefit for intangible capital.

The crucial asymmetry arises from Assumption 2: $\alpha_K > \alpha_S$. When the collateral constraint binds, tangible capital provides greater future collateral value than intangible capital. This creates a pecking-order distortion where constrained firms tilt their investment composition toward tangibles, even when the fundamental marginal products would justify more intangible investment.

V. Equilibrium

V.A. Definition

Definition 1 (Stationary Recursive Equilibrium). *A stationary recursive equilibrium consists of:*

(i) Firm value functions $V(z, K, S, D_{-1})$ *and policy functions* $\{K'(z, K, S, D_{-1}), S'(\cdot), D(\cdot), L(\cdot), H^P(\cdot), H^R(\cdot), I^K(\cdot)\}$,

(ii) Wages (w_L, w_H) *and interest rate* R,

(iii) A stationary distribution of firms $\Psi^*(z, K, S, D_{-1})$,

(iv) Aggregate quantities $\{C, K_{agg}, S_{agg}, L_{agg}, H_{agg}, D_{agg}, Y_{agg}\}$,

such that:

(a) Firms optimize: $V(z, K, S, D_{-1})$ *solves* (15) *and policies satisfy FOCs.*

(b) Household optimizes: $R = 1/\beta$ *from Euler equation* (2).

(c) Labor markets clear:

$$\int L_j d\Psi^* = \bar{L}, \qquad \int (H_j^P + H_j^R) d\Psi^* = \bar{H}.$$

(d) Credit market clears:

$$D_{agg} := \int D_j d\Psi^* = D^H.$$

(e) Goods market clears:

$$C + \int I_j^K d\Psi^* + \zeta c_e N = Y_{agg} := \int Y_j d\Psi^*,$$

where N *is the measure of firms* ($N = \int d\Psi^* = 1$ *by normalization).*

(f) *Free entry: Expected value of entrants equals entry cost:*

$$\int V(z,0,0,0)dF_z(z) = c_e.$$

(g) *Stationarity:* Ψ^* *is invariant under the transition implied by policy functions, exit, and entry.*

VI. Plan for quantitative experiments

Compute different steady-state economies varying $\frac{\bar{H}_t}{\bar{L}_t}$ to see how the changes in aggregates depend on the presence of financial frictions (computing parallel counterfactuals without frictions imposing $\lambda = 0$ for every firm).
Alternative 1. Compute transition dynamics along changes in $\frac{\bar{H}_t}{\bar{L}_t}$, comparing both the case with binding frictions with the counterfactual.
Alternative 2. Play with different pledgeability of intangibles. Holttinen (2025) show that $\alpha_S = 0.134, \alpha_K = 0.381$.

VII. A tractable toy model

To build intuition for the model's key mechanisms, I present a two-period framework that admits analytical characterization. This simplified setting isolates the pecking-order distortion and skill-biased stagnation channels while abstracting from dynamic complexities.

VII.A. Environment

Consider a two-period economy with a representative firm and no aggregate uncertainty. The firm is endowed with initial tangible capital $K_0 > 0$, initial wealth $W_0 \geq 0$, and a lifetime allocation of skilled labor $\bar{H} > 0$. Productivity $z > 0$ is constant and known.

Period 1 (Investment Phase): The firm chooses tangible investment $I^K \geq 0$ and allocates skilled labor between R&D ($H^R \geq 0$) and production for period 2 ($H^P = \bar{H} - H^R$). Intangible capital is produced linearly:

$$S = \Gamma H^R, \quad \Gamma > 0. \tag{26}$$

Period 2 (Production Phase): The firm produces output using accumulated capital and the skilled labor allocated in period 1. No further investment decisions are made.

