Leg mobility
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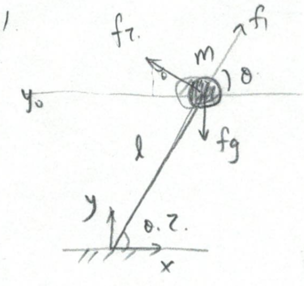
1)

part 1

1.

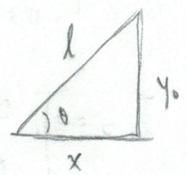


$\tau = f_\tau \cdot l(t)$

$X: f_l \cdot \cos\theta - f_\tau \sin\theta$

$Y: f_l \sin\theta + f_\tau \cos\theta - f_g$ $= mg$

$f_g = mg$



$\cos\theta = \frac{x}{l}$

$\sin\theta = \frac{y_0}{l}$

$l = \sqrt{x^2 + y_0^2}$

2.

$f_x = f_l \cos\theta - f_\tau \sin\theta$

$f_y = f_l \sin\theta + f_\tau \cos\theta - f_g$

$$\begin{cases} f_\tau = \tau / l \\ f_g = mg \\ \cos\theta = x/l \\ \sin\theta = \frac{y_0}{l} \end{cases} \Rightarrow \begin{aligned} f_x &= f_l \cdot \frac{x}{l} - \frac{\tau}{l} \cdot \frac{y_0}{l} \\ f_y &= f_l \cdot \frac{y_0}{l} + \frac{\tau}{l}\frac{x}{l} - mg \end{aligned}$$

3.

$f_y = 0 \quad \therefore \quad f_l \frac{y_0}{l} + \frac{\tau x}{l^2} - mg = 0 \quad \therefore \quad f_l = \left(mg - \frac{\tau x}{l^2}\right)\frac{l}{y_0}$

4.

$$f_x = \left(mg - \frac{\tau x}{l^2}\right)\frac{l}{y_0} \cdot \frac{x}{l} - \frac{\tau}{l} \cdot \frac{y_0}{l}$$

$$= \frac{mgx}{y_0} - \frac{\tau x^2}{l^2 y_0} - \frac{\tau y_0}{l^2} = \frac{mgx}{y_0} - \frac{\tau x^2}{(x^2+y_0^2)y_0} - \frac{\tau y_0}{(x^2+y_0^2)}$$

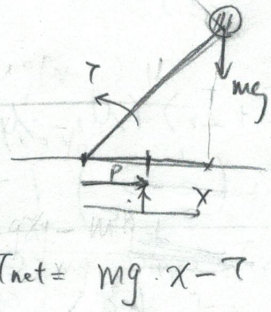
$$= \frac{mgx}{y_0} - \frac{\tau x^2 + \tau y_0^2}{(x^2+y_0^2)\,y_0} = \frac{mgx}{y_0} - \frac{\tau}{y_0}$$

5.

$m\ddot{x} = f_x$

$$m\ddot{x} = \frac{mgx}{y_0} - \frac{\tau}{y_0}$$

$$= \frac{mgx}{y_0} - \frac{mgp}{y_0}$$

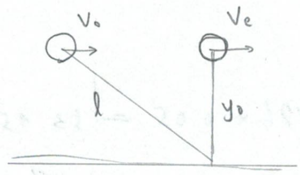


$\tau = m \cdot g \cdot p$

$$m\ddot{x} = \frac{mg(x-p)}{y_0}$$

$\tau_{net} = mg\,x - \tau$

6.)



