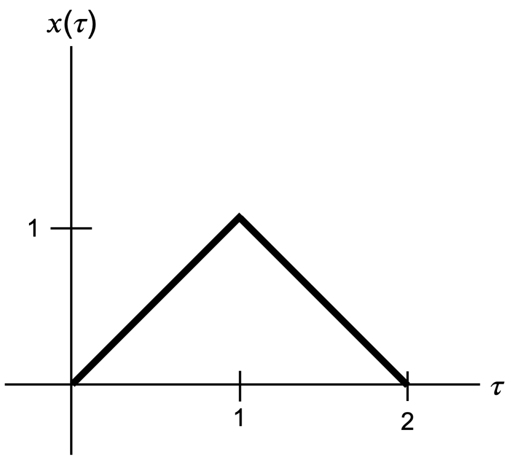
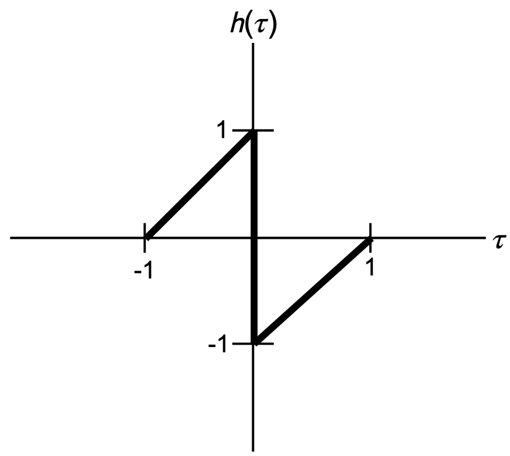
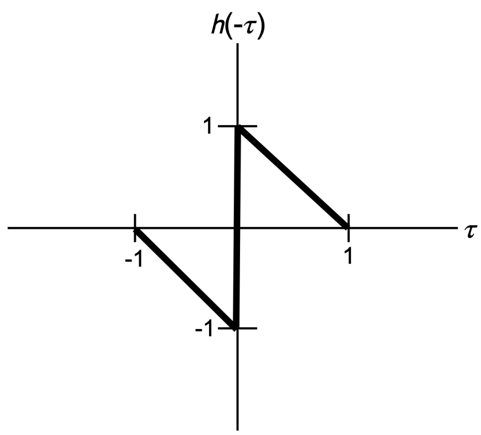
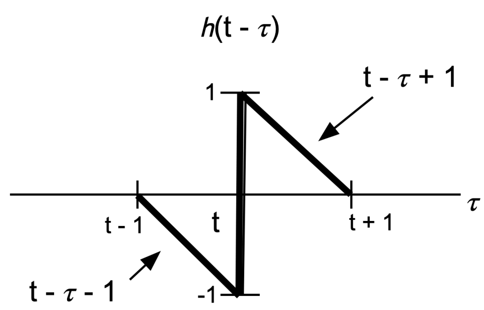
ECE 260: Continuous-Time Signals and Systems
Assignment 3A

