CSc 320: Foundations of Computer Science (Summer 2022)

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Assignment 3

Question 1

$\{q_0, q_1\}$ $\cancel{\{q_1, q_2\}}$ $\{q_2, q_3\}$ $\cancel{\{q_3, q_4\}}$ $\cancel{\{q_4, q_5\}}$ $\cancel{\{q_5, q_6\}}$ $\{q_6, q_7\}$
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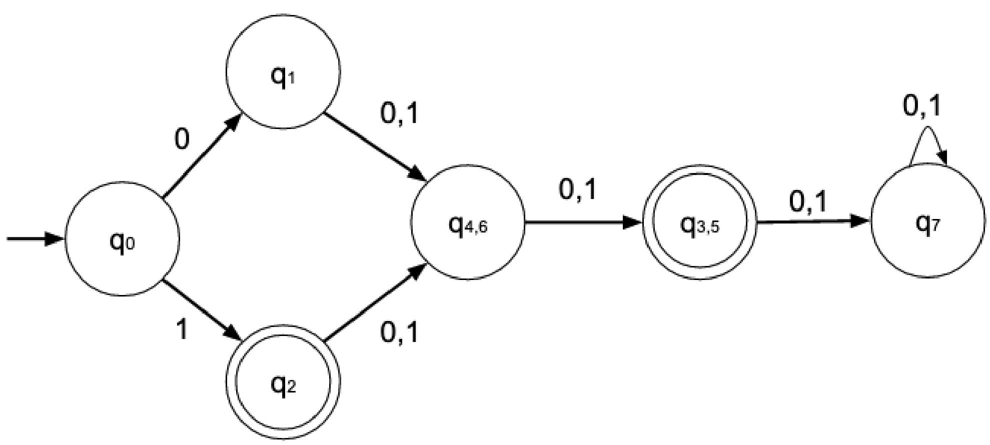
$\cancel{\{q_0, q_7\}}$

$\cancel{\{q_1, q_7\}}$ $\cancel{\{q_2, q_3\}}$ $\{q_3, q_5\}$ $\{q_4, q_6\}$
$\cancel{\{q_2, q_5\}}$

$\{q_3, q_5\}\{q_4, q_6\}$

$q_3 \sim q_5, q_4 \sim q_6$



Question 2
Assume for $L = \{w \in \{0,1\}^* | w = 0^i0101^{2i}, i \geq 0\}$, that L is regular.
Let $s = 0^p0101^{2p} : s \in L$ and $|s| \geq p$
We can split the string s into 3 parts $s = xyz$ satisfying the conditions:

1. $xy^iz \in L$ for each $i \geq 0$
2. $|y| > 0$
3. $|xy| \leq p$

Observe all decomposition's of s:

- $x = 0^\alpha$ where $\alpha \geq 0$
- $y = 0^\beta$ where $\beta \geq 1$ $(\alpha + \beta \leq p)$
- $0^{p-\alpha-\beta}0101^{2p} = 0^{p-\alpha-\beta+1}101^{2p}$

Choose one i such that $xy^iz \notin L$.
Let's pick $i = 2$:

$$\begin{aligned} xy^2z &= 0^\alpha 0^\beta 0^\beta 0^{p-\alpha-\beta+1}101^{2p} \\ &= 0^{p+\beta+1}101^{2p} \end{aligned}$$

$0^{p+\beta+1}101^{2p} \in L$ iff $p + \beta + 1 = 2p$
We can simplify $p + \beta + 1 = 2p$ to $\beta + 1 = p$
From rules 2 and 3:
We have $\alpha = 0$ and $\beta = p$. Since $\beta = p$ and in the case since $i = 2$, $p + 1 \neq p$ which is a contradiction. Thus, the language L is not regular

Question 3
a)
$L_1 = \{w \in \{a,b\}^* | w \text{ contains at least five } as\}$

$$\begin{aligned} L_1 : A &\rightarrow BaBaBaBaBaB \\ B &\rightarrow BB|a|b|\epsilon \end{aligned}$$

b)
$L_2 = \{a^i b^j c^k | i,j,k \geq 0 \text{ and } i = j \text{ or } i = k\}$

$$\begin{aligned} L_2 : A &\rightarrow BC|D \\ B &\rightarrow aBb|\epsilon \\ C &\rightarrow cC|\epsilon \\ D &\rightarrow aDc|E \\ E &\rightarrow bE|\epsilon \end{aligned}$$

c)

