Answers

Chapter 1

Exercise 1A

1a $4x$ **b** $2x$ **c** $-2x$ **d** $-4x$
2a $5a$ **b** $-a$ **c** $-9a$ **d** $-3a$
3a 0 **b** $-y$ **c** $-10a$ **d** $-3b$ **e** $7x$ **f** $-3ab$
g $4pq$ **h** $-3abc$
4a $-6a$ **b** $12a^2$ **c** a^5 **d** a^6
5a $-2a$ **b** 3 **c** a^6 **d** a
6a $2t^2$ **b** 0 **c** t^4 **d** 1
7a $-3x$ **b** $-9x$ **c** $-18x^2$ **d** -2
8a -4 **b** -12 **c** 18 **d** 2
9a $x + 3$ **b** $2y - 3$ **c** $2a - 3$ **d** $8x + 4y$
e $-10t - 5$ **f** $4a - 3a^2$ **g** $-5x^2 - 12x - 3$
h $9a - 3b - 5c$
10a 5 **b** $7m^2$ **c** $-12a$ **d** $-3p^3q^4r$
11a $2x$ **b** $4x$ **c** $-6a$ **d** $-4b$
12a $10a$ **b** $-18x$ **c** $-3a^2$ **d** $6a^3b$
e $-8x^5$ **f** $-6p^3q^4$
13a -2 **b** $3x$ **c** xy **d** $-a^4$ **e** $-7ab^3$ **f** $5ab^2c^6$
14a $6a^5b^6$ **b** $-24a^4b^8$ **c** $9a^6$ **d** $-8a^{12}b^3$
15a 0 **b** -1 **c** 59 **d** 40
16a $3a^2$ **b** $5c^4$ **c** a^2bc^6
17a $2x^5$ **b** $9xy^5$ **c** b^4 **d** $2a^3$

Exercise 1B

1a $3x - 6$ **b** $2x - 6$ **c** $-3x + 6$ **d** $-2x + 6$
e $-3x - 6$ **f** $-2x - 6$ **g** $-x + 2$ **h** $-2 + x$ **i** $-x - 3$
2a $3x + 3y$ **b** $-2p + 2q$ **c** $4a + 8b$ **d** $x^2 - 7x$
e $-x^2 + 3x$ **f** $-a^2 - 4a$ **g** $5a + 15b - 10c$
h $-6x + 9y - 15z$ **i** $2x^2y - 3xy^2$
3a $x + 2$ **b** $7a - 3$ **c** $2x - 4$ **d** $4 - 3a$
e $2 - x$ **f** $2c$ **g** $-x - y$ **h** $x + 4$ **i** $5a - 18b$
j $-2s - 10t$ **k** $x^2 + 17xy$ **l** $16a - b$
4a $x^2 + 5x + 6$ **b** $y^2 + 11y + 28$ **c** $t^2 + 3t - 18$
d $x^2 - 2x - 8$ **e** $t^2 - 4t + 3$ **f** $2a^2 + 13a + 15$
g $3u^2 - 10u - 8$ **h** $8p^2 - 2p - 15$
i $2b^2 - 13b + 21$ **j** $15a^2 - a - 2$
k $-c^2 + 9c - 18$ **l** $2d^2 + 5d - 12$
5a Answers will vary
b **i** Answers will vary
ii Answers will vary

6a $x^2 + 2xy + y^2$ **b** $x^2 - 2xy + y^2$ **c** $x^2 - y^2$
d $a^2 + 6a + 9$ **e** $b^2 - 8b + 16$ **f** $c^2 + 10c + 25$
g $d^2 - 36$ **h** $49 - e^2$ **i** $64 + 16f + f^2$
j $81 - 18g + g^2$ **k** $h^2 - 100$ **l** $i^2 + 22i + 121$
m $4a^2 + 4a + 1$ **n** $4b^2 - 12b + 9$ **o** $9c^2 + 12c + 4$
p $4d^2 + 12de + 9e^2$ **q** $4f^2 - 9g^2$ **r** $9h^2 - 4i^2$
s $25j^2 + 40j + 16$ **t** $16k^2 - 40k\ell + 25\ell^2$
u $16 - 25m^2$ **v** $25 - 30n + 9n^2$
w $49p^2 + 56pq + 16q^2$ **x** $64 - 48r + 9r^2$
7a $t^2 + 2 + \frac{1}{t^2}$ **b** $t^2 - 2 + \frac{1}{t^2}$ **c** $t^2 - \frac{1}{t^2}$
8a $10\,404$ **b** $998\,001$ **c** $39\,991$
9a $a^3 - b^3$ **b** $2x + 3$ **c** $18 - 6a$ **d** $x^2 + 2x - 1$
e $x^3 - 6x^2 + 12x - 8$ **f** $p^2 + q^2 + r^2$

Exercise 1C

1a $2(x + 4)$ **b** $3(2a - 5)$ **c** $a(x - y)$
d $5a(4b - 3c)$ **e** $x(x + 3)$ **f** $p(p + 2q)$
g $3a(a - 2b)$ **h** $6x(2x + 3)$ **i** $4c(5d - 8)$
j $ab(a + b)$ **k** $2a^2(3 + a)$ **l** $7x^2y(x - 2y)$
2a $(p + q)(m + n)$ **b** $(x - y)(a + b)$
c $(x + 3)(a + 2)$ **d** $(a + b)(a + c)$
e $(z - 1)(z^2 + 1)$ **f** $(a + b)(c - d)$
g $(p - q)(u - v)$ **h** $(x - 3)(x - y)$
i $(p - q)(5 - x)$ **j** $(2a - b)(x - y)$
k $(b + c)(a - 1)$ **l** $(x + 4)(x^2 - 3)$
m $(a - 3)(a^2 - 2)$ **n** $(2t + 5)(t^2 - 5)$
o $(x - 3)(2x^2 - a)$
3a $(a - 1)(a + 1)$ **b** $(b - 2)(b + 2)$ **c** $(c - 3)(c + 3)$
d $(d - 10)(d + 10)$ **e** $(5 - y)(5 + y)$
f $(1 - n)(1 + n)$ **g** $(7 - x)(7 + x)$
h $(12 - p)(12 + p)$ **i** $(2c - 3)(2c + 3)$
j $(3u - 1)(3u + 1)$ **k** $(5x - 4)(5x + 4)$
l $(1 - 7k)(1 + 7k)$ **m** $(x - 2y)(x + 2y)$
n $(3a - b)(3a + b)$ **o** $(5m - 6n)(5m + 6n)$
p $(9ab - 8)(9ab + 8)$
4a $(a + 1)(a + 2)$ **b** $(k + 2)(k + 3)$ **c** $(m + 1)(m + 6)$
d $(x + 3)(x + 5)$ **e** $(y + 4)(y + 5)$ **f** $(t + 2)(t + 10)$
g $(x - 1)(x - 3)$ **h** $(c - 2)(c - 5)$ **i** $(a - 3)(a - 4)$
j $(b - 2)(b - 6)$ **k** $(t + 2)(t - 1)$ **l** $(u - 2)(u + 1)$
m $(w - 4)(w + 2)$ **n** $(a + 4)(a - 2)$
o $(p - 5)(p + 3)$ **p** $(y + 7)(y - 4)$ **q** $(c - 3)(c - 9)$
r $(u - 6)(u - 7)$ **s** $(x - 10)(x + 9)$ **t** $(x + 8)(x - 5)$
u $(t - 8)(t + 4)$ **v** $(p + 12)(p - 3)$
w $(u - 20)(u + 4)$ **x** $(t + 25)(t - 2)$

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5a $(3x+1)(x+1)$ **b** $(2x+1)(x+2)$
c $(3x+1)(x+5)$ **d** $(3x+2)(x+2)$
e $(2x-1)(x-1)$ **f** $(5x-3)(x-2)$
g $(5x-6)(x-1)$ **h** $(3x-1)(2x-3)$
i $(2x-3)(x+1)$ **j** $(2x+5)(x-1)$
k $(3x+5)(x-1)$ **l** $(3x-1)(x+5)$
m $(2x+3)(x-5)$ **n** $(2x-5)(x+3)$
o $(6x-1)(x+3)$ **p** $(2x-3)(3x+1)$
q $(3x-2)(2x+3)$ **r** $(5x+3)(x+4)$
s $(5x-6)(x+2)$ **t** $(5x-4)(x-3)$
u $(5x+4)(x-3)$ **v** $(5x-2)(x+6)$
w $(3x-4)(3x+2)$ **x** $(3x-5)(x+6)$
6a $(a-5)(a+5)$ **b** $b(b-25)$ **c** $(c-5)(c-20)$
d $(2d+5)(d+10)$ **e** $(e+5)(e^2+5)$
f $(4-f)(4+f)$ **g** $g^2(16-g)$ **h** $(h+8)^2$
i $(i-18)(i+2)$ **j** $(j+4)(5j-4)$
k $(2k+1)(2k-9)$ **l** $(k-8)(2k^2-3)$
m $(2a+b)(a-2)$ **n** $3m^2n^4(2m+3n)$
o $(7p-11q)(7p+11q)$ **p** $(t-4)(t-10)$
q $(3t-10)(t+4)$ **r** $(5t+4)(t+10)$
s $(5t+8)(t+5)$ **t** $5t(t^2+2t+3)$
u $(u+18)(u-3)$ **v** $(3x-2y)(x^2-5)$
w $(p+q-r)(p+q+r)$ **x** $(2a-3)^2$
7a $3(a-2)(a+2)$ **b** $(x-y)(x+y)(x^2+y^2)$
c $x(x-1)(x+1)$ **d** $5(x+2)(x-3)$
e $y(5-y)(5+y)$ **f** $(2-a)(2+a)(4+a^2)$
g $2(2x-3)(x+5)$ **h** $a(a+1)(a^2+1)$
i $(c+1)(c-1)(c+9)$ **j** $x(x-1)(x-7)$
k $(x-2)(x+2)(x^2+1)$ **l** $(x-1)(x+1)(a-2)$

Exercise 1D

1a 1 **b** 2 **c** $\frac{1}{2}$ **d** $\frac{1}{a}$ **e** $\frac{x}{3y}$ **f** $\frac{3}{a}$
2a 1 **b** $\frac{1}{2}$ **c** $3x$ **d** $\frac{b}{2}$ **e** $\frac{3}{2x}$ **f** $\frac{1}{2a}$ **g** $\frac{4}{b}$ **h** 6
3a $\frac{3x}{2}$ **b** $\frac{3y}{4}$ **c** $\frac{2m}{9}$ **d** $\frac{7n}{10}$ **e** $\frac{3x-2y}{24}$ **f** $\frac{13a}{6}$
g $\frac{b}{15}$ **h** $-\frac{xy}{20}$
4a $\dfrac{2}{a}$ **b** $-\dfrac{1}{x}$ **c** $\dfrac{3}{2a}$ **d** $\dfrac{1}{6x}$ **e** $\dfrac{25}{12a}$ **f** $\dfrac{1}{2x}$
5a $\frac{5x+7}{6}$ **b** $\frac{18x+11}{20}$ **c** $\frac{x+1}{4}$
d $\frac{x}{6}$ **e** $\frac{2x+17}{20}$ **f** $\frac{2x-3}{6}$
6a 2 **b** $\frac{3}{2}$ **c** $\frac{x}{3}$ **d** $\frac{1}{x+y}$ **e** $\frac{3}{2b}$ **f** $\frac{x}{x-2}$ **g** $\frac{a+3}{a+4}$
h $\frac{x+1}{x-1}$ **i** $\frac{x+5}{x+4}$
7a $\frac{2x+1}{x(x+1)}$ **b** $\frac{1}{x(x+1)}$ **c** $\frac{2x}{(x+1)(x-1)}$
d $\frac{5x-13}{(x-2)(x-3)}$ **e** $\frac{x-5}{(x+1)(x-1)}$ **f** $\frac{10}{(x+3)(x-2)}$
8a $\frac{3x}{2(x-1)}$ **b** a **c** $\frac{c+2}{c+4}$ **d** x
9a -1 **b** $\frac{2}{a-b}$ **c** 1 **d** $3-x$
10a $\frac{2}{x^2-1}$ **b** $\frac{3x}{x^2-y^2}$ **c** $\frac{x+1}{(x-2)(x+3)(x+4)}$
d $\frac{x}{(x-1)(x-2)(x-3)}$
11a $\frac{1}{3}$ **b** $\frac{7}{13}$ **c** $\frac{3}{11}$ **d** $\frac{1}{5}$ **e** $\frac{1}{x+2}$ **f** $\frac{t^2-1}{t^2+1}$
g $\frac{ab}{a+b}$ **h** $\frac{x^2+y^2}{x^2-y^2}$ **i** $\frac{x^2}{2x+1}$ **j** $\frac{x-1}{x-3}$

Exercise 1E

1a $x=3$ **b** $p=0$ **c** $a=8$ **d** $w=-1$
e $x=9$ **f** $x=-5$ **g** $x=-16$ **h** $x=-2$
2a $n=4$ **b** $b=-1$ **c** $x=4$ **d** $x=-11$ **e** $a=-\frac{1}{2}$
f $y=2$ **g** $x=\frac{7}{9}$ **h** $x=-\frac{3}{5}$
3a $a=8$ **b** $y=16$ **c** $x=\frac{1}{3}$ **d** $a=\frac{2}{5}$ **e** $y=\frac{3}{2}$
f $x=-8$ **g** $a=7$ **h** $x=-\frac{1}{2}$ **i** $a=-5$ **j** $t=\frac{3}{5}$
k $x=-2$ **l** $x=5$
4a $a=3$ **b** $s=16$ **c** $v=\frac{2}{3}$ **d** $\ell=21$
e $C=35$ **f** $c=-\frac{2}{5}$
5a 6 **b** -4 **c** 17 **d** 65 cents
6a $y=\frac{2}{3}$ **b** $x=15$ **c** $a=-15$ **d** $x=\frac{9}{2}$ **e** $x=6$
f $x=\frac{1}{6}$ **g** $x=\frac{1}{2}$ **h** $x=20$ **i** $x=-\frac{23}{2}$ **j** $x=-\frac{7}{3}$
7a $b=\frac{a+d}{c}$ **b** $n=\frac{t-a+d}{d}$ **c** $r=\frac{p-qt}{t}$
d $v=\frac{3}{u-1}$
8a $x=\frac{19}{6}$ **b** $x=\frac{3}{14}$ **c** $x=-1$ **d** $x=\frac{17}{6}$
9a $a=-11$ **b** $x=2$ **c** $x=-\frac{7}{3}$ **d** $x=-\frac{5}{2}$
10a $a=-\frac{2b}{3}$ **b** $g=\frac{2fh}{5f-h}$
c $y=\frac{2x}{1-x}$ **d** $b=\frac{4a+5}{a-1}$
11a 20 **b** 16 **c** 30 km/h

Exercise 1F

1a $x=3$ or -3 **b** $y=5$ or -5 **c** $a=2$ or -2
d $c=6$ or -6 **e** $t=1$ or -1 **f** $x=\frac{3}{2}$ or $-\frac{3}{2}$
g $x=\frac{1}{2}$ or $-\frac{1}{2}$ **h** $a=2\frac{2}{3}$ or $-2\frac{2}{3}$ **i** $y=\frac{4}{5}$ or $-\frac{4}{5}$
2a $x=0$ or 5 **b** $y=0$ or -1 **c** $c=0$ or -2
d $k=0$ or 7 **e** $t=0$ or 1 **f** $a=0$ or 3 **g** $b=0$ or $\frac{1}{2}$
h $u=0$ or $-\frac{1}{3}$ **i** $x=-\frac{3}{4}$ or 0 **j** $a=0$ or $\frac{5}{2}$
k $y=0$ or $\frac{2}{3}$ **l** $n=0$ or $-\frac{3}{5}$
3a $x=-3$ or -1 **b** $x=1$ or 2 **c** $x=-4$ or -2
d $a=2$ or 5 **e** $t=-2$ or 6 **f** $c=5$ **g** $n=1$ or 8
h $p=-5$ or 3 **i** $a=-2$ or 12 **j** $y=-5$ or 1
k $p=-2$ or 3 **l** $a=-11$ or 12 **m** $c=3$ or 6
n $t=-2$ or 10 **o** $u=-8$ or 7 **p** $k=-4$ or 6
q $h=-25$ or -2 **r** $a=-22$ or 2

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4a $x = -\frac{1}{2}$ or -1 **b** $a = \frac{1}{3}$ or 2 **c** $y = \frac{1}{4}$ or 1
d $x = -5$ or $-\frac{1}{2}$ **e** $x = -1\frac{1}{2}$ or 1 **f** $n = -1$ or $1\frac{2}{3}$
g $b = -\frac{2}{3}$ or 2 **h** $a = -5$ or $1\frac{1}{2}$ **i** $y = -2\frac{1}{2}$ or 3
j $y = -4$ or $\frac{2}{3}$ **k** $x = \frac{1}{5}$ or 5 **l** $t = \frac{3}{4}$ or 3 **m** $t = -\frac{2}{5}$ or 3
n $u = -\frac{4}{5}$ or $\frac{1}{2}$ **o** $x = \frac{1}{5}$ **p** $x = -\frac{2}{3}$ or $\frac{3}{2}$
q $b = -\frac{3}{2}$ or $-\frac{1}{6}$ **r** $k = -\frac{8}{3}$ or $\frac{1}{2}$
5a $x = \frac{1+\sqrt{5}}{2}$ or $\frac{1-\sqrt{5}}{2}$, $x \doteqdot 1.618$ or -0.6180
b $x = \frac{-1+\sqrt{13}}{2}$ or $\frac{-1-\sqrt{3}}{2}$, $x \doteqdot 1.303$ or -2.303
c $a = 3$ or 4
d $u = -1 + \sqrt{3}$ or $-1 - \sqrt{3}$, $u \doteqdot 0.7321$ or -2.732
e $c = 3 + \sqrt{7}$ or $3 - \sqrt{7}$, $c \doteqdot 5.646$ or 0.3542
f $x = -\frac{1}{2}$
g $a = \frac{2+\sqrt{2}}{2}$ or $\frac{2-\sqrt{2}}{2} a \doteqdot 1.707$ or 0.2929
h $x = -3$ or $\frac{2}{5}$
i $b = \frac{-3+\sqrt{17}}{4}$ or $\frac{-3-\sqrt{17}}{4}$, $b \doteqdot 0.2808$ or -1.781
j $c = \frac{2+\sqrt{13}}{3}$ or $\frac{2-\sqrt{13}}{3}$, $c \doteqdot 1.869$ or -0.5352
k $t = \frac{1+\sqrt{5}}{4}$ or $\frac{1-\sqrt{5}}{4}$, $t \doteqdot 0.8090$ or -0.3090
l no solutions
6a $x = -1$ or 2 **b** $a = 2$ or 5 **c** $y = \frac{1}{2}$ or 4 **d** $b = -\frac{2}{5}$ or $\frac{2}{3}$
7a $x = 1 + \sqrt{2}$ or $1 - \sqrt{2}$ **b** $x = 2 + \sqrt{3}$ or $2 - \sqrt{3}$
c $a = 1 + \sqrt{5}$ or $1 - \sqrt{5}$ **d** $m = \frac{2+\sqrt{14}}{5}$ or $\frac{2-\sqrt{14}}{5}$
8a $p = \frac{1}{2}$ or 1 **b** $x = -3$ or 5 **c** $n = 5$
9a 7 **b** 6 and 9 **c** $x = 15$
10a $k = -1$ or 3 **b** $u = \frac{4}{3}$ or 4 **c** $y = 1 + \sqrt{6}$ or $1 - \sqrt{6}$
d $k = \frac{-5+\sqrt{73}}{4}$ or $\frac{-5-\sqrt{73}}{4}$ **e** $a = -\frac{7}{3}$ or 3
f $k = -4$ or 15 **g** $t = 2\sqrt{3}$ or $-\sqrt{3}$
h $m = \frac{1+\sqrt{2}}{3}$ or $\frac{1-\sqrt{2}}{3}$
11a 4 cm **b** 3 cm **c** 55 km/h and 60 km/h

Exercise 1G

1a $x = 3, y = 3$ **b** $x = 2, y = 4$ **c** $x = 2, y = 1$
d $a = -3, b = -2$ **e** $p = 3, q = -1$ **f** $u = 1, v = -2$
2a $x = 3, y = 2$ **b** $x = 1, y = -2$ **c** $x = 4, y = 1$
d $a = -1, b = 3$ **e** $c = 2, d = 2$ **f** $p = -2, q = -3$
3a $x = 2, y = 4$ **b** $x = -1, y = 3$ **c** $x = 2, y = 2$
d $x = 9, y = 1$ **e** $x = 3, y = 4$ **f** $x = 4, y = -1$
g $x = 5, y = 3\frac{3}{5}$ **h** $x = 13, y = 7$
4a $x = -1, y = 3$ **b** $x = 5, y = 2$ **c** $x = -4, y = 3$
d $x = 2, y = -6$ **e** $x = 1, y = 2$ **f** $x = 16, y = -24$
g $x = 1, y = 6$ **h** $x = 5, y = -2$ **i** $x = 5, y = 6$
j $x = 7, y = 5$
5a $x = 1$ & $y = 1$ or $x = -2$ & $y = 4$
b $x = 2$ & $y = 1$ or $x = 4$ & $y = 5$
c $x = 0$ & $y = 0$ or $x = 1$ & $y = 3$
d $x = -2$ & $y = -7$ or $x = 3$ & $y = -2$
e $x = -3$ & $y = -5$ or $x = 5$ & $y = 3$
f $x = 1$ & $y = 6$ or $x = 2$ & $y = 3$
6a 53 and 37
b The pen cost 60 c, the pencil cost 15 c.
c Each apple cost 40 c, each orange cost 60 c.
d 44 adults, 22 children
e The man is 36, the son is 12.
f 189 for, 168 against
7a $x = 5$ & $y = 10$ or $x = 10$ & $y = 5$
b $x = -8$ & $y = -11$ or $x = 11$ & $y = 8$
c $x = \frac{1}{2}$ & $y = 4$ or $x = 2$ & $y = 1$
d $x = 4$ & $y = 5$ or $x = 5$ & $y = 4$
e $x = 1$ & $y = 2$ or $x = \frac{3}{2}$ & $y = \frac{7}{4}$
f $x = 2$ & $y = 5$ or $x = \frac{10}{3}$ & $y = 3$
8a 9 \$20 notes, 14 \$10 notes **b** 5 km/h, 3 km/h

Exercise 1H

1a 1 **b** 9 **c** 25 **d** 81 **e** $\frac{9}{4}$ **f** $\frac{1}{4}$ **g** $\frac{25}{4}$ **h** $\frac{81}{4}$
2a $(x + 2)^2$ **b** $(y + 1)^2$ **c** $(p + 7)^2$ **d** $(m - 6)^2$
e $(t - 8)^2$ **f** $(x + 10)^2$ **g** $(u - 20)^2$ **h** $(a - 12)^2$
3a $x^2 + 6x + 9 = (x + 3)^2$ **b** $y^2 + 8y + 16 = (y + 4)^2$
c $a^2 - 20a + 100 = (a - 10)^2$
d $b^2 - 100b + 2500 = (b - 50)^2$
e $u^2 + u + \frac{1}{4} = = \left(u + \frac{1}{2}\right)^2$ **f** $t^2 - 7t + \frac{49}{4} = \left(t - \frac{7}{2}\right)^2$
g $m^2 + 50m + 625 = (m + 25)^2$
h $c^2 - 13c + \frac{169}{4} = \left(c - \frac{13}{2}\right)^2$
4a $x = -1$ or 3 **b** $x = 0$ or 6 **c** $a = -4$ or -2
d $x = -2 + \sqrt{3}$ or $-2 - \sqrt{3}$ **e** $x = 5 + \sqrt{5}$ or $5 - \sqrt{5}$
f $y = -5$ or 2 **g** $b = -2$ or 7 **h** no solution for y
i $a = \frac{-7+\sqrt{21}}{2}$ or $\frac{-7-\sqrt{21}}{2}$
5a $x = \frac{2+\sqrt{6}}{2}$ or $\frac{2-\sqrt{6}}{2}$
b $x = \frac{-4+\sqrt{10}}{2}$ or $\frac{-4-\sqrt{10}}{2}$
c no solution for x **d** $x = -\frac{3}{2}$ or $\frac{1}{2}$
e $x = \frac{1+\sqrt{5}}{4}$ or $\frac{1-\sqrt{5}}{4}$ **f** $x = \frac{5+\sqrt{11}}{2}$ or $\frac{5-\sqrt{11}}{2}$
6a Answers will vary **b** $a = 3, b = 4$ and $c = 25$
c Answers will vary **d** $A = -5, B = 6$ and $C = 8$

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Chapter 1 review exercise

1a $-6y$ **b** $-10y$ **c** $-16y^2$ **d** -4
2a $-3a^2$ **b** $-a^2$ **c** $2a^4$ **d** 2
3a $2t - 1$ **b** $4p + 3q$ **c** $x - 2y$ **d** $5a^2 - 3a - 18$
4a $-18k^9$ **b** $-2k^3$ **c** $36k^{12}$ **d** $27k^9$
5a $14x - 3$ **b** $-4a + 2b$ **c** $-2a$ **d** $-6x^3 - 10x^2$
e $2n^2 + 11n - 21$ **f** $r^2 + 6r + 9$ **g** $y^2 - 25$
h $6x^2 - 19x + 15$ **i** $t^2 - 16t + 64$ **j** $4c^2 - 49$
k $16p^2 + 8p + 1$ **l** $9u^2 - 12u + 4$
6a $18(a + 2)$ **b** $4(5b - 9)$ **c** $9c(c + 4)$
d $(d - 6)(d + 6)$ **e** $(e + 4)(e + 9)$ **f** $(f - 6)^2$
g $(6 - 5g)(6 + 5g)$ **h** $(h - 12)(h + 3)$
i $(i + 9)(i - 4)$ **j** $(2j + 3)(j + 4)$ **k** $(3k + 2)(k - 3)$
l $(5\ell - 4)(\ell - 2)$ **m** $(2m - 3)(2m + 5)$
n $(n + 1)(m + p)$ **o** $(p + 9)(p^2 + 4)$ **p** $(q - r)(t - 5)$
q $(u^2 + v)(w - x)$ **r** $(x - y)(x + y + 2)$
7a $\frac{3x}{4}$ **b** $\frac{x}{4}$ **c** $\frac{x^2}{8}$ **d** 2 **e** $\frac{13a}{6b}$ **f** $\frac{5a}{6b}$ **g** $\frac{a^2}{b^2}$ **h** $\frac{9}{4}$
i $\frac{x^2 + y^2}{xy}$ **j** $\frac{x^2 - y^2}{xy}$ **k** 1 **l** $\frac{x^2}{y^2}$
8a $\frac{8x - 13}{15}$ **b** $\frac{8x - 13}{(x + 4)(x - 5)}$ **c** $\frac{3x + 13}{10}$
d $\frac{-3x - 13}{(x + 1)(x - 4)}$ **e** $\frac{x - 3}{4}$ **f** $\frac{-2x + 6}{x(x + 3)}$
9a $\frac{3}{5}$ **b** $\frac{2}{x + y}$ **c** $\frac{x + 3}{x - 4}$ **d** $\frac{x + 1}{x^2 + 1}$ **e** $\frac{1}{a + b}$ **f** $\frac{x - 7}{3x - 2}$
10a $x = 4$ **b** $x = \frac{2}{3}$ **c** $x = 46$ **d** $x = 36$ **e** $a = 3$
f $a = 10$ **g** $a = -17$ **h** $a = -42$
11a $a = -7$ or 7 **b** $b = -7$ or 0 **c** $c = -6$ or -1
d $d = -7$ or 1 **e** $e = 2$ or 3 **f** $f = -\frac{3}{2}$ or 2
g $g = \frac{1}{2}$ or 6 **h** $h = -2$ or $\frac{4}{3}$
12a $x = 2 + \sqrt{3}$ or $2 - \sqrt{3}$
b $y = \frac{-3 + \sqrt{21}}{2}$ or $\frac{-3 - \sqrt{21}}{2}$
c $y = -3 + \sqrt{5}$ or $-3 - \sqrt{5}$ **d** $y = \frac{1 + \sqrt{7}}{3}$ or $\frac{1 - \sqrt{7}}{3}$
e $y = \frac{-5 + \sqrt{65}}{4}$ or $\frac{-5 - \sqrt{65}}{4}$
f $y = \frac{3 + \sqrt{13}}{4}$ or $\frac{3 - \sqrt{13}}{4}$
13a $x = -2 + \sqrt{10}$ or $-2 - \sqrt{10}$
b $x = 3 + \sqrt{6}$ or $3 - \sqrt{6}$
c $x = 1 + \sqrt{13}$ or $1 - \sqrt{13}$
d $x = -5 + 3\sqrt{2}$ or $-5 - 3\sqrt{2}$

Chapter 2

Exercise 2A

1a $\frac{3}{10}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$ **d** $\frac{1}{20}$
2a 0.6 **b** 0.27 **c** 0.09 **d** 0.165
3a 25% **b** 40% **c** 24% **d** 65%
4a 32% **b** 9% **c** 22.5% **d** 150%
5a 5×7 **b** 2×3^2 **c** $2 \times 3^2 \times 5$ **d** $2^2 \times 5 \times 11$
6a $\frac{1}{3}$ **b** $\frac{4}{5}$ **c** $\frac{2}{3}$ **d** $\frac{3}{4}$ **e** $\frac{2}{5}$ **f** $\frac{7}{15}$ **g** $\frac{4}{7}$ **h** $\frac{5}{6}$ **i** $\frac{3}{5}$ **j** $\frac{3}{4}$
7a 0.5 **b** 0.2 **c** 0.6 **d** 0.75 **e** 0.04 **f** 0.35
g 0.125 **h** 0.625
8a $\frac{2}{5}$ **b** $\frac{1}{4}$ **c** $\frac{3}{20}$ **d** $\frac{4}{25}$ **e** $\frac{39}{50}$ **f** $\frac{1}{200}$ **g** $\frac{3}{8}$ **h** $\frac{33}{125}$
9a $0.\dot{3}$ **b** $0.\dot{6}$ **c** $0.\dot{1}$ **d** $0.\dot{5}$ **e** $0.\dot{2}\dot{7}$ **f** $0.\dot{0}\dot{9}$
g $0.1\dot{6}$ **h** $0.8\dot{3}$
10a $\frac{3}{4}$ **b** $\frac{7}{10}$ **c** $\frac{5}{6}$ **d** $\frac{4}{15}$ **e** $\frac{5}{18}$ **f** $\frac{1}{24}$ **g** $\frac{5}{6}$ **h** $\frac{1}{75}$
11a 5 **b** 8 **c** $\frac{1}{10}$ **d** $\frac{1}{7}$ **e** $\frac{1}{4}$ **f** 6 **g** $\frac{1}{4}$ **h** $\frac{2}{3}$ **i** 4 **j** $\frac{1}{4}$
12a 60 c **b** 15 kg **c** \$7800 **d** 72 min or $1\frac{1}{5}$ h
13a 0.132 **b** 0.025 **c** 0.3125 **d** 0.3375 **e** $0.58\dot{3}$
f $1.\dot{8}\dot{1}$ **g** $0.1\dot{3}$ **h** $0.2\dot{3}\dot{6}$
14a \$800 **b** \$160 **c** \$120
15a $\frac{14}{15}$ **b** $\frac{5}{11}$ **c** $\frac{1}{2000}$
16a $\frac{1}{11} = 0.\dot{0}\dot{9}$, $\frac{2}{11} = 0.\dot{1}\dot{8}$, …, $\frac{5}{11} = 0.\dot{4}\dot{5}$, $\frac{6}{11} = 0.\dot{5}\dot{4}$, …, $\frac{10}{11} = 0.\dot{9}\dot{0}$. The first digit runs from 0 to 9, the second digit runs from 9 to 0.
b $\frac{1}{7} = 0.\dot{1}4285\dot{7}$, $\frac{2}{7} = 0.\dot{2}8571\dot{4}$, etc. The digits of each cycle are in the same order but start at a different place in the cycle.
17c $3.000\,000\,3 \neq 3$, showing that some fractions are not stored exactly.