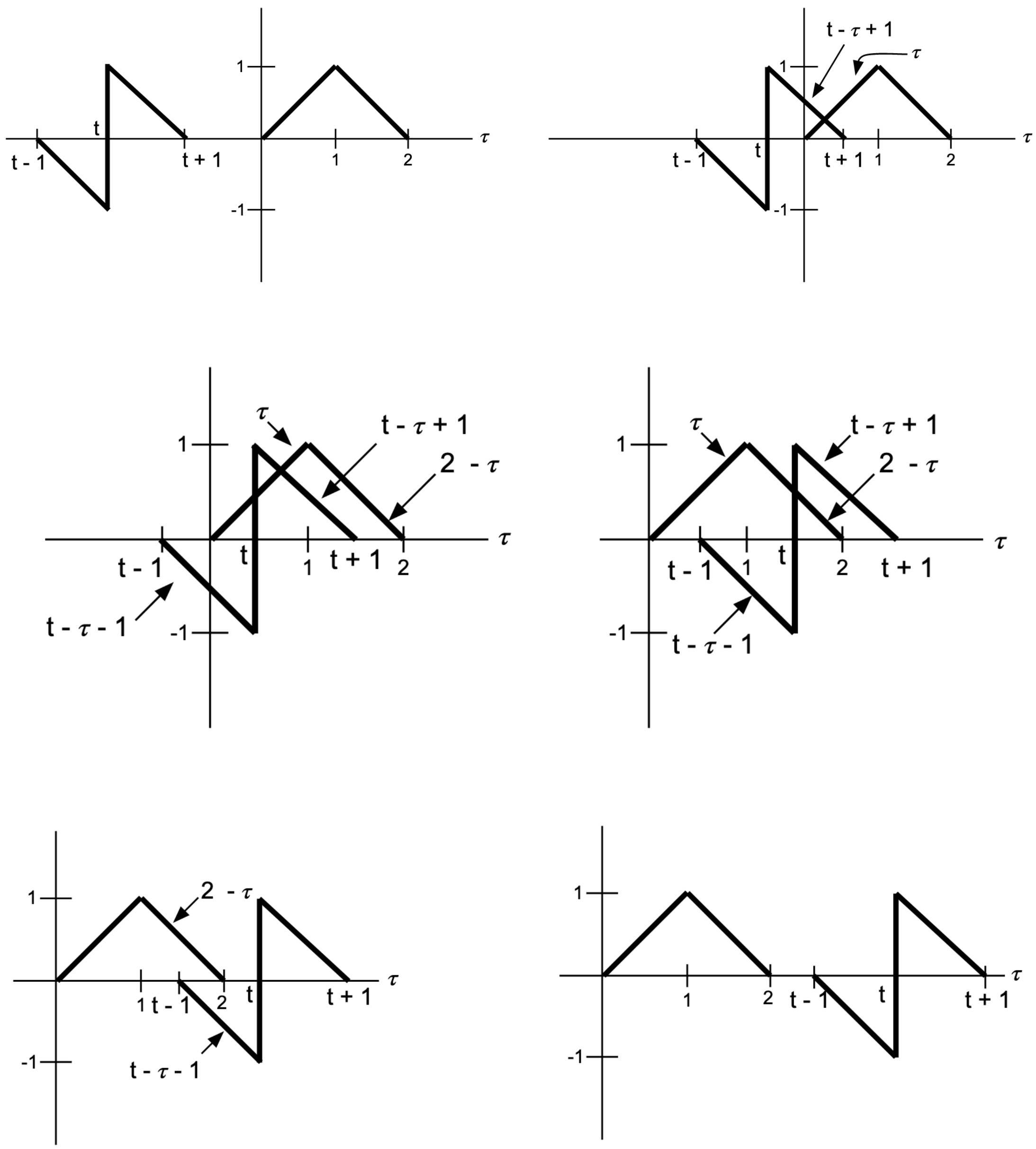
4.1 [**compute convolution**]
(e)











For the case of $t < -1$:

$$x * h(t) = 0$$

For the case of $-1 \leq t < 1$:

$$x * h(t) = \int_0^{t+1} (\tau)(t - \tau + 1) d\tau$$

For the case of $0 \leq t < 1$:

$$x * h(t) = \int_0^{t} (\tau)(t - \tau - 1) d\tau + \int_t^{1} (\tau)(t - \tau + 1) d\tau + \int_1^{t+1} (2 - \tau)(t - \tau + 1) d\tau$$

For the case of $1 \leq t < 2$:

$$x * h(t) = \int_{t-1}^{1} (\tau)(t-\tau-1)d\tau + \int_{1}^{t} (2-\tau)(t-\tau-1)d\tau + \int_{t}^{2} (2-\tau)(t-\tau+1)d\tau$$

