CSc 320: Foundations of Computer Science (Summer 2022)

Alex Holland

Bonus Assignment

Question 1

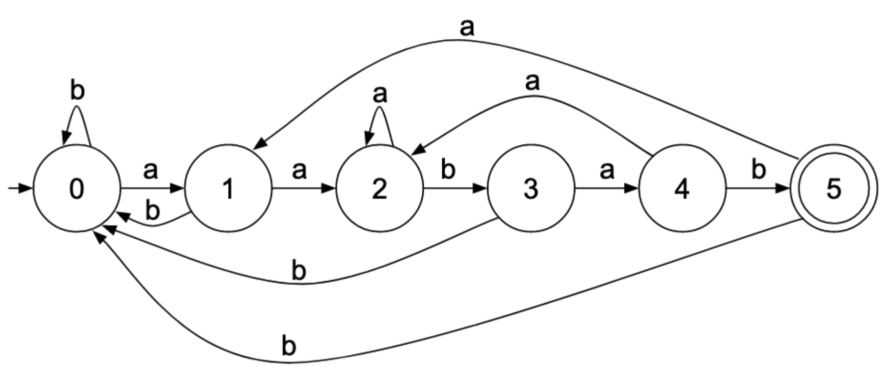
We want to construct the string-matching automaton for the pattern $P = aabab$ and illustrate its operation on the text string $T = aaababaabaababaab$.

First we can compute the transition function δ table. The states are $\{0, 1, 2, 3, 4, 5\}$

The only accepting state is 5.

State	a	b
0	1	0
1	2	0
2	2	3
3	4	0
4	2	5
5	1	0

For the transition table we can construct a state-transition diagram by creating 6 states for the *aabab* transitions, where state 0 is the start state and state 5 is the only accepting state.



With the state-transition diagram for pattern P created, we now have the ability to apply transition rules to strings that are a subset of the alphabet of the pattern P. Note that only the letters a and b are valid in the string-matching automaton.

We can now determine the states of at each index i in the string T.
At $i = 0$ and $T[0] = \emptyset$ we get: δ initially in state 0
At $i = 1$ and $T[1] = a$ we get: $state = \delta(0, a) = 1$
At $i = 2$ and $T[2] = a$ we get: $state = \delta(1, a) = 2$
At $i = 3$ and $T[3] = a$ we get: $state = \delta(2, a) = 2$
At $i = 4$ and $T[4] = b$ we get: $state = \delta(2, b) = 3$
At $i = 5$ and $T[5] = a$ we get: $state = \delta(3, a) = 4$
At $i = 6$ and $T[6] = b$ we get: $state = \delta(4, b) = 5$
At $i = 7$ and $T[7] = a$ we get: $state = \delta(5, a) = 1$
At $i = 8$ and $T[8] = a$ we get: $state = \delta(1, a) = 2$
At $i = 9$ and $T[9] = b$ we get: $state = \delta(2, b) = 3$
At $i = 10$ and $T[10] = a$ we get: $state = \delta(3, a) = 4$
At $i = 11$ and $T[11] = a$ we get: $state = \delta(4, a) = 2$
At $i = 12$ and $T[12] = b$ we get: $state = \delta(2, b) = 3$
At $i = 13$ and $T[13] = a$ we get: $state = \delta(3, a) = 4$
At $i = 14$ and $T[14] = b$ we get: $state = \delta(4, b) = 5$
At $i = 15$ and $T[15] = a$ we get: $state = \delta(5, a) = 1$
At $i = 16$ and $T[16] = a$ we get: $state = \delta(1, a) = 2$
At $i = 17$ and $T[17] = b$ we get: $state = \delta(2, b) = 3$

Every time we reach state 5 we know we have a match. When matching string P to string T we reached state 5 twice. The shift is the number of positions before the pattern occurrences in the string. We calculate the shift as $shift = i - 5$ at $i = 6$ and 14:

$$6 - 5 = 1$$
$$14 - 5 = 9$$

We can summarize our results in the following table, the strings highlighted in *blue* represent strings that match P:

i	$T[i]$	δ
0	$\emptyset$	0
1	a	1
2	a	2
3	a	2
4	b	3
5	a	4
6	b	5
7	a	1
8	a	2
9	b	3
10	a	4
11	a	2
12	b	3
13	a	4
14	b	5
15	a	1
16	a	2
17	b	3

Question 2

We want to compute the prefix function π for the pattern $ababbabbabbababbabb$ that we will denote as P. The prefix function can be defined as $\pi[q] = max\{k : k < q \ and \ P_k P_q\}$, where $\pi[q]$ is the length of the longest prefix of P that is suffix of P_q.