Exercise 2B

1a rational, $\frac{-3}{1}$ **b** rational, $\frac{3}{2}$ **c** irrational **d** rational, $\frac{2}{1}$
e rational, $\frac{3}{1}$ **f** irrational **g** rational, $\frac{2}{3}$ **h** rational, $\frac{9}{20}$
i rational, $\frac{3}{25}$ **j** rational, $\frac{333}{1000}$
k rational, $\frac{1}{3}$ **l** rational, $\frac{22}{7}$ **m** irrational
n rational, $3\frac{7}{50}$ **o** rational, $\frac{0}{1}$
2a 0.3 **b** 5.7 **c** 12.8 **d** 0.1 **e** 3.0 **f** 10.0
3a 0.43 **b** 5.4 **c** 5.0 **d** 0.043 **e** 430 **f** 4300
4a 3.162 **b** 6.856 **c** 0.563 **d** 0.771
e 3.142 **f** 9.870
5a 7.62 **b** 5.10 **c** 3840 **d** 538 000
e 0.740 **f** 0.00806
6a 1 **b** 2 **c** 3 **d** 2 **e** 4 **f** either 1, 2 or 3
7a i closed **ii** open **iii** closed **iv** neither open nor closed
v open **vi** open **vii** closed **viii** neither open nor closed
b i bounded **ii** unbounded **iii** unbounded
iv bounded **v** unbounded **vi** bounded
vii unbounded **viii** bounded

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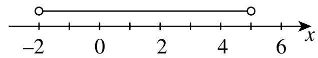
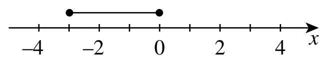
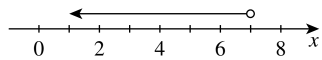
8a $-2 < x < 5$



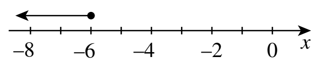
b $-3 \le x \le 0$



c $x < 7$



d $x \le -6$



9a 45.186 **b** 2.233 **c** 0.054 **d** 0.931
e 0.842 **f** 0.111

10a 10, rational **b** $\sqrt{41}$, irrational **c** 8, rational
d $\sqrt{5}$, irrational **e** $\frac{13}{15}$, rational **f** 45, rational

11a 0.3981 **b** 0.05263 **c** 1.425 **d** 5.138
e 0.1522 **f** 25 650 **g** 5.158 **h** 0.7891
i 1.388×10^{14} **j** 1.134 **k** 0.005892 **l** 1.173

12a The passage seems to take $\pi \doteqdot 3$.
b 3 significant figures. **c** Search the internet.
d 7, with a gap of about 0.3 inches.

13a 9.46×10^{15} m **b** 2.4×10^{22} m
c 4.35×10^{17} seconds **d** 1.3×10^{26} m

14a 1.836×10^{3} **b** 6×10^{26}

15 Answers will vary

Exercise 2C

1a 4 **b** 6 **c** 9 **d** 11 **e** 12 **f** 20 **g** 50 **h** 100

2a $2\sqrt{3}$ **b** $3\sqrt{2}$ **c** $2\sqrt{5}$ **d** $3\sqrt{3}$ **e** $2\sqrt{7}$ **f** $2\sqrt{10}$
g $4\sqrt{2}$ **h** $3\sqrt{11}$ **i** $3\sqrt{6}$ **j** $10\sqrt{2}$ **k** $2\sqrt{15}$ **l** $5\sqrt{3}$
m $4\sqrt{5}$ **n** $7\sqrt{2}$ **o** $20\sqrt{2}$ **p** $10\sqrt{10}$

3a $2\sqrt{3}$ **b** $2\sqrt{7}$ **c** $\sqrt{5}$ **d** $-2\sqrt{2}$ **e** $2\sqrt{3} + 3\sqrt{2}$
f $\sqrt{5} - 2\sqrt{7}$ **g** $3\sqrt{6} - 2\sqrt{3}$ **h** $-3\sqrt{2} - 6\sqrt{5}$
i $-4\sqrt{10} + 2\sqrt{5}$

4a $6\sqrt{2}$ **b** $10\sqrt{3}$ **c** $4\sqrt{6}$ **d** $8\sqrt{11}$ **e** $9\sqrt{5}$ **f** $12\sqrt{13}$
g $20\sqrt{3}$ **h** $8\sqrt{6}$

5a $\sqrt{20}$ **b** $\sqrt{50}$ **c** $\sqrt{128}$ **d** $\sqrt{108}$ **e** $\sqrt{125}$ **f** $\sqrt{112}$
g $\sqrt{68}$ **h** $\sqrt{490}$

6a $3\sqrt{2}$ **b** $\sqrt{3}$ **c** $2\sqrt{2}$ **d** $5\sqrt{6}$ **e** $\sqrt{5}$ **f** $2\sqrt{10}$ **g** $4\sqrt{3}$
h $2\sqrt{5}$ **i** $11\sqrt{2}$

7a $4\sqrt{6} + 10\sqrt{3}$ **b** $2\sqrt{2} + 6\sqrt{3}$ **c** $4\sqrt{7} - 10\sqrt{35}$

8a 7 **b** 20 **c** 96

Exercise 2D

1a 3 **b** $\sqrt{6}$ **c** 7 **d** $\sqrt{30}$ **e** $6\sqrt{2}$ **f** $10\sqrt{5}$ **g** $6\sqrt{15}$
h $30\sqrt{14}$ **i** 12 **j** 63 **k** 30 **l** 240

2a $\sqrt{5}$ **b** $\sqrt{7}$ **c** $\sqrt{5}$ **d** 2 **e** $3\sqrt{2}$ **f** $\sqrt{3}$ **g** $2\sqrt{7}$ **h** $5\sqrt{5}$

3a $5 + \sqrt{5}$ **b** $\sqrt{6} - \sqrt{2}$ **c** $2\sqrt{3} - 3$ **d** $2\sqrt{10} - 4$
e $7\sqrt{7} - 14$ **f** $18 - 2\sqrt{30}$

4a $2\sqrt{3}$ **b** $5\sqrt{2}$ **c** $3\sqrt{5}$ **d** $4\sqrt{11}$ **e** 24 **f** $12\sqrt{10}$

5a $2\sqrt{5} - 2$ **b** $3\sqrt{6} + 3\sqrt{2}$ **c** $5\sqrt{3} + 4\sqrt{5}$
d $4\sqrt{3} - 2\sqrt{6}$ **e** $27\sqrt{3} - 9\sqrt{7}$ **f** $21\sqrt{2} - 42$

6a $\sqrt{6} - \sqrt{3} + \sqrt{2} - 1$ **b** $\sqrt{35} + 3\sqrt{5} - 2\sqrt{7} - 6$
c $\sqrt{15} + \sqrt{10} + \sqrt{6} + 2$ **d** $8 - 3\sqrt{6}$
e $4 + \sqrt{7}$ **f** $7\sqrt{3} - 4\sqrt{6}$

7a 4 **b** 2 **c** 1 **d** 7 **e** 15 **f** 29

8a $4 + 2\sqrt{3}$ **b** $6 - 2\sqrt{5}$ **c** $5 + 2\sqrt{6}$ **d** $12 - 2\sqrt{35}$
e $13 - 4\sqrt{3}$ **f** $29 + 12\sqrt{5}$ **g** $33 + 4\sqrt{35}$
h $30 - 12\sqrt{6}$ **i** $55 + 30\sqrt{2}$

9a 2 **b** $\frac{3}{5}$ **c** $2\sqrt{3}$ **d** $\frac{5\sqrt{3}}{2}$ **e** 5 **f** 4

10a 3 **b** 5 **c** 4 **d** 6

11a $\sqrt{3}$ **b** $\frac{6\sqrt{7}}{13}$

12a $a^2 + 2ab + b^2$
b $6 + \sqrt{11} - 2\sqrt{36 - 11} + 6 - \sqrt{11} = 2$ **c** $\sqrt{2}$

Exercise 2E

1a $\frac{\sqrt{3}}{3}$ **b** $\frac{\sqrt{7}}{7}$ **c** $\frac{3\sqrt{5}}{5}$ **d** $\frac{5\sqrt{2}}{2}$ **e** $\frac{\sqrt{6}}{3}$ **f** $\frac{\sqrt{35}}{7}$ **g** $\frac{2\sqrt{55}}{5}$ **h** $\frac{3\sqrt{14}}{2}$

2a $\frac{\sqrt{3} + 1}{2}$ **b** $\frac{\sqrt{7} - 2}{3}$ **c** $\frac{3 - \sqrt{5}}{4}$ **d** $\frac{4 + \sqrt{7}}{9}$
e $\frac{\sqrt{5} + \sqrt{2}}{3}$ **f** $\frac{\sqrt{10} - \sqrt{6}}{4}$ **g** $\frac{2\sqrt{3} - 1}{11}$ **h** $\frac{5 + 3\sqrt{2}}{7}$

3a $\sqrt{2}$ **b** $\sqrt{5}$ **c** $2\sqrt{3}$ **d** $3\sqrt{7}$ **e** $\frac{\sqrt{6}}{2}$ **f** $\frac{\sqrt{15}}{3}$
g $\frac{4\sqrt{6}}{3}$ **h** $\frac{7\sqrt{10}}{5}$

4a $\frac{\sqrt{5}}{10}$ **b** $\frac{\sqrt{7}}{21}$ **c** $\frac{3\sqrt{2}}{10}$ **d** $\frac{2\sqrt{3}}{21}$ **e** $\frac{5\sqrt{2}}{3}$ **f** $\frac{3\sqrt{3}}{4}$
g $\frac{\sqrt{30}}{20}$ **h** $\frac{2\sqrt{77}}{35}$

5a $\frac{3\sqrt{5} - 3}{4}$ **b** $\frac{8\sqrt{2} + 4\sqrt{3}}{5}$ **c** $\frac{5\sqrt{7} + 7}{18}$ **d** $\frac{3\sqrt{15} - 9}{2}$
e $\frac{28 + 10\sqrt{7}}{3}$ **f** $\sqrt{2} + 1$ **g** $2 - \sqrt{3}$ **h** $\frac{7 + 2\sqrt{10}}{3}$
i $8 - 3\sqrt{7}$ **j** $\frac{23 + 6\sqrt{10}}{13}$ **k** $4 - \sqrt{15}$ **l** $\frac{93 + 28\sqrt{11}}{5}$

6a $\sqrt{3} + 1$ **b** $4 - \sqrt{10}$

7a 3 **b** 1 **c** 7 **d** 2

8 Answers will vary

9 $a = -1, b = 2$

10a $x^2 + 2 + \frac{1}{x^2}$ **b i** Answers will vary
ii $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 28 - 2 = 26$

Chapter 2 review exercise

1a rational, $\frac{7}{1}$ **b** rational, $\frac{-9}{4}$ **c** rational, $\frac{3}{1}$ **d** irrational
e irrational **f** rational, $\frac{2}{1}$ **g** rational, $\frac{-4}{25}$ **h** irrational

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2a i 4.12 **ii** 4.1 **b i** 4.67 **ii** 4.7 **c i** 2.83 **ii** 2.8
d i 0.77 **ii** 0.77 **e i** 0.02 **ii** 0.019 **f i** 542.41 **ii** 540
3a 1.67 **b** 70.1 **c** 1.43 **d** 0.200 **e** 0.488
f 0.496 **g** 1.27 **h** 1590 **i** 0.978
4a $2\sqrt{6}$ **b** $3\sqrt{5}$ **c** $5\sqrt{2}$ **d** $10\sqrt{5}$ **e** $9\sqrt{2}$ **f** $4\sqrt{10}$
5a $2\sqrt{5}$ **b** 5 **c** 28 **d** $\sqrt{7}-\sqrt{5}$ **e** $\sqrt{7}$ **f** $3\sqrt{5}$ **g** 4
h $2\sqrt{5}$ **i** $24\sqrt{10}$
6a $\sqrt{3}$ **b** $7\sqrt{2}$ **c** $4\sqrt{2}$ **d** $8\sqrt{6}-6\sqrt{5}$
7a $3\sqrt{7}-7$ **b** $2\sqrt{30}+3\sqrt{10}$ **c** $3\sqrt{5}-5\sqrt{15}$
d $3\sqrt{2}+6$
8a $\sqrt{5}+1$ **b** $13+7\sqrt{3}$ **c** $2\sqrt{35}+4\sqrt{7}-6\sqrt{5}-12$
d 1 **e** 13 **f** $11-4\sqrt{7}$ **g** $7+2\sqrt{10}$ **h** $34-24\sqrt{2}$
9a $\frac{\sqrt{5}}{5}$ **b** $\frac{3\sqrt{2}}{2}$ **c** $\frac{\sqrt{33}}{11}$ **d** $\frac{\sqrt{3}}{15}$ **e** $\frac{5\sqrt{7}}{14}$ **f** $\frac{\sqrt{5}}{15}$
10a $\frac{\sqrt{5}-\sqrt{2}}{3}$ **b** $\frac{3+\sqrt{7}}{2}$ **c** $\frac{2\sqrt{6}+\sqrt{3}}{21}$ **d** $\frac{3-\sqrt{3}}{2}$
e $\frac{\sqrt{11}-\sqrt{5}}{2}$ **f** $\frac{6\sqrt{35}+21}{13}$
11a $\frac{9-2\sqrt{14}}{5}$ **b** $26+15\sqrt{3}$
12 $x = 50$
13 $5\sqrt{5}+2$
14 $p = 5, q = 2$
15 $\frac{7}{3}$

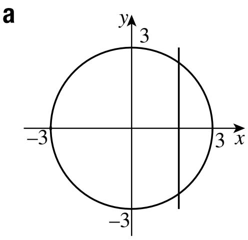
Chapter 3

Exercise 3A

1a 5 **b** 3 **c** -1 **d** 11
2a -3 **b** 5 **c** 0 **d** 5
3a 5 **b** -10 **c** -3 **d** 2
4a $5, -1, -7$ **b** $0, 4, 0$ **c** $16, 8, 0$ **d** $4, 1, \frac{1}{4}$
5a $-4, 4, 12$ **b** $-\frac{1}{3}, 1, \frac{1}{5}$ **c** $-18, 2, -10$ **d** $1, \sqrt{5}, 3$
6a y: $-1, 1, 3$ **b** y: $3, 0, -1, 0, 3$
c $f(x)$: $-3, 0, 1, 0, -3$ **d** $f(x)$: $-15, 0, 3, 0, -3, 0, 15$
7a 8 **b** 2 **c** -6 **d** 4 **e** 11 **f** 6 **g** -35 **h** 4
8a 4 **b** $5\frac{1}{2}$ **c** $5\frac{1}{3}$ **d** $4\frac{1}{3}$
9a $-2-2\sqrt{2}$ **b** $3-2\sqrt{7}$
10a $y = -\dfrac{3}{4}x - \dfrac{5}{4}$ **b** $x = -\dfrac{4}{3}y - \dfrac{5}{3}$ **c** $y = -\dfrac{4}{x}$
d $s = \sqrt[3]{V}, s = \sqrt{\dfrac{A}{6}}$ **e i** $\ell = \dfrac{100}{b}$ **ii** $b = \dfrac{100}{\ell}$
11 $C = 50 + 20x$
12a The square root of a negative is undefined.
b The square root of a negative is undefined.
c Division by zero is undefined.
d Division by zero is undefined.
13a 0 **b** $2-4\sqrt{3}$
14a $2a-4, -2a-4, 2a-2$ **b** $2-x, 2+a, 1-a$
c a^2, a^2, a^2+2a+1 **d** $\frac{1}{a-1}, \frac{1}{-a-1} = -\frac{1}{a+1}, \frac{1}{a}$
15a $5t, 5t-8$ **b** $\sqrt{t}-2, \sqrt{t-2}$
c t^2+2t-2, t^2-2t **d** $-t^2, -t^2+4t-2$
16a $7+h$ **b** $p+q+5$ **c** $2x+h+5$
17a, b, c Answers will vary

Exercise 3B

1 Notice that the y-axis is such a line in every case. Shown below are some other vertical lines that intersect at least twice.

a


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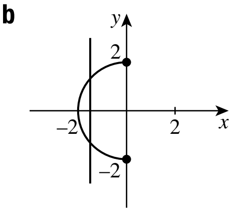

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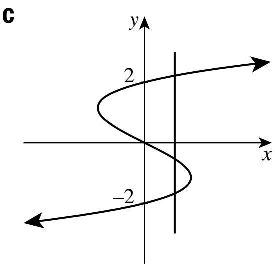

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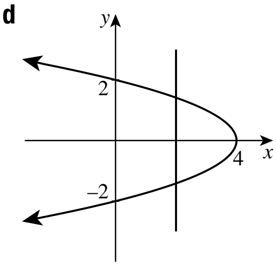

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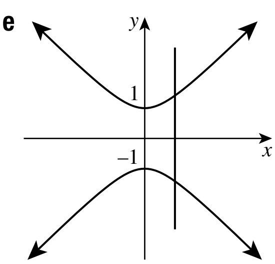
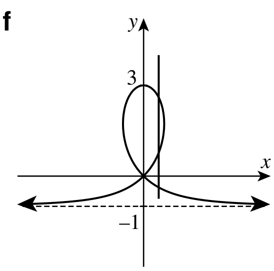
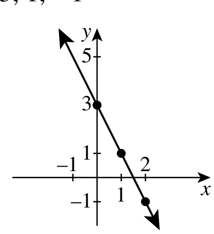
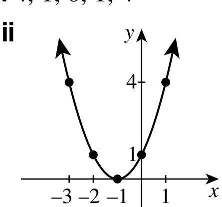


f


2 a, c, f, h
3a domain: all real x, range: $y \geq -1$
b domain: $-2 \leq x \leq 2$, range: $-2 \leq y \leq \sqrt{3}$
c domain: all real x, range: all real y
d domain: $-1 \leq x$, range: all real y
e domain: $-2 \leq x \leq 2$, range: $-3 \leq y \leq 3$
f domain: all real x, range: all real y
g domain: $0 \leq x \leq 2$, range: $-2 \leq y \leq 2$
h domain: all real x, range: $y < 1$
4a i $3, 1, -1$ **b i** $4, 1, 0, 1, 4$

ii


iii domain: all real x,
range: all real y

ii


iii domain: all real x,
range: $y \geq 0$.

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5a $x \neq 0$ **b** $x \neq 3$ **c** $x \neq -1$ **d** $x \neq -2$

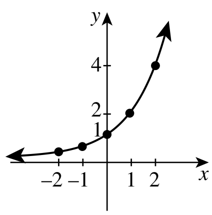
6a $x \geq 0$ **b** $x \geq 2$ **c** $x \geq -3$ **d** $x \geq -5$

7a $(0, 3)$ and $(0, -3)$ **b** $(0, 1)$ and $(0, -1)$
c $(2, 1)$ and $(2, 5)$ **d** $(2, 2)$ and $(2, -2)$

8a all real x **b** all real x **c** $x \neq 4$ **d** $x \neq \frac{1}{2}$
e $x \geq -4$ **f** $x \geq -\frac{1}{2}$ **g** $x \leq 5$ **h** $x \leq 2$
i $x > 0$ **j** $x > -1$ **k** $x < 1$ **l** $x > 1\frac{1}{2}$

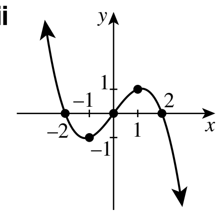
9a i $\frac{1}{4}, \frac{1}{2}, 1, 2, 4$ **b i** $5, 0, -1, 0, 1, 0, -5$

ii



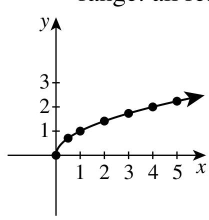
iii domain: all real x, range: $y > 0$

ii



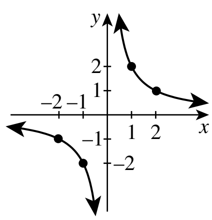
iii domain: all real x, range: all real y

10a $x \geq 0$
b 0, 0.7, 1, 1.4, 1.7, 2, 2.2
c It is the top half of a concave right parabola.



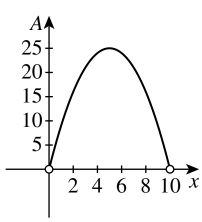
11a $x \neq 0$
b $-\frac{1}{2}, -1, -2, -4,$ $*, 4, 2, 1, \frac{1}{2}$
Division by zero is undefined.

c



12a x **b** $(10 - x)$
c $A = x(10 - x)$
d Both $10 - x > 0$ and $x > 0$. Thus $10 > x$ and $x > 0$. Hence the domain is $0 < x < 10$.

e



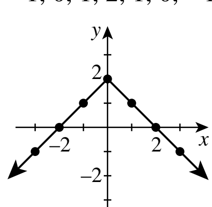
13a $y = 2x + 3$ **b** $y = \dfrac{4}{x}$ **c** $y = \frac{3}{x-2}$

d $y = -2 + \sqrt{9 - x^2}$

14a $x > -2$ **b** $x \neq 2$ and $x \neq -2$
c $x \neq -1$ and $x \neq 0$ **d** $x \neq 2$ and $x \neq 3$
e $x \leq -2$ or $x \geq 2$ **f** $-1 < x < 1$

15a $-1, 0, 1, 2, 1, 0, -1$

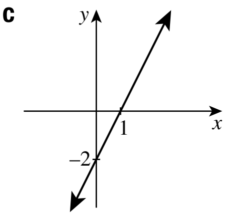
b



Exercise 3C

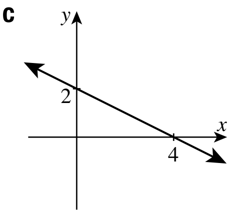
1a $y = -2$ **b** $x = 1$

c

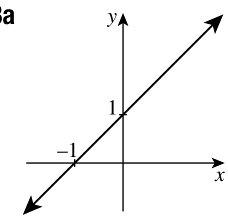


2a $y = 2$ **b** $x = 4$

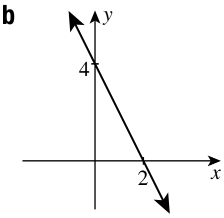
c



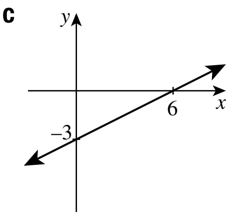
3a



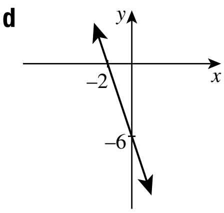
b



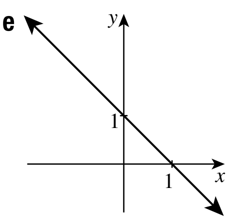
c



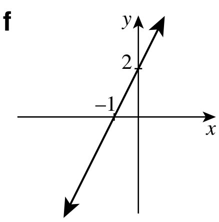
d



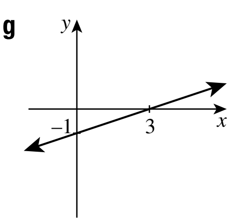
e



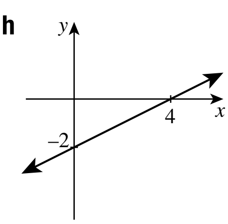
f



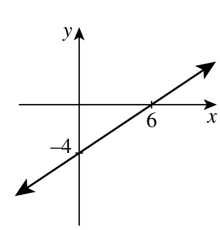
g



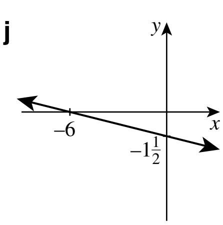
h



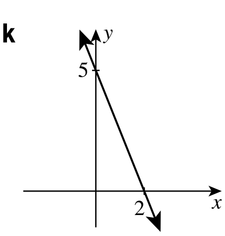
i



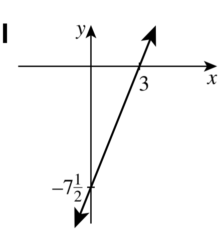
j



k



l



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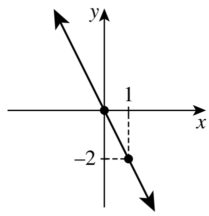
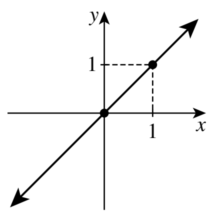
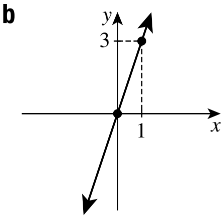
4a When $x = 0$, $y = 0$.
b $(1, -2)$



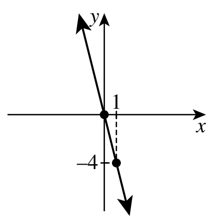
5a



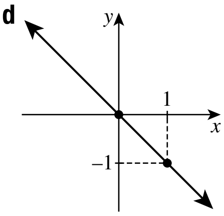
b



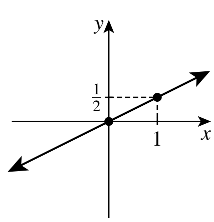
c



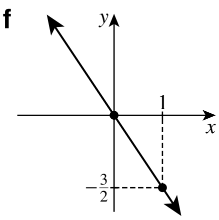
d



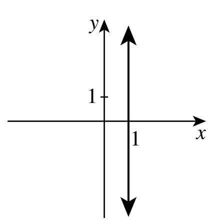
e



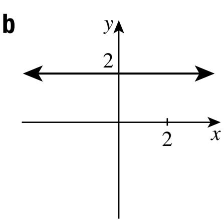
f



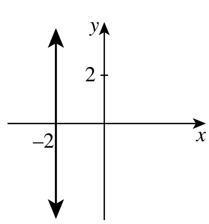
6a



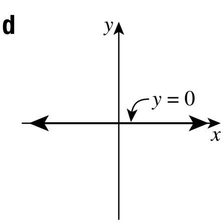
b



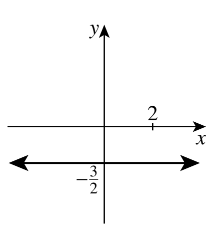
c



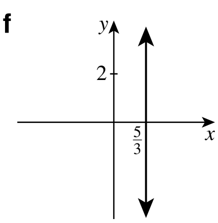
d



e



f

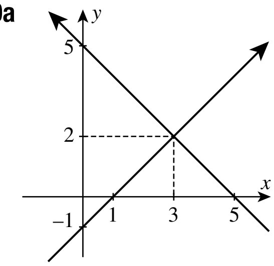


7a a, c, f
b $(1, 0)$ and $(1, 1)$ are on $x = 1$.
$(-2, 0)$ and $(-2, 1)$ are on $x = -2$.
$\left(\frac{5}{3}, 0\right)$ and $\left(\frac{5}{3}, 1\right)$ are on $3x = 5$.

8e $y = 1 - x$ **f** $y = 2x + 2$ **g** $y = \frac{1}{3}x - 1$
h $y = \frac{1}{2}x - 2$ **i** $y = \frac{2}{3}x - 4$ **j** $y = -\frac{1}{4}x - \frac{3}{2}$
k $y = -\frac{5}{2}x + 5$ **l** $y = \frac{5}{2}x - \frac{15}{2}$

9a yes **b** no **c** yes **d** yes **e** yes **f** no

10a

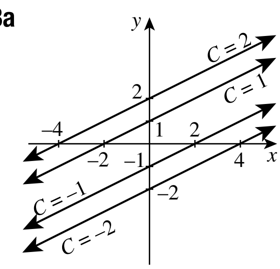


b $(3, 2)$ **c** See 10b

11a $(-1, 3)$ **b** $(1, -2)$ **c** $(-2, -1)$

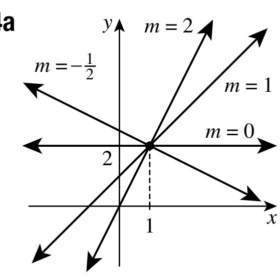
12a $C(n) = 60 + 50n$
b i $D(n) = 10 + 2n$ **ii** $T = C + D$ so $T(n) = 70 + 52n$

13a



b They are parallel.
The value of c gives the y-intercept.