$\Delta KE = \frac{-1}{2} m V_0^2$

$\Delta PE = 0$

$$W = \int Fx\,dx = \frac{-1}{2} m V_0^2$$

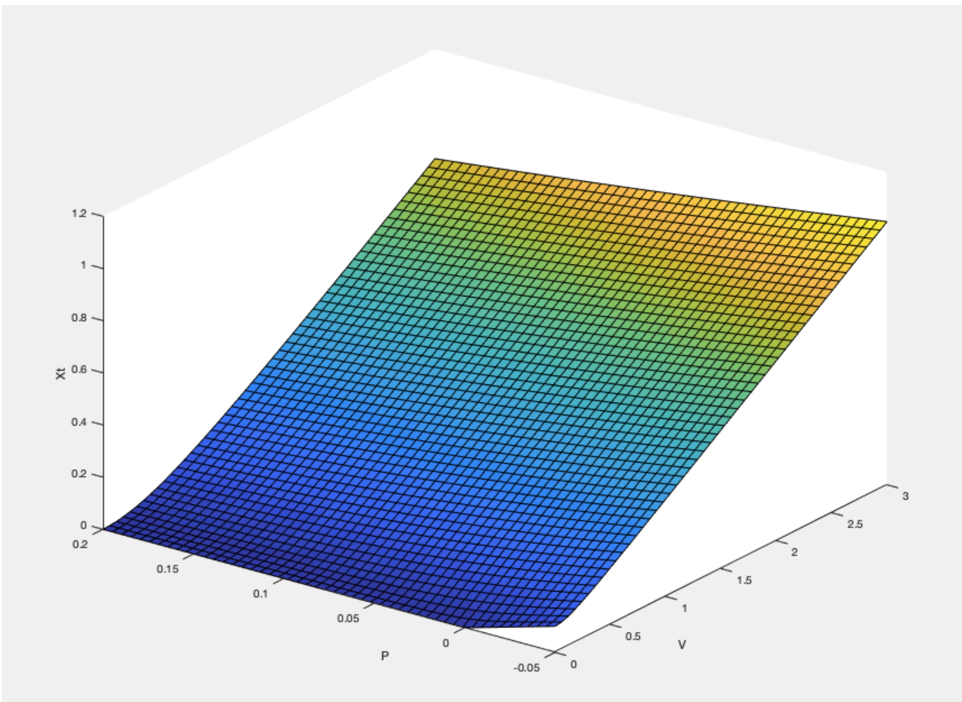
$$-\int_{-X_T}^{0} \frac{mg(x-P)}{y_0}\,dx = \frac{-1}{2} m V_0^2$$

$$\Rightarrow \frac{mg}{y_0} \cdot \left(\frac{1}{2} x^2 - Px\right)\Big|_{-X_T}^{0} = \frac{-1}{2} m V_0^2$$

$$-\frac{mg}{y_0}\left(\frac{-1}{2} X_T^2 + P X_T\right) = \frac{-1}{2} m V_0^2$$

$$\frac{-g}{2y_0} X_T^2 + \frac{Pg}{y_0} X_T - \frac{V_0^2}{2} = 0$$

$$\therefore X_T = \frac{\frac{-Pg}{y_0} \pm \sqrt{\frac{P^2 g^2}{y_0^2} + \frac{g V_0^2}{y_0}}}{g/y_0}$$



A reasonable human foot size is 25cm, so the range for center of pressure is set to be -0.05m to 0.2m. The speed for human walking is around 3.3 m/s. Therefore, the initial horizontal velocity is set in the range of 0~3m/s. When the capture point X_T is greater than the foot length P (0.2m) then the robot should make a step. On the other hand, if X_T is less than P, then the robot can rely on the ankle strategy to resume standing balance.

Problem 2
2.1)
Maxon EC90 48 V

Part number: 500267

J_m = 5.060 x 10^{-4} [kgm^2]

Maximum continuous torque τ_{max} = 0.964 [Nm]

2.2)

Required peak force = 137% Body Weight = 1.37 x 9.81 = 1075.176 N
Gear radius =0.05 m

∴ 1075.18 x 0.05 = 53.76 Nm (Required peak torque) …..(1)

τ_{max} = 0.964 [Nm]

$$\frac{I_{ON}}{I_A} = 1.4 \quad I_A = 4.06A \quad \therefore I_{ON} = 5.684A$$

Torque constant = 0.231 $\frac{Nm}{A}$ $\quad \therefore \tau^*_{max} = 1.313Nm$ …………(2)

