Importance-sampled regression imputation

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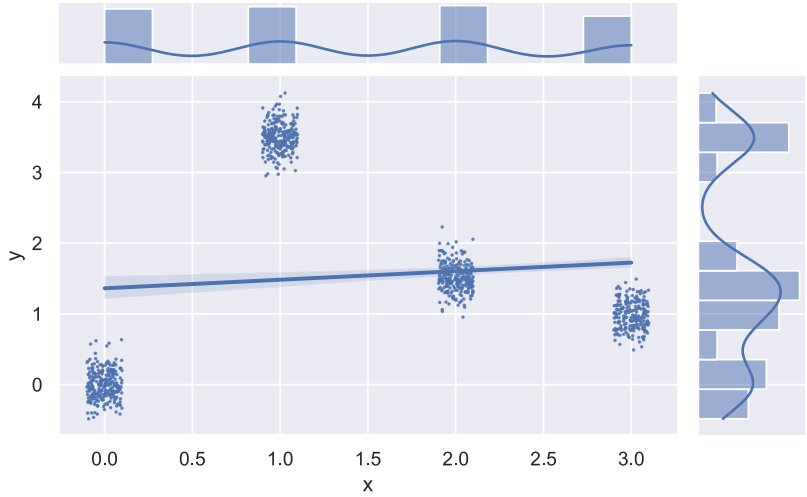
Covariate shift in regression imputation

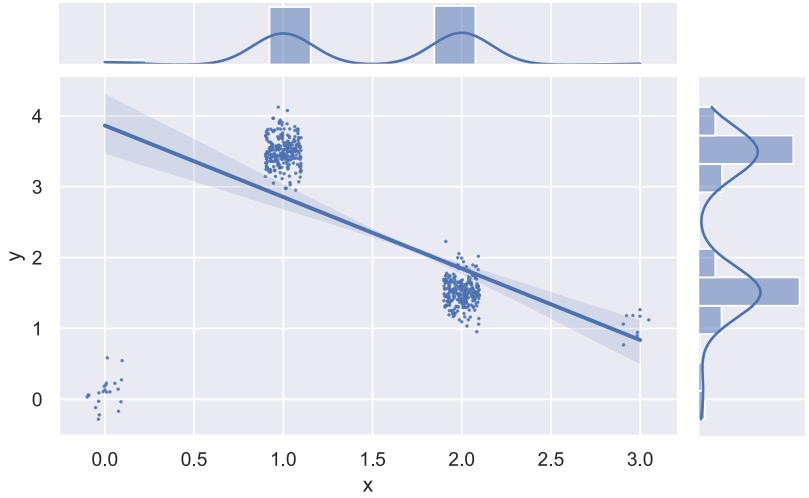
Recap: MAR_normal_nonlinear

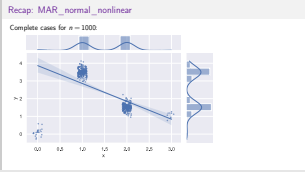
Full data for $n = 1000$:



Recap: MAR_normal_nonlinear

Complete cases for $n = 1000$:





Note that the linear fit is completely off from the fit to the full data (preceding slide) because of the sample bias.

Recap: Performance on MAR_normal_nonlinear

- True mean: 1.50

estimator	mean	SD	SE	bias	RMSE
mean	2.4075	0.0476	0.0015	0.9063	0.9075
ipw_mean	1.4985	0.0851	0.0027	-0.0027	**0.0852**
sn_ipw_mean	1.5070	0.1224	0.0039	0.0057	0.1225
impute_linear	2.4060	0.0583	0.0018	0.9048	**0.9066**

Importance-sampling imputation estimators

- Our linear model is fit to data from the complete case distribution
  - we need it to be fit to the incomplete case distribution
  - or the full data distribution (also common)
- Two new estimators:
  - **impute_IPW_linear:** examples weighted by $\frac{1}{\pi(X_i)}$ so unbiased for full data
  - **impute_IS_linear:** examples weighted by $\frac{1-\pi(X_i)}{\pi(X_i)}$ so unbiased for incomplete data

