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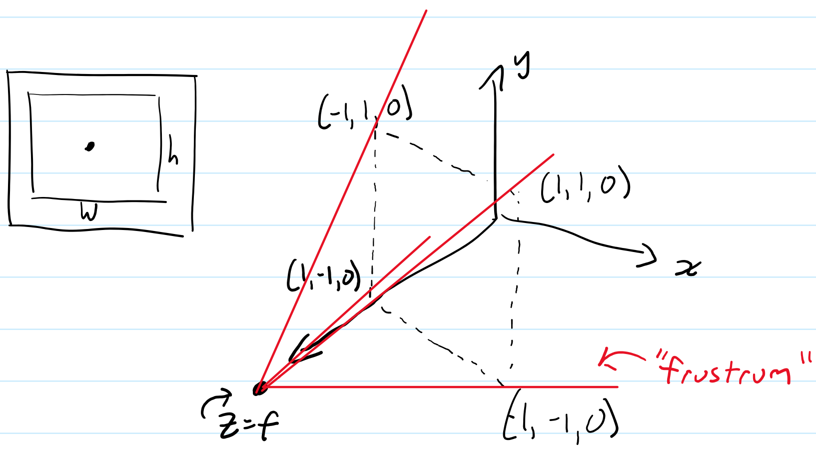
Tuesday, September 26, 2023 8:04 AM

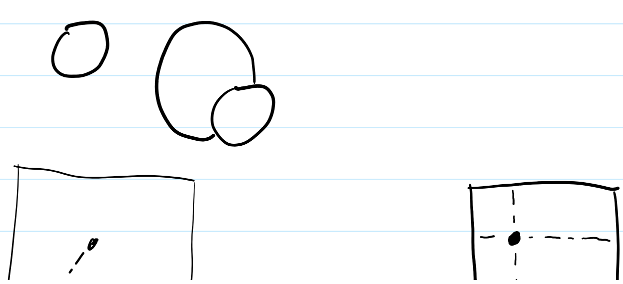
Rendering via ray tracing

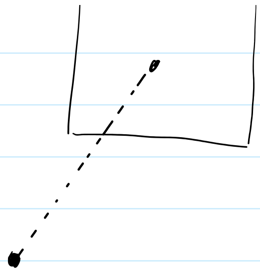
- Simple objects are easy, complex shapes are hard
  - We will focus on simple objects in this course
  - Doesn't scale up well, but things like shadows and reflections are easier

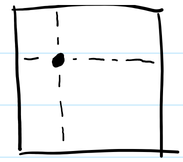


- We shoot a ray from our point z=f, and through our plane, which maps to our (w,h) plane on our canvas
  - Because as the ray covers a larger area as it goes further, we get perspective for free

In the fragment shader, we have to have a math description of these rays so that we can calculate per pixel, as well as descriptions of the shapes and ordering







A ray has 2 properties: where it starts, and its direction

direction: NOT $\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$, but $\begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}$, where $x^2+y^2+z^2=1$ (generally)

We want to normalize our vector: $\frac{\bar{a}}{|\bar{a}|}$

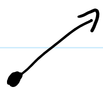
The origin is at $\bar{v}$, and the direction is $\bar{w}$.
We have 6 important numbers: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ of each (we can assume 1 and 0 for each w)

To check collision on sphere:

We have C_x, C_y, C_z and r

can be a vec4 in fragment shader

Pixel at $(x, y, 0)$
Camera at $(0,0,f) = \bar{v}$
$(x, y, -f) = \bar{w}$ (if you normalized it)



$\bar{v} + t \cdot \bar{w} \mid t > 0$ every possible point on our line from $\bar{v}$ in direction $\bar{w}$

$[v_x + t \cdot \bar{w}_x, v_y + t \cdot w_y, v_z + t \cdot w_z, 1]$

Is there any value of $t > 0$ where this coordinate hits the sphere?

$$(x - C_x)^2 + (y - C_y)^2 + (z - C_z)^2 - r^2 = 0$$

We need to compare this to our coordinate

$x = v_x + t \cdot \bar{w}_x$

$y = v_y + t \cdot \bar{w}_y$

$z = v_z + t \cdot \bar{w}_z$

$\bar{v}' = \bar{v} - \bar{C}$

$x = v'_x + t \cdot \bar{w}_x$

$y = v'_y + t \cdot \bar{w}_y$ "normalize"

$z = v'_z + t \cdot \bar{w}_z$

$x^2 + y^2 + z^2 - r^2 = 0$

$(v'_x + t \cdot \bar{w}_x)^2 + (v'_y + t \cdot \bar{w}_y)^2 + (v'_z + t \cdot \bar{w}_z)^2 - r^2 = 0$

${v'_x}^2 + 2(t \cdot \bar{w}_x) + t^2 \cdot \bar{w}_x^2$ (and for each coordinate)

$t^2(w_x^2 + w_y^2 + w_z^2) + t(2v'_x w_x + 2v'_y w_y + 2v'_z w_z) + ({v'_x}^2 + {v'_y}^2 + {v'_z}^2 - r^2) = 0$

$A^2t^2 + Bt + C = 0$, but A is always 1. Now solve for t

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-B \pm \sqrt{B^2 - 4C}}{2} = -\frac{B}{2} \pm \sqrt{\left(\frac{B}{2}\right)^2 - C}$$

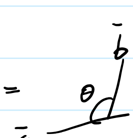
If t is a non-real root, we've missed

$\frac{B}{2} = \bar{v}' \cdot \bar{w}$, $C = v' \cdot v' - r^2$

$$t = -(\bar{v}' \cdot \bar{w}) \pm \sqrt{(\bar{v}' \cdot \bar{w})^2 - (v' \cdot v' - r^2)}$$

$\bar{a} \cdot \bar{b} = a_x b_x + a_y b_y + a_z b_z$

length $\rightarrow$

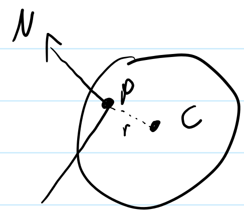
$= |\bar{a}||\bar{b}| \cdot \cos(\theta)$, $\theta =$ 

If a and b are right angles, dot product is 0. If opposite, negative. If the same, max value.

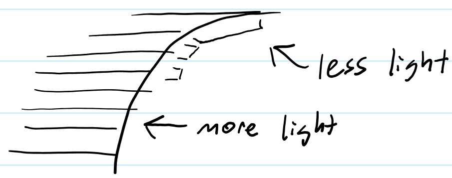
$P = \bar{v} + tw'$ (our point on the sphere that we've collided with)

$P = \bar{v} + t w'$ (our point on the sphere that we've collided with)

Lighting



$\frac{P - C}{r}$ is our normalized normal



$\max(0, \bar{N} \cdot \bar{L}) + A$
(Lambertian lighting)