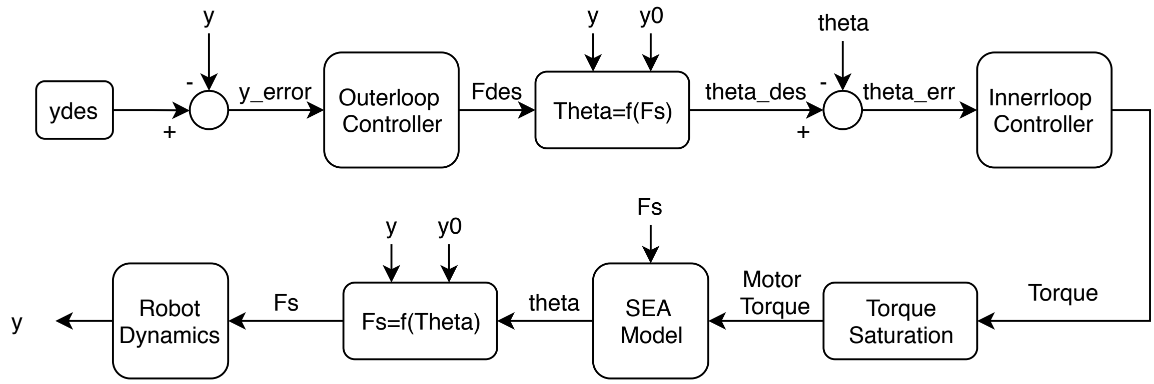
(1) / (2) = 40.9 ∴ Choose N =40

2.3)

$$F_s = M\ddot{y} + Mg \qquad F_s = k\Delta l = k[\frac{r\theta_m}{N} + (y_0 - y)]$$

$$J_m\ddot{\theta}_m = \tau_m - \tau_{ext} = \tau_m - \frac{rF_s}{N} \qquad \therefore \ddot{\theta}_m = \frac{\tau_m}{J_m} - \frac{rF_s}{NJ_m}$$

2.4)



Notes:

1) Outer-loop controller:
   Outer-loop controller is a PID controller.
   a) Purpose: The outer_loop controller is aimed to let y track y_des(constant).
   b) Input: y_des, y
   c) Output: F_des
   d) Equation: The gains are as follow, Kp=16000, Kd=100, Ki=7000.

$$F_{des} = K_p \cdot y\_error + K_s \cdot \frac{dy\_error}{dt} + K_i \cdot \int y\_error \cdot dt$$

2) Theta=f(Fs):

From the previous part of this problem, we know that Fs is a function of theta. Therefore, it is possible to obtain θ_{des}as a function of Fs_des.

a) Purpose: Transform Fs_des to theta_des.
b) Input: F_des, y0, y
c) Output: theta_des
d) Equation:

$$\theta_{des} = [\frac{F_{sdes}}{k} - (y_0 - y)] \cdot \frac{N}{r}$$

3) Inner-loop controller:

Inner-loop controller is also a PID controller.

a) Purpose: The inner_loop controller is aimed to let θ track θ_{des}(not a constant) .
b) Input: theta_des, theta.
c) Output: Motor torque
d) Equation: The gains are as follow, Kp=2, Kd=3, Ki=0.

$$Motor\_torque = K_p \cdot \theta\_error + K_s \cdot \frac{d\theta\_error}{dt} + K_i \cdot \int \theta\_error \cdot dt$$

4) SEA Model:

SEA model is the motor dynamics second order equation.

a) Purpose: SEA dynamics.
b) Input: Fs, Motor torque.
c) Output: theta
d) Equation:

$$\ddot{\theta}_m = \frac{\tau_m}{J_m} - \frac{rF_s}{NJ_m}$$

5) Fs=(Theta) :

From the previous part of this problem, we know that Fs is a function of theta.

a) Purpose: Transform θ to Fs.
b) Input: θ, y0, y
c) Output: Fs
d) Equation:

