ECE 260: Continuous-Time Signals and Systems

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Assignment 2A

2.1 **[notation]**

(a)

$$\mathcal{H}x(t) = t^2 + 1$$
$$(\mathcal{H}x)(t) = [(t)^2] + 1$$

(b)

$$\mathcal{G}\mathcal{H}y(t)$$
$$[\mathcal{G}(\mathcal{H}y)](t)$$

(c)

$$\mathcal{H}x + y$$
$$(\mathcal{H}x) + y$$

(d)

$$x\mathcal{H}\mathcal{G}y$$
$$(x)[\mathcal{H}(\mathcal{G}y)]$$

2.2 **[notation]**

(a) the output of system $\mathcal{H}$ when its input is y:

$$\mathcal{H}y$$

(b) the output of system $\mathcal{H}$ evaluated at $2t - 1$ when the input to the system is x:

$$\mathcal{H}x(2t - 1)$$

(c) the output of system $\mathcal{H}$ evaluated at t when the input to the system is ax:

$$\mathcal{H}\{ax\}(t)$$

(d) the output of system $\mathcal{H}$ evaluated at $5t$ when the input to the system is $x + y$:

$$\mathcal{H}\{x + y\}(5t)$$

(e) the derivative of the output of the system $\mathcal{H}$ when its input is ax:

$$\mathcal{D}\mathcal{H}(ax)$$

(f) the output of the system $\mathcal{H}$ when its input is the derivative of ax:

$$\mathcal{H}\mathcal{D}(ax)$$

(g) the sum of: 1) the output of the system $\mathcal{H}$ when its input is x; and 2) the output of the system $\mathcal{H}$ when its input is y:

$$\mathcal{H}x + \mathcal{H}y$$

(h) the output of the system $\mathcal{H}$ when its input is $x + y$:

$$\mathcal{H}(x + y)$$

(i) the derivative of x evaluated at $5t - 3$:

$$\mathcal{D}x(5t - 3)$$

3.1 **[time/amplitude transformations]**
(f)
$y(t) = x(7[t + 3])$
We must do the following transformations:

1. time shift left by 21
2. time scale by compressing horizontally by a factor of 7

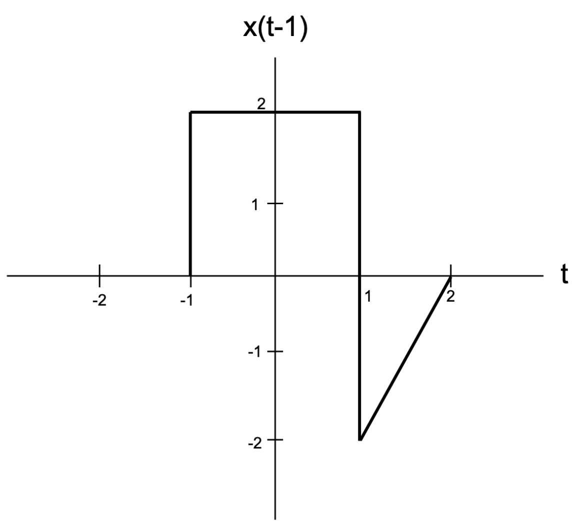
3.2 **[time transformation]**
$x_2(t)$ is generated from $x_1(t)$ from the following transformations:

1. time shift left by 1
2. time scaling by 4
3. time reversal

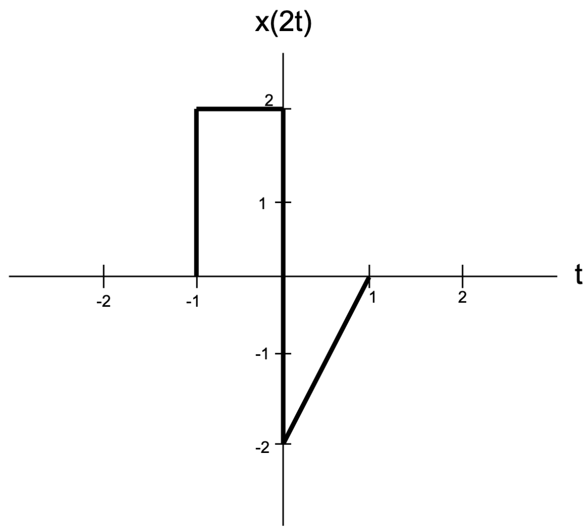
Which we can represent as:

$$x_2(t) = x_1(-4t - 1)$$

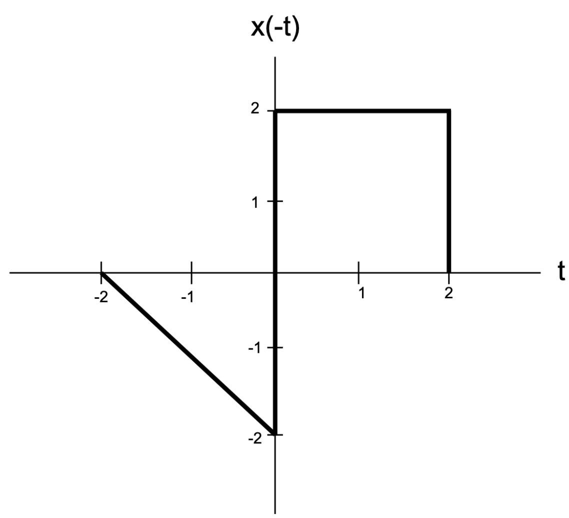
3.4 **[time/amplitude transformations]**
(a)