For the case of $2 \leq t < 3$:

$$x * h(t) = \int_{t-1}^{2} (2-\tau)(t-\tau-1)d\tau$$

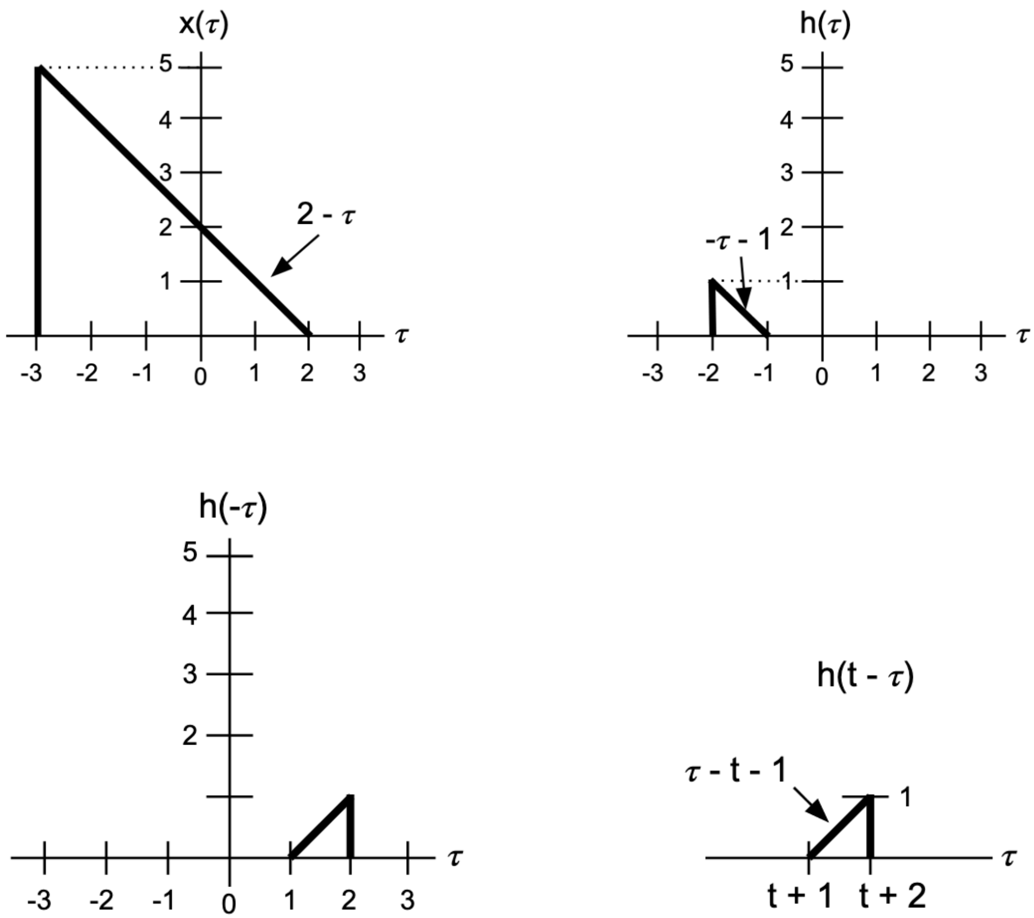
For the case of $t \geq 3$:

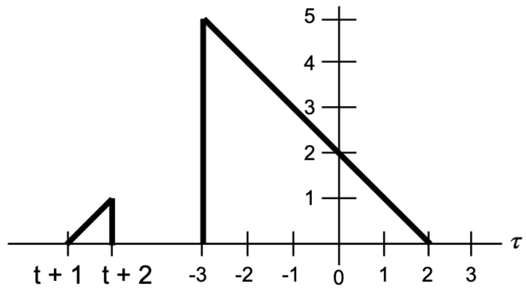
$$x * h(t) = 0$$

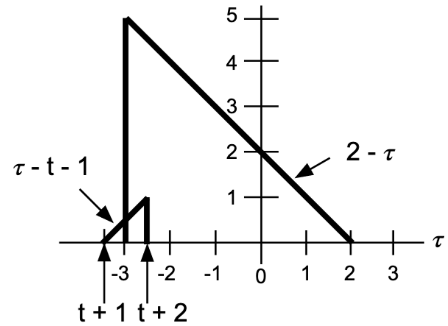
Combining the above results we get:

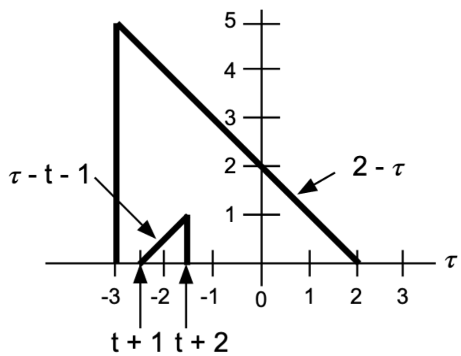
$$x * h(t) = \begin{cases} \int_0^{t+1}(\tau)(t-\tau+1)d\tau & -1 \leq t < 0 \\ \int_0^{t}(\tau)(t-\tau-1)d\tau + \int_t^{1}(\tau)(t-\tau+1)d\tau + \int_1^{t+1}(2-\tau)(t-\tau+1)d\tau & 0 \leq t < 1 \\ \int_{t-1}^{1}(\tau)(t-\tau-1)d\tau + \int_1^{t}(2-\tau)(t-\tau-1)d\tau + \int_t^{2}(2-\tau)(t-\tau+1)d\tau & 1 \leq t < 2 \\ \int_{t-1}^{2}(2-\tau)(t-\tau-1)d\tau & 2 \leq t < 3 \\ 0 & otherwise \end{cases}$$

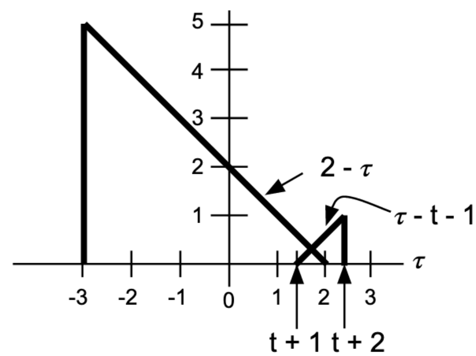
(f)

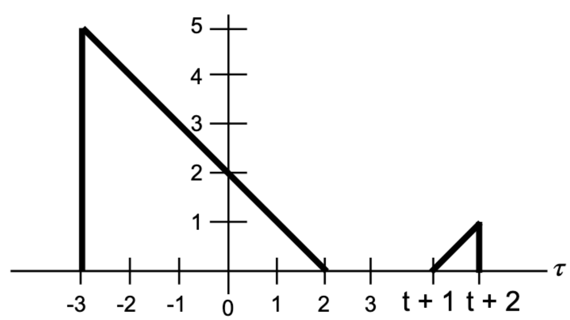












For the case of $t \leq -5$:

$$x * h(t) = 0$$

For the case of $-5 \leq t < 4$:

$$x * h(t) = \int_{-3}^{t+2} (2 - \tau)(t - \tau - 1) d\tau$$

For the case of $-4 \leq t < 0$:

$$x * h(t) = \int_{t+1}^{t+2} (2 - \tau)(t - \tau - 1) d\tau$$

For the case of $0 \leq t < 1$:

$$x * h(t) = \int_{t+1}^{2} (2-\tau)(t-\tau-1)d\tau$$

For the case of $t \geq 1$:

$$x * h(t) = 0$$

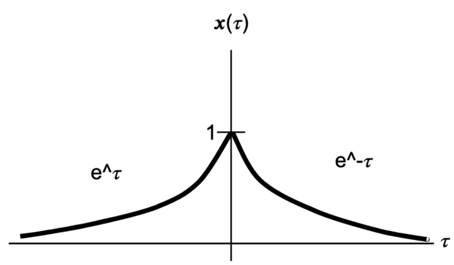
Combining the above results we get:

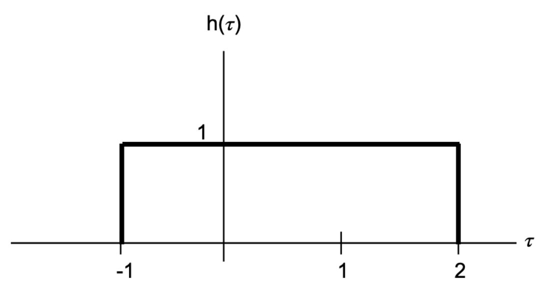
$$x * h(t) = \begin{cases} \int_{-3}^{t+2} (2-\tau)(t-\tau-1)d\tau & -5 \leq t < 4 \\ \int_{t+1}^{t+2} (2-\tau)(t-\tau-1)d\tau & -4 \leq t < 0 \\ \int_{t+1}^{2} (2-\tau)(t-\tau-1)d\tau & 0 \leq t < 1 \\ 0 & otherwise \end{cases}$$

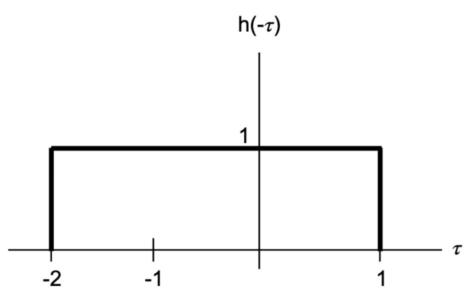
4.3 [**compute convolution**]

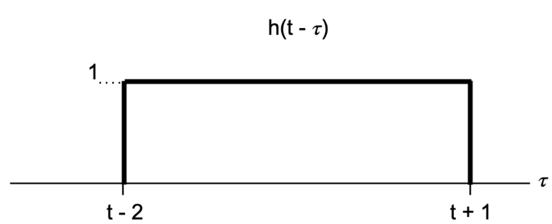
(b) Using graphical method, we want to compute $x * h$ for each pair of function x and h for:

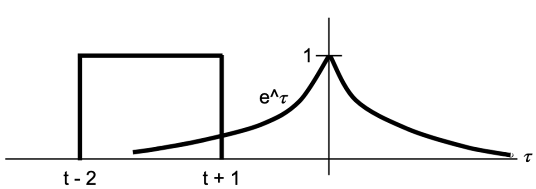
$$x(t) = e^{-|t|} \text{ and } h(t) = rect(\frac{1}{3}[t - \frac{1}{2}])$$

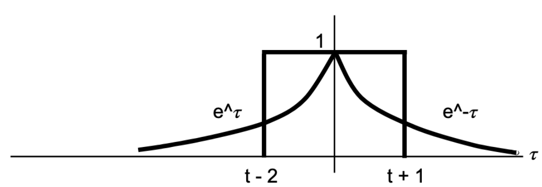


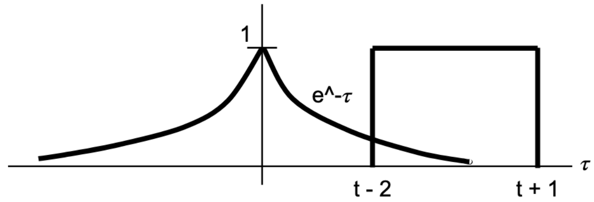












For the case of $t < -1$:

$$x * h(t) = \int_{t-2}^{t+1} e^{\tau} d\tau$$

For the case of $-1 \leq t < 2$:

$$x * h(t) = \int_{t-2}^{0} e^{\tau} d\tau + \int_{0}^{t+1} e^{-\tau} d\tau$$

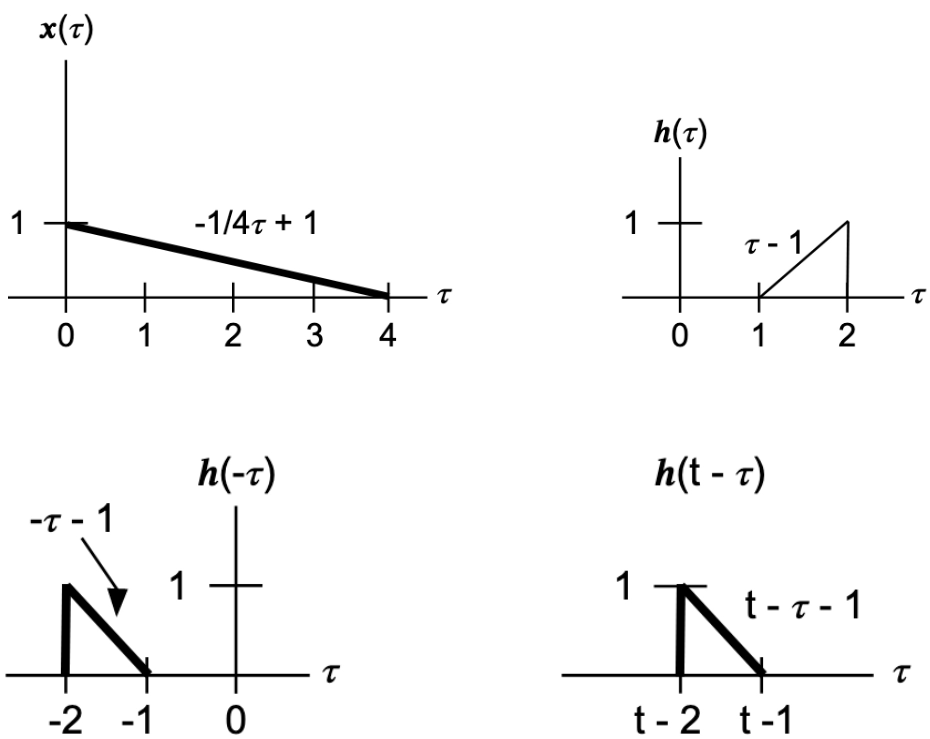
For the case of $t \geq 2$:

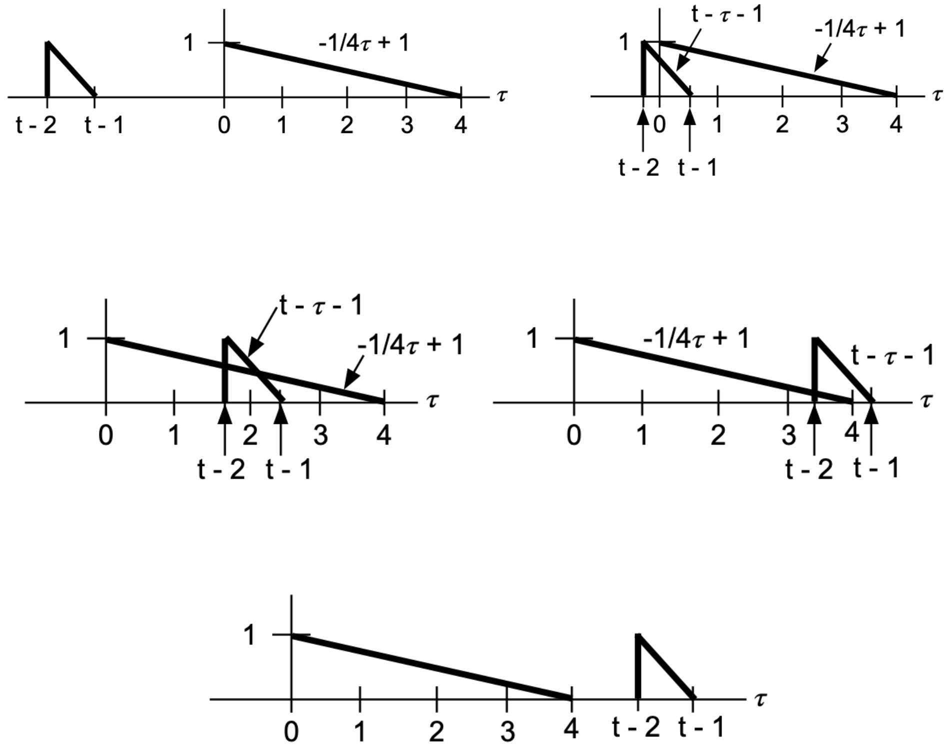
$$x * h(t) = \int_{t-2}^{t+1} e^{-\tau} d\tau$$