$$F_s = k\Delta l = k[\frac{r\theta_m}{N} + (y_0 - y)]$$

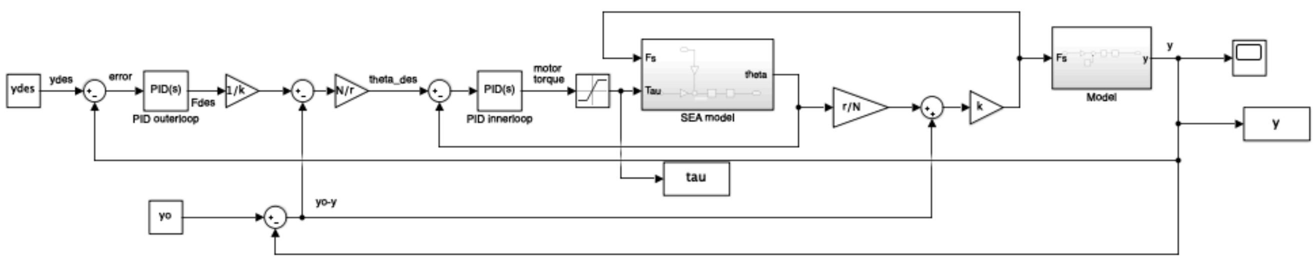
6) Robot Dynamics:

Robot dynamics is the second order equation that describes the movement of the system.

a) Purpose: Robot dynamics.
b) Input: Fs.
c) Output: y
d) Equation:

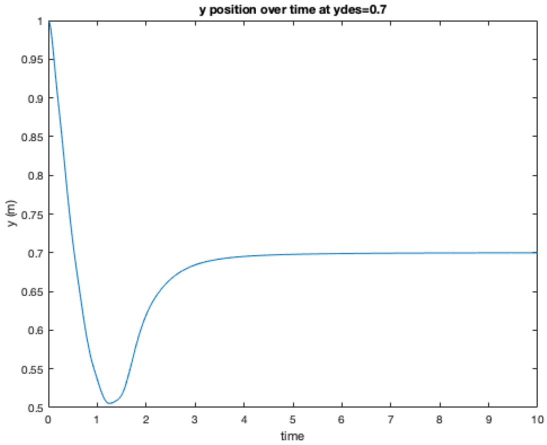
$$\ddot{y} = \frac{F_s}{M} - g$$

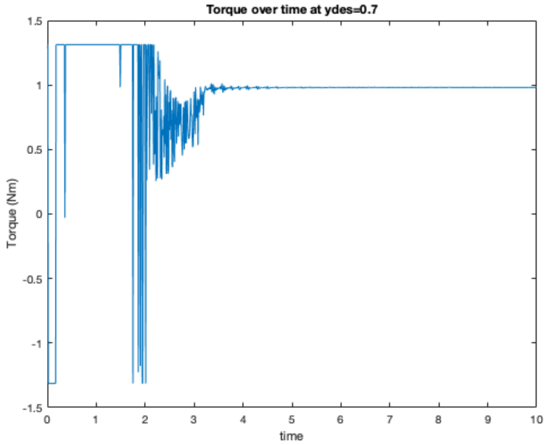
Simulink



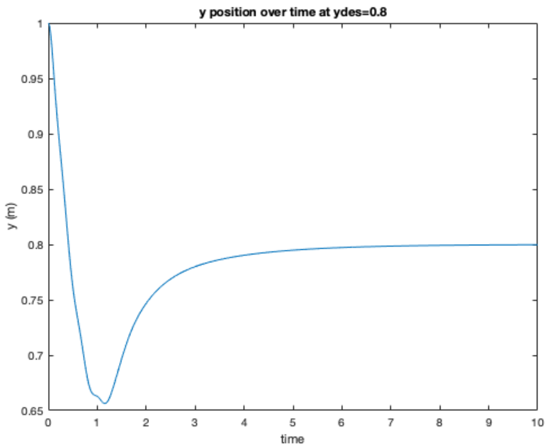
2.5)

