The Quadratic Equations

Set 1

4 Find the two solutions for each equation. Check by substitution to ensure your answers are correct.

a $x(x-5)=0$
b $x(x+7)=0$
c $2x(x+1)=0$
d $5a(a-2)=0$
e $(x-2)(x-1)=0$
f $4q(q+5)=0$
g $(y+3)(y+4)=0$
h $(x-7)(x-3)=0$
i $(t+3)(t+2)=0$
j $(y+3)(y+4)=0$
k $(t+3)(t+2)=0$
l $(a-5)(a-2)=0$
m $(a-6)(a+6)=0$
n $(y+8)(y-7)=0$
o $(n+1)(n-1)=0$
P $(a+1)(2a-1)=0$
q $(3x+2)(x-5)=0$
s $(4x-1)(2x+1)=0$
t $(3a-4)(2a-1)=0$
v $6x(5x-3)=0$
w $(9y+1)(7y+2)=0$

2 After factorising the left-hand side of each equation, solve the following.

a $x^2+3x=0$
b $9m^2-5m=0$
c $y^2+2y=0$
d $2x^2-11x+12=0$
e $9m^2-3n=0$
f $4x^2+8x=0$
g $x^2-4=0$
h $a^2-49=0$
i $4x^2-8x=0$
j $5x^2+16x+3=0$
k $n^2-100=0$
l $y^2-36=0$
m $a^2-64=0$
n $a^2-5a+6=0$
o $y^2+12y+35=0$
P $a^2-10a+21=0$
q $x^2-10x+16=0$
r $m^2-11m+24=0$
t $x^2+2x-35=0$
u $a^2-4a-45=0$
v $x^2+x-56=0$
w $y^2-8y+7=0$
x $a^2+9a-10=0$

3 Factorise and solve the following.

a $2x^2+x-1=0$
b $3x^2+7x+2=0$
c $3x^2+17x+10=0$
d $2x^2-11x+12=0$
e $2x^2-x-10=0$
f $2x^2-11x-21=0$
g $4x^2+21x+5=0$
h $4x^2-19x-5=0$
i $4x^2-21x+5=0$
j $5x^2+16x+3=0$
k $2x^2+13x+5=0$
l $7x^2+48x-7=0$
m $4x^2-4x-3=0$
n $6x^2-x-1=0$
o $9x^2+9x+2=0$
P $10x^2+9x+2=0$
q $12x^2-7x+1=0$
r $10x^2-13x+4=0$

4 Gather all the terms to one side of each equation and then solve.

a $x^2=3x$
b $m^2=8m$
c $x^2=-5x$
d $x^2=5x-4$
e $a^2=2a+15$
f $y^2=3y-2$
g $m^2=9m-18$
h $n^2=7n+18$
i $h^2=4h+32$
j $x^2+x=2$
k $y^2+2y=3$
l $x^2-7x=-10$
m $y^2+3y=18$
n $t^2+3t=28$
o $y^2+7x=15$
P $2x^2+x=1$
q $2x^2-x=15$
r $4m^2-3m=6$
s $3x^2=13x-14$
t $5p^2=17p-6$
u $2x^2=11x-5$

Answers (Set 1)

1
a 0.5 b 0,-7 c 0,-1 d 0.2 e 0,-5 f 0.7 g 2.1 h 7.3
i 5.2 j -3,-4 k -3,-2 l -9,-5 m 6,-6 n -8,-7 o -1,1 p -1,½
g -3,-5 s 0.3 t ½,-½ u ½,-½ v 0.3 w -⁹⁄₇,-²⁄₇ x ½,-½
z 0.3 b 5.0 c 0,-2 d 0,-2 e -½,-¹⁄₃ f 0,-2 g 2,-2 h 7,-7
i 6,-6 j 1,-1 k 10,-10 l 8,-8 m -1,-2 n 2,3 o -5,-7 p 7,3
g 8,2 r 8,3 s 4,-5 t 5,-7 u 9,-5 v 7,-8 w 7,1 x 1,-10
i ½,-1 b -½,-2 c -⁵⁄₃,-5 d 4,1½ e 2½,-2 f 7,-1½ g -1,-5 h 5,-1
j 5,¼ k 1½,-8 l ⅓,-7 m 1½,-⅓ n 1⅓,-⅓ o -⅓,-½ p -⅓,-¼
4 a 0.3 b 0.8 c 0,-5 d 4.1 e 5,-3 f 2,1 g 6,3 h 9,-2
i 8,-1 j 1,-2 k 1,-3 l 5.2 m 3,-6 n 4,-7 o 3,-5 P ½,-1
q 3,-2½ r 2,-1 s 2½,2 t 3,⅔ u 5,½

