Econ 212a: Business Cycles
Lecture 8
Positive Analysis of the New Keynesian Model

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Recap

- Last class we introduced Calvo pricing frictions into our monetary model
- This gave us the (log-linearized) **three-equation NK model**

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- Last class we introduced Calvo pricing frictions into our monetary model
- This gave us the (log-linearized) **three-equation NK model**
    - Four equations if you don't write it in "gap" form (sometimes better)
- Today, we do the "positive analysis" of the model
    - solve it for various shocks: monetary, TFP, demand, cost-push
- Bonus:
    - discuss critiques of the model
    - beyond the NK model: what's next in monetary economics?

Recap

Last class

- Two main equations:

$$x_t = \mathbb{E}_t\left[x_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^n\right) \tag{DIS}$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right] \tag{NKPC}$$

  - $x_t = y_t - y_t^n$ is the output gap, with $y_t^n = \frac{1+\phi}{\alpha+\phi+\sigma(1-\alpha)} a_t$
  - $r_t^n = \rho + \sigma\left(y_{t+1}^n - y_t^n\right)$ is the RBC real interest rate (or 'natural rate')

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- What do these equations imply for the dynamic effect of macro shocks?

- Phillips Curve comes from aggregated price-setting problem

$$\pi_t = \lambda \widehat{\omega}_t + \beta \mathbb{E}_t \left[\pi_{t+1}\right]$$

where rmc is $\widehat{\omega}_t = \widehat{\omega}_t - \widehat{\omega}_t{}^n = \left(\frac{\alpha+\phi}{1-\alpha} + \sigma\right) x_t$, so slope of NKPC is

$$\kappa \equiv \underbrace{\frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{1}{1+\frac{\epsilon\alpha}{1-\alpha}}}_{\text{Sens. of inflation to rmc } (\lambda)} \underbrace{\left(\frac{\alpha+\phi}{1-\alpha} + \sigma\right)}_{\text{Sens. of rmc to gap}} \tag{1}$$

Real rigidities and the NK model

- Phillips Curve comes from aggregated price-setting problem

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- Intuition: inflation is less sensitive to output ($\kappa \downarrow$) when there is:
    1. More nominal rigidity ($\theta \uparrow$)
    2. More *real* rigidity ($\epsilon \uparrow$, $\alpha \uparrow$). If marginal cost is more sensitive to own price, it is more costly to deviate from everyone else's price.
    3. Less sensitivity of rmc to output: more elastic labor supply $\phi \downarrow$, less curvature over consumption $\sigma \downarrow$, less decreasing returns $\alpha \downarrow$ (so overall α ambiguous)

Reminder: forward-looking properties

- We can iterate (NKPC) to find

$$\pi_t = \kappa \mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k x_{t+k} \right] \tag{2}$$

  - Inflation is forward looking (though prices aren't), depends on future marginal costs and therefore future output gaps.
- Similarly, iterating (DIS) **and** assuming $\lim_{k \to \infty} \mathbb{E}_t [x_{t+k}] = 0$

$$x_t = -\sigma^{-1} \mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(r_{t+k} - r_{t+k}^n \right) \right] \tag{3}$$

  - level of output determined by intertemp. subst. wrt present and future rates
- Monetary transmission mechanism: lower r_t now or in future $\to$ boost output via (3) and inflation via (2)

Solving the model

- Plug in (MP), after some manipulation, obtain [check!]

$$x_t(\sigma + \phi_x + \kappa\phi_\pi) - (\sigma(1+\beta) + \beta\phi_x + \kappa)\mathbb{E}_t[x_{t+1}] + \beta\sigma\mathbb{E}_t[x_{t+2}] = -(\rho_t - r_t^n) + \beta\mathbb{E}_t\left[\rho_{t+1} - r_{t+1}^n\right]$$