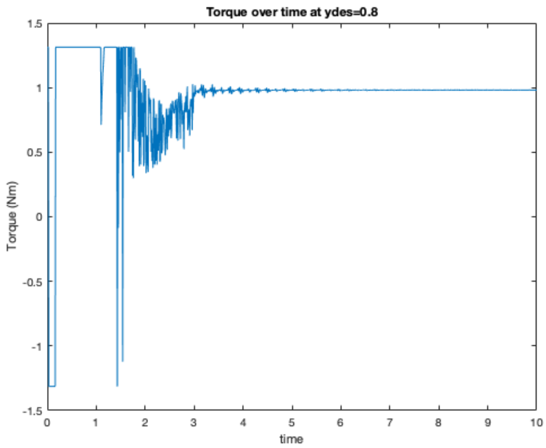
1) ydes =0.7



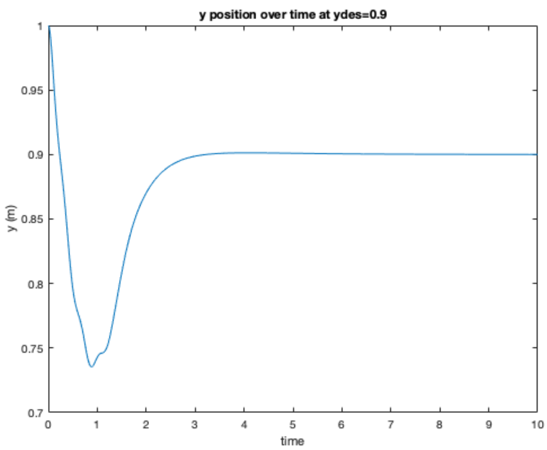


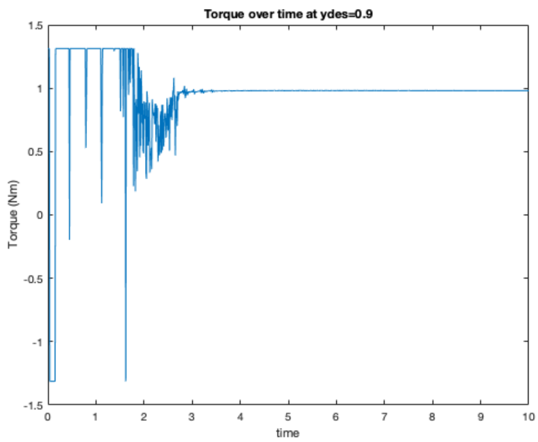
2) ydes=0.8





3) ydes=0.9





There are some oscillations at the SEA (inner-loop), but it dies out very fast. With all three desired y positions, the system all come to stable eventually. In the first condition when ydes= 0.7, the system takes more time to become stable, since there is a 0.3 position error at t=0. This can be seen as an impulse. With this logic, the system gets to stable situation at the third condition when ydes=0.9.

2.6)

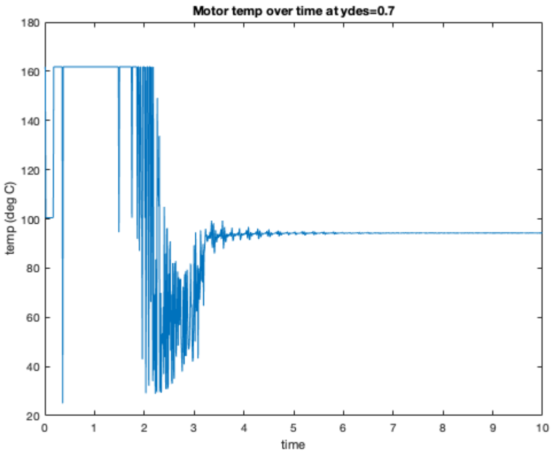
The thermal motor dynamic is provided as :

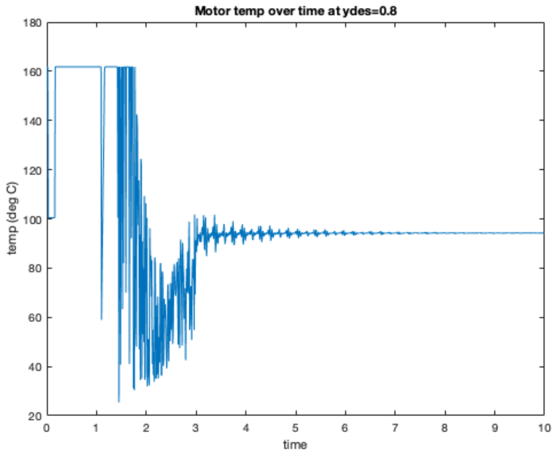
$$\Delta T_W = \frac{(R_{th1} + R_{th2}) \cdot R \cdot I_{mot}{}^2}{1 - \alpha_{Cu} \cdot (R_{th1} + R_{th2}) \cdot R \cdot I_{mot}{}^2}$$

