Econ 212a: Business Cycles
Lecture 3
RBC Model, part II

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Recap

Last week, we ...

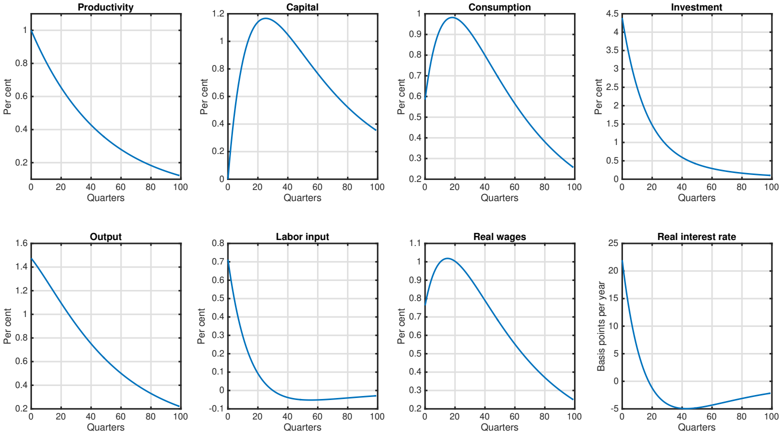
- introduced the RBC model, as: **NGM + labor + technology shocks**
- derived **first order conditions** for the solution to the planning problem
- imposed **balanced growth preferences** and de-trended the model
- calibrated the model to hit **steady state moments** ($A = 1$) and heuristics
- saw that **RBC business cycles** do look a bit like the ones in the data!

Today, we want to:

- understand the **mechanisms** underlying the RBC model
- **decentralize** the planner's solution as competitive equilibrium

Understanding RBC transmission

Impulse response to technological innovation (KR fig 10)



- Q: Why does labor increase? Why does consumption increase?

Interpreting the impulse response

For both c_t and N_t, we have income (aka wealth) & substitution effects

- $c_t \uparrow$ from **wealth effect**: economy more productive, so agent feels richer
  - mitigated initially by **substitution effect** from higher MPK
  - over time MPK falls, as $A \searrow$ and $K \nearrow$, which reduces substitution effect
  - this leads to the hump-shape in c_t

- $N_t \uparrow$ due to **substitution effect** (higher MPL *and* higher MPK):
  - mitigated by **wealth effect**
  - over time MPK falls, which reduces substitution, $N_t \searrow$ quickly

- Next: Explore role of: shock persistence ρ, Frisch elasticity, EIS...

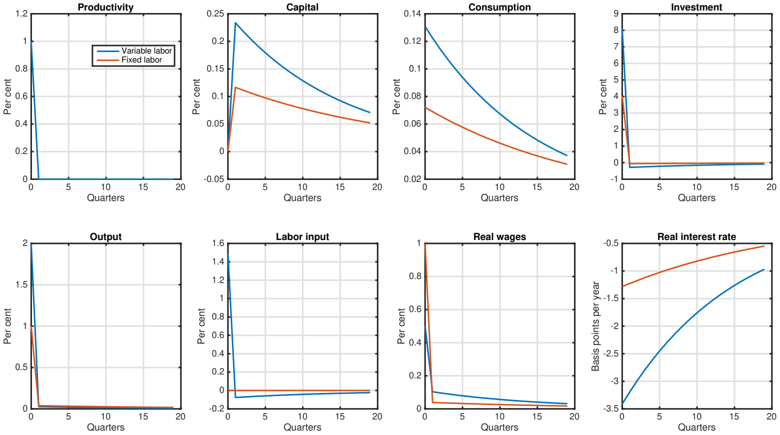
Role of shock persistence

Imagine shock was purely transitory ...

- Weaker wealth effect
  - $\rightarrow$ $c_t \uparrow$ by **less** than before
  - $\rightarrow$ $N_t \uparrow$ by **more** than before
- Also: I, K still increase since agents want to save
  - causes MPK and r to fall!

Need persistent shock to get sizable c_t response (and for r_t to rise)!

