Econ 212a: Business Cycles
Lecture 2
RBC Model, part I

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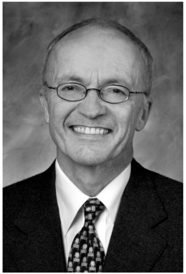
Recap

- Last class, we discussed business-cycle regularities
  - business cycle = comovement of Y, C, I, N + their volatility patterns
- We started discussing the real business cycle (RBC) model
  - neoclassical growth model (NGM) + endogenous labor + technology shocks
  - we measured technology shocks as Solow residuals
- **Today:** Solve + calibrate RBC. How well does it capture the business cycle?

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2004



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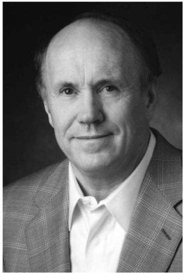
Finn E. Kydland

Prize share: 1/2



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Edward C. Prescott

Prize share: 1/2

"Kydland and Prescott demonstrated how **variations in technological development...** can lead to **short-run fluctuations**. In so doing, they offered a **new and operational paradigm for macroeconomic analysis** based on microeconomic foundations."

RBC: planning problem

- Recall we wrote the production function as
$$Y_t = A_t F\left(K_{t-1}, (1+g)^t N_t\right)$$
Sometimes write $X_t \equiv (1+g)^t$. We also call A_t **TFP** (total factor productivity).
- A_t is stochastic. We think of it as following a *Markov process*, e.g. AR(1)
$$\log A_t = \rho \log A_{t-1} + \epsilon_t \tag{1}$$
- The RBC model in a nutshell...
    - take the NGM with endogenous labor supply
    - assume stochastic proces for A_t, say (1)
    - see if macroeconomic aggregates (C_t, I_t, N_t, K_t) behave anything like in the data
- We start with the **planning problem**. We will later see that just like the NGM, the RBC model is frictionless, so planner's solution = comp. equilibrium.

One key difference with NGM: uncertainty...

- Consider **state space** S
- One state $s_t \in S$ realized every period
    - In baseline model: TFP is function of state, $A = A(s_t)$
    - s_t follows a Markov chain, and A inherits its properties
    - $\pi(s_t|s_{t-1})$ transition matrix
- Histories of shocks $s^t = (s_0, s_1 \ldots s_t) \in S^t$, probability:

$$\pi(s^t) \equiv \pi(s_t|s_{t-1})\pi(s_{t-1}|s_{t-2}) \cdots \pi(s_1|s_0)$$

- Agent have **rational expectations:** evaluate histories s^t with π
- A *consumption plan* is a set of functions $C_t(s^t)$, with C_t: $S^t \to \mathbb{R}$
    - similarly for all other variables

Planner's problem setup

- Planner's problem: choose $\left\{C_t\left(s^t\right), N_t\left(s^t\right), I_t\left(s^t\right), K_t\left(s^t\right)\right\}$ to solve

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) U\left(C_t\left(s^t\right), N_t\left(s^t\right)\right) \\ \text{s.t.} \quad & C_t\left(s^t\right) + I_t\left(s^t\right) = A\left(s_t\right) F\left(K_{t-1}\left(s^{t-1}\right), X_t N_t\left(s^t\right)\right) \\ & K_t\left(s^t\right) = (1-\delta) K_{t-1}\left(s^{t-1}\right) + I_t\left(s^t\right) \end{aligned}$$

- Notes:
    - Closed economy with no government
    - Timing matters: capital K pre-determined in production F
    - Utility is decreasing in labor N (a "bad", with $U_N < 0$)
        - sometimes written as a function of leisure $L = L^{max} - N$

Simplifying notation...

- Sometimes, drop subscripts, eg write $C\left(s^t\right)$, $K\left(s^t\right)$, or drop s^t, write C_t, K_t
- Also, we write conditional expectation of consumption as

$$\mathbb{E}_t\left[C_{t+1}\right] = \mathbb{E}\left[C_{t+1}|s^t\right] = \mathbb{E}_t\left[C_{t+1}(s^{t+1})\right] = \sum_{s_{t+1}} \pi(s_{t+1}|s_t)C_{t+1}(s^{t+1})$$

  and similarly for other variables

- For example, the objective function can then be written as

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)\right]$$

Solution methods

- An optimum can be characterized as solution to ...
  - Bellman equation
  - first order conditions (e.g. Euler equation)
- Start with first approach, for simplicity in special case $X_t = 1$.
- Then, discuss second approach.

