Econ 212a: Business Cycles
Lecture 7
The New Keynesian Model

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Recap

- We introduced money into the RBC model
  - understood **long-run effects** of monetary policy
  - for **short-run** we noticed that monopolistic competition + **rigid prices** can make a big difference
- Today, we introduce **sticky prices**: not flexible but not fully rigid either
  - using Calvo (1983) approach
  - together with model ingredients from last time: **New-Keynesian model**
- A warning: the derivation of the Calvo pricing model is algebra heavy ...
  - not everybody needs to know all the details of the derivation
  - will do my best to explain intuition, but skip some of the algebra

References: Gali textbook, Woodford textbook (more advanced). We will work with nonlinear equations since they are more insightful.

What we have and what we still need

- Assume preferences $U(C, N) = u(C) - v(N) + h(m)$
  - NK model: $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$, $v(N) = \psi \frac{N^{1+\phi}}{1+\phi}$. $h(m)$ irrelevant with interest rate rule!
- **What we have:** 3 equations, 5 unknowns: $C_t, \pi_t, i_t, N_t, W_t/P_t$

$$C_t^{-\sigma} = \beta \mathbb{E}\left[C_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}}\right]$$

$$\psi C_t^{\sigma} N_t^{\phi} = \frac{W_t}{P_t}$$

$$i_t = \rho_t + \phi_\pi \pi_t$$

- **What's still missing?**
  - firm pricing: How high is P_t given W_t? (and so, how high is W_t/P_t?)
  - aggregate production technology: How much N_t produces C_t?
- Next: Recap the steps to get both for flex prices. Then Calvo ...

Flex price recap

Prelude: Per period profits D_{it}

- Notation: profits are now D_t (for dividends) and gross inflation is $\Pi_t = 1 + \pi_t$
- From last time: For given price p, profits are

$$D_t(p) = p \cdot y_t(p) - \left(1 + \tau^W\right) W_t \cdot n_t(p)$$

where with CES demand and Cobb Douglas production:

$$y_t(p) = \left(\frac{p}{P_t}\right)^{-\epsilon} C_t \qquad \text{and} \qquad y_t(p) = A_t f\left(n_t(p)\right) = A_t n_t(p)^{1-\alpha}$$

and so

$$D_t(p) = p^{1-\epsilon} \cdot P_t^{\epsilon} C_t - \left(1 + \tau^W\right) W_t \cdot f^{-1}\left(A_t^{-1} p^{-\epsilon} P_t^{\epsilon} C_t\right)$$

- Define marginal cost like last time:

$$MC_t(p) \equiv \left(1 + \tau^W\right) W_t \cdot \frac{1}{A_t f'\left(f^{-1}\left(A_t^{-1} p^{-\epsilon} P_t^{\epsilon} C_t\right)\right)} \tag{1}$$

Then:

$$D_t'(p) = -\left(\epsilon - 1\right) p^{-\epsilon-1} \cdot P_t^{\epsilon} C_t \left(p - \frac{\epsilon}{\epsilon - 1} MC_t(p)\right) \tag{2}$$

Four steps to get to our two conditions

1. **Optimal price setting**

$$p_t^* = \underbrace{\frac{\epsilon}{\epsilon - 1}}_{\text{monopoly markup}} \cdot \underbrace{MC_t(p_t^*)}_{\text{nominal marginal cost}} \quad \Rightarrow \quad \left(\frac{p_t^*}{P_t}\right)^{1+\frac{\epsilon\alpha}{1-\alpha}} = \underbrace{\frac{\epsilon}{\epsilon - 1} \cdot \left(1 + \tau^w\right) \frac{W_t}{P_t} \cdot \frac{1}{A_t f'\left(f^{-1}\left(A_t^{-1} C_t\right)\right)}}_{\text{avg. real marginal cost } \omega_t}$$

2. **Evolution of aggregate price index** as function of p_t^*

$$P_t = \left(\int (p_t^*)^{1-\epsilon}\, di\right)^{\frac{1}{1-\epsilon}} \quad \Rightarrow \quad P_t = p_t^*$$

3. Get **equation for real marginal cost** [condition 1]

$$\omega_t = 1 \quad \Rightarrow \quad \frac{W_t}{P_t} = \frac{\epsilon - 1}{\epsilon} \cdot \frac{A_t f'\left(f^{-1}\left(A_t^{-1} C_t\right)\right)}{1 + \tau^w}$$

