Laborator 6 /

TEMĂ:

23) $f(x) = \frac{e^x}{e-1}$, $0 \le x \le 1$

i) Dacă $x < 0$ atunci: $F(x) = 0$

ii) Dacă $0 \le x \le 1$ atunci:

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{x} \frac{e^t}{e-1}\,dt = \frac{e^x - 1}{e-1}$$

iii) Dacă $x > 1$ atunci:

$$F(x) = \int_{0}^{1} \frac{e^x}{e-1}\,dx = 1$$

Din i, ii, iii $\Rightarrow$ $F(x) = \begin{cases} 0, & x < 0 \\ \frac{e^x - 1}{e-1}, & x \in [0,1] \\ 1, & x \cancel{\ge} 1 \end{cases}$

Pt $x \in [0,1]$:

$F(x) = u \Leftrightarrow \frac{e^x - 1}{e-1} = u \Leftrightarrow e^x = u(e-1) + 1 \Leftrightarrow$

$\Rightarrow x = \ln[u(e-1)+1] \Rightarrow X = \ln[U(e-1)+1]$

24) $f(x) = \begin{cases} \frac{x-2}{2}, & 2 \le x \le 3 \\ \frac{2 - \frac{x}{3}}{2}, & 3 \le x \le 6 \end{cases}$

i) Dacă $x < 2$: $F(x) = 0$

ii) Dacă $x \in [2,3)$:

$$F(x) = \int_{2}^{x} \frac{t-2}{3}\,dt = \int_{2}^{x} \left(\frac{t}{2} - 1\right)dt = \frac{(x-2)^2}{4}$$

iii) Dacă $x \in [3,6]$:

$$F(x) = \int_{2}^{3} \frac{t-2}{2}\,dt + \int_{3}^{x} \frac{2 - \frac{t}{3}}{2}\,dt = \frac{t^2}{4}\Big|_2^3 - t\Big|_2^3 + \int_{3}^{x} \frac{6-6}{6} =$$

$$= \frac{1}{4} + \int_{3}^{x} \left(1 - \frac{1}{6}\right)dt = \frac{-x^2}{12} + x - 2$$

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iv) Dacă $x \geq 6$:

$$F(x) = \int_2^3 \frac{t-2}{2}\,dt + \int_3^6 \frac{6-t}{6}\,dt + \int_6^x \frac{2-\frac{t}{3}}{2}\,dt =$$

$$= \frac{1}{4} + t\Big|_3^6 - \frac{t^2}{12}\Big|_3^6 + 0 = 1$$

Din i), ii), iii), iv) $\Rightarrow$

$$\Rightarrow F(x) = \begin{cases} 0, & x < 2 \\ \left(\frac{x-2}{2}\right)^2, & 2 \leq x < 3 \\ -\frac{x^2}{12} + x - 2, & 3 \leq x < 6 \\ 1, & x \geq 6 \end{cases}$$

Pt $x \in [2,3)$: $\Rightarrow [0, \frac{1}{4})$

$F(x) = u \Leftrightarrow \frac{(x-2)^2}{4} = u \Leftrightarrow x^2 - 4x + 4(1-u) = 0 \Rightarrow$

$\Rightarrow x = 1 + \sqrt{u} \Rightarrow X = 1 + \sqrt{u}$

$-\frac{6^2}{12} + 6 - 2 = -\frac{1}{2} + 4 = \frac{7}{2}$

Pt $x \in [3,6)$: $\Rightarrow -\frac{3^2}{12} + 3 - 2 = -\frac{1}{4} + 1 = \frac{3}{4}$

$F(x) = u \Leftrightarrow -\frac{x^2}{12} + x - 2 = u \Leftrightarrow -\frac{x^2}{12} + x - (2+u) = 0 \Leftrightarrow$

$\Leftrightarrow x^2 - 12x + 12(2+u) = 0 \Leftrightarrow x = 6 + 2\sqrt{3(1-u)} \Rightarrow$

$\Rightarrow X = 6 + 2\sqrt{3(1-u)},\ u \in [\frac{3}{4}, \frac{4}{4})$

Cum $1 - U \sim Unif(0,1) \Rightarrow X = 6 + 2\sqrt{3U}$,

25) $f(x) = \begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$

i) Dacă $x < 0$:

$$F(x) = \int_0^x e^{2t}\,dt = \frac{1}{2} e^{2t}\Big|_0^x = \frac{e^{2x} - 1}{2}$$

ii) Dacă $x \geq 0$:

$$F(x) = \int_0^x e^{-2t}\,dt = -\frac{1}{2} e^{-2t}\Big|_0^x = \frac{1 - e^{-2x}}{2}$$

Din i, ii) $\Rightarrow$

$$F(x) = \begin{cases} \dfrac{e^{2x}-1}{2}, & x < 0 \\ \dfrac{1-e^{-2x}}{2}, & x \geq 0 \end{cases}$$

Pt $x < 0$ $\Rightarrow$ $u \leq 0$

$$F(x) = u \Leftrightarrow \frac{e^{2x}-1}{2} = u \Leftrightarrow 2x = \ln(2u+1) \Rightarrow X = \frac{1}{2}\ln(2U+1)$$

Pt $x \geq 0 \Rightarrow u \geq 0$

$$F(x) = u \Leftrightarrow 1 - e^{-2x} = 2u \Leftrightarrow X = \frac{-1}{2}\ln(1-2u)$$

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