Solution method 1: Bellman equation

- To set up Bellman, need to figure out the states of the model
  - economy at date t depends on entire history s^t ...
  - ... but only two objects really matter: current state s and capital $K_{t-1}(s^{t-1})$
- With s, K as states, Bellman is:

$$V(K,s) = \max_{C,N,K'} \left\{ U(C,N) + \beta \sum_{s'} \pi(s'|s)\, V(K',s') \right\} \tag{2}$$

$$C + K' = (1-\delta)K + A(s)F(K,N)$$

- $\rightarrow$ Solution is recursive and takes form of policy rules:

$$C = \Phi^C(K,s) \quad N = \Phi^N(K,s) \quad K' = \Phi^K(K,s)$$

- In special cases, can guess and verify analytical solution
- In other cases, can solve (2) on computer (we'll do something else)

Exercise: the Brock-Mirman model

- Assume that

$$F(K,N) = K^{\alpha} N^{1-\alpha} \qquad \delta = 1 \qquad U(C,N) = \log C - v(N)$$

  Note: $\delta = 1$ is unrealistic! But very useful special case...

- Then guess $V(K,s) = a(s) + b \log K$ for some function $a(s)$, constant b
- Verifying the guess, we find policies:

$$\begin{aligned}
K' &= \Phi^K(K,s) = \alpha\beta A(s) F\left(K,\overline{N}\right) \\
C &= \Phi^C(K,s) = (1-\alpha\beta) A(s) F\left(K,\overline{N}\right) \\
N &= \Phi^N(K,s) = \overline{N}
\end{aligned}$$

  where $\overline{N}$ is solution to $v'\left(\overline{N}\right)\overline{N} = \frac{1-\alpha}{1-\alpha\beta}$ (and $b = \frac{\alpha}{1-\alpha\beta}$)

- Labor is **constant**, even though marginal product $AF_N(K,N)$ moves. Why?

Solution method 2: First order conditions

- Many ways to derive them (eg Lagrangian)... here: **variational arguments**
- **Intratemporal optimality:** Work more in t, consume increased production

$$-U_N(C_t, N_t) = U_C(C_t, N_t) \cdot A_t X_t F_N(K_{t-1}, X_t N_t) \tag{3}$$

- **Intertemporal optimality:** Consume less in t, save, consume more in $t+1$

$$U_C(C_t, N_t) = \beta \mathbb{E}_t \left[\left(A_{t+1} F_K(K_t, X_{t+1} N_{t+1}) + 1 - \delta \right) U_C(C_{t+1}, N_{t+1}) \right] \tag{4}$$

- Finally, a **transversality condition:**

$$\lim_{t \to \infty} \beta^t \mathbb{E}_0 \left[U_C(C_t, N_t) K_{t-1} \right] = 0 \tag{5}$$

- With **resource constraints**, these are necessary and sufficient for optimum

$$C_t + I_t = A_t F(K_{t-1}, X_t N_t) \tag{6}$$

$$K_t = (1 - \delta) K_{t-1} + I_t \tag{7}$$

Two helpful objects

- Will "decentralize" the optimum from the planning problem as a competitive equilibrium next class
- Only then will "wages" and "interest rates" appear
- However, useful to define these objects already now:

$$W_t = A_t X_t F_N\left(K_{t-1}, X_t N_t\right) = MPL_t$$
$$R_t = 1 + r_t = A_t F_K\left(K_{t-1}, X_t N_t\right) + 1 - \delta = MPK_t + 1 - \delta$$