Combining the above results we get:

$$x * h(t) = \begin{cases} \int_{t-2}^{t+1} e^{\tau} d\tau & -1 \leq t < 2 \\ \int_{t-2}^{0} e^{\tau} d\tau + \int_{0}^{t+1} e^{-\tau} d\tau & -1 \leq t < 2 \\ \int_{t-2}^{t+1} e^{-\tau} d\tau & t \geq 2 \end{cases}$$

(g)





For the case of $t < 1$:

$$x * h(t) = 0$$

For the case of $1 \leq t < 2$:

$$x * h(t) = \int_0^{t-1} (-\frac{1}{4}\tau + 1)(t - \tau - 1) d\tau$$

For the case of $2 \leq t < 5$:

$$x * h(t) = \int_{t-2}^{t-1} (-\frac{1}{4}\tau + 1)(t - \tau - 1) d\tau$$

For the case of $5 \leq t < 6$:

$$x * h(t) = \int_{t-2}^{4} (-\frac{1}{4}\tau + 1)(t - \tau - 1) d\tau$$

For the case of $t \geq 6$:

$$x * h(t) = 0$$

Combining the above results we get:

$$x * h(t) = \begin{cases} \int_0^{t-1} (-\frac{1}{4}\tau + 1)(t - \tau - 1) d\tau & 1 \leq t < 2 \\ \int_{t-2}^{t-1} (-\frac{1}{4}\tau + 1)(t - \tau - 1) d\tau & 2 \leq t < 5 \\ \int_{t-2}^{4} (-\frac{1}{4}\tau + 1)(t - \tau - 1) d\tau & 5 \leq t < 6 \\ 0 & otherwise \end{cases}$$

4.5 [manipulation of expressions involving convolution]

Let x, y, h and v be function such that $y = x * h$ and

$$v(t) = \int_{-\infty}^{\infty} x(-\tau - b)h(\tau + at)d\tau$$

where a and b are real constants. We want to express v in terms of y.

Let $\delta = -\tau - b$, then $\tau = -\delta - b$ and $d\tau = -d\delta$

$$\begin{aligned}
v(t) &= \int_{-\infty}^{\infty} x(-\tau - b)h(\tau + at)d\tau \\
&= \int_{\infty}^{-\infty} x(\delta)h((-\delta - b) + at)d\delta \\
&= \int_{\infty}^{-\infty} x(\delta)h(at - b - \delta)d\delta \\
&= y(at - b)
\end{aligned}$$

