Econ 212a: Business Cycles
Lecture 1
Business Cycle Facts and RBC

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Stanford

Course Intro

Welcome to Econ 212

- Adrien Auclert
    - aauclert@stanford.edu
    - Lectures: Spring quarter, TTh, 1:30pm–3:20pm, in 380-380W
    - My part: 4 April to 5 May
    - Office Hours: Wednesdays 4:30-5:30pm, Landau 348
- Teaching Assistant: Cedomir Malgieri
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    - Sections: Fridays 1:30pm-3:20pm, Lathrop 299
    - Office Hours: Mondays 5pm-7pm, Econ 149

Some details on course

- Syllabus has detailed class plan
- Main material is in the slides (plus what you will learn in section)
  - I will post slides online ahead of class
  - Syllabus references provide background reading
- Please ask questions!

Course requirements for my half

You will be graded based on:

1. 4 Problem Sets (10% of your grade each)
    - Posted on Thursdays, due the following Friday
    - You can work in groups but submit your own solution
2. A Final Exam (60% of your grade)
    - 1h45min in class, closed-book exam
    - Will take place on the last Friday (May 5) during section

The topics we cover

- We'll learn the central building blocks of modern business cycle models
  - The Real Business Cycle (RBC) model
  - The New Keynesian (NK) model

- Coherent conceptual frameworks that help us answer
  - **Positive questions**: why do we see expansions and recessions? What causes inflation? How do monetary and fiscal policy work?
  - **Normative questions**: what should be the objectives of monetary and fiscal policy, and how can we achieve them?

- Prerequisites: basic knowledge and familiarity with
  1. Dynamic programming
  2. The neoclassical growth model
  3. General equilibrium analysis
  4. Time series analysis

Let’s see...

- Why do **you** think we have business cycles?

Answers to key questions are not settled yet!

- Why do we see business cycles?
  - technology / supply shocks? sentiments? demand shocks? financial shocks? misallocation?
  - shocks, or amplification mechanisms?
- If you believe demand shocks matter, why do they matter?
  - sticky prices? sticky wages?
  - are those even allocative? e.g. wages of new hires vs continuing workers
- Still: RBC and NK models...
  - are extremely useful as organizing framework
  - allow us to build intuitions that work in general equilibrium
  - are "workhorse models", i.e. commonly built upon

30-second history of business-cycle macro

- 18-19th century: casual observation of business cycles (e.g. Hume, Marx)
- 1930s: much improved measurement (e.g. Mitchell, Burns, Kuznets, Stone)
- Keynes ties everything together in his **General Theory**, "birth of macro"
  - formalized by young followers: Hicks & Modigliani (IS-LM), Samuelson
  - crucial role for **fiscal policy** to stabilize cycles
  - rests on **ad hoc decision rules** (e.g. "consumption function") ...
  - and **ad hoc expectation formation**
- Implications for econometrics (Tinbergen): estimating Keynesian models
- Looking for "microfoundations": Friedman, Modigliani, Hayashi, Tobin, ...
  - "which questions are the ad-hoc decision rules an answer to?"
- Looking for better formalization of expectations: Muth, Lucas, Prescott

30-second history of business-cycle macro

- Move away from fiscal policy:
  - Marginal propensity to spend out of current income much smaller!
  - Monetarism (Friedman and Schwartz): Fed policy too tight during Great Depression. Monetary policy crucial to stabilize cycles!
  - Barro: Ricardian Equivalence
- Move away from monetary policy:
  - If firms anticipate monetary policy, they'd change their prices!
  - "Rational expectations" (Muth, Lucas, Prescott): monetary policy less powerful.
- Culminated in **Real Business Cycle (RBC) model** (Kydland Prescott 1982)
- Response: **New Keynesian (NK) model** (Mankiw Romer book 1991)
  - "RBC + sticky prices"
- **Unification of macro since GFC:** financial frictions, integration with asset pricing, role of behavioral frictions, role of heterogeneity / MPCs, "big data"

Course Outline

1. Real Business Cycles
    - What is the “business cycle?” Facts and comovements
    - Derivation of RBC model
    - Calibration
    - Successes and challenges
2. The New Keynesian Model
    - Nominal rigidities: empirical motivation
    - Derivation of NK model
    - Optimal policy

Business Cycle Facts

What describes the business cycle?