Purely transitory shock (blue line, KR figure 9)



But is it clear that c_t is even positive on impact ?

- The elasticity of intertemporal substitution (EIS) is ν^{-1} in $U = \frac{\left(Ce^{v(N)}\right)^{1-\nu}-1}{1-\nu}$
- Greater EIS strengthens the substitution effect on c_t
- With persistent shock (rising MPK), a higher EIS leads c_t to **fall** on impact

Need EIS that is not too large to even get positive c_t response!

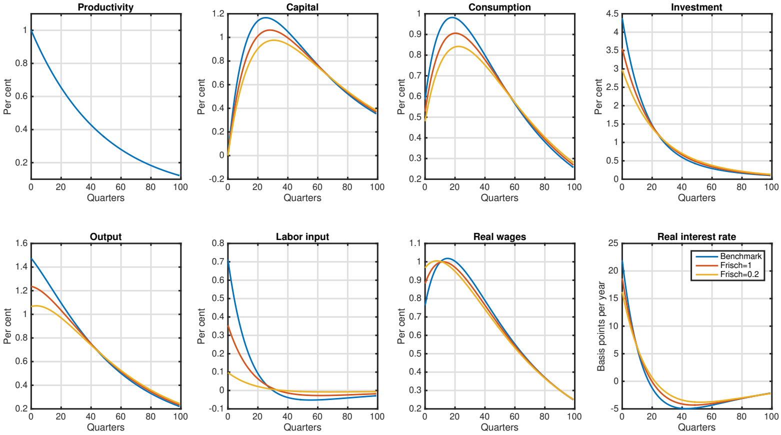
Role of Frisch elasticity

- Recall that model assumed Frisch $= \frac{1-0.2}{0.2} = 4$
  - that is … huge! If you earn 10% more for a year, do you work 40% more?
  - the Chetty approved™ range is 0.2–1
- What happens with lower elasticity of labor supply? Extend $U = \log C + \psi \log(1-N)$ to

$$U(C,N) = \log C + \frac{\psi}{1-\eta}\left((1-N)^{1-\eta} - 1\right) \Rightarrow \text{Frisch} = \frac{1}{\eta} \times \frac{1-N}{N}$$

- Note that when $\eta \to \infty$, we go to a model with fixed labor, so η matters!
- So how well does model do with realistic η?

Varying the Frisch elasticity



Summary

For the RBC model to produce a “realistic” business cycle, we need ...

- a **persistent TFP shock**
  - otherwise c_t responds too little and r falls!
- a **low EIS**
  - otherwise c_t falls!
- a **high Frisch elasticity**
  - otherwise hours respond too little!

Decentralizing the RBC model

The two welfare theorems

(1) Any competitive equilibrium (CE) is Pareto-efficient.

(2) Any Pareto-efficient allocation can be "decentralized" as a CE with transfers.

- Here CE assumes **no market failures**
  - e.g. no externalities, financial constraints, information frictions, ...
- Note: **transfers are key with many agents**. Imagine agents $j = 1, \ldots, J$
  - get Pareto-efficient allocation by max. $\sum \lambda^j U^j$ for some Pareto weights $\lambda^j \geq 0$
  - "as if" we had a representative agent with utility $U = \sum \lambda^j U^j$
  - the associated CE may require transfers to high λ^j agents

Next: we apply the second welfare theorem to our RBC economy!

Introducing prices

Debreu approach: claims on all goods after all histories $\{s^t\}$ are traded at $t = 0$. All prices are contingent on s^t. **Notation:**

- $Q_t(s^t) \equiv$ price of a unit of consumption good at date t after history s^t
  - normalize $Q_0 = 1$: date-0 good is numeraire
- $w_t(s^t) \equiv$ real wage (labor services at date t for goods at date t)
- $r_t(s^t) \equiv$ rental rate (capital services at date t for goods at date t)

Markets: consumption good, labor services, capital services in each period

Allow for **heterogeneity:** $j \in J$ agents and $m \in M$ firms

- all agents have same utility U, all firms same technology $AF(k, n)$

  [can relax this: it becomes more interesting but more complicated!]

