Package 'ConvergenceConcepts'

September 18, 2019

Type Package

Title Seeing Convergence Concepts in Action

Version 1.2.2

Date 2019-09-18

Maintainer Pierre Lafaye De Micheaux <lafaye@unsw.edu.au>

Description This is a pedagogical package, designed to help students understanding convergence of random variables. It provides a way to investigate interactively various modes of convergence (in probability, almost surely, in law and in mean) of a sequence of i.i.d. random variables. Visualisation of simulated sample paths is possible through interactive plots. The approach is illustrated by examples and exercises through the function 'investigate', as described in
Lafaye de Micheaux and Liquet (2009) <doi:10.1198/tas.2009.0032>.
The user can study his/her own sequences of random variables.

License GPL (>= 2)

LazyLoad yes

Depends R (>= 2.5.0), tcltk, tkrplot, lattice, grDevices

Suggests

NeedsCompilation no

Author Pierre Lafaye De Micheaux [aut, cre],
Benoit Liquet [aut]

Repository CRAN

Date/Publication 2019-09-18 09:10:16 UTC

R topics documented:

`check.convergence` *Check convergence*

Description

This function enables one to investigate the four classical modes of convergence on simulated data: in probability, almost surely, in r-th mean and in law.

Usage

```
check.convergence(nmax,M,genXn,argsXn=NULL,mode="p",epsilon=0.05,r=2,nb.sp=10,
  density=FALSE,densfunc=dnorm,probfunc=pnorm,tinf=-3,tsup=3,plotfunc=plot,...)
```

Arguments

`nmax`	number of points in each sample path.
`M`	number of sample paths to be generated.
`genXn`	a function that generates the Xn-X values, or only the Xn values in the law case.
`argsXn`	a list of arguments to `genXn`.
`mode`	a character string specifying the mode of convergence to be investigated, must be one of "p" (default), "as", "r" or "L".
`epsilon`	a numeric value giving the interval endpoint.
`r`	a numeric value (r>0) if convergence in r-th mean is to be studied.
`nb.sp`	number of sample paths to be drawn on the left plot.
`density`	if density=TRUE, then the plot of the density of X and the histogram of Xn is returned. If density=FALSE, then the plot of the distribution function F(t) of X and the empirical distribution Fn(t) of Xn is returned.
`densfunc`	function to compute the density of X.
`probfunc`	function to compute the distribution function of X.
`tinf`	lower limit for investigating convergence in law.
`tsup`	upper limit for investigating convergence in law.
`plotfunc`	R function used to draw the plot: for example `plot` or `points`.
`...`	optional arguments to `plotfunc`.

Details

The objective of this function is to investigate graphically the convergence of some random variable Xn to some random variable X. In order to use it, you should be able to provide generators of Xn and X (or of Xn-X). The four modes of convergence that you can try are: in probability, almost surely, in r-th mean and in law. For the convergence in law, we compute

$$\hat{(l)}_n(t) = |\hat{F}_n(t) - F(t)|$$

for ten values equally distributed between `tinf` and `tsup`.

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

criterion, generate, investigate, law.plot2d, law.plot3d, p.as.plot, visualize.crit, visualize.sp

Examples

```
## Not run:

######################## Exercise 3 ##############################
# Let X1, X2, ..., Xn be independent random variables such that #
# P[Xn=sqrt(n)]=1/n and P[Xn=0]=1-1/n                           #
# Does Xn converges to 0 in 2-th mean? in probability?         #
#################################################################

       options(example.ask=FALSE)

       pnotrgen<-function(n){rbinom(n,1,1/(1:n))*sqrt(1:n)}

       check.convergence(nmax=1000,M=500,genXn=pnotrgen,mode="r",r=2)
       legend(100,6,legend=expression(hat(e)["n,2"]),lty=1)
       tt3.1 <<- check.convergence(nmax=1000,M=500,genXn=pnotrgen,mode="p")


## End(Not run)
```

`criterion` *Convergence criterion computation*

Description

This function computes the values of the criterion convergence function for convergence in probability, almost surely or in r-th mean, given the sample paths.