VII.B. Technology

Capital Stocks. Period 2 tangible capital is:

$$K = K_0 + I^K. \tag{27}$$

Intangible-Skill Composite. The intangible capital S from period 1 combines with the allocated production labor $H^P = \bar{H} - H^R$ to form:

$$Q(S, H^P) = \left[\omega S^\rho + (1-\omega)(H^P)^\rho\right]^{1/\rho}, \quad \omega \in (0,1),\ \rho < 0. \tag{28}$$

The elasticity of substitution $\sigma_Q = 1/(1-\rho) < 1$ captures complementarity.

Production. Period 2 output is:

$$Y = zK^\alpha Q(S, H^P)^{1-\alpha}, \quad \alpha \in (0,1). \tag{29}$$

For tractability, I abstract from unskilled labor and normalize away depreciation.

VII.C. Financial Constraint

The firm may borrow only against tangible assets. At the end of period 1, the pledgeable capital stock equals $K_0 + I^K$. Thus:

$$D \leq \alpha_K(K_0 + I^K), \quad \alpha_K \in (0,1), \tag{30}$$

where D denotes borrowing in period 1. Intangible capital is non-pledgeable: $\alpha_S = 0$.

Period 1 Budget Constraint:

$$I^K + w_H H^R \leq W_0 + D. \tag{31}$$

If the constraint binds, $D = \alpha_K(K_0 + I^K)$, so:

$$I^K + w_H H^R \leq W_0 + \alpha_K(K_0 + I^K), \tag{32}$$

$$I^K(1-\alpha_K) + w_H H^R \leq W_0 + \alpha_K K_0. \tag{33}$$

Define *effective wealth*:

$$W_0' \equiv W_0 + \alpha_K K_0,$$

so the constraint becomes:

$$I^K(1-\alpha_K) + w_H H^R \leq W_0'. \tag{34}$$

Interpretation. Each additional unit of tangible investment increases borrowing capacity by α_K, effectively reducing its cost to $(1-\alpha_K)$. R&D yields no such benefit. The firm's effective resources include its financial wealth W_0 plus the borrowing capacity from its initial capital stock ($\alpha_K K_0$).

VII.D. Firm Problem

The firm chooses (I^K, H^R) to maximize net output:

$$\max_{I^K \geq 0,\, H^R \in [0,\bar{H}]} \quad Y - I^K - w_H \bar{H} \tag{35}$$

subject to the budget constraint (33), where:

$$Y = z(K_0 + I^K)^\alpha Q(\Gamma H^R, \bar{H} - H^R)^{1-\alpha}, \tag{36}$$

$$Q(\Gamma H^R, \bar{H} - H^R) = \left[\omega(\Gamma H^R)^\rho + (1-\omega)(\bar{H} - H^R)^\rho\right]^{1/\rho}. \tag{37}$$

Note that total skilled labor cost $w_H \bar{H}$ is fixed, so the firm's choice concerns only the allocation of $\bar{H}$ between R&D and production.

1. Unconstrained Solution

When W_0' is sufficiently large that constraint (33) does not bind, the first-order conditions are:

$$\frac{\partial Y}{\partial I^K} = \alpha z K^{\alpha-1} Q^{1-\alpha} = 1, \tag{38}$$

$$\frac{\partial Y}{\partial H^R} = (1-\alpha) z K^{\alpha} Q^{-\alpha} \left[\frac{\partial Q}{\partial S}\Gamma - \frac{\partial Q}{\partial H^P} \right] = 0. \tag{39}$$

From (39), the optimal allocation balances the marginal product of intangibles (via R&D) against the marginal product of direct production labor:

$$\frac{\partial Q}{\partial S}\Gamma = \frac{\partial Q}{\partial H^P}. \tag{40}$$

This is an interior solution exploiting the complementarity structure (28).

2. Constrained Solution

When constraint (33) binds, form the Lagrangian:

$$\mathcal{L} = Y - I^K - w_H \bar{H} - \mu[I^K(1-\alpha_K) + w_H H^R - W_0'], \tag{41}$$

where $\mu > 0$ is the shadow value of resources.