- W_t will be the real wage, R_t the gross ex-post return on capital

RBC Model: Calibration

Calibration philosophy

- We'd like to know how well the RBC model describes business cycles
- **Calibration** = picking the model's **functional forms** and **parameter values**
- Main idea: Match
  - **long-run patterns** (e.g. growth rates, "big ratios"). Prescott's original approach.
  - **micro evidence** (more recent)
- Then we can simulate model and validate its business cycle predictions
- See Hansen–Heckman "the Empirical Foundations of Calibration" for more

Next: How to pick functional forms for the model, then parameters

Functional forms, part I: Impose balanced growth

- Our model has growing X_t. "Balanced growth" = $C_t/Y_t, K_t/Y_t, I_t/Y_t$ fluctuate around constant long-run trend, with constant hours per person N_t
- Along our optimum, this is only possible for a specific utility function

$$U(C,N) = \frac{\left[C \cdot e^{v(N)}\right]^{1-\nu} - 1}{1-\nu} \tag{8}$$

where $v(N)$ is a decreasing function of N.

- Known as **King-Plosser-Rebelo** (KPR) preferences.
  - great exercise to derive these! Hint: Set $A = 1$ in (3) and argue that $U(C,N)$ must be of the form $\mathcal{U}(Ce^{v(N)})$. Then use (4) to show (8).
- Case $\nu = 1$: *separable* preferences

$$U(C,N) = \log C - v(N) \tag{9}$$

How should we choose $v(N)$? Define **Frisch elasticity** of labor

$$\text{Frisch} \equiv \left.\frac{d\log N}{d\log W}\right|_{U_C=const} = \left[\frac{d\log(-U_N)}{d\log N}\right]^{-1} \overset{\text{if } \nu=1}{=} \frac{v'(N)}{v''(N)N}$$

- Prescott's favorite specification (log C, log leisure)

$$U(C,N) = \log C + \psi \log(1-N) \qquad \Rightarrow \qquad \text{Frisch} = \frac{1-N}{N} \tag{10}$$

- More common today: constant-Frisch specification

$$U(C,N) = \log C - \psi \frac{N^{1+\phi}}{1+\phi} \qquad \Rightarrow \qquad \text{Frisch} = \phi^{-1} \tag{11}$$

- New Keynesian literature considers more general 'MaCurdy' prefs,

$$U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \psi \frac{N^{1+\phi}}{1+\phi} \tag{12}$$

Detrending the model

- With KPR preferences (8), we can detrend the model
- Define $c_t \equiv \frac{C_t}{X_t}$, $k_{t-1} \equiv \frac{K_{t-1}}{X_t}$, $i_t \equiv \frac{I_t}{X_t}$
- Then, given k_{-1}, $\{c_t, N_t, i_t, k_t\}_{t\geq 0}$ are solution to (check this!)

$$\begin{aligned} \max \quad & \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \widetilde{\beta}^t U(c_t, N_t) \right] \\ \text{s.t.} \quad & c_t + i_t = A_t F(k_{t-1}, N_t) \\ & (1+g) k_t = (1-\delta) k_{t-1} + i_t \end{aligned}$$

where $\widetilde{\beta} \equiv \beta (1+g)^{1-\nu}$.

- It's **"as if" there was no long-run growth**, $X_t = 1$
  - except that $\widetilde{\beta}$ replaces β and $1+g$ shows up in capital accumulation
  - given that we pick $\tilde{\beta}$ anyway, common to just start with $g = 0$

Calibration of all parameters except ψ, β

- Cobb-Douglas production $F(k, N) = k^{\alpha} N^{1-\alpha}$, $\alpha \simeq \frac{1}{3}$
    - since labor share more or less constant
    - for depreciation rate δ King-Rebelo (KR) use 10% p.a. (on high end)
- Separable preferences (9). Traditional RBC literature uses (10)
    - more common today to use (11) to get plausible Frisch elasticity ϕ
    - ϕ turns out to be crucial to success of RBC model!
- Persistence ρ and std. dev. σ_{ϵ} of innovations to productivity

$$\log A_t = \rho \log A_{t-1} + \epsilon_t \quad \epsilon_t \sim N\left(0, \sigma_{\epsilon}^2\right)$$

    - King-Rebelo find $\rho = 0.979$ and $\sigma_{\epsilon} = 0.0072$ (quarterly), growth $g = 1.6\%$ p.a.

