Econ 212a: Business Cycles
Lecture 5
Money and Prices

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Recap + motivation

- Last two weeks: business cycles + RBC interpretation
- **But** conspicuously **absent**: inflation, money, monetary policy ...
- This is what we start to look into this week
- **Problem:** first, we first need to answer a fundamental question ...

The fundamental question about money

Q: Money is just a piece of paper. So why does it have any value at all?

- value of money = inverse of the aggregate price level

1. Money is a **medium of exchange**
   - solves "double coincidence of wants" problem in decentralized trades
2. Money is a **store of value**
   - can hold onto it to save, but requires price stability
3. Money is a **unit of account**
   - all prices, debts, income, taxes stated in the same currency, easy to compare
   - requires some rigidity / non-state-contingency of prices, contracts, taxes

- All three **interact**, and depend a lot on **coordination** ...

  $\rightarrow$ questions of equilibrium multiplicity / "determinacy" will become relevant

Side remark: Which of these functions does Bitcoin serve?

Two approaches to modeling money

- Foundational approach, typically capturing one of the three reasons
  - think hard about **fundamental drivers** (double coincidence, information, commitment, spatial separation, ...)
  - examples: store of value (Samuelson 1958, Bewley 1979), medium of exchange (Townsend 1980, Kiyotaki Wright 1989), unit of account (Doepke Schneider 2017, Gopinath Stein 2021)
- In our class, we take a shortcut
  - put money in the utility (Sidrauski 1967), capturing (2) in reduced form
  - alternative: "cash in advance" models, OLG models
  - next time: also give money unit of account role through sticky prices
- With this model, we'll start analyzing **monetary policy**
  - today: **medium / long run effects.** later: short run, w/ price rigidity
  - will also focus mostly on **money supply** as policy tool. later: interest rates

Roadmap

1. Money in an RBC model
2. The effects of monetary policy
3. Empirical evidence on money and inflation [optional]

Money in an RBC model

- Representative agent now also values **real money balances** $m_t \equiv \frac{M_t}{P_t}$

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U \left(C_t, N_t, \frac{M_t}{P_t} \right) \right]$$

$M_t \equiv$ nominal money balances

$P_t \equiv$ aggregate price level at t, $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ rate of inflation

- Also allow for risk-free nominal bond B_t. Budget constraint:

$$\begin{aligned} & P_t C_t + P_t K_t + B_t + M_t + T_t \\ & \leq P_t \left(1 - \delta\right) K_{t-1} + R_t K_{t-1} + W_t N_t + \left(1 + i_{t-1}\right) B_{t-1} + M_{t-1} \end{aligned} \tag{1}$$

T_t nominal taxes, i_t nominal interest rate

- Assume $M_t \geq 0$ (household can't *issue* money), no-Ponzi for $\{B_t\}$

Real budget constraint

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U \left(C_t, N_t, \frac{M_t}{P_t} \right) \right]$$

- Convert the **nominal** budget constraint into a **real** one:

$$\begin{aligned} & C_t + K_t + b_t + m_t + \tau_t \\ & \leq (1 - \delta + r_t) K_{t-1} + w_t N_t + \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1} + \frac{1}{1 + \pi_t} m_{t-1} \end{aligned} \tag{2}$$

with:

$b_t \equiv \frac{B_t}{P_t}$ real value of bonds

$\tau_t \equiv \frac{T_t}{P_t}$ real value of taxes

$r_t \equiv \frac{R_t}{P_t}$ real rental rate

$w_t \equiv \frac{W_t}{P_t}$ real wage

Solving the model: the opportunity cost of money

This model has two more choice variables: b_t and m_t, two more FOCs:

- **Variation for** ***m*****:** save 1 more unit in money, consume tomorrow

$$U_{C,t} = \beta \mathbb{E}_t \left[U_{C,t+1} \cdot \frac{1}{1+\pi_{t+1}} \right] + U_{m,t}$$

- **Variation for** ***b*****:** save 1 more unit in bonds, consume tomorrow

$$U_{C,t} = \beta \mathbb{E}_t \left[U_{C,t+1} \cdot \frac{1+i_t}{1+\pi_{t+1}} \right] \tag{3}$$

