ECE 260: Continuous-Time Signals and Systems

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Assignment 3B

4.11 **[find impulse response]**

(a)

$$\begin{aligned}
\mathcal{H}x(t) &= \int_{-\infty}^{t+1} x(\tau)d\tau,\ x(\tau) = \delta(\tau) \\
h(t) &= \int_{-\infty}^{t+1} \delta(\tau)d\tau \\
&= \begin{cases} 1 & t > -1 \\ 0 & t < -1 \end{cases} \\
&= u(t+1)
\end{aligned}$$

(b)

$$\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau+5)e^{\tau-t+1}u(t-\tau-2)d\tau$$

$$\begin{aligned}
x(\tau) &= \delta(\tau) \\
\tau + 5 &= 0 \\
\tau &= -5
\end{aligned}$$

$$\begin{aligned}
h(t) &= \int_{-\infty}^{\infty} \delta(\tau+5)e^{\tau-t+1}u(t-\tau-2)d\tau \\
&= e^{-5-t+1}u(t-(-5)-2) \\
&= e^{-(t+4)}u(t+3)
\end{aligned}$$

(c)

$$\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)v(t-\tau)d\tau$$

$$\begin{aligned}
h(t) &= \int_{-\infty}^{t} \delta(\tau)v(t-\tau)d\tau \\
\lambda &= \tau - t \\
\tau &= \lambda + t \\
\tau &= d\lambda
\end{aligned}$$

$$\begin{aligned}
h(t) &= \int_{-\infty}^{0} \delta(\lambda+t)v(t-(\lambda+t))d\lambda \\
&= \int_{-\infty}^{0} \delta(\lambda+t)v(-\lambda)d\lambda \\
&= \int_{-\infty}^{0} \delta(\lambda+t)v(t)d\lambda \\
&= v(t)\int_{-\infty}^{0} \delta(\lambda+t)d\lambda \\
&= v(t)u(t)
\end{aligned}$$

4.12 **[impulse response and series/parallel interconnection]**
(a)

$$\begin{aligned}
v(t) &= x * h_1(t) \\
y(t) &= v * [h_2 + h_3](t) + x(t)
\end{aligned}$$

Combining the results we get:

$$\begin{aligned}
y(t) &= v * [h_2 + h_3](t) + x(t) \\
&= [x * h_1] * [h_2 + h_3](t) + x(t) \\
&= [h_1 * [h_2 + h_3]](t) + x(t) \\
&= x * [h_1 * h_2 + h_1 * h_3](t) + x * \delta(t) \\
&= x * [h_1 * h_2 + h_1 * h_3 + \delta](t)
\end{aligned}$$

Thus:

$$h(t) = h_1 * h_2(t) + h_1 * h_3(t) + \delta(t)$$

(b)
We want to determine the impulse response h in the specific case that:

$$h_1(t) = \delta(t+1),\ h_2(t) = \delta(t),\ h_3(t) = \delta(t)$$

$$\begin{aligned} h_1(t) &= \delta(t+1), h_2(t) = \delta(t), \text{ and } h_3(t) = \delta(t) \\ h(t) &= h_1 * h_2(t) + h_1 * h_3(t) + \delta(t) \end{aligned}$$

$$\begin{aligned} h(t) &= \delta(t+1) * \delta(t) + \delta(t+1) * \delta(t) + \delta(t) \\ &= \delta(t+1) + \delta(t+1) + \delta(t) \\ &= 2\delta(t+1) + \delta(t) \end{aligned}$$

4.13 **[convolution, impulse response, system interconnection]**
(b)

$$h_1(t) = \delta(t+1) \text{ and } h_2(t) = \delta(t+1)$$

$$\begin{aligned} h(t) &= h_1 * h_2 \\ &= \delta(t+1) + \delta(t+1) \\ &= \int_{-\infty}^{\infty} \delta(\tau+1)\delta(t-\tau+1)d\tau \\ &= \delta(t-\tau+1)\big|_{\tau=-1} \\ &= \delta(t-(-1)+1) \\ &= \delta(t+2) \end{aligned}$$

$$\begin{aligned} y(t) &= x * h(t) \\ &= u(t) * \delta(t+2) \\ &= \int_{-\infty}^{\infty} u(\tau)\delta(t-\tau+2)d\tau \\ &= u(\tau)\big|_{\tau=t+2} \\ &= u(t+2) \end{aligned}$$

(c)

$$h_1(t) = e^{-3t}u(t) \text{ and } h_2(t) = \delta(t)$$

$$\begin{aligned} h(t) &= h_1(t) * h_2(t), \quad t-\tau = 0, \ t = \tau \\ &= e^{-3t}u(t) * \delta(t) \\ &= \int_{-\infty}^{\infty} e^{-3t}u(\tau)\delta(t-\tau)d\tau \\ &= e^{-3t}u(t) \end{aligned}$$

We want to consider $y(t)$ at $t < 0$ and $t \geq 0$. For $t \geq 0$:

$$\begin{aligned}
y(t) &= x * h(t) \\
&= h * x(t) \\
&= \int_{-\infty}^{\infty} e^{-3\tau} u(\tau) u(t-\tau) d\tau \\
&= \int_{0}^{t} e^{-3\tau} d\tau \\
&= -\frac{1}{3} e^{-3\tau}\Big|_0^t \\
&= -\frac{1}{3} e^{-3t} + \frac{1}{3}
\end{aligned}$$

When $t < 0$, $y(t)$ is 0, so the convolution of $y(t)$ can be represented as:

$$\begin{aligned}
y(t) &= \begin{cases} -\frac{1}{3} e^{-3t} + \frac{1}{3} & t \geq 0 \\ 0 & t < 0 \end{cases} \\
&= [-\frac{1}{3} e^{-3t} + \frac{1}{3}] u(t)
\end{aligned}$$

4.14 **[causality, memory]**
(a)

$$h(t) = (t+1)u(t-1)$$
$$u(t-1) = \begin{cases} 1 & t \geq 1 \\ 0 & t < 1 \end{cases}$$
$$h(t) = 0, \ t < 0$$

Since $h(t) \neq 0$ for all $t \neq 0$, and $h(t) = 0$ for all $t < 0$, the system is causal and has memory
(f)

$$h(t) = e^{-3|t|}$$
$$e^{-3|t|}, \ -\infty < t < \infty$$

Since $h(t) \neq 0$ for all $t \neq 0$, and $h(t) \neq 0$ for all $t < 0$, the system has memory but is not causal.
(g)

$$h(t) = 3\delta(t)$$
$$t\delta(t) = \begin{cases} 3 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Since $h(t) = 0$ for all $t \neq 0$, and $h(t) = 0$ for all $t < 0$, the system is memoryless and causal.

4.15 **[BIBO stability]**
(a)

$$\begin{aligned} u &= at \\ \frac{du}{dt} &= a \\ \frac{1}{a}du &= dt \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} |h(t)|dt &= \int_{-\infty}^{\infty} |e^{at}u(-t)|dt \\ &= \int_{-\infty}^{0} e^{at}dt \\ &= [\frac{1}{a}e^{at}]\Big|_{-\infty}^{0} \\ &= \frac{1}{a}[e^{at}]\Big|_{-\infty}^{0} \\ &= \frac{1}{a}[e^{a+0} - e^{a(-\infty)}] \\ &= \frac{1}{a}[1-0] \\ &= \frac{1}{a} \\ &\text{which is} < \infty \end{aligned}$$

$\therefore$ the system is BIBO stable
(b)

$$\begin{aligned} h(t) &= \frac{1}{t}u(t-1 \\ \int_{-\infty}^{\infty} |h(t)| &= \int_{-\infty}^{\infty} |\frac{1}{t}u(t-1)|dt \\ &= \int_{-\infty}^{1} 0dt + \int_{1}^{\infty} \frac{1}{t}dt \\ &= \int_{1}^{\infty} \frac{1}{t}dt \\ &= [lnt]\Big|_{1}^{\infty} \\ &= ln\infty - ln1 \\ &= \infty \end{aligned}$$

$\therefore$ the system is not BIBO stable

4.16 **[inverse system]**

$h_1(t) = \frac{1}{2}\delta(t-1),\ h_2(t) = 2\delta(t+1)$. We can determine if the systems are inverses if $h_1(t) * h_2(t) = \delta(t)$.

$$\begin{aligned}
h_1 * h_2(t) &= \int_{-\infty}^{\infty} h_1(\tau)h_2(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} \frac{1}{2}\delta(\tau-1)2\delta(t-\tau+1)d\tau \\
&= \int_{-\infty}^{\infty} \delta(\tau-1)\delta(t-\tau+1)d\tau,\ t-\tau+1=0,\ \tau=t+1 \\
&= \delta(\tau-1)\big|_{\tau=t+1} \\
&= \delta(t+1-1) \\
&= \delta(t)
\end{aligned}$$

$\therefore$ the systems are inverses of each other

4.17 **[system function, eigenfunction]**
(a)
We want to find the response y of the LTI system with system function H to the input x.

$$h(s) = \frac{1}{s+1} \text{ for } Re(s) > -1 \text{ and } x(t) = 10 + 4cos(3t) + 2sin(5t)$$

Knowing that $ae^{j\theta} + a^*e^{-j\theta} = 2Re(ae^{j\theta})$ we can determine x(t):

$$\begin{aligned}
x(t) &= 10 + 4[\frac{1}{2}(e^{j3t} + e^{-j3t})] + 2[\frac{1}{2j}(e^{j5t} - e^{-j5t})] \\
&= 10 + 2(e^{j3t} + e^{-j3t}) + \frac{1}{j}(e^{j5t} - e^{-j5t}) \\
&= 10 + 2e^{j3t} + 2e^{-j3t} - je^{j5t} + je^{-j5t}
\end{aligned}$$

We have an LTI system with system function H to the input x, so:

$$\begin{aligned}
y(t) &= H(0)(10) + H(j3)(2e^{j3t}) + H(-j3)(2e^{-j3t}) + H(j5)(-je^{j5t}) + H(-j5)(je^{-j5t}) \\
&= 1(10) + \frac{1}{1+j3}(2e^{j3t}) + \frac{1}{1-j3}(2e^{-j3t}) + \frac{1}{1+j5}(-je^{j5t}) + \frac{1}{1-j4}(je^{-j5t}) \\
&= 10 + \frac{2}{1+j3}e^{j3t} + (\frac{2}{1+j3})^*(e^{j3t})^* +^* (\frac{-j}{1+j5})^*(e^{j5t})^*
\end{aligned}$$

Since $ae^{j\theta} + a^*e^{-j\theta} = 2Re(ae^{j\theta})$ we get:

$$= 10 + 2Re(\frac{2}{1+j3}e^{j3t}) + 2Re(\frac{-j}{1+j5}e^{j5t})$$

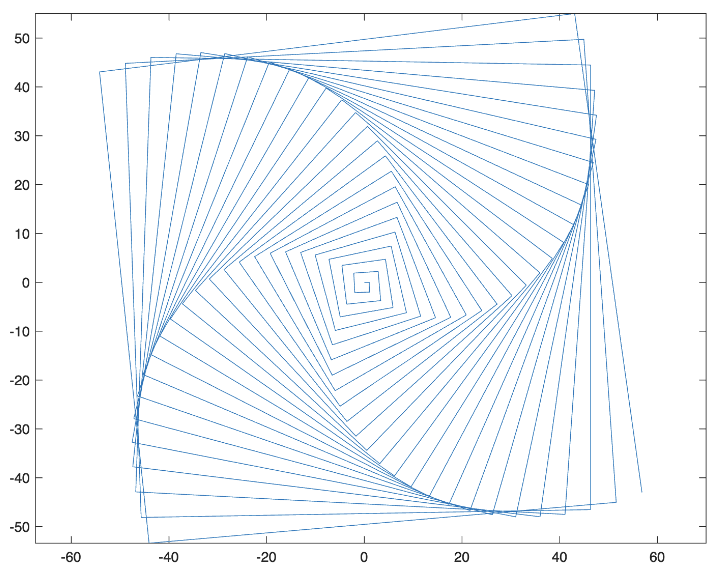
D.108 **[graphic patterns]**

(a)

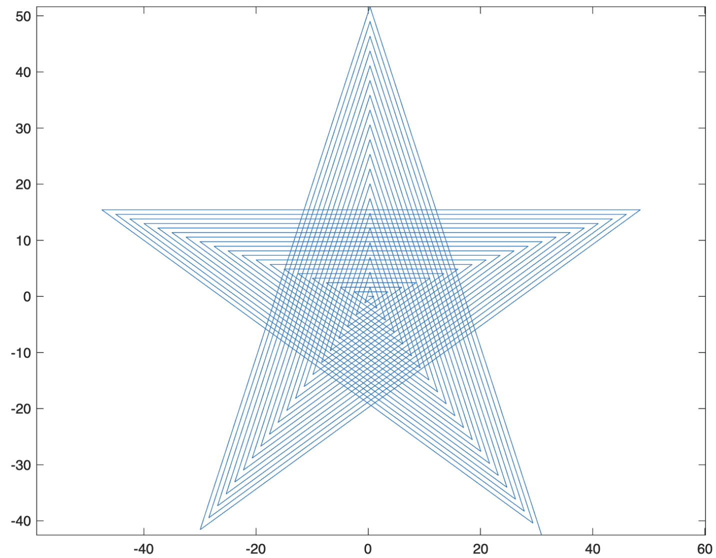
```
function drawpattern(n, theta)
    angle = theta * (pi / 180);
    p = [0 0]';
    prime = p';
    m = [cos(angle) sin(angle); -sin(angle) cos(angle)];
    for i = 1 : (n - 1)
        p = p + m ^ (i - 1) * [i 0]';
        prime = [prime; p'];
    end
    plot(prime(:, 1), prime(:, 2));
    axis('equal');
end
```

(b)

$n = 100$ and $\theta = 89$



$n = 100$ and $\theta = 144$



$n = 100$ and $\theta = 154$