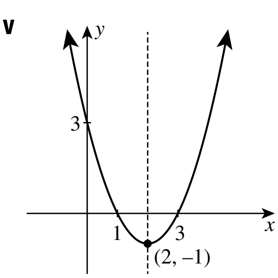
14a



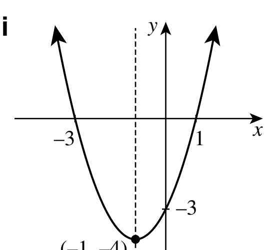
b $(1, 2)$
c Answers will vary

Exercise 3D

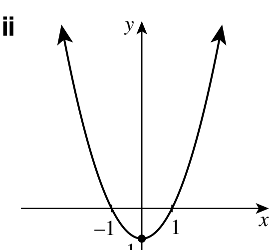
1a i $y = 3$
ii $x = 1, 3$
iii $x = 2$
iv $(2, -1)$
v



b i



ii



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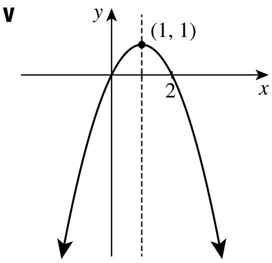
2a i $y = 0$

ii $x = 0, 2$

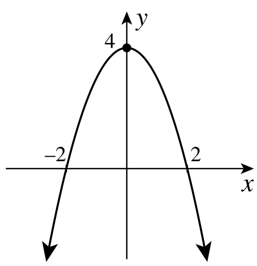
iii $x = 1$

iv $(1, 1)$

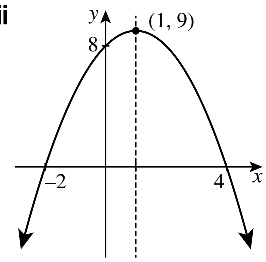
v



b i



ii

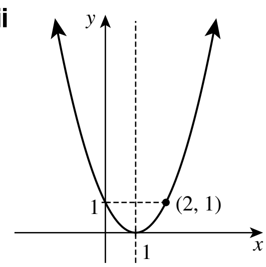


3a i $(0, 1)$

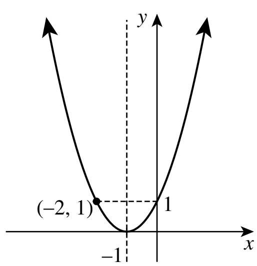
ii $(1, 0)$

iv $(2, 1)$

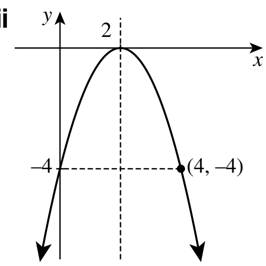
iii



b i



ii



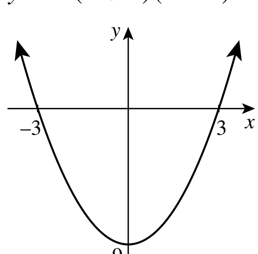
4a $y = (x - 4)(x - 6)$ **b** $y = x(x - 3)$

c $y = (x + 3)(x - 5)$ **d** $y = (x + 6)(x + 1)$

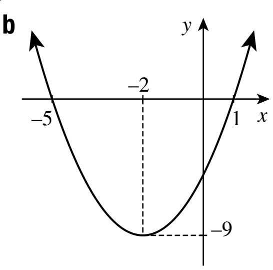
5a $y = x(x - 3)$ **b** $y = (x + 2)(x - 1)$

c $y = -(x + 1)(x - 3)$ **d** $y = -(x + 2)(x + 5)$

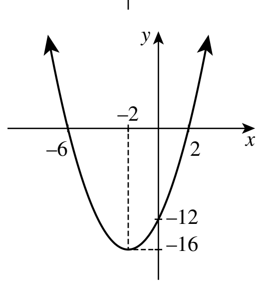
6a



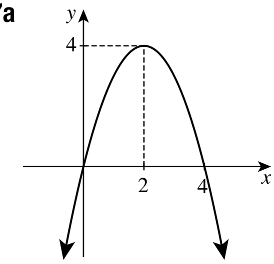
b



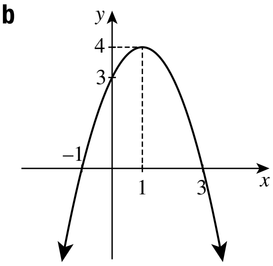
c



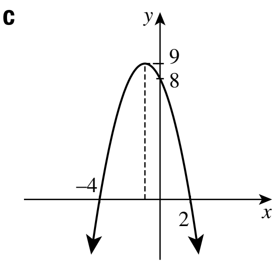
7a



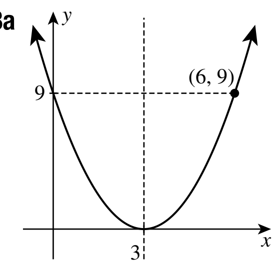
b



c

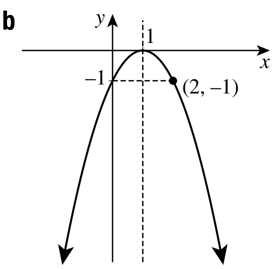


8a



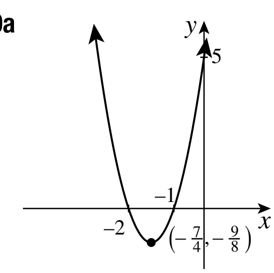
$y = (x - 3)^2$

b

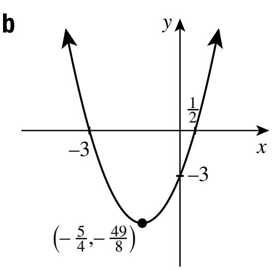


$y = -(x - 1)^2$

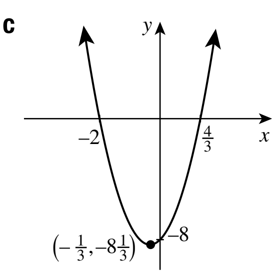
9a



b



c



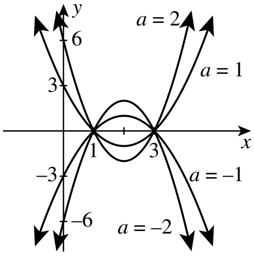
10a $y = (x + 1)(x - 2)$ **b** $y = -(x + 3)(x - 2)$

c $y = 3(x + 2)(x - 4)$ **d** $y = -\frac{1}{2}(x - 2)(x + 2)$

11a $y = 2(x - 1)(x - 3)$ **b** $y = -2(x + 2)(x - 1)$

c $y = -3(x + 1)(x - 5)$ **d** $y = \frac{1}{4}(x + 2)(x + 4)$

12a



b $(1, 0)$ and $(3, 0)$

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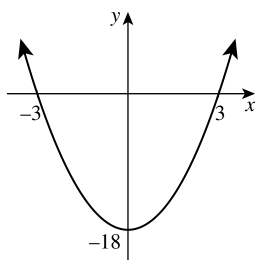
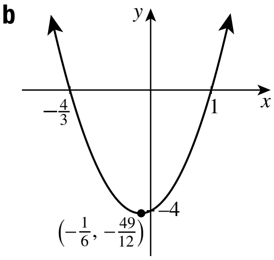
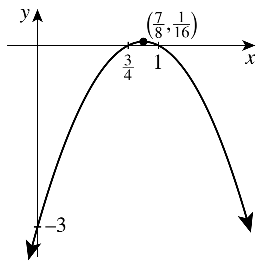
13a



b



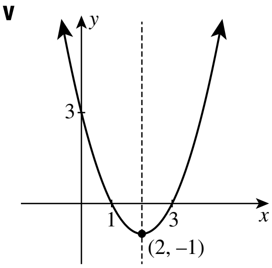
c



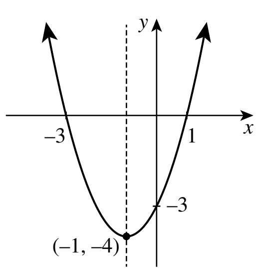
14a $y = 3(x-2)(x-8)$ **b** $y = -(x-2)(x-8)$
c $y = \frac{4}{3}(x-2)(x-8)$ **d** $y = -\frac{20}{7}(x-2)(x-8)$
15a $f(x) = (x-4)(x+2)$, so the axis is $x = 1$.
b i Both $f(1+h) = h^2 - 9$ and $f(1-h) = h^2 - 9$.
ii The parabola is symmetric in the line $x = 1$.

Exercise 3E

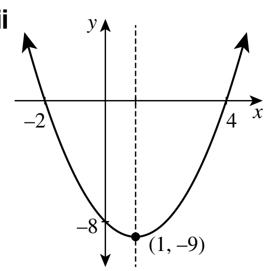
1a i $a = 1$, concave up
ii $y = 3$
iii $x = 1, 3$
iv $x = 2$, $V(2, -1)$
v



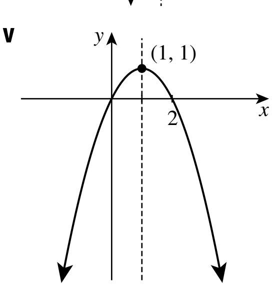
b i



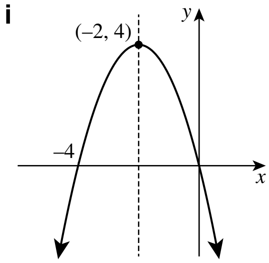
ii



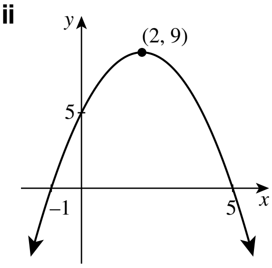
2a i $a = -1$
ii $y = 0$
iii $x = 0, 2$
iv $x = 1$, $V(1, 1)$
v



b i

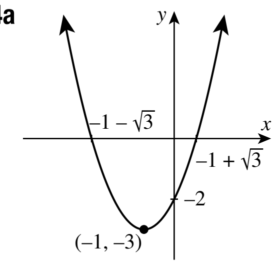


ii

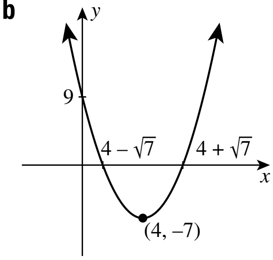


3a $f(x) = (x-2)^2 + 1$ **b** $f(x) = (x+3)^2 + 2$
c $f(x) = (x-1)^2 + 7$ **d** $f(x) = (x-5)^2 - 24$
e $f(x) = (x+1)^2 - 6$ **f** $f(x) = (x+2)^2 - 5$

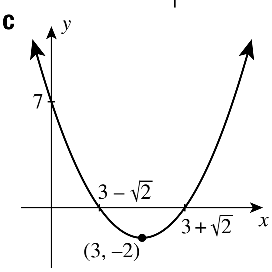
4a



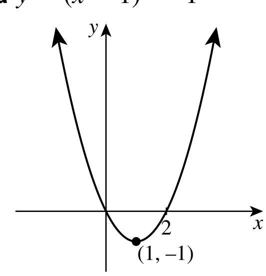
b



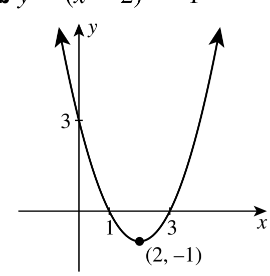
c



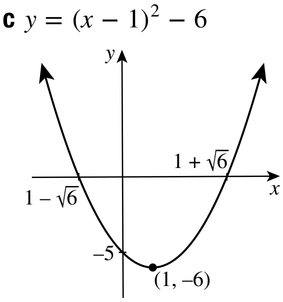
5a $y = (x-1)^2 - 1$



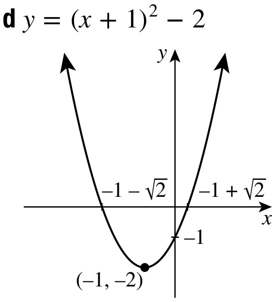
b $y = (x-2)^2 - 1$



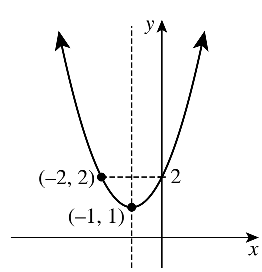
c $y = (x-1)^2 - 6$



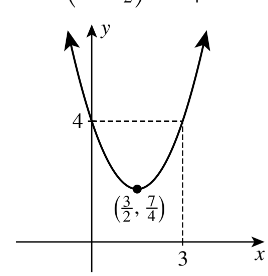
d $y = (x+1)^2 - 2$



e $y = (x+1)^2 + 1$



f $y = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$



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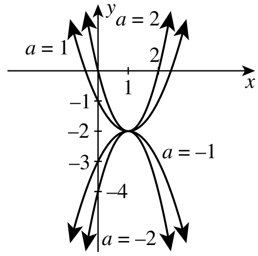
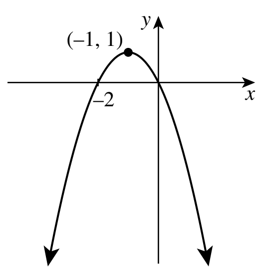
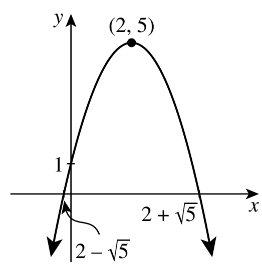
6a $x = 1, 3$ **b** $x = -3, 1$ **c** $x = -1, 2$
7a $y = (x - 1)^2 + 2$ **b** $y = (x + 2)^2 - 3$
c $y = -(x - 3)^2 + 5$ **d** $y = -(x - 2)^2 - 1$
8a $y = (x - 2)^2 + 5$ **b** $y = x^2 - 3$
c $y = (x + 1)^2 + 7$ **d** $y = (x - 3)^2 - 11$
9a



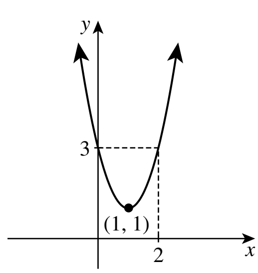
b $(1, -2)$ **c** $a > 0$
d The vertex is below the x-axis.
Thus the parabola will only intersect the x-axis if it is concave up.
10a $V = (3, -5)$, concave up, two x-intercepts.
b $V = (-1, 3)$, concave down, two x-intercepts.
c $V = (-2, -1)$, concave down, no x-intercepts.
d $V = (4, 3)$, concave up, no x-intercepts.
e $V = (-1, 0)$, concave up, one x-intercept.
f $V = (3, 0)$, concave down, one x-intercept.
11a $y = -(x + 1)^2 + 1$



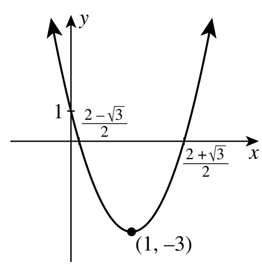
b $y = -(x - 2)^2 + 5$



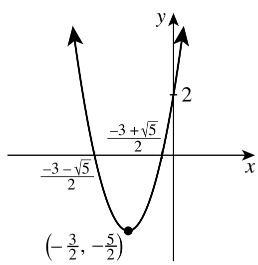
c $y = 2(x - 1)^2 + 1$



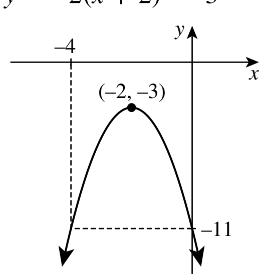
d $y = 4(x - 1)^2 - 3$



e $y = 2\left(x + \frac{3}{2}\right)^2 - \frac{5}{2}$



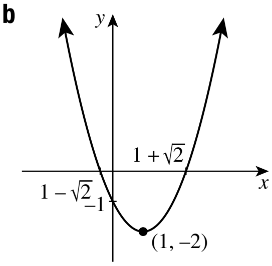
f $y = -2(x + 2)^2 - 3$



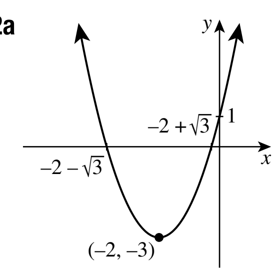
12a $f(x) = \left(x + 1 + \sqrt{2}\right)\left(x + 1 - \sqrt{2}\right)$
b $f(x) = \left(x - 2 + \sqrt{3}\right)\left(x - 2 - \sqrt{3}\right)$
c $f(x) = -\left(x + 1 + \sqrt{5}\right)\left(x + 1 - \sqrt{5}\right)$
13 Put $h = -4$ and $k = 2$ into the formula $y = a(x - h)^2 + k$.
a $y = (x + 4)^2 + 2$ **b** $y = 3(x + 4)^2 + 2$
c $y = \frac{7}{8}(x + 4)^2 + 2$ **d** $y = -\frac{1}{8}(x + 4)^2 + 2$
14a Answer in question
b $y = a\left(x + \dfrac{b}{2a}\right)^2 - \dfrac{b^2 - 4ac}{4a}$ with vertex
$\left(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a}\right)$ and axis of symmetry $x = -\dfrac{b}{2a}$.
c $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Exercise 3F

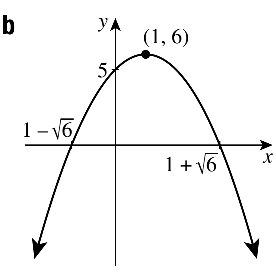
1a i concave up **ii** $y = -1$
iii $x = 1$ **iv** $= (1, -2)$
v $\Delta = 8$ **vi** $\Delta > 0$
vii $x = 1 - \sqrt{2}, 1 + \sqrt{2} \doteqdot -0.41, 2.41.$
b



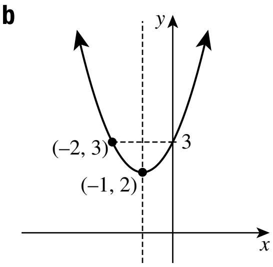
2a



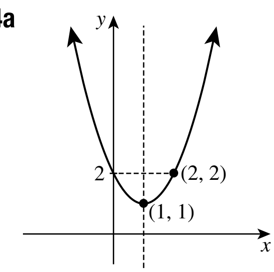
b



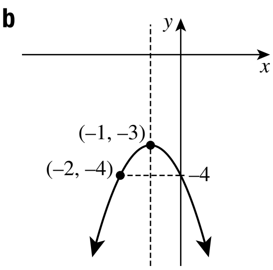
3a i concave up
ii $y = 3$
iii $x = -1$
iv $= (-1, 2)$
v $\Delta = -8$
vi $\Delta < 0$
b



4a



b



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5a $-1+\sqrt{3}$ or $-1-\sqrt{3}$, 0.73 or -2.73

b $2+\sqrt{3}$ or $2-\sqrt{3}$, 3.73 or 0.27

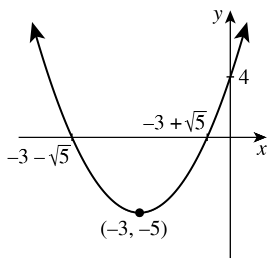
c $\frac{1}{2}\left(-3+\sqrt{17}\right)$ or $\frac{1}{2}\left(-3-\sqrt{17}\right)$, 0.56 or -3.56

d $-1+\sqrt{5}$ or $-1-\sqrt{5}$, 1.23 or -3.23

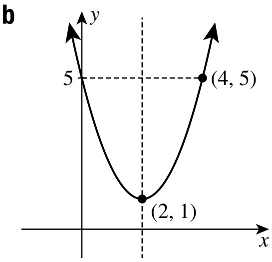
e $\frac{1}{3}\left(1+\sqrt{7}\right)$ or $\frac{1}{3}\left(1-\sqrt{7}\right)$, 1.22 or -0.55

f $\frac{1}{2}\left(-2+\sqrt{6}\right)$ or $\frac{1}{2}\left(-2-\sqrt{6}\right)$, 0.22 or -2.22

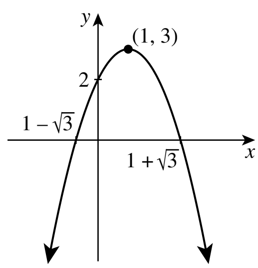
6a



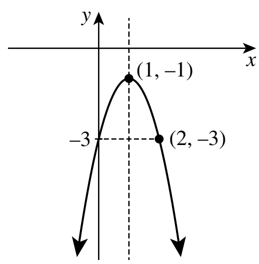
b



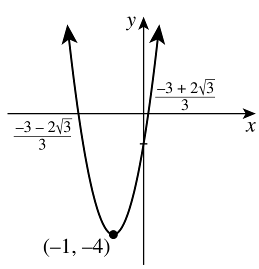
c



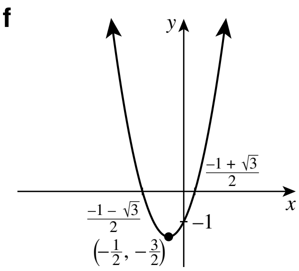
d



e



f



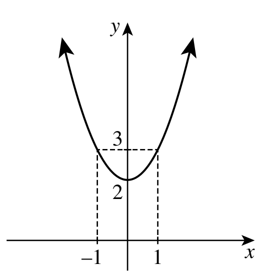
7a $x=-1, 4$ **b** $x=2, 3$ **c** $x=-2, 6$

8a i $\Delta=-8<0$

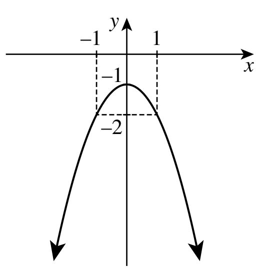
ii Both equal $(0, 2)$.

iii $(1, 3)$

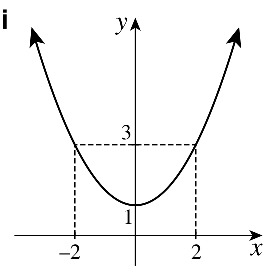
iv $(-1, 3)$



b i



ii



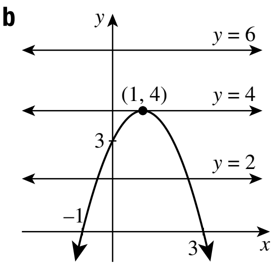
9a $\Delta=17$, two zeroes **b** $\Delta=0$, one zero

c $\Delta=-7$, no zeroes

10a i $x=1-\sqrt{2}$ and $x=-1+\sqrt{2}$

ii $x=1$ **iii** There are none.

b



c $k<4$

11a $3+2\sqrt{2}, 3-2\sqrt{2}$ **b** $1+\sqrt{5}, 1-\sqrt{5}$

c $\frac{5+\sqrt{10}}{3}, \frac{5-\sqrt{10}}{3}$

12a $\left(x-3+\sqrt{5}\right)\left(x-3-\sqrt{5}\right)$

b $\left(x+1+\sqrt{2}\right)\left(x+1-\sqrt{2}\right)$

c $\left(x-\frac{3+\sqrt{5}}{2}\right)\left(x-\frac{3-\sqrt{5}}{2}\right)$

13a Answer in question

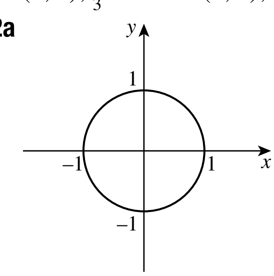
b Compare to answer to question 6

Exercise 3G

1a $(0, 0)$, 4 units **b** $(0, 0)$, 7 units

c $(0, 0)$, $\frac{1}{3}$ units **d** $(0, 0)$, 1.2 units

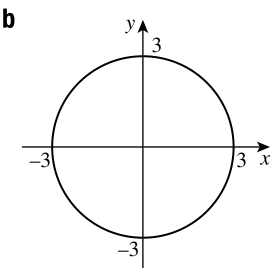
2a



$-1 \le x \le 1$

$-1 \le y \le 1$

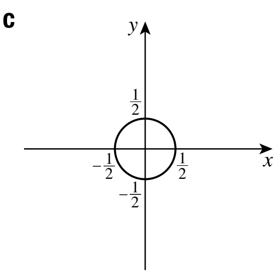
b



$-3 \le x \le 3$

$-3 \le y \le 3$

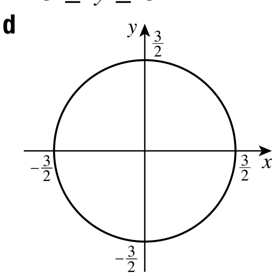
c



$-\frac{1}{2} \le x \le \frac{1}{2}$

$-\frac{1}{2} \le y \le \frac{1}{2}$

d



$-\frac{3}{2} \le x \le \frac{3}{2}$

$-\frac{3}{2} \le y \le \frac{3}{2}$

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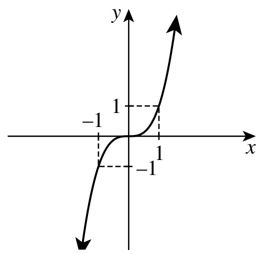
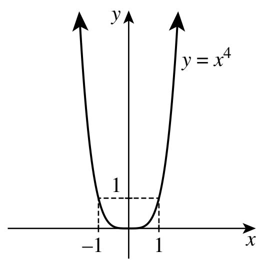
3a $-3.375, -1, -0.125, 0, 0.125, 1, 3.375$

b

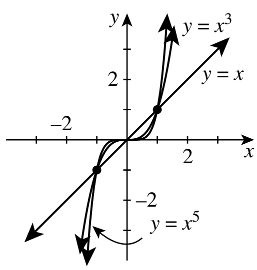


4a $5.0625, 1, 0.0625, 0, 0.0625, 1, 5.0625$

b



5a



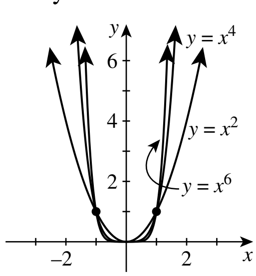
b $(-1, -1)$, $(0, 0)$ and $(1, 1)$

c i $y = x^5$ **ii** $y = x$ **d i** $y = x^5$ **ii** $y = x$

e In each case, the result is the same curve.

f Every index is odd.

6a



b $(-1, 1)$, $(0, 0)$ and $(1, 1)$

c i $y = x^6$ **ii** $y = x^2$ **d i** $y = x^6$ **ii** $y = x^2$

e In each case, the result is the same curve.

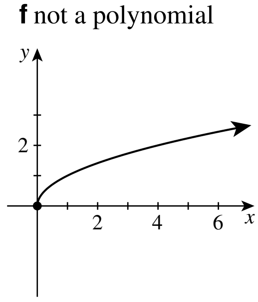
f Every index is even.

7a degree 1, coefficient 2 **b** degree 3, coefficient 0

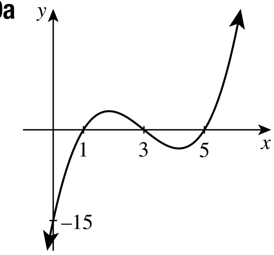
c not a polynomial **d** not a polynomial

e degree 3, coefficient -1 **f** not a polynomial

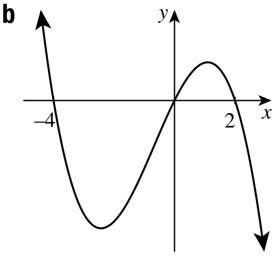
8a $0, 0.5, 1, 1.5, 23, 2.5$ **b**



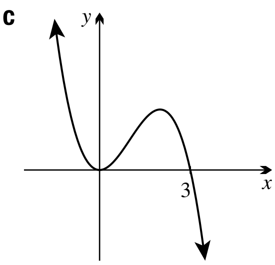
9a



b



c

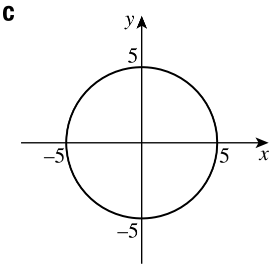


10a $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 5$ **c** $x^2 + y^2 = 25$

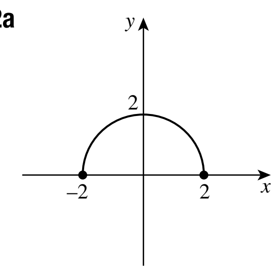
d $x^2 + y^2 = 10$

11a 5 or -5, 4.9 or -4.9, 4.6 or -4.6, 4 or -4, 3 or -3, 0

c

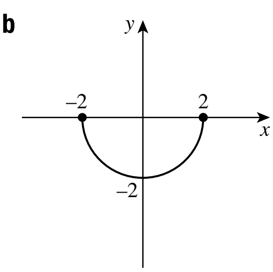


12a



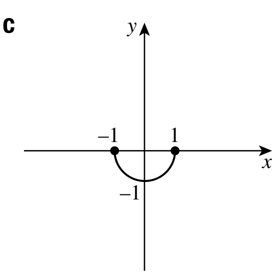
domain: $-2 \le x \le 2$,
range: $0 \le y \le 2$

b



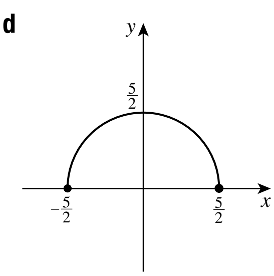
domain: $-2 \le x \le 2$,
range: $-2 \le y \le 0$

c



domain: $-1 \le x \le 1$,
range: $-1 \le y \le 0$

d



domain: $-\frac{5}{2} \le x \le \frac{5}{2}$,
range: $0 \le y \le \frac{5}{2}$

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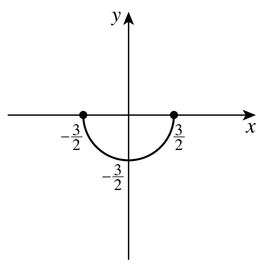
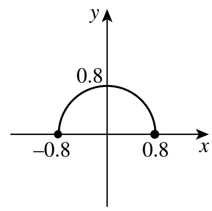
e



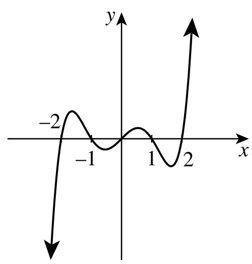
domain: $-\frac{3}{2} \le x \le \frac{3}{2}$,
range: $-\frac{3}{2} \le y \le 0$

f

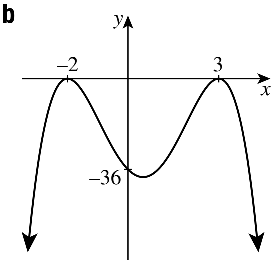


domain: $-0.8 \le x \le 0.8$,
range: $0 \le y \le 0.8$

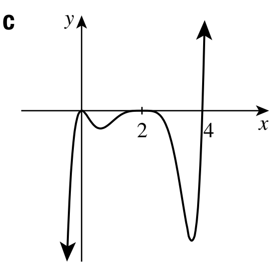
13a



b

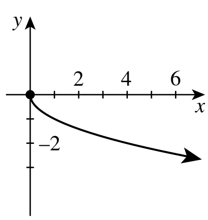


c

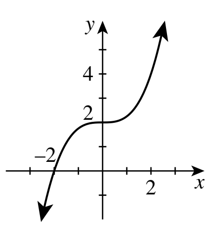


14a $0, -0.5, -1, -1.5, -2, -2.5$

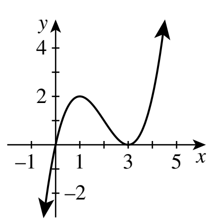
b



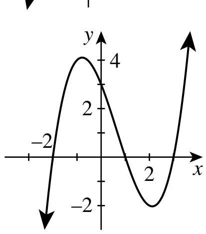
15a i



ii



iii

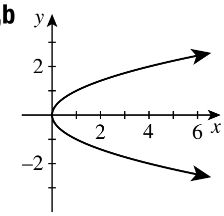


b Answer in question **c** Answer in question

16a $y = -3(x+1)(x-1)(x-4)$

b $y = -(x+1)^2(x-1)^3(x-3)^2$

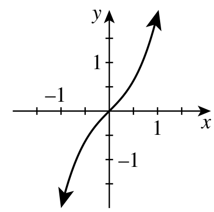
17a,b



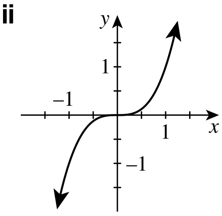
c It is a concave right parabola.

d In both cases, squaring gives $x = y^2$. This is the result of swapping x and y in $y = x^2$.