Set 2

4 What number must be inserted to complete the square?

a $x^2+6x+\ldots^2=(x+\ldots)^2$
b $x^2+8x+\ldots^2=(x+\ldots)^2$
c $x^2-2x+\ldots^2=(x-\ldots)^2$
d $x^2-4x+\ldots^2=(x-\ldots)^2$
e $x^2+3x+\ldots^2=(x+\ldots)^2$
f $x^2-7x+\ldots^2=(x-\ldots)^2$
g $x^2+11x+\ldots^2=(x+\ldots)^2$
h $x^2-x+\ldots^2=(x-\ldots)^2$
i $x^2+\dfrac{5x}{2}+\ldots^2=(x+\ldots)^2$
j $x^2-\dfrac{2x}{3}+\ldots^2=(x-\ldots)^2$

2 Solve the following equations, leaving your answers in surd form.

a $(x-2)^2=3$
b $(x+1)^2=2$
c $(x+5)^2=5$
d $(x-1)^2=10$
e $(x-3)^2=7$
f $(x+2)^2=11$
g $(x+3)^2=8$
h $(x+10)^2=12$
i $(x-3)^2=18$
j $(x+\tfrac{1}{2})^2=5$
k $(x-\tfrac{1}{2})^2=3$
l $(x+1\tfrac{1}{2})^2=12$
m $(x-1)^2=2\tfrac{1}{2}$
n $(x+3)^2=4\tfrac{1}{2}$
o $(x-\tfrac{1}{3})^2=\tfrac{5}{9}$

3 Solve the following equations by completing the square. Also find approximations for your answers, correct to two decimal places.

a $x^2+2x-1=0$
b $x^2-2x-5=0$
c $x^2-4x-8=0$
d $x^2+6x-8=0$
e $x^2-6x+2=0$
f $x^2+4x+1=0$
g $x^2+10x=5$
h $x^2+2x=4$
i $x^2-12x=1$
j $x^2+5x+2=0$
k $x^2+7x-3=0$
l $x^2+x-3=0$
m $x^2+9x+3=0$
n $x^2+3x-5=0$
o $x^2-11x+5=0$
P $x^2-x=3$
q $-x^2+3x=2$
r $x^2-5x+1=0$
s $2x^2-4x-1=0$
t $2x^2+3x-4=0$
u $2x^2-8x+1=0$
v $3x^2+2x-3=0$
w $5x^2-4x-3=0$
x $4x^2-x-2=0$

Answers (Set 2)

1
z a 3 b 4 c 1 d 2 e ½ f ⁷⁄₂ g 11/2 h ½ i ⁵⁄₄ j ⅓
z a 2±√5 b -1±√2 c -5±√5 d 1±√10 e 3±√7 f -2±√11 g 3±√2 h ½±√5
g -3±2√2 h -10±2√5 i 3±3√2 j -½±√5 k ½±√3 l -1½±2√3
m 1±√(5/2) n -3±³√⁵⁄₂ o ⅓±√5/3

3 a 0.41,-2.41 b 3.45,-1.45 c 5.46,-1.46 d 1.12,-7.12 e 5.65,0.35 f -0.27,-3.73
g 0.48,-10.48 h 1.24,-3.24 i 12.08,-0.08 i -0.44,-4.56 k 0.41,-7.41 l 1.30,-2.30
c 7.77,-0.77 t 0.85,-2.35 o 10.52,0.48 P 2.30,-1.30 q 0.56,-3.56 r 5.19,-0.19
u 3.87,0.13 v 0.72,-1.39 w 1.27,-0.47 x 0.84,-0.59

Set 3

1 Use the quadratic formula to solve the following equations. All have rational answers.

a $x^2 + 5x + 6 = 0$	b $x^2 + 6x + 8 = 0$
c $x^2 + 10x + 9 = 0$	d $x^2 - 3x - 10 = 0$
e $x^2 - 2x - 15 = 0$	f $x^2 + 4x - 12 = 0$
g $x^2 - 9x + 14 = 0$	h $x^2 - 8x + 12 = 0$
i $3x^2 + 7x + 2 = 0$	k $2x^2 + 11x + 5 = 0$
m $2x^2 - 5x - 3 = 0$	n $5x^2 - 9x - 2 = 0$
p $6x^2 + 7x + 2 = 0$	o $3x^2 - 5x + 2 = 0$
	q $6x^2 + 7x - 3 = 0$