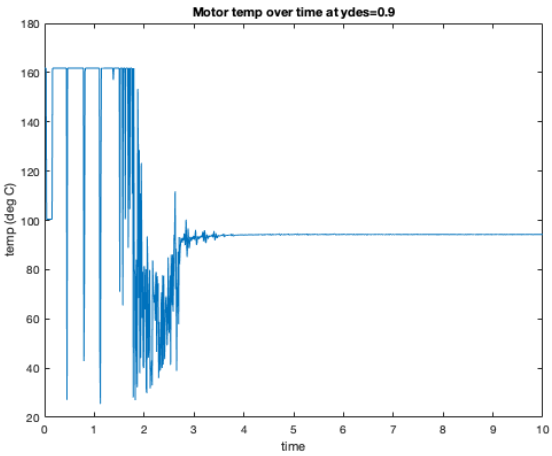
Matlab Code:

```
R1=1.748;
R2=1.82;
R=0.844;
a=0.0039;
torque_c=0.231;

initial_temp=25;
winding_temp=initial_temp+(R1+R2)*R*(tau/torque_c).^2./(1-a*(R1+R2)*R*(tau/torque_c.^2));
figure
plot(tout, winding_temp)

title(['Motor temp over time at ydes=',num2str(ydes)])
xlabel('time')
ylabel('temp (deg C)')
```

Results:







The acceptable temperature bound is less than 125$^\circ C$. The final steady state temperature is under 100$^\circ C$, which is in the acceptable bound. There are some differences between the target heights, when ydes is closer to y0, the thermal dynamics come to stable faster. This is expected, since the motor get to steady state faster. This is unrealistic since the temperature goes to around 160 $^\circ C$ before coming stable. 160$^\circ C$ is relative high, and for a real humanoid, the motor will be actuated at most of the time, so it will maintain at 160$^\circ C$ for most of the time.

Problem 3
3.1)

FP: Swing Foot point. The tip of the swing foot. (Position, Velocity, Acceleration)
TR: Trunk. The bottom of the trunk. (Position, Velocity, Acceleration)

3.2)

Plan for FP:
The FP plan is modified based on the nominal swing foot point plan (desired) and current swing foot point (position and velocity). Noted that, here we will use the current position and velocity of the stance foot (q1to q5 states)to calculate the position and velocity of swing foot point.Then using PD-feedback plus nominal acceleration as feedforward term to compute the desired acceleration of the foot point at each time step.

Plan for TR:
The TR plan is similar to that of FP plan. The TR plan is also modified based on the nominal trunk state(position and velocity) and current trunk state(position and velocity). It use a nominal trunk acceleration as the feedforward term plus PD-feeback. TR plan is more straight forward that we take the position and velocity of q3 state, and no additional calculation is needed.

3.3)

$$GRF_x - \mu FRG_y \le 0 \quad - GRF_x - \mu FRG_y \le 0$$

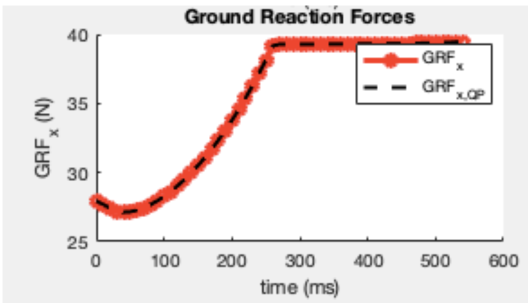
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} GRF_x \\ GRF_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore a_{11} = 1 \quad a_{12} = -\mu \quad a_{21} = -1 \quad a_{22} = -\mu$$

```
mu=0.8;  % friction coefficient

QP.Aineq = [zeros(1,10) 1 -mu;zeros(1,10) -1 -mu];
QP.bineq = [0;0];
```

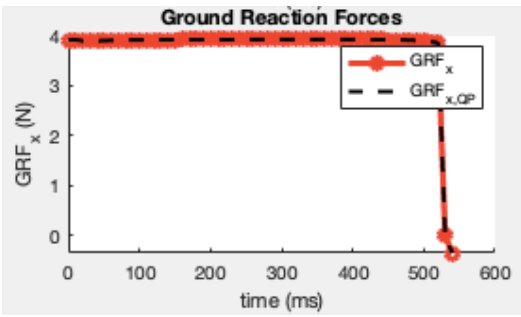
3.4)