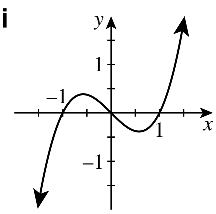
18a i



ii



iii



b i 1st and 3rd

ii In each case, the result is the same curve.

iii Every index is odd.

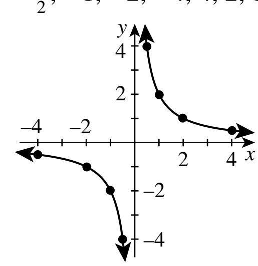
c The slope: $x^3 + x$ is upwards, x^3 is horizontal, $x^3 - x$ is downwards.

d Answers will vary

Exercise 3H

1a $-\frac{1}{2}, -1, -2, -4, 4, 2, 1, \frac{1}{2}$

b



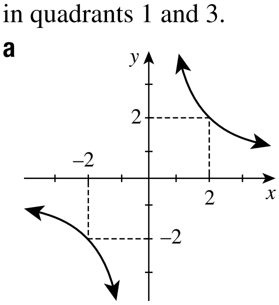
c 1st and 3rd

d the x-axis $(y = 0)$ and the y-axis $(x = 0)$

e domain: $x \ne 0$, range: $y \ne 0$

2 In each case, the domain is $x \ne 0$, the range is $y \ne 0$. The asymptotes are $y = 0$ and $x = 0$. The branches are in quadrants 1 and 3.

a



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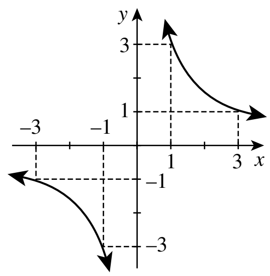
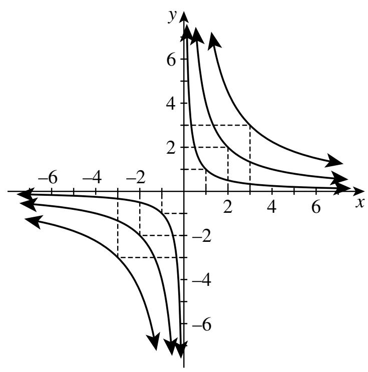
b



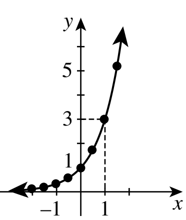
3



a 1st and 3rd
b the x-axis ($y = 0$) and the y-axis ($x = 0$)
c $x \neq 0, y \neq 0$
d $(1, 1)$ and $(-1, -1)$ on $y = \frac{1}{x}$
$(2, 2)$ and $(-2, -2)$ on $y = \frac{4}{x}$
$(3, 3)$ and $(-3, -3)$ on $y = \frac{9}{x}$
The values are the square roots of the numerator.

4a 0.1, 0.2, 0.3, 0.6, 1, 1.7, 3, 5.2, 9

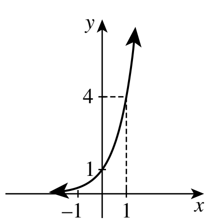
b



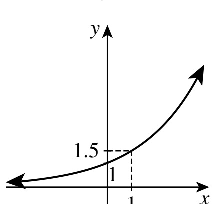
c (0, 1) **d** 3, the base **e** the x-axis ($y = 0$)
f domain: all real x, range: $y > 0$

5 In each case, the domain is all real x, the range is $y > 0$. The asymptote is $y = 0$. The y-intercept is (0, 1). At $x = 1$, $y =$ the base.

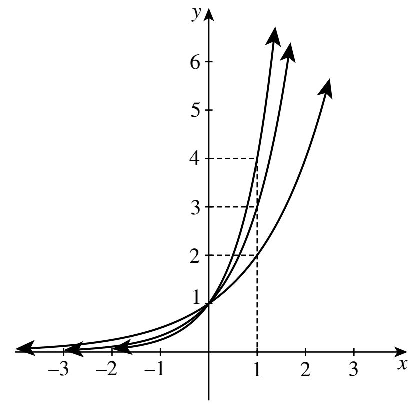
a



b

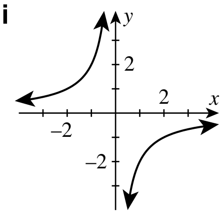


6

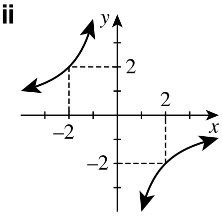


a (0, 1) **b** the x-axis ($y = 0$) **c** all real x, $y > 0$
d No answer required.
e $y = 4^x$, it has the greater base.
f $y = 4^x$ again, it has the greater base.

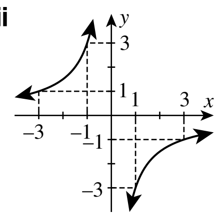
7a i



ii

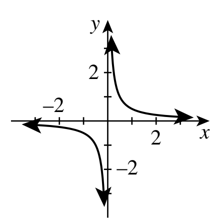


iii

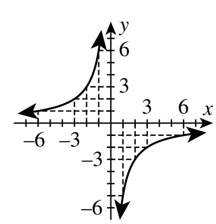


b i quadrants 2 and 4
ii The minus sign has caused the quadrants to change.

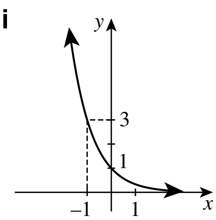
8a $y = \dfrac{1}{2x}$



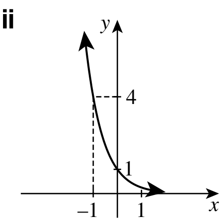
b $y = \dfrac{6}{x}$



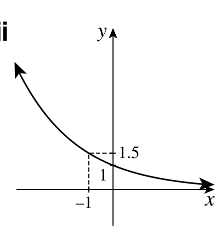
9a i



ii



iii



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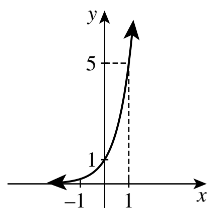
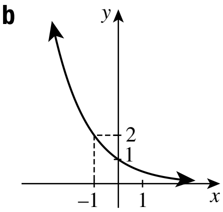
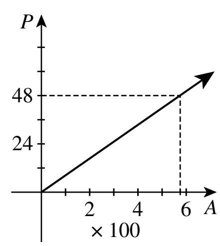
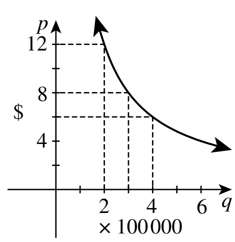
b i No: it is (0, 1). **ii** No: it is the x-axis. **iii** $x = -1$
iv In Questions 4 and 5, the y-values grow. In these questions they decay away.
v The minus sign has caused the changes.

10a


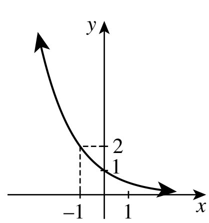
b


11a $P = kA$
b $k = \frac{1}{12}$
c $55\frac{2}{3}$L
d 1 bucket, 4 tins

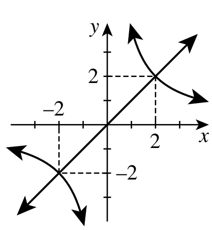


12a $T = 2400000$ **b** 300000 **c** Sales will halve.
d


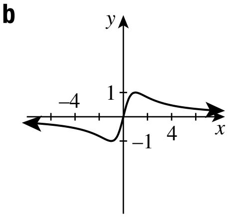
13a $y \to 0$ as $x \to -\infty$.
b $y \to 0$ as $x \to \infty$.
c $y \to 0$ as $x \to \infty$ and as $x \to -\infty$, $y \to \infty$ as $x \to 0^+$, $y \to -\infty$ as $x \to 0^-$.
14 No, because the only points that satisfy the equation lie on the x- and y-axes.

15a


b $\left(\frac{1}{2}\right)^x = \left(2^{-1}\right)^x$
so $\left(\frac{1}{2}\right)^x = 2^{-x}$

16a (c, c) and $(-c, -c)$
b


17 4 m × 12 m or 6 m × 8 m
18a $-\frac{16}{65}, -\frac{8}{17}, -\frac{4}{5}, -1, -\frac{4}{5}, 0, \frac{4}{5}, 1, \frac{4}{5}, \frac{8}{17}, \frac{16}{65}$

b


c x-axis ($y = 0$)
d (0, 0)

Exercise 3I

1a Vertical line test: Yes. It is a function.
b Horizontal line test: No. Many-to-one
c 10:00 pm on Saturday to 10:00 pm on Sunday
d 3 ft and 4 ft
e i 10:00 pm, 6:00 am, 10:30 am and 3:30 pm
ii 11:00 pm, 4:45 am and 1:00 pm
iii Never
f 0, 1, 2, 3 and 4
2a It passes the vertical line test, so it is a function. Also, it fails the horizontal line test, so it is many-to-one.
b 1°C
c It was never 20°C. It was 8°C at 01:00 am, 8:00 am and 10:30 pm on the first day, and at about 3:30 pm on the second day.
d 0, 2, 3, 4, 5 (Whether 1 is omitted depends on how accurately you are supposed to read the graph.)
3a i Vertical line test: No. Horizontal line test: Yes.
ii Vertical line test: No. Horizontal line test: No.
iii Vertical line test: Yes. Horizontal line test: No.
iv Vertical line test: No. Horizontal line test: No.
v Vertical line test: Yes. Horizontal line test: Yes.
vi Vertical line test: No. Horizontal line test: Yes.
b iii, v
c i, v, vi
d v
e i One-to-many
ii Many-to-many
iii Many-to-one
iv Many-to-many
v One-to-one
vi One-to-many
4a one-to-many **b** many-to-many
c one-to-many **d** many-to-one
e one-to-one **f** many-to-many
5a i When $y = 0$, $x = 2$ or -2
ii When $y = 3$, $x = 0$ or 6
iii When $y = 0$, $x = 1$ or 0 or -1
iv When $y = 2$, $x = 1$ or -1
b i–iv They are all one-to-many, because x and y are reversed.

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6a i $x = \frac{1}{3}y + \frac{1}{3}$ **ii** $x = -\frac{1}{2}y + \frac{5}{2}$

iii $x = \frac{1}{2}\sqrt[3]{y}$ **iv** $x = \frac{5}{y}$

b i–iv They are all one-to-one also, because x and y are reversed.

7a When $x = 3$, $y = 4$ or -6. When $y = -1$, $x = 8$ or -2

b When $x = 0$, $y = 3$ or -3. When $y = 0$, $x = 2$ or -2

c When $x = 2$, $y = \sqrt{3}$ or $-\sqrt{3}$. When $y = 0$, $x = 1$ or -1

8a It passes neither test, and is thus many-to-many.

b Vertical line test: Yes, Horizontal line test: No. It is many-to-one, and therefore a function.

9a It is a function, but it may be one-to-one or many-to-one.

b If there are two of more students with the same preferred name, it is many-to-one. Otherwise it is one-to-one.

10a …, −270°, 90°, 450°, …

b one-to-many

c many-to-one

11a Probably many-to-many, but just possibly one-to-one.

b The condition to be one-to-one is that every flat has no more than one occupant, and in this case, every inhabitant is mapped to himself, that is, $f(x) = x$, for every inhabitant x. Otherwise the relation is many-to-many.

c The relation is then the *empty relation*, which is discussed later in Section 4E. This empty relation is a one-to-one function, because it trivially passes the vertical and horizontal lines tests.

12a many-to-one

b one-to-many

c one-to-one

d one-to-one (trivially because the graph has only one point)

e many-to-many

f one-to-many (factor as $x = (y - 2)(y - 3)$)

g many-to-one (factor as $y = x(x - 3)(x - 4)$)

h one-to-one

i one-to-one

j one-to-one

k many-to-many

l one-to-one

Chapter 3 review exercise

1a not a function **b** function **c** function **d** not a function

2a $-2 \le x \le 0$, $-2 \le y \le 2$ **b** all real x, all real y

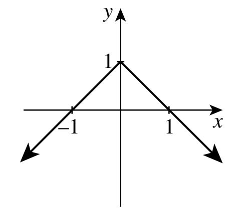
c $x \ne 0$, $y \ne 0$ **d** $x = 2$, all real y

3a 21, −4 **b** 5, −15

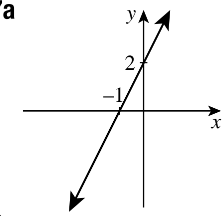
4a $x \ne 2$ **b** $x \ge 1$ **c** $x \ge -\frac{2}{3}$ **d** $x < 2$

5a $2a + 2$, $2a + 1$ **b** $a^2 - 3a - 8$, $a^2 - 5a - 3$

6 −2, −1, 0, 1, 0, −1, −2



7a



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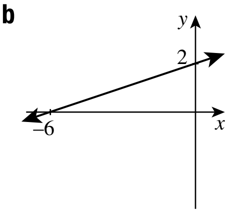
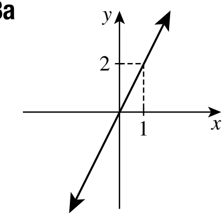


8a



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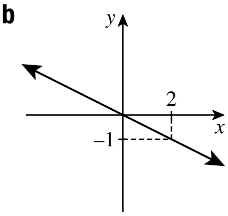
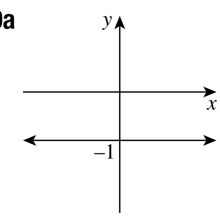


9a



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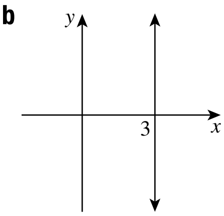
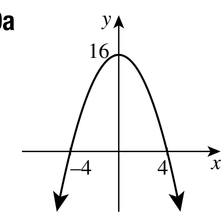
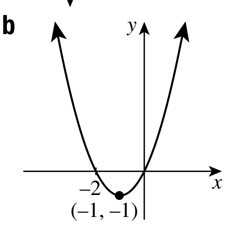


10a



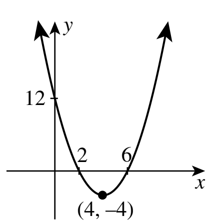
domain: all real x,
range: $y \le 16$

b



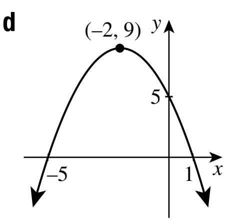
domain: all real x,
range: $y \ge -1$

c



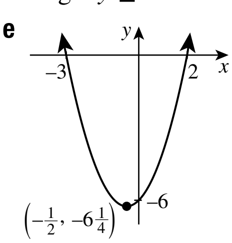
domain: all real x,
range: $y \ge -4$

d



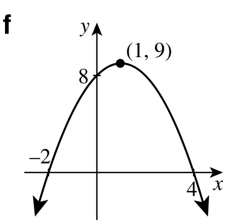
domain: all real x,
range: $y \le 9$

e



domain: all real x,
range: $y \ge -6\frac{1}{4}$

f



domain: all real x,
range: $y \le 9$

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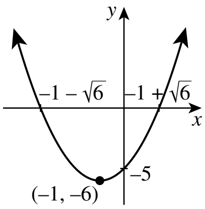
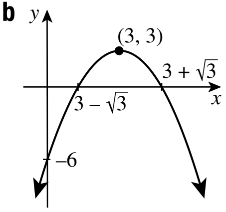
11a



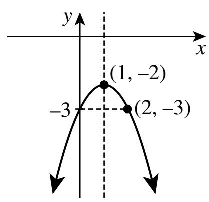
$y = (x + 1)^2 - 6$

b



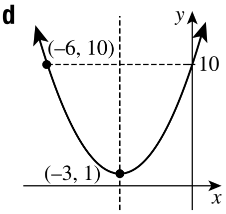
$y = -(x - 3)^2 + 3$

c



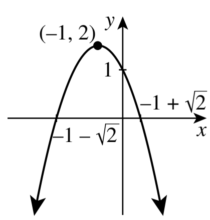
$y = -(x - 1)^2 - 2$

d

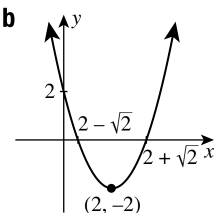


$y = (x + 3)^2 + 1$

12a



b



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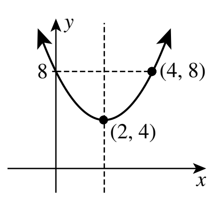

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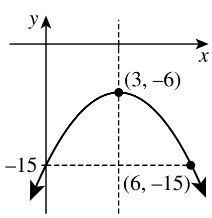
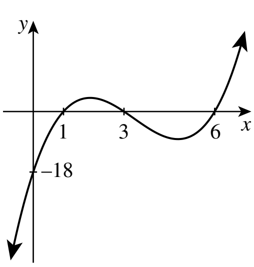


13a



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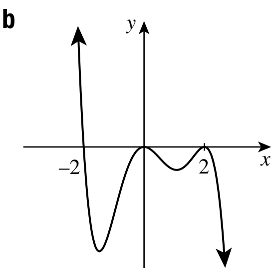
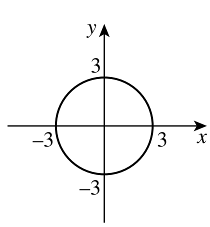
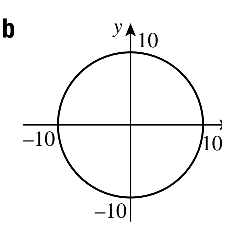


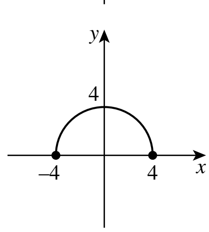
14a



b

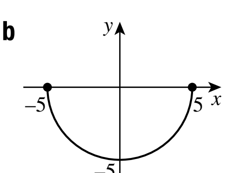


15a



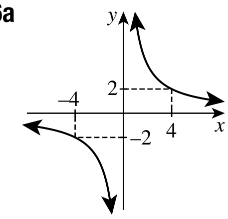
domain: $-4 \le x \le 4$,
range: $0 \le y \le 4$

b



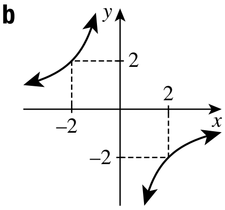
domain: $-5 \le x \le 5$,
range: $-5 \le y \le 0$

16a



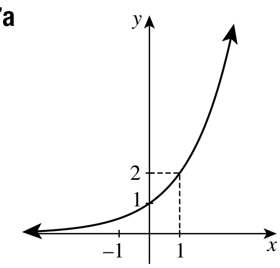
domain: $x \neq 0$,
range: $y \neq 0$

b



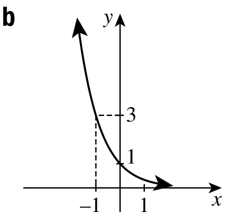
domain: $x \neq 0$,
range: $y \neq 0$

17a



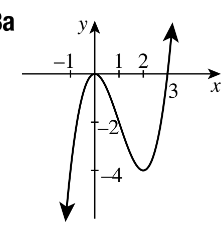
domain: all real x,
range: $y > 0$

b



domain: all real x,
range: $y > 0$

18a



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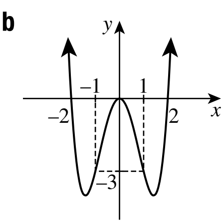

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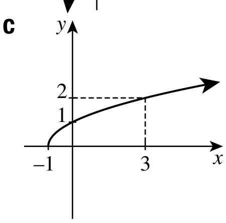
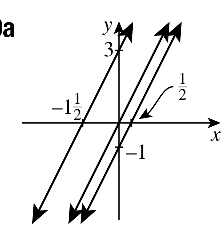


19a



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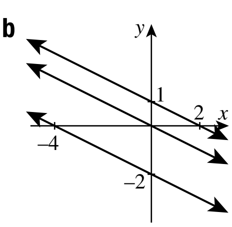

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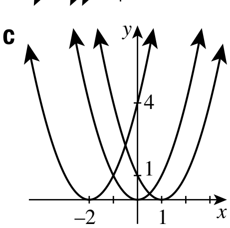

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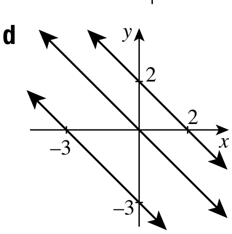

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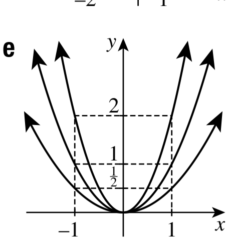

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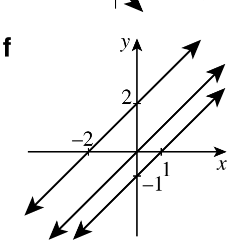

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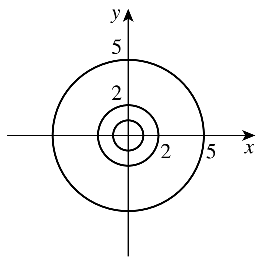
g



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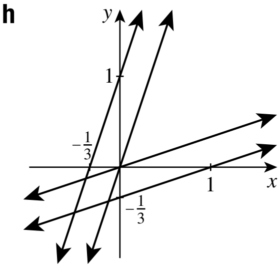

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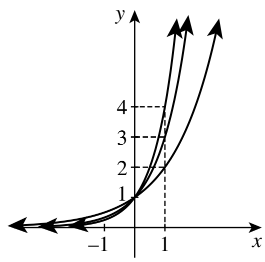

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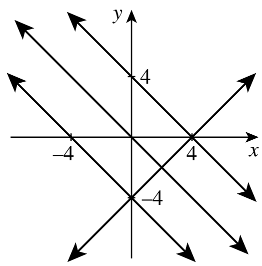

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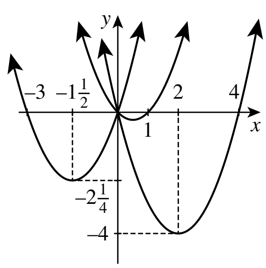

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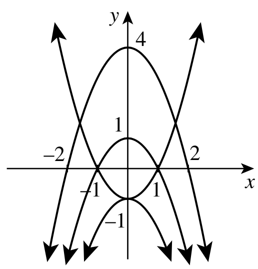

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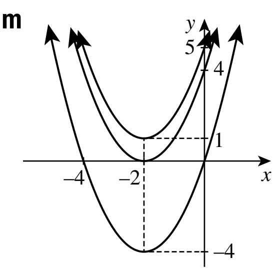

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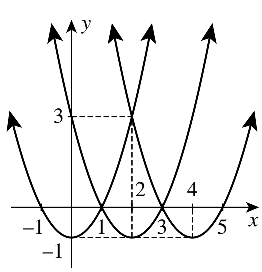

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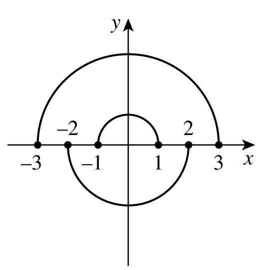

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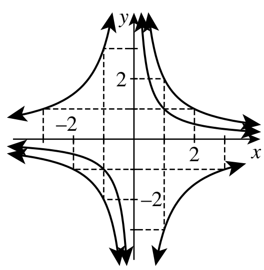

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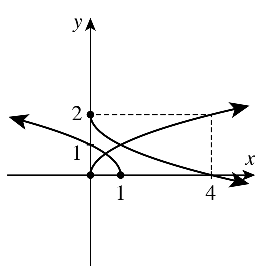

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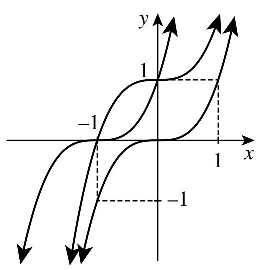

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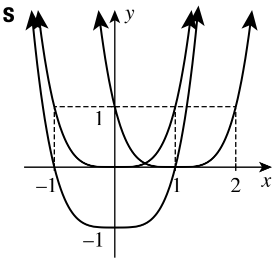

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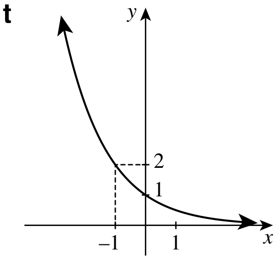


All three are the same.

20a one-to-one **b** many-to-many
c one-to-many **d** many-to-one

21a It is probably a many-to-one function, but it is possibly a one-to-one function

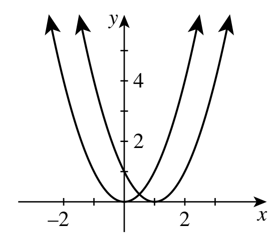
b If every person was born in a different country, the function is one-to-one. Otherwise it is many-to-one.

Chapter 4

Exercise 4A

1a x^2: 4, 1, 0, 1, 4, 9
$(x-1)^2$: 9, 4, 1, 0, 1, 4

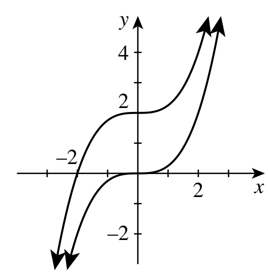
b $y = x^2$, $V = (0, 0)$
$y = (x-1)^2$, $V = (1, 0)$



c Here x is replaced by $(x-1)$, so it is a shift right by 1 unit.

2a $\frac{1}{4}x^3$: $-6\frac{3}{4}$, -2, $-\frac{1}{4}$, 0, $\frac{1}{4}$, 2, $6\frac{3}{4}$
$\left(\frac{1}{4}x^3 + 2\right)$: $-4\frac{3}{4}$, 0, $1\frac{3}{4}$, 2, $2\frac{1}{4}$, 4, $8\frac{3}{4}$

b (0, 0) and (0, 2)

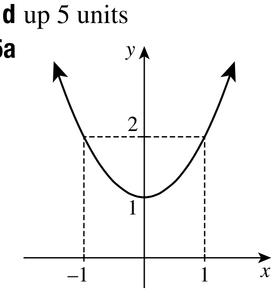


c The second equation is also $y - 2 = \frac{1}{4}x^3$. Here y is replaced by $(y-2)$, so it is a shift up by 2 units.

3a up 2 units **b** down 5 units **c** left 4 units
d right 3 units

4a right 2 units **b** left 3 units **c** down 4 units
d up 5 units

5a



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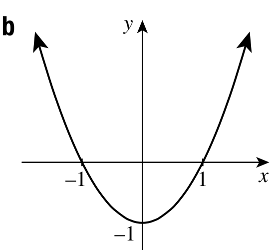

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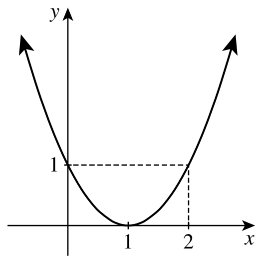
c



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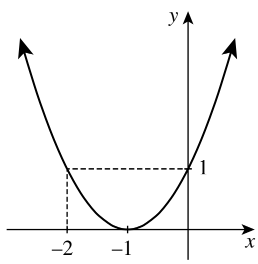
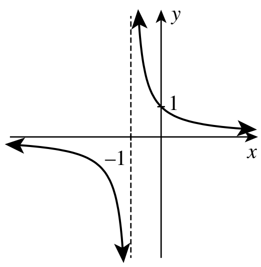


6a



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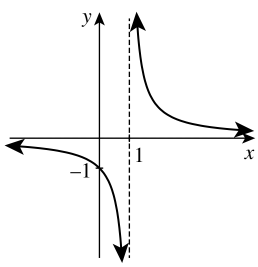

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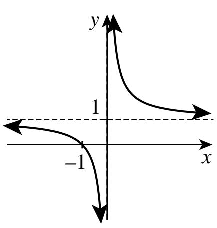

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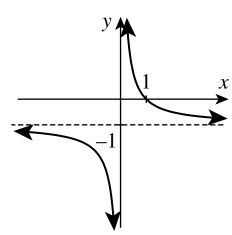
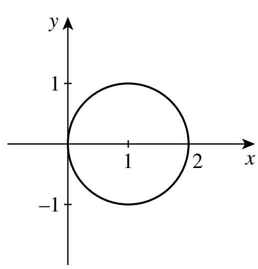


7a



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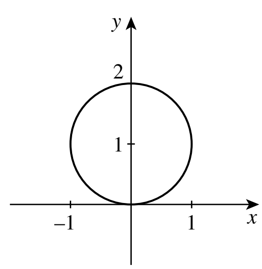
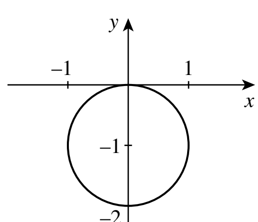


c



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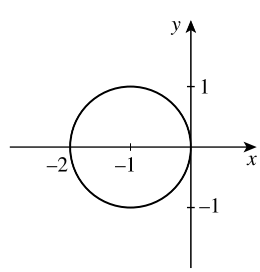
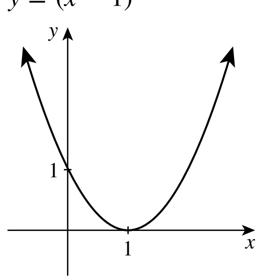
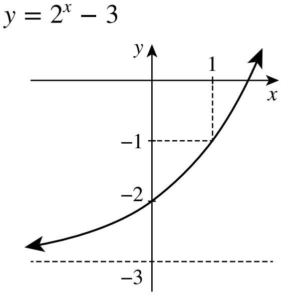


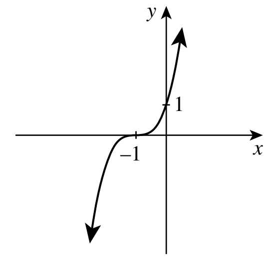
8a $y = (x - 1)^2$



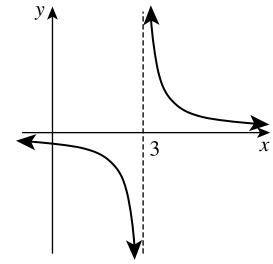
b $y = 2^x - 3$



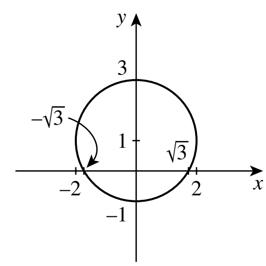
c $y = (x + 1)^3$



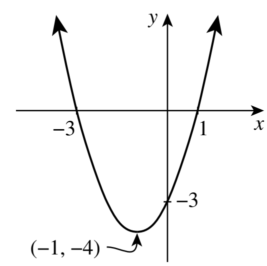
d $y = \dfrac{1}{x - 3}$



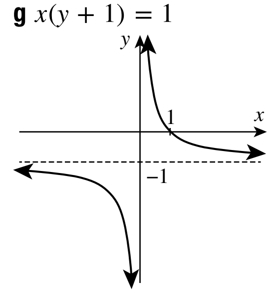
e $x^2 + (y - 1)^2 = 4$



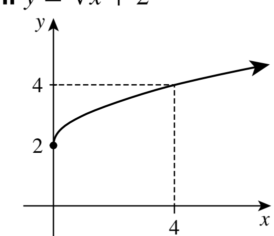
f $y = (x + 1)^2 - 4$



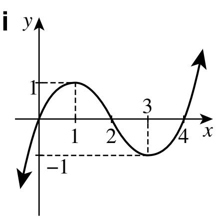
g $x(y + 1) = 1$



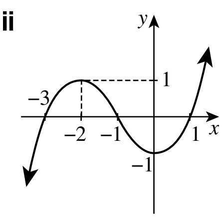
h $y = \sqrt{x} + 2$



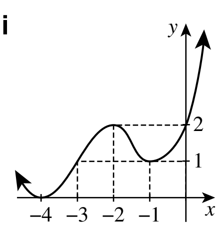
9a i



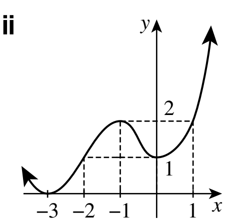
ii



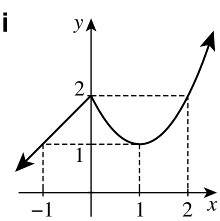
b i



ii



c i



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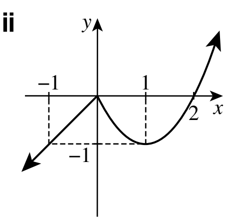
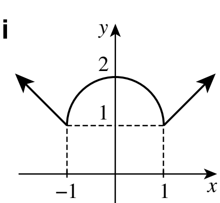


d i



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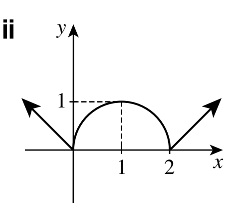

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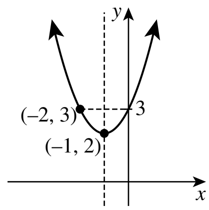
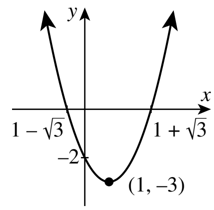
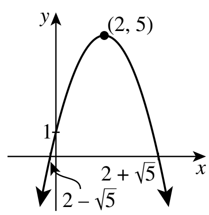
10a $y = (x+1)^2 + 2$
This is $y = x^2$ shifted left 1 and up 2.