2 Solve the following, leaving your answers in surd form. (Remember: A surd is an expression involving a square root.)

a $x^2 + 4x + 2 = 0$	b $x^2 + 3x + 1 = 0$	c $x^2 + 5x + 3 = 0$
d $x^2 + x - 1 = 0$	e $x^2 + 2x - 2 = 0$	f $x^2 + 4x - 1 = 0$
g $x^2 - 2x - 1 = 0$	h $x^2 - 7x + 2 = 0$	i $x^2 - 6x + 3 = 0$
j $x^2 - 10x - 9 = 0$	k $x^2 - 8x + 3 = 0$	l $x^2 - 5x + 7 = 0$
m $2x^2 + 6x + 1 = 0$	n $2x^2 + 3x - 1 = 0$	o $2x^2 - 7x + 4 = 0$
p $3x^2 + 10x + 2 = 0$	q $3x^2 - 9x + 2 = 0$	r $5x^2 + 4x - 2 = 0$
s $4x^2 - x + 1 = 0$	t $3x^2 - 3x - 1 = 0$	u $4x^2 - 3x - 2 = 0$
v $2x^2 + 11x - 5 = 0$	w $2x^2 - 9x + 8 = 0$	x $5x^2 + 2x - 1 = 0$

3 Use the formula to solve the following and give the answers to two decimal places.

a $x^2 - 4x + 1 = 0$	b $x^2 - 6x + 3 = 0$	c $x^2 + 8x - 5 = 0$
d $x^2 + 9x + 1 = 0$	e $x^2 + 2x - 5 = 0$	f $x^2 + 3x - 1 = 0$
g $x^2 + 2 = 0$	h $x^2 - 7x = 2$	i $x^2 = 6x - 11$
i $2x^2 + x - 2 = 0$	k $2x^2 - 5x - 2 = 0$	l $3x^2 + 9x + 5 = 0$
m $2x^2 = 7x - 2$	n $5x^2 - 3x = 4$	o $6x^2 = x + 3$

Answers

1 a $-2,-3$ b $-2,-4$ c $-1,-9$ d $5,-2$ e $5,-3$ f $2,-6$ g $7,2$ h $6,2$
 i $5,1$ j $-\tfrac13,-2$ k $-\tfrac12,-5$ l $-\tfrac13,-2$ m $3,-\tfrac12$ n $2,-\tfrac15$ o $1,\tfrac23$ p $-\tfrac12,-\tfrac13$
 q $\tfrac13,-1\tfrac12$ r $1\tfrac14,\tfrac14$

2 a $\frac{-2\pm\sqrt6}{2}$ b $\frac{-3\pm\sqrt5}{2}$ c $\frac{-5\pm\sqrt{13}}{2}$ d $\frac{-1\pm\sqrt5}{2}$ e $\frac{-2\pm\sqrt{12}}{2}$ f $\frac{-4\pm\sqrt{...}}{2}$ g $1\pm\sqrt2$
 h $\frac{7\pm\sqrt{41}}{2}$ i $\frac{6\pm\sqrt{24}}{2}$ k $\frac{8\pm\sqrt{52}}{2}$
 m $\frac{-6\pm\sqrt{28}}{4}$ n $\frac{-3\pm\sqrt{17}}{4}$ o $\frac{7\pm\sqrt{17}}{4}$ p $\frac{-10\pm\sqrt{76}}{6}$ q $\frac{9\pm\sqrt{57}}{6}$ r $\frac{-4\pm\sqrt{56}}{10}$
 s no solutions t $\frac{3\pm\sqrt{41}}{6}$ u $\frac{3\pm\sqrt{41}}{4}$ v $\frac{-11\pm\sqrt{161}}{4}$ w $\frac{9\pm\sqrt{17}}{6}$ x $\frac{-2\pm\sqrt{24}}{10}$

3 a $3.73, 0.27$ b $5.45, 0.55$ c $0.58, -8.58$ d $-0.11, -8.89$ e $1.45, -3.45$ f $0.30, -3.30$
 g no solutions h $7.27, -0.27$ i no solutions j $-0.78, -1.28$ k $2.85, -0.35$ l $-0.74, -2.26$
 m $3.19, 0.31$ n $1.24, -0.64$ o $0.80, -0.63$

Set 4

By evaluating $b^2 - 4ac$ for each equation, determine how many solutions it will have.