Calibration or ψ, β: steady state optimality

Pin down ψ, β using long-run averages. Imagine no shocks, $A = 1$...

- From (3)–(7) at steady-state we have

$$v'(N)\,N = (1-\alpha)\,\frac{Y}{C} \tag{13}$$

$$1 + g = \beta R = \beta\left(\alpha\frac{Y}{K} + 1 - \delta\right) \tag{14}$$

$$\frac{I}{Y} = (g+\delta)\,\frac{K}{Y} \tag{15}$$

- From (14), we get

$$\beta = \frac{1+g}{1+r} \simeq 0.988/\text{quarter} \qquad \text{and} \qquad \frac{K}{Y} = \frac{\alpha}{r+\delta} \simeq 8$$

- From (15) follows $\frac{I}{Y} \simeq 23\%$, and therefore $\frac{C}{Y} = 1 - \frac{I}{Y}$
- From (13) and Prescott's (10), we get $\psi = (1-\alpha)\,\frac{1}{\frac{C}{Y}}\frac{1-N}{N}$. What is N?

Calibration: final step

$$\psi = (1-\alpha)\frac{1}{\frac{C}{Y}}\frac{1-N}{N}$$

- Prescott calibration is $N = 0.2$
  - 16 hours per day, 7 days per week: 112 hours of time endowment
  - average weekly hours of employees = 35
  - employment-population ratio $\simeq 0.65$
  - so $N \simeq \frac{0.65\times 35}{112} \simeq 0.2$
- Summary of benchmark calibration parameters (King-Rebelo)

g	β	ψ	α	δ	ρ	σ_ϵ
0.004	0.988	3.48	0.333	0.025	0.979	0.0072

$F(K,N) = K^\alpha N^{1-\alpha} \quad U(C,N) = \log C + \psi \log(1-N)$

RBC Model: Results

Results

- Simulate artificial data from model, HP filter it, compare second moments

Table 3
Business cycle statistics for basic RBC model[a,b]

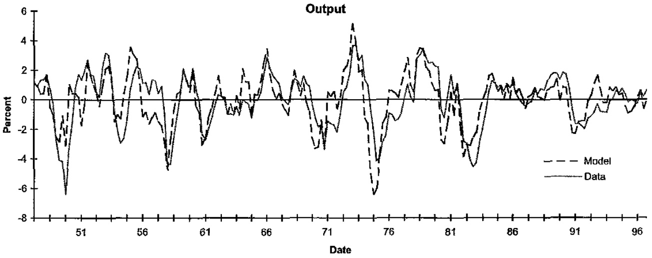
	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

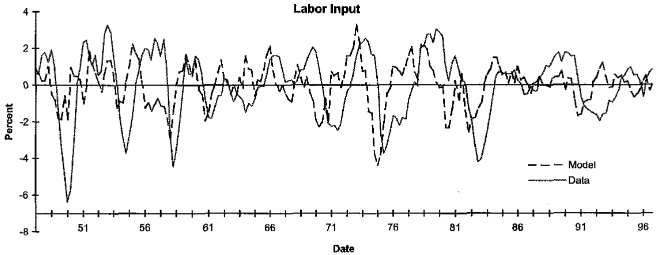
Table 1
Business cycle statistics for the US Economy

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	–0.35
A	0.98	0.54	0.74	0.78

- Surprisingly good performance for such a simple model?
  - Prescott: *"account for more than half of the fluctuations in the postwar period, with a best point estimate near 75%"* $\left[\left(\frac{1.39}{1.81}\right)^2 = 0.77\right]$
- Another way to look at it: feed actual Solow residual into model & compare

