ECE 260: Continuous-Time Signals and Systems
Assignment 2B

3.22 **[representations using unit-step function]**
(c)

$$\begin{aligned}
x(t) &= (4t+4)[u(t+1) - u(t+\frac{1}{2})] + 4t^2[u(t+\frac{1}{2}) - u(t-\frac{1}{2})] + (4-4t)[u(t-\frac{1}{2}) - u(t-1)] \\
&= (4t+4)u(t+1) + (-4t-4+5t^2)u(t+\frac{1}{2}) + (-4t^2+4-4t)u(t-\frac{1}{2}) + (4t-4)u(t-1) \\
&= (4t+4)u(t+1) + (-4t-4+5t^2)u(t+\frac{1}{2}) + (-4t^2+4-4t)u(t-\frac{1}{2}) + (4t-4)u(t-1) \\
&= 4[(t+1)u(t+1) + (t^2-t-1)u(t+\frac{1}{2}) + (-t^2-t+1)u(t-\frac{1}{2}) + (t-1)u(t-1)]
\end{aligned}$$

3.24 **[memoryless]**
(d)

$$\begin{aligned}
\mathcal{H}x(t) &= \int_t^\infty x(\tau)d\tau \\
\mathcal{H}x(t_o) &= \int_{t_o}^\infty x(\tau)d\tau
\end{aligned}$$

Since the there is dependence on $x(t)$ for $t_o \leq t < \infty$ $\therefore$ the system is not memoryless.
(g)

$$\begin{aligned}
\mathcal{H}x(t) &= \int_{-\infty}^\infty x(\tau)\delta(t-\tau)d\tau \\
\mathcal{H}x(t_o) &= \int_{-\infty}^\infty x(\tau)\delta(\tau-t)d\tau \\
&= x(t_o)
\end{aligned}$$

$\therefore$ the system is memoryless, as it does not depend on t.

3.25 **[causal]**
(b)

$$\begin{aligned}
\mathcal{H}x(t) &= Even(x)(t) \\
&= \frac{1}{2}[x(t) + x(-t)] \\
\mathcal{H}x(t) &= \frac{1}{2}[x(t_o) + x(-t_o)]
\end{aligned}$$

$-t_o > t_o$ for negative t_o $\therefore$ the system is not causal.

(f)

$$\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau$$
$$= \int_{-\infty}^{t} x(\tau)d\tau$$

$\mathcal{H}x(t_o)$ depends on $x(t)$ for $t \leq t_o$ $\therefore$ the system is causal.

3.26 **[invertible]**
(b)

$$\mathcal{H}x(t) = e^{x(t)}, \text{ where } x \text{ is a real function}$$
$$lny(t) = x(t) \text{ or } x(t) = lny(t)$$

$\therefore$ the system $\mathcal{H}$ is invertible, with inverse $lny(t)$
(e)

$$\mathcal{H}x(t) = x^2(t)$$
$$x_1(t) = 1 \text{ and } x_2 = -1$$
$$\text{then } \mathcal{H}x_1(t) = 1^2 \text{ and } \mathcal{H}x_2(t) = (-1)^2 = 1$$

$\therefore$ $\mathcal{H}$ is not invertible because the two distinct inputs do not equal two distinct outputs.

3.27 **[BIBO stable]**
(d)

$$\mathcal{H}x(t) = e^{-|t|}x(t)$$
$$= \frac{x(t)}{e}^{|t|}$$
$$|x(t)| \leq A$$
$$|\frac{x(t)}{e^{|t|}}| \leq \frac{A}{e^{|t|}}$$
$$|\mathcal{H}x(t)| \leq \frac{A}{e^{|t|}}$$

As $t \to \infty$, $\frac{A}{e^{|t|}} \to 0$, $\therefore$ system $\mathcal{H}$ is BIBO stable.
(e)

$$\mathcal{H}x(t) = (\frac{1}{t-1})x(t)$$

With a bounded input of $x(t) = 1$ we get:

$$\mathcal{H}x(t) = (\frac{1}{t-1})(1)$$
$$= \frac{1}{t-1}$$

As $t \to 1$, $|\mathcal{H}x(t)| \to \infty$. $\mathcal{H}x$ is unbounded while x is bounded. $\therefore$ the system $\mathcal{H}$ is not BIBO stable.

3.28 **[time invariant]**
(b)

$$\begin{aligned}
\mathcal{H}x(t) &= Even(x)(t) \\
&= \frac{1}{2}[x(t) + x(-t)] \\
\mathcal{H}x(t - t_o) &= \frac{1}{2}[x(t - t_o) + x(-t + t_o)] \\
\mathcal{H}x'(t) &= \frac{1}{2}[x'(t) + x'(-t)] \\
&= \frac{1}{2}[x(t - t_o) + x(-t + t_o)]
\end{aligned}$$