Usage

```
criterion(data,epsilon=0.05,mode="p",r=2)
```

Arguments

`data`	matrix containing the sample paths of Xn-X values.
`epsilon`	a numeric value giving the interval endpoint.
`mode`	a character string specifying the mode of convergence to be investigated, must be one of "p" (default), "as" or "r".
`r`	a numeric value (r>0) if convergence in r-th mean is to be studied.

Details

The `data` matrix contains the

$$X_n - X$$

values. If mode="p", `criterion` approximates

$$p_n = P[|X_n - X| > \epsilon]$$

. If mode="as", `criterion` approximates

$$a_n = P[\exists k \geq n; |X_k - X| > \epsilon]$$

. If mode="r", `criterion` approximates

$$e_{n,r} = E|X_n - X|^r$$

. The approximations are based on the frequentist approach.

Value

`crit`	the vector of criterion values.

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

check.convergence, generate, investigate, law.plot2d, law.plot3d, p.as.plot, visualize.crit, visualize.sp

Examples

```
myrbinom <- function(n,alpha){rbinom(n,1,1/(1:n))*((1:n)**alpha)}
data <- generate(nmax=1000,M=500,myrbinom,args=list(alpha=0.5))$data
critr1 <- criterion(data,mode="r",r=1)$crit
```

---

generate *Generation of sample paths*

Description

This function generates the sample paths of a sequence of random variables.

Usage

```
generate(randomgen,nmax=1000,M=500,argsgen=NULL)
```

Arguments

nmax	number of points in each sample path.
M	number of sample paths to be generated.
randomgen	a function that generates the Xn-X values.
argsgen	a list of arguments to randomgen.

Value

data	matrix containing in each row a sample path of Xn-X values.

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

check.convergence, criterion, investigate, law.plot2d, law.plot3d, p.as.plot, visualize.crit, visualize.sp

Examples

```
myrbinom<-function(n,alpha){rbinom(n,1,1/(1:n))*((1:n)**alpha)}
data <- generate(randomgen=myrbinom,argsgen=list(alpha=0.5))$data
```

---

investigate *Investigate examples and exercises from the paper*

Description

This function investigates the convergence for all the examples and exercises from the article cited in references.

Usage

```
investigate()
```

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

check.convergence, criterion, generate, law.plot2d, law.plot3d, p.as.plot, visualize.crit, visualize.sp

Examples

```
investigate()
```

`law.plot2d` *Interactive 2D convergence in law*

Description

This function enables one to draw the interactive 2D plot to investigate convergence in law.

Usage

```
law.plot2d(data,density=FALSE,densfunc=dnorm,probfunc=pnorm,tinf=-5,tsup=5)
```

Arguments

`data`	matrix containing the sample paths of Xn values.
`density`	if density=TRUE, then the plot of the density of X and the histogram of Xn is returned. If density=FALSE, then the plot of the distribution function F(t) of X and the empirical distribution Fn(t) of Xn is returned.
`densfunc`	function to compute the density of X.
`probfunc`	function to compute the distribution function of X.
`tinf`	lower limit for investigating convergence in law.
`tsup`	upper limit for investigating convergence in law.

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

`check.convergence`, `criterion`, `generate`, `investigate`, `law.plot3d`, `p.as.plot`, `visualize.crit`, `visualize.sp`

Examples

```
rand <- function(n){(cumsum(rchisq(n,df=1))-(1:n))/sqrt(2*(1:n))}
data <- generate(randomgen=rand,nmax=1000,M=500)$data
law.plot2d(data)
```

`law.plot3d` *Static 3D convergence in law*

Description

This function enables one to draw the static 3D plot to investigate convergence in law.

Usage

```
law.plot3d(data,probfunc,tinf=-5,tsup=5)
```

Arguments

`data`	matrix containing the sample paths of Xn values.
`probfunc`	function to compute the distribution function of X.
`tinf`	lower limit for investigating convergence in law.
`tsup`	upper limit for investigating convergence in law.