1 $x^2 + 4x + 3 = 0$	2 $x^2 + 4x + 4 = 0$	3 $x^2 + 4x + 5 = 0$
4 $x^2 - x - 2 = 0$	5 $x^2 - x = 0$	6 $x^2 - x + 2 = 0$
7 $4x^2 - 12x + 9 = 0$	8 $4x^2 - 12x + 7 = 0$	9 $4x^2 - 12x + 11 = 0$
10 $5x^2 - x + 7 = 0$	11 $5x^2 - x - 7 = 0$	12 $9x^2 + 6x + 1 = 0$

Answers

1	2	3	4	5	6	7	8	9	10	11	12
2	1	0	2	2	0	1	2	0	0	2	1

Set 5

1 Solve the following quadratic equations. (Give irrational answers to 2 dec. pl. if necessary)

a $x^2 + 7x + 6 = 0$	b $x^2 - 8x + 12 = 0$	c $x^2 + 5x - 24 = 0$
d $x^2 - 3x + 1 = 0$	e $x^2 + 3x - 3 = 0$	f $x^2 + 4x + 2 = 0$
g $x^2 + 8x = 0$	h $x^2 - 10x = 0$	i $5x^2 - 10x = 0$
j $x^2 - 81 = 0$	k $x^2 - 121 = 0$	l $4x^2 - 9 = 0$
m $2x^2 + 4x + 1 = 0$	n $3x^2 - x - 1 = 0$	o $2x^2 - 5x + 1 = 0$
p $2x^2 + 6x + 4 = 0$	q $3x^2 + 15x + 18 = 0$	r $2x^2 - 5x + 1 = 0$
s $x^2 = 6x + 27$	t $x^2 = 13x - 36$	u $2x^2 - 5x = 12$
v $25 = 10x - x^2$	w $36 = 13x - x^2$	x $2 - 9x = -5x^2$

2 Rearrange each equation below to the form $ax^2 + bx + c = 0$, and solve.

a $x^2 + 9x = 2x - 12$	b $x^2 + 20 = 8x + 5$	c $x^2 - 4x + 10 = 2x + 2$
d $3x^2 - 3x + 1 = 2x^2 - 6$	e $4x^2 + 5x = 3x^2 - 2x$	f $x^2 + 3x - 10 = 3x - 1$
g $x^2 + 5x = 3x + 1$	h $x^2 + 7 = 5 - 4x$	i $x^2 = 4(x + 8)$
j $x(x + 5) = 6$	k $x(x - 7) = 18$	l $x^2 = 4(x + 8)$
m $(m - 1)^2 = 4$	n $(x + 3)^2 = 9$	o $(x + 5)^2 = 11$
p $(2a + 1)^2 = 16$	q $(5y - 3)^2 = 7$	r $(6n - 7)^2 = 3$
s $x = \dfrac{2x + 15}{x}$	t $x = \dfrac{3x + 28}{x}$	u $1 = \dfrac{2 - x^2}{x}$
v $x = \dfrac{5x - 3}{x}$	w $\dfrac{3(x + 1)}{x} = x$	x $2(x + 2) = \dfrac{1}{x}$

Answers

1 a $-1,-6$ b $6,2$ c $3,-8$ d $2.62, 0.38$ e $0.79, -3.79$ f $-0.59, -3.41$
 g $0,-8$ h $10,0$ i $2,0$ j $9,-9$ k $11,-11$ l $1\tfrac12,-1\tfrac12$
 m $-0.29,-1.71$ n $0.77,-0.43$ o $2.28, 0.22$ p $-1,-2$ q $-2,-3$ r $2.28, 0.22$
 s $9,-3$ t $9,4$ u $4,-1\tfrac12$ v $5,5$ w $9,4$ x $1.54, 0.26$

2 a $-3,-4$ b $6,2$ c $3,-4$ d $2.62, 0.38$ e $0,-7$ f $3,-3$
 g $\frac{-2\pm\sqrt6}{2}$ h $-2\pm\sqrt6$ i $4,2$ k $9,-2$ l $1.54, 0.26$
 m $3,-1$ n $0,-6$ o $-5\pm\sqrt{11}$ p $1\tfrac12,-2\tfrac12$ q $\frac{3\pm\sqrt7}{5}$ r $\frac{7\pm\sqrt3}{6}$
 s $5,-3$ t $7,-4$ u $1,-2$ v $\frac{5\pm\sqrt{13}}{2}$ w $\frac{3\pm\sqrt{11}}{2}$ x $\frac{-4\pm\sqrt{24}}{4} = \frac{-2\pm\sqrt6}{2}$