

$$a^{[1]} = x \in \mathbb{R}^{n_1} \quad \text{--- (3.1)}$$

$$a^{[l]} = \sigma(W^{[l]} a^{[l-1]} + b^{[l]}) \in \mathbb{R}^{n_l}, \quad \text{for } l = 2 \sim L. \quad \text{--- (3.2)}$$

Define $z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$. Then we have $a^{[l]} = \sigma(z^{[l]})$.

Define $\delta^{[l]} = \frac{\partial a^{[l]}}{\partial z^{[l]}}$. Then $\delta^{[l]} = \sigma'(z^{[l]})$.

$$\Rightarrow \delta^{[L]} = \frac{\partial a^{[L]}}{\partial z^{[L+1]}} \cdot \frac{\partial z^{[L+1]}}{\partial z^{[L]}}, \quad \text{for } l = L-1$$

$$= \delta^{[L]} \cdot W^{[L]} \text{diag}(\sigma'(z^{[L-1]}))$$

$$\Rightarrow \text{For } l = L-2, \dots, 2$$

$$\delta^{[l]} = \frac{\partial a^{[l]}}{\partial z^{[l+1]}} \cdot \frac{\partial z^{[l+1]}}{\partial z^{[l]}} = \sigma'(z^{[l]}) - (W^{[l+1]})^T \delta^{[l+1]}$$

$$\text{For } n_1=1, \quad z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$$

$$\text{Then } \nabla_a a^{[1]}(x) = \frac{\partial a^{[1]}}{\partial x} = \frac{\partial a^{[1]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial x}$$

$$= \delta^{[2]} (W^{[2]})^T$$