- The business cycle is defined as the **volatility** and **comovement pattern** of aggregate economic variables
    - e.g. output, consumption, investment, employment, wages, interest rates...
- To study volatility and comovement, need covariance-stationary processes
- Thus, we need to **de-trend** variables first

Detrending

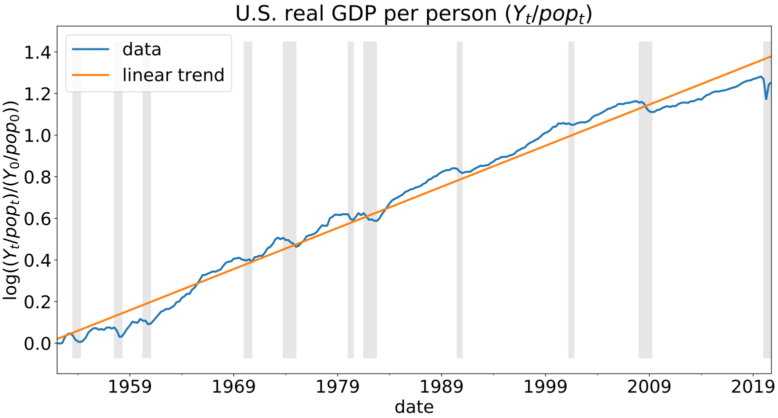
How to de-trend?

- Start with time series of aggregate variable $\mathbf{x} = (x_t)_{t=1}^{T}$ (eg $\log Y_t/pop_t$)
- Goal: decompose into “trend” and “cyclical” components

$$x_t = x_t^g + x_t^c \tag{1}$$

- Q: Can we usefully separate trend and cycle?
- Following are graphs from King-Rebelo (1999) handbook chapter (“KR”), updated to today (stop before Covid)

GDP per person vs a linear trend



De-trend with care...

- Generally, if trend is stochastic

$$x_t^g = const + x_{t-1}^g + \epsilon_t^g$$

$$x_t^c = \rho x_{t-1}^c + \epsilon_t^c$$

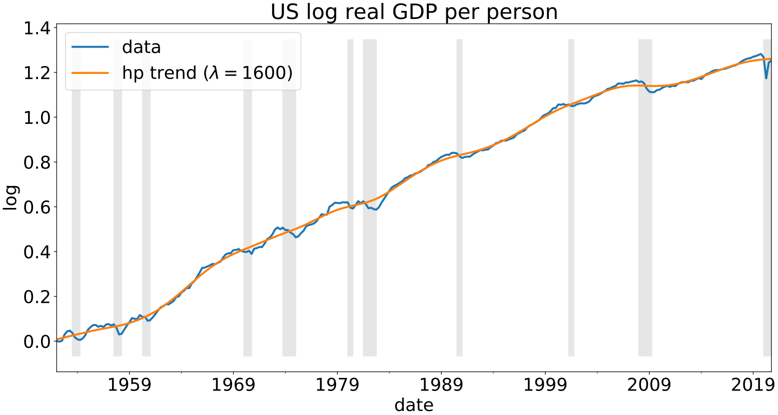
  can only separate x_t^g, x_t^c if $\epsilon_t^g, \epsilon_t^c$ perfectly correlated or uncorrelated

- See Stock Watson 1988 JEP: “without a more precise definition, one economist’s "trend" can be another’s "cycle."”

- What’s more realistic?

- Conventional in RBC literature to assume trend is approx. deterministic (ϵ_t^g small). Then, can use filter: linear, HP filter, band-pass filter, Hamilton filter...