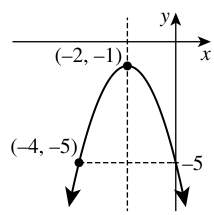
b $y = (x-1)^2 - 3$
This is $y = x^2$ shifted right 1 and down 3.



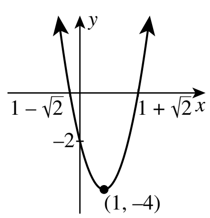
c $y = -(x-2)^2 + 5$
This is $y = -x^2$ shifted right 2 and up 5.



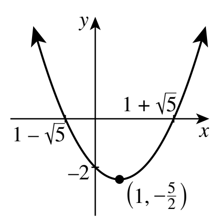
d $y = -(x+2)^2 - 1$
This is $y = -x^2$ shifted left 2 and down 1.



e $y = 2(x-1)^2 - 4$
This is $y = 2x^2$ shifted right 1 and down 4.



f $y = \frac{1}{2}(x-1)^2 - \frac{5}{2}$
This is $y = \frac{1}{2}x^2$ shifted right 1 and down $\frac{5}{2}$.

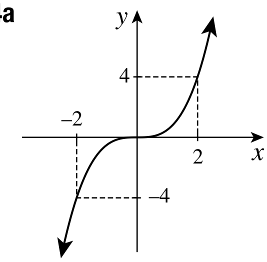


11a the parabola $y = x^2$ translated right 2, $y = (x-2)^2$
b the hyperbola $xy = 1$ translated right 2, $(x-2)y = 1$ or $y = \frac{1}{x-2}$
c the parabola $y = x^2$ translated right 2, down 1, $y + 1 = (x-2)^2$
d the hyperbola $xy = 1$ translated right 2, down 1, $(x-2)(y+1) = 1$ or $y + 1 = \frac{1}{x-2}$

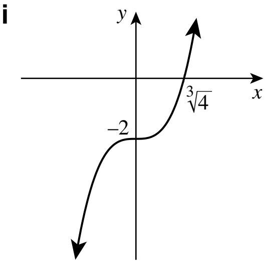
12a $r = 2$, $(-1, 0)$ **b** $r = 1$, $(1, 2)$
c $r = 3$, $(1, 2)$ **d** $r = 5$, $(-3, 4)$
e $r = 3$, $(5, -4)$ **f** $r = 6$, $(-7, 1)$

13a the circle $x^2 + y^2 = 1$ translated right 2, up 3, $(x-2)^2 + (y-3)^2 = 1$
b the circle $x^2 + y^2 = 4$ translated left 2, down 1, $(x+2)^2 + (y+1)^2 = 4$
c the circle $x^2 + y^2 = 10$ translated left 1, up 1, $(x+1)^2 + (y-1)^2 = 10$
d the circle $x^2 + y^2 = 5$ translated right 2, down 1, $(x-2)^2 + (y+1)^2 = 5$

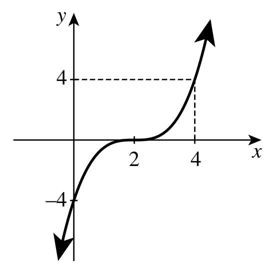
14a



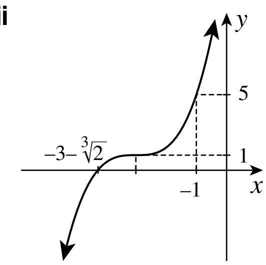
i



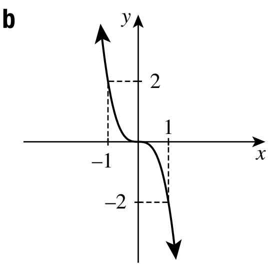
ii



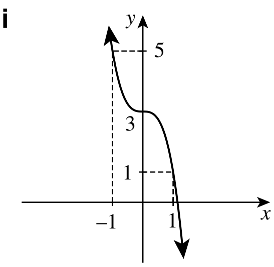
iii



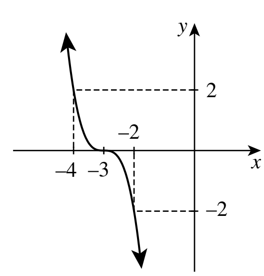
b



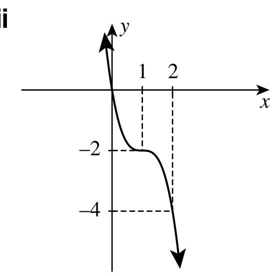
i



ii

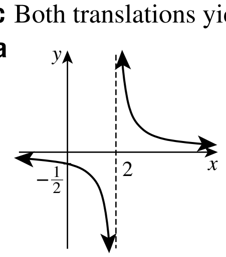


iii

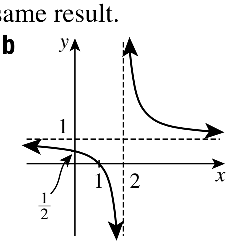


15a $x + 2y - 2 = 0$ **b** $x + 2y - 2 = 0$
c Both translations yield the same result.

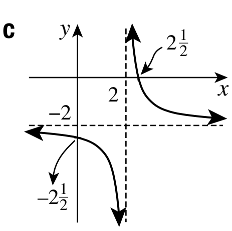
16a



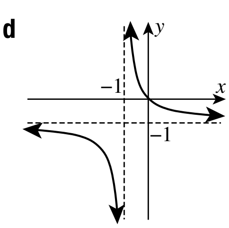
b



c



d



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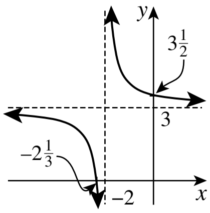
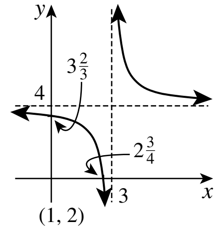
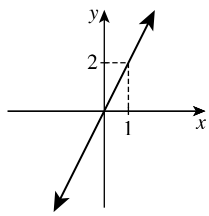
e



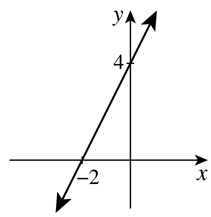
f



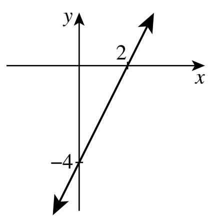
17a From $y = 2x$:



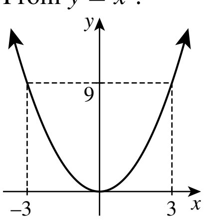
i shift up 4 (or left 2)



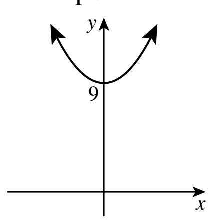
ii shift down 4 (or right 2)



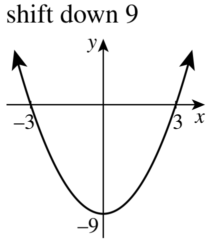
b From $y = x^2$:



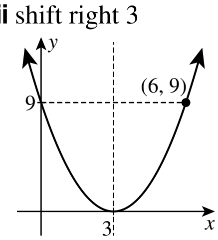
i shift up 9



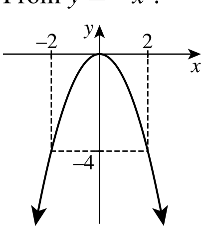
ii shift down 9



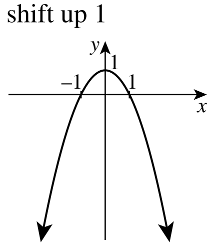
iii shift right 3



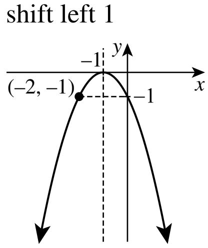
c From $y = -x^2$:



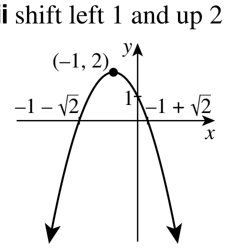
i shift up 1



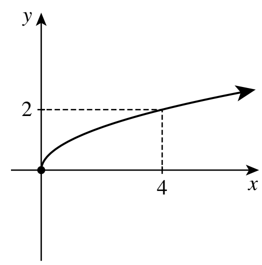
ii shift left 1



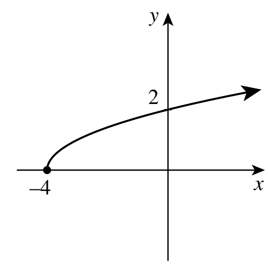
iii shift left 1 and up 2



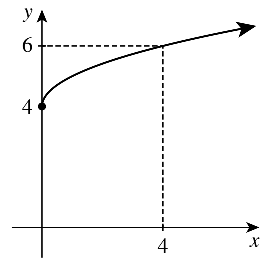
d From $y = \sqrt{x}$:



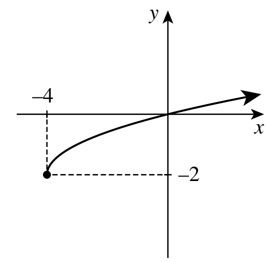
i shift left 4



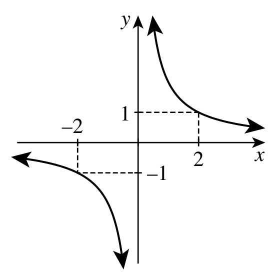
ii shift up 4



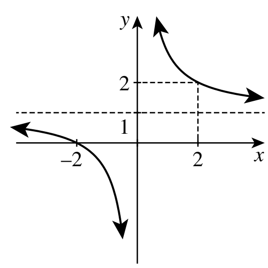
iii shift left 4 and down 2



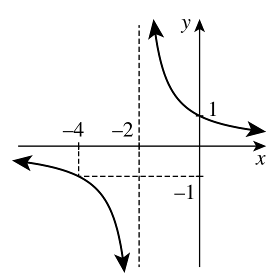
e From $y = \dfrac{2}{x}$:



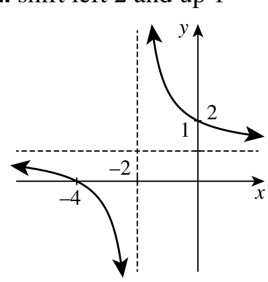
i shift up 1



ii shift left 2



iii shift left 2 and up 1



18a $(x - h)^2 + (y - k)^2 = r^2$

b Answer same as 18a

Exercise 4B

1a Answers will vary

b $y = x^2 - 2x$: 8, 3, 0, −1, 0, 3, 8

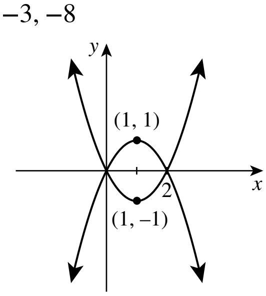
$y = 2x - x^2$: −8, −3, 0, 1, 0, −3, −8

c $y = x^2 - 2x$:

$V = (1, -1)$

$y = 2x - x^2$: $V = (1, 1)$

d Here y is replaced with $-y$, so it is a reflection in the x-axis.



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2a Answers will vary

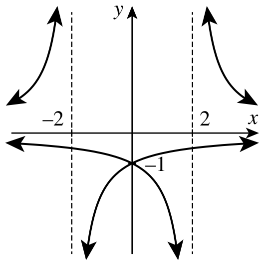
b $y = \frac{2}{x-2}$: $-\frac{1}{3}, -\frac{2}{5}, -\frac{1}{2}, -\frac{2}{3}, -1, -2, *, 2, 1$

$y = -\frac{2}{x+2}$: $1, 2, *, -2, -1, -\frac{2}{3}, -\frac{1}{2}, -\frac{2}{5}, -\frac{1}{3}$

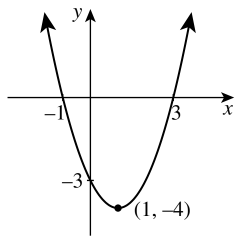
c $y = \frac{2}{x-2}$: $x = 2$

$y = -\frac{2}{x+2}$: $x = -2$

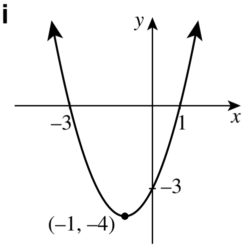
d Here x is replaced with $-x$, so it is a reflection in the y-axis.



3a

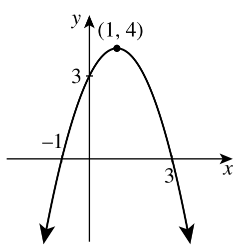


b i



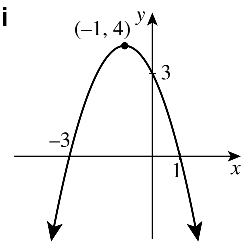
$y = x^2 + 2x - 3$

ii



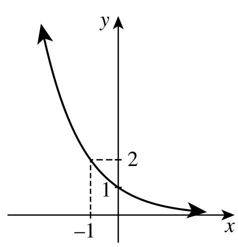
$y = -x^2 + 2x + 3$

iii

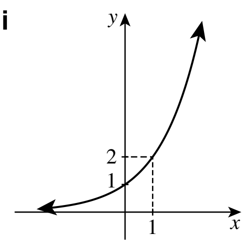


$y = -x^2 - 2x + 3$

4a

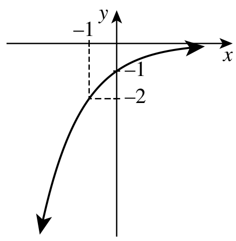


b i



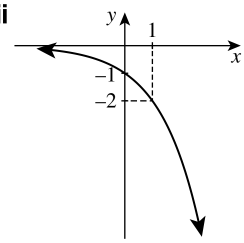
$y = 2^x$

ii



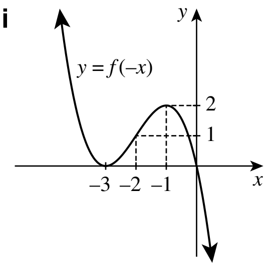
$y = -2^{-x}$

iii

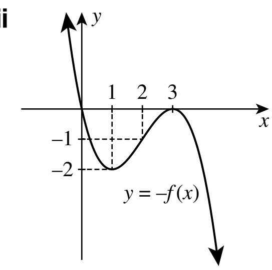


$y = -2^x$

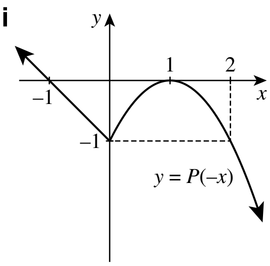
5a i



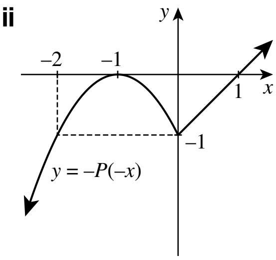
ii



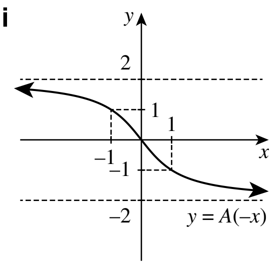
b i



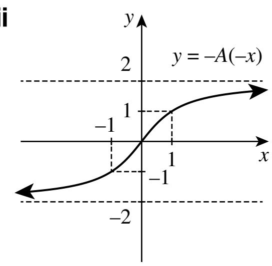
ii



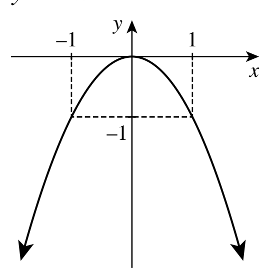
c i



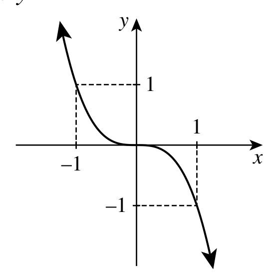
ii



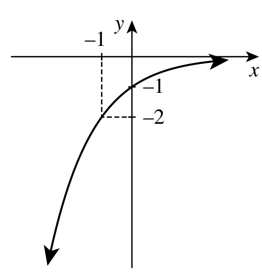
6a $y = -x^2$



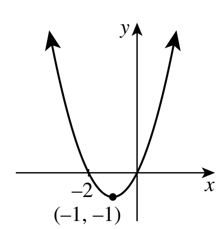
b $y = -x^3$



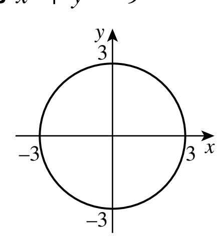
c $y = -2^{-x}$



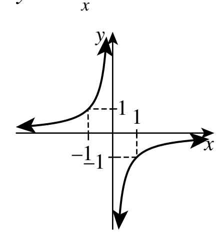
d $y = x^2 + 2x$



e $x^2 + y^2 = 9$



f $y = -\frac{1}{x}$



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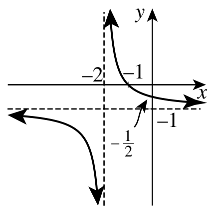
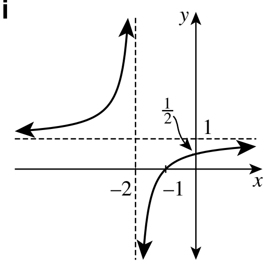
7a

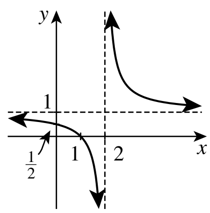


b i



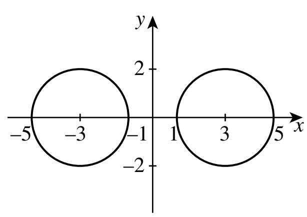
Reflect in the x-axis: $y = 1 - \frac{1}{x+2}$

ii



Rotate by180°: $y = 1 - \frac{1}{2-x}$

8a



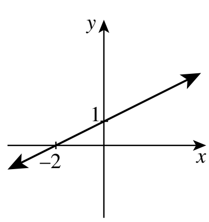
b Reflect in the y-axis.

c You will need to use $(-x-3)^2 = (x+3)^2$.

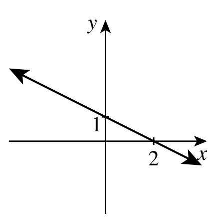
9a Answers will vary

b The circle is symmetric in both axes.

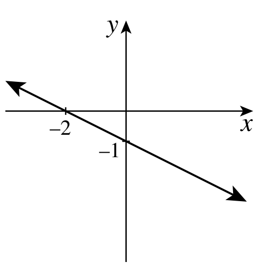
10a From $y = \frac{1}{2}x + 1$:



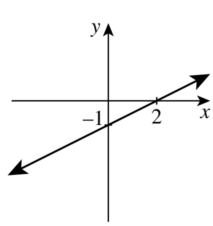
i reflect in the y-axis



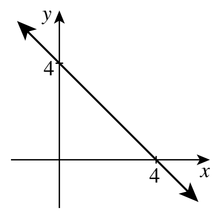
ii reflect in the x-axis



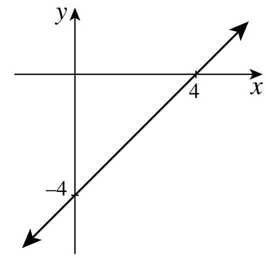
iii rotate by 180°



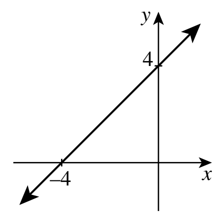
b From $y = 4 - x$:



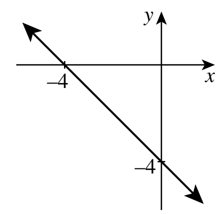
i reflect in the x-axis



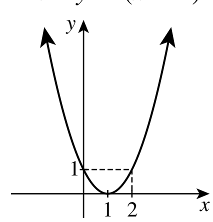
ii reflect in the y-axis



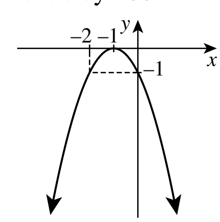
iii rotate by 180°



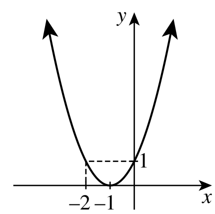
c From $y = (x-1)^2$:



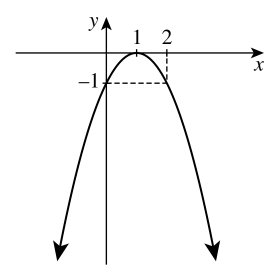
i rotate by 180°



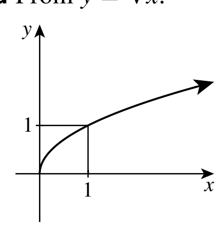
ii reflect in the y-axis



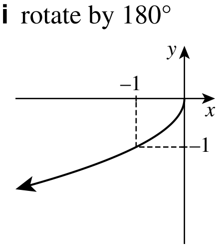
iii reflect in the x-axis



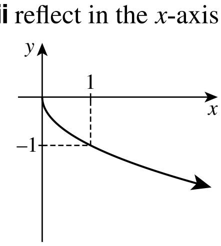
d From $y = \sqrt{x}$:



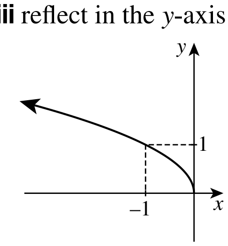
i rotate by 180°



ii reflect in the x-axis



iii reflect in the y-axis



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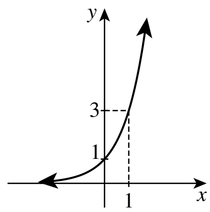
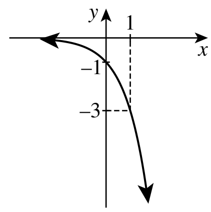
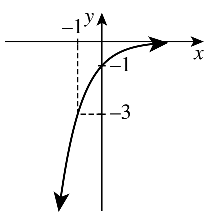
e From $y = 3^x$:



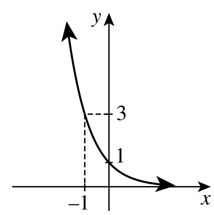
i reflect in the x-axis



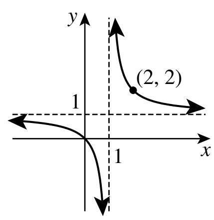
ii rotate by 180°



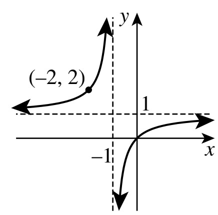
iii reflect in the y-axis



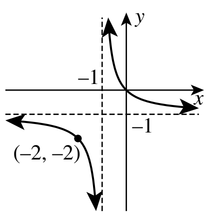
f From $y = 1 + \frac{1}{x - 1}$:



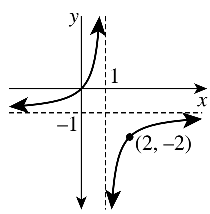
i reflect in the y-axis



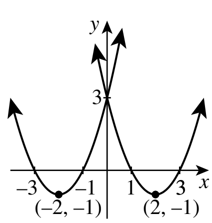
ii rotate by 180°



iii reflect in the x-axis

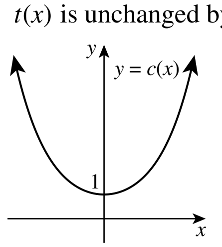


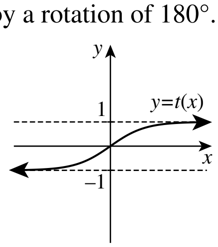
11a



b Reflect in the y-axis. **c** Shift left 4 units.
d $(x + 4)^2 - 4(x + 4) + 3 = x^2 + 4x + 3$
e a iii, b iii, c ii, f ii

12a $c(x)$ is the same when reflected in the y-axis.
b $t(x)$ is unchanged by a rotation of 180°.
c



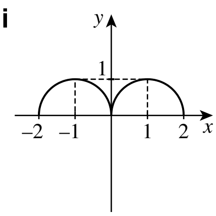


13a i $y = (x - 2)^2$ **ii** $y = (x + 2)^2$
b i $y = (x + 1)^2$ **ii** $y = x^2$
c Yes: the answer depends on the order.
d The order is irrelevant when the shift is parallel with the axis of reflection.

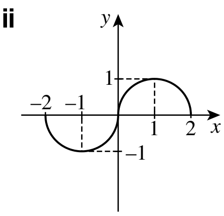
Exercise 4C

1a even **b** neither **c** odd **d** neither **e** odd **f** even

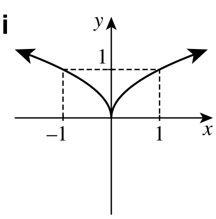
2a i



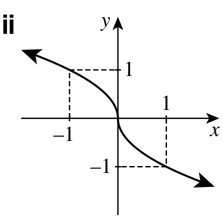
ii



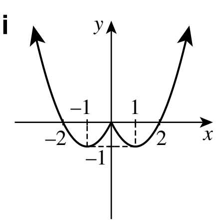
b i



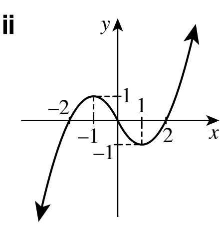
ii



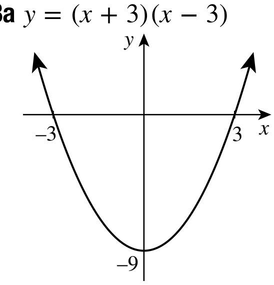
c i

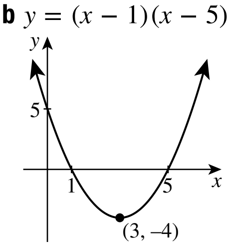


ii



3a $f(-x) = x^4 - 2x^2 + 1$
b $f(-x) = f(x)$, so it is even.
4a $g(-x) = -x^3 + 3x$
b $-g(x) = -(x^3 - 3x) = g(-x)$, so it is odd.
5a $h(-x) = -x^3 + 3x^2 - 2$
b $-h(x) = -x^3 - 3x^2 + 2$. Since $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$, it is neither.
6a even **b** neither **c** odd **d** even **e** neither **f** odd **g** odd **h** neither
7a … if all powers of x are odd.
b … if all powers of x are even.
8a $y = (x + 3)(x - 3)$ **b** $y = (x - 1)(x - 5)$





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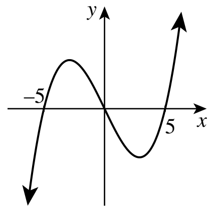
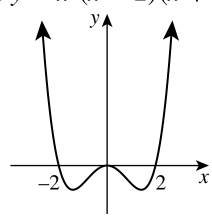
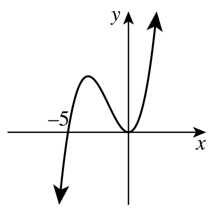
c $y = x(x - 5)(x + 5)$



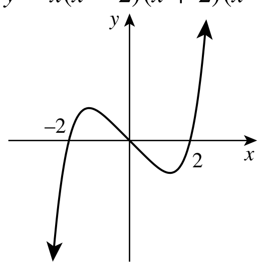
d $y = x^2(x - 2)(x + 2)$



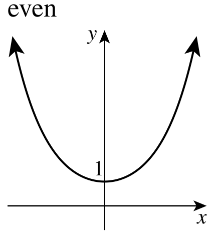
e $y = x^2(x + 5)$



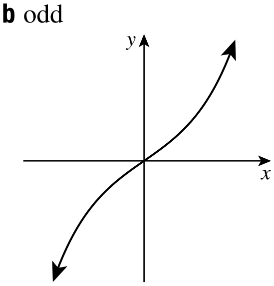
f $y = x(x - 2)(x + 2)(x^2 + 4)$



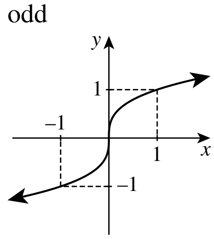
9a even



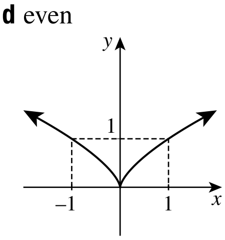
b odd



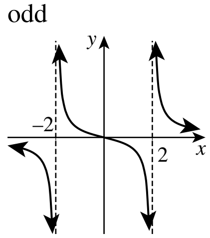
c odd



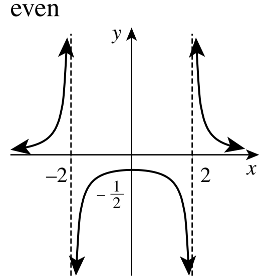
d even



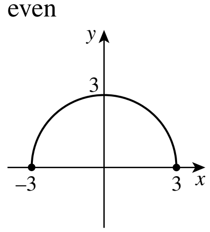
e odd



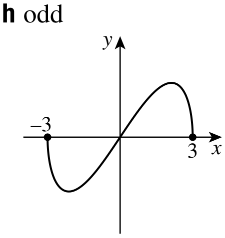
f even



g even



h odd



10a neither **b** neither **c** even **d** even **e** odd
f even **g** odd **h** neither

11a Either show that the equation is unchanged when x is replaced by $-x$. Or use the fact that the circle graph has line symmetry in the y-axis.

b Either show that the equation is unchanged when x is replaced by $-x$, and when x and y are replaced by $-x$ and $-y$. Or use the fact that that the circle graph has line symmetry in the x-axis and in the y-axis.