Actual vs simulated series





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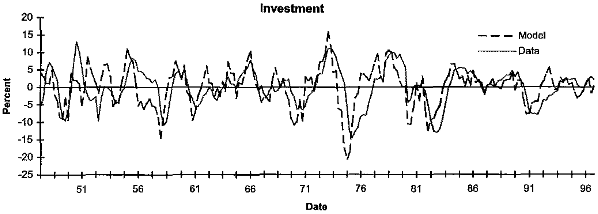
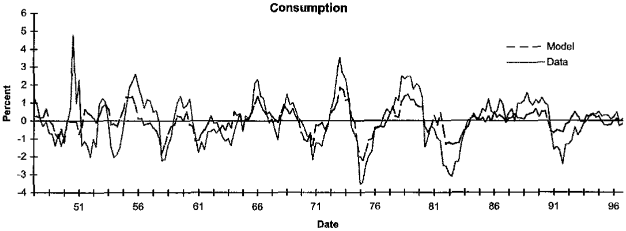




Main takeaways from exercise

- Success:
  - I 3 times more volatile than Y
  - C smoother than Y (too smooth?)
  - all quantities comove with output as in data (correlation too high?)
- Failures:
  - too little *amplification*: model $\frac{\sigma_Y}{\sigma_A} = 1.47$, vs data $\frac{\sigma_Y}{\sigma_A} = 1.94$
  - The model's N does not move enough, and leads N in the data
  - prices (W and r) are much too procyclical
- Does RBC model do well **for the right reasons ?**
  - Summers 1986: *"Extremely bad theories can predict remarkably well..."* (his examples: Ptolemaian astronomy, Lamarckian Biology)

$\rightarrow$ How can we know? Let's study the "transmission mechanism"

The RBC impulse response

Law of motion and impulse response

- Fundamental concept: the *impulse responses* of model to shocks (here, ϵ_t)
- Recall the policy functions of the model are $C_t = \Phi^C(K_{t-1}, A_t)$, $K_t = \Phi^K(K_{t-1}, A_t)$, where the Φ^C, Φ^K depend on model parameters
- Given these, the **state-space law of motion** of the model is

$$\begin{aligned} K_t &= \Phi^K(K_{t-1}, A_t) \\ C_t &= \Phi^C(K_{t-1}, A_t) \\ \log A_t &= \rho \log A_{t-1} + \epsilon_t \end{aligned}$$

- Definition: the **impulse response** of C to A is the expected time path of C, following an innovation to the process for A, starting from the steady state
- We can obtain this impulse response by simulating the state-space LOM and taking averages across paths of $\{\epsilon_t\}$ to get $\mathbb{E}[C_t]$. It's pretty complicated!

- Loglinearizing the policy functions, denoting log deviations from steady state $k_t = dK_t/K = d\log K_t$, etc, we get:

$$\begin{aligned} k_t &= \eta_{kk} k_{t-1} + \eta_{ka} a_t \\ c_t &= \eta_{ck} k_{t-1} + \eta_{ca} a_t \\ a_t &= \rho a_{t-1} + \epsilon_t \end{aligned} \tag{LOM}$$

where $\eta_{kk} = \frac{\partial \Phi^K}{\partial K}$, $\eta_{ck} = \frac{K}{C}\frac{\partial \Phi^C}{\partial K}$, etc, and ϵ_t is iid with $\mathbb{E}\left[\epsilon_t\right] = 0$, $\mathrm{Var}\left(\epsilon_t\right) = \sigma_\epsilon^2$

- This is known as the **linear state-space law of motion**
- In this linearized model, the impulse response of c to a is:

$$\mathrm{irf}_t^c \equiv \mathbb{E}_0\left[c_t | \epsilon_0 = 1, k_{-1} = 0, a_{-1} = 0\right] - \mathbb{E}_0\left[c_t | \epsilon_0 = 0, k_{-1} = 0, a_{-1} = 0\right] \tag{16}$$

This is *much* simpler to get!