4. What about **production**? Labor demand of a firm with price p is [condition 2]

$$n_t(p) = f^{-1}\left(y_t(p)\right) = \left(\frac{p}{P_t}\right)^{-\epsilon/(1-\alpha)} A_t^{-1/(1-\alpha)} C_t^{1/(1-\alpha)} \quad \Rightarrow \quad C_t = A_t N_t^{1-\alpha}$$

Calvo pricing

- Calvo restricts firms from resetting prices freely ...
- Each firm can only reset its price with probability $1-\theta$ ("Calvo fairy")
    - iid over time and across firms
    - average price duration is $(1-\theta)\sum_{s\geq 0} s\theta^{s-1} = \frac{1}{1-\theta}$

Q: Intuitively, how will Calvo pricing change our four steps?

- **Q:** which price does a resetting firm choose?
- Objective: identical for all firms resetting at date t

$$p_t^* \equiv \arg\max_p \mathbb{E}_t \sum_{k\geq 0} \underbrace{\theta^k}_{\Pr(\text{no adj. until } t+k)} \cdot \underbrace{\beta^k \frac{u'(C_{t+k})}{u'(C_t)}}_{\text{stochastic discount factor}} \cdot \underbrace{\frac{D_{t+k}(p)}{P_{t+k}}}_{\text{real dividends}}$$

- First order condition:

$$\mathbb{E}_t \sum_{k\geq 0} \theta^k \cdot \beta^k \frac{u'(C_{t+k})}{u'(C_t)} \cdot \frac{D'_{t+k}(p_t^*)}{P_{t+k}} = 0$$

$$\Leftrightarrow \qquad \mathbb{E}_t \sum_{k\geq 0} (\theta\beta)^k \cdot u'(C_{t+k}) \cdot P_{t+k}^{\epsilon-1} C_{t+k} \left(p_t^* - \frac{\epsilon}{\epsilon - 1} MC_{t+k}(p_t^*) \right) = 0$$

$$\Leftrightarrow \qquad p_t^* = \frac{\mathbb{E}_t \left[\sum_{k\geq 0} (\theta\beta)^k \cdot u'(C_{t+k}) C_{t+k} P_{t+k}^{\epsilon-1} \cdot \frac{\epsilon}{\epsilon-1} MC_{t+k}(p_t^*) \right]}{\mathbb{E}_t \left[\sum_{k\geq 0} (\theta\beta)^k \cdot u'(C_{t+k}) C_{t+k} P_{t+k}^{\epsilon-1} \right]}$$

Part 1: Price as markup over weighted average MC

$$p_t^* = \frac{\mathbb{E}_t\left[\sum_{k\geq 0}(\theta\beta)^k \cdot u'(C_{t+k})\, C_{t+k} P_{t+k}^{\epsilon-1} \cdot \frac{\epsilon}{\epsilon-1} MC_{t+k}(p_t^*)\right]}{\mathbb{E}_t\left[\sum_{k\geq 0}(\theta\beta)^k \cdot u'(C_{t+k})\, C_{t+k} P_{t+k}^{\epsilon-1}\right]} \tag{3}$$

- Optimal price p_t^* is markup over **weighted average future** marginal cost
  - generalizes the flexible-price price $p_t^* = \frac{\epsilon}{\epsilon-1} MC_t(p_t^*)$
- (3) is not a closed-form solution since $MC_{t+k}(p_t^*)$ depends on p_t^* (see (1))
- Just like before, we can write

$$\frac{\epsilon}{\epsilon-1} MC_t(p) = \left(\frac{p}{P_t}\right)^{-\frac{\epsilon\alpha}{1-\alpha}} \cdot P_t \cdot \omega_t$$

Part 1: Getting to an explicit solution for p_t^*

- That gives us our explicit solution:

$$\left(\frac{p_t^*}{P_t}\right)^{1+\frac{\epsilon\alpha}{1-\alpha}} = \frac{\mathbb{E}_t\left[\sum_{k\geq 0}(\theta\beta)^k \cdot u'\left(C_{t+k}\right)C_{t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\epsilon/(1-\alpha)} \cdot \omega_{t+k}\right]}{\mathbb{E}_t\left[\sum_{k\geq 0}(\theta\beta)^k \cdot u'\left(C_{t+k}\right)C_{t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\epsilon-1}\right]}$$