12a Suppose $f(0) = c$. Then since $f(x)$ is odd, $f(0) = -f(0) = -c$. So $c = -c$, and hence $c = 0$.

b No. A counter-example is $y = x^2 + 1$.

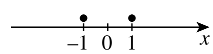
13a i–ii Answers will vary

b i–ii Answers will vary

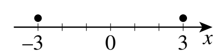
Exercise 4D

1a 3 **b** 3 **c** 3 **d** 3 **e** 7 **f** 1 **g** 16 **h** –3

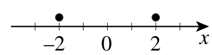
2a $x = 1$ or -1



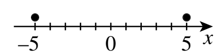
b $x = 3$ or -3



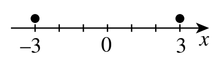
c $x = 2$ or -2



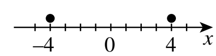
d $x = 5$ or -5



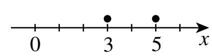
e $x = -3$ or 3



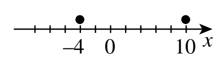
f $x = -4$ or 4



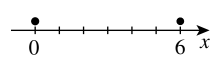
3a $x = 3$ or 5



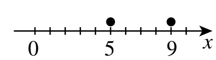
b $x = -4$ or 10



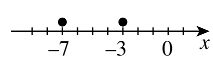
c $x = 0$ or 6



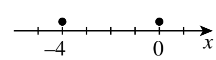
d $x = 5$ or 9



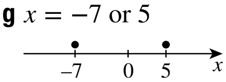
e $x = -3$ or -7



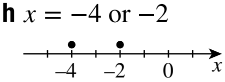
f $x = -4$ or 0



g $x = -7$ or 5

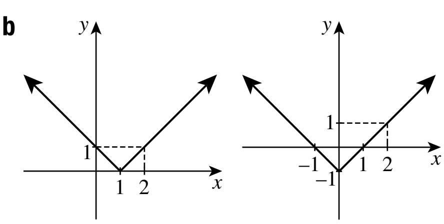


h $x = -4$ or -2



4a For $|x - 1|$: 3, 2, 1, 0, 1, 2
For $|x| - 1$: 1, 0, –1, 0, 1, 2

b



The two graphs overlap for $x > 1$.

c The first is $y = |x|$ shifted right 1 unit, the second is $y = |x|$ shifted down 1 unit.

5a LHS = RHS = 15 **b** LHS = RHS = 3
c LHS = RHS = 9 **d** LHS = RHS = 10
e $-3 < 3$ **f** $-3 \le -3$

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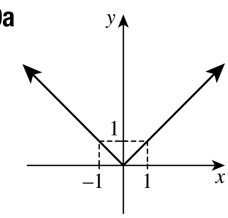
6a LHS = 2, RHS = −2 **b** LHS = 2, RHS = −2
c LHS = 0, RHS = 4 **d** LHS = 1, RHS = −1
e LHS = 3, RHS = 1 **f** LHS = 8, RHS = −8

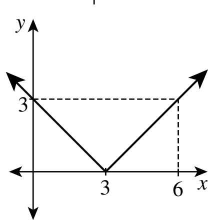
7a $x = 5$ or -5 **b** $x = -2$ or 1 **c** $x = 6$ or -5
d no solution **e** no solution **f** $x = -\frac{2}{5}$ **g** $x = \frac{5}{3}$
h $x = \frac{1}{3}$ or 2 **i** $x = -2$ or $\frac{2}{5}$

8a i $|1 - 2x| = |2x - 1|$ **ii** $x = -1$ or 2
b i $x = 1$ or 2 **ii** $x = -\frac{1}{3}$ or 1

9a

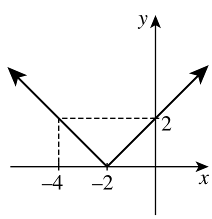


b i



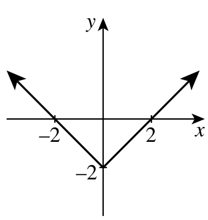
shift right 3,
$y = \begin{cases} x - 3, & \text{for } x \ge 3, \\ 3 - x, & \text{for } x < 3. \end{cases}$

ii



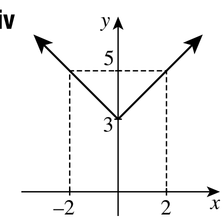
shift left 2,
$y = \begin{cases} x + 2, & \text{for } x \ge -2, \\ -x - 2, & \text{for } x < -2. \end{cases}$

iii



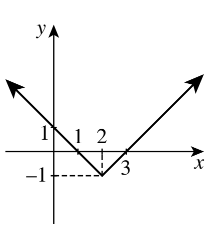
shift down 2,
$y = \begin{cases} x - 2, & \text{for } x \ge 0, \\ -x - 2, & \text{for } x < 0. \end{cases}$

iv



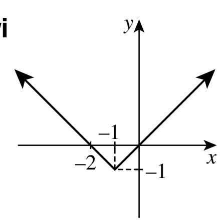
shift up 3,
$y = \begin{cases} x + 3, & \text{for } x \ge 0, \\ 3 - x, & \text{for } x < 0. \end{cases}$

v



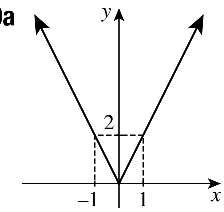
shift right 2, down 1,
$y = \begin{cases} x - 3, & \text{for } x \ge 2, \\ 1 - x, & \text{for } x < 2. \end{cases}$

vi



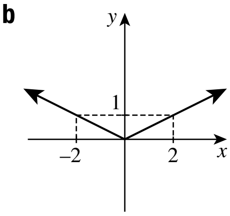
shift left 1, down 1,
$y = \begin{cases} x, & \text{for } x \ge -1, \\ -x - 2, & \text{for } x < -1. \end{cases}$

10a



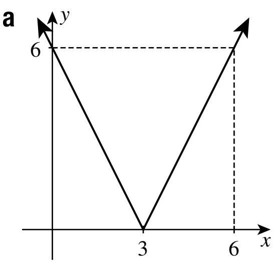
$y = \begin{cases} 2x, & \text{for } x \ge 0, \\ -2x, & \text{for } x < 0. \end{cases}$

b

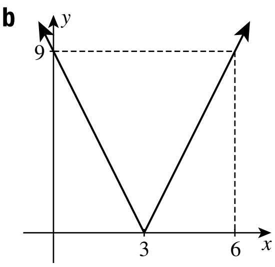


$y = \begin{cases} \frac{1}{2}x, & \text{for } x \ge 0, \\ -\frac{1}{2}x, & \text{for } x < 0. \end{cases}$

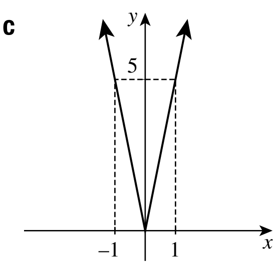
11a



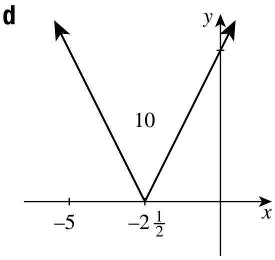
b



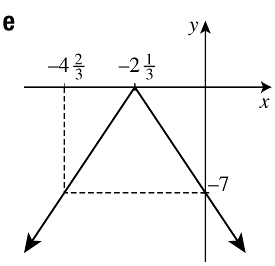
c



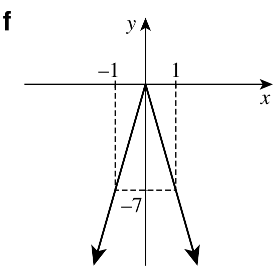
d



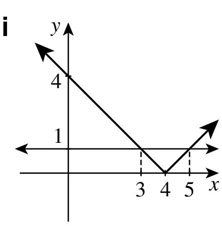
e



f

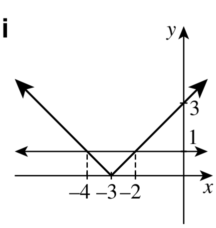


12a i



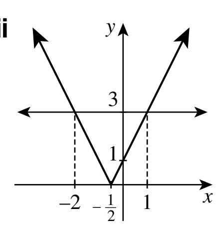
ii The x-coordinates of the points of intersection give: $x = 3$ or 5

b i



$x = -4$ or -2

ii



$x = -2$ or 1

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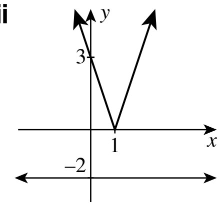
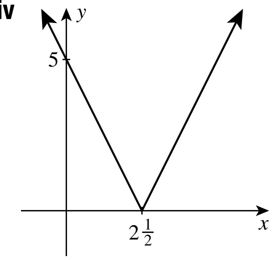
iii



no solution

iv



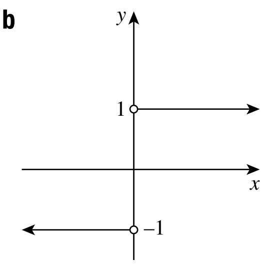
$x = 2\frac{1}{2}$

13a Answers will vary

b The graph is symmetric in the y-axis.

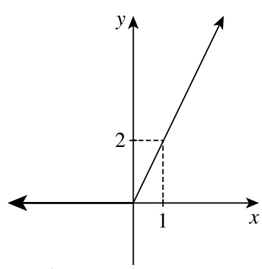
14a $x = 0$

b



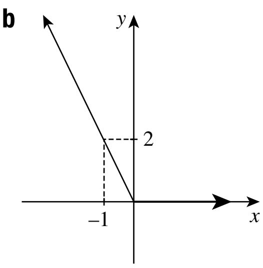
c $y = \begin{cases} 1, & \text{for } x \geq 0, \\ -1, & \text{for } x < 0. \end{cases}$

15a



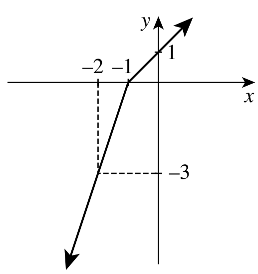
$y = \begin{cases} 2x, & \text{for } x \geq 0, \\ 0, & \text{for } x < 0. \end{cases}$

b



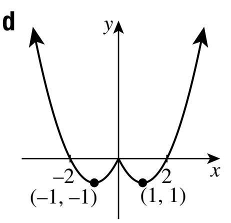
$y = \begin{cases} 0, & \text{for } x \geq 0, \\ -2x, & \text{for } x < 0. \end{cases}$

c



$y = \begin{cases} x + 1, & \text{for } x \geq -1, \\ 3x + 3, & \text{for } x < -1. \end{cases}$

d



$y = \begin{cases} x^2 - 2x, & \text{for } x \geq 0, \\ x^2 + 2x, & \text{for } x < 0. \end{cases}$

Exercise 4E

1a i 4 **ii** 7 **iii** 3 **iv** −4 **b i** $x + 4$ **ii** $x + 6$ **c** $x = -4$

2a $F(F(0)) = 0$, $F(F(7)) = 28$,
$F(F(-3)) = -12$, $F(F(-11)) = -44$

b $F(F(x)) = 4x$, $F(F(F(x))) = 8x$ **c** $x = 8$

3a $g(g(0)) = 0$, $g(g(4)) = 4$, $g(g(-2)) = -2$,
$g(g(-9)) = -9$

b $g(g(x)) = 2 - (2 - x) = x$

c $g(g(g(x))) = g(x)$

4a $h(h(0)) = -20$, $h(h(5)) = 25$,
$h(h(-1)) = -29$, $h(h(-5)) = -65$

b $h(h(x)) = 9x - 20$, $h(h(h(x))) = 27x - 65$

5a $f(g(7)) = 12$, $g(f(7)) = 13$, $f(f(7)) = 9$,
$g(g(7)) = 19$

b i $2x - 2$ **ii** $2x - 1$ **iii** $x + 2$ **iv** $4x - 9$

c Shift 1 unit to the left (or shift two units up).

d Shift 1 unit up (or shift $\frac{1}{2}$ left).

6a $\ell(q(-1)) = -2$, $q(\ell(-1)) = 16$,
$\ell(\ell(-1)) = -7$, $q(q(-1)) = 1$

b i $x^2 - 3$ **ii** $(x - 3)^2$ **iii** $x - 6$ **iv** x^4

c i Domain: all real x, range: $y \geq -3$

ii Domain: all real x, range: $y \geq 0$

d It is shifted 3 units to the right.

e It is shifted 3 units down.

7a $F(G(25)) = 20$, $G(F(25)) = 10$,
$F(F(25)) = 400$, $G(G(25)) = \sqrt{5}$

b $4\sqrt{x}$ **c** $\sqrt{4x} = 2\sqrt{x}$

d Answers will vary **e** Domain: $x \geq 0$, range: $y \geq 0$

8a $f\left(h\left(-\frac{1}{4}\right)\right) = 4$, $h\left(f\left(-\frac{1}{4}\right)\right) = 4$,
$f\left(f\left(-\frac{1}{4}\right)\right) = -\frac{1}{4}$, $h\left(h\left(-\frac{1}{4}\right)\right) = -\frac{1}{4}$

b i Both sides equal $-\frac{1}{x}$, for all $x \neq 0$.

ii Both sides equal x, for all $x \neq 0$.

c Domain: $x \neq 0$, range $y \neq 0$.

d It is reflected in the y-axis (or in the x-axis).

9a $f(g(x)) = -5 - \sqrt{x}$.
Domain: $x \geq 0$, range: $y \leq -5$. Take the graph of $y = \sqrt{x}$, reflect it in the y-axis, then shift down 5.

b $f(x) = -5 - |x|$, which is negative for all x, so $g(f(x)) = \sqrt{-5 - |x|}$ is never defined.

10a $g(f(-x)) = g(-f(x)) = -g(f(x))$

b $g(f(-x)) = g(-f(x)) = g(f(x))$

c $g(f(-x)) = g(f(x))$

11a $g(f(x)) = 7$ for all x, $f(g(x)) = 4$ for all x

b $g(f(x)) = g(x)$, $f(g(x)) = g(x)$

12a $g(f(x)) = 10x + 15 + b$, $f(g(x)) = 10x + 2b + 3$

b $b = 12$

13a i Translation down a **ii** Translation right a

b i Reflection in the x-axis **ii** Reflection in the y-axis

14a $g(f(x)) = 2ax + 3a + b$, $f(g(x)) = 2ax + 2b + 3$

b First, $2a = 1$, so $a = \frac{1}{2}$. Secondly, $3a + b = 0$, so $b = -1\frac{1}{2}$.

c Answers will vary

15a $f(g(0)) = -3$, $g(f(0)) = 3$, $f(g(-2)) = 3$,
$g(f(-2)) = 1$

b i $x^2 + x - 3$ **ii** $x^2 - x - 3$

16a All real y and $y \geq -1$.

b $x^2 + 2x + 1 = (x + 1)^2$, Range: $y \geq 0$

c $x^2 + 4x + 3 = (x + 1)(x + 3)$, Range: $y \geq -1$

d −1 and −3.

e Answers will vary

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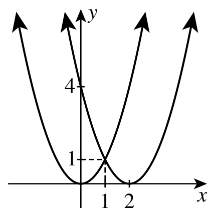
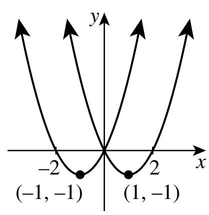
Chapter 4 review exercise

1a x^2 : 4, 1, 0, 1, 4, 9, 16
$(x - 2)^2$: 16, 9, 4, 1, 0, 1, 4
b $y = x^2$, $V = (0, 0)$
$y = (x - 2)^2$, $V = (2, 0)$
c Here x is replaced by $(x - 2)$, so it is a shift right by 2 units.

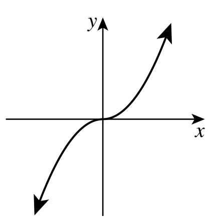


2a Replace x with $-x$.
b $y = x^2 - 2x$:
15, 8, 3, 0, −1, 0, 3
$y = x^2 + 2x$:
3, 0, −1, 0, 3, 8, 15
c $y = x^2 - 2x$: $(1, -1)$
$y = x^2 + 2x$: $(-1, -1)$

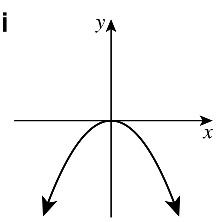


3a 7 **b** 4 **c** 5 **d** 3 **e** −3 **f** 12
4a $x = -5$ or 5 **b** $x = -6$ or 6 **c** $x = -2$ or 6
d $x = -5$ or -1 **e** $x = -1$ or 4 **f** $x = -1$ or $3\frac{2}{3}$
5a Shift $y = x^2$ up by 5 units.
b Shift $y = x^2$ down by 1 unit.
c Shift $y = x^2$ right by 3 units.
d Shift $y = x^2$ left by 4 units and up by 7 units.
6a $y = (x - 1)^2$ **b** $y = x^2 - 2$
c $y = (x + 1)^2 + 5$ **d** $y = (x - 4)^2 - 9$
7a $C(0, 0)$, $r = 1$ **b** $C(-1, 0)$, $r = 2$
c $C(2, -3)$, $r = \sqrt{5}$ **d** $C(0, 4)$, $r = 8$
8a $y = -x^3 + 2x + 1$ **b** $y = -x^2 + 3x + 4$
c $y = -2^{-x} - x$ **d** $y = \sqrt{9 - x^2}$
9a neither **b** odd **c** even

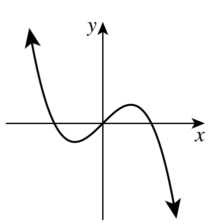
10a i



ii



b i



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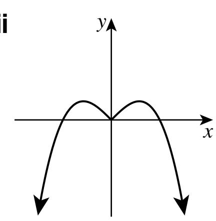
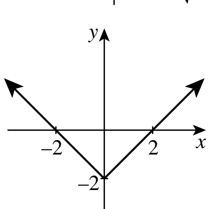
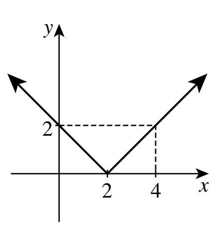


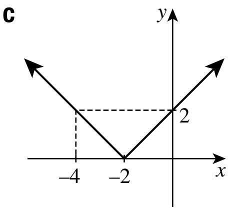
11a



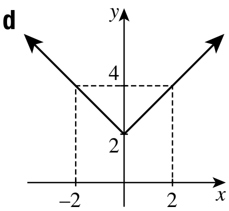
b



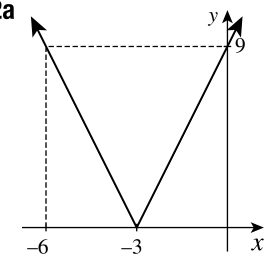
c



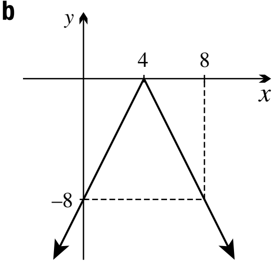
d



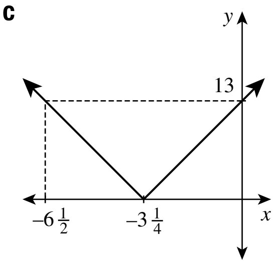
12a



b



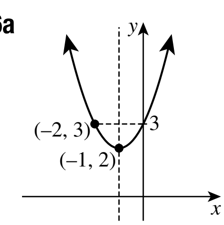
c



13a 5 or −5 **b** 1 or −9 **c** no solutions **d** 12 or −2
e 1 or −8 **f** 4 or $\frac{4}{3}$ **g** $\frac{-2}{7}$ **h** 5 or −5

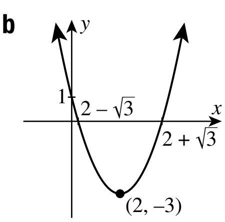
14a neither **b** even **c** odd **d** odd
15a $y = (x - 1)^2 + 4$, $V = (1, 4)$
b $y = (x + 2)^2 - 7$, $V = (-2, -7)$
c $y = 2(x + 2)^2 + 3$, $V = (-2, 3)$
d $y = -(x - 3)^2 + 10$, $V = (3, 10)$

16a



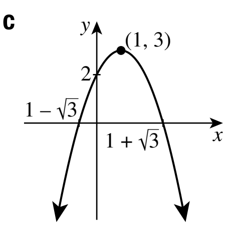
$y = (x + 1)^2 + 2$

b



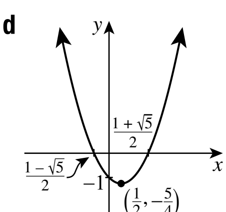
$y = (x - 2)^2 - 3$

c



$y = 3 - (x - 1)^2$

d



$y = \left(x - \frac{1}{2}\right)^2 - \frac{5}{4}$

17a $C(0, 1)$, $r = 2$ **b** $C(-3, 0)$, $r = 1$
c $C(2, -3)$, $r = 4$ **d** $C(4, -7)$, $r = 10$

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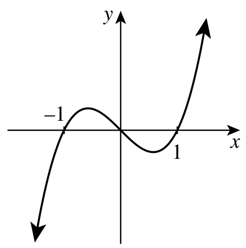
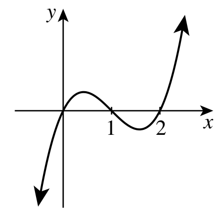
18a Answers will vary

b



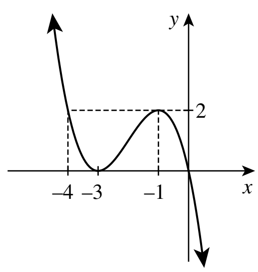


19a 13 **b** 7 **c** 93 **d** 327 **e** $5a^2 + 13$ **f** $25a^2 - 20a + 7$

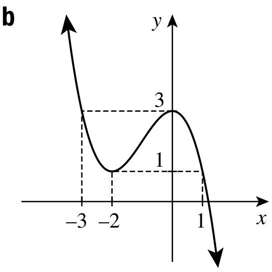
20a $f(g(x))$ has domain $x \geq 0$ and range $y \geq -1$, $g(f(x))$ has domain $x \geq 1$ and range $y \geq 0$.

b $g((x))$ has domain all real x and range $0 < y \leq 1$, $g(f(x))$ has domain all real x, $x \neq 0$ and range $y > 1$.

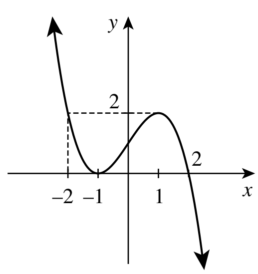
21a



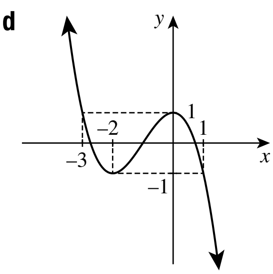
b



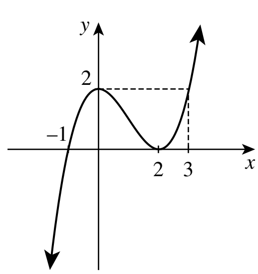
c



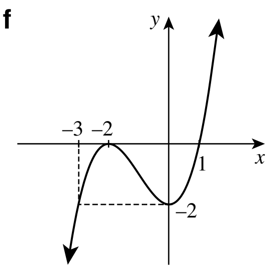
d



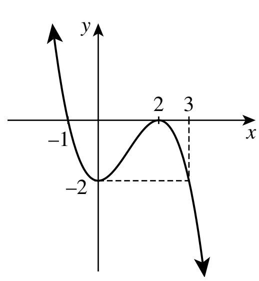
e



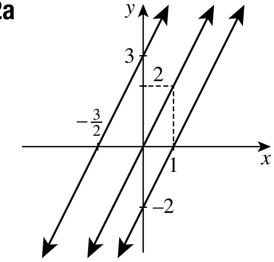
f



g



22a



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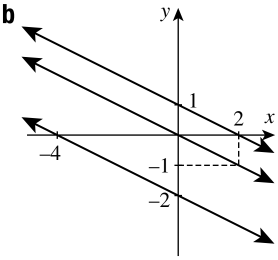

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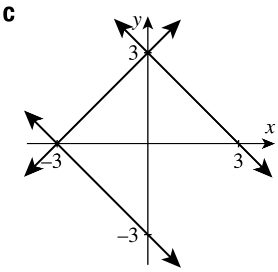
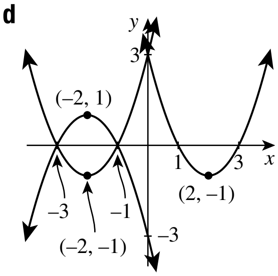
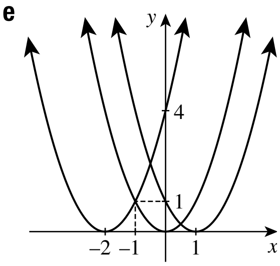


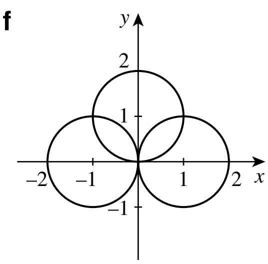
d



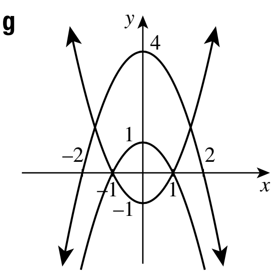
e



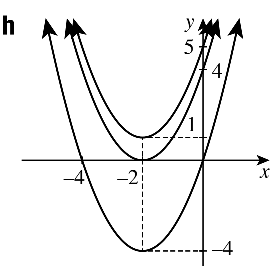
f



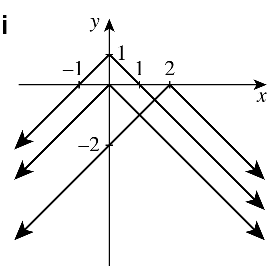
g



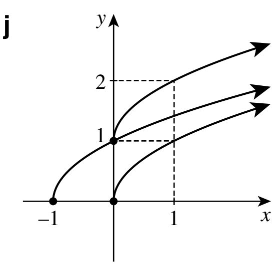
h



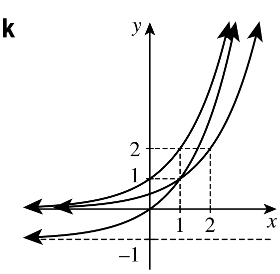
i



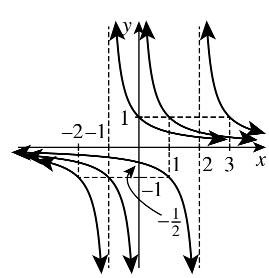
j



k



l



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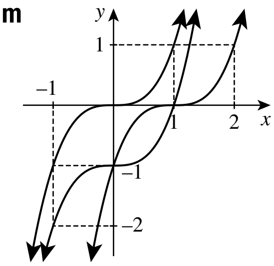
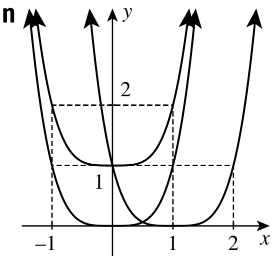
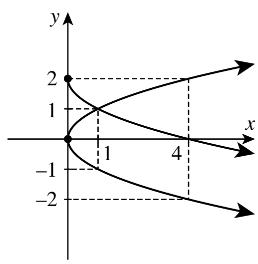
m



n



o



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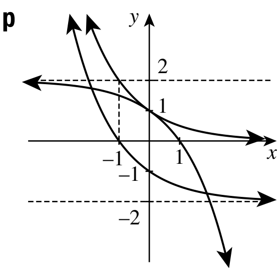


Chapter 5

Exercise 5A

1a $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{4}{5}$ **d** $\frac{4}{5}$ **e** $\frac{3}{5}$ **f** $\frac{4}{3}$

2a 0.4067 **b** 0.4848 **c** 0.7002 **d** 0.9986
e 0.0349 **f** 0.8387 **g** 0.0175 **h** 0.9986

3a 1.5697 **b** 0.8443 **c** 4.9894 **d** 0.9571
e 0.6833 **f** 0.1016 **g** 0.0023 **h** 0.0166

4a 76° **b** 46° **c** 12° **d** 27°
e No such angle — cos θ cannot exceed 1. **f** 39° **g** 60°
h No such angle — sin θ cannot exceed 1.