Now, setup household + firm max. problems, s.t. Q, w, r; impose market clearing

Household problem

$$
\begin{aligned}
\max \quad & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) U\left(c_t^j\left(s^t\right), n_t^j\left(s^t\right)\right) \\
\text{s.t.} \quad & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} Q_t\left(s^t\right)\left\{c_t^j\left(s^t\right) + i_t^j\left(s^t\right)\right\} \\
& \quad = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} Q_t\left(s^t\right)\left\{w_t\left(s^t\right) n_t^j\left(s^t\right) + r_t\left(s^t\right) k_{t-1}^j\left(s^{t-1}\right)\right\} \\
& k_t^j\left(s^t\right) = (1-\delta) k_{t-1}^j\left(s^{t-1}\right) + i_t^j\left(s^t\right)
\end{aligned} \tag{1}
$$

Denote by λ^j the (single!) multiplier on the budget constraint for agent j. Three key FOCs:

$$\beta^t \pi\left(s^t\right) U_C\left(c_t^j\left(s^t\right), n_t^j\left(s^t\right)\right) = \lambda^j Q_t\left(s^t\right) \tag{2}$$

$$\beta^t \pi\left(s^t\right) U_N\left(c_t^j\left(s^t\right), n_t^j\left(s^t\right)\right) = -\lambda^j Q_t\left(s^t\right) w_t\left(s^t\right) \tag{3}$$

$$\sum_{s_{t+1}} Q_{t+1}\left(s^t, s_{t+1}\right)\left(1 - \delta + r_{t+1}\left(s^t, s_{t+1}\right)\right) = Q_t\left(s^t\right) \tag{4}$$

Firm problem

- Firm m's problem: choose $\left\{k^m_{t-1}\left(s^t\right), n^m_t\left(s^t\right)\right\}$ to solve

$$\max \quad \sum_{t=0}^{\infty} \sum_{s^t \in S^t} Q_t\left(s^t\right) \Pi^m_t\left(s^t\right)$$

$$\begin{aligned}\Pi^m_t\left(s^t\right) \equiv & A\left(s_t\right) F\left(k^m_{t-1}\left(s^t\right), n^m_t\left(s^t\right)\right) \\ & - w_t\left(s^t\right) n^m_t\left(s^t\right) - r_t\left(s^t\right) k^m_{t-1}\left(s^t\right)\end{aligned}$$

- Note problem is static: FOCs

$$r_t\left(s^t\right) = A\left(s_t\right) F_K\left(k^m_{t-1}\left(s^t\right), n^m_t\left(s^t\right)\right) \tag{5}$$

$$w_t\left(s^t\right) = A\left(s_t\right) F_N\left(k^m_{t-1}\left(s^t\right), n^m_t\left(s^t\right)\right) \tag{6}$$

- Constant returns to scale $\Rightarrow \Pi^m_t\left(s^t\right) = 0$
  - justifies not including firm ownership in agent's budget constraints

    note: firms can choose their capital k^m_{t-1} at each node s^t
    (aggregate supply of capital is fixed at $t-1$, but can be reshuffled via rental market at t)

Competitive equilibrium

Definition

Given initial k_{-1}^{j}, a competitive equilibrium is a set of allocations $\left\{c_t^j, n_t^j, k_t^j\right\}_{j\in J}$ $\{k_t^m, n_t^m\}_{m\in M}$, and prices $\{Q_t, r_t, w_t\}$ such that

1. Each household $j \in J$ maximizes utility, implying (2)–(4)
2. Each firm $m \in M$ maximizes profits, implying (5)–(6)
3. Markets clear:

$$\sum_{m\in M} n_t^m\left(s^t\right) = \sum_{j\in J} n_t^j\left(s^t\right)$$

$$\sum_{m\in M} k_{t-1}^m\left(s^t\right) = \sum_{j\in J} k_{t-1}^j\left(s^{t-1}\right)$$

$$\sum_{j\in J}\left\{c_t^j\left(s^t\right) + i_t^j\left(s^t\right)\right\} = \sum_{m\in M} A\left(s_t\right) F\left(k_{t-1}^m\left(s^t\right), n_t^m\left(s^t\right)\right)$$