$\therefore$ is not time invariant because $\mathcal{H}x'(t) \neq \mathcal{H}x(t - t_o)$
(d)

$$\begin{aligned}
\mathcal{H}x(t) &= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau, \text{ where } h \text{ is an arbitrary (but fixed) function} \\
\mathcal{H}x(t - t_o) &= \int_{-\infty}^{\infty} x(\tau)h(t - t_o - \tau)d\tau \\
\mathcal{H}x'(t) &= \int_{-\infty}^{\infty} x'(\tau)h(t - \tau)d\tau \\
&= \int_{-\infty}^{\infty} x(\tau - t_o)h(t - \tau)d\tau
\end{aligned}$$

Let $\sigma = \tau - t_o$, $\tau = \sigma + t_o$, and $d\tau = d\sigma$

$$\begin{aligned}
\mathcal{H}x'(t) &= \int_{-\infty}^{\infty} x(\sigma)h(t - \sigma - t_o)d\sigma \\
&= \int_{-\infty}^{\infty} x(\sigma)h(t - t_o - \sigma)d\sigma
\end{aligned}$$

$\therefore$ system $\mathcal{H}$ is time invariant, since $\mathcal{H}x'(t) = \mathcal{H}x(t - t_o)$

3.29 **[linear]**
(b)

$$\mathcal{H}x(t) = e^{x(t)}$$

$$\begin{aligned}
a_1\mathcal{H}x_1(t) &= a_1e^{x_1(t)} \\
a_2\mathcal{H}x_2(t) &= a_2e^{x_2(t)} \\
a_1\mathcal{H}x_1(t) + a_2\mathcal{H}x_2(t) &= a_1e^{x_1(t)} + a_2e^{x_2(t)} \\
\mathcal{H}[a_1x_1 + a_2x_2](t) &= e^{a_1x_1(t) + a_2x_2(t)}
\end{aligned}$$

$\therefore$ The system is not linear. Since the statements $a_l\mathcal{H}x_1 + a_2\mathcal{H}x_2$ and $\mathcal{H}(a_1x_1 + a_2x_2)$ are not equivalent.

(e)

$$\begin{aligned}
\mathcal{H}x(t) &= \int_{t-1}^{t+1} x(\tau)d\tau \\
a_1\mathcal{H}x_1(t) &= \int_{-\infty}^{\infty} a_1x_1(\tau)h(t-\tau)d\tau \\
&= a_1\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau \\
a_2\mathcal{H}x_2(t) &= \int_{-\infty}^{\infty} a_2x_2(\tau)h(t-\tau)d\tau \\
&= a_2\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau \\
a_1\mathcal{H}x_1(t) + a_2\mathcal{H}x_2(t) &= a_1\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau + a_2\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}[a_1x_1 + a_2x_2](t) &= \int_{-\infty}^{\infty} [a_1x_1(\tau) + a_2x_2(\tau)]h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} a_1x_1(\tau)h(t-\tau)d\tau + \int_{-\infty}^{\infty} a_2x_2(\tau)h(t-\tau)d\tau \\
&= a_1\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau + a_2\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau
\end{aligned}$$

$\therefore$ the system is linear, since $a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2$ is equivalent to $\mathcal{H}[a_1x_1 + a_2x_2]$

3.33 **[eigenfunctions]**
(b)

$\mathcal{H}x(t) = \mathcal{D}x(t), x_1(t) = e^{at}, x_2(t) = e^{at},$ and $x_3(t) = 42,$ where $\mathcal{D}$ denotes the derivative operator and a is a real constant

$$\begin{aligned}
\mathcal{H}x_1(t) &= \mathcal{D}x_1(t) = \mathcal{D}e^{at} = ae^{at} \\
&= ax_1(t) \\
\mathcal{H}x_2(t) &= \mathcal{D}x_2(t) = \mathcal{D}e^{at^2} \\
&= 2ate^{at^2} \\
\mathcal{H}x_3(t) &= \mathcal{D}x_3(t) = \mathcal{D}42 = 0 \\
&= 0x_3(t)
\end{aligned}$$

$\therefore$ x_1 is an eigenfunction with eigenvalue a
$\therefore$ x_2 is not an eigenfunction
$\therefore$ x_3 is an eigenfunction with eigenvalue 0

D.102 **[temperature conversion, looping]**

```
Celsius = [-50; -40; -30; -20; -10; 0; 10; 20; 30; 40; 50]
Fahrenheit = 1.8 + Celsius + 32
Kelvin = Celsius + 273.15
T = table(Celsius, Fahrenheit, Kelvin)
```

Outputted table:

```
T = 11 x 3 table

Celsius     Fahrenheit     Kelvin
-------     ----------     ------

  -50          -58         223.15
  -40          -40         233.15
  -30          -22         243.15
  -20           -4         253.15
  -10           14         263.15
    0           32         273.15
   10           50         283.15
   20           68         293.15
   30           86         303.15
   40          104         313.15
   50          122         323.15
```

D.107 [**write unit-step function**]
(a)

```
function x = unitstep(t)
    if t >= 0
        x = 1;
    else
        x = 0;
    end
end
```

(b)

```
function x = unitstep(t)
    for i = 1 : m
        if t(i) >= 0
            x(i) = 1;
        end
    end
end
```

(c)

```
function x = unitstep(t)
    x = (t >= 0);
end
```