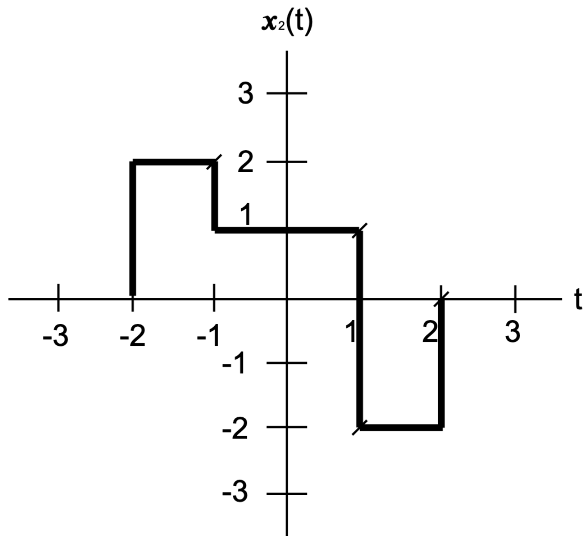
4.6 [convolution property proof]
(a) Consider the convolution $y = x * h$. Assuming that he convolution y exists, we want to prove that if x is periodic, then y is periodic.

If x is periodic then $x(t) = x(t + T)$
Let $\delta = \tau + T$, and so $\tau = \delta - T$ and $d\delta = d\tau$

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \\
&= \int_{-\infty}^{\infty} x(\tau + T)h(t - \tau)d\tau \\
&= \int_{-\infty}^{\infty} x(\delta - \tau + T)h(t - (\delta - T))d\delta \\
&= \int_{-\infty}^{\infty} x(\delta)h(t - \delta + T)d\delta \\
&= \int_{-\infty}^{\infty} x(\delta)h((t + T) - \delta)d\delta \\
&= y(t + T)
\end{aligned}$$

$\therefore$ y is Periodic.

4.9 [meaning of LTI]
Consider a LTI system whose response to the function $x_1(t) = u(t) - u(t - 1)$ is the function y_1. We want to determine the response y_2 of the system to the input x_2 shown in the following figure in terms of y_1



Since we know that $x_1(t) = u(t) - u(t-1)$, we can represent x_2 in terms of x_1:

$$\begin{aligned}
&v_1(t) = 2u(t+2)\\
&v_2(t) = u(t+1)\\
&v_3(t) = u(t)\\
&v_4(t) = 2u(t-1)\\
&x_2(t) = v_1(t) + v_2(t) + v_3(t) + v_4(t)\\
\\
&x_2(t) = 2u(t+2) + u(t+1) + u(t) - 2u(t-1)
\end{aligned}$$

And so we can represent x_2 in terms of x_1 as follows:

$$x_2(t) = 2x_1(t+2) + x_1(t+1) + x_1(t) - 2x_1(t-1)$$

Since it is an LTI system whose response to the function $x_1(t)$ is the function y_1, we get the following response:

$$\begin{aligned}
&2y_1(t+2) + y_1(t+1) + y_1(t) - 2y_1(t-1)\\
\therefore\ &y_2(t) = 2y_1(t+2) + y_1(t+1) + y_1(t) - 2y_1(t-1)
\end{aligned}$$

D.103 [plot, abds, angle, complex numbers]
The following program and output plots $|F(\omega)|$ and $argF(\omega)$ for ω in the interval $[-10, 10]$, where F denotes the complex-values function of a real variable given by:

$$F(\omega) = \frac{1}{j\omega + 1}$$

```
w = linspace(-10, 10, 500);
% f denotes the complex-values function of a real variable
f = 1 ./ (j * w + 1);
% Gets the phase angle
fAngle = angle(f);
```

```
% Gets the absolute value of f
fAbs = abs(f);
omeg = linspace(-10, 10, 500);
% Plot arg(F(w))
subplot(2, 1, 2);
plot(omeg, fAngle);
title('argF(\omega) for \omega in the interval [-10, 10]');
xlabel('\omega');
ylabel('argF(\omega)');
% Plot |F(w)|
subplot(2, 1, 1);
plot(omeg, fAbs);
title('|F(\omega)| for \omega in the interval [-10, 10]');
xlabel('\omega');
ylabel('|F(\omega)|');
```