Taking Out the Trend: The HP Filter



Definition of HP filter

- Compute the trend component by penalizing changes in growth rate:

$$\min_{(x_t^g)_{t=1}^T} \sum_{t=1}^{T} \left(x_t - x_t^g\right)^2 + \lambda \sum_{t=2}^{T-1} \left(\left(x_{t+1}^g - x_t^g\right) - \left(x_t^g - x_{t-1}^g\right)\right)^2 \tag{2}$$

- **Idea:**
  - trend should be close to actual series (first term)...
  - but smooth, i.e. its growth rate stable over time (second term)
  - value of $\lambda > 0$ measures trade-off between these two objectives
  - as $\lambda \to 0$, $\mathbf{x^g}$ gets closer to the initial series
  - as $\lambda \to \infty$, $\mathbf{x^g}$ gets closer to linear trend through $(x_t)_{t=1}^T$

- The solution to (2) takes the linear form

$$\mathbf{x^g} = \mathbf{G}\mathbf{x} \qquad \mathbf{x^c} = (\mathbf{I} - \mathbf{G})\,\mathbf{x}$$

where $\mathbf{G} = (\mathbf{I} + \lambda \mathbf{K}'\mathbf{K})^{-1}$ is a $T \times T$ matrix, and $\mathbf{K}$ is the $(T-2) \times T$ matrix

$$\mathbf{K} \equiv \begin{pmatrix} 1 & -2 & 1 & & & & & \\ & 1 & -2 & 1 & & & & \\ & & & \ddots & \ddots & & & \\ & & & & 1 & -2 & 1 & \\ & & & & & 1 & -2 & 1 \end{pmatrix}$$

- Why? Note that $\mathbf{Kx^g}$ is the vector of second derivatives,

$$\mathbf{Kx^g} = \left(x_2^g - 2x_1^g + x_0^g,\; x_3^g - 2x_2^g + x_1^g,\; \ldots\right)$$

Implementation

- So, (2) can be rewritten in vector form as

$$\min_{\mathbf{x}^g} \frac{1}{2} \left\{ (\mathbf{x}^g - \mathbf{x})' (\mathbf{x}^g - \mathbf{x}) + \lambda \left(\mathbf{K}\mathbf{x}^g\right)' \mathbf{K}\mathbf{x}^g \right\}$$

  which has the standard solution

$$\mathbf{x}^g - \mathbf{x} + \lambda \mathbf{K}'\mathbf{K}\mathbf{x}^g = 0 \quad \Leftrightarrow \quad \left(\mathbf{I} + \lambda \mathbf{K}'\mathbf{K}\right) \mathbf{x}^g = \mathbf{x}$$

- Note that $\mathbf{I} + \lambda \mathbf{K}'\mathbf{K}$ is positive definite, so invertible.
- **Implication:** all observations in $\mathbf{x}$ are used to determine $\mathbf{x}^g$
  - as we add observations we update the past trend, use with care!

Some properties of HP filter and alternatives

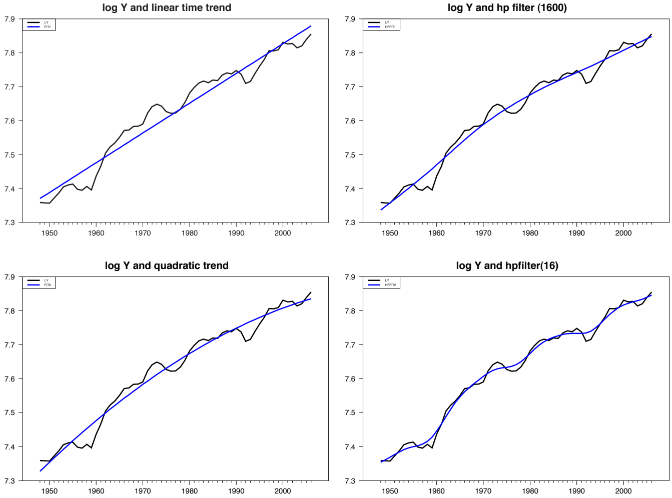
- Standard practice for quarterly data is $\lambda = 1600$
  - Chosen by Hodrick and Prescott on basis of optimality properties
  - Rule of thumb (Ravn-Uhlig) is to adjust by fourth power of ratio of observation frequencies, eg

$$\lambda^{year} = \frac{1}{4^4}\lambda^Q = 6.25$$

- The HP filter is a “high-pass” filter
  - With $\lambda = 1600$, takes out cycles moving slower than 32 quarters
- Alternatives:
  - band-pass filter, filters out high-frequency components too (Baxter King 1999)
  - see Hamilton (2018) for critique: “Why You Should Never Use the Hodrick-Prescott Filter”