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

`check.convergence`, `criterion`, `generate`, `investigate`, `law.plot2d`, `p.as.plot`, `visualize.crit`, `visualize.sp`

Examples

```
rand <- function(n){(cumsum(rchisq(n,df=1))-(1:n))/sqrt(2*(1:n))}
data <- generate(randomgen=rand,nmax=300,M=2000)$data
law.plot3d(data,pnorm)
```

p.as.plot *Interactive plot for convergence in probability and almost surely*

Description

This function enables one to draw the interactive plots used to study convergence in probability and almost surely.

Usage

```
p.as.plot(data,critp,critas,epsilon=0.05,nb.sp=10,mode="p")
```

Arguments

data	matrix containing the sample paths of Xn-X values.
critp	vector containing the criterion values for convergence in probability.
critas	vector containing the criterion values for convergence almost surely.
epsilon	a numeric value giving the interval endpoint.
nb.sp	number of sample paths to be drawn on the left plot.
mode	a character string specifying the mode of convergence to investigate, must be one of "p" (default), "as" or "r".

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

check.convergence, criterion, generate, investigate, law.plot2d, law.plot3d, visualize.crit, visualize.sp

Examples

```
myrbinom <- function(n,alpha){rbinom(n,1,1/(1:n))*((1:n)**alpha)}
data <- generate(randomgen=myrbinom,nmax=1000,M=500,argsgen=list(alpha=0.5))$data
critp <- criterion(data,epsilon=0.05,"p")$crit
critas <- criterion(data,epsilon=0.05,"as")$crit
p.as.plot(data,critp,critas,epsilon=0.05,nb.sp=10,mode="p")
```

`visualize.crit` *Visualisation of convergence criterion curves*

Description

This function enables one to visualize the convergence criterion curve.

Usage

```
visualize.crit(crit,plotfunc=plot,...)
```

Arguments

`crit`	vector containing the criterion values.
`plotfunc`	R function used to draw the plot: for example `plot` or `points`.
`...`	optional arguments to `plotfunc`.

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

`check.convergence`, `criterion`, `generate`, `investigate`, `law.plot2d`, `law.plot3d`, `p.as.plot`, `visualize.sp`

Examples

```
myrbinom <- function(n,alpha){rbinom(n,1,1/(1:n))*((1:n)**alpha)}
data <- generate(randomgen=myrbinom,nmax=1000,M=500,argsgen=list(alpha=0.5))$data
critr1 <- criterion(data,epsilon=0.05,mode="r",r=2)$crit
visualize.crit(critr1,main="Convergence in 2-th mean?",col="red")
legend(0,5,legend=expression(hat(e)["n,2"]),lty=1,col="red")
```

visualize.sp *Visualisation of sample paths*

Description

This function enables one to visualize the sample paths.

Usage

```
visualize.sp(data,epsilon=0.05,nb.sp=10,plotfunc=plot,...)
```

Arguments

data	matrix containing the sample paths of Xn-X values.
epsilon	a numeric value giving the interval endpoint.
nb.sp	number of sample paths to draw on the plot.
plotfunc	R function used to draw the plot: for example plot or points.
...	optional arguments to plotfunc.

Author(s)

P. Lafaye de Micheaux and B. Liquet

References

Lafaye de Micheaux, P. (<plafaye@club.fr>), Liquet, B. "Understanding Convergence Concepts: a Visual-Minded and Graphical Simulation-Based Approach", The American Statistician, 63:2, 173–178, (2009).

See Also

check.convergence, criterion, generate, investigate, law.plot2d, law.plot3d, p.as.plot, visualize.crit

Examples

```
myrand <- function(n,...){cumsum(rnorm(n,...))/(1:n)}
data <- generate(randomgen=myrand,nmax=1000,M=500)$data
visualize.sp(data,main="Ten sample paths",col="grey")
```

Index