Combining the two:

$$i_t U_{C,t} = (1+i_t)\, U_{m,t} \qquad \Rightarrow \qquad \boxed{\frac{U_{m,t}}{U_{C,t}} = \frac{i_t}{1+i_t}} \tag{4}$$

Intuition: $\frac{i_t}{1+i_t}$ is **opportunity cost of holding money**

agent is willing to hold money because of $U_{m,t} > 0$, even if $i_t > 0$, so return on money is strictly worse than return on bonds

Alternative derivation of the opportunity cost [will skip in class]

- Define **nominal wealth** for our agent as

$$\mathcal{W}_t = (1 + i_{t-1})\, B_{t-1} + M_{t-1}$$

hence saving required for wealth $\mathcal{W}_{t+1}$ tomorrow is

$$\frac{\mathcal{W}_{t+1}}{1+i_t} = B_t + \frac{M_t}{1+i_t} = B_t + M_t - \frac{i_t}{1+i_t} M_t$$

- Use this to rewrite the nominal budget constraint (1)

$$P_t C_t + P_t K_t + \frac{i_t}{1+i_t} M_t + \frac{\mathcal{W}_{t+1}}{1+i_t} + T_t$$
$$\leq P_t (1-\delta)\, K_{t-1} + R_t K_{t-1} + W_t N_t + \mathcal{W}_t$$

- Here, $\frac{i_t}{1+i_t}$ shows up as relative price of $\frac{M_t}{P_t}$ in terms of c_t as claimed

- Ex: standard preferences $U(C, N, m) = \log C - v(N) + \theta \log m$. Then:

$$\frac{U_{m,t}}{U_{C,t}} = \frac{i_t}{1+i_t} \qquad \Rightarrow \qquad \frac{M_t}{P_t} = \theta C_t \left(\frac{i_t}{1+i_t}\right)^{-1}$$

- Special case of **money demand** function

$$\frac{M_t}{P_t} = L\left(\underbrace{C_t}_{+}, N_t, \underbrace{i_t}_{-}\right)$$

- **Money** is **supplied** by the government, whose nominal budget constraint is

$$P_t g_t + (1 + i_{t-1}) B_{t-1}^s = B_t^s + \left(M_t^s - M_{t-1}^s\right) + T_t \tag{5}$$

and in real terms:

$$g_t + \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1}^s = b_t^s + \frac{M_t^s - M_{t-1}^s}{P_t} + \tau_t$$

Equilibrium in the RBC model with money

- Firm optimality is as before:

$$w_t = A_t F_N\left(K_{t-1}, N_t\right) \tag{6}$$

$$r_t = A_t F_K\left(K_{t-1}, N_t\right) \tag{7}$$

- Goods market clearing + Market clearing for bonds and money

$$C_t + g_t + K_t - (1-\delta) K_{t-1} = A_t F\left(K_{t-1}, N_t\right) \tag{8}$$

$$M_t = M_t^s \qquad B_t = B_t^s \tag{9}$$

- Given processes for $\{A_t, g_t, M_t^s, T_t\}$, equilibrium is a set of prices $\{r_t, w_t, i_t, P_t\}$ and quantities $\{C_t, K_t, N_t, B_t, M_t\}$ such that all FOCs + market clearing conditions, budget constraints are satisfied

The effects of monetary policy

- This is a first model in which money M_t has a value, price level P_t
- Can study monetary policy! For now: choosing path of money supply $\{M_t\}$
- Ask two questions:
    1. effects on **real economy** $\{C_t, K_t, N_t, r_t, w_t\}$?
    2. effects on **inflation**?

Money is neutral in this model:

- If $\{P_t\}$ is equilibrium nominal price given $\{M_t^s\}$ then for any $\kappa > 0$:
  - $\{\kappa P_t\}$ is equilibrium nominal price given $\{\kappa M_t^s\}$
  - **same** real allocation $\{C_t, K_t, N_t, r_t, w_t\}$

    [exercise: check that all FOC's work with this allocation!]
- **Level** of nominal money balances M_t does not matter for real stuff!