Decentralization

- Suppose all hh's start with same k_{-1}.
- In equilibrium, all firms and households behave the same at all t
- Define $K_t = \sum_m k_t^m$, $N_t = \sum_m n_t^m$, $C_t = \sum_j c_t^j$
- Can easily check that planning problem FOCs are satisfied here
- So what are Q_t, w_t, r_t that decentralize the RBC planning solution?
- We can find w_t, r_t from either household or firms' FOCs, e.g.

$$r_t(s^t) = A(s_t) \cdot F_K\left(K_{t-1}(s^t), N_t(s^t)\right) = MPK_t$$

$$w_t(s^t) = A(s_t) \cdot F_N\left(K_{t-1}(s^t), N_t(s^t)\right) = MPL_t$$

$$Q_t(s^t) = \beta^t \pi\left(s^t\right) \frac{U_C\left(C_t(s^t), N_t(s^t)\right)}{U_C\left(C_0, N_0\right)}$$

Implications for asset pricing [optional]

Implications for asset pricing

- With $Q_t(s^t)$, we can price assets!
- Given state s^t, what's price of 1 consumption unit in state $s^{t+1} = (s^t, s_{t+1})$?

$$q_t(s^t, s_{t+1}) = \frac{Q_{t+1}\left(s^{t+1}\right)}{Q_t\left(s^t\right)} = \beta\pi\left(s_{t+1}|s^t\right)\frac{U_C\left(C_{t+1}\left(s^{t+1}\right), N_{t+1}\left(s^{t+1}\right)\right)}{U_C\left(C_t\left(s^t\right), N_t\left(s^t\right)\right)} \tag{7}$$

- (7) is the heart of **consumption based asset pricing** in finance:
  - fundamental determinants: discounting, probability, risk aversion
  - any asset with state-dependent payoff $x_{t+1}\left(s_{t+1}|s^t\right)$ has time-t price

$$p_t^x\left(s^t\right) = \sum_{s_{t+1}} q_t\left(s^t, s_{t+1}\right) x_{t+1}\left(s_{t+1}|s^t\right) = \mathbb{E}_t\left[\beta\frac{U_{C,t+1}}{U_{C,t}}x_{t+1}\right] \tag{8}$$

  - implication of no arbitrage (equilibrium $\Rightarrow$ no arbitrage)

Example: Risk-free rate

- Example: **risk-free bond** paying one unit in every future state. Price

$$p_t^f\left(s^t\right) = \sum_{s_{t+1}} q_t\left(s^t, s_{t+1}\right) = \beta \mathbb{E}_t\left[\frac{U_{C,t+1}}{U_{C,t}}\right]$$

- The gross risk-free rate $R_t^f = 1/p_t^f$ is given by

$$R_t^f = \left(\beta \mathbb{E}_t\left[\frac{U_{C,t+1}}{U_{C,t}}\right]\right)^{-1}$$

- This will turn out to be a very important price

- What is expected return on more general assets?
- Express (8) in terms of returns $R^x_{t+1} \equiv \frac{x_{t+1}(s_{t+1}|s^t)}{p^x_t(s^t)}$

$$U_{C,t} = \beta \mathbb{E}_t \left[R^x_{t+1} U_{C,t+1} \right] \tag{9}$$

- Can rewrite (9) as

$$1 = \mathbb{E}_t \left[\beta R^x_{t+1} \frac{U_{C,t+1}}{U_{C,t}} \right] = \mathbb{E}_t \left[\beta \frac{U_{C,t+1}}{U_{C,t}} \right] \mathbb{E}_t \left[R^x_{t+1} \right] + \text{Cov}_t \left(\beta \frac{U_{C,t+1}}{U_{C,t}}, R^x_{t+1} \right)$$