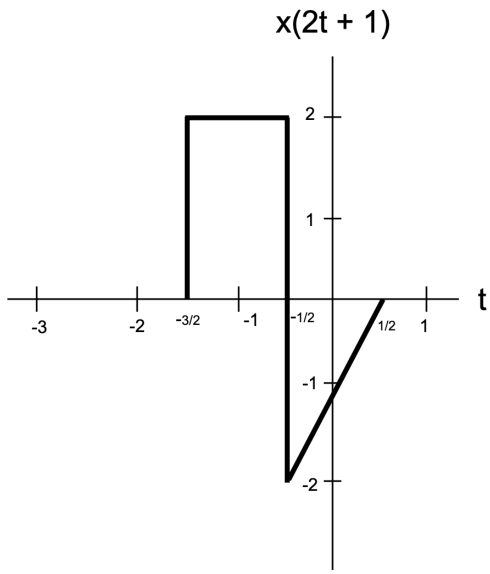
(b)



(c)



(d)



3.6 **[periodicity]**
(e)

$$\begin{aligned}
x(t) &= cos(14t-1) + cos(77t-3) \\
T_1 &= \frac{2\pi}{14} \\
T_2 &= \frac{2\pi}{77} \\
\frac{T_1}{T_2} &= \frac{\frac{2\pi}{14}}{\frac{2\pi}{77}} \\
&= \frac{11}{2}
\end{aligned}$$

$\therefore$ the function x is periodic and $T = 2T_1 = \frac{2\pi}{7}$

(f)

$$\begin{aligned}
x(t) &= cos(et) + sin(42t) \\
T_1 &= \frac{2\pi}{e} \\
T_2 &= \frac{2\pi}{42} \\
\frac{T_1}{T_2} &= \frac{\frac{2\pi}{e}}{\frac{2\pi}{42}} \\
&= \frac{42}{e}
\end{aligned}$$

$\frac{42}{e}$ is irrational $\therefore$ the function x is not periodic

(g)

$$
\begin{aligned}
x(t) &= |sin(\pi t)| \\
T &= \frac{2\pi}{\pi} \\
&= 2
\end{aligned}
$$

The absolute value of sine cut's the period in half.

$$
\begin{aligned}
T &= \frac{2}{2} \\
&= 1
\end{aligned}
$$

$\therefore$ the function x is periodic and $T = 1$

3.9 **[even/odd symmetry]**
(c)

$$
\begin{aligned}
x(t) &= |t^3| \\
x(t) &= x(-t) \\
|t|^3 &= |-t|^3 \\
t^3 &= t^3
\end{aligned}
$$

$\therefore$ the function is even

(d)

$$
\begin{aligned}
x(t) &= cos(2\pi t)sin(2\pi t) \\
x(-t) &= (cos2\pi(-t))(sin2\pi(-t)) \\
&= cos2\pi t(-sin2\pi t) \\
&= -cos(2\pi t)sin(2\pi t)
\end{aligned}
$$

$\therefore$ the function is odd

3.10 **[symmetry and sums/products]**
(b)
We want to prove that the sum of two odd functions is odd.
Let $x_1(t)$ and $x_2(t)$ be odd functions.

$$
\begin{aligned}
x_1(-t) &= -x_1(t) \\
x_2(-t) &= -x_2(t)
\end{aligned}
$$

$$
\begin{aligned}
(x_1 + x_2)(t) &= x_1(t) + x_2(t) \\
(x_1 + x_2)(-t) &= x_1(-t) + x_2(-t) \\
&= -x_1(t) - x_2(t) \\
&= -(x_1 + x_2)(t)
\end{aligned}
$$

$\therefore$ the sum of two odd functions is odd.

3.17 **[even/odd decomposition, signal properties]**

$$\begin{aligned} h_e(t) &= t[u(t) - u(t-1)] + u(t-1) \text{ for } t \geq 0 \\ &= tu(t) + (-t+1)u(t-1) \end{aligned}$$

Since h is causal and has the even part h_e we can determine h as follows:

Since $h_e(t)$ is even, for all $t < 0$:

$$\begin{aligned} h_e(t) &= h_e(-t) \\ &(-t)u(-t) + (t+1)u(-t-1) \end{aligned}$$

Since $h_o(t) = -h_e(t)$, then for $t < 0$:

$$\begin{aligned} h_o(t) &= -h_e(t) \\ &= -[(-t)u(-t) + (t+1)u(-t-1)] \\ &= tu(-t) + (-t-1)u(-t-1) \end{aligned}$$

