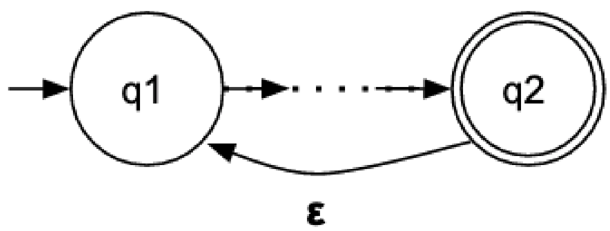
CSc 320: Foundations of Computer Science (Summer 2022)

Alex Holland

Assignment 2

Question 1

We want to show that given any regular language L, L^+ is regular. We can do this by taking a DFA D that recognizes L and constructing an NFA N^+. If DFA $D = (Q, \Sigma, \delta, q_0, F)$, then by definition $L^+ = L^* - \epsilon$. Thus the empty string ϵ is not contained within L or L^+. Every element in L^+ is accepted by D since L^+ contains at least one combination of strings that are already in L. If we create a DFA with a defined start and accept state such as q_1 and q_2 respectively, we can introduce an ϵ for the N^+ that goes from q_2 to q_1. Note that this transition will work for all DFA's to NFA's with states between q_1 and q_2.



Every element of L^+ contains elements of L, we have N^+ that accepts L^+. Therefore L^+ is a regular language.

Question 2
We can represent the states in the Transition Table:

δ	0	1	ϵ
q_1	$\{q_2\}$	$\{q_2\}$	$\emptyset$
q_2	$\{q_5\}$	$\{q_3\}$	$\emptyset$
q_3	$\{q_3\}$	$\{q_3\}$	$\emptyset$
q_5	$\{q_6\}$	$\{q_1\}$	$\emptyset$
q_6	$\{q_6\}$	$\{q_6\}$	$\emptyset$
q_7	$\{q_2\}$	$\{q_2\}$	$\emptyset$

The states, start state, and final states are the same for the equivalent NFA A_N.
Thus, $Q_N = Q_D$, $q_N = q_D$, $F_N = F_D$

Question 3
Following the construction presented in class, given NFA $A_N = (Q_N, \Sigma, \delta_N, q_N, F_N)$ we can prove that for every NFA there exists an equivalent DFA. $Q_D \subseteq P(Q_N) = \{\{q_0\}, \{q_1\}, \{q_2\}, \{q_3\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_3\}, \{q_1, q_2\}, \{q_1, q_3\}, \{q_2, q_3\}, \{q_0, q_1, q_2\}, \{q_0, q_1, q_3\}, \{q_1, q_2, q_3\}, \{q_0, q_1, q_2, q_3, \}\}$

δ_N	a	b
$\emptyset$	$\emptyset$	$\emptyset$
$\{q_0\}$	$\{q_2, q_3\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_3\}$	$\emptyset$
$\{q_2\}$	$\{q_2, q_3\}$	$\{q_2\}$
$\{q_3\}$	$\emptyset$	$\emptyset$
$\{q_0, q_1\}$	$\{q_2, q_3\}$	$\{q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_2, q_3\}$	$\{q_1, q_2\}$
$\{q_1, q_3\}$	$\{q_3\}$	$\emptyset$
$\{q_2, q_3\}$	$\{q_2, q_3\}$	$\{q_2\}$

$Q_D \subseteq P(Q_N) = \{\emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_3\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_3\}, \{q_1, q_2\}, \{q_1, q_3\}, \{q_2, q_3\}, \{q_0, q_1, q_2\}, \{q_0, q_1, q_3\}, \{q_0, q_2, q_3\}, \{q_1, q_2, q_3\}, \{q_0, q_1, q_2, q_3\}\}$

$Q_D = \{\emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_3\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_3\}, \{q_2, q_3\}, \{q_0, q_1, q_2, q_3\}\}$

q_D is the start state and is $\{q_0, q_2\}$

The accept states in column a are $F_D = \{q_2, q_3\}, \{q_3\}$

Question 4
$L_1 = \{wa | w \in \{a, b\}^*\} \cup \{\epsilon\}$ is not recognized by A. Since ϵ is not accepted. E.g. abaϵ

$L_2 = L((a \cup b)^* a)$ is recognized by A. Since any number of a and b's is followed by an a is accepted. E.g. aba

$L_3 = L((a \cup b)^*)$ is not recognized by A. Since if there is any number of a,b's is followed by a b, then the string is not accepted. E.g. ab