- The equilibrium expected excess return (= risk premium) on asset x is

$$\frac{\mathbb{E}_t\left[R^x_{t+1}\right] - R^f_t}{R^f_t} = -\mathrm{Cov}_t\left(\beta\frac{U_{C,t+1}}{U_{C,t}}, R^x_{t+1}\right) \tag{10}$$

- Intuition:
  - high risk premium: if asset pays off in good times (low $U_{C,t+1}$)
  - low risk premium: if asset pays off in bad times (high $U_{C,t+1}$)
  - variance of returns $\neq$ risk
- Note: what happens when we linearize the model?
  - Since Cov involves the product of two small terms, is is 0 to first order!
  - Therefore $\mathbb{E}_t\left[R^x_{t+1}\right] = R^f_t$: all date-$t$ expected returns equal the risk-free rate at t
  - This is why our irfs only show the risk-free rate: it's “the” expected return...
  - Need higher-order or “global” solutions for asset pricing to matter

Equity risk premium

- How well does (10) perform empirically?
- Assume MaCurdy preferences with risk aversion σ, hence $\frac{U_{C,t+1}}{U_{C,t}} = \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$
- Write $\rho \equiv -\log\beta$, $g_{t+1} \equiv \log\frac{C_{t+1}}{C_t}$ and $r^x_{t+1} \equiv \log R^x_{t+1}$
- Assume $\left(g_{t+1}, r^x_{t+1}\right)$ jointly normal. Then (9) is

$$\mathbb{E}_t\left[e^{-\rho-\sigma g_{t+1}+r^x_{t+1}}\right] = 1 \quad \Leftrightarrow \quad -\rho-\sigma\mathbb{E}_t\left[g_{t+1}\right]+\mathbb{E}_t\left[r^x_{t+1}\right]+\frac{1}{2}\mathrm{Var}\left(r^x_{t+1}-\sigma g_{t+1}\right) = 0$$

- Apply this to $x =$ risk-free bond and subtract

$$\begin{aligned}\mathbb{E}_t\left[r^x_{t+1}\right] - r^f_t &= \frac{1}{2}\mathrm{Var}\left(\sigma g_{t+1}\right) - \frac{1}{2}\mathrm{Var}\left(r^x_{t+1}-\sigma g_{t+1}\right)\\ &= \sigma\mathrm{Cov}\left(r^x_{t+1}, g_{t+1}\right) - \frac{1}{2}\mathrm{Var}\left(r^x_{t+1}\right)\end{aligned}$$

Equity premium puzzle

- So under these assumptions, another way to express (10) is

$$\begin{aligned}\log \mathbb{E}_t\left[R^x_{t+1}\right] - r^f_t &= \sigma \mathrm{Cov}\left(r^x_{t+1}, g_{t+1}\right)\\ &= \sigma \mathrm{sd}_t\left(r^x_{t+1}\right)\mathrm{sd}_t\left(g_{t+1}\right)\mathrm{Corr}_t\left(r^x_{t+1}, g_{t+1}\right)\end{aligned}$$

- From Campbell (2003)

	$\log \mathbb{E}_t\left[R^x_{t+1}\right] - r^f_t$	$\mathrm{Cov}\left(r^x_{t+1}, g_{t+1}\right)$	**Implied** σ	σ if Corr $= 1$
USA 1947-1993	8.07%	0.0354%	**240**	49
AUL 1970–1998	3.88%	0.0640%	**58**	8
CAN 1970–1999	3.96%	0.0694%	**59**	12
FRA 1973–1998	8.30%	-0.0631%	**<0**	12
GER 1978–1998	8.67%	0.0145%	**599**	17

Equity premium puzzle: conclusion

- CCAPM theory is mostly a *qualitative* success
  - … but quantitatively, requires implausibly large values of σ
- **Equity premium puzzle**
  - very robust, huge literature
  - many other puzzles, many solutions also
- Broadly speaking, explanations fall in two categories:
  1. Change preferences: recursive prefs, habit formation, uncertainty aversion…
  2. Drop complete markets/add trading costs
- See Monika and Martin’s classes for more