- We can write numerator and denominator recursively …

- We write

$$\left(\frac{p_t^*}{P_t}\right)^{1+\frac{\epsilon\alpha}{1-\alpha}} = \frac{F_t}{H_t} \tag{4}$$

where we define F_t and H_t as

$$\begin{aligned}
F_t &\equiv u'\left(C_t\right)C_t \cdot \omega_t + \theta\beta\mathbb{E}_t\left[\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon/(1-\alpha)} F_{t+1}\right] \\
&= u'\left(C_t\right)C_t \cdot \omega_t + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon/(1-\alpha)} F_{t+1}\right] && (5) \\
H_t &\equiv u'\left(C_t\right)C_t + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon-1} H_{t+1}\right] && (6)
\end{aligned}$$

- This concludes part 1, our description of **optimal price setting** p_t^*.
- Next: part 2, the law of motion of P_t ....

Part 2: Law of motion of P_t

- Mass $1-\theta$ of firms reset each period, all choosing same p_t^*. Thus:

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)\left(p_t^*\right)^{1-\epsilon} \tag{7}$$

- Rewrite (7) in terms of inflation

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\Pi_t \frac{p_t^*}{P_t}\right)^{1-\epsilon} \tag{8}$$

    - here $\frac{p_t^*}{P_{t-1}} = \Pi_t \frac{p_t^*}{P_t}$ is 'reset price inflation' (useful concept)

- Solving for Π_t we find

$$\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} = \left(\frac{p_t^*}{P_t}\right)^{1-\epsilon} \tag{9}$$

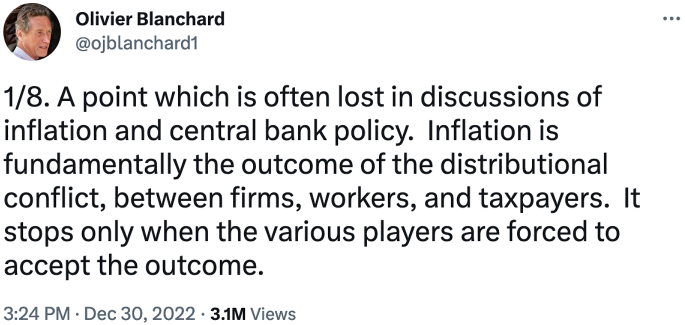
- This allows us to go from p_t^* to inflation rates Π_t !

Part 3: Equation for ω_t

- Combine this with (4) to link inflation and real marginal cost ω_t:

$$\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} = \left(\frac{p_t^*}{P_t}\right)^{1-\epsilon} = \left(\frac{F_t}{H_t}\right)^{\frac{1-\epsilon}{1+\frac{\epsilon\alpha}{1-\alpha}}}$$

where recall that ω_t determines F_t. This is the **Calvo Phillips Curve**. Intuition:



- Due to Calvo pricing: firms reset at different $t \rightarrow$ have different prices!
- This leads to **price dispersion** which misallocates labor! Recall:

$$n_t(p_i) = \left(\frac{p_i}{P_t}\right)^{-\epsilon/(1-\alpha)} A_t^{-1/(1-\alpha)} C_t^{1/(1-\alpha)}$$

and so aggregate labor demand is

$$N_t = \int n_t(p_i) di = \int \left(\frac{p_i}{P_t}\right)^{-\epsilon/(1-\alpha)} di \cdot A_t^{-1/(1-\alpha)} C_t^{1/(1-\alpha)}$$

Rearranging, we find

$$C_t = Y_t = \frac{A_t}{\Delta_t} N_t^{1-\alpha} \tag{10}$$

where $\triangle_t \equiv \left[\int_{i=0}^{1} \left(\frac{p_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di\right]^{1-\alpha}$ measures the effect of **price dispersion** on misallocation

- The price dispersion term $\triangle_t \equiv \left[\int_{i=0}^{1} \left(\frac{p_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di\right]^{1-\alpha}$ depends on past inflation...