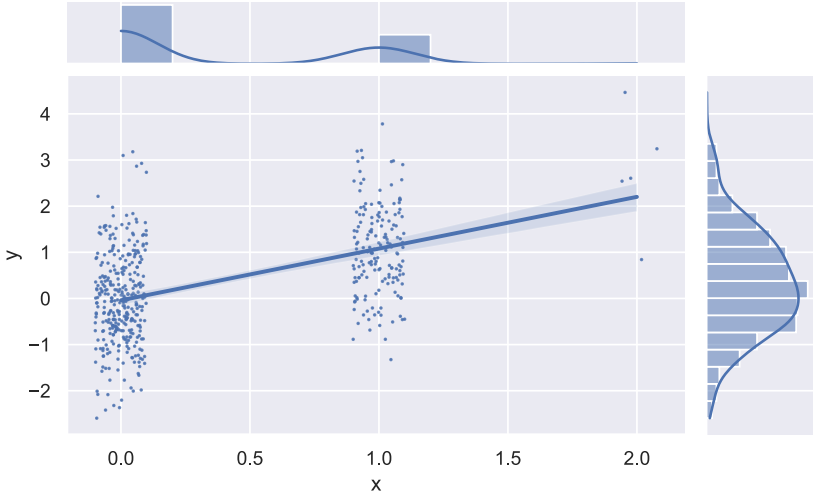
Performance on MAR_normal_nonlinear

- True mean: 1.50

estimator	mean	SD	SE	bias	RMSE
mean	2.4075	0.0476	0.0015	0.9063	0.9075
ipw_mean	1.4985	0.0851	0.0027	-0.0027	0.0852
sn_ipw_mean	1.5070	0.1224	0.0039	0.0057	0.1225
impute_linear	2.4060	0.0583	0.0018	0.9048	**0.9066**
impute_ipw_linear	1.9895	0.0777	0.0025	0.4883	**0.4944**
impute_is_linear	1.5005	0.0466	0.0015	-0.0007	**0.0466**

Recap: SeaVan1 distribution illustrated

(X_i, Y_i) for which $R_i = 1$, i.e. the complete cases.



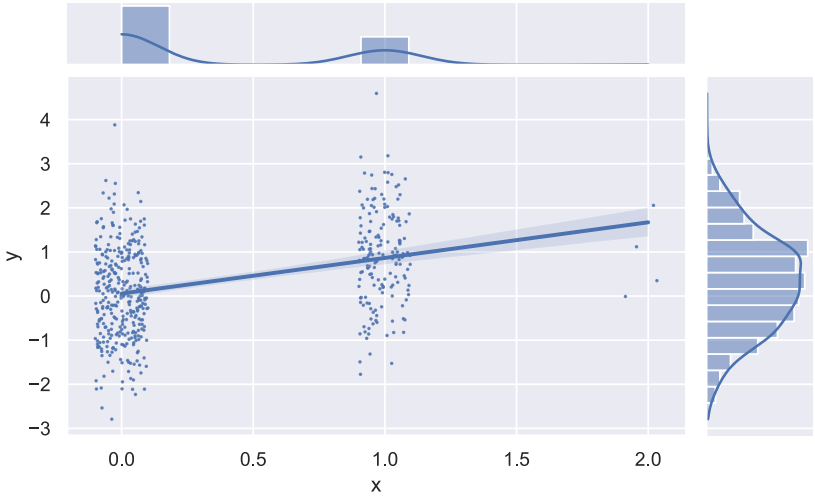
Performance on SeaVan1

- Fit $\hat{f}(x) = a + bx$ to the complete cases.

estimator	mean	SD	SE	bias	RMSE
mean	0.3564	0.0515	0.0016	-0.6431	0.6452
ipw_mean	1.0127	0.2968	0.0094	0.0132	0.2971
sn_ipw_mean	0.9906	0.1890	0.0060	-0.0089	0.1892
impute_linear	1.0022	0.0781	0.0025	0.0027	**0.0782**
impute_ipw_linear	1.0039	0.1439	0.0046	0.0044	**0.1440**
impute_is_linear	1.0047	0.1529	0.0048	0.0052	**0.1530**

MAR: "SeaVan2" distribution illustrated

- Complete cases in sample of size $n = 1000$



Performance on SeaVan2

- Fit $\hat{f}(x) = a + bx$ to the complete cases.

estimator	mean	SD	SE	bias	RMSE
mean	0.3425	0.0493	0.0007	-0.3244	0.3282
ipw_mean	0.6655	0.1939	0.0027	-0.0014	0.1939
sn_ipw_mean	0.6594	0.1446	0.0020	-0.0075	0.1448
impute_linear	0.9364	0.0792	0.0011	0.2695	**0.2809**
impute_ipw_linear	0.6750	0.1503	0.0021	0.0081	**0.1505**
impute_is_linear	0.6677	0.1561	0.0022	0.0008	**0.1561**

Caveat on results

- The importance-sampled regression imputation estimators seem promising.
- The estimators rely on knowing the importance weights $p(x)/q(x)$.
- Performance may be significantly worse when we use estimates $\hat{p}(x)/\hat{q}(x)$.
- This is something we can explore in homeworks and projects.

References

Resources

- Terminology was based on [CFV17].

References I

[CFV17] Victor Chernozhukov and Iván Fernández-Val, *Treatment effects*, Econometrics—MIT Course 14.382, Cambridge MA, 2017, MIT OpenCourseWare.