The first-order conditions become:

$$\frac{\partial Y}{\partial I^K} = 1 + \mu(1-\alpha_K), \tag{42}$$

$$\frac{\partial Y}{\partial H^R} = \mu w_H. \tag{43}$$

Key Observation. Comparing (38) with (42): the constrained firm requires a higher marginal product of capital, implying lower I^K.

Comparing (39) with (43): the constrained firm requires a positive marginal product of R&D labor (instead of zero), implying lower H^R (and correspondingly higher H^P) than the unconstrained firm.

VII.E. Pecking-Order Distortion

Before stating the main proposition, I establish that the constrained firm produces a lower intangible-skill composite.

Lemma 1 (Suboptimal Allocation)**.** *The constrained firm produces a lower intangible-skill composite than the unconstrained firm: $Q_c < Q_u$.*

Proof. Define $\tilde{Q}(H^R; \bar{H}) \equiv Q(\Gamma H^R, \bar{H} - H^R)$ as the intangible-skill composite viewed as a function of the R&D allocation for given $\bar{H}$. The unconstrained firm chooses H_u^R to satisfy (40), which is equivalent to maximizing $\tilde{Q}$ over $H^R \in [0, \bar{H}]$:

$$H_u^R = \arg\max_{H^R \in [0,\bar{H}]} \tilde{Q}(H^R; \bar{H}).$$

The constrained firm chooses $H_c^R < H_u^R$ (as established below in Proposition 1). Since $\tilde{Q}$ is strictly concave in H^R and $H_c^R \neq H_u^R$, we have $\tilde{Q}(H_c^R; \bar{H}) < \tilde{Q}(H_u^R; \bar{H})$, i.e., $Q_c < Q_u$. $\square$

Proposition 1 (Pecking Order)**.** *Under the collateral constraint* (30) *with* $\alpha_K \in (0,1)$ *and* $\alpha_S = 0$*, constrained firms* ($\mu > 0$) *exhibit:*

(i) Lower R&D investment: $H_c^R < H_u^R$

(ii) Lower intangible capital: $S_c < S_u$

(iii) Higher production labor: $H_c^P > H_u^P$

(iv) Lower tangible investment: $I_c^K < I_u^K$

where subscripts c and u denote constrained and unconstrained firms respectively.

Proof. **Parts (i)–(iii):** From (43), the constrained firm requires $\partial Y/\partial H^R = \mu w_H > 0$, while the unconstrained firm has $\partial Y/\partial H^R = 0$ at optimum. Since

$$\frac{\partial Y}{\partial H^R} = (1-\alpha)zK^\alpha Q^{-\alpha}\left[\frac{\partial Q}{\partial S}\Gamma - \frac{\partial Q}{\partial H^P}\right],$$

and the term $(1-\alpha)zK^\alpha Q^{-\alpha} > 0$, the constrained firm has

$$\frac{\partial Q}{\partial S}\Gamma > \frac{\partial Q}{\partial H^P}$$

at its optimum, whereas the unconstrained firm has equality. Computing the partial derivatives of (28):

$$\frac{\partial Q}{\partial S} = \omega S^{\rho-1}Q^{1-\rho}, \qquad \frac{\partial Q}{\partial H^P} = (1-\omega)(H^P)^{\rho-1}Q^{1-\rho}.$$

The condition $\frac{\partial Q}{\partial S}\Gamma > \frac{\partial Q}{\partial H^P}$ implies a lower S/H^P ratio than the unconstrained optimum. Given the constraint $H^P = \bar{H} - H^R$ and $S = \Gamma H^R$, this requires $H_c^R < H_u^R$. Parts (ii) and (iii) follow immediately.

Part (iv): Define $f(I^K; Q) \equiv \alpha z(K_0 + I^K)^{\alpha-1}Q^{1-\alpha}$, which is decreasing in I^K (since $\alpha - 1 < 0$) and increasing in Q.

The unconstrained firm satisfies $f(I_u^K; Q_u) = 1$.