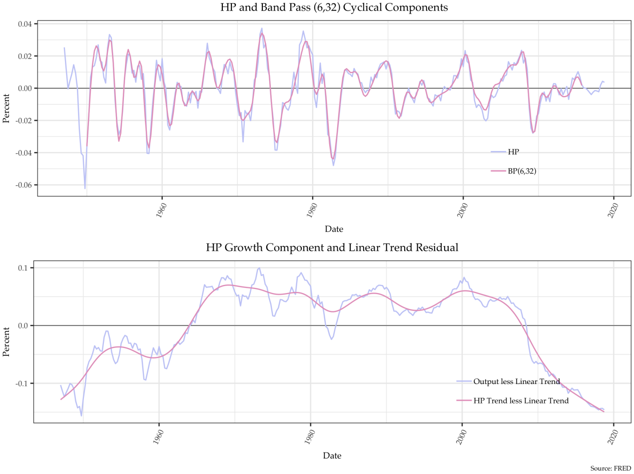
US output: detrending matters

Business cycles more severe & persistent with linear trend



US output: detrending matters

Band-pass & HP similar, HP and linear trend very distinct



Business Cycle Facts

Overview: Business Cycle Facts (original KR chapter)

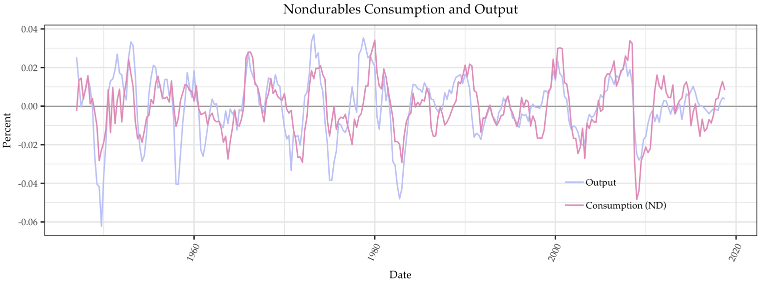
Table 1
Business cycle statistics for the US Economy

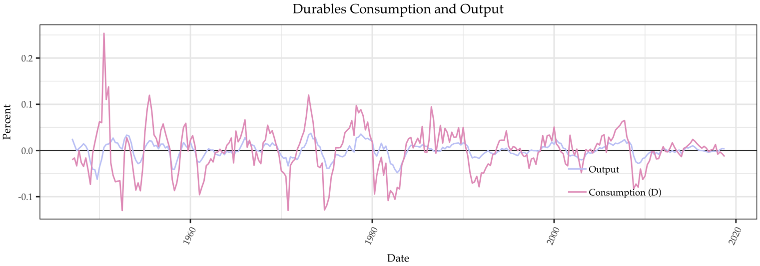
	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	−0.35
A	0.98	0.54	0.74	0.78

[a] All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson (1999), who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

- N as volatile as Y, C less volatile than Y, I more volatile than Y
- Strong comovement of Y, C, I, N: that's the "business cycle"

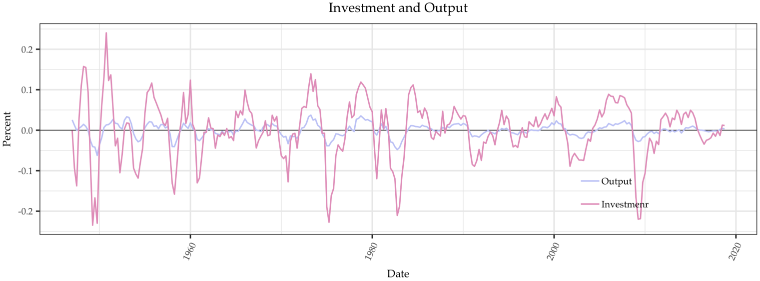
(1) ND consumption less volatile, D consumption more volatile, highly correlated