For $t < 0$ we can determine $h_o(t) = -h_e(t)$:

$$\begin{aligned} h_o(t) &= -h_e(t) \\ &= -[(-t)u(-t) + (t+1)u(-t-1)] \\ &= tu(t) + (-t-1)u(t-1) \end{aligned}$$

$\therefore$ we can determine $h(t) = h_o(t) + h_e(t)$:

$$\begin{aligned} h(t) &= h_o(t) + h_e(t) \\ &= [tu(t) + (-t+1)u(t-1)] + [tu(t) + (-t+1)u(t-1)] \\ &= (2t)[u(t) - u(t-1)] + 2u(t-1) \end{aligned}$$

3.18 **[signal properties]**
(b) We are given the following properties regarding a function x:

- $x(t) = t - 1$ for $0 \leq t \leq 1$;
- the function v is casual, where $v(t) = x(t-1)$; and
- the function w is odd, where $w(t) = x(t) + 1$.

Since $v(t) = x(t-1)$ is causal, we get

$$\begin{aligned} v(t) &= 0 \text{ for } t < 0 \\ x(t-1) &= 0 \text{ for } t < 0 \\ x(t) &= 0 \text{ for } t + 1 < 0 \\ x(t) &= 0 \text{ for } t < -1 \end{aligned}$$

As stated, $w(t) = x(t) + 1$ is odd, so x is shifted up by 1.

$\therefore$ we can conclude that the piecewise function $x(t)$ for all t is

$$= \begin{cases} 0 & t < -1 \\ t - 1 & -1 \leq t \leq 1 \\ -2 & t > 1 \end{cases}$$

3.20 **[properties of delta functions]**
(a)

$$\begin{aligned}\int_{-\infty}^{\infty} sin(2t+\frac{\pi}{4})\delta(t)dt &= [sin(2t+\frac{\pi}{4}]\big|_{t=0}\\ &= sin(\frac{\pi}{4})\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

(b)

$$\begin{aligned}\int_{-\infty}^{t} cos(\tau)\delta(\tau+\pi)d\tau &= \begin{cases} cos(\tau)\big|_{t=-\pi} & \text{for } t>-\pi \\ 0 & \text{for } t<-\pi\end{cases}\\ &= \begin{cases} cos(-\pi) & \text{for } t>-\pi \\ 0 & \text{for } t<-\pi\end{cases}\\ &= \begin{cases} -1 & \text{for } t>-\pi \\ 0 & \text{for } t<-\pi\end{cases}\\ &= -u(t+\pi)\end{aligned}$$

(c)

$$\int_{-\infty}^{\infty} x(t)\delta(at-b)dt, \text{ where } a \text{ and } b \text{ are real constants and } a\neq 0$$

$$\begin{aligned}&= \begin{cases} \int_{-\infty}^{\infty} x(\frac{\lambda}{a})\delta(\lambda-b)(\frac{1}{a})d\lambda & \text{for } a>0 \\ \int_{\infty}^{-\infty} x(\frac{\lambda}{a})\delta(\lambda-b)(\frac{1}{a})d\lambda & \text{for } a<0\end{cases}\\ &= \begin{cases} \frac{1}{a}\int_{-\infty}^{\infty} x(\frac{\lambda}{a})\delta(\lambda-b)d\lambda & \text{for } a>0 \\ -\frac{1}{a}\int_{\infty}^{-\infty} x(\frac{\lambda}{a})\delta(\lambda-b)d\lambda & \text{for } a<0\end{cases}\\ &= \frac{1}{|a|}\int_{-\infty}^{\infty} x(\frac{\lambda}{a})\delta(\lambda-b)d\lambda\\ &= \frac{1}{|a|}[x(\frac{\lambda}{a})]\big|_{\lambda=b}\\ &= \frac{1}{|a|}x(\frac{b}{a})\end{aligned}$$

(f)

$$\begin{aligned}\int_{0}^{\infty} \tau^2 cos(\tau)\delta(\tau+42)d\tau &= \int_{0}^{\infty} 0 d\tau\\ &= 0\end{aligned}$$

D.101 **[MATLAB identifiers]**
(a) $4ever$ - invalid, first character must be a letter
(b) $\$rich\$$ - invalid, first character must be a letter
(c) $foobar$ - valid
(d) foo_bar - valid
(e) $\_foobar$ - invalid, first character must be a letter

D.106 [**MATLAB expressions**

(a)

```
v = [0 1 2 3 4 5]
2*v-3
ans = -3 -1 1 3 5 7
```

(b)

```
v = [0 1 2 3 4 5]
1./(v+1)
ans = 1.0000 0.5000 0.3333 0.2500 0.2000 0.1667
```

(c)

```
v = [0 1 2 3 4 5]
v.^5 - 3
ans = -3 -2 29 240 1021 3122
```

(d)

```
v = [0 1 2 3 4 5]
abs(v) + v.^4
ans = 0 2 18 84 260 630
```