For the constrained firm, evaluate f at I_u^K with Q_c:

$$f(I_u^K; Q_c) = \alpha z(K_0 + I_u^K)^{\alpha-1}Q_c^{1-\alpha} < \alpha z(K_0 + I_u^K)^{\alpha-1}Q_u^{1-\alpha} = 1,$$

where the inequality uses $Q_c < Q_u$ (Lemma 1) and $(1-\alpha) > 0$.

The constrained firm's FOC (42) requires $f(I_c^K; Q_c) = 1+\mu(1-\alpha_K) > 1$. Since $f(\cdot; Q_c)$ is strictly decreasing in I^K and $f(I_u^K; Q_c) < 1 < f(I_c^K; Q_c)$, we must have $I_c^K < I_u^K$. □

Remark. *The asymmetry arises from differential collateral value. Each unit of* I^K *reduces the effective resource constraint by* $(1-\alpha_K)$ *because it generates* α_K *in borrowing capacity. In contrast,* H^R *provides no collateral benefit since* $\alpha_S = 0$*. This creates an implicit subsidy for tangible investment relative to R&D.*

VII.F. Underexploitation of Complementarity

Lemma 2 (Complementarity). *The intangible-skill composite (28) with $\rho < 0$ satisfies:*

$$\frac{\partial^2 Q}{\partial S\,\partial H^P} = \omega(1-\omega)(1-\rho)S^{\rho-1}(H^P)^{\rho-1}Q^{1-2\rho} > 0. \tag{44}$$

Proof. Direct differentiation of (28). All terms are positive when $\rho < 0$ since $(1-\rho) > 0$ and $(1-2\rho) > 0$. □

Corollary 1 (Underexploitation of Skilled Labor). *Despite having higher production labor ($H_c^P > H_u^P$), the constrained firm has lower marginal product of that labor:*

$$\frac{\partial Q}{\partial H^P}(S_c, H_c^P) < \frac{\partial Q}{\partial H^P}(S_u, H_u^P). \tag{45}$$

Proof. The marginal product of production labor can be rewritten as:

$$\frac{\partial Q}{\partial H^P} = (1-\omega)(H^P)^{\rho-1}Q^{1-\rho} = (1-\omega)\left(\frac{Q}{H^P}\right)^{1-\rho}. \tag{46}$$

Since $\rho < 0$, we have $1-\rho > 1$, so (46) is strictly increasing in the ratio Q/H^P.

From Lemma 1, $Q_c < Q_u$. From Proposition 1(iii), $H_c^P > H_u^P$. Therefore:

$$\frac{Q_c}{H_c^P} < \frac{Q_u}{H_u^P},$$

which implies $\frac{\partial Q}{\partial H^P}(S_c, H_c^P) < \frac{\partial Q}{\partial H^P}(S_u, H_u^P)$. □

Economic Interpretation. The constrained firm overallocates skilled labor to production (H^P) relative to intangibles (S) because of the financial distortion. However, due to complementarity, the productivity of that labor is limited by the low stock of intangibles. This creates a *double inefficiency*: wrong allocation *and* underexploitation of the resources allocated to production.

VII.G. Skill-Biased Stagnation

Consider a comparative static exercise: an increase in the skilled labor endowment $\bar{H}$.

1. Unconstrained Firm Response

For the unconstrained firm, the optimality condition (40) determines the allocation of $\bar{H}$ between H^R and H^P. Using the expressions for marginal products:

$$\omega\Gamma(\Gamma H^R)^{\rho-1} = (1-\omega)(\bar{H} - H^R)^{\rho-1}.$$

This condition pins down the *ratio* $H^R/(\bar{H} - H^R)$ independently of $\bar{H}$. Therefore, the R&D share $\phi^R \equiv H^R/\bar{H}$ is constant:

$$\frac{dH_u^R}{d\bar{H}} = \phi_u^R, \qquad \frac{dH_u^P}{d\bar{H}} = 1 - \phi_u^R. \tag{47}$$

Both margins expand proportionally, fully exploiting the complementarity.

