CSc 320: Foundations of Computer Science (Summer 2022)

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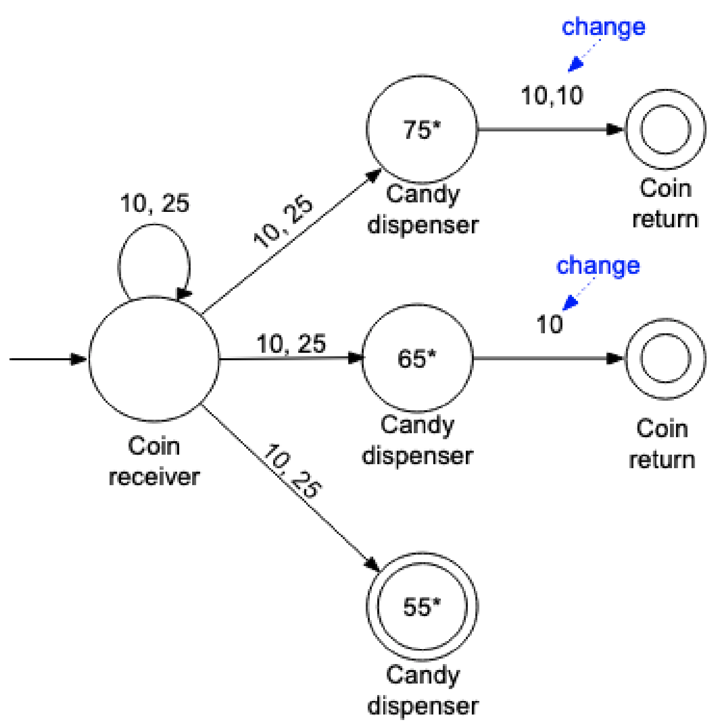
Assignment 1

Question 1



∗ indicates the total coin amount (¢) that the user has inputted into the candy machine.

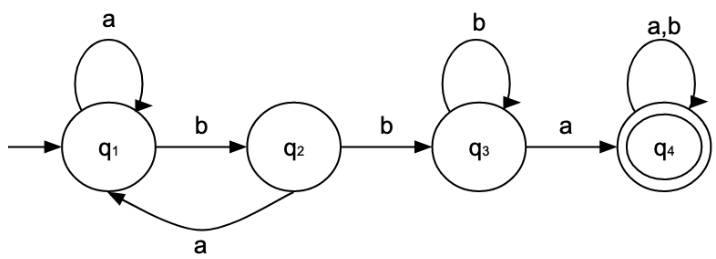
Informal state diagram assumptions:

- The user does not input an amount that the machine cannot provide change for. E.g. the user cannot input 60¢ because the machine can not give change in 5¢ amounts.
- Only one candy can be bought and dispensed at a time.
- The user does not input excessive coin amounts that exceed the cost of the candy (55¢). E.g. the user wont input more then three 25¢ coins because it is already enough to purchase a candy.
- $10, 10$ indicates a change amount of 20¢.

Question 2

$L_A = \{w \in \Sigma^* | w \text{ contains the substring } bba\}$.

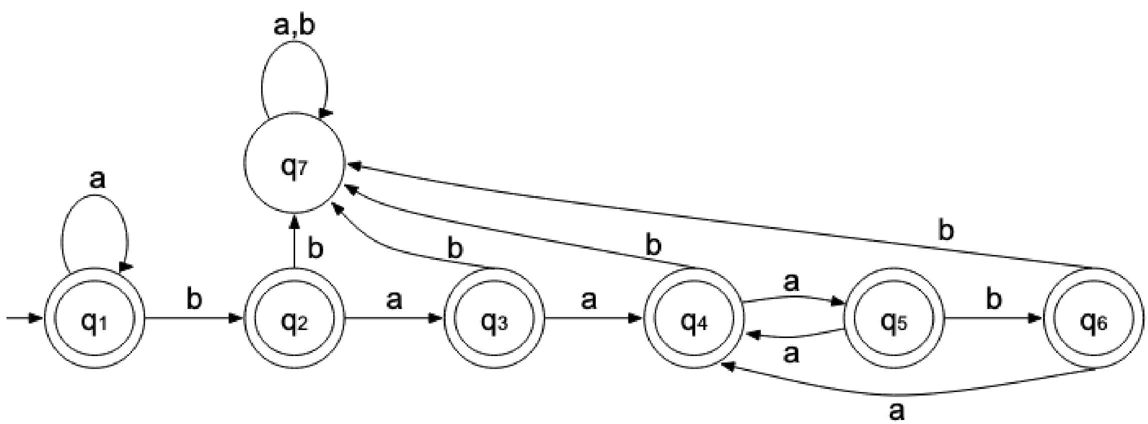
State diagram:



Transition Table:

δ	a	b
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_4	q_3
q_4	q_4	q_4

$L_B = \{w \in \Sigma^* | w \text{ each pair of consecutive } bs \text{ in } w \text{ is separated by a substring of } as \text{ that is of length } 3i, i > 0\}$.