- Write this in the sequence space:

$$\mathbf{F}^2\left(A\mathbf{L}^2 + B\mathbf{L} + C\mathbf{I}\right)\mathbf{x} = -(\mathbf{I} - \beta\mathbf{F})(\boldsymbol{\rho} - \mathbf{r}^n) \tag{4}$$

- Plug in (MP), after some manipulation, obtain [check!]

$$\begin{aligned} x_t\left(\sigma+\phi_x+\kappa\phi_\pi\right)-\left(\sigma\left(1+\beta\right)+\beta\phi_x+\kappa\right)\mathbb{E}_t\left[x_{t+1}\right]+\beta\sigma\mathbb{E}_t\left[x_{t+2}\right] \\ =-\left(\rho_t-r_t^n\right)+\beta\mathbb{E}_t\left[\rho_{t+1}-r_{t+1}^n\right] \end{aligned}$$

- Write this in the sequence space:

$$\mathbf{F}^2\left(A\mathbf{L}^2+B\mathbf{L}+C\mathbf{I}\right)\mathbf{x}=-\left(\mathbf{I}-\beta\mathbf{F}\right)\left(\boldsymbol{\rho}-\mathbf{r}^n\right) \tag{4}$$

- Let λ_1,λ_2 be the (possibly complex) roots of $P\left(X\right)=AX^2+BX+C$
- Rewrite (4) as

$$A\mathbf{F}^2\left(\mathbf{L}-\lambda_1\mathbf{I}\right)\left(\mathbf{L}-\lambda_2\mathbf{I}\right)\mathbf{x}=A\left(\mathbf{I}-\lambda_1\mathbf{F}\right)\left(\mathbf{I}-\lambda_2\mathbf{F}\right)\mathbf{x}=-\left(\mathbf{I}-\beta\mathbf{F}\right)\left(\boldsymbol{\rho}-\mathbf{r}^n\right)$$

- If $|\lambda_1|<1$ and $|\lambda_2|<1$, this is invertible and the unique solution is:

$$\mathbf{x}=-\frac{1}{A}\left(\mathbf{I}-\lambda_1\mathbf{F}\right)^{-1}\left(\mathbf{I}-\lambda_2\mathbf{F}\right)^{-1}\left(\mathbf{I}-\beta\mathbf{F}\right)\left(\boldsymbol{\rho}-\mathbf{r}^n\right) \tag{5}$$

- So let us consider the roots of:

$$P(X) = (\sigma + \phi_x + \kappa\phi_\pi) X^2 - (\sigma(1+\beta) + \beta\phi_x + \kappa) X + \beta\sigma$$

- We see that $P(0) = \beta\sigma > 0$ and

$$P(1) = (\sigma + \phi_x + \kappa\phi_\pi) - (\sigma(1+\beta) + \beta\phi_x + \kappa) + \beta\sigma$$

- Assume that $P(1) > 0$, ie:

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_x > 0 \tag{6}$$

then, since $\arg\min P = \frac{\sigma(1+\beta)+\beta\phi_x+\kappa}{2(\sigma+\phi_x+\kappa\phi_\pi)} < 1$, we indeed have $|\lambda_1| < 1$ and $|\lambda_2| < 1$.

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- On other hand, if $P(1) < 0$ then we have $\lambda_1 < 1$, $\lambda_2 > 1$ and indeterminacy
- This is the **Taylor principle**: need sufficient response to either inflation and/or the output gap to rule out sunspots (eg $\phi_\pi > 1$ is enough)
  - cf similar principle in Lecture 5 with flexible prices

- Given Taylor principle, apply partial fraction decomposition to (5), find:

$$\mathbf{x} = -\frac{1}{\sigma + \phi_x + \kappa\phi_\pi}\left(\alpha \sum_{k\geq 0} \lambda_1^k \mathbf{F}^k + (1-\alpha)\sum_{k\geq 0}\lambda_2^k \mathbf{F}^k\right)(\boldsymbol{\rho} - \mathbf{r}^n) \tag{7}$$

where $\alpha \in (0, 1)$. So, similar to Lecture 5:

- $\rho_{t+k} > r^n_{t+k}$ is 'tight' monetary policy, implying $x_t < 0$
- Nominal interest rate usually 'low' in this situation
- $\rho_{t+k} < r^n_{t+k}$ is 'loose' monetary policy, implying $x_t > 0$

Consequences

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  - $\rho_{t+k} < r^n_{t+k}$ is 'loose' monetary policy, implying $x_t > 0$

- When setting $\rho_{t+k} = r^n_{t+k}$ at all k, obtain $x_t = 0$ at all t; then also $\pi_t = 0$
  - Later: when the flex price is also first best, $\pi_t = x_t = 0$ achieves highest welfare

  - So can already anticipate the **divine coincidence** result: no conflict between achieving zero inflation and zero output gap

Propagation of shocks in the NK model

Propagation of shocks

- Next we hit the model with several shocks to see how it responds
- Monetary, TFP, demand, government spending, cost-push shocks

Monetary policy shock: basic idea (iid case)

- Suppose

$$\rho_t = r_t^n + \epsilon_t^m$$

where ϵ_t^m is iid mean-0 monetary policy shock. Since

$$\mathbb{E}_t\left[\rho_{t+k} - r_{t+k}^n\right] = \mathbb{E}_t\left[\epsilon_{t+k}^m\right] = 0 \quad k > 0$$

we know the solution features $\mathbb{E}_t\left[x_{t+1}\right] = \mathbb{E}_t\left[\pi_{t+1}\right] = 0$

- Using (DIS)-(NKPC), we find

$$x_t = -\sigma^{-1}\left(i_t - r_t^n\right) \qquad \pi_t = \kappa x_t \tag{8}$$

  - (can also solve for x_t and π_t as a function of ϵ_t^m)
- (8) is very intuitive:
  - Tightening shock ($\epsilon_t^m \uparrow$) increases the *real* interest rate (=i_t here)
  - This lower aggregate demand via intertemporal substitution
  - This lowers marginal costs, and creates deflation

Monetary policy in persistent case

- More generally, suppose persistent shock $0 < \rho_\nu < 1$

$$\rho_t = r_t^n + \nu_t$$

$$\nu_t = \rho_\nu \nu_{t-1} + \epsilon_t^m$$

Since now

$$\mathbb{E}_t \left[\rho_{t+k} - r_{t+k}^n\right] = \mathbb{E}_t \left[\nu_{t+k}\right] = \rho_\nu^k \nu_t$$

  - The solution has the form $x_t = \Psi_x \nu_t$ and $\pi_t = \Psi_\pi \nu_t$
  - Hence also $\mathbb{E}_t \left[x_{t+1}\right] = \rho_\nu \Psi_x \nu_t$ and $\mathbb{E}_t \left[\pi_{t+1}\right] = \rho_\nu \Psi_\pi \nu_t$