2. Constrained Firm Response

The binding constraint implies:

$$I_c^K(1-\alpha_K) + w_H H_c^R = W_0'.$$

Differentiating:

$$(1-\alpha_K)\,dI_c^K + w_H\,dH_c^R = 0.$$

Because both constrained FOCs depend on

$$Q_c = Q(\Gamma H_c^R, \bar{H} - H_c^R),$$

which varies with $\bar{H}$, the pair (I_c^K, H_c^R) generally adjusts with $\bar{H}$.

However, the constraint imposes a strong wedge: additional skilled labor primarily enters production. Formally, the constrained R&D response satisfies:

$$0 \le \frac{dH_c^R}{d\bar{H}} < \phi_u^R, \qquad \frac{dH_c^P}{d\bar{H}} = 1 - \frac{dH_c^R}{d\bar{H}}. \tag{48}$$

Proposition 2 (Skill-Biased Stagnation)**.** *When the skilled labor endowment increases ($\bar{H}\uparrow$):*

(i) The unconstrained firm increases both H^R and H^P proportionally, building higher S and exploiting complementarity.

(ii) The constrained firm expands H^P strictly more than H^R:

$$0 \le \frac{dH_c^R}{d\bar{H}} < \phi_u^R.$$

(iii) The intangible-skill composite grows faster for the unconstrained firm:

$$\frac{dQ_u}{d\bar{H}} > \frac{dQ_c}{d\bar{H}} > 0. \tag{49}$$

(iv) The output gap between firms widens:

$$\frac{d(Y_u - Y_c)}{d\bar{H}} > 0. \tag{50}$$

Proof. Parts (i)–(ii) follow from the derived responses.

Part (iii): By the envelope theorem, since the unconstrained firm maximizes Q over the allocation of $\bar{H}$:

$$\frac{dQ_u}{d\bar{H}} = \left.\frac{\partial Q}{\partial H^P}\right|_{(S_u, H_u^P)}.$$

For the constrained firm:

$$\frac{dQ_c}{d\bar{H}} = \frac{\partial Q}{\partial S}\Gamma\frac{dH_c^R}{d\bar{H}} + \frac{\partial Q}{\partial H^P}\left(1 - \frac{dH_c^R}{d\bar{H}}\right).$$

Since the unconstrained firm satisfies $\frac{\partial Q}{\partial S}\Gamma = \frac{\partial Q}{\partial H^P}$, we can write:

$$\frac{dQ_u}{d\bar{H}} = \left.\frac{\partial Q}{\partial H^P}\right|_{(S_u, H_u^P)} = \left.\frac{\partial Q}{\partial S}\right|_{(S_u, H_u^P)}\Gamma = \left[\frac{\partial Q}{\partial S}\Gamma\phi_u^R + \frac{\partial Q}{\partial H^P}(1-\phi_u^R)\right]_{(S_u, H_u^P)}.$$

From Corollary 1, $\frac{\partial Q}{\partial H^P}(S_u, H_u^P) > \frac{\partial Q}{\partial H^P}(S_c, H_c^P)$. Moreover, from the constrained firm's FOC, $\frac{\partial Q}{\partial S}(S_c, H_c^P)\Gamma > \frac{\partial Q}{\partial H^P}(S_c, H_c^P)$. Since $\frac{dH_c^R}{d\bar{H}} < \phi_u^R$ (part ii), the constrained firm places less weight on the higher-valued margin (R&D) and more weight on the lower-valued margin (production), establishing (49).