5a 41°25′ **b** 63°26′ **c** 5°44′ **d** 16°42′ **e** 46°29′ **f** 57°25′

6a 13 **b** 19 **c** 23 **d** 88

7a 53° **b** 41° **c** 67° **d** 59°

8a $\frac{12}{13}$ **b** $\frac{5}{12}$ **c** $\frac{13}{12}$ **d** $\frac{5}{12}$ **e** $\frac{13}{12}$ **f** $\frac{13}{5}$

9a 6 and 17 **b i** $\frac{15}{17}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{4}$ **iv** $\frac{17}{8}$ **v** $\frac{5}{3}$ **vi** $\frac{15}{8}$

10a $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{\sqrt{3}}$ **c** $\frac{1}{\sqrt{2}}$ **d** 2 **e** $\sqrt{2}$ **f** $\sqrt{3}$

11a 19.2 **b** 21.6 **c** 30.3 **d** 8.3

12a 29.78 **b** 10.14 **c** 16.46 **d** 29.71

13a 36°2′ **b** 68°38′ **c** 34°44′ **d** 38°40′ **e** 54°19′ **f** 70°32′

14a Answers will vary **b** Answers will vary

15a Answers will vary **b** 3 **c** $\frac{1}{3}\sqrt{5}$, $\frac{2}{3}$ **d** Answers will vary

16a i $\frac{1}{2}\sqrt{22}$ **ii** $\frac{3}{2}\sqrt{2}$ **b** Answers will vary

17a 1 **b** $\frac{1}{2}$ **c** 4 **d** 1

18a–d Answers will vary

Exercise 5B

1 2.65 m
2 63°
3 55 km
4 038° T
5 13.2 m
6 2.5 m
7 77 km
8 23 m
9 73°
10 21.3 m
11 11°
12a 46° **b** 101° T
13a Answers will vary **b** 67 km
14a $\angle PQR = 360° - (200° + 70°) = 90°$
(using co-interior angles on parallel lines and the fact that a revolution is 360°)
b $110° + 39° = 149°$ T
15a 5.1 cm **b** 16 cm **c** $PQ = 18 \sin 40°$, 63°25′
16a–c Answers will vary
17 457 m

Exercise 5C

1a 15 cm **b** 17 cm **c** 28°

2a i 90° **ii** 90° **iii** 90°
b i $\sqrt{2}$ **ii** $\sqrt{3}$
c i 35° **ii** 35°

3a i $2\sqrt{5}$ cm **ii** $2\sqrt{6}$ cm **b** 90° **c** 66°

4a i 90° **ii** 90° **iii** 90°
b i 2 cm **ii** $2\sqrt{2}$ cm
c i 72° **ii** 65°

5a i 90° **ii** 90° **b** 27°

6a $3\sqrt{2}$ cm **b** 43°

7a $BQ = 30 \tan 72°$ **b** 145 m

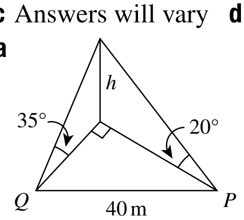
8a Answers will vary **b** 16 m **c** 21°

9a Answers will vary **b** 76 m **c** 14°

10a 1 cm **b** $\sqrt{2}$ cm **c** $\sqrt{2}$ **d** 70°32′

11a $h \cot 55°$ **b** It is the angle between south and east.
c Answers will vary **d** 114 m

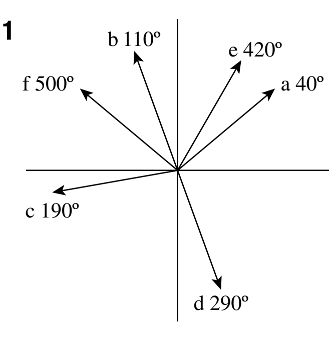
12a



b 13 metres

Exercise 5D

1



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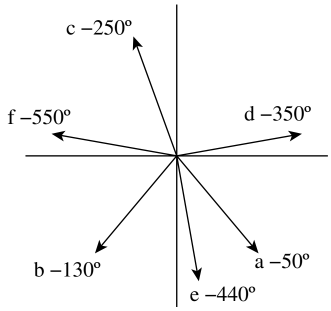
2



3a $-320°$ **b** $-250°$ **c** $-170°$ **d** $-70°$
e $-300°$ **f** $-220°$

4a $310°$ **b** $230°$ **c** $110°$ **d** $10°$ **e** $280°$ **f** $170°$

5a $70°, 430°, -290°, -650°$
b $100°, 460°, -260°, -620°$
c $140°, 500°, -220°, -580°$
d $200°, 560°, -160°, -520°$
e $240°, 600°, -120°, -480°$
f $340°, 700°, -20°, -380°$

6a $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$,
$\operatorname{cosec}\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$
b $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$,
$\operatorname{cosec}\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$
c $\sin\theta = -\frac{2}{\sqrt{5}}$, $\cos\theta = -\frac{1}{\sqrt{5}}$, $\tan\theta = 2$,
$\operatorname{cosec}\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = \frac{1}{2}$
d $\sin\theta = -\frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = -\frac{5}{12}$,
$\operatorname{cosec}\theta = -\frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = -\frac{12}{5}$

7a–c All six trigonometric functions are sketched in Section 5E.

8a **i** 0.5 **ii** −0.5 **iii** 0.95 **iv** 0.95 **v** 0.59 **vi** 0.81
vii −0.89 **viii** 0.45 **ix** −0.81 **x** 0.59
b **i** $30°, 150°$ **ii** $120°, 240°$ **iii** $64°, 116°$
iv $53°, 307°$ **v** $53°, 127°$ **vi** $143°, 217°$
vii $204°, 336°$ **viii** $107°, 253°$
c $45°, 225°$

Exercise 5E

1a + **b** + **c** − **d** − **e** + **f** − **g** − **h** + **i** − **j** +
k − **l** − **m** − **n** + **o** + **p** −

2a $10°$ **b** $30°$ **c** $50°$ **d** $20°$ **e** $80°$ **f** $70°$
g $70°$ **h** $80°$ **i** $10°$ **j** $20°$

3a $-\tan 50°$ **b** $\cos 50°$ **c** $-\sin 40°$ **d** $\tan 80°$
e $-\cos 10°$ **f** $-\sin 40°$ **g** $-\cos 5°$ **h** $\sin 55°$
i $-\tan 35°$ **j** $\sin 85°$ **k** $-\cos 85°$ **l** $\tan 25°$

4a 0 **b** −1 **c** 0 **d** 0 **e** 1 **f** 1 **g** −1 **h** undefined
i 0 **j** 0 **k** undefined **l** 0

5a $\frac{\sqrt{3}}{2}$ **b** $\frac{\sqrt{3}}{2}$ **c** $-\frac{\sqrt{3}}{2}$ **d** $-\frac{\sqrt{3}}{2}$ **e** $\frac{1}{\sqrt{2}}$ **f** $-\frac{1}{\sqrt{2}}$
g $-\frac{1}{\sqrt{2}}$ **h** $\frac{1}{\sqrt{2}}$ **i** $\frac{1}{\sqrt{3}}$ **j** $-\frac{1}{\sqrt{3}}$ **k** $\frac{1}{\sqrt{3}}$ **l** $-\frac{1}{\sqrt{3}}$

6a $-\frac{1}{2}$ **b** 1 **c** $-\frac{1}{2}$ **d** $\frac{1}{\sqrt{2}}$ **e** $\sqrt{3}$ **f** $-\frac{\sqrt{3}}{2}$ **g** −1 **h** $\frac{1}{2}$
i $-\frac{1}{\sqrt{2}}$ **j** $-\frac{\sqrt{3}}{2}$ **k** $-\frac{1}{2}$ **l** $-\sqrt{3}$

7a 2 **b** $-\sqrt{2}$ **c** $-\frac{1}{\sqrt{3}}$ **d** $\sqrt{3}$ **e** $\frac{2}{\sqrt{3}}$ **f** $-\frac{2}{\sqrt{3}}$

8a 1 **b** −1 **c** undefined **d** undefined **e** 0 **f** undefined

9a $60°$ **b** $20°$ **c** $30°$ **d** $60°$ **e** $70°$ **f** $10°$
g $50°$ **h** $40°$

10a $\frac{1}{2}$ **b** $-\frac{\sqrt{3}}{2}$ **c** $\sqrt{3}$ **d** $\frac{1}{\sqrt{2}}$ **e** $-\frac{1}{\sqrt{3}}$ **f** $-\frac{1}{\sqrt{2}}$ **g** $\sqrt{3}$
h $-\frac{\sqrt{3}}{2}$ **i** $\frac{1}{\sqrt{2}}$ **j** $-\frac{1}{2}$ **k** $-\frac{1}{2}$ **l** 1

11 All six graphs are many-to-one.

12a 0.42 **b** −0.91 **c** 0.91 **d** −0.42
e 0.49 **f** 0.49

13a −0.70 **b** −1.22 **c** −0.70 **d** −0.52
e 1.92 **f** −0.52

14a–c Answers will vary

15a $-\sin\theta$ **b** $\cos\theta$ **c** $-\tan\theta$ **d** $\sec\theta$ **e** $\sin\theta$
f $-\sin\theta$ **g** $-\cos\theta$ **h** $\tan\theta$

Exercise 5F

1a $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$
b $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$
c $\sin\theta = -\frac{7}{25}$, $\cos\theta = -\frac{24}{25}$, $\tan\theta = \frac{7}{24}$
d $\sin\theta = -\frac{21}{29}$, $\cos\theta = \frac{20}{29}$, $\tan\theta = -\frac{21}{20}$

2a $y = 12$, $\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$, $\tan\alpha = \frac{12}{5}$
b $r = 3$, $\sin\alpha = \frac{2}{3}$, $\cos\alpha = -\frac{\sqrt{5}}{3}$, $\tan\alpha = -\frac{2}{\sqrt{5}}$
c $x = -4$, $\sin\alpha = -\frac{3}{5}$, $\cos\alpha = -\frac{4}{5}$, $\tan\alpha = \frac{3}{4}$
d $y = -3$, $\sin\alpha = -\frac{3}{\sqrt{13}}$, $\cos\alpha = \frac{2}{\sqrt{13}}$, $\tan\alpha = -\frac{3}{2}$

3a **i** $\sin\theta = -\frac{4}{5}$ **ii** $\tan\theta = -\frac{4}{3}$
b **i** $\sin\theta = \frac{5}{13}$ **ii** $\cos\theta = -\frac{12}{13}$

4a **i** $\cos\theta = -\frac{3}{4}$ **ii** $\tan\theta = \frac{\sqrt{7}}{3}$,
or $\cos\theta = \frac{3}{4}$ or $\tan\theta = -\frac{\sqrt{7}}{3}$
b **i** $\sin\theta = \frac{\sqrt{15}}{4}$ **ii** $\tan\theta = -\sqrt{15}$,
or $\sin\theta = -\frac{\sqrt{15}}{4}$ or $\tan\theta = \sqrt{15}$

5a $2\sqrt{2}$ **b** $-\frac{3}{4}$ **c** $-\frac{\sqrt{3}}{2}$ **d** $\frac{3}{\sqrt{13}}$ **e** $\frac{9}{41}$ **f** $\frac{1}{2}$

6a $\frac{1}{\sqrt{10}}$ or $-\frac{1}{\sqrt{10}}$ **b** $\frac{1}{\sqrt{5}}$ or $-\frac{1}{\sqrt{5}}$ **c** $\frac{4}{5}$ or $-\frac{4}{5}$
d $\frac{\sqrt{5}}{2}$ or $-\frac{\sqrt{5}}{2}$ **e** $\frac{12}{5}$ or $-\frac{12}{5}$ **f** $\frac{\sqrt{3}}{\sqrt{7}}$ or $-\frac{\sqrt{3}}{\sqrt{7}}$

7a $-\frac{3}{4}$ **b** $-\frac{15}{17}$ **c** $-\frac{\sqrt{15}}{4}$ **d** $\frac{35}{37}$ **e** $-\frac{21}{20}$ **f** $\frac{\sqrt{11}}{6}$

8a $\sqrt{2}$ or $-\sqrt{2}$ **b** $\frac{15}{8}$ or $-\frac{15}{8}$ **c** $\frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$ **d** $\frac{6}{5}$ or $-\frac{6}{5}$

9a $-\frac{3}{\sqrt{5}}$ **b** $-\frac{41}{9}$ **c** $-\frac{15}{8}$ **d** $\frac{\sqrt{7}}{\sqrt{3}}$

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10a $\frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$ **b** $-\frac{3}{2\sqrt{10}} = \frac{3\sqrt{10}}{20}$ **c** 1 **d** $\frac{12}{13}$

11 $\cos\theta = -\dfrac{\sqrt{q^2 - p^2}}{q}$, $\tan\theta = -\dfrac{p}{\sqrt{q^2 - p^2}}$

12 $\sin\alpha = \dfrac{k}{\sqrt{1 + k^2}}$ or $-\dfrac{k}{\sqrt{1 + k^2}}$,
$\sec\alpha = \sqrt{1 + k^2}$ or $-\sqrt{1 + k^2}$

13a Answers will vary **b** $\sin x = \dfrac{2t}{1 + t^2}$, $\tan x = \dfrac{2t}{1 - t^2}$

Exercise 5G

1a–f Answer is in question
2a $\operatorname{cosec}\theta$ **b** $\cot\alpha$ **c** $\tan\beta$ **d** $\cot\phi$
3a 1 **b** 1 **c** 1
4a–c Answers will vary
5a $\cos\theta$ **b** $\operatorname{cosec}\alpha$ **c** $\cot\beta$ **d** $\tan\phi$
6a 1 **b** $\sin^2\beta$ **c** $\sec^2\phi$ **d** 1
7a $\cos^2\beta$ **b** $\operatorname{cosec}^2\phi$ **c** $\cot^2 A$ **d** -1
8a $\cos^2\theta$ **b** $\tan^2\beta$ **c** $\cot^2 A$ **d** 1
9a–c Answers will vary
10a $\cos^2\alpha$ **b** $\sin^2\alpha$ **c** $\sin A$ **d** $\cos A$
11a–j Answers will vary
12a–f Answers will vary
13a–h Answers will vary

Exercise 5H

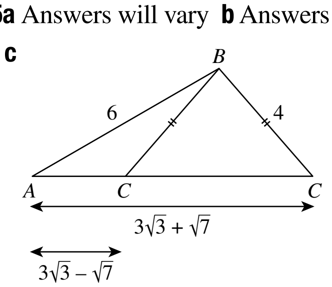
1a $\theta = 60°$ or $120°$ **b** $\theta = 30°$ or $150°$
c $\theta = 45°$ or $225°$ **d** $\theta = 60°$ or $240°$
e $\theta = 135°$ or $225°$ **f** $\theta = 120°$ or $300°$
g $\theta = 210°$ or $330°$ **h** $\theta = 150°$ or $210°$
2a $\theta = 90°$ **b** $\theta = 0°$ or $360°$ **c** $\theta = 90°$ or $270°$
d $\theta = 180°$ **e** $\theta = 0°$ or $180°$ or $360°$ **f** $\theta = 270°$
3a $x \doteqdot 65°$ or $295°$ **b** $x \doteqdot 7°$ or $173°$ **c** $x \doteqdot 82°$ or $262°$
d $x \doteqdot 222°$ or $318°$ **e** $x \doteqdot 114°$ or $294°$
f $x \doteqdot 140°$ or $220°$
4a $\alpha \doteqdot 5°44'$ or $174°16'$ **b** $\alpha \doteqdot 95°44'$ or $264°16'$
c $\alpha = 135°$ or $315°$ **d** $\alpha = 270°$ **e** no solutions
f $\alpha = 120°$ or $240°$ **g** $\alpha = 150°$ or $330°$
h $\alpha \doteqdot 18°26'$ or $198°26'$
5a $x \doteqdot -16°42'$ or $163°18'$ **b** $x = 90°$ or $-90°$
c $x = 45°$ or $-45°$ **d** $x \doteqdot -135°34'$ or $-44°26'$
6a $\theta = 60°, 300°, 420°$ or $660°$
b $\theta = 90°, 270°, 450°$ or $630°$
c $\theta = 210°, 330°, 570°$ or $690°$
d $\theta = 22°30', 202°30', 382°30'$ or $562°30'$
7a $x = 15°, 75°, 195°$ or $255°$
b $x = 30°, 120°, 210°$ or $300°$
c $x = 67°30', 112°30', 247°30'$ or $292°30'$
d $x = 135°$ or $315°$
8a $\alpha = 75°$ or $255°$ **b** $\alpha = 210°$ or $270°$
c $\alpha = 300°$ **d** $\alpha = 210°$ or $300°$
9a $\theta = 45°$ or $225°$ **b** $\theta = 135°$ or $315°$
c $\theta = 60°$ or $240°$ **d** $\theta = 150°$ or $330°$

Exercise 5I

1a 8.2 **b** 4.4 **c** 4.9 **d** 1.9 **e** 9.2 **f** 3.5
2a 14.72 **b** 46.61 **c** 5.53
3a 49° **b** 53° **c** 43° **d** 20° **e** 29° **f** 42°
4a $5\,\text{cm}^2$ **b** $19\,\text{cm}^2$ **c** $22\,\text{cm}^2$
5b $b \doteqdot 10.80\,\text{cm}$, $c \doteqdot 6.46\,\text{cm}$
6b 97 cm
7a $49°46'$ **b** $77°53'$ **c** $3.70\,\text{cm}^2$
8 42°, 138°
9 62°, 118°
10a $69°2'$ or $110°58'$ **b** 16.0 cm or 11.0 cm
11 317 km
12a Answers will vary **b** 9 m
13a 32 **b** $\frac{5}{7}$
14a 16 m **b** 11.35 m **c** 3.48 m
15a 30° or 150° **b** $17°27'$ or $162°33'$
c No solutions, because $\sin\theta = 1.2$ is impossible.
16a $3\sqrt{6}$ **b** $3\sqrt{2}$ **c** $2\sqrt{6}$ **d** $6\sqrt{2}$
17 11.0 cm

Exercise 5J

1a 3.3 **b** 4.7 **c** 4.0 **d** 15.2 **e** 21.9 **f** 24.6
2a 39° **b** 56° **c** 76° **d** 94° **e** 117° **f** 128°
3a $\sqrt{13}$ **b** $\sqrt{7}$
4a $\sqrt{10}$ **b** $\sqrt{21}$
5a $44°25'$ **b** $101°32'$ **c** $\frac{7}{32}$
6 11.5 km
7 167 nautical miles
8 20°
9a $101°38'$ **b** $78°22'$
10 $13°10'$, 120°
11a Answers will vary **b** Answers will vary
12a 19 cm **b** $\frac{37}{38}$
13a $\angle DAP = \angle DPA = 60°$ (angle sum of isosceles triangle), so ΔADP is equilateral. Hence $AP = 3\,\text{cm}$.
b $3\sqrt{7}\,\text{cm}$ **c** Answers will vary
14 3 or 5
15a Answers will vary **b** Answers will vary
c



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Exercise 5K

1a 28.3 **b** 17.3 **c** 12.5 **d** 36.2 **e** 12.6 **f** 23.2
2a 59° **b** 55° **c** 40° **d** 37° **e** 52° **f** 107°
3a 26 cm **b** 28 cm **c** 52° **d** 62°
4a Answers will vary **b** 28 m
5a $\angle ACP + 31° = 68°$ (exterior angle of ΔACP)
b Answers will vary **c** 6 cm
6a 11.6 cm **b** 49°
7a 44°25′ **b** 10 cm^2
8 Answer is in question
9a Answers will vary **b** 36 cm **c** Answers will vary
10a PQ is inclined at 26° to a north–south line through Q, because of alternate angles on parallel lines. Then $\angle PQR = 26° + 90°$.
b 112 nautical miles
11a 46°59′ or 133°1′ **b** 66.4 m or 52.7 m
12a $\angle PJK = \angle PBQ = 20°$ (corresponding angles on parallel lines), but $\angle PJK = \angle PAJ + \angle APJ$ (exterior angle of triangle), so $\angle APJ = 20° - 5° = 15°$.
b Answers will vary **c** Answers will vary **d** 53 m
13a 38 tan 68° **b** 111 m
14a Answers will vary **b** 131 m
15a Answers will vary **b** 108 km
c $\angle ACB \doteqdot 22°$, bearing $\doteqdot$ 138° T
16a Answers will vary **b** Answers will vary **c** 34 m
d Answers will vary
17 P_1 by 2.5 min
18 50.4 m
19a $x \cot 27°$ **b** Answers will vary **c** Answers will vary

Chapter 5 review exercise

1a 0.2924 **b** 0.9004 **c** 0.6211 **d** 0.9904
2a 17°27′ **b** 67°2′ **c** 75°31′ **d** 53°8′
3a 10.71 **b** 5.23 **c** 10.36 **d** 15.63
4a 45°34′ **b** 59°2′ **c** 58°43′ **d** 36°14′
5a $\sqrt{3}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$ **d** 1 **e** 2 **f** $\frac{2}{\sqrt{3}}$
6 6.25 metres
7 65°
8a Answers will vary **b** 114 km **c** 108° T
9 All six trigonometric graphs are drawn on page 175.
10a −cos 55° **b** −sin 48° **c** tan 64° **d** sin 7°
11a $\sqrt{3}$ **b** $-\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$ **d** $-\frac{1}{\sqrt{3}}$
12a 0 **b** −1 **c** undefined **d** −1
13a $y = 3$, $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$
b $x = -2\sqrt{5}$, $\sin\theta = -\frac{\sqrt{5}}{5}$, $\cos\theta = -\frac{2\sqrt{5}}{5}$, $\tan\theta = \frac{1}{2}$
14a $\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$ **b** $\cos\beta = \frac{5}{7}$, $\tan\beta = \frac{2\sqrt{6}}{5}$
c $\sin\alpha = -\frac{9}{41}$, $\cos\alpha = \frac{40}{41}$ **d** $\cos\beta = -\frac{5}{7}$, $\tan\beta = -\frac{2\sqrt{6}}{5}$
15a $\sec\theta$ **b** $\tan\theta$ **c** $\tan\theta$ **d** $\cos^2\theta$ **e** 1 **f** $\cot^2\theta$
16a–f Answers will vary
17a $x = 60°$ or $300°$ **b** $x = 90°$ **c** $x = 135°$ or $315°$
d $x = 90°$ or $270°$ **e** $x = 30°$ or $210°$
f $x = 0°$, $180°$ or $360°$ **g** $x = 225°$ or $315°$
h $x = 150°$ or $210°$ **i** $x = 45°$, $135°$, $225°$ or $315°$
j $x = 30°$, $150°$, $210°$ or $330°$ **k** $x = 15°$ or $135°$
l $\tan x = -\sqrt{3}$, $x = 120°$ or $300°$
18a $\sin\theta = 0$ or $-\frac{1}{2}$, $\theta = 0°, 180°, 210°, 330°$ or $360°$
b $\cos\theta = -1$ or 2, $\theta = 180°$ **c** $\tan\theta = \frac{1}{2}$ or −3,
$\theta = 26°34′, 108°26′, 206°34′$ or $288°26′$
19a 8.5 **b** 10.4 **c** 7.6 **d** 8.9
20a 27 cm^2 **b** 56 cm^2
21a 57°55′ **b** 48°33′ **c** 24°29′ **d** 150°26′
22 28 cm^2
23a $\frac{5\sqrt{3}}{3}$ cm **b** 30° or 150°
24a Answers will vary **b** 48 m
25a Answers will vary **b** 31.5 m
26a Answers will vary **b** 316 nautical miles **c** 104° T
27a 10 tan 77° **b** 45 m
28a 1.612 m **b** 1.758 m **c** 23°28′
29a Answers will vary **b** Answers will vary **c** 129 m

Chapter 6

Exercise 6A

1a (2, 7) **b** (5, 6) **c** (2, −2) **d** $\left(0, 3\frac{1}{2}\right)$
e $\left(-5\frac{1}{2}, -10\right)$ **f** (4, 0)
2a 5 **b** 13 **c** 10 **d** $\sqrt{8} = 2\sqrt{2}$ **e** $\sqrt{80} = 4\sqrt{5}$ **f** 13
3a $M(1, 5)$ **b** $PM = MQ = 5$
4a $PQ = QR = \sqrt{17}$, $PR = \sqrt{50} = 5\sqrt{2}$
b Answers will vary
5a $AB = 15$, $BC = 20$ and $AC = 25$
b LHS $= AB^2 + BC^2 = 15^2 + 20^2 = 625 =$ RHS
6a $AB = \sqrt{58}$, $BC = \sqrt{72} = 6\sqrt{2}$, $CA = \sqrt{10}$
b AB: $\left(1\frac{1}{2}, 1\frac{1}{2}\right)$, BC: (0, 1), CA: $\left(-1\frac{1}{2}, 4\frac{1}{2}\right)$
7a 13 **b** $\sqrt{41}$ **c** (5, −3) **d** Answers will vary
8a (1, 6) **b** (1, 6) **c** The diagonals bisect each other.
d parallelogram
9a All sides are $5\sqrt{2}$. **b** rhombus
10a $XY = YZ = \sqrt{52} = 2\sqrt{13}$, $ZX = \sqrt{104} = 2\sqrt{26}$
b $XY^2 + YZ^2 = 104 = ZX^2$ **c** 26 square units
11a Each point is $\sqrt{17}$ from the origin.
b $\sqrt{17}, 2\sqrt{17}, 2\pi\sqrt{17}, 17\pi$
12 (5, 2)
13a $S(-5, -2)$
b i $P = (4, -14)$ **ii** $P = (-1, -17)$ **iii** $P = (7, -7)$
c $B = (0, 7)$ **d** $R = (12, -9)$
14a $A(3, 5)$ and $B(5, 7)$ will do.
b $C(0, 0)$ and $D(6, 8)$ will do.

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15a ABC is an equilateral triangle.
b PQR is a right triangle.
c DEF is none of these.
d XYZ is an isosceles triangle.
16a $(x-5)^2+(y+2)^2=45$
b $(x+2)^2+(y-2)^2=74$

Exercise 6B

1a i 2 **ii** $\frac{3}{4}$ **iii** $-1\frac{1}{2}$ **b i** $-\frac{1}{2}$ **ii** $-\frac{4}{3}$ **iii** $\frac{2}{3}$
2a $-1, 1$ **b** $2, -\frac{1}{2}$ **c** $\frac{1}{2}, -2$ **d** $-\frac{1}{2}, 2$
e $3, -\frac{1}{3}$ **f** $-\frac{7}{10}, \frac{10}{7}$
3a Vertical **b** Horizontal **c** Neither
d Horizontal **e** Neither **f** Vertical
4a 3 **b** $\frac{1}{2}$ **c** parallelogram
5a $m_{AB}=m_{CD}=\frac{1}{2}, m_{BC}=m_{DA}=-\frac{1}{5}$.
b $m_{AB}=2, m_{CD}=-3$
6a 0.27 **b** -1.00 **c** 0.41 **d** 3.08
7a 45° **b** 120° **c** 76° **d** 30°
8a less **b** equal **c** less **d** more
9a $m_{AB}=m_{CD}=-\frac{1}{2}, m_{BC}=m_{DA}=2$
b $m_{AB}=m_{BC}=-1$ **c** $AB=BC=2\sqrt{5}$
10 In each case, show that each pair of opposite sides is parallel.
a Show also that two adjacent sides are equal.
b Show also that two adjacent sides are perpendicular.
c Show that it is both a rhombus and a rectangle.
11a $-2, -\frac{7}{3}$, non-collinear **b** $\frac{2}{3}, \frac{2}{3}$, collinear
12 The gradients of AB, BC and CD are all $\frac{1}{3}$.
13 $m_{AB}=\frac{1}{2}, m_{BC}=-2$ and $m_{AC}=0$, so $AB \perp BC$.
14a $m_{PQ}=4, m_{QR}=-\frac{1}{4}$ and $m_{PR}=-\frac{5}{3}$, so $PQ \perp QR$. Area $=8\frac{1}{2}$ square units
b $m_{XY}=\frac{7}{3},\ m_{YZ}=\frac{2}{5}$ and $m_{XZ}=-\frac{5}{2}$, so $XZ \perp YZ$. Area $=14\frac{1}{2}$ square units
15a $A(0,0)$ and $B(1,3)$ will do.
b $A(1,1)$ and $B(1,4)$ will do.
16a -5 **b** 5
17a $A(-2,0),\ B(0,6),\ m=3,\ \alpha \doteqdot 72°$
b $A(2,0), B(0,1), m=-\frac{1}{2},\ \alpha \doteqdot 153°$
c $A(-4,0),\ B(0,-3), m=-\frac{3}{4},\ \alpha \doteqdot 143°$
d $A(3,0), B(0,-2), m=\frac{2}{3},\ \alpha \doteqdot 34°$
18a $P=(2,-1),\ Q=(-1,4),\ R=(-3,2),\ S=(0,-3)$
b $m_{PQ}=m_{RS}=-\frac{5}{3}$ and $m_{PS}=m_{QR}=1$
19a They all satisfy the equation, or they all lie 5 units from O.
b The centre $O(0,0)$ lies on AB.
c $m_{AC}=\frac{1}{2}, m_{BC}=-2$
20a 3.73 **b** 1 **c** 2.41 **d** 0.32
21 $a=-\frac{1}{2}$
22 $k=2$ or -1

Exercise 6C

1a not on the line **b** on the line **c** on the line
2a (4, 0) and (0, 3) **b** (1.5, 0) and (0, −6)
c (8, 0) and (0, −4)
3 Check the points in your answer by substitution. (0, 8), (3, 7) and (6, 6) will do.
4a $x=1,\ y=2$ **b** $x=0,\ y=-4$ **c** $x=5,\ y=0$
5a $m=4,\ b=-2$ **b** $m=\frac{1}{5},\ b=-3$
c $m=-1,\ b=2,$ **d** $m=-\frac{5}{7},\ b=0$
6a $y=-3x+5$ **b** $y=-3x-\frac{2}{3}$ **c** $y=-3x$
7a $y=5x-4$ **b** $y=-\frac{2}{3}x-4$ **c** $y=-4$
8a $x-y+3=0$ **b** $2x+y-5=0$
c $x-5y-5=0$ **d** $x+2y-6=0$
9a $m=1,\ b=3$ **b** $m=-1,\ b=2$ **c** $m=\frac{1}{3},\ b=0$
d $m=-\frac{3}{4},\ b=\frac{5}{4}$
10a $m=1,\ \alpha=45°$ **b** $m=-1,\ \alpha=135°$
c $m=2,\ \alpha \doteqdot 63°26'$ **d** $m=-\frac{3}{4},\ \alpha \doteqdot 143°8'$
11 The sketches required are clear from the intercepts.
a $A(3,0), B(0,5)$ **b** $A(-3,0), B(0,6)$
c $A(-4,0), B\left(0, 2\frac{2}{5}\right)$
12a $y=2x+4,\ 2x-y+4=0$
b $y=-x,\ x+y=0$
c $y=-\frac{1}{3}x-4,\ x+3y+12=0$
13a i $y=-2x+3$ **ii** $y=\frac{1}{2}x+3$
b i $y=\frac{5}{2}x+3$ **ii** $y=-\frac{2}{5}x+3$
c i $y=-\frac{3}{4}x+3$ **ii** $y=\frac{4}{3}x+3$
14a $-3,\ \frac{1}{2},\ -3,\ \frac{1}{2}$, parallelogram
b $\frac{4}{3}, -\frac{3}{4}, \frac{4}{3}, -\frac{3}{4}$, rectangle
15 The gradients are $\frac{5}{7}$, $\frac{2}{5}$ and $-\frac{7}{5}$, so the first and last are perpendicular.
16a $x=3,\ x=0,\ y=-7,\ y=-2$
b $y=0,\ y=-4x+12,\ y=2x+12$
17a $x-y+3=0$ **b** $-\sqrt{3}x+y+1=0$
c $x-\sqrt{3}y-2\sqrt{3}=0$ **d** $x+y-1=0$
18a They are about 61° and 119°.
b It is isosceles. (The two interior angles with the x-axis are equal.)
19a $k=-\frac{1}{3}$ **b** $k=3$