$$
\begin{aligned}
\triangle_t^{\frac{1}{1-\alpha}} &= \theta \int_{i=0}^{1} \left(\frac{p_{i,t-1}^{nr}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di + (1-\theta)\left(\frac{p_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} \\
&= \theta \int_{i=0}^{1} \left(\frac{p_{i,t-1}^{nr}}{P_{t-1}} \frac{P_{t-1}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di + (1-\theta)\left(\frac{p_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} \\
&= \theta \Pi_t^{\frac{\epsilon}{1-\alpha}} \triangle_{t-1}^{\frac{1}{1-\alpha}} + (1-\theta)\left(\frac{p_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} \\
&= \theta \Pi_t^{\frac{\epsilon}{1-\alpha}} \triangle_{t-1}^{\frac{1}{1-\alpha}} + (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{(1-\alpha)(\epsilon-1)}} \qquad (11)
\end{aligned}
$$

Taking stock: We arrived at our 5 equations

1. **Euler Equation**

$$u'\left(C_t\right) = \beta\mathbb{E}\left[u'\left(C_{t+1}\right)\frac{1+i_t}{\Pi_{t+1}}\right] \tag{12}$$

2. **Labor supply**

$$\psi C_t^{\sigma} N_t^{\phi} = \frac{W_t}{P_t} \tag{13}$$

3. **Taylor rule:** $i_t = g\left(\Pi_t; C_t; \rho_t\right)$ for some $g(\cdot)$
4. Equation for real wage (+ (5),(6))

$$\omega_t = \frac{\epsilon}{\epsilon-1}\frac{1+\tau^W}{1-\alpha}\frac{W_t}{P_t}A_t^{-\frac{1}{1-\alpha}}C_t^{\frac{\alpha}{1-\alpha}} \qquad \frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} = \left(\frac{F_t}{H_t}\right)^{\frac{1-\epsilon}{1+\frac{\epsilon\alpha}{1-\alpha}}} \tag{14}$$

5. Aggregate production (+ (11))

$$C_t = Y_t = \frac{A_t}{\Delta_t}N_t^{1-\alpha}$$

Zero inflation steady state ...

- Turns out that the model can't generate very high inflation rates:
  - if reset price is ∞, $p_t^* = \infty$, then

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} \quad \Rightarrow \quad \Pi_t = \overline{\Pi} \equiv \theta^{-\frac{1}{\epsilon-1}}$$

    typical calibration: $\theta = 0.85$, $\epsilon = 6 \Rightarrow \overline{\Pi} = 1.03$ (!) that's very low ...

- So typically we focus on zero steady state inflation ...

$$1 + i = \beta^{-1}$$

$$C = Y = N^{1-\alpha}$$

$$\omega = \Pi = F = H = \Delta = 1$$

$$\psi C^{\sigma} N^{\phi} = \frac{\epsilon - 1}{\epsilon} \frac{1 - \alpha}{1 + \tau^{w}} \frac{Y}{N}$$

- Note: infinite number of steady states indexed by Π. Taylor rule picks $\Pi = 1$.

The log-linearized New-Keynesian model

Solving the model: loglinearization

- Log-linearizing around zero inflation steady state ...
- **Euler Equation** (12):

$$c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right) \tag{15}$$

- Price dispersion from (11): $\widehat{\Delta_t} = \theta\widehat{\Delta_{t-1}} = 0$
  - so price dispersion is a *second* order term!
- Marginal costs (14) + (13) gives

$$\widehat{\omega}_t = \left(\frac{\alpha+\phi}{1-\alpha}\right)y_t + \sigma c_t - \left(1 + \frac{\alpha+\phi}{1-\alpha}\right)a_t \tag{16}$$

- Next we linearize the pricing equations

Loglinear solution to price equations

Loglinearize (9)

$$\pi_t = \frac{(1-\theta)}{\theta}\left(p_t^* - p_t\right) = \frac{(1-\theta)}{\theta}\frac{1}{1+\frac{\epsilon\alpha}{1-\alpha}}\left(f_t - h_t\right) \tag{17}$$

where

$$\begin{aligned} f_t &= (1-\theta\beta)\left((1-\sigma)\,c_t + \widehat{\omega}_t\right) + \theta\beta\mathbb{E}_t\left[\frac{\epsilon}{1-\alpha}\pi_{t+1} + f_{t+1}\right] \\ h_t &= (1-\theta\beta)\,(1-\sigma)\,c_t + \theta\beta\mathbb{E}_t\left[(\epsilon-1)\,\pi_{t+1} + h_{t+1}\right] \end{aligned}$$