Part (iv): From $Y = zK^\alpha Q^{1-\alpha}$:

$$\frac{dY}{d\bar{H}} = (1-\alpha)zK^\alpha Q^{-\alpha}\frac{dQ}{d\bar{H}} = (1-\alpha)\frac{Y}{Q}\frac{dQ}{d\bar{H}}.$$

Define the "output-to-composite" ratio $\psi \equiv Y/Q = zK^\alpha Q^{-\alpha}$. Then:

$$\frac{d(Y_u - Y_c)}{d\bar{H}} = (1-\alpha)\left[\psi_u \frac{dQ_u}{d\bar{H}} - \psi_c \frac{dQ_c}{d\bar{H}}\right].$$

From part (iii), $\frac{dQ_u}{d\bar{H}} > \frac{dQ_c}{d\bar{H}}$. Using (46):

$$\frac{dQ}{d\bar{H}} \geq \frac{\partial Q}{\partial H^P} = (1-\omega)\left(\frac{Q}{H^P}\right)^{1-\rho}.$$

Therefore:

$$\begin{aligned}\frac{dY}{d\bar{H}} &\geq (1-\alpha)zK^\alpha Q^{-\alpha}(1-\omega)\left(\frac{Q}{H^P}\right)^{1-\rho} \\ &= (1-\alpha)(1-\omega)zK^\alpha Q^{1-\alpha-\rho}(H^P)^{-(1-\rho)}. \end{aligned} \tag{51}$$

From Proposition 1: $K_u > K_c$, $Q_u > Q_c$, and $H_u^P < H_c^P$. Since $\alpha > 0$, $1-\alpha-\rho > 0$ (as $\rho < 0$), and $1-\rho > 0$:

- $K_u^\alpha > K_c^\alpha$
- $Q_u^{1-\alpha-\rho} > Q_c^{1-\alpha-\rho}$
- $(H_u^P)^{-(1-\rho)} > (H_c^P)^{-(1-\rho)}$

All three factors favor the unconstrained firm, so $\frac{dY_u}{d\bar{H}} > \frac{dY_c}{d\bar{H}}$, establishing (50). □

Economic Intuition. The skill-biased stagnation arises because constrained firms cannot finance the R&D needed to build intangibles that would complement the additional skilled labor in production. While they can deploy more H^P, the lack of complementary S means this labor is underexploited. Unconstrained firms, by contrast, expand both margins and realize the full gains from complementarity.

A Data Construction and Cleaning

AA. Data Sources and Sample Construction

The empirical analysis uses Portuguese firm-level data from two administrative sources for the period 2011–2022:

(i) **SCIE (Sistema de Contas Integradas das Empresas):** Balance sheet and income statement data covering all Portuguese firms required to file annual accounts. Variables include assets, liabilities, equity, revenue, costs, investment flows, and R&D expenditures.

(ii) **QP (Quadros de Pessoal):** Matched employer-employee data containing firm characteristics and worker-level information. I aggregate worker data to construct firm-level skill composition measures, defining skilled workers as those with tertiary education (university degree or higher).

Before the cleaning procedure, I exclude public-owned firms and also non-incorporated businesses ((*empresas individuais*), because balance sheet data is only available for incorporated firms. The initial merged dataset contains 2,329,807 firm-year observations spanning 2011–2022.

AB. Price Deflators

All monetary variables are deflated to constant 2020 prices using three price indices:

- **GDP deflator** (base year 2020=100): Applied to revenue, costs, wages, and R&D expenditures. Source: FRED (Federal Reserve Economic Data), series PRTGDPDEFQISMEI_NBD20200101.
- **GFCF deflator** (Gross Fixed Capital Formation, 2020=100): Applied to investment and disinvestment flows. Source: EU KLEMS-INTAN database, Portuguese data.
- **Capital deflator** (2020=100): Applied to balance sheet stocks. Source: EU KLEMS-INTAN database, Portuguese data.

For 2022, GFCF and capital deflators are extrapolated using the 2021–2022 growth rate of the GDP deflator, as EU KLEMS-INTAN data availability ends in 2021.

AC. Data Cleaning Procedure

The cleaning procedure follows four sequential steps to ensure data quality before constructing intangible capital stocks:

Step 1: Structural Problems. Remove observations with missing firm identifiers or duplicate firm-year observations. This quality control check drops 0 observations.