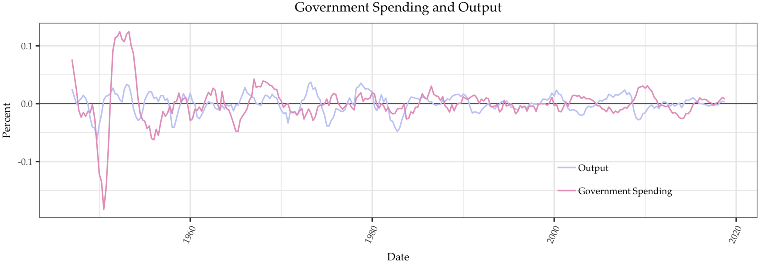




Source: FRED

(2) Investment highly procyclical and Government Spending countercyclical





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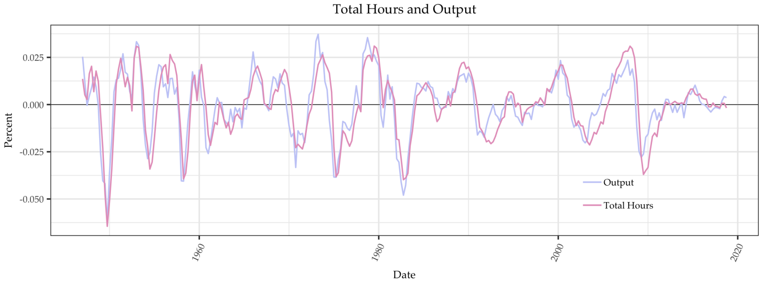
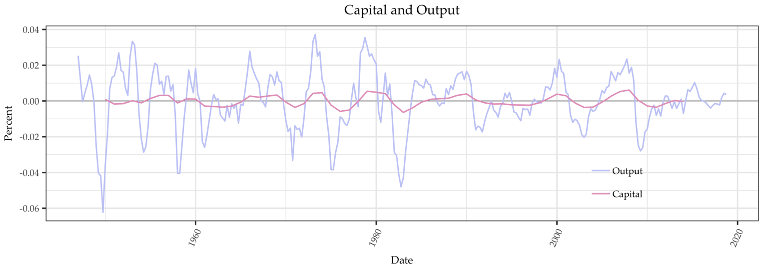
(3) Hours track output, capital mildly procyclical





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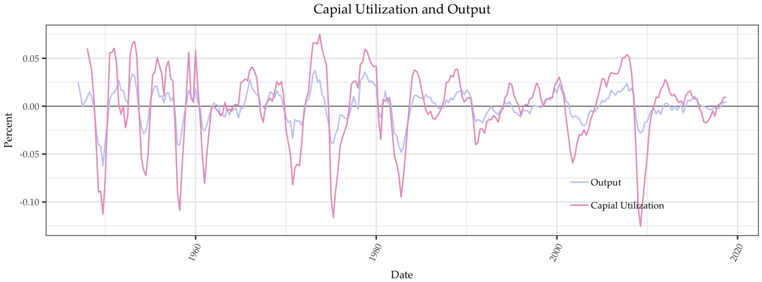
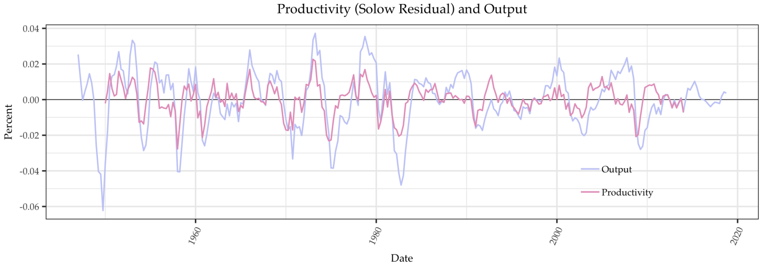
(4) Utilization strongly procyclical, Solow residual procyclical