and so

$$\begin{aligned} f_t - h_t &= (1-\theta\beta)\,\widehat{\omega}_t + \theta\beta\mathbb{E}_t\left[\left(1+\frac{\epsilon\alpha}{1-\alpha}\right)\pi_{t+1} + f_{t+1} - h_{t+1}\right] \\ &= (1-\theta\beta)\,\widehat{\omega}_t + \theta\beta\mathbb{E}_t\left[\left(\frac{(1-\theta)}{\theta}+1\right)\left(f_{t+1} - h_{t+1}\right)\right] \\ &= (1-\theta\beta)\,\widehat{\omega}_t + \beta\mathbb{E}_t\left[f_{t+1} - h_{t+1}\right] \end{aligned}$$

where he have used (17) once. Use again to find

$$\pi_t = \frac{(1-\theta)\,(1-\theta\beta)}{\theta}\frac{1}{1+\frac{\epsilon\alpha}{1-\alpha}}\widehat{\omega}_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right] \tag{18}$$

Reduction using the natural allocation

- Summing up (15), (16), (18) we have found:

$$c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right)$$

$$\widehat{\omega}_t = \left(\frac{\alpha+\phi}{1-\alpha}\right) y_t + \sigma c_t - \left(1 + \frac{\alpha+\phi}{1-\alpha}\right) a_t$$

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{1}{1+\frac{\epsilon\alpha}{1-\alpha}} \widehat{\omega}_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right]$$

- Write often in terms of "gaps" from flexible price allocation
  - flexible prices $\Rightarrow \widehat{\omega}_t = 0$ and so we must have with $c_t = y_t$

$$y_t^n = \frac{1+\phi}{\alpha+\phi+\sigma(1-\alpha)} a_t \qquad \text{output gap: } x_t \equiv y_t - y_t^n$$

  - **natural (real) interest rate:**

$$r_t^n = \rho + \sigma\left(y_{t+1}^n - y_t^n\right)$$

1. **Euler equation** in terms of output gap

$$x_t = \mathbb{E}_t\left[x_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^n\right) \tag{DIS}$$

2. **Phillips curve** in terms of output gap

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right] \tag{NKPC}$$

   where the slope parameter is

$$\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{1}{1+\frac{\epsilon\alpha}{1-\alpha}} \left(\frac{\alpha+\phi}{1-\alpha} + \sigma\right)$$

3. Monetary policy rule, e.g. **Taylor rule**

$$i_t = \rho_t + \phi_\pi \pi_t + \phi_x x_t \tag{MP}$$

- (DIS), (NKPC) and (MP) are the canonical ’3-equation New Keynesian model’

Forward-looking properties

- We can iterate (NKPC) to find

$$\pi_t = \kappa \mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k x_{t+k} \right] \tag{19}$$

  - inflation is forward looking, depends on future marginal costs / output gaps

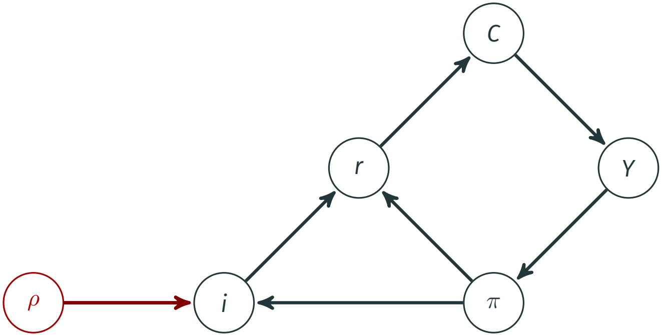
- Similarly, iterating (DIS) **and** assuming $\lim_{k\to 0} \mathbb{E}_t [x_{t+k}] = 0$

$$x_t = -\sigma^{-1} \mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(r_{t+k} - r_{t+k}^n \right) \right] \tag{20}$$

  - output gap determined by present and future interest rates. Why?
  - ex: monetary policy lowers rates now, boosts output via (20), inflation via (19)

- Realistic?

Monetary policy with Taylor rule



What's next

- Two more classes plus a review session
  - See canvas for past exam examples (focus on 2018 & 2019)
- We'll study positive + normative aspects of NK model, e.g.
  - effects of monetary policy, demand & TFP shocks
  - effects of government purchases
  - optimal monetary policy