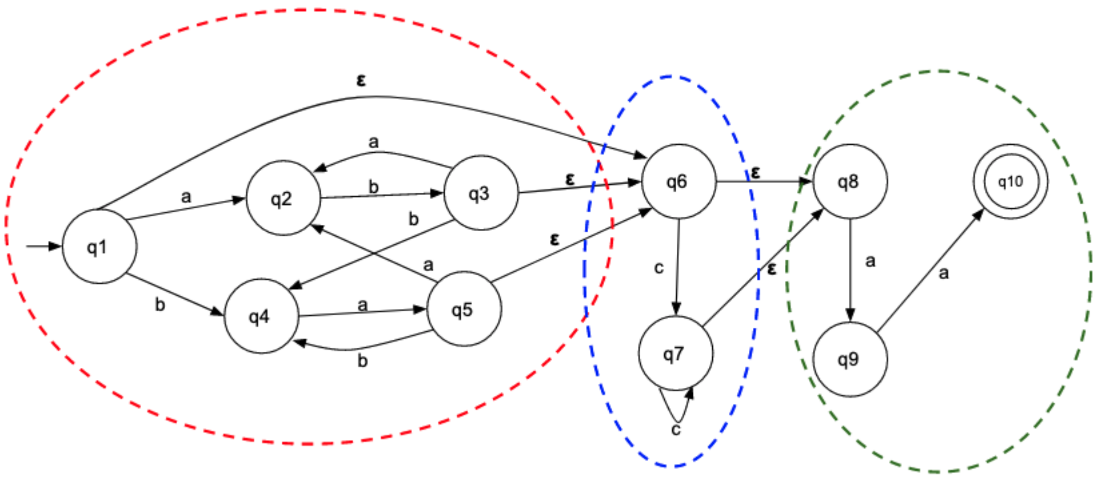
$L_4 = L((a \cup b)^* \cup b)L(a)$ is recognized by A. Since any number of a,b's is followed by b, followed by a is accepted. E.g. aba

$L_5 = \{wa | w \in \{a, b\}^*\}$ is recognized by A. Since any number of a,b's is followed by b. E.g. ababa

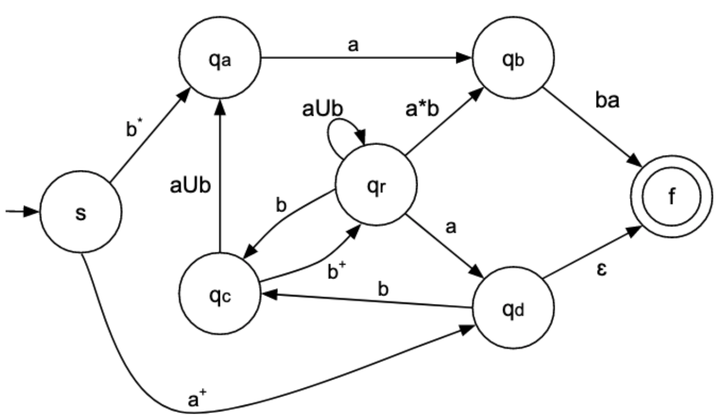
$L_6 = \{wbaa | w \in \{a, b\}^*\}$ is recognized by A.

Question 5
(a)
In $L(R_1)$ any strings that start with any number of a and b's, follwed by any number of c's, followed by aa.
E.g. $abccaa$, $bbbcaa$, aa.

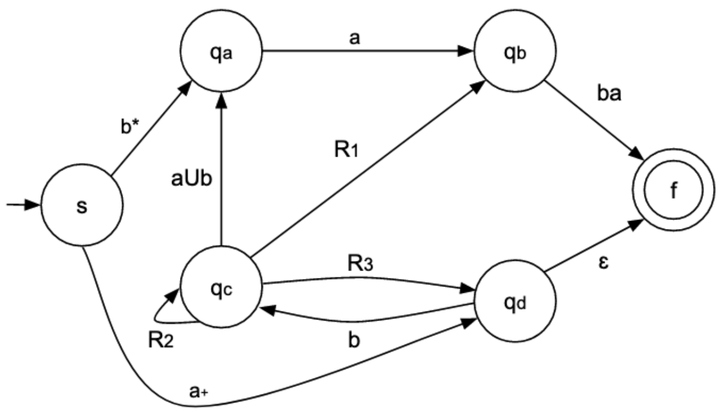
(b)
State diagram for M_1 with $L(R_1) = L(M_1)$
The red circle indicates the portion: $(ab \cup ba)^*$
The blue circle indicates the portion: $(c)^*$
The green circle indicates the portion: aa

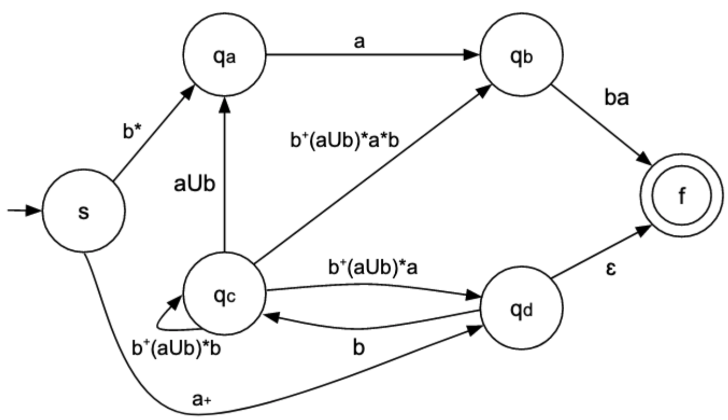


Question 6
With the following GNFA, we want to determine the state diagram after ripping out state q_r



The state that goes into q_r is q_c. The states that go out of q_r is q_b, q_c, and q_d.
The transitions from q_c to q_d can be done by going through $b^+ \to a \cup b$ (any number of times) $\to a$.
The transitions from q_c to q_c can be done by goring through $b^+ \to a \cup b$ (any number of times) $\to b$
The transitions from q_c to q_b can be done by goring through $b^+ \to a \cup b$ (any number of times) $\to a^*b$





after ripping out q_r we get that $R_1 = b^+(a \cup b)^*a^*b$, $R_2 = b^+(a \cup b)^*b$, and $R_3 = b^+(a \cup b)^*a$