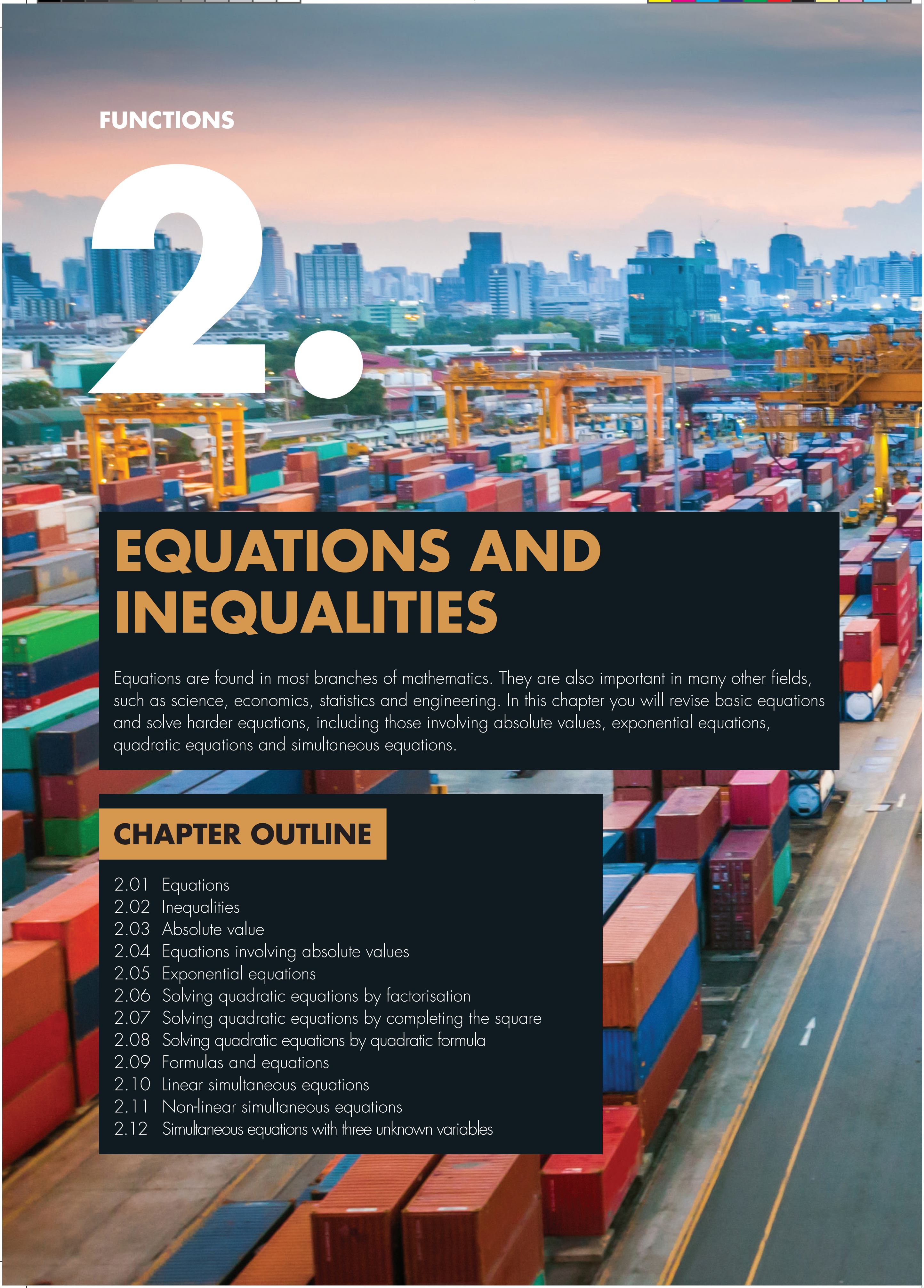


FUNCTIONS

2. EQUATIONS AND INEQUALITIES

Equations are found in most branches of mathematics. They are also important in many other fields, such as science, economics, statistics and engineering. In this chapter you will revise basic equations and solve harder equations, including those involving absolute values, exponential equations, quadratic equations and simultaneous equations.

CHAPTER OUTLINE

IN THIS CHAPTER YOU WILL:

- solve equations and inequalities
- understand and use absolute values in equations
- solve simple exponential equations
- solve quadratic equations using 3 different methods
- understand how to substitute into and rearrange formulas
- solve linear and non-linear simultaneous equations

iStockphoto/primeimages

TERMINOLOGY

absolute value $|x|$ is the absolute value of x, its size without sign or direction. Also the distance of x from 0 on the number line in either direction

equation A mathematical statement that has a pronumeral or unknown number and an equal sign. An equation can be solved to find the value of the unknown number, for example, $3x + 1 = 7$

exponential equation An equation where the unknown pronumeral is the power or index, for example, $2^x = 8$

inequality A mathematical statement involving an inequality sign with an unknown pronumeral, for example, $x - 7 \le 12$

quadratic equation An equation involving x^2 in which the highest power of x is 2

simultaneous equations 2 or more equations that can be solved together to produce a solution that makes each equation true at the same time

PROBLEM

The age of Diophantus at his death can be calculated from his epitaph:

Diophantus passed one-sixth of his life in childhood, one-twelfth in youth, and one-seventh more as a bachelor; five years after his marriage a son was born who died four years before his father at half his father's final age. How old was Diophantus?

WS

Equations

2.01 Equations

EXAMPLE 1

Solve each equation.

a $4y - 3 = 8y + 21$

b $2(3x + 7) = 6 - (x - 1)$

Solution

a

$$\begin{aligned}
4y - 3 &= 8y + 21 \\
4y - 4y - 3 &= 8y - 4y + 21 \\
-3 &= 4y + 21 \\
-3 - 21 &= 4y + 21 - 21 \\
-24 &= 4y \\
\frac{-24}{4} &= \frac{4y}{4} \\
-6 &= y \\
y &= -6
\end{aligned}$$

b

$$\begin{aligned}
2(3x + 7) &= 6 - (x - 1) \\
6x + 14 &= 6 - x + 1 \\
&= 7 - x \\
6x + x + 14 &= 7 - x + x \\
7x + 14 &= 7 \\
7x + 14 - 14 &= 7 - 14 \\
7x &= -7 \\
\frac{7x}{7} &= \frac{-7}{7} \\
x &= -1
\end{aligned}$$

ISBN 9780170413152

When an equation involves fractions, multiply both sides of the equation by the common denominator of the fractions.

EXAMPLE 2

Solve:

a $\frac{m}{3} - 4 = \frac{1}{2}$

b $\frac{x+1}{3} + \frac{x}{4} = 5$

Solution

a

$$\frac{m}{3} - 4 = \frac{1}{2}$$
$$6\left(\frac{m}{3}\right) - 6(4) = 6\left(\frac{1}{2}\right)$$
$$2m - 24 = 3$$
$$2m - 24 + 24 = 3 + 24$$
$$2m = 27$$
$$\frac{2m}{2} = \frac{27}{2}$$
$$m = \frac{27}{2}$$
$$= 13\tfrac{1}{2}$$

b

$$\frac{x+1}{3} + \frac{x}{4} = 5$$
$$12\left(\frac{x+1}{3}\right) + 12\left(\frac{x}{4}\right) = 12(5)$$
$$4(x + 1) + 3x = 60$$
$$4x + 4 + 3x = 60$$
$$7x + 4 = 60$$
$$7x + 4 - 4 = 60 - 4$$
$$7x = 56$$
$$\frac{7x}{7} = \frac{56}{7}$$
$$x = 8$$

DID YOU KNOW?

History of algebra

Algebra was known in ancient civilisations. Many equations were known in Babylon, although general solutions were difficult because symbols were not used in those times.

Diophantus, around 250 CE, first used algebraic notation and symbols (e.g. the minus sign). He wrote a treatise on algebra in his *Arithmetica*, comprising 13 books. Only six of these books survived. About 400 CE, Hypatia of Alexandria wrote a commentary on them.

Hypatia was the first female mathematician on record, and was a philosopher and teacher. She was the daughter of Theon, who was also a mathematician and who ensured that she had the best education.

In 1799 **Carl Friedrich Gauss** proved the Fundamental Theorem of Algebra: that every algebraic equation involving a power of x has at least one solution, which may be a real number or a non-real number.

Exercise 2.01 Equations

Solve each equation.

1 $t+4=-1$

2 $z+1.7=-3.9$

3 $y-3=-2$

4 $w-2.6=4.1$

5 $5=x-7$

6 $1.5x=6$

7 $5y=\frac{1}{3}$

8 $\frac{b}{7}=5$

9 $-2=\frac{n}{8}$

10 $\frac{r}{6}=\frac{2}{3}$

11 $2y+1=19$

12 $33=4k+9$

13 $7d-2=12$

14 $-2=5x-27$

15 $\frac{y}{3}+4=9$

16 $\frac{x}{2}-3=7$

17 $\frac{m}{5}+7=11$

18 $3x+5=17$

19 $4a+7=-21$

20 $7y-1=20$

21 $3(x+2)=15$

22 $-2(3a+1)=8$

23 $7t+4=3t-12$

24 $x-3=6x-9$

25 $2(a-2)=4-3a$

26 $5b+2=-3(b-1)$

27 $3(t+7)=2(2t-9)$

28 $2+5(p-1)=5p-(p-2)$

29 $3.7x+1.2=5.4x-6.3$

30 $\frac{b}{5}=\frac{2}{3}$

31 $\frac{5x}{4}=\frac{11}{7}$

32 $\frac{x}{3}-4=8$

33 $\frac{5+x}{7}=\frac{2}{7}$

34 $\frac{y}{2}=-\frac{3}{5}$

35 $\frac{x}{9}-\frac{2}{3}=7$

36 $\frac{w-3}{2}=5$

37 $\frac{2t}{5}-\frac{t}{3}=2$

38 $\frac{x}{4}+\frac{1}{2}=4$

39 $\frac{x}{5}-\frac{x}{2}=\frac{3}{10}$

40 $\frac{x+4}{3}+\frac{x}{2}=1$

41 $\frac{p-3}{2}+\frac{2p}{3}=2$

42 $\frac{t+3}{7}+\frac{t-1}{3}=4$

43 $\frac{x+5}{9}-\frac{x+2}{5}=1$

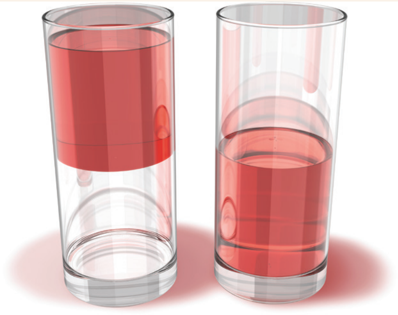
44 $\frac{q-1}{3}-\frac{q-2}{4}=2$

45 $\frac{x+3}{5}+2=\frac{x+7}{2}$

COULD THIS BE TRUE?

Half full = half empty

$\therefore$ full = empty



Shutterstock.com/imagineerinx

ISBN 9780170413152

2.02 Inequalities

$>$ means greater than.

$<$ means less than.

$\geq$ means greater than or equal to.

$\leq$ means less than or equal to.

Solving inequalities

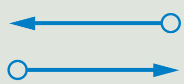
The **inequality sign reverses** when:

- multiplying by a negative
- dividing by a negative
- taking the reciprocal of both sides.

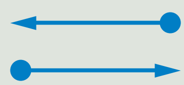
On the number plane, we graph inequalities using arrows and circles (open for greater than and less than and closed in for greater than or equal to and less than or equal to).

Inequalities on a number line

$<$ and $>$



$\leq$ and $\geq$



WS Inequalities on a number line

EXAMPLE 3

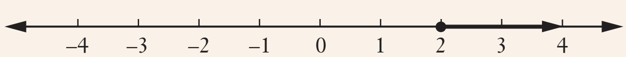
Solve each inequality and show its solution on a number line.

a $5x + 7 \geq 17$ **b** $3t - 2 > 5t + 4$ **c** $1 < 2z + 7 \leq 11$

Solution

a

$$5x + 7 \geq 17$$
$$5x + 7 - 7 \geq 17 - 7$$
$$5x \geq 10$$
$$\frac{5x}{5} \geq \frac{10}{5}$$
$$x \geq 2$$



ISBN 9780170413152

b $3t - 2 > 5t + 4$

$3t - 5t - 2 > 5t - 5t + 4$

$-2t - 2 > 4$

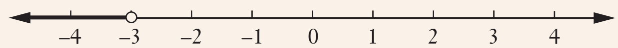
$-2t - 2 + 2 > 4 + 2$

$-2t > 6$

$\frac{-2t}{-2} < \frac{6}{-2}$

$t < -3$

Remember to change the inequality sign when dividing by -2.

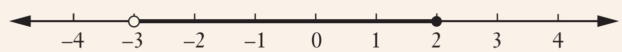


c $1 < 2z + 7 \leq 11$

$1 - 7 < 2z + 7 - 7 \leq 11 - 7$

$-6 < 2z \leq 4$

$-3 < z \leq 2$



Exercise 2.02 Inequalities

1 Solve each equation and plot the solution on a number line.

a $x + 4 > 7$ **b** $y - 3 \leq 1$

2 Solve:

a $5t > 35$ **b** $3x - 7 \geq 2$ **c** $2(p + 5) > 8$

d $4 - (x - 1) \leq 7$ **e** $3y + 5 > 2y - 4$ **f** $2a - 6 \leq 5a - 3$

g $3 + 4y \geq -2(1 - y)$ **h** $2x + 9 < 1 - 4(x + 1)$ **i** $\frac{a}{2} \leq -3$

j $8 > \frac{2y}{3}$ **k** $\frac{b}{2} + 5 < -4$ **l** $\frac{x}{3} - 4 > 6$

3 Solve and plot each solution on a number line.

a $3 < x + 2 < 9$ **b** $-4 \leq 2p < 10$ **c** $2 < 3x - 1 < 11$

d $-6 \leq 5y + 9 \leq 34$ **e** $-2 < 3(2y - 1) < 7$

ISBN 9780170413152

2.03 Absolute value

The **absolute value** of a number is the size of the number without the sign or direction. So absolute value is always positive or zero.

We write the absolute value of x as $|x|$.

For example, $|4| = 4$ and $|-3| = 3$.

We can also define $|x|$ as the distance of x from 0 on the number line.

If x is positive, then its absolute value is itself.

If $x = 0$, then its absolute value is 0.

If x is negative, then its absolute value is its opposite, $-x$. Because x is already negative, the effect of the negative sign in front of it is to make it positive; for example, $-(-5) = 5$.

Absolute value

$$|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

$|4| = 4$ since $4 \geq 0$.

$|-3| = -(-3)$ since $-3 < 0$

$= 3$

Properties of absolute value

Property	Example
$\lvert ab\rvert = \lvert a\rvert \times \lvert b\rvert$	$\lvert 2 \times -3\rvert = \lvert 2\rvert \times \lvert -3\rvert = 6$
$\lvert a\rvert^2 = a^2$	$\lvert -3\rvert^2 = (-3)^2 = 9$
$\sqrt{a^2} = \lvert a\rvert$	$\sqrt{(-5)^2} = \lvert -5\rvert = 5$
$\lvert -a\rvert = \lvert a\rvert$	$\lvert -7\rvert = \lvert 7\rvert = 7$
$\lvert a - b\rvert = \lvert b - a\rvert$	$\lvert 2 - 3\rvert = \lvert 3 - 2\rvert = 1$
$\lvert a + b\rvert \leq \lvert a\rvert + \lvert b\rvert$	$\lvert 2 + 3\rvert = \lvert 2\rvert + \lvert 3\rvert$ but $\lvert -3 + 4\rvert < \lvert -3\rvert + \lvert 4\rvert$

ISBN 9780170413152

EXAMPLE 4

a Evaluate $|2| - |-1| + |-3|^2$.

b Show that $|a + b| \le |a| + |b|$ when $a = -2$ and $b = 3$.

c Write expressions for $|2x - 4|$ without the absolute value signs.

Solution

a
$$\begin{aligned}|2| - |-1| + |-3|^2 &= 2 - 1 + 3^2\\ &= 10\end{aligned}$$

b
$$\begin{aligned}\text{LHS} &= |a + b| & \text{RHS} &= |a| + |b|\\ &= |-2 + 3| & &= |-2| + |3|\\ &= |1| & &= 2 + 3\\ &= 1 & &= 5\end{aligned}$$

Note: LHS means left-hand side and RHS means right-hand side.

Since $1 < 5$,

$|a + b| \le |a| + |b|$

c
$|2x - 4| = 2x - 4$ when $2x - 4 \ge 0$ i.e. when $2x \ge 4$

i.e. when $x \ge 2$

$$\begin{aligned}|2x - 4| &= -(2x - 4) \text{ when } 2x - 4 < 0\\ &= -2x + 4\end{aligned}$$

i.e. when $2x < 4$

i.e. when $x < 2$

CLASS DISCUSSION

ABSOLUTE VALUE

Are these statements true? If so, are there some values for which the expression is undefined (values of x or y that the expression cannot have)?

1 $\dfrac{x}{|x|} = 1$

2 $|2x| = 2x$

3 $|2x| = 2|x|$

4 $|x| + |y| = |x + y|$

5 $|x|^2 = x^2$

6 $|x|^3 = x^3$

7 $|x + 1| = |x| + 1$

8 $\dfrac{|3x - 2|}{3x - 2} = 1$

9 $\dfrac{|x|}{x^2} = 1$

10 $|x| \ge 0$

Discuss absolute value and its definition in relation to these statements.

ISBN 9780170413152

Exercise 2.03 Absolute value

1 Evaluate:

a $|7|$ **b** $|-5|$ **c** $|-6|$ **d** $|0|$

e $|2|$ **f** $|-11|$ **g** $|-2||3|$ **h** $3|-8|$

i $|-5|^2$ **j** $|-5|^3$

2 Evaluate:

a $|3| + |-2|$ **b** $|-3| - |4|$ **c** $|-5 + 3|$

d $|2 \times -7|$ **e** $|-3| + |-1|$ **f** $5 - |-2| \times |6|^2$

g $|-2 + 5 \times -1|$ **h** $3|-4|$ **i** $2|-3| - 3|-4|$

j $|5 - 7| + 4|-2|$

3 Evaluate $|a - b|$ if:

a $a = 5$ and $b = 2$ **b** $a = -1$ and $b = 2$ **c** $a = -2$ and $b = -3$

d $a = 4$ and $b = 7$ **e** $a = -1$ and $b = -2$

4 Write an expression for:

a $|a|$ when $a > 0$ **b** $|a|$ when $a < 0$ **c** $|a|$ when $a = 0$

d $|3a|$ when $a > 0$ **e** $|3a|$ when $a < 0$ **f** $|3a|$ when $a = 0$

g $|a + 1|$ when $a > -1$ **h** $|a + 1|$ when $a < -1$ **i** $|x - 2|$ when $x > 2$

5 Show that $|a + b| \le |a| + |b|$ when:

a $a = 2$ and $b = 4$ **b** $a = -1$ and $b = -2$ **c** $a = -2$ and $b = 3$

d $a = -4$ and $b = 5$ **e** $a = -7$ and $b = -3$

6 Show that $\sqrt{x^2} = |x|$ when:

a $x = 5$ **b** $x = -2$ **c** $x = -3$

d $x = 4$ **e** $x = -9$

7 Use the definition of absolute value to write each expression without the absolute value signs.

a $|x + 5|$ **b** $|b - 3|$ **c** $|a + 4|$

d $|2y - 6|$ **e** $|3x + 9|$ **f** $|4 - x|$

g $|2k + 1|$ **h** $|5x - 2|$ **i** $|a + b|$

8 Find values of x for which $|x| = 3$.

9 Simplify $\frac{|n|}{n}$ where $n \ne 0$.

10 Simplify $\frac{x-2}{|x-2|}$ and state which value x cannot be.

ISBN 9780170413152

WS

Absolute value equations and inequalities

2.04 Equations involving absolute values

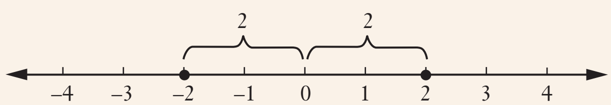
On a number line, $|x|$ means the distance of x from 0 in either direction.

EXAMPLE 5

Solve $|x| = 2$.

Solution

$|x| = 2$ means the distance of x from zero is 2 (in either direction).



$x = \pm 2$

CLASS DISCUSSION

ABSOLUTE VALUE AND THE NUMBER LINE

What does $|a - b|$ mean as a distance along the number line?

Select different values of a and b to help with this discussion.

EXAMPLE 6

Solve:

a $|x + 4| = 7$ **b** $|2x - 3| = 9$

Solution

a This means that the distance from $x + 4$ to 0 is 7 in either direction.

So $x + 4 = \pm 7$

$$\begin{aligned} x + 4 &= 7 \\ x + 4 - 4 &= 7 - 4 \\ x &= 3 \end{aligned} \qquad \text{or} \qquad \begin{aligned} x + 4 &= -7 \\ x + 4 - 4 &= -7 - 4 \\ x &= -11 \end{aligned}$$

So $x = 3$ or -11.

ISBN 9780170413152

Checking your answer:

$$\begin{aligned} \text{LHS} &= |3 + 4| \\ &= |7| \\ &= 7 \\ &= \text{RHS} \end{aligned} \qquad \begin{aligned} \text{LHS} &= |-11 + 4| \\ &= |-7| \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

b $|2x - 3| = 9$

$$\begin{aligned} 2x - 3 &= 9 \\ 2x &= 12 \\ x &= 6 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 2x - 3 &= -9 \\ 2x &= -6 \\ x &= -3 \end{aligned}$$

So $x = 6$ or -3.

Checking your answer:

$$\begin{aligned} \text{LHS} &= |2 \times 6 - 3| \\ &= |9| \\ &= 9 \\ &= \text{RHS} \end{aligned} \qquad \begin{aligned} \text{LHS} &= |2 \times (-3) - 3| \\ &= |-9| \\ &= 9 \\ &= \text{RHS} \end{aligned}$$

Exercise 2.04 Equations involving absolute values

1 Solve:

a $|x| = 5$ **b** $|y| = 8$ **c** $|x| = 0$

2 Solve:

a $|x + 2| = 7$ **b** $|n - 1| = 3$ **c** $9 = |2x + 3|$

d $|7x - 1| = 34$ **e** $\left|\frac{x}{3}\right| = 4$

3 Solve:

a $|8x - 5| = 11$ **b** $|5 - 3n| = 1$ **c** $16 = |5t + 4|$

d $21 = |9 - 2y|$ **e** $|3x + 2| - 7 = 0$

ISBN 9780170413152

2.05 Exponential equations

The word **exponent** means the power or index of a number.

So an **exponential equation** involves an unknown index or power; for example, $2^x = 8$.

EXAMPLE 7

Solve:

a $3^x = 81$ **b** $5^{2k-1} = 25$ **c** $8^n = 4$

Solution

a

$$3^x = 81$$
$$3^x = 3^4$$
$$\therefore x = 4$$

b

$$5^{2k-1} = 25$$
$$5^{2k-1} = 5^2$$
$$\therefore 2k - 1 = 2$$
$$2k = 3$$
$$\frac{2k}{2} = \frac{3}{2}$$
$$k = 1\tfrac{1}{2}$$

c It is hard to write 8 as a power of 4 or 4 as a power of 8, but both can be written as powers of 2.

$$8^n = 4$$
$$(2^3)^n = 2^2$$
$$2^{3n} = 2^2$$
$$\therefore 3n = 2$$
$$\frac{3n}{3} = \frac{2}{3}$$
$$n = \frac{2}{3}$$

To solve other equations involving indices, we do the opposite or inverse operation. For example, squares and square roots are inverse operations, and cubes and cube roots are inverse operations.

EXAMPLE 8

Solve:

a $x^2 = 9$ **b** $5n^3 = 40$

Solution

a There are two possible numbers whose square is 9.

$$x^2 = 9$$
$$x = \pm\sqrt{9}$$
$$\therefore x = \pm 3$$

b

$$5n^3 = 40$$
$$\frac{5n^3}{5} = \frac{40}{5}$$
$$n^3 = 8$$
$$n = \sqrt[3]{8}$$
$$n = 2$$

ISBN 9780170413152

INVESTIGATION

SOLUTIONS FOR EQUATIONS INVOLVING x^n

Investigate equations of the type $x^n = k$ where k is a constant; for example, $x^n = 9$.

Look at these questions.

1 What is the solution when $n = 0$?

2 What is the solution when $n = 1$?

3 How many solutions are there when $n = 2$?

4 How many solutions are there when $n = 3$?

5 How many solutions are there when n is even?

6 How many solutions are there when n is odd?

Exercise 2.05 Exponential equations

1 Solve:

a $2^n = 16$ **b** $3^y = 243$ **c** $2^m = 512$ **d** $10^x = 100\ 000$

e $6^m = 1$ **f** $4^x = 64$ **g** $4^x + 3 = 19$ **h** $5(3^x) = 45$

i $4^x = 4$ **j** $\frac{6^k}{2} = 18$

2 Solve:

a $3^{2x} = 81$ **b** $2^{5x-1} = 16$ **c** $4^{x+3} = 4$ **d** $3^{n-2} = 1$

e $7^{2x+1} = 7$ **f** $3^{x-3} = 27$ **g** $5^{3y+2} = 125$ **h** $7^{3x-4} = 49$

i $2^{4x} = 256$ **j** $9^{3a+1} = 9$

3 Solve:

a $4^m = 2$ **b** $27^x = 3$ **c** $125^x = 5$ **d** $\left(\frac{1}{49}\right)^k = 7$

e $\left(\frac{1}{1000}\right)^k = 100$ **f** $16^n = 8$ **g** $25^x = 125$ **h** $64^n = 16$

i $\left(\frac{1}{4}\right)^{3k} = 2$ **j** $8^{x-1} = 4$

4 Solve:

a $2^{4x+1} = 8^x$ **b** $3^{5x} = 9^{x-2}$ **c** $7^{2k+3} = 7^{k-1}$

d $4^{3n} = 8^{n+3}$ **e** $6^{x-5} = 216^x$ **f** $16^{2x-1} = 4^{x-4}$

g $27^{x+3} = 3^x$ **h** $\left(\frac{1}{2}\right)^x = \left(\frac{1}{64}\right)^{2x+3}$ **i** $\left(\frac{3}{4}\right)^x = \left(\frac{27}{64}\right)^{2x-3}$

ISBN 9780170413152

5 Solve:

a $4^m = \sqrt{2}$ **b** $\left(\frac{9}{25}\right)^{k+3} = \sqrt{\frac{3}{5}}$ **c** $\frac{1}{\sqrt{2}} = 4^{2x-5}$

d $3^k = 3\sqrt{3}$ **e** $\left(\frac{1}{27}\right)^{3n+1} = \frac{\sqrt{3}}{81}$ **f** $\left(\frac{2}{5}\right)^{3n+1} = \left(\frac{5}{2}\right)^{-n}$

g $32^{-x} = \frac{1}{16}$ **h** $9^{2b+5} = 3^b\sqrt{3}$ **i** $81^{x+1} = \sqrt{3^x}$

6 Solve, giving exact answers:

a $x^3 = 27$ **b** $y^2 = 64$ **c** $n^4 = 16$

d $x^2 = 20$ **e** $p^3 = 1000$ **f** $2x^2 = 50$

g $6y^4 = 486$ **h** $w^3 + 7 = 15$ **i** $6n^2 - 4 = 92$

7 Solve and give the answer correct to 2 decimal places:

a $p^2 = 45$ **b** $x^3 = 100$ **c** $n^5 = 240$

d $2x^2 = 70$ **e** $4y^3 + 7 = 34$ **f** $\frac{d^4}{3} = 14$

g $\frac{k^2}{2} - 3 = 7$ **h** $\frac{x^3 - 1}{5} = 2$ **i** $2y^2 - 9 = 20$

8 Solve:

a $x^{-1} = 5$ **b** $a^{-3} = 8$ **c** $y^{-5} = 32$ **d** $x^{-2} + 1 = 50$

e $2n^{-1} = 3$ **f** $a^{-3} = \frac{1}{8}$ **g** $x^{-2} = \frac{1}{4}$ **h** $b^{-1} = \frac{1}{9}$

i $x^{-2} = 2\frac{1}{4}$ **j** $b^{-4} = \frac{16}{81}$

PUZZLE

Test your logical thinking and that of your friends.

1 How many months have 28 days?

2 If I have 128 sheep and take away all but 10, how many do I have left?

3 A bottle and its cork cost \$1.10 to make. If the bottle costs \$1 more than the cork, how much does each cost?

4 What do you get if you add 1 to 15 four times?

5 On what day of the week does Good Friday fall in 2030?

ISBN 9780170413152

2.06 Solving quadratic equations by factorisation

Quadratic equations by factorising

A **quadratic equation** is an equation involving a square. For example, $x^2 - 4 = 0$.

When solving quadratic equations by factorising, we use a property of zero.

For any real numbers a and b, if $ab = 0$ then $a = 0$ or $b = 0$.

EXAMPLE 9

Solve:

a $x^2 + x - 6 = 0$ **b** $y^2 - 7y = 0$ **c** $3a^2 - 14a = -8$

Solution

a

$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$\therefore\ x + 3 = 0 \text{ or } x - 2 = 0$$
$$x = -3 \text{ or } x = 2$$

So the solution is $x = -3$ or 2.

b

$$y^2 - 7y = 0$$
$$y(y - 7) = 0$$
$$\therefore\ y = 0 \text{ or } y - 7 = 0$$
$$y = 7$$

So the solution is $y = 0$ or 7.

c First we make the equation equal to zero so we can factorise and use the rule for zero.

$$3a^2 - 14a = -8$$
$$3a^2 - 14a + 8 = -8 + 8$$
$$3a^2 - 14a + 8 = 0$$
$$(3a - 2)(a - 4) = 0$$
$$\therefore\ 3a - 2 = 0 \text{ or } a - 4 = 0$$
$$3a = 2 \text{ or } a = 4$$
$$\frac{3a}{3} = \frac{2}{3}$$
$$a = \frac{2}{3}$$

So the solution is $a = \frac{2}{3}$ or 4.

Exercise 2.06 Solving quadratic equations by factorisation

Solve each quadratic equation.

1 $y^2 + y = 0$	**2** $b^2 - b - 2 = 0$	**3** $p^2 + 2p - 15 = 0$
4 $t^2 - 5t = 0$	**5** $x^2 + 9x + 14 = 0$	**6** $q^2 - 9 = 0$
7 $x^2 - 1 = 0$	**8** $a^2 + 3a = 0$	**9** $2x^2 + 8x = 0$
10 $4x^2 - 1 = 0$	**11** $3x^2 + 7x + 4 = 0$	**12** $2y^2 + y - 3 = 0$
13 $8b^2 - 10b + 3 = 0$	**14** $x^2 - 3x = 10$	**15** $3x^2 = 2x$
16 $2x^2 = 7x - 5$	**17** $5x - x^2 = 0$	**18** $y^2 = y + 2$
19 $8n = n^2 + 15$	**20** $12 = 7x - x^2$	**21** $m^2 = 6 - 5m$
22 $x(x + 1)(x + 2) = 0$	**23** $(y - 1)(y + 5)(y + 2) = 0$	**24** $(x + 3)(x - 1) = 32$
25 $(m - 3)(m - 4) = 20$		

WS
Completing the square

2.07 Solving quadratic equations by completing the square

Not all trinomials will factorise, so other methods need to be used to solve quadratic equations.

EXAMPLE 10

Solve:

a $(x + 3)^2 = 11$

b $(y - 2)^2 = 7$

Solution

a
$$\begin{aligned}(x+3)^2 &= 11\\ x+3 &= \pm\sqrt{11}\\ x+3-3 &= \pm\sqrt{11}-3\\ x &= \pm\sqrt{11}-3\end{aligned}$$

b
$$\begin{aligned}(y-2)^2 &= 7\\ y-2 &= \pm\sqrt{7}\\ y-2+2 &= \pm\sqrt{7}+2\\ y &= \pm\sqrt{7}+2\end{aligned}$$

To solve a quadratic equation such as $x^2 - 6x + 3 = 0$, which will not factorise, we can use the method of **completing the square**.

We use the perfect square:

$$a^2 + 2ab + b^2 = (a + b)^2$$

ISBN 9780170413152

EXAMPLE 11

Complete the square on $a^2 + 6a$.

Solution

Compare with $a^2 + 2ab + b^2$: $2ab = 6a$

$$b = 3$$

To complete the square: $a^2 + 2ab + b^2 = (a + b)^2$

$$a^2 + 2a(3) + 3^2 = (a + 3)^2$$

$$a^2 + 6x + 9 = (a + 3)^2$$

Completing the square

To complete the square on $a^2 \pm pa$, divide p by 2 and square it.

$$a^2 \pm pa + \left(\frac{p}{2}\right)^2 = \left(a \pm \frac{p}{2}\right)^2$$

EXAMPLE 12

Solve by completing the square:

a $x^2 - 6x + 3 = 0$

b $y^2 + 2y - 7 = 0$ (correct to 3 significant figures)

Solution

a

$$x^2 - 6x + 3 = 0$$

$$x^2 - 6x = -3$$

$$x^2 - 6x + 9 = -3 + 9 \qquad \left(\frac{6}{2}\right)^2 = 3^2 = 9$$

$$(x - 3)^2 = 6$$

$$\therefore x - 3 = \pm\sqrt{6}$$

$$x = \pm\sqrt{6} + 3$$

The 3rd line shows the 'completing the square' step in both solutions.

b

$$y^2 + 2y - 7 = 0$$

$$y^2 + 2y = 7$$

$$y^2 + 2y + 1 = 7 + 1 \qquad \left(\frac{2}{2}\right)^2 = 1^2 = 1$$

$$(y + 1)^2 = 8$$

$$\therefore y + 1 = \pm\sqrt{8}$$

$$y = \pm\sqrt{8} - 1$$

$$y \approx 1.83 \text{ or } -3.83$$

ISBN 9780170413152

Exercise 2.07 Solving quadratic equations by completing the square

1 Solve and give exact solutions:

a $(x+1)^2 = 7$ **b** $(y+5)^2 = 5$ **c** $(a-3)^2 = 6$
d $(x-2)^2 = 13$ **e** $(2y+3)^2 = 2$

2 Solve and give solutions correct to one decimal place:

a $(h+2)^2 = 15$ **b** $(a-1)^2 = 8$ **c** $(x-4)^2 = 17$
d $(y+7)^2 = 21$ **e** $(3x-1)^2 = 12$

3 Solve by completing the square, giving exact solutions in simplest surd form:

a $x^2 + 4x - 1 = 0$ **b** $a^2 - 6a + 2 = 0$ **c** $y^2 - 8y - 7 = 0$
d $x^2 + 2x - 12 = 0$ **e** $p^2 + 14p + 5 = 0$ **f** $x^2 - 10x - 3 = 0$
g $y^2 + 20y + 12 = 0$ **h** $x^2 - 2x - 1 = 0$ **i** $n^2 + 24n + 7 = 0$

4 Solve by completing the square and writing answers correct to 3 significant figures:

a $x^2 - 2x - 5 = 0$ **b** $x^2 + 12x + 34 = 0$ **c** $q^2 + 18q - 1 = 0$
d $x^2 - 4x - 2 = 0$ **e** $b^2 + 16b + 50 = 0$ **f** $x^2 - 24x + 112 = 0$
g $r^2 - 22r - 7 = 0$ **h** $x^2 + 8x + 5 = 0$ **i** $a^2 + 6a - 1 = 0$

WS Quadratic formula

WS Quadratic equations

WS Problems involving quadratic equations

The quadratic formula

2.08 Solving quadratic equations by quadratic formula

Completing the square is difficult with harder quadratic equations such as $2x^2 - x - 5 = 0$. Completing the square on a general quadratic equation gives the following formula.

The quadratic formula

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof

Solve $ax^2 + bx + c = 0$ by completing the square.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{bx}{a} = -\frac{c}{a}$$

ISBN 9780170413152

Completing the square:

$$x^2+\frac{bx}{a}+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$=\frac{-4ac+b^2}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{-4ac+b^2}{4a^2}}$$

$$=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Excel worksheet: The quadratic formula

Excel spreadsheet: The quadratic formula

Solving algebraic equations

EXAMPLE 13

a Solve $x^2 - x - 2 = 0$ by using the quadratic formula.

b Solve $2y^2 - 9y + 3 = 0$ by formula and give your answer correct to 2 decimal places.

Solution

a $a = 1, b = -1, c = -2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-2)}}{2(1)}$$

$$=\frac{1\pm\sqrt{1+8}}{2}$$

$$=\frac{1\pm\sqrt{9}}{2}$$

$$=\frac{1\pm3}{2}$$

$$=2 \text{ or } -1$$

b $a = 2, b = -9, c = 3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(3)}}{2(2)}$$

$$=\frac{9\pm\sqrt{81-24}}{4}$$

$$=\frac{9\pm\sqrt{57}}{4}$$

$$\approx 4.14 \text{ or } 0.36$$

Exercise 2.08 Solving quadratic equations by quadratic formula

1 Solve by formula, correct to 3 significant figures where necessary:

a $y^2 + 6y + 2 = 0$ **b** $2x^2 - 5x + 3 = 0$ **c** $b^2 - b - 9 = 0$

d $2x^2 - x - 1 = 0$ **e** $-8x^2 + x + 3 = 0$ **f** $n^2 + 8n - 2 = 0$

g $m^2 + 7m + 10 = 0$ **h** $x^2 - 7x = 0$ **i** $x^2 + 5x = 6$

ISBN 9780170413152

2 Solve by formula, leaving the answer in simplest surd form:

a $x^2 + x - 4 = 0$	**b** $3x^2 - 5x + 1 = 0$	**c** $q^2 - 4q - 3 = 0$
d $4h^2 + 12h + 1 = 0$	**e** $3s^2 - 8s + 2 = 0$	**f** $x^2 + 11x - 3 = 0$
g $6d^2 + 5d - 2 = 0$	**h** $x^2 - 2x = 7$	**i** $t^2 = t + 1$

CLASS INVESTIGATION

FAULTY PROOF

Here is a proof that 1 = 2. Can you see the fault in the proof?

$$x^2 - x^2 = x^2 - x^2$$
$$x(x - x) = (x + x)(x - x)$$
$$x = x + x$$
$$x = 2x$$
$$\therefore \quad 1 = 2$$

2.09 Formulas and equations

Sometimes substituting values into a formula involves solving an equation.

EXAMPLE 14

a The formula for the surface area of a rectangular prism is given by $S = 2(lb + bh + lh)$. Find the value of b when $S = 180$, $l = 9$ and $h = 6$.

b The volume of a cylinder is given by $V = \pi r^2 h$. Evaluate the radius r, correct to 2 decimal places, when $V = 350$ and $h = 6.5$.

Solution

a

$$\begin{aligned} S &= 2(lb + bh + lh) \\ 180 &= 2(9b + 6b + 9 \times 6) \\ &= 2\,(15b + 54) \\ &= 30b + 108 \\ 72 &= 30b \\ \frac{72}{30} &= \frac{30b}{30} \\ 2.4 &= b \end{aligned}$$

b

$$\begin{aligned} V &= \pi r^2 h \\ 350 &= \pi r^2(6.5) \\ \frac{350}{6.5\pi} &= \frac{\pi r^2(6.5)}{6.5\pi} \\ \frac{350}{6.5\pi} &= r^2 \\ \sqrt{\frac{350}{6.5\pi}} &= r \\ 4.14 &= r \end{aligned}$$

ISBN 9780170413152

Exercise 2.09 Formulas and equations

1 Given that $v = u + at$ is the formula for the velocity of a particle at time t, find the value of t when $u = 17.3$, $v = 100.6$ and $a = 9.8$.

2 The sum of an arithmetic series is given by $S = \frac{n}{2}(a + l)$. Find l if $a = 3$, $n = 26$ and $S = 1625$.

3 The formula for finding the area of a triangle is $A = \frac{1}{2}bh$. Find b when $A = 36$ and $h = 9$.

4 The area of a trapezium is given by $A = \frac{1}{2}h(a + b)$. Find the value of a when $A = 120$, $h = 5$ and $b = 7$.

5 Find the value of y when $x = 3$, given the straight line equation $5x - 2y - 7 = 0$.

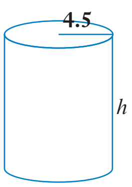
6 The area of a circle is given by $A = \pi r^2$. Find r correct to 3 significant figures if $A = 140$.

7 The area of a rhombus is given by the formula $A = \frac{1}{2}xy$ where x and y are its diagonals. Find the value of x correct to 2 decimal places when $y = 7.8$ and $A = 25.1$.

8 The simple interest formula is $I = Prn$. Find n if $r = 0.145$, $P = 150$ and $I = 326.25$.

9 The gradient of a straight line is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$. Find y_1 when $m = -\frac{5}{6}$, $y_2 = 7$, $x_2 = -3$ and $x_1 = 1$.

10 The surface area of a cylinder is given by the formula $S = 2\pi r(r + h)$. Evaluate h correct to 1 decimal place if $S = 232$ and $r = 4.5$.



11 The formula for body mass index is $\text{BMI} = \frac{w}{h^2}$. Evaluate:

a the BMI when $w = 65$ and $h = 1.6$

b w when $\text{BMI} = 21.5$ and $h = 1.8$

c h when $\text{BMI} = 19.7$ and $w = 73.8$.

12 A formula for depreciation is $D = P(1 - r)^n$. Find r if $D = 12\,000$, $P = 15\,000$ and $n = 3$.

13 The x value of the midpoint is given by $x = \frac{x_1 + x_2}{2}$. Find x_1 when $x = -2$ and $x_2 = 5$.

14 Given the height of a particle at time t is $h = 5t^2$, evaluate t when $h = 23$.

15 If $y = x^2 + 1$, evaluate x when $y = 5$.

16 If the surface area of a sphere is $S = 4\pi r^2$, evaluate r to 3 significant figures when $S = 56.3$.

17 The area of a sector of a circle is $A = \frac{1}{2}r^2\theta$. Evaluate r when $A = 24.6$ and $\theta = 0.45$.

ISBN 9780170413152

18 If $y = \frac{2}{x^3 - 1}$, find the value of x when $y = 3$.

19 Given $y = \sqrt{2x + 5}$, evaluate x when $y = 4$.

20 The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Evaluate r to 1 decimal place when $V = 150$.

INVESTIGATION

BODY MASS INDEX

Body mass index (BMI) is a formula that is used by health professionals to screen for weight categories that may lead to health problems.

The formula for BMI is $\text{BMI} = \frac{m}{h^2}$ where m is the mass of a person in kg and h is the height in metres.

For adults over 20, a BMI under 18.5 means that the person is underweight and over 25 is overweight. Over 30 is considered obese.

Shutterstock.com/Istvan Csak

The BMI may not always be a reliable measurement of body fat. Can you think of some reasons?

Is it important where the body fat is stored? Does it make a difference if it is on the hips or the stomach?

Research more about BMI generally.

ISBN 9780170413152

2.10 Linear simultaneous equations

You can solve two equations together to find one solution that satisfies both equations. Such equations are called **simultaneous equations** and there are two ways of solving them. The **elimination method** adds or subtracts the equations. The **substitution method** substitutes one equation into the other.

EXAMPLE 15

Solve simultaneously using the elimination method:

a $3a + 2b = 5$ and $2a - b = -6$

b $5x - 3y = 19$ and $2x - 4y = 16$

Solution

a

$$3a + 2b = 5 \quad [1]$$
$$2a - b = -6 \quad [2]$$

[2] × 2: $4a - 2b = -12 \quad [3]$

[1] + [3]: $3a + 2b = 5 \quad [1]$

$$7a = -7$$
$$a = -1$$

Substitute $a = -1$ in [1]:

$$3(-1) + 2b = 5$$
$$-3 + 2b = 5$$
$$2b = 8$$
$$b = 4$$

Check that the solution is correct by substituting back into *both* equations.

$\therefore$ Solution is $a = -1$, $b = 4$

b

$$5x - 3y = 19 \quad [1]$$
$$2x - 4y = 16 \quad [2]$$

[1] × 4: $20x - 12y = 76 \quad [3]$

[2] × 3: $6x - 12y = 48 \quad [4]$

[3] − [4]: $14x = 28$

$$x = 2$$

Substitute $x = 2$ in [2]:

$$2(2) - 4y = 16$$
$$4 - 4y = 16$$
$$-4y = 12$$
$$y = -3$$

$\therefore$ Solution is $x = 2, y = -3$

Exercise 2.10 Linear simultaneous equations

Solve each pair of simultaneous equations.

1 $a - b = -2$ and $a + b = 4$

2 $5x + 2y = 12$ and $3x - 2y = 4$

3 $4p - 3q = 11$ and $5p + 3q = 7$

4 $y = 3x - 1$ and $y = 2x + 5$

5 $2x + 3y = -14$ and $x + 3y = -4$

6 $7t + v = 22$ and $4t + v = 13$

7 $4x + 5y + 2 = 0$ and $4x + y + 10 = 0$

8 $2x - 4y = 28$ and $2x - 3y = -11$

9 $5x - y = 19$ and $2x + 5y = -14$

10 $5m + 4n = 22$ and $m - 5n = -13$

11 $4w_1 + 3w_2 = 11$ and $3w_1 + w_2 = 2$

12 $3a - 4b = -16$ and $2a + 3b = 12$

13 $5p + 2q + 18 = 0$ and $2p - 3q + 11 = 0$

14 $7x_1 + 3x_2 = 4$ and $3x_1 + 5x_2 = -2$

15 $9x - 2y = -1$ and $7x - 4y = 9$

16 $5s - 3t - 13 = 0$ and $3s - 7t - 13 = 0$

17 $3a - 2b = -6$ and $a - 3b = -2$

18 $3k - 2h = -14$ and $2k - 5h = -13$

PROBLEM

A group of 39 people went to see a play. There were both adults and children in the group. The total cost of the tickets was $939, with children paying $17 each and adults paying $29 each. How many in the group were adults and how many were children? (Hint: let x be the number of adults and y the number of children.)

2.11 Non-linear simultaneous equations

In simultaneous equations involving **non-linear equations** there may be more than one set of solutions. When solving these, you need to use the substitution method.

Non-linear simultaneous equations

EXAMPLE 16

Solve each pair of equations simultaneously using the substitution method:

a $xy = 6$ and $x + y = 5$

b $x^2 + y^2 = 16$ and $3x - 4y - 20 = 0$

ISBN 9780170413152

Solution

a

$$xy = 6 \qquad [1]$$

$$x + y = 5 \qquad [2]$$

From [2]: $y = 5 - x \qquad [3]$

Substitute [3] in [1]:

$$x(5 - x) = 6$$

$$5x - x^2 = 6$$

$$0 = x^2 - 5x + 6$$

$$0 = (x - 2)(x - 3)$$

$$\therefore\ x = 2 \text{ or } x = 3$$

Substitute $x = 2$ in [3]: $y = 5 - 2 = 3$

Substitute $x = 3$ in [3]: $y = 5 - 3 = 2$

Solutions are $x = 2, y = 3$ and $x = 3, y = 2$

b

$$x^2 + y^2 = 16 \qquad [1]$$

$$3x - 4y - 20 = 0 \qquad [2]$$

From [2]:

$$3x - 20 = 4y$$

$$\frac{3x - 20}{4} = y \qquad [3]$$

Substitute [3] into [1]:

$$x^2 + \left(\frac{3x - 20}{4}\right)^2 = 16$$

$$x^2 + \left(\frac{9x^2 - 120x + 400}{16}\right) = 16$$

$$16x^2 + 9x^2 - 120x + 400 = 256$$

$$25x^2 - 120x + 144 = 0$$

$$(5x - 12)^2 = 0$$

$$\therefore\ 5x - 12 = 0$$

$$x = 2.4$$

Substitute $x = 2.4$ into [3]:

$$y = \frac{3(2.4) - 20}{4}$$

$$= -3.2$$

So the solution is $x = 2.4, y = -3.2$

ISBN 9780170413152

Exercise 2.11 Non-linear simultaneous equations

Solve each pair of simultaneous equations.

1 $y = x^2$ and $y = x$

2 $y = x^2$ and $2x + y = 0$

3 $x^2 + y^2 = 9$ and $x + y = 3$

4 $x - y = 7$ and $xy = -12$

5 $y = x^2 + 4x$ and $2x - y - 1 = 0$

6 $y = x^2$ and $6x - y - 9 = 0$

7 $x = t^2$ and $x + t - 2 = 0$

8 $m^2 + n^2 = 16$ and $m + n + 4 = 0$

9 $xy = 2$ and $y = 2x$

10 $y = x^3$ and $y = x^2$

11 $y = x - 1$ and $y = x^2 - 3$

12 $y = x^2 + 1$ and $y = 1 - x^2$

13 $y = x^2 - 3x + 7$ and $y = 2x + 3$

14 $xy = 1$ and $4x - y + 3 = 0$

15 $h = t^2$ and $h = (t + 1)^2$

16 $x + y = 2$ and $2x^2 + xy - y^2 = 8$

17 $y = x^3$ and $y = x^2 + 6x$

18 $y = |x|$ and $y = x^2$

19 $y = x^2 - 7x + 6$ and $24x + 4y - 23 = 0$

20 $x^2 + y^2 = 1$ and $5x + 12y + 13 = 0$

2.12 Simultaneous equations with three unknown variables

WS Simultaneous equations

Three equations can be solved simultaneously to find 3 unknown pronumerals.

EXAMPLE 17

Solve simultaneously: $a - b + c = 7$, $a + 2b - c = -4$ and $3a - b - c = 3$.

Solution

$$a - b + c = 7 \quad [1]$$
$$a + 2b - c = -4 \quad [2]$$
$$3a - b - c = 3 \quad [3]$$

[1] + [2]:

$$\begin{array}{r} a - b + c = 7 \\ a + 2b - c = -4 \\ \hline 2a + b \quad = 3 \end{array} \quad [4]$$

[1] + [3]:

$$\begin{array}{r} a - b + c = 7 \\ 3a - b - c = 3 \\ \hline 4a - 2b \quad = 10 \end{array}$$

or

$$2a - b = 5 \quad [5]$$

ISBN 9780170413152

[4] + [5]:

$$\begin{aligned} 2a + b &= 3 \\ 2a - b &= 5 \\ \hline 4a &= 8 \\ a &= 2 \end{aligned}$$

Substitute $a = 2$ in [4]:

$$\begin{aligned} 2(2) + b &= 3 \\ 4 + b &= 3 \\ b &= -1 \end{aligned}$$

Substitute $a = 2$ and $b = -1$ in [1]:

$$\begin{aligned} 2 - (-1) + c &= 7 \\ 2 + 1 + c &= 7 \\ 3 + c &= 7 \\ c &= 4 \end{aligned}$$

$\therefore$ solution is $a = 2, b = -1, c = 4$

Exercise 2.12 Simultaneous equations with three unknown variables

Solve each set of simultaneous equations.

1 $x = -2$, $2x - y = 4$ and $x - y + 6z = 0$

2 $a = -2$, $2a - 3b = -1$ and $a - b + 5c = 9$

3 $2a + b + c = 1$, $a + b = -2$ and $c = 7$

4 $a + b + c = 0$, $a - b + c = -4$ and $2a - 3b - c = -1$

5 $x + y - z = 7$, $x + y + 2z = 1$ and $3x + y - 2z = 19$

6 $2p + 5q - r = 25$, $2p - 2q - r = -24$ and $3p - q + 5r = 4$

7 $2x - y + 3z = 9$, $3x + y - 2z = -2$ and $3x - y + 5z = 14$

8 $x - y - z = 1$, $2x + y - z = -9$ and $2x - 3y - 2z = 7$

9 $3h + j - k = -3$, $h + 2j + k = -3$ and $5h - 3j - 2k = -13$

10 $2a - 7b + 3c = 7$, $a + 3b + 2c = -4$ and $4a + 5b - c = 9$

ISBN 9780170413152

2. TEST YOURSELF

Qz Practice quiz

For Questions **1** to **3**, select the correct answer **A**, **B**, **C** or **D**.

1 Find the exact solution of $x^2 - 5x - 1 = 0$.

A $\frac{-5\pm\sqrt{29}}{2}$ **B** $\frac{5\pm\sqrt{21}}{2}$ **C** $\frac{5\pm\sqrt{29}}{2}$ **D** $\frac{-5\pm\sqrt{21}}{2}$

2 If $S = 4\pi r^2$, find the value of r when $S = 200$ (there may be more than one answer).

A $5\sqrt{\frac{2}{\pi}}$ **B** $\sqrt{\frac{200}{\pi}}$ **C** $10\sqrt{\frac{2}{\pi}}$ **D** $\sqrt{\frac{50}{\pi}}$

3 Solve the simultaneous equations $x - y = 7$ and $x + 2y = 1$.

A $x = 5, y = 2$ **B** $x = 5, y = -2$ **C** $x = -5, y = -2$ **D** $x = -5, y = 2$

4 Solve:

a $8 = 3b - 22$ **b** $\frac{a}{4} - \frac{a+2}{3} = 9$ **c** $4(3x + 1) = 11x - 3$

d $3p + 1 \le p + 9$

5 The compound interest formula is $A = P(1 + r)^n$. Find, correct to 2 decimal places:

a A when $P = 1000$, $r = 0.06$ and $n = 4$

b P when $A = 12\,450$, $r = 0.055$ and $n = 7$.

6 Solve each pair of simultaneous equations.

a $x - y + 7 = 0$ and $3x - 4y + 26 = 0$ **b** $xy = 4$ and $2x - y - 7 = 0$

7 Solve:

a $3^{x+2} = 81$ **b** $16^y = 2$

8 Solve $|3b - 1| = 5$.

9 The area of a trapezium is given by $A = \frac{1}{2}h(a + b)$. Find:

a A when $h = 6$, $a = 5$ and $b = 7$

b b when $A = 40$, $h = 5$ and $a = 4$.

10 Solve $2x^2 - 3x + 1 = 0$.

11 Solve $-2 < 3y + 1 \le 10$ and plot the solution on a number line.

12 Solve, correct to 3 significant figures:

a $x^2 + 7x + 2 = 0$ **b** $y^2 - 2y - 9 = 0$ **c** $3n^2 + 2n - 4 = 0$

ISBN 9780170413152

13 The surface area of a sphere is given by $A = 4\pi r^2$. Evaluate to 1 decimal place:

a A when $r = 7.8$ **b** r when $A = 102.9$

14 Solve the simultaneous equations $x^2 + y^2 = 16$ and $3x + 4y - 20 = 0$.

15 The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Evaluate to 2 significant figures:

a V when $r = 8$ **b** r when $V = 250$

16 For each equation, decide if it has:

A 2 solutions **B** 1 solution **C** no solutions.

a $x^2 - 6x + 9 = 0$ **b** $|2x - 3| = 7$ **c** $x^2 - x - 5 = 0$

d $2x^2 - x + 4 = 0$ **e** $3x + 2 = 7$

17 Solve simultaneously $a + b = 5$, $2a + b + c = 4$, $a - b - c = 5$.

18 Solve $9^{2x+1} = 27^x$.

19 Solve:

a $2(3y - 5) > y + 5$ **b** $3^{2x-1} = 27$ **c** $5x^3 - 1 = 39$

d $|5x - 4| = 11$ **e** $8^{x+1} = 4^x$ **f** $27^{2x-1} = 9$

2. CHALLENGE EXERCISE

1 Find the value of y if $a^{3y-5} = \frac{1}{a^2}$.

2 The solutions of $x^2 - 6x - 3 = 0$ are in the form $a + b\sqrt{3}$. Find the values of a and b.

3 **a** Factorise $x^5 - 9x^3 - 8x^2 + 72$.

b Hence or otherwise solve $x^5 - 9x^3 - 8x^2 + 72 = 0$.

4 Solve the simultaneous equations $y = x^3 + x^2$ and $y = x + 1$.

5 Find the value of b if $x^2 - 8x + b$ is a perfect square. Hence solve $x^2 - 8x - 1 = 0$ by completing the square.

6 Considering the definition of absolute value, solve $\frac{|x-3|}{3-x} = x$, where $x \neq 3$.

7 Solve $x^{\frac{3}{2}} = \frac{1}{8}$.

8 Find the solutions of $x^2 - 2ax - b = 0$ by completing the square.

9 Solve $3x^2 = 8(2x - 1)$ and write the solution in the simplest surd form.

10 Solve $|2x - 1| = 5 - x$ and check solutions.

ISBN 9780170413152