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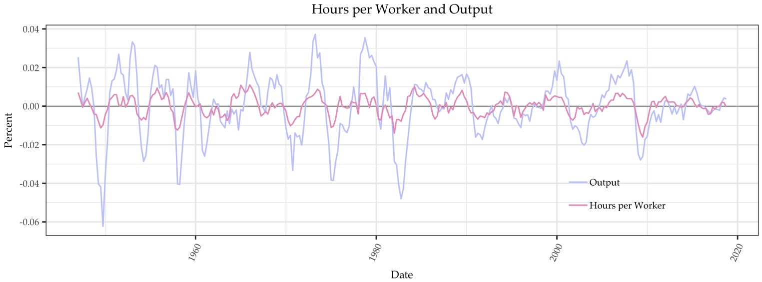
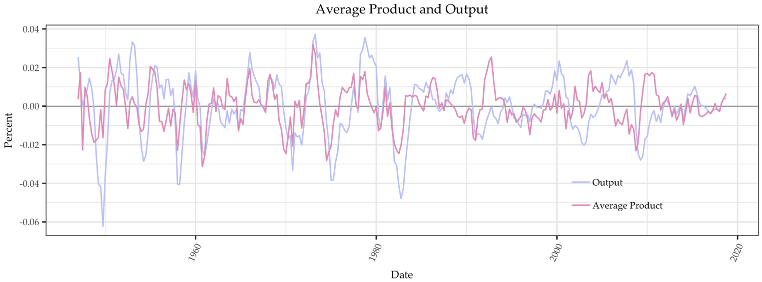
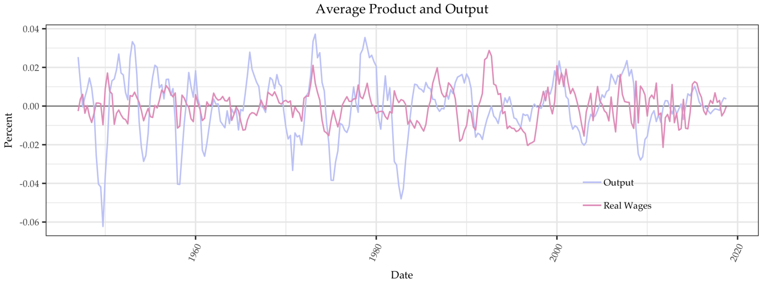
(5) Employment more procyclical than hours per worker



Source: FRED

(6) Average product (Y/N) and real wage not very cyclical





Comovements at different horizons

- Can also characterize comovement at different horizons (beyond simple correlation)
  - which variables lead output vs lag behind?
- Define "cross correlogram" for $k = -3, ..., 0, ..., +3$

$$\rho\left(X_t^c, Y_{t+k}^c\right)$$

  - if positive and highest for $k < 0$, then X is procyclical and **lags**
  - if positive and highest for $k > 0$, then X is procyclical and **leads**
- Does not give us a sense of the amplitudes (for this, look at std. deviation)
- See Stock-Watson (1999) table 2, Cooley-Prescott (1995) table 1.1

Comovements with consumption (original KR)

- Output and consumption components: $\rho\left(X_t^c, Y_{t+k}^c\right)$ vs k

	std dev. %	-3	-2	-1	0	1	2	3
Y	1.66	0.33	0.66	**0.91**	1	**0.91**	0.66	0.33
C	1.26	0.21	0.51	0.76	0.9	0.89	0.75	0.53
C^{ND}	**0.78**	0.29	0.55	**0.75**	**0.87**	**0.85**	0.71	0.49
C^D	**4.66**	0.09	0.42	0.70	0.85	**0.86**	0.73	0.53

  - output (Y) highly auto-correlated: cycles are **persistent**
  - non-durables and services (C^{ND}): **smoother, positive comovement**
  - durables (C^D): fluctuate more than Y, positive comovement, leads
  - somewhat consistent with consumption smoothing:
    - consumption of services of durables $\neq$ durable good expenditure
    - purchasing a durable is similar to an investment

Comovements with I, X, M, G

- Output and other components of spending

	std dev. %	-3	-2	-1	0	1	2	3
Y	1.66	0.33	0.66	0.91	1	0.91	0.66	0.33
I	**4.97**	0.32	0.61	**0.82**	**0.89**	**0.83**	0.65	0.41
X	**4.76**	0.50	0.48	0.40	**0.27**	0.09	-0.11	-0.29
M	**4.42**	0.27	0.54	0.72	**0.78**	0.70	0.53	0.34
G	2.49	0.21	0.21	0.19	**0.15**	0.03	-0.10	-0.20

  - Investment I **three times more volatile than** Y, highly procyclical, leads
  - Exports X and imports M volatile; X not much comovement, M more
  - Government spending G: not much comovement

