SIMULAREA V. A.

$F \to$ c.d.f.
$f \to$ p.d.f.

Construiți un algoritm pentru simularea v.a. X în următoarele cazuri:

1) $F(x) = x^n, \; x \in (0,1), \; n \in \mathbb{N}^*$ fixat

2) $F(x) = \dfrac{x^2 + x}{2}, \; x \in [0,1]$

3) $F(x) = 1 - e^{-\alpha \cdot x^{\beta}}, \; x \in (0,\infty), \; \alpha, \beta > 0$ fixate

4) $$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \le x < 1 \\ \frac{1}{2}, & 1 \le x < 2 \\ \frac{x}{4}, & 2 \le x < 4 \\ 1, & x > 4 \end{cases}$$

5) $F(x) = 1 - e^{-\frac{x^2}{b}}, \; x > 0, \; b > 0$ fixat

6) $F(x) = \dfrac{1}{\pi} \operatorname{arctg} x + \dfrac{1}{2}, \; x \in \mathbb{R}$

7) $F(x) = \dfrac{2}{\pi} \operatorname{arctg}(e^x), \; x \in \mathbb{R}$

8) $F(x) = 1 - \dfrac{kx^2 + 2kx + 2}{2} \cdot e^{-kx}, \; x \in [0,1), \; k > 0$ fixat

9) $$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{\alpha(\alpha + x)}, & 0 \le x < \alpha \\ 1 - \frac{\alpha^2}{x(\alpha + x)}, & x \ge \alpha \end{cases}$$ $\alpha > 0$ fixat

10) $$F(x) = \begin{cases} 0, & x < \frac{1}{e} \\ \frac{1 + \ln x}{2}, & x \in [\frac{1}{e}, e) \\ 1, & x \in [e, \infty) \end{cases}$$

11) $F(x) = \sqrt{\frac{x-1}{2}}$; $x \in (1,3)$

12) a) $X = \max\{X_1, X_2 \dots X_n\}$ unde $X_1, X_2 \dots X_n \sim Exp(\alpha)$ independente

b) $X = \min\{X_1, X_2 \dots X_n\}$ — idem —

13). Aceeași cerință pt. $X_1, X_2 \dots X_n$ i.i.d. repartizate:

a) $Pois(\lambda)$

b) $Bern(p)$

c) $Binom(n, p)$

14) $X = \sum_{i=1}^{n} X_i$, $X_1, X_2 \dots X_n$ i.i.d. repartizate

a) $Exp(\lambda)$

b) $Pois(\lambda)$

c) $Binom(n, p)$

d) $Geom(p)$

e) $NegBin(a_i, p)$

15) $X = X_1 \mid (X_1 + X_2 = s)$ condiționat, s fixat, $X_1 \sim Binom(n, p)$

$X_2 \sim Binom(m, p)$

X_1, X_2 independente

16) $X \mid Y = \lambda \sim Pois(\lambda)$ și $Y \sim Exp(1)$

17) $X \mid Y = y \sim \mathcal{N}(0, \sqrt{y})$ și $Y \sim Exp(1)$

18) $X = X_1 \cdot X_2$, unde $X_1 \sim Beta(\alpha_1, \beta_1)$, $X_2 \sim Beta(\alpha_2, \beta_2)$

X_1, X_2 independente

19) a) $X = \frac{X_1 - X_2}{\sqrt{2}}$; b) $X = \frac{X_1 + X_2 - 2X_3}{\sqrt{6}}$; c) $X = \frac{X_1 + X_2 + X_3}{\sqrt{3}}$

unde X_1, X_2, X_3 i.i.d $\sim N(0, 1)$

20) $X = \dfrac{U}{U+V}$, unde $U, V \sim Exp(1)$
U, V independente

21) $X = X_1 \cdot X_2 + X_3 \cdot X_4$ unde $X_1, X_2, X_3, X_4 \sim N(0,1)$
independente

22) $X = \sqrt{X_1^2 + X_2^2}$ unde $X_1, X_2 \sim N(0,1)$
independente

23) $f(x) = \dfrac{e^x}{e-1}, \quad 0 \le x \le 1$

24) $f(x) = \begin{cases} \dfrac{x-2}{2}, & 2 \le x \le 3 \\ \dfrac{2 - \frac{x}{3}}{2}, & 3 \le x \le 6 \end{cases}$

25) $f(x) = \begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$

26) $f(x) = \frac{1}{2} x^2 \cdot e^{-x}, \quad x > 0$

27) $f(x) = \dfrac{10^6}{336} \cdot x(1-x)^3, \quad 0.8 < x < 1$

28) $f(x) = \begin{cases} \frac{1}{a} \ln\left(\frac{a}{x}\right), & 0 < x < a, \\ 0, & \text{în rest} \end{cases}$ $a > 0$ fixat

29) $f(x) = \begin{cases} \frac{1}{x}, & x \in [1-c, 1+c] \\ 0, & \text{în rest} \end{cases}$ când $c = \dfrac{e-1}{e+1}$

30) $f(x) = |x| \cdot e^{-x^2}, \quad x \in \mathbb{R}$

31) $f(x) = \begin{cases} \frac{1}{2}, & 0 \le x \le 1 \\ \frac{1}{4}, & 2 \le x \le 4 \\ 0, & \text{în rest} \end{cases}$

32) $f(x) = \dfrac{1}{\pi(1+x^2)}, \quad x \in \mathbb{R}$

33) $f(x) = 2\alpha \cdot e^{-\alpha x}\left(1 - e^{-\alpha x}\right)$, $x > 0$, $\alpha > 0$ fixat

34) $f(x) = \dfrac{2}{\pi} \cdot \dfrac{1}{e^{x} + e^{-x}}$, $x \in \mathbb{R}$

35) $$f(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{x(2\alpha + x)}{\alpha(\alpha + x)^2}, & 0 < x \le \alpha \\ \dfrac{\alpha^2(\alpha + 2x)}{x^2(\alpha + x)^2}, & x > \alpha \end{cases}$$ $\alpha > 0$ fixat

36) $$f(x) = \begin{cases} 0, & x \le 0 \text{ și } x \ge 3 \\ \dfrac{\theta}{2}, & 0 < x < 1 \\ \dfrac{1}{2}, & 1 \le x \le 2 \\ \dfrac{1-\theta}{2}, & 2 < x < 3 \end{cases}$$ $\theta \in [0, 1]$ fixat

37) $f(x) = \dfrac{1}{\beta}\left[1 - \left|\dfrac{x - \alpha}{\beta}\right|\right]$, $\alpha - \beta < x < \alpha + \beta$
$\alpha \in \mathbb{R}$, $\beta > 0$ fixate

38) $f(x) = \dfrac{1}{n!} \cdot x^{n} \cdot e^{-x}$, $x \ge 0$, $n \in \mathbb{N}$ fixat

39) $f(x) = \dfrac{1}{2\beta} \cdot e^{-\frac{|x - \alpha|}{\beta}}$, $x \in \mathbb{R}$, $\alpha \in \mathbb{R}$, $\beta > 0$ fixate

40) $f(x) = k \cdot \left(1 + \dfrac{a}{2}x\right)^{\left(\frac{4}{a^2} - 1\right)} \cdot e^{-\frac{2x}{a}}$, $-\dfrac{2}{a} \le x < \infty$
$a > 0$ fixat

unde $\boxed{k = \dfrac{b^{2b^2 - 1}}{e^{b^2} \cdot \Gamma(b^2)}}$ iar $\boxed{b = \dfrac{2}{a}}$