The prefix is the left-most end of the pattern. The suffix is the right-most end of the pattern. We can determine a list of prefixes and suffixes for the pattern $ababbabbabbababbabb$:
The list of prefixes is: $a, ab, aba, abab, ababb, ababba, ababbab, ababbabb, ababbabba, ababbabbab, ababbabbabb, ...$
The list of suffixes is: $b, bb, abb, babb, bbabb, abbabb, babbabb, ababbabb, bababbabb, bbababbabb, ...$

We want to find the the longest possible suffix of P which is also a the longest possible prefix of P_q. This is a subpattern of the string that appears in both the prefix and suffix.

For substring of length 1 which is a: $\pi[1] = 0$
For substring of length 2 which is ab: $\pi[2] = 0$
For substring of length 3 which is aba: $\pi[3] = 1$, because it starts and ends with a
For substring of length 4 which is $abab$: $\pi[4] = 2$, because it starts and ends with ab
For substring of length 5 which is $ababb$: $\pi[5] = 0$, because it starts with a and ends with b
For substring of length 6 which is $ababba$: $\pi[6] = 1$, because it starts and ends with a
For substring of length 7 which is $ababbab$: $\pi[7] = 2$, because it starts and ends with ab
For substring of length 7 which is $ababbabb$: $\pi[8] = 0$, because it starts with a and ends with b
For substring of length 7 which is $ababbabba$: $\pi[9] = 1$, because it starts and ends with a
For substring of length 7 which is $ababbabbab$: $\pi[10] = 2$, because it starts and ends with ab
For substring of length 7 which is $ababbabbabb$: $\pi[11] = 0$, because it starts with a and ends with b
For substring of length 7 which is $ababbabbabba$: $\pi[12] = 1$, because it starts and ends with a
For substring of length 7 which is $ababbabbabbab$: $\pi[13] = 2$, because it starts and ends with ab
For substring of length 7 which is $ababbabbabbaba$: $\pi[14] = 3$, because it starts and ends with aba
For substring of length 7 which is $ababbabbabbabab$: $\pi[15] = 4$, because it starts and ends with $abab$
For substring of length 7 which is $ababbabbabbababb$: $\pi[16] = 5$, because it starts and ends with $ababb$
For substring of length 7 which is $ababbabbabbababba$: $\pi[17] = 6$, because it starts and ends with $ababba$
For substring of length 7 which is $ababbabbabbababbab$: $\pi[18] = 7$, because it starts and ends with $ababbab$
For substring of length 7 which is $ababbabbabbababbabb$: $\pi[19] = 8$, because it starts and ends with $ababbabb$

Here we can see that that the longest possible suffix is $ababbabb$ as it appears in both the left-most and right-most part of the pattern P.

The results of the above calculations can be shown in the table of values π for the pattern P:

i	$P[i]$	$\pi[i]$
1	a	0
2	b	0
3	a	1
4	b	2
5	b	0
6	a	1
7	b	2
8	b	0
9	a	1
10	b	2
11	b	0
12	a	1
13	b	2
14	a	3
15	b	4
16	b	5
17	a	6
18	b	7
19	b	8

The prefix function π can be determined by scanning substrings of P of size q from left-to-right starting from an index $i = 1$ and determining if the prefix is the same as the suffix. For example, a substring of P of length 4 which would be *abab*, *ab* appears at both the start and end of the substring so by the prefix function $\pi[q] = max\{k : k < q \text{ and } P_k P_q\}$ we can determine that $\pi[4] = 2$. We continue this process for all indexes of P (from $i = 1$ to $i = 19$).