$\mu = 0.1$



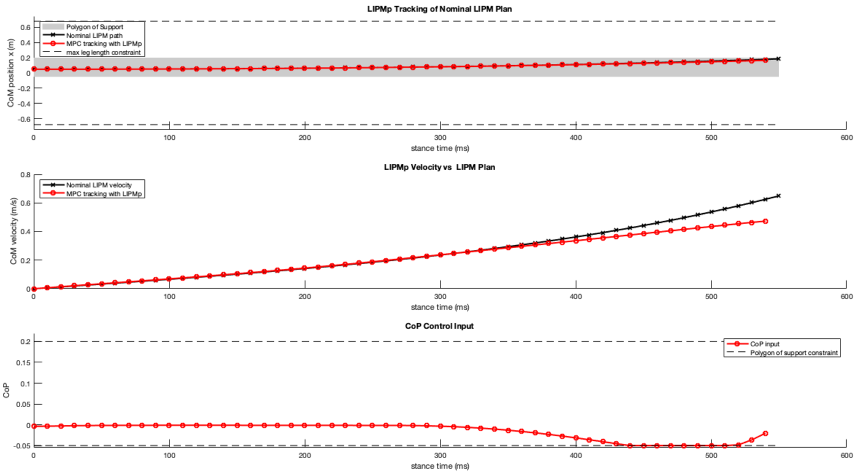
(fig 3.1, GRF_x)

$\mu = 0.01$



(fig 3.2, GRF_x)

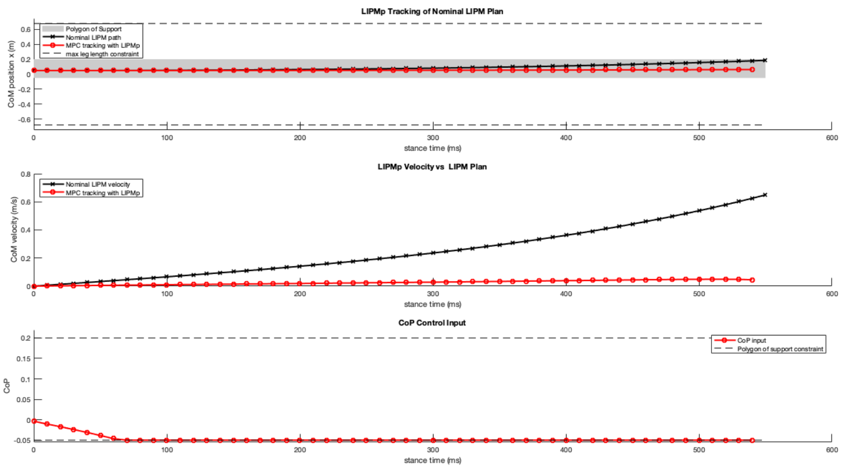
$\mu = 0.1$



(fig 3.3, Tracking performance when $\mu = 0.1$)

The ground reaction force is limited to 40N and it immensely affects the tracking performance after 300ms. It is expected by looking at GRF_x that the horizontal force reaches it limitation at 250 ms.