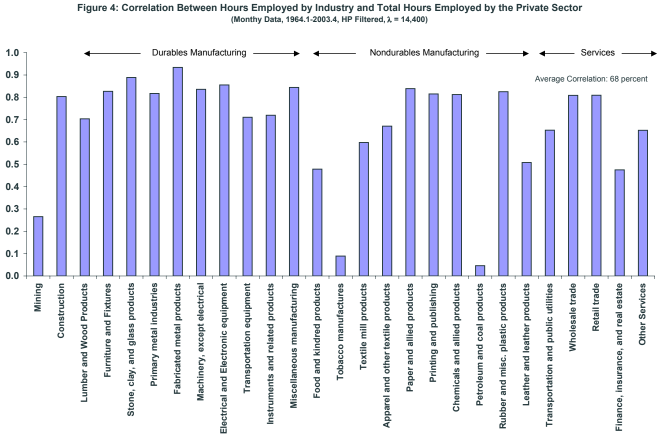
Comovements with N, e, A

- Output, hours $N = e \times \frac{N}{e}$ where e is employment, and TFP

	std dev. %	-3	-2	-1	0	1	2	3
Y	1.66	0.33	0.66	0.91	1	0.91	0.66	0.33
N	**1.61**	0.63	0.85	0.94	**0.88**	0.67	0.36	0.03
e	1.39	0.72	0.89	0.92	0.81	0.57	0.24	-0.07
$\frac{N}{e}$	0.37	0.05	0.38	0.66	0.82	0.80	0.64	0.40
A	**2.29**	-0.03	0.27	0.56	**0.77**	0.86	0.82	0.68

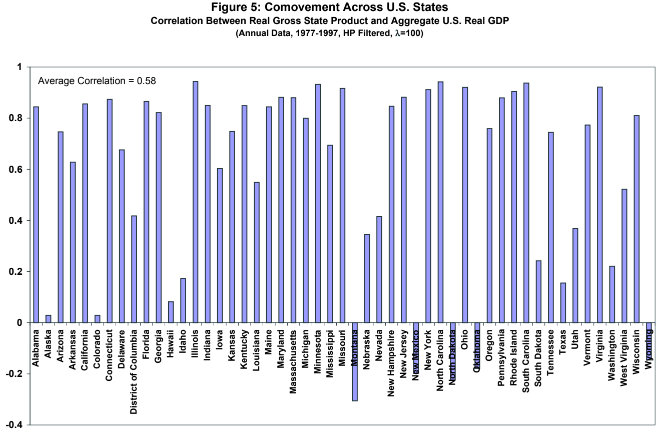
  - Total hours N **as volatile as** Y.
  - Productivity A (measured as Solow residual) is procyclical

Not all industries are similarly procyclical



Source: Rebelo (2005)

Not all states are similarly procyclical



Source: Rebelo (2005)

Modern approaches to business cycle facts

- Angeletos Collard Dellas (2020 AER): Identify "Main business cycle shock" from VAR that explains maximal variation in unemployment

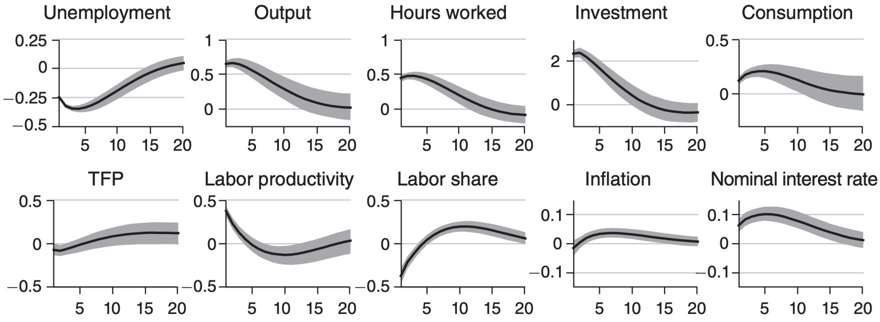


FIGURE 1. IMPULSE RESPONSE FUNCTIONS TO THE MBC SHOCK

Notes: Impulse Response Functions of all the variables to the identified MBC shock. Horizontal axis: time horizon in quarters. Shaded area: 68 percent Highest Posterior Density Interval (HPDI).

