# Recitation 1

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# Announcements

# Why is Linear Algebra important?

Linear Algebra ...

- Appears in ALL applied math, including *data science*
- Is solvable. If you can write it down, you can solve it!
  - (not true for other math, e.g differential equations, integration)
- Is *fundamental* to understanding tools in machine learning

Relevant applications we will cover in the class:

- Linear Regression
- Principal Component Analysis
- Gradient Descent

# Concept Review: Vector Spaces

## Definition (Vector space)

$V$ is a vector space over field $F$ when

1. Closure under Addition: $x + y \in V$
2. Sum is commutative $(x + y = y + x)$ and associative $x + (y + z) = (x + y) + z$
3. Additive Identity (in $V$): $0 \in V$ $(x + 0 = x)$
4. Additive Inverse: $(\forall x, \exists -x$ s.t $x + (-x) = 0$
5. Closure under Scalar Multiplication: $\alpha x \in V$
6. Multiplicative Identity (in $F$): $\alpha x \in V$
7. Compatibility in Multiplication: $\alpha(\beta x) = (\alpha\beta)x$
8. Distributivity: $(\alpha + \beta)x = \alpha x + \beta \cdot \vec{y}$ and $\alpha(x + y) = \alpha x + \alpha \cdot y$.

Notice that "vectors" are not explicitly defined.
Optional: prove (after class) that $\mathbb{R}^3$ with standard definitions for addition and scalar multiplication is vector space over field $\mathbb{R}$ .

# Concept Review: Vector Spaces

In this class,

- Our field is always $\mathbb{R}$. ($\mathbb{C}$ is also a field)
- Standard definitions for vector addition, and scalar multiplication
- But, V is (usually) $\mathbb{R}^n$, or (sometimes) $\mathbb{R}^{n \times n}$

Everything in Linear Algebra is in a vector space.

# Concept Review: Subspaces

**Definition (Subspace)**

A subset $S$ of a vector space $V$ is a *subspace* if it is closed under addition and scalar multiplication.

1. Closure under Addition: $x + y \in V$
2. Closure under Scalar Multiplication: $\alpha x \in V$

A subset is also a vector space!
Everything in linear algebra is in a vector space.
Anything *interesting* in linear algebra is in a subspace.

# Questions 1: Subspaces, Span

# Solutions 1, Linear Independence, Bases,