Obtaining the IRF in the linear model and certainty equivalence

- Why? Applying $\mathbb{E}_0$ to (LOM), we get:

$$\begin{aligned}\mathbb{E}_0\left[k_t\right] &= \eta_{kk}\mathbb{E}_0\left[k_{t-1}\right] + \eta_{ka}\mathbb{E}_0\left[a_t\right] \\ \mathbb{E}_0\left[c_t\right] &= \eta_{ck}\mathbb{E}_0\left[k_{t-1}\right] + \eta_{ca}\mathbb{E}_0\left[a_t\right] \\ \mathbb{E}_0\left[a_t\right] &= \rho\mathbb{E}_0\left[a_{t-1}\right] + \epsilon_0 \mathbf{1}_{t=0}\end{aligned} \tag{PF-LOM}$$

  This is a system of regular (i.e. non-stochastic) difference equations

- Using our definition (16), we obtain:

$$\mathrm{irf}_t^c \equiv \mathbb{E}_0\left[c_t|\epsilon_0 = 1\right] - \mathbb{E}_0\left[c_t|\epsilon_0 = 0\right] = \mathbb{E}_0\left[c_t|\epsilon_0 = 1\right]$$

$\rightarrow$ can get irf_t^c by simulating (PF-LOM) **once** given $k_{-1} = 0$, $a_{-1} = 0$, and $\epsilon_0 = 1$

- In words: the irf_t^c is the deterministic solution to (LOM), assuming that productivity a_t realizes at its expected value as of date 0, $\mathbb{E}_0\left[a_t|\epsilon_0 = 1\right] = \rho^t$
- This property is called **certainty equivalence.**

MA(∞) representation of the solution

- An alternative route to irf^c_t: obtain the MA(∞) representation of (LOM)
- Iterating $a_t = \rho a_{t-1} + \epsilon_t$ backward, we get $a_t = \sum_{j=0}^{\infty} \rho^j \epsilon_{t-j}$, the MA($\infty$) representation of productivity on its innovations.
- Then, iterating (LOM) backward, we get

$$c_t = \sum_{j=0}^{\infty} \gamma^c_j \epsilon_{t-j} \tag{MA}$$

  this is the **MA(∞) representation of consumption** on the innovations to a

- We have the following important result:

$$\mathrm{irf}^c_t = \gamma^c_t$$

  with a linear LOM, impulse response and MA(∞) coeffs are the same thing

Proof that impulse response and MA coefficients are the same

- Why? Let's calculate

$$\mathbb{E}_0\left[c_t | \{\epsilon_0, \epsilon_{-1}, \cdots\}\right] = \sum_{j=0}^{\infty} \gamma_j^c \mathbb{E}_0\left[\epsilon_{t-j}\right] = 0 + \cdots + 0 + \gamma_t^c \epsilon_0 + \gamma_{t+1}^c \epsilon_{-1} + \cdots$$

so, since e.g. $\{\epsilon_{-1}, \epsilon_{-2}, \ldots\} = \{0, 0, \ldots\}$ gives us the steady state, we find

$$\mathbb{E}_0\left[c_t | \epsilon_0, k_{-1} = 0, a_{-1} = 0\right] = \gamma_t^c \epsilon_0$$

- So, applying our definition:

$$\begin{aligned}
\mathrm{irf}_t^c &= \mathbb{E}_0\left[c_t | \epsilon_0 = 1, k_{-1} = 0, a_{-1} = 0\right] - \mathbb{E}_0\left[c_t | \epsilon_0 = 0, k_{-1} = 0, a_{-1} = 0\right] \\
&= \gamma_t^c \times 1 - \gamma_t^c \times 0 \\
&= \gamma_t^c
\end{aligned}$$

Voilà!

Impulse response and MA coefficients

- Bottom line: so far, we have two equivalent ways of obtaining irf's:
    1. Iterate forward on (LOM) from date 0, assuming all future ϵ_t's realize at 0
    2. Get the MA(∞) representation
- Neiter requires simulations. Instead, we can *use* the irf's to simulate c_t!
    - How? Draw paths for $\{\epsilon_{t-j}\}$, then simulate paths for c_t by applying (MA)
- In fact, to obtain 2nd moments of the model, we don't even need simulation
    - Instead, we can directly use (MA) again, e.g.

$$\mathrm{Cov}\left(c_t, c_{t+k}\right) = \sigma_\epsilon^2 \sum_{j=0}^{\infty} \mathrm{irf}_j^c \mathrm{irf}_{j+k}^c$$

e.g. $\mathrm{Var}\left(c_t\right)$ is just σ_ϵ^2 times the area under the squared impulse response of c

- We can then use these moments to estimate the model parameters! (eg by SMM, maximum likelihood, Bayesian estimation with priors, etc.)