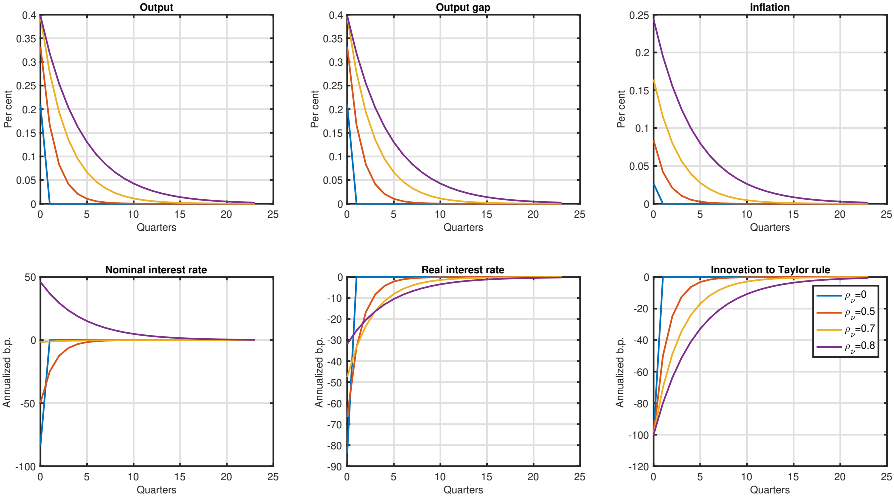
- Using (DIS)-(NKPC), check that

$$\pi_t = -\kappa \Lambda \nu_t \qquad \text{and} \qquad x_t = -\left(1 - \beta\rho_\nu\right) \Lambda \nu_t$$

where $\Lambda = \frac{1}{\kappa(\phi_\pi - \rho_\nu) + (1 - \beta\rho_\nu)(\phi_x + (1 - \rho_\nu)\sigma)} > 0$.

  - Same intuition, nominal rate may fall instead of rise

Monetary policy shocks, varying persistence ρ_ν



Solving the model: technology shocks

- Assume $\varepsilon_t^m = 0$, and turn on technology shocks

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

These create changes in natural rate: recall

$$\begin{aligned} r_t^n &= \rho + \sigma \frac{1+\phi}{\alpha + \phi + \sigma (1-\alpha)} \mathbb{E}_t \left[a_{t+1} - a_t \right] \\ &= \rho - \sigma \frac{1+\phi}{\alpha + \phi + \sigma (1-\alpha)} (1-\rho_a) a_t \end{aligned}$$

  - Transitory $a_t \uparrow$ raises desired savings, leads to $r_t^n \downarrow$ (no capital!)

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  - Transitory $a_t \uparrow$ raises desired savings, leads to $r_t^n \downarrow$ (no capital!)
- Suppose first that $\rho_t = r_t^n$: then we know $x_t = \pi_t = 0$
  - Monetary policy 'tracks' the natural allocation
  - Hence $y_t = y_t^n$: effect of tech shocks *same as under flexible prices*
  - (Careful: output $\neq$ output gap)

Solving the Model: Technology Shocks

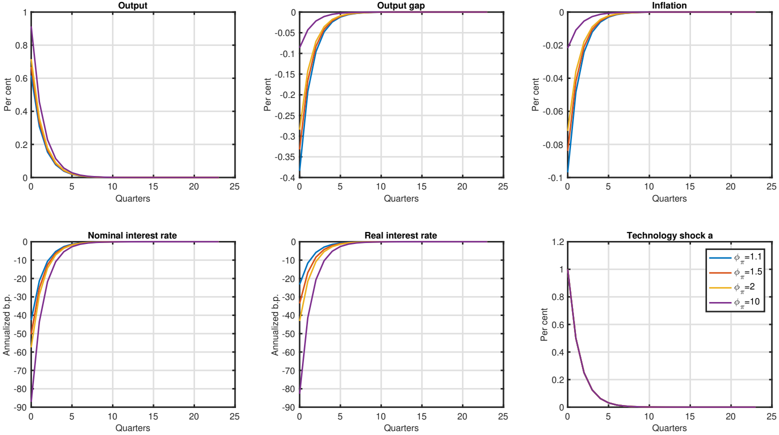
- Suppose now $\rho_t = \rho$
  - Central bank does not respond to direct effect of shock
  - But responds indirectly through effect of shock on π and x

- Then $\rho_t - r_t^n \propto a_t$
  - Positive technology shock leads mp to be too tight
  - Equivalently, we say it does not 'accommodate' the shock
  - This leads creates some deflation and a negative output gap
  - Output rises in general, but employment ambiguous:

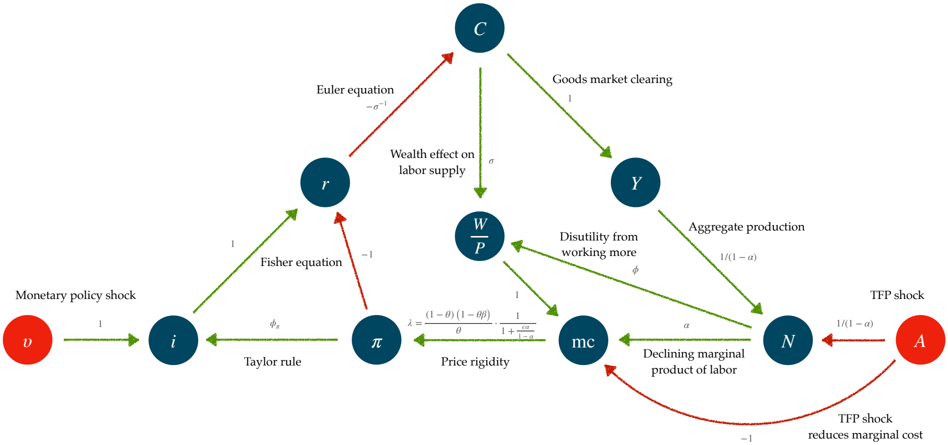
- Consider for example the case $\sigma = 1$, so $n_t^n = 0$ and $y_t^n = a_t$
  - Then clearly $n_t = y_t - a_t = y_t - y_t^n = x_t < 0$
  - Contractionary technology shock (for employment)
  - As in Gali (1999) and Basu, Fernald, Kimball (2006)

Technology Shocks, varying mp responsiveness ϕ_π

- Increasing responsiveness ϕ_π gets allocation closer to flex price



Taking stock: drawing the NK model



The role of the natural rate

- Lesson: effects of all shocks in model depend on mon. pol. response
  - Specifically, extent to which it accommodates the shock
  - A key indicator of stance of mp is natural rate of interest r_t^n
  - Many shocks have this simple reduced form

The role of the natural rate

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  - Many shocks have this simple reduced form
- Example: 'impatience shocks'
  - Assume household preferences are

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \xi_t \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\phi}}{1+\phi} \right) \right\}$$

  - Increase in ξ_t / ξ_{t+1} raises MUC, lowers desired savings at t vs $t+1$
  - Assume follows an AR(1) in logs: $z_t = \log \xi_t = \rho_z z_{t-1} + \epsilon_t^z$
  - Euler equation:

$$\xi_t C_t^{-\sigma} = \beta R_t \mathbb{E}_t \left[\xi_{t+1} C_{t+1}^{-\sigma} \right] \tag{9}$$

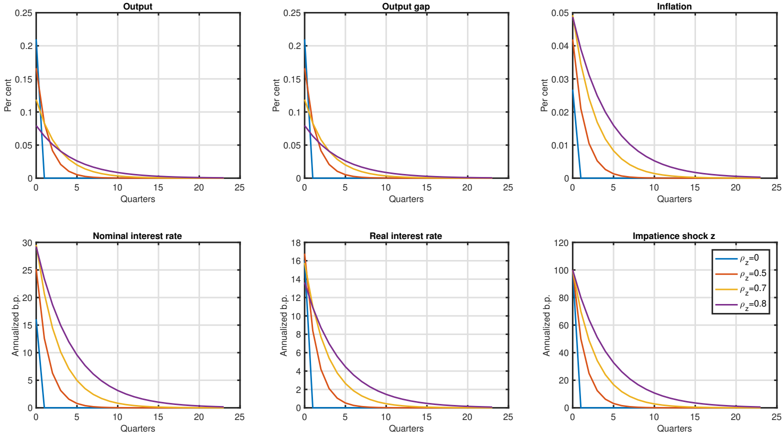
Impatience shock

- Equation (9) in loglinear form

$$c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho - (1-\rho_z)\, z_t\right)$$