Step 2: Missing or Negative Values. Drop observations with missing or negative values in variables that should exist and be non-negative. Since all monetary variables are deflated to constant 2020 prices, negative values indicate data errors rather than economic phenomena. Checked variables include:

- Tangible fixed assets: buildings, machinery, equipment
- Balance sheet intangibles: excludes goodwill
- Revenue, production value, wagebill
- Total debt, long-term debt, short-term debt, interest expenses
- Tangible investment
- Intangible construction inputs: R&D expenditures, advertising, training

This step drops 165,494 observations.

Step 3: Missing Economic Activity. Drop observations with value zero in core variables (number of workers, physical (tangible) capital, revenue, production, wagebill), indicating absence of genuine economic activity. This step drops 330,414 observations. Note that physical capital is defined as tangible fixed assets (buildings, machinery, equipment) following standard practice in production function estimation, excluding inventories and other working capital items.

Step 4: Panel Structure. Require at least two consecutive observations per firm to enable construction of intangible capital stocks via the perpetual inventory method. This step drops 74,806 observations.

Final Sample. The sequential cleaning procedure excludes 570,714 observations (15.0%), yielding a final analytical sample of **1,759,093 firm-year observations** with 75.50% retention rate. All observations have complete data for balance sheet variables, positive economic activity, and sufficient time length for capital stock construction.

AD. Intangible Capital Construction

Following Peters and Taylor (2017), I construct intangible capital stocks using the perpetual inventory method (PIM). Intangible capital comprises two components:

Knowledge Capital (from R&D). Accumulated from reported R&D expenditures:

$$K_{j,t}^{knowledge} = (1 - \delta_R) K_{j,t-1}^{knowledge} + RD_{j,t}, \tag{52}$$

where $\delta_R = 0.15$ is the depreciation rate for knowledge capital and $RD_{j,t}$ denotes real R&D expenditures in year t. Initial stocks are set to zero ($K_{j,2011}^{knowledge} = 0$).

Organization Capital (from SG&A). Accumulated from selling, general, and administrative expenses:

$$K_{j,t}^{org} = (1 - \delta_{SGA}) K_{j,t-1}^{org} + SGA_{j,t}, \tag{53}$$

where $\delta_{SGA} = 0.20$ is the depreciation rate for organization capital and $SGA_{j,t} = \text{Advertising}_{j,t} + \text{Training}_{j,t}$. Initial stocks are again set to zero.

Total Intangible Capital. The sum of internally created intangibles (knowledge + organization capital) and externally acquired intangibles reported on balance sheets:

$$K_{j,t}^{intangible} = K_{j,t}^{knowledge} + K_{j,t}^{org} + K_{j,t}^{external}. \tag{54}$$

Total capital is defined as $K_{j,t}^{total} = K_{j,t}^{physical} + K_{j,t}^{intangible}$, where physical capital is tangible fixed assets from balance sheets.

AE. Winsorization

To limit the influence of extreme outliers while preserving genuine economic variation, I apply *asymmetric* winsorization to key real variables in the clean sample:

- Winsorization threshold: 99.5th percentile (top 0.5% only)
- Variables winsorized: Total capital, physical capital, intangible capital, revenue, production, wagebill, total debt, long-term debt, short-term debt, tangible investment, number of workers
- Lower tail: No winsorization applied, preserving variation at the bottom of distributions

The asymmetric approach reflects the research design: the model focuses on binding financial constraints and complementarity between intangibles and skills, where genuine economic variation at lower values (small firms, low capital) is theoretically informative, while extremely large values likely reflect measurement error.

AF. Final Dataset Structure

The final analysis-ready dataset contains 303,336 firm-year observations with:

- All monetary variables in constant 2020 prices
- Constructed intangible capital stocks (knowledge, organization, total)
- Firm-level skill composition (share of workers with tertiary education)
- Quality flags for all excluded observations (enabling robustness checks)
- Winsorized versions of key variables (suffix `_w`)

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