Transition Table:

δ	a	b
q_1	q_1	q_2
q_2	q_3	q_7
q_3	q_4	q_7
q_4	q_5	q_7
q_5	q_4	q_6
q_6	q_4	q_7
q_7	q_7	q_7

Assumptions:

- "each pair of consecutive bs in w is separated by a substring of as" means that if we have 2 bs next to each other as a pair, they will be separated by a substring of $3i, i > 0$ as. E.g. $baaab$ is an accepted string.

Question 3

The state diagram $F1$ recognizes strings that have a maximum of one '1' symbol and unlimited '0' symbols. If more then one '1' symbol in the string ω is used in the $F1$, the input will become stuck in the transition state C.

Examples of strings (ω) accepted by $F1$:

$$0,\ 1,\ 01,\ 10,\ 00,\ 100,\ 010,\ 001$$

Examples of strings (ω) not accepted by $F1$:

$$11,\ 110,\ 011,\ 101,\ 111,\ 1110,\ 1101,\ 0111$$

Question 4

$\omega_1 = \epsilon$

The automation stays in $q1$ state (initial state), which is an accepted state. Hence, ω_1 is accepted by D.

$\omega_2 = 0111$

$$\begin{gathered}\text{Start in } q_1\text{, read } 0\\ \text{Start in } q_3\text{, read } 1\\ \text{Start in } q_4\text{, read } 1\\ \text{Start in } q_3\text{, read } 1\\ \text{In } q_4\end{gathered}$$

The string ω_2 is accepted in D.

$\omega_3 = 100000100$

$$\begin{gathered}\text{Start in } q_1\text{, read } 1\\ \text{Start in } q_4\text{, read } 0\\ \text{Start in } q_2\text{, read } 0\\ \text{Start in } q_4\text{, read } 0\\ \text{Start in } q_2\text{, read } 0\\ \text{Start in } q_4\text{, read } 0\\ \text{Start in } q_2\text{, read } 1\\ \text{Start in } q_3\text{, read } 0\\ \text{Start in } q_1\text{, read } 0\\ \text{In } q_3\end{gathered}$$

The string ω_3 is not accepted in D.

$\omega_4 = 1010110$

$$
\begin{gathered}
\text{Start in } q_1\text{, read } 1 \\
\text{Start in } q_4\text{, read } 0 \\
\text{Start in } q_2\text{, read } 1 \\
\text{Start in } q_3\text{, read } 0 \\
\text{Start in } q_1\text{, read } 1 \\
\text{Start in } q_4\text{, read } 1 \\
\text{Start in } q_3\text{, read } 0 \\
\text{In } q_1
\end{gathered}
$$

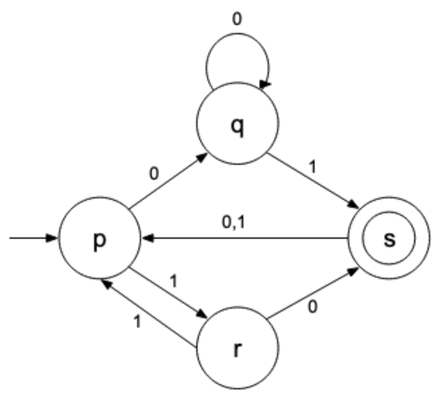
The string ω_4 is accepted in D.

$\omega_5 = 1111111$

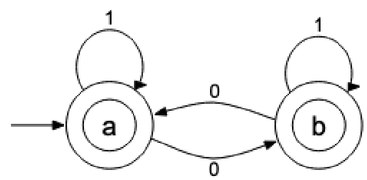
$$
\begin{gathered}
\text{Start in } q_1\text{, read } 1 \\
\text{Start in } q_4\text{, read } 1 \\
\text{Start in } q_3\text{, read } 1 \\
\text{Start in } q_4\text{, read } 1 \\
\text{Start in } q_3\text{, read } 1 \\
\text{Start in } q_4\text{, read } 1 \\
\text{Start in } q_3\text{, read } 1 \\
\text{In } q_4
\end{gathered}
$$

The string ω_5 is accepted in D.

Question 5
$M = (\{p, q, r, s\}, \{0, 1\}, \delta_M, p, \{s\})$



$N = (\{a, b\}, \{0, 1\}, \delta_N, a, \{a, b\})$



$A = (\{p, q, r, s, a, b\}, \Sigma, \delta, (p, a), \{(p, a), (q, a), (r, a), (s, a), (p, b), (q, b), (r, b)(s, b)\})$

Transition Table:

δ_A	0	1
(p, a)	(q, b)	(r, a)
(r, a)	(s, b)	(p, a)
(s, a)	(p, b)	(p, a)
(q, a)	(q, b)	(s, a)
(p, b)	(q, a)	(r, b)
(r, b)	(s, a)	(p, b)
(s, b)	(p, a)	(p, b)
(q, b)	(q, a)	(s, b)

Regular languages are closed under union, and since N accepts all strings (all of it's states are accept states), determining $L(A) = L(M) \cup L(N)$ must mean that all states of A are accept states.

State Diagram: