Mobility HW 1

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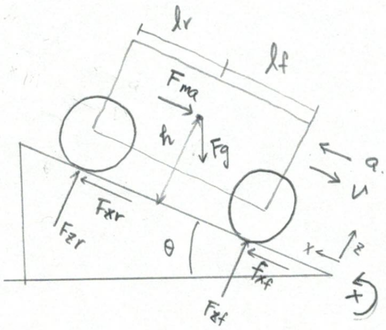
Problem 1

1.)



At tipping over

$$\begin{cases} F_{zr} = F_{xr} = 0 \\ F_g = mg \\ F_{ma} = -m\dot{v} = -ma \quad (\text{imaginary force}) \\ F_{zf} = F_g \cos\theta \\ -F_{ma}\cdot h - F_g \sin\theta \cdot h + F_g \cos\theta\, l_f = 0. \\ -F_g \sin\theta + F_{xf} = m\ddot{x} \end{cases}$$

2.)

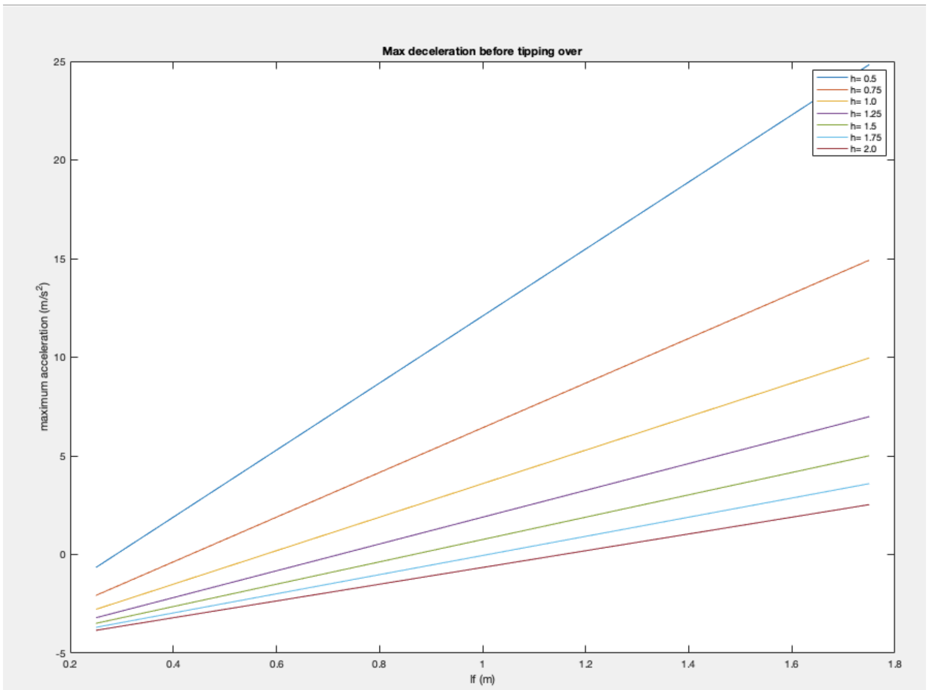
$$\left.\begin{array}{l} F_g \cos\theta\, l_f - (\underset{-ma}{F_{ma}} + \underset{-mg\sin\theta}{F_g \sin\theta})\cdot h \geq 0 \\ -mg\cos\theta\, l_f + mah + mg\sin\theta\cdot h \geq 0 \end{array}\right\}$$

$$g\left(\cos\theta \frac{l_f}{h} - \sin\theta\right) \leq a \leq 0 \quad (\text{deceleratio}$$

↳ in magnitude

$$a \leq g\left(\cos\theta \frac{l_f}{h} - \sin\theta\right)$$

1.3)



1.4)

A) l_f=1.75 m, h=0.5m

B) Yes, the answer in A is expected. The car is more stable if the center of gravity is lower. Also, if the center of gravity is further back at the car (l_f is longer), the moment arm for the ground reaction force in the front wheel is longer which provides more torque to compensate for the tip over.

C) I believe the answer is it depends. If the vehicle I am designing is a daily driving car, then it is impossible to have l_f=1.75 m and l_r=0.25 m. Since the passenger will be seated evenly for maximum comfort, in the section between front and rear wheel, it is almost impossible to design the center of gravity of the whole system(car+passengers) to be such further among the vehicle. However, if the vehicle I am designing is an unmanned field robot, it is more likely to set the center of gravity in this desired condition.

Problem 2

2.1

$$R_{c\,\text{rigid}} = \frac{\left(3F_z/\sqrt{d_w}\right)^{\frac{2n+2}{2n+1}}}{(3-n)^{\frac{2n+2}{2n+1}}(n+1)(k_c+bk_\phi)^{\frac{1}{2n+1}}}$$

$$F_z: \begin{cases} 4 \text{ wheels}: W/4 \\ 6 \text{ wheels}: W/6 \end{cases}$$

$$\text{TOTAL } R_c \text{ ratio}: \frac{4 \cdot R_{c4}}{6 \cdot R_{c6}} = \frac{2}{3}\cdot\left(\frac{3}{2}\right)^{\frac{2n+2}{2n+1}} = \left(\frac{3}{2}\right)^{\frac{1}{2n+1}} > 1$$

Ans: 6-wheeled

2.2

Contact area: $A_4 = 1.2 A_6$

$$\tau = c + \sigma \tan\phi$$

$$H = cA + \sigma A \tan\phi = cA + W\tan\phi$$

$$\text{Area/per tire} \begin{cases} 4: 1.2\,A_6/4 \\ 6: A_6/6 \end{cases}$$

$$\therefore \frac{H_4}{H_6} = \frac{(cA_4 + \frac{W}{4}\tan\phi)\cdot 4}{(cA_6 + \frac{W}{6}\tan\phi)\cdot 6} = \frac{c(1.2A_6) + W\tan\phi}{c\,A_6 + W\tan\phi} > 1$$

Ans: 4-wheeled

2.3

$$DP = H - \Sigma R$$

A_6: area per tire

$$H_6 = cA_6 + \frac{W}{6}\tan\phi$$

$$\Sigma R = \frac{\left(3(\frac{W}{6})/\sqrt{d_w}\right)^{2n+2/2n+1}}{(3-n)^{\frac{2n+2}{2n+1}}(n+1)(k_c+bk_\phi)^{\frac{1}{2n+1}}} = R_c$$

$$A_6 \cdot P_6 = W/6$$

$$P_6 = \left(\frac{k_c}{b} + k_\phi\right) z_{rw}^{\,n}, \quad z_{rw} = \left(\frac{3\cdot W/6}{(3-n)(k_c+k_\phi b)\sqrt{d}}\right)^{2/2n+1}$$

$$\Rightarrow A_6 = \frac{W/6}{P_6}$$

$$\Rightarrow DP = c \cdot \frac{W/6}{P_6} + \frac{W}{6}\tan\phi - R_c$$

2.4

	n	k_c	k_ϕ	c	ϕ
DRY SAND	1.1	0.1	3.9	0.15	28°
CLAY	0.5	12	3	0.2	38°

$$\left.\begin{array}{l} z_{sand} > z_{clay} \\ P_{sand} > P_{clay} \end{array}\right\} A_{6\,clay} > A_{6\,sand}$$

$$R_{c\,sand} > R_{c\,clay}$$

$$DP = c\,A + \frac{W}{6}\tan\phi - R_c \qquad \therefore D_{P\,clay} > D_{P\,sand}$$

2.5 AIR RESISTANCE

BULLDOZING EFFECTS

DISPLACING RESISTANCE

2.6)

W_{Gross}: total weight

n: number of wheels per axle

m: total number of axles

b: average unloaded tire width(in)

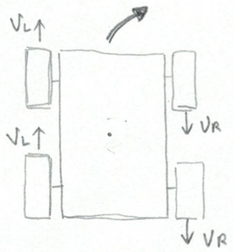
d: average unloaded tire diatmeter(in)

delta: average tire deflection(in)

There is no an absolute choice for MMP. As shown in the lecture slides, if the vehicle will be operated in a temperate climate, wet, and fine-grain soil, I will choose MMP 100 over MMP 40. Since the ideal MMP for this situation is 120, and 100 is closer to the optimal value, choosing MMP=100 is more reasonable.

Problem 3

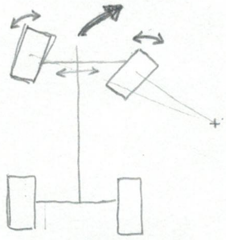
3.1) Skid



$V_L +$, $V_R -$

All parts of the vehicle stay rigid.

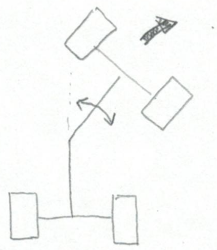
Ackerman



single-axle

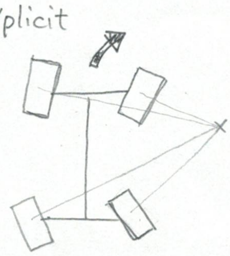
Front wheels can move to turn the front wheels.

Actuated Body



The front part of the vehicle turns.

Explicit



Both front and rear wheels (independently) turn as the vehicle turns

3.2)

	Advantages	Disadvantages
Skid	Simple mechanism, minimal radius of turn	Maximum friction loss
Ackerman	Less friction loss and good space arrangement for human driving vehicles.	Accumulated error increases as turning steps increases.
Actuated Body	Easy mechanism with small radius of turning, higher strength in both front and rear axles	Bad space arrangement for human driving vehicles, since the space between two axles is not rigid.
Explicit	Low accumulated error after multiple turning steps	Complicated mechanism, i.e. turning axles at both front and rear.

3.3)

A) Vehicles will undergo larger impact in higher speed. For example, passengers in cars feel larger bounces when they drive over a bump on the road in a higher speed. Therefore, by implementing semi-suspension systems, the unwilling response can be reduced immensely.

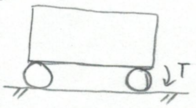
B) Steep climbs and deep drops are situations that all-terrain vehicles come across. With semi-active suspension systems, vehicles can response as early as possible and try to their maintain balance.

C) A semi-active suspension changes the stiffness and damping capacity of the suspension system, which can provide the "optimal" driving experience to the passenger inside the vehicle. The "optimal" driving experience is not a universal standard but varies as a driver chooses different driving modes for their vehicle.

3.4.) $V = 2 ft/s = 0.6096 m/s$

$m = 100\ lb = 45.3 kg$

$d_w = 20\ in = 0.508 m$



Assumption

- 4 wheels.
- car tires on hard ground
- slope $= 0°$

$\Rightarrow$ SELECTION

AC Gear Motor 6660N11

SPEED @ Continuous operating torque : 168 rpm @ 60 in-lbm

SPEED REDUCER 5887K11 SPEED RATIO : 5 : 1

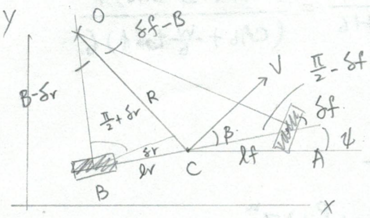
TORQUE REQUIRED $= 45.43 \times 0.254 \times 10 = 115 N\cdot m$

REV REQUIRED $= 0.6096 / \pi \cdot 0.508 = 0.38\ rev/s$

$= 22.8\ rpm$

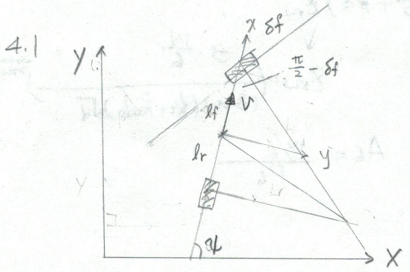
Problem 4

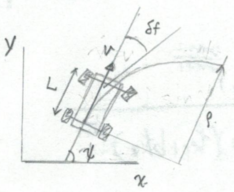
4.



NO STEER REAR WHEEL $\Rightarrow \delta r = 0$

4.1





$\dot{X} = V\cos\psi$

$\dot{Y} = V\sin\psi$

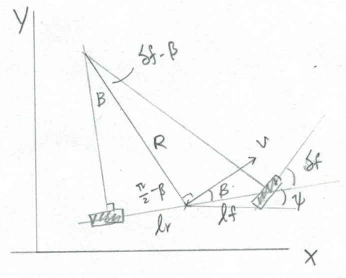
$\dot{\psi} = \frac{V\cos\beta}{l_f + l_r}(\tan\delta_f - \tan\delta_r)$

$\beta = 0 \Rightarrow$ measured from rear axle

$\delta_r = 0 \Rightarrow$ no rear steering

$\therefore \dot{\psi} = \frac{V}{l_f + l_r}(\tan\delta_f)$ ※

4.2



$\dot{X} = V\cos(\beta + \psi)$

$\dot{Y} = V\sin(\beta + \psi)$

$\dot{\psi} = \frac{V}{R} = \frac{V}{l_r}\cdot\sin\beta$ ※

$\frac{\sin\beta}{l_r} = \frac{\sin(\frac{\pi}{2})}{R}$

$\therefore R = \frac{l_r}{\sin\beta}$

4.3

A) SLIP ANGLE

FRONT $\frac{V_y + l_f\cdot\dot{\psi}}{V_x} - \delta_f$

REAR: $\frac{V_y - l_r\cdot\dot{\psi}}{V_x}$

V_y = lateral speed

$\Rightarrow$ same as the original eq.

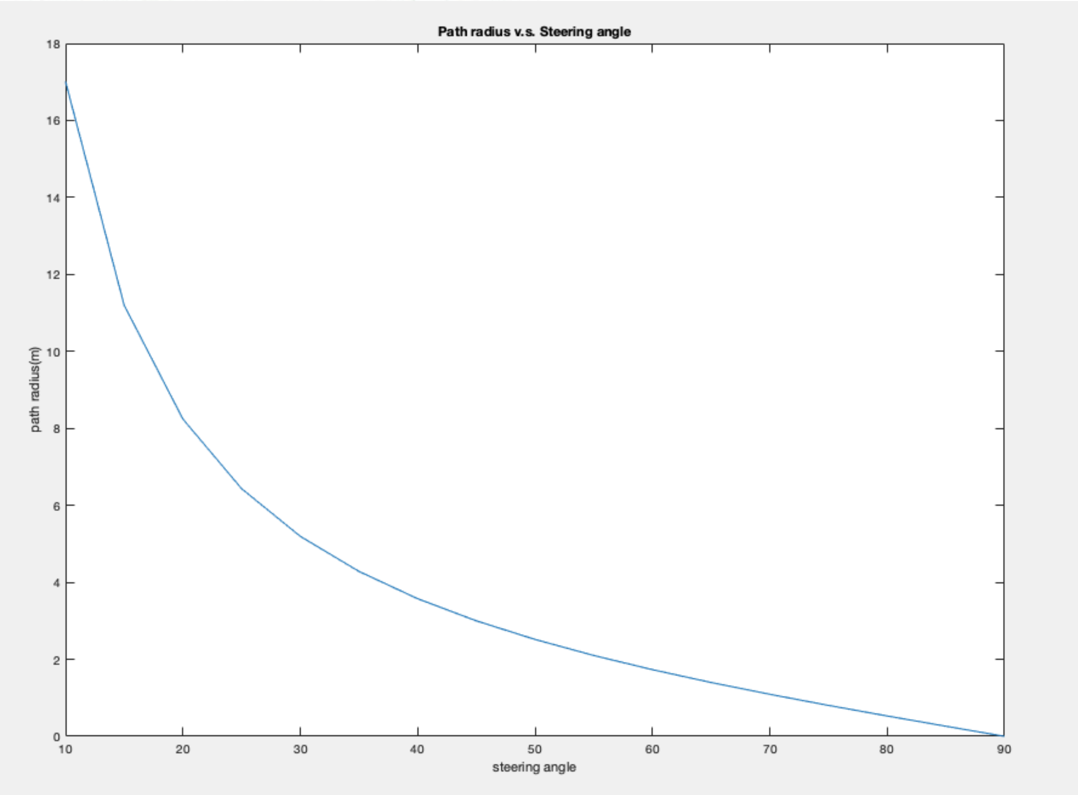
B) Tire slip: It is the angle between the direction the tire is moving and the wheel's orientation.
Vehicle slip: The angle between the vehicle is moving toward and the direction the vehicle is pointing at.

Yes, they are related. The direction the vehicle is moving toward is determined by the steering angle and the tire slip angle. In most of the studies in the bicycle models, we set tire slip=0 and vehicle slip for some values, so it is possible to have one without the other.

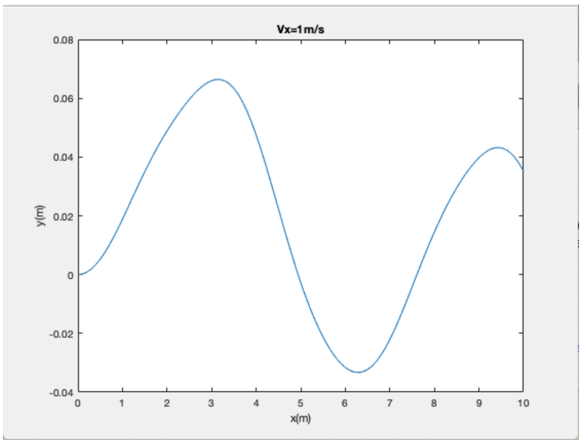
4.4)

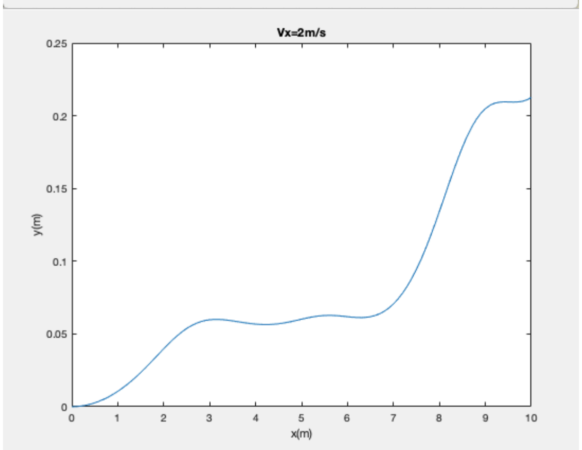
A) ko

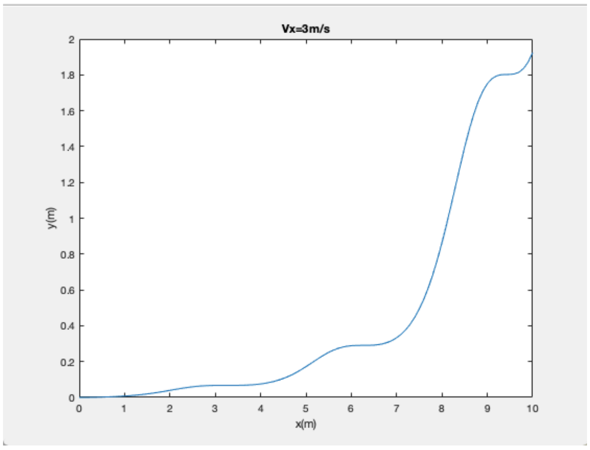
$$\frac{V}{R} = \dot{\psi} = \frac{V}{l_f+l_r} \cdot \tan\delta_f \quad \therefore \quad R = (l_f+l_r)/\tan\delta_f$$

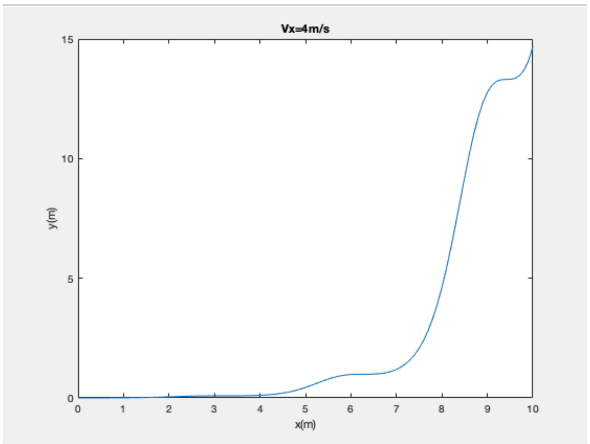


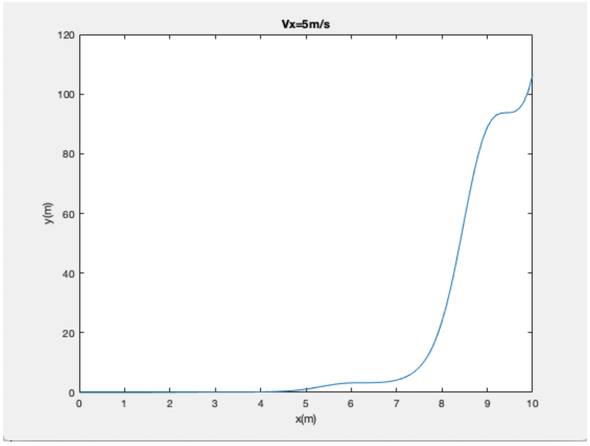
B)





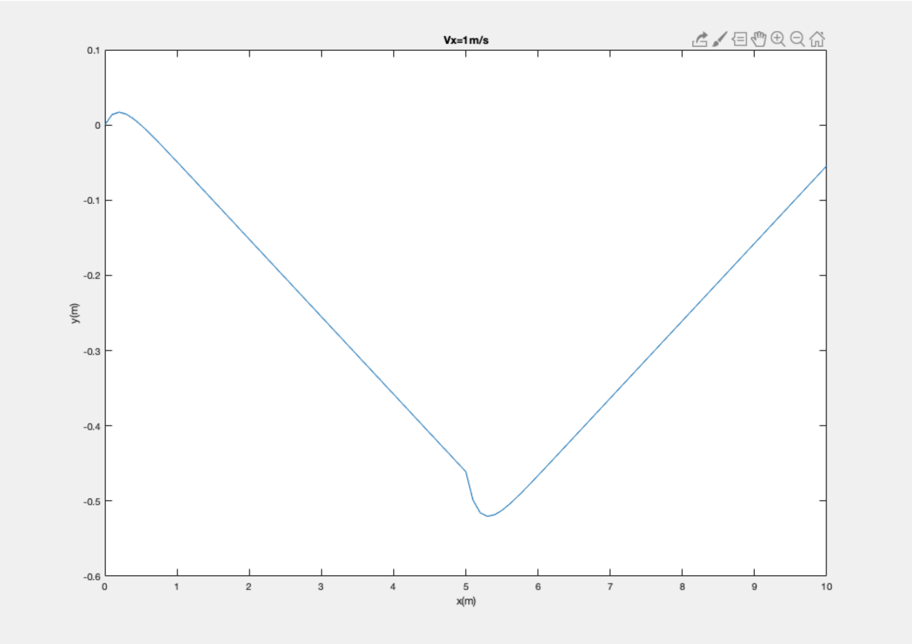






The results are expected. When the velocity along x axis (Vx) is low, the system is able to follow the input sinusoid waves. However, when Vx increases the system becomes unstable. A vehicle is susceptive to a higher centrifugal force when making turns in higher speeds. When the tire tractions are not sufficient for the vehicle to turn, the vehicle slips and the system becomes unstable.

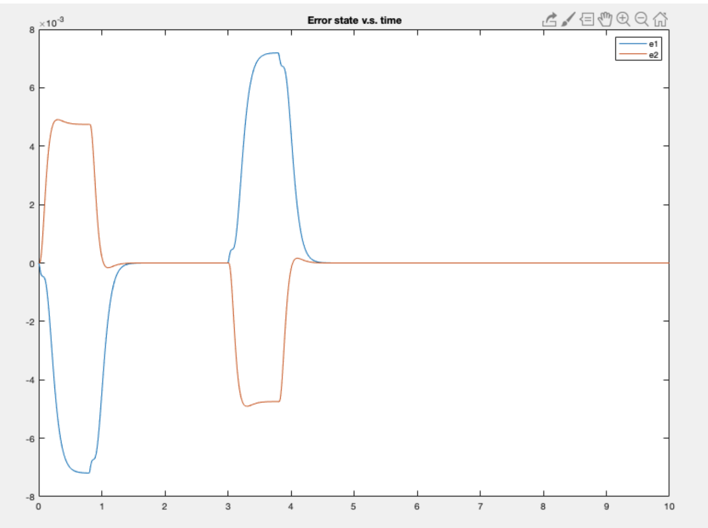
C)



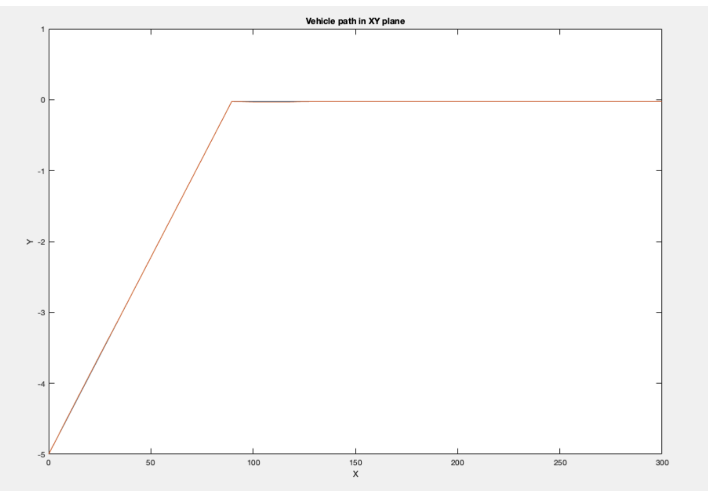
A square wave in steering angle represents a sharp turn in driving, which in most cases is not the way a vehicle is operated.

Problem 5

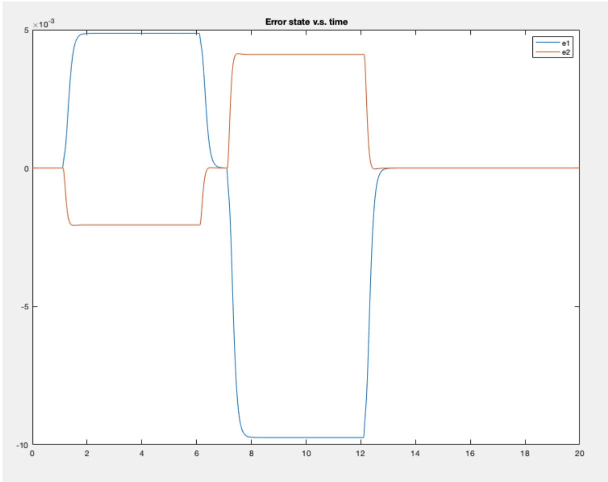
5.2)



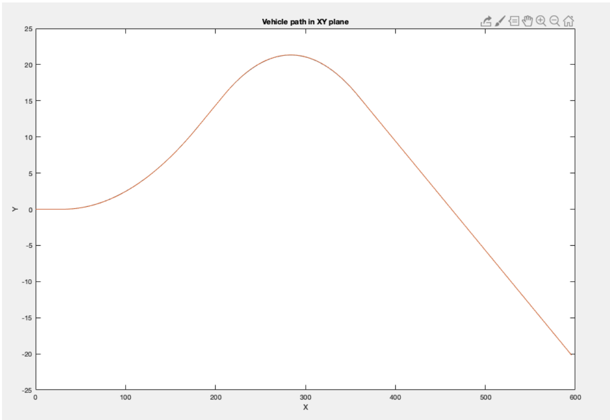
5.3)



5.4)

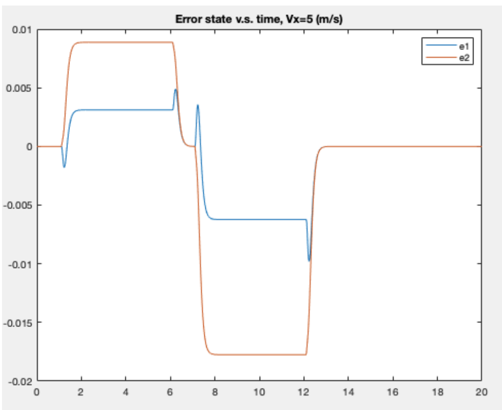


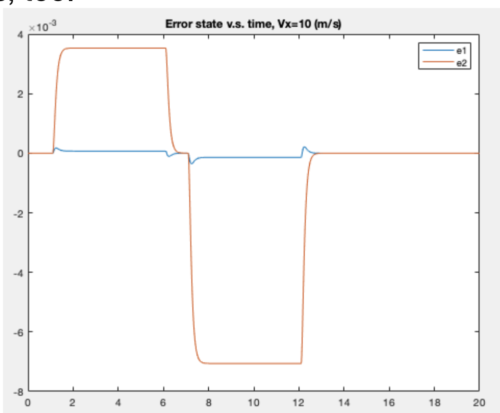
5.5)

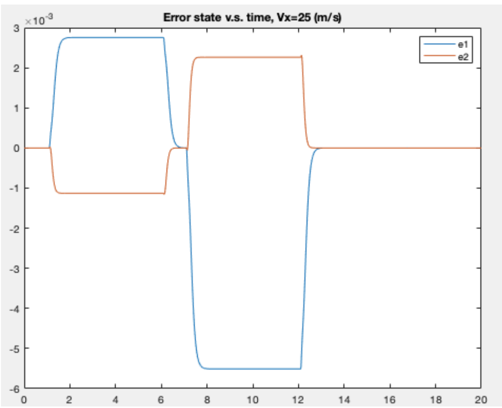


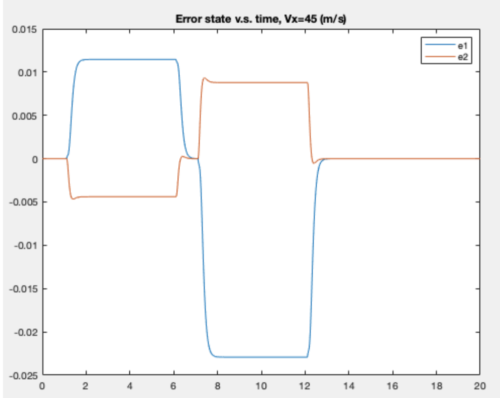
5.6)

The effect on e1 is very straight forward that as Vx increases, e1 increases. On the other hand, the effect on e2 is less intuitive. As Vx increases from 5 m/s to 40 m/s, not only does the magnitude of e2 increases, the sign changes, too.

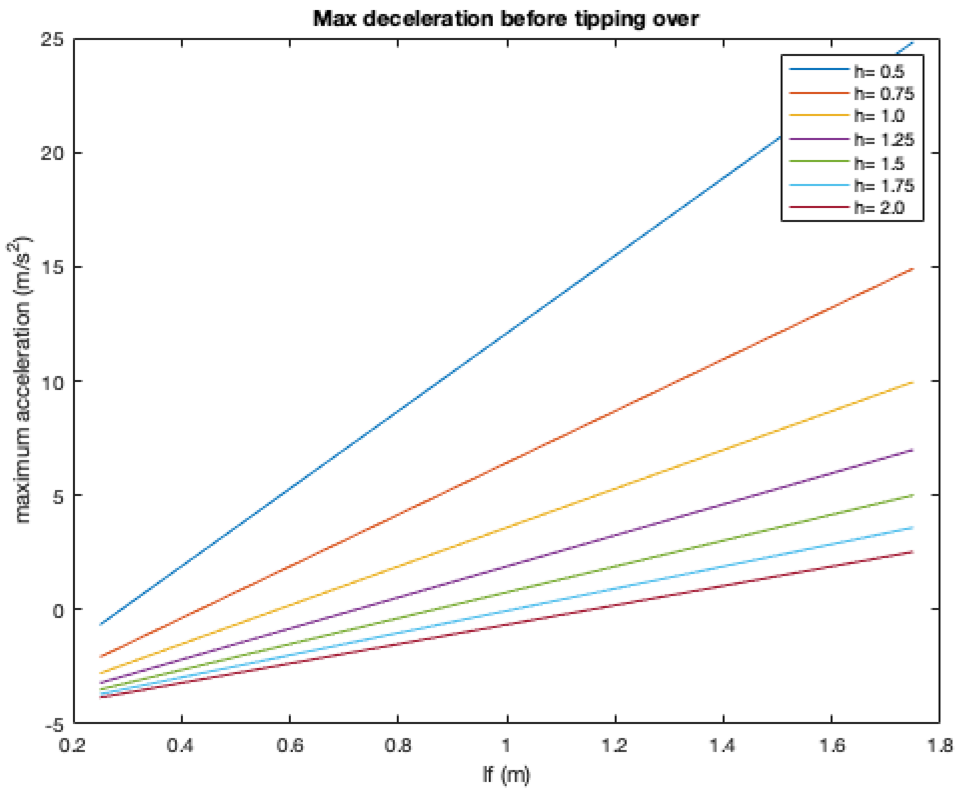








```
%1.3 %1.4
%model parameters:
m=100;
theta=30/180*pi();
L=2;
g=9.81;

%memory space to store ans:
ansa=zeros();
anslf=zeros();
ansh=zeros();
i=1;
j=1;


for h=0.5:0.25:2  %various h
    for lf=0.25:0.05:1.75  %various lf
        %deceleration according to given h, lf
        a=g*(cos(theta)*(lf/h)-sin(theta));
        %store the ans:
        anslf(i,j)=lf;
        ansa(i,j)=a;
        i=i+1;
    end
    i=1;
    ansh(j,1)=h;
    j=j+1;

end


for j=1:1:7
    plot(anslf(:,j), ansa(:,j))
    xlabel('lf (m)');
    ylabel('maximum acceleration (m/s^2)');
    title('Max deceleration before tipping over');

    num=(j-1)*0.25+0.5;
    hold on


end
legend('h= 0.5','h= 0.75','h= 1.0','h= 1.25','h= 1.5','h= 1.75','h=
 2.0')
```



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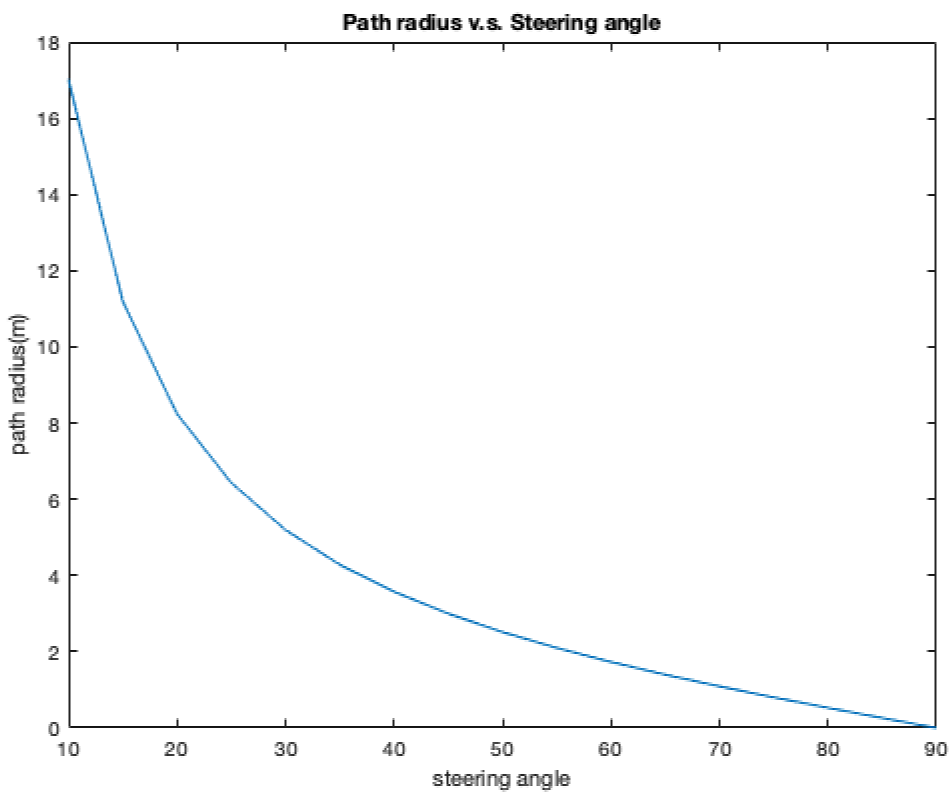
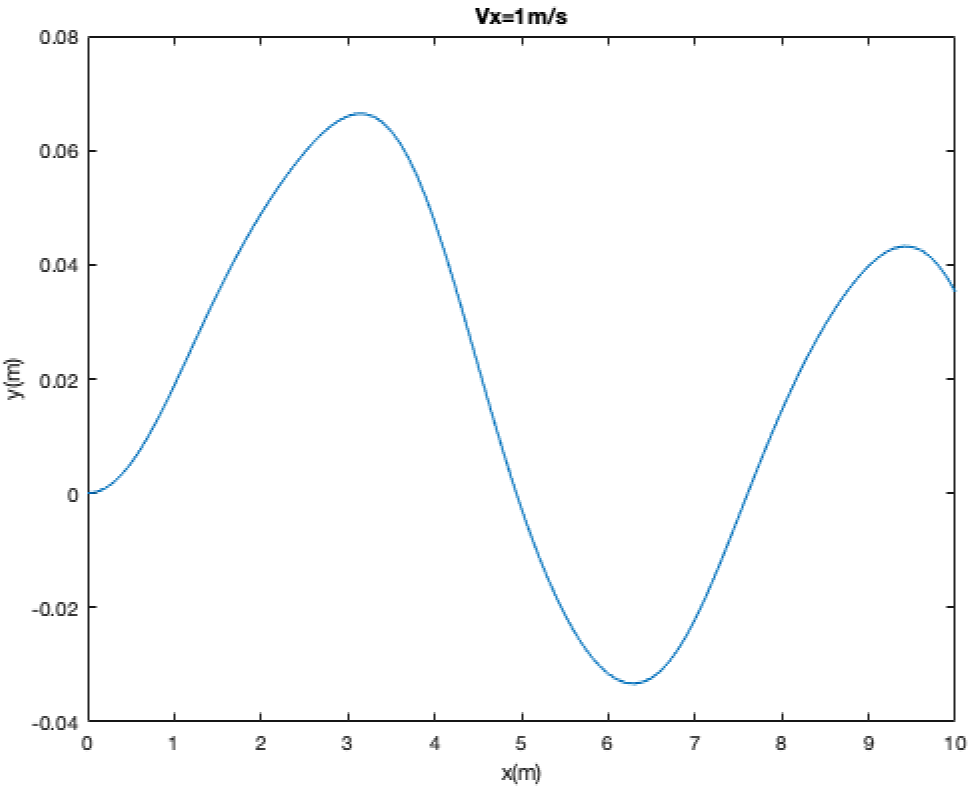
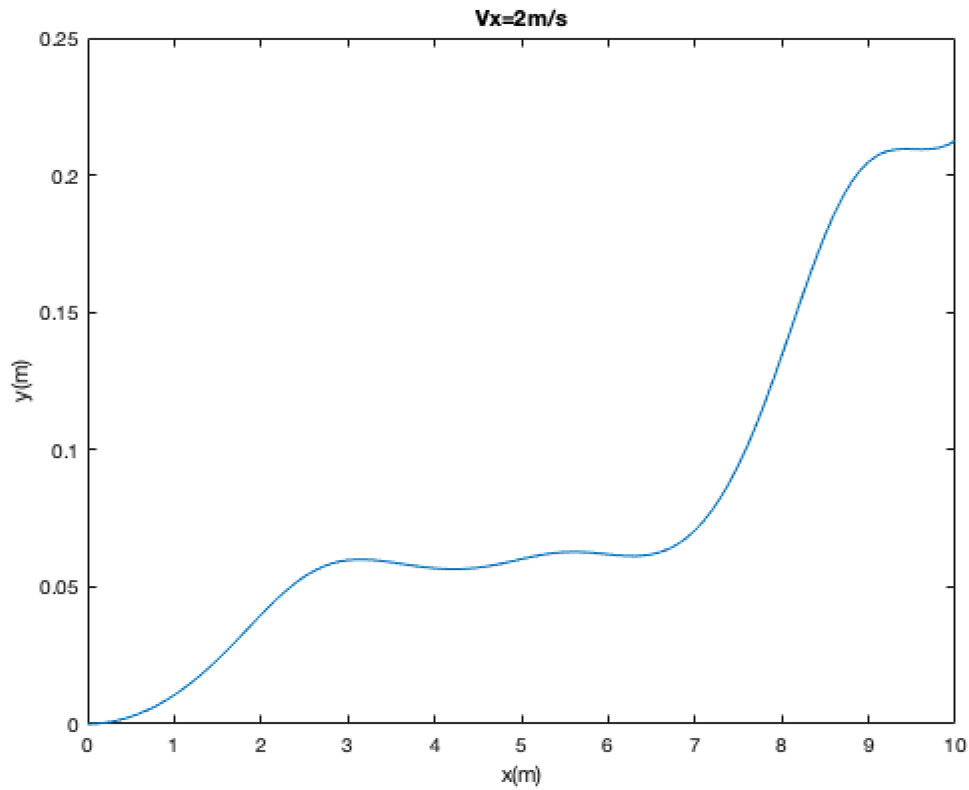
```
function ode45_ex

close all
clear all


lr=1.5;
lf=1.5;

radius=zeros();
steer=zeros();
i=1;

for steering_angle=10:5:90
    steer(i,1)=steering_angle;
    steering_angle_rad=steering_angle*pi()/180;
    radius(i,1)=(lf+lr)/tan(steering_angle_rad);
    i=i+1;
end
plot (steer(:,1), radius(:,1))
ylabel('path radius(m)');
xlabel('steering angle');
title('Path radius v.s. Steering angle')
```

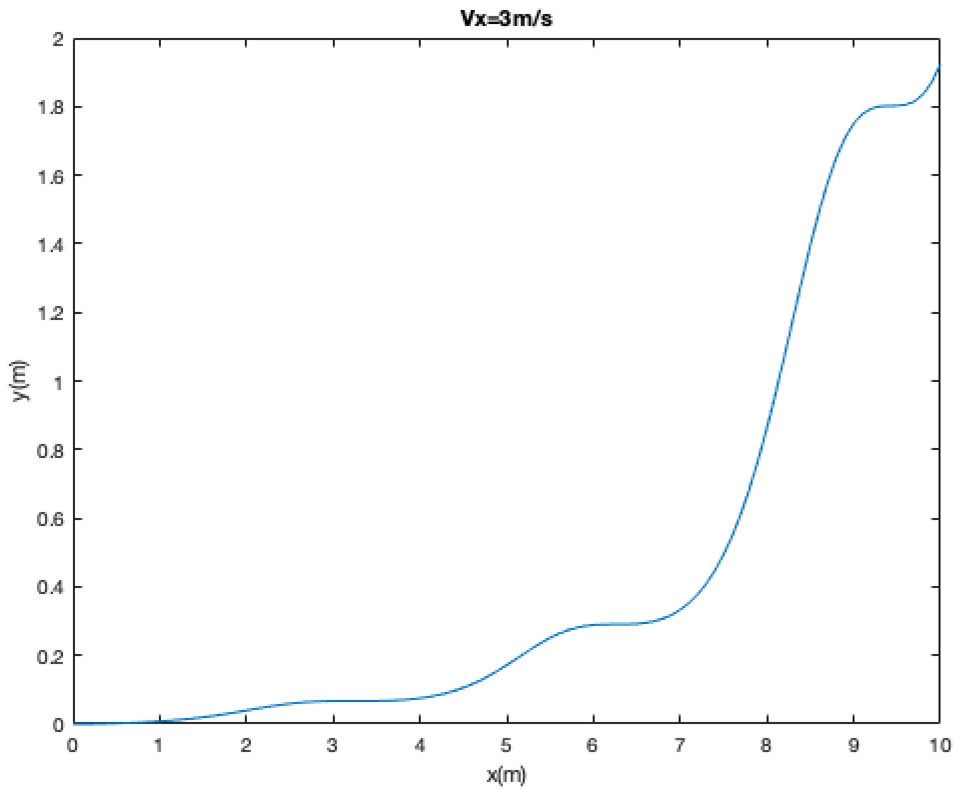



```
%B
```

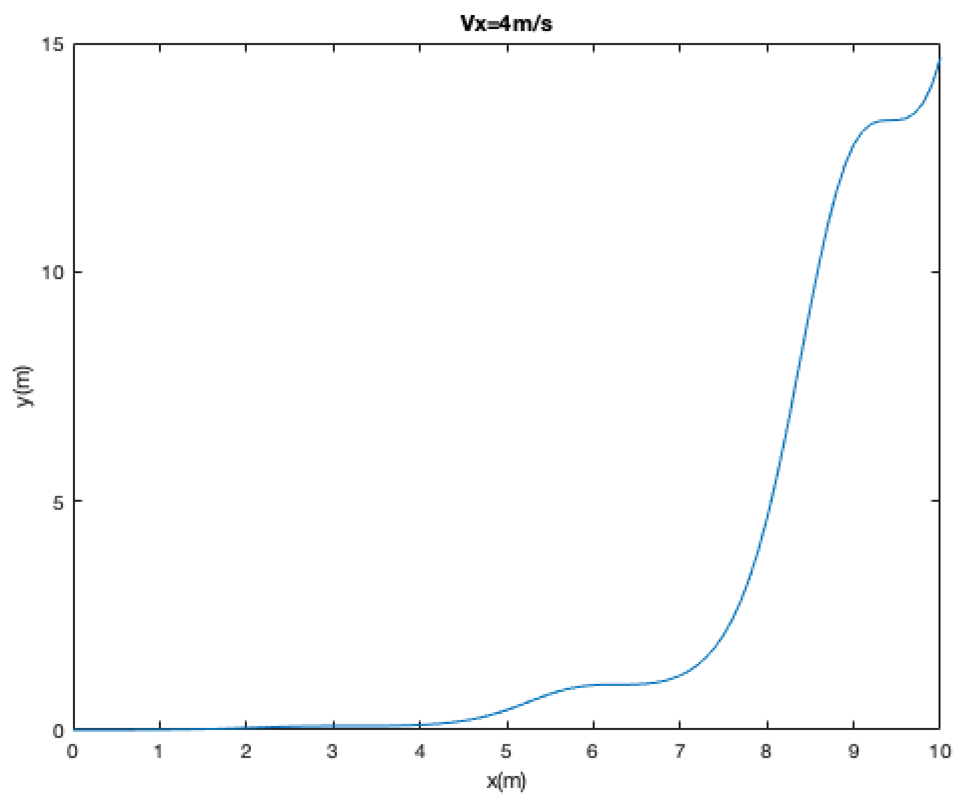
```
%Initial Condition:
xo=[0;0;0];


%timespan:
t=0:0.1:10;
x=zeros();
y=zeros();

for Vx=1:1:5
    [t,x]=ode45(@sys,t,xo);
    figure
    plot(t, x(:,3))
    ylabel('y(m)');
    xlabel('x(m)');
    title(['Vx=',num2str(Vx), 'm/s'])


end



function  dx = sys(t, x)

%Parameters:

%Vx=4; %(m/s)
m=50; %(kg)
Iz=100; %(kg-m^2);
Caf=8000; %(N/rad)
Car=8000;  %(N/rad)

%sinusoid input
steerangle=5*sin(t);
steerangle=steerangle*pi()/180;

A=-(cos(steerangle)*Caf+Car)/m*Vx;
B=(-(cos(steerangle)*Caf*lf+lr*Car)/(m*Vx))-Vx;
C=(-cos(steerangle)*Caf*lf+lr*Car)/Iz*Vx;
D=-(cos(steerangle)*Caf*lf*lf+lr*lr*Car)/Iz*Vx;
E=Caf*cos(steerangle)/m;
F=lf*Caf*cos(steerangle)/Iz;

u=steerangle;
dx(1,:)=A*x(1)+C*x(2)+E*u;
dx(2,:)=B*x(1)+C*x(2)+F*u;
dx(3,:)=x(1);

end
```


Vx=1m/s
0.08
0.06
0.04
0.02
0
-0.02
-0.04
y(m)
0 1 2 3 4 5 6 7 8 9 10
x(m)

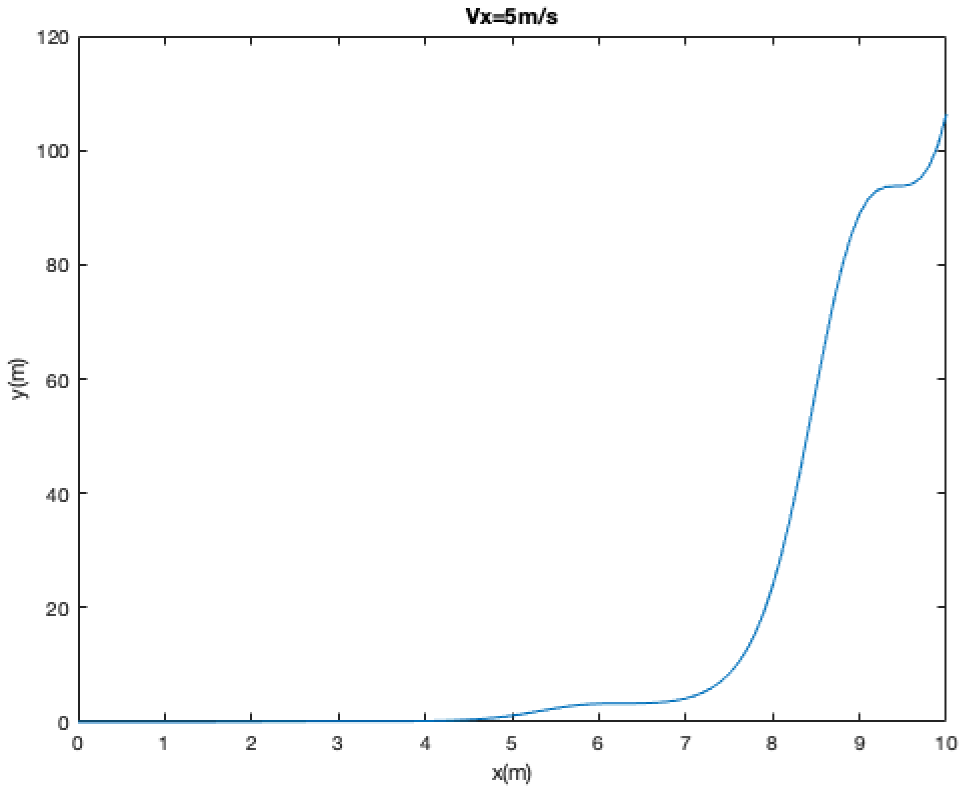

Vx=2m/s
0.25
0.2
0.15
0.1
0.05
0
y(m)
0 1 2 3 4 5 6 7 8 9 10
x(m)



Vx=3m/s
y(m)
x(m)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
1
2
3
4
5
6
7
8
9
10



Vx=4m/s
y(m)
x(m)
0
5
10
15
0
1
2
3
4
5
6
7
8
9
10




```
end
```

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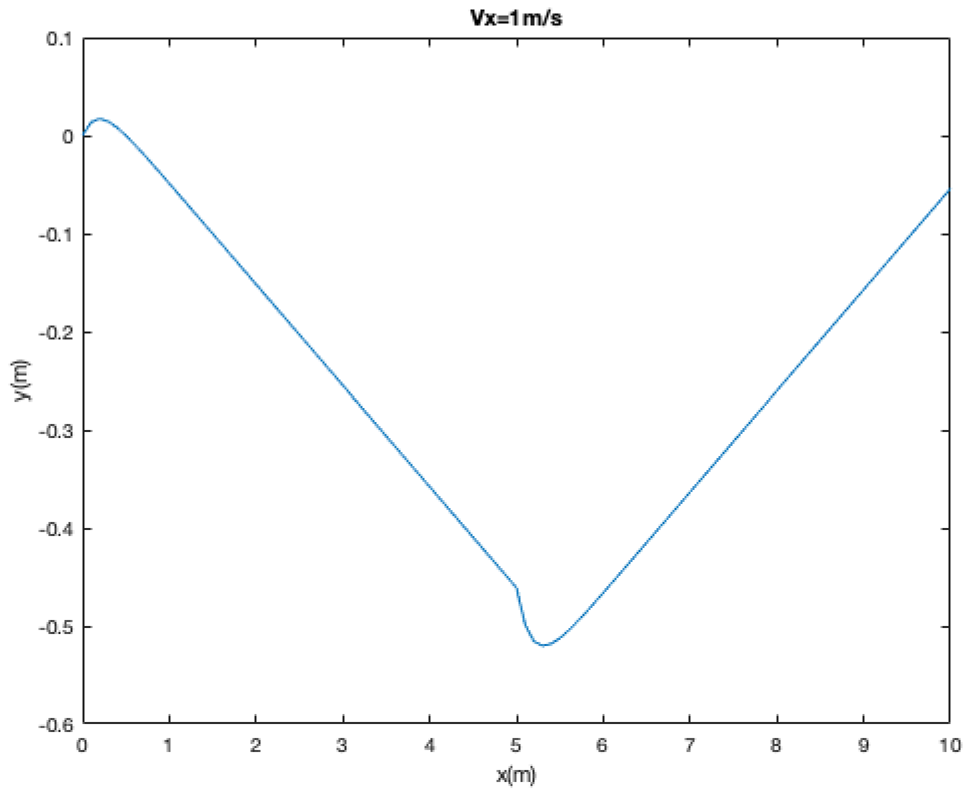
```
function ode45_ex
close all
clear all


lr=1.5;
lf=1.5;

%C
%Initial Condition:
xo=[0;0;0];


%timespan:
t=0:0.1:10;

Vx=1;
[t,x]=ode45(@sys,t,xo);
plot(t, x(:,3))
ylabel('y(m)');
xlabel('x(m)');
title(['Vx=',num2str(Vx), 'm/s'])




function  dx = sys(t, x)

%Parameters:

%Vx=4; %(m/s)
m=50; %(kg)
Iz=100; %(kg-m^2);
Caf=8000; %(N/rad)
Car=8000;  %(N/rad)

%square input
Amp=5; %steering angle in degree
yd=Amp-2*Amp.*heaviside(t-5)+2*Amp.*heaviside(t-10); %input square
 wave
steerangle=5*yd;
steerangle=steerangle*pi()/180;


A=-(cos(steerangle)*Caf+Car)/m*Vx;
B=(-(cos(steerangle)*Caf*lf+lr*Car)/(m*Vx))-Vx;
C=(-cos(steerangle)*Caf*lf+lr*Car)/Iz*Vx;
D=-(cos(steerangle)*Caf*lf*lf+lr*lr*Car)/Iz*Vx;
E=Caf*cos(steerangle)/m;
F=lf*Caf*cos(steerangle)/Iz;

u=steerangle;
```

```
dx(1,:)=A*x(1)+C*x(2)+E*u;
dx(2,:)=B*x(1)+C*x(2)+F*u;
dx(3,:)=x(1);

end


end
```



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```
close all
clear all

%Initial Condition:

xo=[0;0; 0;0];

%timespan:
dt=0.01;
ts=0:dt:10;


%desire path

Vx=30;
X=zeros();
Y=zeros();
X(1)=0;
Y(1)=-5;

 for step=1:10/dt+1
    if  ts(step)>=0 && ts(step)<3
        theta(step)=atan(5/90);
    else
        theta(step) =0;
    end
   if step>=2
        X(step)=X(step-1)+Vx*cos(theta(step))*dt;
        Y(step)=Y(step-1)+Vx*sin(theta(step))*dt;
   end

 end

[t,x]=ode45(@sys,ts,xo);

plot (t, x(:,1))
hold on
plot (t,x(:,3))
title ('Error state v.s. time')
legend('e1','e2')



for step= 1: 10/dt+1
   xr_e(step)=x(step,1)*sin(theta(step));
   yr_e(step)=x(step,1)*cos(theta(step));

end

figure
plot(X(1,:), Y(1,:))
hold on
```

```
plot (X(1,:)+xr_e(1,:), Y(1,:)-yr_e(1,:))
xlabel('X')
ylabel ('Y')

title ('Vehicle path in XY plane')


xlim([0 300])
ylim([-5 1])

function  dx = sys(t, x)

%Parameters:
Vx=30; %(m/s)
m=1573; %(kg)
Iz=2873; %(kg-m^2);
lf=1.1; %(m)
lr=1.58;  %(m)
Caf=80000; %(N/rad)
Car=80000;  %(N/rad)


A=[0 1 0 0; 0 -(2*Caf+2*Car)/(m*Vx) (2*Caf+2*Car)/m -
(2*Caf*lf-2*Car*lr)/(m*Vx); 0 0 0 1; 0 -(2*Caf*lf-2*Car*lr)/(Iz*Vx)
 (2*Caf*lf-2*Car*lr)/Iz -(2*Caf*lf*lf+2*Car*lr*lr)/(Iz*Vx)];
B1=[0; 2*Caf/m; 0; 2*Caf*lf/Iz];
B2=[0;-((2*Caf*lf-2*Car*lr)/(m*Vx))-Vx; 0; -(2*Caf*lf*lf+2*Car*lr*lr)/
(Iz*Vx)];


%R=1;
%Q=[1 0 0 0; 0 110 0 0; 0 0 1 0; 0 0 0 110];
%[K,S1,P1] = lqr(A,B1,Q,R);
p=[-25, -15, -20+i, -20-i];  % -25 -22 -20 -15
K=place(A,B1,p);

Ap=A-B1*K;     %Ap matrix after u=-Kx(t)
eig (Ap);


dt=0.8; %time for the turnning
dphi=atan(5/90);
dphi_des_val=dphi/dt;

dphi_des =  dphi_des_val*heaviside(t-0)-dphi_des_val*heaviside(t-dt)-
dphi_des_val*heaviside(t-3)+dphi_des_val*heaviside(t-3-dt);

dx = Ap*x+ B2*dphi_des;

end
```

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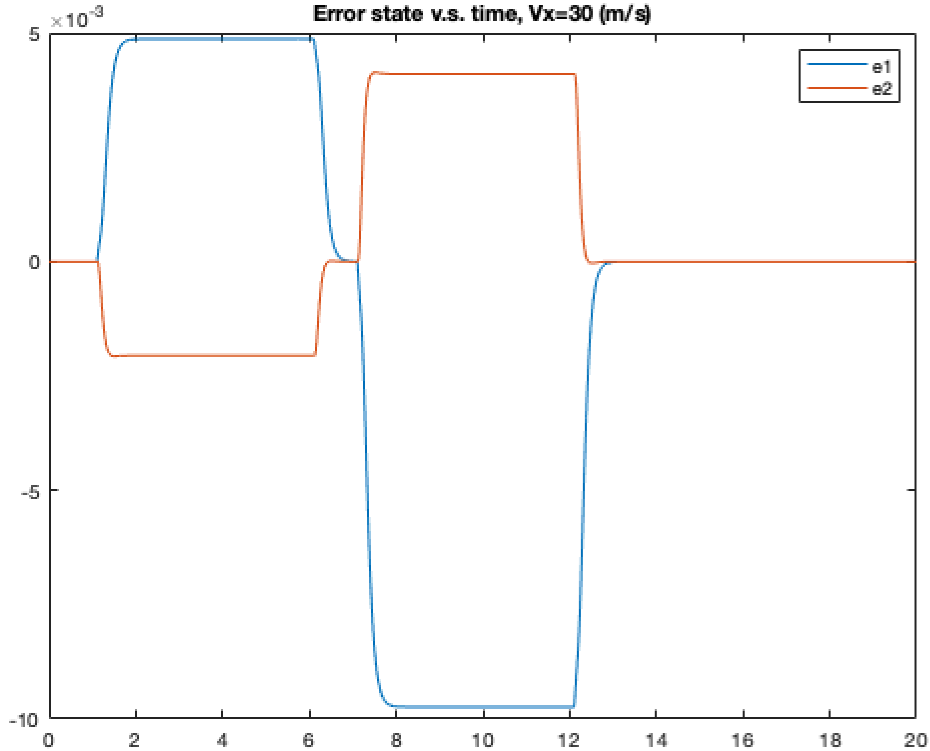
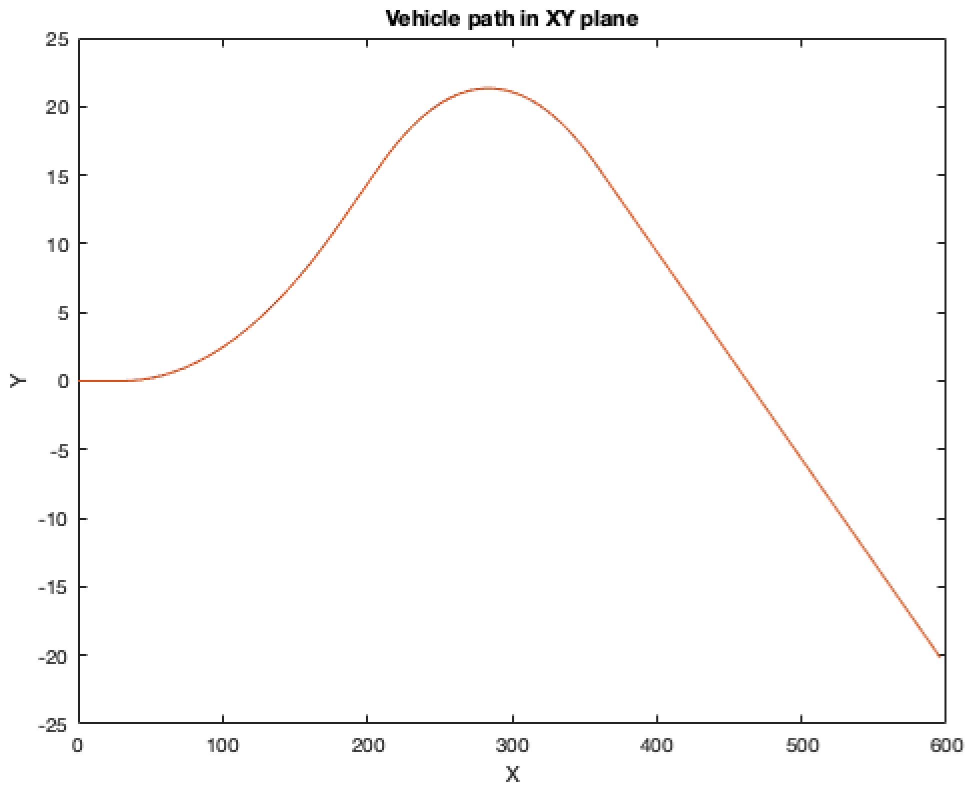
```
%Initial Condition:

xo=[0;0;0;0];

%timespan:
dt=0.01;
ts=0:dt:20;


Vx=30;
omega1=30/1000;
omega2=-30/500;
X=zeros();
Y=zeros();
X(1)=0;
Y(1)=0;


 for step=1:20/dt+1
    if  ts(step)>=0 && ts(step)<1
        theta(step)=0;
    elseif ts(step)>=1 && ts(step) <6
        theta(step)=omega1*(ts(step)-1);
    elseif ts(step)>=6 && ts(step)<7
        theta(step)=omega1*5;
    elseif ts(step)>=7 && ts(step)<12
        theta(step)=omega1*5+omega2*(ts(step)-7);
    else
        theta(step)=omega1*5+omega2*5;
    end
   if step>=2
        X(step)=X(step-1)+Vx.*cos(theta(step))*dt;
        Y(step)=Y(step-1)+Vx.*sin(theta(step))*dt;
    end

 end

[t,x]=ode45(@sys,ts,xo);



for step= 1: 20/dt+1

   xr_e(step)=x(step,1)*sin(theta(step));
   yr_e(step)=x(step,1)*cos(theta(step));

end
figure
plot (t, x(:,1))
hold on
plot (t,x(:,3))
title ('Error state v.s. time, Vx=30 (m/s)')
legend('e1','e2')
```

```
figure

 plot(X(1,:), Y(1,:))
 hold on
 plot (X(1,:)+xr_e(1,:), Y(1,:)-yr_e(1,:))
 xlabel('X')
 ylabel ('Y')
 title ('Vehicle path in XY plane')

function  dx = sys(t, x)

%Parameters:
Vx=30; %(m/s)
m=1573; %(kg)
Iz=2873; %(kg-m^2);
lf=1.1; %(m)
lr=1.58;  %(m)
Caf=80000; %(N/rad)
Car=80000;  %(N/rad)


A=[0 1 0 0; 0 -(2*Caf+2*Car)/(m*Vx) (2*Caf+2*Car)/m -
(2*Caf*lf-2*Car*lr)/(m*Vx); 0 0 0 1; 0 -(2*Caf*lf-2*Car*lr)/(Iz*Vx)
 (2*Caf*lf-2*Car*lr)/Iz -(2*Caf*lf*lf+2*Car*lr*lr)/(Iz*Vx)];
B1=[0; 2*Caf/m; 0; 2*Caf*lf/Iz];
B2=[0;-((2*Caf*lf-2*Car*lr)/(m*Vx))-Vx; 0; -(2*Caf*lf*lf+2*Car*lr*lr)/
(Iz*Vx)];


p=[-25, -10, -20+i, -20-i];
K=place(A,B1,p);

Ap=A-B1*K;     %Ap matrix after u=-Kx(t)

dt=0.1; %time for the turnning

dphi_des1=30/1000;
dphi_des2=30/500;

dphi_des =  -dphi_des1*heaviside(t-1-dt)+dphi_des1*heaviside(t-6-
dt)+dphi_des2*heaviside(t-7-dt)-dphi_des2*heaviside(t-12-dt);


dx = Ap*x+ B2*dphi_des;



end
```


Error state v.s. time, Vx=30 (m/s)
×10^-3
e1
e2


Vehicle path in XY plane
Y
X

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