1) Let $z = b_1 + w_1x_1 + w_2x_2$. Then $h(z) = \sigma(z)$,

By $\sigma(z) = \frac{1}{1+e^{-z}}$, then $\sigma'(z) = e^{-z}/(1+e^{-z})^2 = 1/(1+e^{-z})\left(1 - \frac{1}{1+e^{-z}}\right) = \sigma(z)(1-\sigma(z))$

Consider MSE loss of SGD is $L = \frac{1}{2}\|h-y\|^2$.

Hence, $\frac{\partial L}{\partial z} = \frac{\partial L}{\partial h}\cdot\frac{\partial h}{\partial z} = (h-y)h(1-h) = \phi(h)$

So, $\partial L/\partial b = \frac{\partial L}{\partial z}\cdot\frac{\partial z}{\partial b} = \phi(h)\cdot 1$

$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial z}\cdot\frac{\partial z}{\partial w_1} = \phi(h)\cdot x_1$

$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial z}\cdot\frac{\partial z}{\partial w_2} = \phi(h)\cdot x_2$

$\Rightarrow z^0 = b^0 + w_1^0x_1 + w_2^0x_2 = 4 + 5\cdot 1 + 6\cdot 2 = 21$; $h^0 = \sigma(21)$,

$\theta' = \theta^0 - \alpha\nabla L$

$= (4,5,6) - \alpha(\phi(h^0), \phi'(h^0), 2\phi'(h^0))$

$1 - 4\sigma + 4\sigma^2$

2) a) $\sigma(x) = \frac{1}{1+e^{-x}}$ $\quad\sigma'(x) = \sigma(x)(1-\sigma(x))$

Hence $\sigma'' = \sigma'(1-\sigma) - \sigma\sigma'$

$= \sigma'(1-2\sigma) = \sigma(1-\sigma)(1-2\sigma)$

$\sigma^{(3)} = \sigma''(1-\sigma) - (\sigma')^2 - (\sigma')^2 - \sigma\sigma''$

$= \sigma''(1-2\sigma) - 2\sigma(1-\sigma)$

$= \sigma(1-\sigma)(1-6\sigma+6\sigma^2)$

b) $\sigma(x) = \frac{1}{1+e^{-x}} = \frac{e^{\frac{x}{2}}}{e^{\frac{x}{2}} + e^{\frac{x}{2}}}$

$= \frac{1}{2}\left(\frac{e^{\frac{x}{2}} - e^{-\frac{x}{2}}}{e^{\frac{x}{2}} + e^{-\frac{x}{2}}} + 1\right) = \frac{1}{2}\left(\arctan\left(\frac{x}{2}\right) + 1\right)$

3)