- Natural rate is $r_t^n = \rho + (1-\rho_z)\, z_t$
- Since ξ_t does not affect MRS of C and N, it does not influence y_t^n
- $z_t \uparrow$ shock is a pure positive 'demand' shock:
  - It has no effect on the natural level of output
  - If monetary policy does not tighten by raising r, leads to an inflationary boom
- This transmission from desired consumption to output in GE is sometimes called the *aggregate demand channel*

Impatience shocks, varying persistence ρ_z



Fiscal policy: government spending

- We now consider the effects of government spending shocks
- Assume a positive spending level $G_t > 0$. New resource constraint:

$$C_t + G_t = Y_t$$

- Euler equation and Phillips curve in terms of rmcs are unchanged:

$$c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \bar{r}\right)$$
$$\pi_t = \lambda\widehat{\omega}_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right]$$

- Let tilde variables denote *level* deviations from the *zero inflation s.s.*
  - $\widetilde{g}_t \equiv \frac{dG_t}{Y} = \mathcal{G}\frac{dG_t}{G}$ where $\mathcal{G} \equiv \frac{G}{Y}$
  - $\widetilde{c}_t \equiv \frac{dC_t}{Y} = (1-\mathcal{G})\,\frac{dC_t}{C}$
  - This implies $\widetilde{c}_t + \widetilde{g}_t = \widetilde{y}_t$
- Marginal costs $\widehat{\omega}_t$ now depend on $\widetilde{g}_t$. Why?

Relating spending to marginal costs

- Real marginal costs:

$$\omega_t \propto \frac{1}{A_t\,(1-\alpha)\,N_t^{-\alpha}}\frac{W_t}{P_t} \propto \frac{1}{A_t}\,(N_t)^{\alpha+\phi}\,C_t^{\sigma} \propto \frac{1}{A_t}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{\alpha+\phi}{1-\alpha}} C_t^{\sigma}$$

- Hence, assuming A_t is constant

$$\begin{aligned}\widehat{\omega}_t &= \left(\frac{\alpha+\phi}{1-\alpha}\right)\frac{dY_t}{Y} + \sigma\frac{dC_t}{C} + \mathrm{cst} \\ &= \left(\frac{\alpha+\phi}{1-\alpha}\right)(\widetilde{c}_t + \widetilde{g}_t) + \frac{\sigma}{(1-\mathcal{G})}\widetilde{c}_t + \mathrm{cst}\end{aligned}$$

- Conditional on consumption, $G_t \uparrow$ pushes costs up

New Keynesian model with government spending

- Neoclassical multiplier Γ solves $\widehat{\omega}_t^n = 0$

$$\widetilde{y}_t^n = \frac{\frac{\sigma}{(1-\mathcal{G})}}{\frac{\alpha+\phi}{1-\alpha} + \frac{\sigma}{(1-\mathcal{G})}} \widetilde{g}_t \equiv \Gamma \widetilde{g}_t$$

  - So $\Gamma \in (0, 1)$, reflecting wealth effect [Baxter King 1993]

- Can rewrite IS and PC as

$$\begin{aligned} \widetilde{c}_t &= \mathbb{E}_t \left[\widetilde{c_{t+1}}\right] - \widetilde{\sigma} \left(i_t - \mathbb{E}_t \left[\pi_{t+1}\right] - \rho\right) \\ \pi_t &= \kappa \left(\widetilde{y}_t - \Gamma \widetilde{g}_t\right) + \beta \mathbb{E}_t \left[\pi_{t+1}\right] \\ &= \kappa \left(\widetilde{c}_t + (1 - \Gamma) \widetilde{g}_t\right) + \beta \mathbb{E}_t \left[\pi_{t+1}\right] \end{aligned}$$

  where $\widetilde{\sigma} \equiv \sigma^{-1} (1 - \mathcal{G})$ and $\kappa \equiv \lambda \left(\frac{\alpha+\phi}{1-\alpha} + \widetilde{\sigma}^{-1}\right)$

- Allows simple analysis of fiscal multipliers [Woodford 2011]