How to discipline business cycle models?

Imagine I have a business cycle model. How can I discipline it?

- Check comovement & volatility patterns
- Target "main business cycle" impulse responses, or VAR evidence more generally
- Estimate the model in time series data, either as VAR or using Bayesian methods
- Compare model to identified responses to specific shocks (e.g. monetary policy, exchange rates, oil shocks, carbon price shocks, ...)

RBC Model: Setup

Making sense of the business cycle

- How can we model the comovement and volatility patterns that make up the "business cycle"?
- It's easy to tell stories ... but not trivial to formalize them!
- Our starting point & main idea of the RBC model: technology shocks!
    - easy to dismiss them ... what about dot-com boom? global financial crisis? ... Covid-induced reduction in labor force? ...
- Later in this class: demand and monetary policy
- See 2nd year macro classes for more realistic shocks and propagation

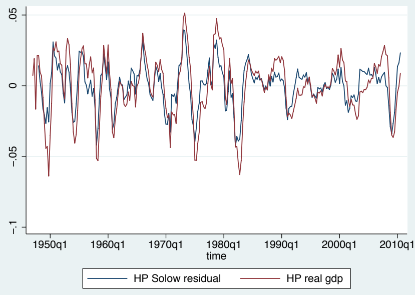
Measuring technology shocks

- How can we measure "technology shocks"? Write production function as

$$Y_t = A_t F\left(K_{t-1}, (1+g)^t N_t\right)$$

If F Cobb-Douglas, then A_t is simply de-trended Solow residual

$$\log A_t = \log Y_t - \alpha \log K_{t-1} - (1-\alpha) \log N_t - (1-\alpha)\, t \log (1+g)$$



Note: Will later see that Solow residuals overstate role of technology shocks...

Modeling technology shocks

- Model A_t as *exogenous Markov process*, for example AR(1)

$$\log A_t = \rho \log A_{t-1} + \epsilon_t \tag{3}$$

- The RBC model in a nutshell...
  - take the neoclassical growth model (NGM), with endogenous labor supply
  - assume that A_t behaves stochastically, say according to (3)
  - see if macroeconomic aggregates (C_t, I_t, N_t, K_t) behave anything like in the data
- We start with the **planning problem**. We will later see that just like the NGM, the RBC model is frictionless, so planner's solution = comp. equilibrium.

RBC model: planning problem

Representing uncertainty

One key difference with NGM: uncertainty...

- Consider **state space** S
- One state $s_t \in S$ realized every period
  - In baseline model: TFP is function of state, $A = A(s_t)$
  - s_t follows a Markov chain, and A inherits its properties
  - $\pi(s_t|s_{t-1})$ transition matrix
- Histories of shocks $s^t = (s_0, s_1 \dots s_t) \in S^t$, probability:

$$\pi(s^t) \equiv \pi(s_t|s_{t-1})\pi(s_{t-1}|s_{t-2}) \cdots \pi(s_1|s_0)$$

- Agent have **rational expectations:** evaluate histories s^t with π
- A *consumption plan* is a set of functions $C_t(s^t)$, with C_t: $S^t \to \mathbb{R}$
  - similarly for all other variables

Planner's problem setup

- Planner's problem: choose $\left\{C_t\left(s^t\right), N_t\left(s^t\right), I_t\left(s^t\right), K_t\left(s^t\right)\right\}$ to solve

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right) U\left(C_t\left(s^t\right), N_t\left(s^t\right)\right) \\ \text{s.t.} \quad & C_t\left(s^t\right) + I_t\left(s^t\right) = A\left(s_t\right) F\left(K_{t-1}\left(s^{t-1}\right), N_t\left(s^t\right)\right) \\ & K_t\left(s^t\right) = (1-\delta) K_{t-1}\left(s^{t-1}\right) + I_t\left(s^t\right) \end{aligned}$$

- Note: closed economy with no government, K pre-determined in F
- Sometimes, drop subscripts, eg write $C\left(s^t\right)$, $K\left(s^t\right)$, or drop s^t, write C_t, K_t
- Utility is a function of labor N (a bad, with $U_N < 0$)
    - sometimes written as a function of leisure $L = L^{max} - N$