With **separability** $U(C_t, N_t, m_t) = u(C_t, N_t) + h(m_t)$ get even stronger result:

- **entire path** of $\{M_t\}$ does not matter for real allocation!
- Money is "**super-neutral**"; **"neoclassical dichotomy"**

Proof of neoclassical dichotomy

- With separable utility, m_t does not influence RBC FOC's:

$$-\frac{u_N(C_t,N_t)}{u_C(C_t,N_t)} = w_t$$

$$u_C(C_t,N_t) = \beta\mathbb{E}_t\left[u_C(C_{t+1},N_{t+1})(1-\delta+r_{t+1})\right]$$

$$C_t + g_t + K_t - (1-\delta)K_{t-1} = A_tF(K_{t-1},N_t)$$

$$F_N(K_t,N_t) = w_t \quad F_K(K_t,N_t) = r_t$$

  [only difference to previous lecture: gov spending g_t]

- Equations (3) and (4) determine the monetary side $\{i_t, P_t\}$ given M_t

$$u_C(C_t,N_t) = \beta\mathbb{E}_t\left[u_C(C_{t+1},N_{t+1})\frac{1+i_t}{1+\pi_{t+1}}\right] \tag{10}$$

$$h'(m_t) = u_C(C_t,N_t)\frac{i_t}{1+i_t} \tag{11}$$

Prices and inflation

- We have seen that, under separable preferences, M_t is super-neutral:
  - real allocation independent of money supply $\{M_t\}$!
  - result of assuming **price flexibility**
- Yet, model has interesting predictions for **prices** P_t & **inflation** $\pi_t = \frac{P_t}{P_{t-1}} - 1$

  $\rightarrow$ solve (10) and (11)
  - interpret as “long-run implications of monetary policy”
  - focus on economy *without risk*, real allocation in *steady state*
  - keep separable preferences $U = u\left(C_t, N_t\right) + h(m_t)$

Can restate (10) and (11) as:

- **Money demand equation:**

$$h'\left(\frac{M_t}{P_t}\right) = u_{C,ss} \cdot \frac{i_t}{1+i_t} \qquad \Leftrightarrow \qquad \frac{M_t}{P_t} = L\left(\frac{i_t}{1+i_t}\right) \tag{12}$$

- **Fisher equation:**

$$\underbrace{\frac{1+i_t}{1+\pi_{t+1}}}_{(1+i_t)\frac{P_t}{P_{t+1}}} = 1 + r_{ss} = \beta^{-1} \tag{13}$$

Note: does this mean raising rates increases inflation?

How can we solve these equations for $\{P_t\}$ as function of path $\{M_t\}$?

- Combine to eliminate i_t:

$$h'\left(\frac{M_t}{P_t}\right) = u_{C,ss} \cdot \frac{\beta^{-1}P_{t+1} - P_t}{\beta^{-1}P_{t+1}}$$

  and solve for P_{t+1} as function of P_t and M_t

$$P_{t+1} = \beta \frac{P_t}{1 - \frac{1}{u_{c,ss}} h'\left(\frac{M_t}{P_t}\right)}$$

- Can plot this as law of motion for given M_t ...

Unique steady state: if $M_t = M$, then $P = M \cdot L\,(1-\beta)^{-1}$

- similar if stable money growth $M_{t+1} = M_t \cdot (1+\pi)$, $P_t = M_t \cdot L \left(1 - \frac{\beta}{1+\pi}\right)^{-1}$
- constant inflation rate π, real money $m = L\left(1 - \frac{\beta}{1+\pi}\right) \equiv \mu(\pi)$
- [an aside: get **Laffer curve for seignorage**: real change in money

$$\frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} + \underbrace{\frac{\pi_t}{1+\pi_t}\frac{M_{t-1}}{P_{t-1}}}_{\text{seignorage revenue}}$$

- steady state seignorage $\frac{\pi}{1+\pi}\mu(\pi)$ typically hump shaped]

Outside of steady state: $P_t \to 0$ or $P_t \to \infty$. Either way, $\log P_t$ explodes!