- Assume Taylor rule $i_t = \bar{r} + \phi \pi_t$ and AR(1) fiscal shock

$$\widetilde{g}_t = \rho \widetilde{g_{t-1}} + \epsilon_t$$

- Conjecture $\mathbb{E}_t \left[\widetilde{c_{t+1}} \right] = \rho \widetilde{c}_t$ and $\mathbb{E}_t \left[\pi_{t+1} \right] = \rho \pi_t$. Then

$$\begin{aligned} (1-\rho)\, \widetilde{c}_t \qquad &= -\widetilde{\sigma} \left(\phi - \rho \right) \pi_t \\ (1-\beta\rho)\, \pi_t \quad &= \kappa \left(\widetilde{c}_t + (1-\Gamma)\, \widetilde{g}_t \right) \end{aligned}$$

- Solve for $\widetilde{c}_t$:

$$\widetilde{c}_t = \frac{-\left(1-\Gamma\right)}{\frac{(1-\beta\rho)(1-\rho)}{\kappa\widetilde{\sigma}(\phi-\rho)} + 1} \widetilde{g}_t$$

New Keynesian model with government spending

- Solve for output:

$$\widetilde{y}_t = \frac{1-\rho+\Gamma\frac{\kappa\widetilde{\sigma}(\phi-\rho)}{1-\beta\rho}}{1-\rho+\frac{\kappa\widetilde{\sigma}(\phi-\rho)}{1-\beta\rho}}\widetilde{g}_t$$

  and real rate

$$r_t = \bar{r} + (\phi-\rho)\,\pi_t$$

- **Conclusion**: the fiscal multiplier is
  1. Between Γ and 1 provided $\frac{\kappa\widetilde{\sigma}(\phi-\rho)}{1-\beta\rho} > 0$
     - Real rate increases: monetary policy tightens
  2. Exactly equal to 1 if $\phi = \rho$, in which case r is *constant*
  3. Larger than 1 when $\phi < \rho$ (example: zero lower bound!)
     - Real rate declines: monetary policy accommodates
     - Approaches infinity as $\phi \to \rho - (\kappa\widetilde{\sigma})^{-1}(1-\beta\rho)(1-\rho)$

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- One idea: as “cost-push shock” υ_t to NKPC

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$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \left[\pi_{t+1}\right] + \upsilon_t$$

- This shock **breaks divine coincidence!** E.g. assume υ_t is iid, then:

$$x_t = -\sigma^{-1} \left(\rho_t - \rho\right)$$
$$\pi_t = \kappa x_t + \upsilon_t$$

- Trade-off fighting inflation with creating a negative output gap x_t !
- How should we optimally resolve this? $\rightarrow$ need optimal policy analysis!

Taking stock

- We have a positive model to analyze:
  - Monetary policy
  - Fiscal policy
- Do well descriptively, with solid microfoundations
- Can be used to study welfare and optimal policy

Bonus slides 1: Critiques of the NK model

What could possibly be wrong here?

$$x_t = \mathbb{E}_t [x_{t+1}] - \sigma^{-1} (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \tag{DIS}$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t [\pi_{t+1}] \tag{NKPC}$$

$$i_t = \rho_t + \phi_\pi \pi_t + \phi_x x_t \tag{MP}$$

Q: What are the most unrealistic features of this model?

Euler equation: Inertia

- Euler equation predicts that consumption **growth** < 0 when MP eases since

$$\mathbb{E}_t[c_{t+1}] - c_t = \sigma^{-1}\left(i_t - \mathbb{E}_t \pi_{t+1} - \rho\right)$$

- Very different empirically: c, y have much more **inertia** ("hump shape")
  - Q: what do you think causes this inertial response?

