CSc 320: Foundations of Computer Science (Summer 2022)

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Assignment 4

Question 1

Consider the language $L = \{a^n b^m a^n b^m | m, n \geq 0\}$ Assume that L is a context-free language.
If we can prove that this string cannot be pumped, then L is not a context free language.
Let p be the pumping length for L
Choose string $s = a^p b^p a^p b^p,\ s \in L,\ |s| \geq p$

According to PL:

1. for each $i \geq 0,\ uv^i xy^i z \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$

From 2: $vy \neq \epsilon$, i.e. $v \neq \epsilon$ or $y \neq \epsilon$

Fome 3: $|vxy| \leq p$, yields the following cases:

- $vxy = a...a \Longrightarrow uv^2xy^2z = a^k b^p a^l b^p$ with $k > p$ or $l > p$
- $vxy = b...b \Longrightarrow uv^2xy^2z = a^p b^k a^p b^l$ with $k > p$ or $l > p$
- $vxy = a...ab...b \Longrightarrow$ either $wv^2xy^2z = a^p b^p a^k b^l$ or $wv^2xy^2z = a^k b^l a^p b^p$ with $k > p$ or $l > p$
- $vxy = b...ba...a \Longrightarrow uv^2xy^2z = a^p b^k a^l b^p$ with $k > p$ or $l > p$

There is no rewriting of s into $s = uvxyz$ and $uv^2xy^2z \in L$. Therefore string s cannot be pumped without violating the pumping lemma for CFL, hence L is not a context-free language

Question 2
Let L_1 and L_2 be two decidable Languages. M_1 and M_2 be the Turing machines that decides L_1 and L_2 respectively. There exists a Turing machine M such that, $L(M) = L_1 \cup L_2$.

We can create a description of M for an input of w as follows:

1. Run M_1 on w. If M_1 accepts, **accept**.
2. Else run M_2 on w. If M_2 accepts, **accept**, Else **reject**."

M Accepts w if either M_1 or M_2 accepts it, or if both M_1 and M_2 rejects, then M rejects. If $w \in L_1 \cup L_2$. Then w is in either one of or both L_1 and L_2. Likewise, if $w \notin L_1 \cup L_2$, then both M_1 and M_2 reject w, so M rejects $w \notin L_1 \cup L_2$. If $w \in L_1$, then M_1 accepts w, and M will then accept w, regardless of the result of M_2. M_1 rejects w if $w \in L_2$ and $w \notin L_1$ since M_1 is a decider, and since $w \in L_2$, M_2 would accept w, so M would also then accept w. Therefore, there exists a Turing machine M that recognizes $L_1 \cup L_2$, $\therefore$ the class of decidable languages is closed under the union operation.

Question 3
Consider language $L = \{a^n b^n c^n | n \geq 0\}$. The following is an implementation-level description of a Turing machine that decides L.

"On input string w:

0. Scan the tape from left to right to make sure that the tape input is in the form $a^n b^n c^n$, $n \geq 0$, if it is not, **reject**.
1. Return tape head to left-hand end of tape.
2. Repeat the following steps until no more unmarked a's are left on the tape
   - Scan the tape for the first unmarked (leftmost) a, mark it.
   - Scan right until an unmarked b is found in the tape and mark it. If there are no b's, **reject**.
   - Scan right until an unmarked c is found in the tap and mark it. If there are no c's, **reject**.
   - Move the tape head back to the left-hand side of the tape, and go to step 2.
3. If the tape contains any unmarked b's or c's, **reject**. Otherwise, **accept**."