Determinacy

- In monetary economies, often to hard to rule out multiplicity of equilibria
- So we usually apply a different concept: **local determinacy**
- An equilibrium is *locally determinate* if no other *bounded* equilibrium exists
  - i.e. locally determinate if the sequence $\log \mathbf{P}$ is in ℓ^∞ (or even ℓ^2)
- Let's show that when $\log \mathbf{M}$ is in ℓ^2 there is a unique $\log \mathbf{P}$ in ℓ^2
  - in other words that the economy with exogenous M_t is **locally determinate**

General solution for price level in log-linearized setting

- Let's log-linearize (12) and (13). If $\eta \equiv$ steady state elasticity of L

$$\hat{M}_t - \hat{P}_t = -\eta \Delta i_t \qquad \text{where} \quad \Delta i_t = i_t - i_{ss}, \quad i_{ss} = r^f_{ss} \equiv \beta^{-1} - 1$$

$$i_t = r^f_{ss} + \hat{P}_{t+1} - \hat{P}_t$$

- Combining the two equations, get

$$\hat{M}_t = -\eta \left(\hat{P}_{t+1} - \hat{P}_t \right) + \hat{P}_t$$

ie, in sequence space, $\hat{\mathbf{M}} = (1+\eta)\hat{\mathbf{P}} - \eta \mathbf{F} \hat{\mathbf{P}} = (1+\eta)\left(\mathbf{I} - \frac{\eta}{1+\eta}\mathbf{F} \right)\hat{\mathbf{P}}$

- See section notes : $\mathbf{I} - \frac{\eta}{1+\eta}\mathbf{F}$ is invertible in ℓ^2, so unique solution

$$\hat{P}_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k \hat{M}_{t+k}$$

- Local determinacy: unique path $\{\hat{P}_t\}$ in ℓ^2 for arbitrary path $\{\hat{M}_t\}$ in ℓ^2

Nominal interest rate as policy instrument

- What is M_t in the data? Not quite clear ...
  - central bank picks quantity of **reserves** using open market operations ($\approx$ M0)
  - but households typically use (demand) **deposits** for payment ($\approx$ M1)
- This highlights an **issue**: central bank cannot directly control M1
  - for example, fiscal stimulus naturally increases M1
  - in addition, movements in activity C_t, N_t shift demand for M1
- This is why modern central banks use **nominal rate** i_t as tool, not M_t
- How can we make sense of that? Prices more / less stable than if M_t is picked? Determinacy?
- Note: M_t is then endogenously determined to satisfy (12). Only (13) remains.

If i_t follows **exogenous** path:

- then $\pi_{t+1} = i_t - r^f_{ss} \rightarrow$ price level $\hat{P}_0$ not determined $\rightarrow$ **indeterminacy**
- in stochastic setting, this is $\mathbb{E}_t\left[\pi_{t+1}\right] = i_t - r^f_{ss}$. Can shift inflation π_{t+1} by arbitrary mean-zero ϵ_{t+1} . Sunspot equilibria.

If i_t follows **endogenous** Taylor rule: $i_t = \rho_t + \phi_\pi \pi_t$ with $\phi_\pi > 1$

- then $\pi_{t+1} = \rho_t + \phi_\pi \pi_t - r^f_{ss}$ and so

$$\pi_t = \frac{1}{\phi_\pi} \sum_{s=0}^{\infty} \left(\frac{1}{\phi_\pi}\right)^s \left(r^f_{ss} - \rho_{t+s}\right)$$