Nonlinear perfect-foresight approach

- There is a third useful way to get the irf's, which is what we will use
- Idea: assume perfect foresight from the very beginning. Then, (3)—(7) read

$$-U_N(C_t, N_t) = U_C(C_t, N_t) \cdot A_t F_N(K_{t-1}, N_t)$$
$$U_C(C_t, N_t) = \beta\left(A_{t+1} F_K(K_t, N_{t+1}) + 1 - \delta\right) U_C(C_{t+1}, N_{t+1})$$
$$C_t + K_t - (1-\delta) K_{t-1} = A_t F(K_{t-1}, N_t)$$

- Starting from steady state K_{-1} and a given known path for $\{A_t\}_{t=0}^{\infty}$, these equations (plus the TVC) give us a unique path for C_t, which we can write

$$C_t\left(\{A_s\}_{s=0}^{\infty}\right)$$

- For any $\sigma > 0$, let us define $c_t^{NL}(\sigma) \equiv \log C_t\left(\{\exp(\sigma \rho^s)\}\right)$
- This is a "nonlinear impulse response". **Result**: we have $\mathrm{irf}_t^c = \left(c_t^{NL}\right)'(0)$

Sequence-space Jacobian

- This result implies:

$$\text{irf}_t^c = \frac{d \log C_t \left(\{\exp(\sigma\rho^s)\}\right)}{d\sigma} = \frac{A}{C}\sum_{s=0}^{\infty}\frac{\partial C_t}{\partial A_s}\rho^s$$

- Let $G_{t,s} \equiv \frac{\partial C_t}{\partial A_s}$. This is a "sequence-space Jacobian". Write as matrix $\mathbf{G}^c$, then

$$\begin{pmatrix} \text{irf}_0^c & \text{irf}_1^c & \text{irf}_2^c & \cdots \end{pmatrix}' = \frac{A}{C}\mathbf{G}^c \begin{pmatrix} 1 & \rho & \rho^2 & \cdots \end{pmatrix}'$$

- More generally, if the MA(∞) for TFP is $a_t = \sum_{j=0}^{\infty}\gamma_j^a \epsilon_{t-j}$ then we have

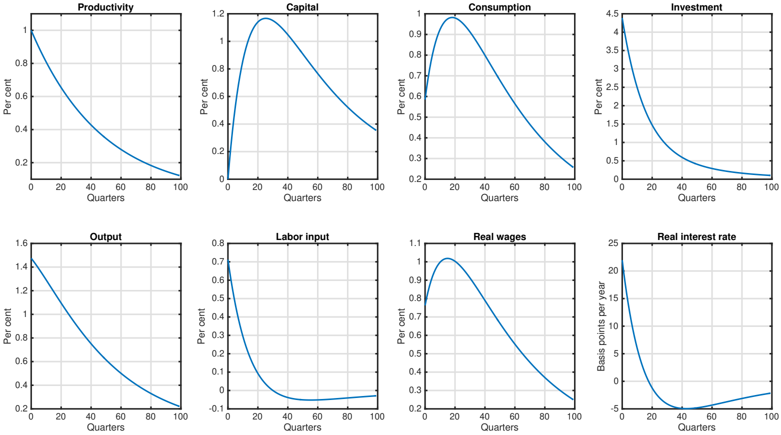
$$\begin{pmatrix} \text{irf}_0^c & \text{irf}_1^c & \text{irf}_2^c & \cdots \end{pmatrix}' = \frac{A}{C}\mathbf{G}^c \begin{pmatrix} \gamma_0^a & \gamma_1^a & \gamma_2^a & \cdots \end{pmatrix}'$$

  The **G** matrix maps one MA representation into another! Bottom line:

$$\mathbf{G} \to \text{irf} \to \text{MA} \to \text{simulations, second moments, estimation....}$$

- How do we get **G**? Analytically, or numerically with SSJ (cf notes+section)

Impulse response to technological innovation (KR fig 10)



- Q: Why does labor increase? Why does consumption increase?
- See next class!