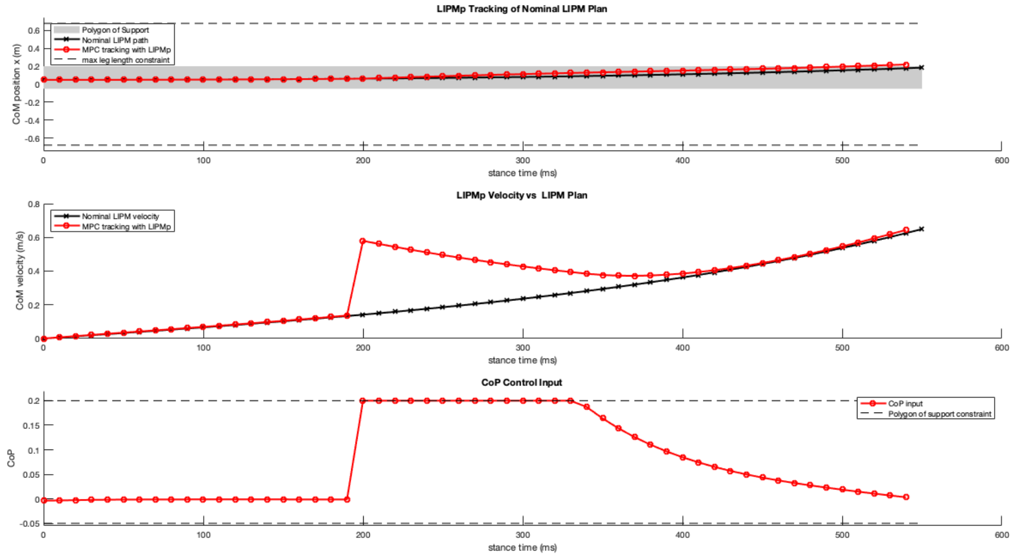
$\mu = 0.01$



(fig 3.4, Tracking performance when $\mu = 0.01$)

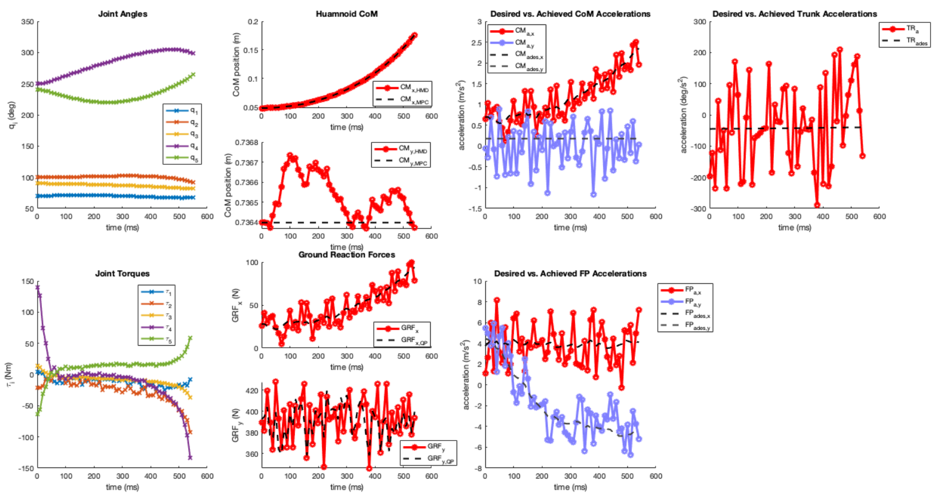
When μ is relatively small, the friction between the stance foot and the ground is insufficient. With low friction, there will not be enough horizontal force to push the body forward to track the desired position and velocity. It is similar to someone walking on ice that slip at his or her stance foot, when trying to push hard to run forward.

3.5)

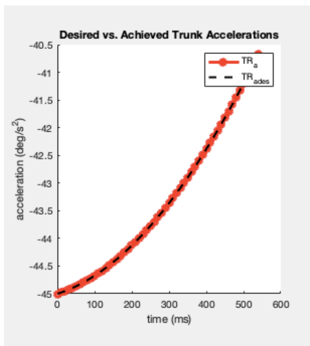


(fig 3.5, Tracking performance under 20Ns disturbance.)

Yes, the humanoid somehow get back to its intended plan after the disturbance. As shown in figure 3.5, LIPM velocity deviates from the LIPM velocity plan at t=200ms due to the disturbance. This deviation is compensated by the ankle strategy from the stance leg. As one can see in the "COP control Input" in figure 5.3, the center of pressure was pushed to the tip of the stance foot at 200ms by the disturbance. This generated an additional torque to maintain the stability of the model.



(fig 3.6, Behavior under noises.)



(fig 3.7, Behavior goals.)

The "trends" of the behaviors under noises are similar to that without noises. However, when the magnitude of the noises present during the simulation are relatively substantial to the overall trend, the model cannot perfectly track the goals. For example, figure 3.7 is the desired and achieved accelerations of the trunk without any noise. In figure 3.6, the noises in achieved trunk acceleration are too big (0 +/-200) for the model to track the desired acceleration(-43 +/- 2).