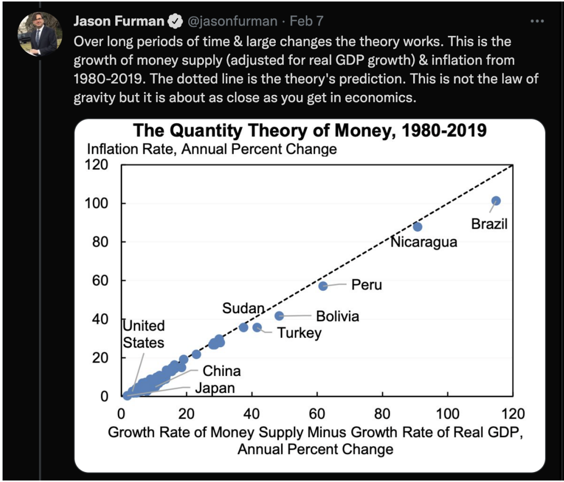
- loose interest rates $\rho_t \downarrow$ lead to inflation $\pi_t \uparrow$
- **determinacy** iff $\phi_\pi > 1$, that is, i_t responds aggressively to inflation

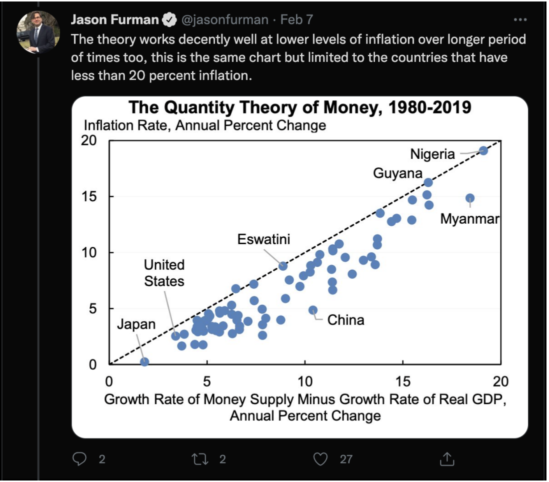
Empirical evidence on money and inflation [optional]

In long run, money growth and inflation are very correlated

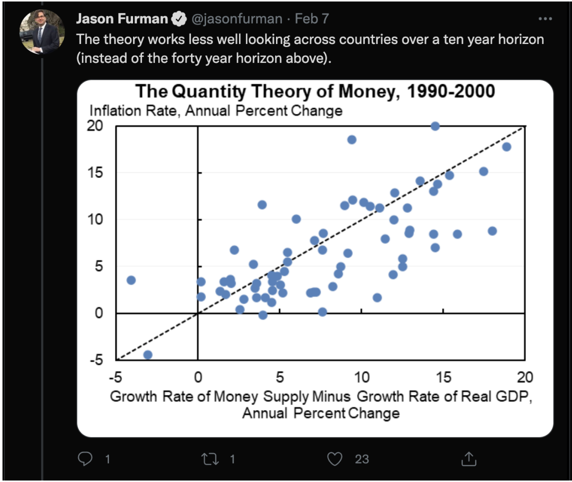
- Useful twitter economics...
  https://twitter.com/jasonfurman/status/1490765989775962115

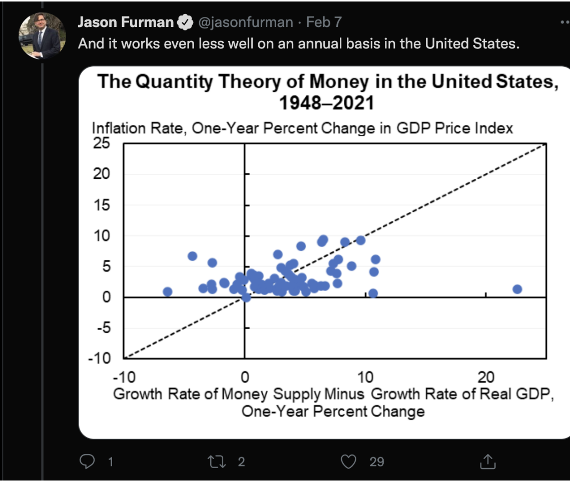
Money growth and inflation very correlated over long horizons





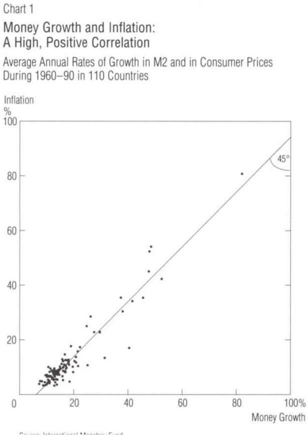
… but much less at shorter horizons





Money growth and inflation the data

- Money growth and inflation tend to be positively correlated
- Cross-country evidence: McCandless-Weber 1995