Lets consider $A \rightarrow BC|D$
Leftmost derivation 1:

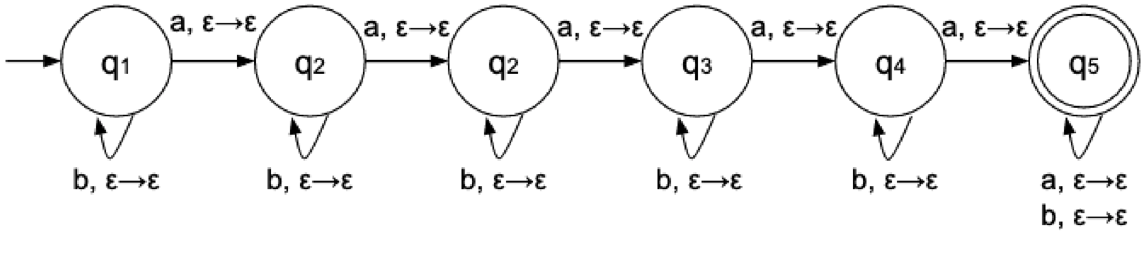
$$\begin{aligned} A &\rightarrow BC \\ &\rightarrow \epsilon C \;\; (since\ B \rightarrow \epsilon) \\ &\rightarrow \epsilon\epsilon \;\;\; (since\ C \rightarrow \epsilon) \\ &\rightarrow \epsilon \end{aligned}$$

Leftmost derivation 2:

$$\begin{aligned} A &\rightarrow D \\ &\rightarrow E \;\; (since\ D \rightarrow E) \\ &\rightarrow \epsilon \;\; (since\ E \rightarrow \epsilon) \end{aligned}$$

Since we can achieve two strings that are in the grammar of L_2 with two different leftmost derivations. The grammar for b) is inherently ambiguous.

Question 4
$L_1 = \{w \in \{a,b\}^* | w \text{ contains at least five } as\}$
L_1 is a regular language, so the language has a corresponding DFA that we can convert into the following PDA. As such, we can use the same states and transitions for both a DFA and PDA and we are not required to push or pop anything from a stack.



Question 5

$S \to ASA|AS|0A|\epsilon$
$A \to 001|\epsilon$

Add new start symbol
$S_0 \to S$
$S \to ASA|AS|0A|\epsilon$
$A \to 001|\epsilon$

Eliminate epsilon rules
$S_0 \to S$
$S \to ASA|AS|0A|\epsilon|S|SA|0$
$A \to 001$

$S_0 \to S|\epsilon$
$S \to ASA|AS|0A|S|SA|0|AA|A$
$A \to 001$

$S_0 \to S|\epsilon$
$S \to ASA|AS|0A|SA|0|AA|A$
$A \to 001$

Eliminate unit rules
$S_0 \to ASA|AS|0A|SA|0|AA|A|\epsilon$
$S \to ASA|AS|0A|SA|0|AA|A$
$A \to 001$

$S_0 \to ASA|AS|0A|SA|0|AA|001|\epsilon$
$S \to ASA|AS|0A|SA|0|AA|001$
$A \to 001$

Convert remaining rules
$S_0 \to AW|AS|0A|SA|0|AA|001|\epsilon$
$S \to AW|AS|0A|SA|0|AA|001$
$A \to 001$
$W \to SA$

$$
\begin{aligned}
&S_0 \rightarrow AW|AS|XA|SA|0|AA|001|\epsilon \\
&S \rightarrow AW|AS|XA|SA|0|AA|001 \\
&A \rightarrow 001 \\
&W \rightarrow SA \\
&X \rightarrow 0
\end{aligned}
$$

$$
\begin{aligned}
&S_0 \rightarrow AW|AS|XA|SA|0|AA|XY|\epsilon \\
&S \rightarrow AW|AS|XA|SA|0|AA|XY \\
&A \rightarrow XY \\
&W \rightarrow SA \\
&X \rightarrow 0 \\
&Y \rightarrow 01
\end{aligned}
$$

$$
\begin{aligned}
&S_0 \rightarrow AW|AS|XA|SA|0|AA|XY|\epsilon \\
&S \rightarrow AW|AS|XA|SA|0|AA|XY \\
&A \rightarrow XY \\
&W \rightarrow SA \\
&X \rightarrow 0 \\
&Y \rightarrow XZ \\
&Z \rightarrow 1
\end{aligned}
$$

G is now in Chomsky Normal Form.