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- One solution: habits! e.g. utility $u(c_t - \gamma\bar{c}_{t-1})$ where $\bar{c}_{t-1} = c_{t-1}$ is average consumption, but not internalized by agents (external habit). Then:

$$(c_t - \gamma c_{t-1}) - \mathbb{E}_t[c_{t+1} - \gamma c_t] = -\sigma^{-1}(i_t - \mathbb{E}_t\pi_{t+1} - \rho)$$

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- Other obvious issues with Euler equation:
  - investment, net exports, etc, should all be in x!

NKPC: Credible disinflation

- NKPC has very similar issue: Jay Powell is promising to lower the rate of inflation. What does that require for output?

$$x_t = \kappa^{-1}\left(\pi_t - \beta \mathbb{E}_t \pi_{t+1}\right)$$

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    - no backward looking terms!
- Solutions:
    1. Fraction of backward-looking firms (Gali and Gertler 1999).
    2. Indexation (Christiano, Eichenbaum, Evans 2005): for passive firms, prices automatically increase by amount of past inflation.
    3. Information frictions (Mankiw and Reis 2002, Angeletos Huo 2021).
- Another issue with NKPC: *wage* inflation very volatile

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- Yes. They're called "medium scale DSGE models"
  - see: Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007)

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    - Calvo prices and inflation indexation, Calvo wages, capital and investment adjustment costs, habit formation in consumption, variable capital utilization, fixed costs in production, strategic complementarity in price setting

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- **Seven** shocks:
  - TFP, risk premium shock, investment specific technology shocks, wage markup shocks, price markup shocks, government spending shock, monetary policy shock

Minnesota Critique: Chari et al. (2009)

- Chari, Kehoe, and McGrattan (2009) argue medium-scale NK models are not suitable for quantitative policy analysis.
- Main critique: Too many shocks and parameters!
  - e.g. wage + price markup shocks are basically inserting exogenous labor wedge into model.
  - not 'primitive, interpretable shocks', but critical to quantitative model fit (explain almost 90% of inflation).
  - also do not like indexation, generally think NK has not figured out inflation persistence.
- However, lots of new research on this topic supports NK predictions.

Bonus slides 2: Beyond NK ...

What's next in business cycle macro?

- **Financial frictions:** No banking sector, no housing.
  - → Bernanke Gertler Gilchrist (1999), Iacoviello (2005), Gertler Karadi (2011)
- **Household heterogeneity:** Model has tiny MPCs. Changes monetary and fiscal policy propagation.
  - → Werning (2015), Kaplan Moll Violante (2018), Auclert (2019), Auclert Rognlie Straub (2018, 2020)
- **Firm heterogeneity:** No investment, no lumpy investment, no balance sheet.
  - → Khan Thomas (2007) vs. Bachmann Caballero Engel (2013), Winberry (2021), Ottonello Winberry (2020)
- **Price-setting:** Calvo seems off! What if firms pay cost to change price?
  - → "Menu cost" models: Nakamura Steinsson (2010), Alvarez Le Bihan Lippi (2016), Auclert, Rognlie, Rigato, Straub (2023). Facts: Bils Klenow (2004), Nakamura Steinsson (2008), Klenow Malin (2010)

What's next in business cycle macro?

- **Labor market frictions:** No unemployment here, no wage rigidities
  - → Erceg Henderson Levin (2000), Gertler Trigari (2009), Blanchard Gali (2010), Christiano, Eichenbaum and Trabandt (2016)
- **Information:** All agents have perfect information here
  - → Mackowiak Wiederholt (2009, 2015), many Angeletos papers, e.g. with Lian (2018), or with Huo (2020)
- **Behavioral macro:** All agents are rational here
  - → Gabaix (2017, 2020), Farhi Werning (2019), Laibson Maxted Moll (2021)
- **Empirical macro:** Many moments hard to calibrate. Need well identified empirical work!
  - → Nakamura Steinsson (2018), Chodorow-Reich (2020)
- **Open economy:** Model has closed economy
  - Gali Monacelli (2005), Schmitt-Grohe-Uribe (2017), Auclert Souchier et al (2021)