- Time-Series evidence: Marcet-Nicolini (2005)

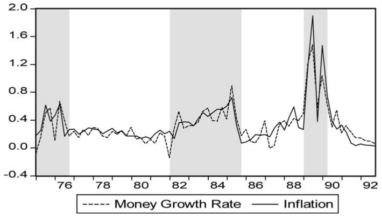


Fig. 1(a). Argentina.

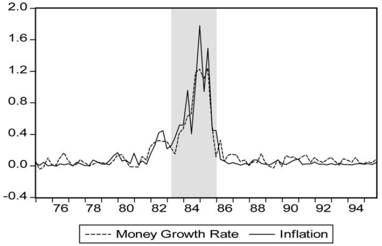


Fig. 1(b). Bolivia.

- Classic paper: Cagan (1956)

Country	Beginning	End	P_T/P_0	Av Monthly Inf rate (%)	Av Monthly M Growth (%)
Austria	Oct. 1921	Aug. 1922	70	47	31
Germany	Aug. 1922	Nov. 1923	1.0×10^{10}	322	314
Greece	Nov. 1943	Nov. 1944	4.7×10^{6}	365	220
Hungary 1	Mar. 1923	Feb. 1924	44	46	33
Hungary 2	Aug. 1945	Jul. 1946	3.8×10^{27}	19,800	12,200
Poland	Jan. 1923	Jan. 1924	699	82	72
Russia	Dec. 1921	Jan. 1924	1.2×10^{5}	57	49

- P_T/P_0: Price level in the last month of hyperinflation divided by the price level in the first month.
- Timing of price stabilization coincides with decline in money growth

Hyperinflations

Table A2 **Total Note Circulation of Austrian Crowns (in thousands of crowns)**

Year	Month	Circulation	Year	Month	Circulation
1919	January	—		May	397,829,313
	February	—		June	549,915,678
	March	4,687,056		July	786,225,601
	April	5,577,851		August	1,353,403,632
	May	5,960,003		September	2,277,677,738
	June	7,397,692		October	2,970,916,607
	July	8,391,405		November	3,417,786,498
	August	9,241,135		December	4,080,177,238
	September	9,781,112	1923	January	4,110,551,163
	October	10,819,310		February	4,207,991,722
	November	11,193,670		March	4,459,117,216
	December	12,134,474		April	4,577,382,333
1920	January	13,266,878		May	4,837,042,081
	February	14,292,809		June	5,432,619,312
	March	15,457,749		July	5,684,133,721
	April	15,523,832		August	5,894,786,367
	May	15,793,805		September	6,225,109,352
	June	16,971,344		October	6,607,839,105
	July	18,721,495		November	6,577,616,341
	August	20,050,281		December	7,125,755,190
	September	22,271,686	1924	January	6,735,109,000
	October	25,120,385		February	7,364,441,000
	November	28,072,331		March	7,144,901,000
	December	30,645,658		April	7,135,471,000
1921	January	34,525,634		May	7,552,620,000
	February	38,352,648		June	7,774,958,000
	March	41,067,299		July	7,995,647,000
	April	45,036,723		August	5,894,786,367
	May	45,583,194		September	7,998,509,000
	June	49,685,140		October	8,213,003,000
	July	54,107,281		November	8,072,021,000
	August	58,533,766		December	8,387,767,000
	September	70,170,798	1925	January	7,902,217,000
1922	January	227,015,925		February	7,957,242,000
	February	259,931,138		March	7,897,792,000
	March	304,063,642		April	7,976,420,000
	April	346,697,776			

Source: Young [36, vol. 2, p. 292].

Money growth vs inflation in the US

- All of those examples: large inflations + no independent monetary policy
- Money growth much less of a predictor otherwise (e.g. partly driven by fiscal policy, credit growth, also matters whether rich vs poor hold money)