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Exercise 6D

1 $3x - y - 4 = 0$
2a $6x - y + 19 = 0$ **b** $2x + y - 3 = 0$
c $2x - 3y + 25 = 0$ **d** $7x + 2y = 0$
3a $3x + 5y - 13 = 0$ **b** $3x + 5y - 18 = 0$
c $3x + 5y = 0$ **d** $3x + 5y + 20 = 0$
4a $2x - y - 1 = 0$ **b** $x + y - 4 = 0$
c $5x + y = 0$ **d** $x + 3y - 8 = 0$ **e** $4x + 5y + 8 = 0$
5a $y = 2x + 1$ **b** $y = -\frac{1}{2}x + 6$ **c** $y = \frac{1}{5}x - 8$
d $y = \frac{3}{7}x + 9$ **e** $y = \frac{5}{2}x + 10$
6a 3 **b** $3x - y - 5 = 0$
7a $2,\ 2x - y - 2 = 0$ **b** $-2,\ 2x + y - 1 = 0$
c $\frac{1}{3},\ x - 3y + 13 = 0$ **d** $2,\ 2x - y + 2 = 0$
e $-\frac{1}{4},\ x + 4y + 4 = 0$ **f** $1,\ x - y - 3 = 0$
8a $-\frac{3}{2}$ **b i** $3x + 2y + 1 = 0$ **ii** $2x - 3y - 8 = 0$
9a $2x - 3y + 2 = 0$ **b** $2x - 3y - 9 = 0$
10a $4x - 3y - 8 = 0$ **b** $4x - 3y + 11 = 0$
11a $M(3, -1)$ **b** Answers vary.
c i No, the first two intersect at $(-4, 7)$, which does not lie on the third.
ii They all meet at $(5, 4)$.
12a i $y = -2x + 5$ **ii** $y = \frac{1}{2}x + 6$
b i $y = 2\frac{1}{2}x - 8\frac{1}{2}$ **ii** $y = -\frac{2}{5}x + 4\frac{1}{5}$
c i $y = -1\frac{1}{3}x + 3$ **ii** $y = \frac{3}{4}x + 6\frac{1}{2}$
13a $x - y - 1 = 0$ **b** $\sqrt{3}x + y + \sqrt{3} = 0$
c $x - y\sqrt{3} - 4 - 3\sqrt{3} = 0$
d $x + \sqrt{3}y + 2 + 5\sqrt{3} = 0$
14a i $x - 3 = 0$ **ii** $y + 1 = 0$ **b** $3x + 2y - 6 = 0$
c i $x - y + 4 = 0$ **ii** $\sqrt{3}x + y - 4 = 0$
d $x\sqrt{3} + y + 6\sqrt{3} = 0$
15 $\ell_1 \parallel \ell_2$, and $\ell_3 \parallel \ell_4$ so there are two pairs of parallel sides. The vertices are $(-2, -1),\ (-4, -7),\ (1, -2),\ (3, 4)$.
16 $m_{BC} \times m_{AC} = -1$ so $BC \perp AC$.
AB: $y = x - 1$, BC: $y = \frac{1}{2}x + 2$,
AC: $y = 2 - 2x$
17a $m_{AC} = \frac{2}{3}$, $\theta \doteqdot 34°$ **b** $2x - 3y - 2 = 0$ **c** $D(4, 2)$
d $m_{AC} \times m_{BD} = \frac{2}{3} \times -\frac{3}{2} = -1$, hence they are perpendicular. **e** isosceles
f area $= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times \sqrt{52} \times \sqrt{52} = 26$
g $E(8, -4)$
18a $4y = 3x + 12$ **b** $ML = MP = 5$ **c** $N(4, 6)$
d Answers will vary **e** $x^2 + (y - 3)^2 = 25$
19 $k = 2\frac{1}{2}$
20a $\mu = 4$ **b** $\mu = -9$

Exercise 6E

1a i $1, -1$ **ii** The product of their gradients is -1.
b i $1, -1$ **ii** The product of their gradients is -1.
2a i $M = (4, 5)$ **ii** $OM = PM = QM = \sqrt{41}$
iii OM, PM and QM are three radii of the circle.
b $M = (p, q)$, $OM = PM = QM = \sqrt{p^2 + q^2}$
3a i $P(5, 2)$ and $Q(4, 1)$ **ii, iii** Answers will vary
iv $AC = 2\sqrt{2}$ and $PQ = \sqrt{2}$
b $P(a + b, c)$, $Q(b, c)$, $y = c$ and so $Q(b, c)$ lies on $y = c$. Also, $AC = 2a$ and $PQ = a$ so $PQ = \frac{1}{2}AC$.
4a $P = \left(\frac{1}{2}(a_1 + b_1), \frac{1}{2}(a_2 + b_2)\right)$,
$Q = \left(\frac{1}{2}(b_1 + c_1), \frac{1}{2}(b_2 + c_2)\right)$,
$R = \left(\frac{1}{2}(c_1 + d_1), \frac{1}{2}(c_2 + d_2)\right)$,
$S = \left(\frac{1}{2}(d_1 + a_1), \frac{1}{2}(d_2 + a_2)\right)$.
b Both midpoints are,
$\left(\frac{1}{4}(a_1 + b_1 + c_1 + d_1), \frac{1}{4}(a_2 + b_2 + c_2 + d_2)\right)$.
c Part **b** shows that its diagonals bisect each other, so it is a parallelogram.
5 Answers will vary
6a $\frac{x}{3} + \frac{y}{4} = 1$ and $4y = 3x$, thus $C = \left(\frac{48}{25}, \frac{36}{25}\right)$.
b $OA = 3, AB = 5, OC = \frac{12}{5}, BC = \frac{16}{5}, AC = \frac{9}{5}$
c i Answers will vary **ii** Answers will vary
7a $AB = BC = CA = 2a$ **b** $AB = AD = 2a$
c $BD = 2a\sqrt{3}$
8a AB and DC have gradient $\dfrac{b}{a}$; AD and BC have gradient $\dfrac{d}{c}$.
b Both the midpoints are $(a + c,\ b + d)$.
c The midpoints coincide.
9a i $P = (1, 4)$, $Q = (-1, 0)$ and $R = (3, 2)$,
BQ: $x - y + 1 = 0$, CR: $y - 2 = 0$, AP: $x = 1$
ii The medians intersect at $(1, 2)$.
b i $P(-3a, 3c - 3b)$, $Q(3a, 3c + 3b)$, $R(0, 0)$
ii The median passing through B is
$3a(y + 6b) = (c + 3b)(x + 6a)$.
The median passing through A is
$-3a(y - 6b) = (c - 3b)(x - 6a)$.
iii The medians intersect at $(0, 2c)$.
10a gradient $AB = 0$, gradient $BC = \dfrac{c}{b + a}$, gradient $CA = \dfrac{c}{b - a}$
b perpendicular bisector of AB: $x = 0$,
of BC: $c(c - y) = (b + a)(x - b + a)$,
of AC: $c(c - y) = (b - a)(x - b - a)$
c They all meet at $(0, \frac{c^2 + b^2 - a^2}{c})$.
d Any point on the perpendicular bisector of an interval is equidistant from the endpoints of that interval.

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Chapter 6 review exercise

1a $\left(8, 6\frac{1}{2}\right)$ **b** $-\dfrac{5}{12}$ **c** 13
2a $AB = 5, BC = \sqrt{2}, CA = 5$ **b** isosceles
3a $P(3, 7), Q(6, 5), R(3, -3), S(0, -1)$
b PQ and RS have gradient $-\frac{2}{3}$,
QR and SP have gradient $\frac{8}{3}$. **c** parallelogram
4a $C(-1, 1), r = \sqrt{45} = 3\sqrt{5}$ **b** $PC = \sqrt{53}$, no
5a $m_{LM} = -2, m_{MN} = -\frac{8}{9}, m_{NL} = \frac{1}{2}$
b $m_{LM} \times m_{NL} = -1$
6a -1 **b** $a = 8$ **c** $Q(7, -4)$
d $d^2 = 16$, so $d = 4$ or -4.
7a $2x + y - 5 = 0$ **b** $2x - 3y + 9 = 0$
c $x + 7y = 0$ **d** $3x + y + 8 = 0$ **e** $x\sqrt{3} - y - 2 = 0$
8a $b = -\frac{7}{6},\ m = \frac{5}{6},\ \alpha \doteqdot 39°48'$
b $b = \frac{3}{4}, m = -1, \alpha = 135°$
9a $8x - y - 24 = 0$ **b** $5x + 2y - 21 = 0$
10a No; $m_{LM} = -\frac{1}{3}$ and $m_{MN} = -\frac{5}{12}$.
b Yes; they all pass through $(2, 5)$.
11a Yes; the 2nd and 3rd lines have gradients $\frac{3}{2}$ and $-\frac{2}{3}$ and are perpendicular.
b Trapezium; the 1st and 3rd lines are parallel.
12a $A = (6, 0), B = (0, 7\frac{1}{2})$ **b** $22\frac{1}{2}$ square units
13a $m_{AB} = -\frac{3}{4}, AB = 10, M(6, 5)$ **b** Answers will vary
c $C(15, 17)$ **d** $AC = BC = 5\sqrt{10}$ **e** 75 units^2
f $\sin\theta = \frac{3}{5},\ \theta \doteqdot 36°52'$

Chapter 7

Exercise 7A

1a The factors are $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243$.
b Population in 2010 = 810 000, population in 2020 = 2 430 000, so the decade was 2010–2020.
2a 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096
b i 1, 3, 9, 27, 81, 243, 729
ii 1, 5, 25, 125, 625, 3125
iii 1, 6, 36, 216 **iv** 1, 7, 49, 343
v 1, 10, 100, 1000, 10 000, 100 000, 1 000 000
vi 1, 20, 400, 8 000, 160 000, 3 200 000, 64 000 000
c i 1, 4, 16, 64, 256, 1024, 4096
ii 1, 8, 64, 512, 4096
d 1, 9, 81, 729 **e** 1, 25, 625
3a 8 **b** 64 **c** 81 **d** 729 **e** $\frac{4}{9}$ **f** $\frac{8}{27}$ **g** $\frac{81}{10000}$ **h** $\frac{16}{49}$
i $\frac{5}{9}$ **j** 1
4a 1 **b** 1 **c** $\frac{1}{5}$ **d** $\frac{1}{11}$ **e** $\frac{1}{36}$ **f** $\frac{1}{100}$ **g** $\frac{1}{27}$ **h** $\frac{1}{125}$
i $\frac{1}{32}$ **j** $\frac{1}{1000000}$
5a 5 **b** 11 **c** $\frac{7}{2}$ or $3\frac{1}{2}$ **d** $\frac{2}{7}$ **e** $\frac{4}{3}$ or $1\frac{1}{3}$ **f** $\frac{23}{10}$ or $2\frac{3}{10}$
g $\frac{1}{10}$ or 0.1 **h** 10 **i** 100 **j** 50
6a $\frac{1}{25}$ **b** 25 **c** 125 **d** 16 **e** 1 000 000 **f** $\frac{9}{4}$ **g** $\frac{81}{16}$
h $\frac{16}{81}$ **i** $\frac{25}{4}$ **j** 1
7a 2^{14} **b** a^{15} **c** 7^{-8} **d** x^2 **e** $9^0 = 1$ **f** $a^0 = 1$
g 5^{-3} **h** 8
8a 7^5 **b** a^{-2} **c** x^{12} **d** x^{-12} **e** 2^{16} **f** 1 **g** y^{11} **h** y^{-11}
9a x^{15} **b** x^{15} **c** z^{14} **d** a^{-6} **e** a^{-6} **f** 5^{-28} **g** y^{10} **h** 2^{16}
10a $x = 2$ **b** $x = 4$ **c** $x = 3$ **d** $x = 6$ **e** $x = -1$
f $x = -1$ **g** $x = -2$ **h** $x = -3$ **i** $x = -1$
j $x = -1$ **k** $x = 0$ **l** $x = 0$
11a $9x^2$ **b** $125a^3$ **c** $64c^6$ **d** $81s^4t^4$
e $49x^2y^2z^2$ **f** $\frac{1}{x^5}$ **g** $\frac{9}{x^2}$ **h** $\frac{y^2}{25}$ **i** $\frac{49a^2}{25}$ **j** $\frac{27x^3}{8y^3}$
12a $3\,\text{km}^3$ **b** $(10^3 \times 10^3)^3 = 10^{18}$ **c** 3×10^{18}
13a $\frac{1}{9}$ **b** $\frac{1}{x}$ **c** $\frac{1}{b^2}$ **d** $-\frac{1}{a^4}$ **e** $\frac{1}{7x}$ **f** $\frac{7}{x}$ **g** $-\frac{9}{x}$ **h** $\frac{1}{9a^2}$
i $\frac{3}{a^2}$ **j** $\frac{4}{x^3}$
14a x^{-1} **b** $-x^{-2}$ **c** $-12x^{-1}$ **d** $9x^{-2}$ **e** $-x^{-3}$ **f** $12x^{-5}$
g $7x^{-3}$ **h** $-6x^{-1}$ **i** $\frac{1}{6}x^{-1}$ **j** $-\frac{1}{4}x^{-2}$
15a $\frac{2}{3}$ **b** $\frac{3}{7}$ **c** $\frac{3}{8}$ **d** $\frac{4}{25}$ **e** $\frac{27}{1000}$ **f** $\frac{9}{400}$ **g** 5
h $\frac{5}{12}$ **i** $\frac{4}{9}$ **j** $\frac{4}{25}$ **k** $\frac{8}{125}$ **l** 400
16a $x = 2$ **b** $x = -1$ **c** $x = -2$ **d** $x = -3$
e $x = \frac{10}{13}$ **f** $x = 2$ **g** $x = \frac{1}{3}$ **h** $x = \frac{9}{8}$
17a 2^{x+3} **b** 3^{x+1} **c** 7^{-x} **d** 5^{2x-3} **e** 10^{6x}
f 5^{-8x} **g** 6^{14x} **h** 2^{3x-4}
18a x^6y^4 **b** $\frac{y}{x^2}$ **c** $\frac{21a^3}{x}$ **d** $\frac{1}{3st^2}$ **e** $\frac{7x}{y^2}$ **f** $\frac{5b^{10}}{4a^6}$ **g** $\frac{s^6}{y^9}$
h $\frac{c^2}{5d^3}$ **i** $27x^8y^{17}$ **j** $\frac{2a^7}{y^{15}}$ **k** $5s^5$ **l** $\frac{250x^8}{y^{12}}$
19a $x^2 + 2 + \frac{1}{x^2}$ **b** $x^2 - 2 + \frac{1}{x^2}$ **c** $x^4 - 2 + \frac{1}{x^4}$
20a 2^{x+1} **b** 2^{x+1} **c** 3^{x+1} **d** 3^{x+1} **e** 2^{x+2} **f** 2^{x+5}
g 5^{x+3} **h** 3^{x+4} **i** 2^{x-1} **j** 3^{x-2}
21a $x = -1$ **b** $x = 6$ **c** $x = 8$ **d** $x = -1$
e $x = -4$ **f** $x = 2$
22a Take the reciprocal: 5.97×10^{26}
b $5.73 \times 10^{-45}\text{m}^3$ **c** $2.9 \times 10^{17}\text{kg/m}^3$

Exercise 7B

1a 5 **b** 6 **c** 10 **d** 3 **e** 4 **f** 10 **g** 3 **h** 2 **i** 10 **j** 1000
2a 125 **b** 27 **c** 9 **d** 4 **e** 8 **f** 27 **g** 81 **h** 32 **i** 8 **j** 16
3a $\frac{1}{7}$ **b** $\frac{1}{2}$ **c** $\frac{5}{7}$ **d** $\frac{3}{2}$ **e** $\frac{1}{8}$ **f** $\frac{1}{125}$ **g** $\frac{8}{27}$ **h** $\frac{27}{1000}$

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4a $28\,561$ **b** 109.5 **c** 1.126×10^{15} **d** 15 **e** 2.154
f 2.031 **g** 7.225×10^{-11} **h** 0.1969

5a x **b** x^6 **c** $x^{3\frac{1}{2}}$ **d** x **e** $x^{\frac{1}{2}}$ **f** $x^{-4\frac{1}{2}}$ **g** x^2 **h** x^{-4} **i** x^6

6a $2^1 = 2$ **b** $2^0 = 1$ **c** $2^3 = 8$ **d** $3^{-1} = \frac{1}{3}$ **e** $25^{\frac{1}{2}} = 5$
f $7^0 = 1$ **g** $3^{-3} = \frac{1}{27}$ **h** $3^{-2} = \frac{1}{9}$ **i** $9^2 = 81$

7a $x = \frac{1}{2}$ **b** $x = \frac{1}{2}$ **c** $x = \frac{1}{4}$ **d** $x = \frac{1}{6}$ **e** $x = \frac{1}{2}$ **f** $x = \frac{1}{3}$

8a $\sqrt{x}$ **b** $\sqrt[3]{x}$ **c** $7\sqrt{x}$ **d** $\sqrt{7x}$ **e** $15\sqrt[4]{x}$ **f** $\sqrt{x^3}$ or $\left(\sqrt{x}\right)^3$
g $6\sqrt{x^5}$ or $6\left(\sqrt{x}\right)^5$ **h** $\sqrt[3]{x^4}$ or $\left(\sqrt[3]{x}\right)^4$

9a $x^{\frac{1}{2}}$ **b** $3x^{\frac{1}{2}}$ **c** $(3x)^{\frac{1}{2}}$ **d** $12x^{\frac{1}{3}}$ **e** $9x^{\frac{1}{6}}$ **f** $x^{\frac{3}{2}}$ **g** $x^{\frac{9}{2}}$ **h** $25x^{\frac{6}{5}}$

10a $\frac{1}{5}$ **b** $\frac{1}{10}$ **c** $\frac{1}{5}$ **d** $\frac{1}{2}$ **e** $\frac{1}{8}$ **f** $\frac{1}{9}$ **g** $\frac{1}{27}$ **h** $\frac{1}{343}$

11a 2 **b** 5 **c** 7 **d** 3 **e** 8 **f** 27 **g** $\frac{27}{8}$ **h** $\frac{4}{25}$

12a $9xy^3$ **b** $35b$ **c** $3s^{\frac{1}{2}}$ **d** $x^{1\frac{1}{2}}y^{2\frac{1}{2}}$ **e** a **f** $a^{-1}b^2$
g $2xy^{-2}$ **h** p^2q^{-6} **i** x^7

13a $x^{-\frac{1}{2}}$ **b** $12x^{-\frac{1}{2}}$ **c** $-5x^{-\frac{1}{2}}$ **d** $15x^{-\frac{1}{3}}$ **e** $-4x^{-\frac{2}{3}}$
f $x^{1\frac{1}{2}}$ **g** $5x^{-1\frac{1}{2}}$ **h** $8x^{2\frac{1}{2}}$

14a 9 **b** -3 **c** $\frac{1}{20}$ **d** $\frac{3}{10}$

15a $\$6000 \times (1.03)^0 = \$6000$
b $\$6000 \times (1.03)^1 = \$6180$
c i $\$6000 \times (1.03)^5 \doteqdot \$6960$
ii $\$6000 \times (1.03)^{\frac{1}{2}} \doteqdot \$6090$
iii $\$6000 \times (1.03)^{\frac{7}{2}} \doteqdot \$6650$

16a 5.765×10^6 **b** 1.261×10^1 **c** 8.244×10^{-1}
d 7.943×10^{-3} **e** 8.825×10^0 **f** 2.595×10^1
g 7.621×10^{-2} **h** 5.157×10^4

17a $x + 2 + x^{-1}$ **b** $x - 2 + x^{-1}$ **c** $x^5 - 2 + x^{-5}$

18a $x = -\frac{1}{2}$ **b** $x = -\frac{1}{4}$ **c** $x = \frac{2}{3}$ **d** $x = -\frac{2}{3}$ **e** $x = \frac{3}{2}$
f $x = -\frac{3}{2}$ **g** $x = \frac{3}{4}$ **h** $x = -\frac{4}{3}$ **i** $x = -\frac{1}{2}$ **j** $x = -\frac{2}{3}$

19a $3^{\frac{1}{3}} > 2^{\frac{1}{2}}$ **b** $2^{\frac{1}{2}} > 5^{\frac{1}{5}}$ **c** $7^{\frac{3}{2}} < 20$ **d** $5^{\frac{1}{5}} < 3^{\frac{1}{3}}$

Exercise 7C

1a because $2^3 = 8$. **b** because $5^2 = 25$.
c because $10^3 = 1000$. **d** so $\log_7 49 = 2$.
e so $\log_3 81 = 4$. **f** so $\log_{10} 100\,000 = 5$.

2a $x = a^y$ **b** $x = \log_a y$

3a $10^x = 10\,000, x = 4$ **b** $10^x = 1000, x = 3$
c $10^x = 100, x = 2$ **d** $10^x = 10, x = 1$
e $10^x = 1, x = 0$ **f** $10^x = \frac{1}{10}, x = -1$
g $10^x = \frac{1}{100}, x = -2$ **h** $10^x = \frac{1}{1000}, x = -3$

4a $3^x = 9, x = 2$ **b** $5^x = 125, x = 3$ **c** $7^x = 49, x = 2$
d $2^x = 64, x = 6$ **e** $4^x = 64, x = 3$ **f** $8^x = 64, x = 2$
g $8^x = 8, x = 1$ **h** $8^x = 1, x = 0$ **i** $7^x = \frac{1}{7}, x = -1$
j $12^x = \frac{1}{12}, x = -1$ **k** $11^x = \frac{1}{121}, x = -2$
l $6^x = \frac{1}{36}, x = -2$ **m** $4^x = \frac{1}{64}, x = -3$
n $8^x = \frac{1}{64}, x = -2$ **o** $2^x = 64, x = -6$
p $5^x = \frac{1}{125}, x = -3$

5a $x = 7^2 = 49$ **b** $x = 9^2 = 81$ **c** $x = 5^3 = 125$
d $x = 2^5 = 32$ **e** $x = 4^3 = 64$ **f** $x = 100^3 = 1\,000\,000$
g $x = 7^1 = 7$ **h** $x = 11^0 = 1$ **i** $x = 13^{-1} = \frac{1}{13}$
j $x = 7^{-1} = \frac{1}{7}$ **k** $x = 10^{-2} = \frac{1}{100}$ **l** $x = 12^{-2} = \frac{1}{144}$
m $x = 5^{-3} = \frac{1}{125}$ **n** $x = 7^{-3} = \frac{1}{343}$ **o** $x = 2^{-5} = \frac{1}{32}$
p $x = 3^{-4} = \frac{1}{81}$

6a $x^2 = 49, x = 7$ **b** $x^3 = 8, x = 2$ **c** $x^3 = 27, x = 3$
d $x^4 = 10\,000, x = 10$ **e** $x^2 = 10\,000, x = 100$
f $x^6 = 64, x = 2$ **g** $x^2 = 64, x = 8$ **h** $x^1 = 125, x = 125$
i $x^1 = 11, x = 11$ **j** $x^{-1} = \frac{1}{17}, x = 17$
k $x^{-1} = \frac{1}{6}, x = 6$ **l** $x^{-1} = \frac{1}{7}, x = 7$ **m** $x^{-2} = \frac{1}{9}, x = 3$
n $x^{-2} = \frac{1}{49}, x = 7$ **o** $x^{-3} = \frac{1}{8}, x = 2$
p $x^{-2} = \frac{1}{81}, x = 9$

7a $a^x = a, x = 1$ **b** $x = a^1 = a$ **c** $x^1 = a, x = a$
d $a^x = \frac{1}{a}, x = -1$ **e** $x = a^{-1} = \frac{1}{a}$ **f** $x^{-1} = \frac{1}{a}, x = a$
g $a^x = 1, x = 0$ **h** $x = a^0 = 1$
i $x^0 = 1$ where x can be any positive number.

8a 1 **b** -1 **c** 3 **d** -2 **e** -5 **f** $\frac{1}{2}$ **g** $-\frac{1}{2}$ **h** 0

9a 1 and 2 **b** 2 and 3 **c** 0 and 1 **d** 3 and 4
e 5 and 6 **f** 9 and 10 **g** -1 and 0 **h** -2 and -1

10a 1 and 2 **b** 0 and 1 **c** 3 and 4 **d** 0 and 1 **e** 3 and 4
f 4 and 5 **g** 2 and 3 **h** 1 and 2 **i** -1 and 0
j -2 and -1

11a 0.301 **b** 1.30 **c** 2.00 **d** 20.0 **e** 3.16 **f** 31.6
g 0.500 **h** 1.50 **i** 3 **j** 6 **k** 1000 **l** $1\,000\,000$
m -0.155 **n** -2.15 **o** 0.700 **p** 0.00708

12a $\log_{10} 45 \doteqdot 1.7$ **b** $10^{1.7} \doteqdot 50$

13a $7^x = \sqrt{7}, x = \frac{1}{2}$ **b** $11^x = \sqrt{11}, x = \frac{1}{2}$
c $x = 9^{\frac{1}{2}} = 3$ **d** $x = 144^{\frac{1}{2}} = 12$ **e** $x^{\frac{1}{2}} = 3, x = 9$
f $x^{\frac{1}{2}} = 13, x = 169$ **g** $6^x = \sqrt[3]{6}, x = \frac{1}{3}$
h $9^x = 3, x = \frac{1}{2}$ **i** $x = 64^{\frac{1}{3}} = 4$ **j** $x = 16^{\frac{1}{4}} = 2$
k $x^{\frac{1}{3}} = 2, x = 8$ **l** $x^{\frac{1}{6}} = 2, x = 64$ **m** $8^x = 2, x = \frac{1}{3}$

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n $125^x = 5,\ x = \frac{1}{3}$ **o** $x = 7^{\frac{1}{2}}$ or $\sqrt{7}$ **p** $x = 7^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{7}}$
q $x^{-\frac{1}{2}} = \frac{1}{7},\ x = 49$ **r** $x^{-\frac{1}{2}} = \frac{1}{20},\ x = 400$
s $4^x = \frac{1}{2},\ x = -\frac{1}{2}$ **t** $27^x = \frac{1}{3},\ x = -\frac{1}{3}$
u $x = 121^{-\frac{1}{2}} = \frac{1}{11}$ **v** $x = 81^{-\frac{1}{4}} = \frac{1}{3}$ **w** $x^{-\frac{1}{4}} = \frac{1}{2},\ x = 16$
x $x^{-\frac{1}{4}} = 2,\ x = \frac{1}{16}$

Exercise 7D

1a $\log_6 36 = 2$ **b** $\log_5 25 = 2$ **c** $\log_2 8 = 3$
2a $\log_6 6 = 1$ **b** $\log_{15} 15 = 1$ **c** $\log_{10} 100 = 2$
d $\log_{12} 144 = 2$ **e** $\log_{10} 1000 = 3$ **f** $\log_6 36 = 2$
3a $\log_3 3 = 1$ **b** $\log_4 4 = 1$ **c** $\log_2 8 = 3$
d $\log_5 25 = 2$ **e** $\log_3 81 = 4$ **f** $\log_2 32 = 5$
4a 1 **b** 2 **c** 3 **d** 2 **e** 0 **f** −2 **g** −3 **h** 2 **i** 0
5a $3\log_a 2$ **b** $4\log_a 2$ **c** $6\log_a 2$ **d** $-\log_a 2$
e $-3\log_a 2$ **f** $-5\log_a 2$ **g** $\frac{1}{2}\log_a 2$ **h** $-\frac{1}{2}\log_a 2$
6a $2\log_2 3$ **b** $2\log_2 5$ **c** $1 + \log_2 3$ **d** $1 + \log_2 5$
e $1 + 2\log_2 3$ **f** $2 + \log_2 5$ **g** $1 - \log_2 3$
h $-1 + \log_2 5$
7a 3.90 **b** 3.16 **c** 3.32 **d** 5.64 **e** 0.58
f −0.74 **g** −0.58 **h** 6.22
8a 3 **b** 5 **c** 1.3 **d** n
9a 100 **b** 7 **c** 3.6 **d** y
10a 2 **b** 15 **c** −1 **d** 6
11a $3\log_a x$ **b** $-\log_a x$ **c** $\frac{1}{2}\log_a x$ **d** $-2\log_a x$
e $-2\log_a x$ **f** $2\log_a x$ **g** $8 - 8\log_a x$ **h** $\log_a x$
12a $\log_a y + \log_a z$ **b** $\log_a z - \log_a y$ **c** $4\log_a y$
d $-2\log_a x$ **e** $\log_a x + 3\log_a y$
f $2\log_a x + \log_a y - 3\log_a z$ **g** $\frac{1}{2}\log_a y$
h $\frac{1}{2}\log_a x + \frac{1}{2}\log_a z$
13a 1.30 **b** −0.70 **c** 2.56 **d** 0.15 **e** 0.45
f −0.50 **g** 0.54 **h** −0.35
14a $6x$ **b** $-x - y - z$ **c** $3y + 5$ **d** $2x + 2z - 1$
e $y - x$ **f** $x + 2y - 2z - 1$ **g** $-2z$ **h** $3x - y - z - 2$
15a $10 = 3^{\log_3 10}$ **b** $3 = 10^{\log_{10} 3}$ **c** $0.1 = 2^{\log_2 0.1}$

Exercise 7E

1a–c Answer is in question
2a 2.807 **b** 4.700 **c** −3.837 **d** 7.694
e 0.4307 **f** 1.765 **g** 0.6131 **h** 0.2789
i −2.096 **j** −7.122 **k** 2.881 **l** 7.213
m 0.03323 **n** 578.0 **o** −687.3
3a $x = \log_2 15 \doteqdot 3.907$
b $x = \log_2 5 \doteqdot 2.322$
c $x = \log_2 1.45 \doteqdot 0.5361$
d $x = \log_2 0.1 \doteqdot -3.322$
e $x = \log_2 0.0007 \doteqdot -10.48$
f $x = \log_3 10 \doteqdot 2.096$
g $x = \log_3 0.01 \doteqdot -4.192$
h $x = \log_5 10 \doteqdot 1.431$ **i** $x = \log_{12} 150 \doteqdot 2.016$
j $x = \log_8 \frac{7}{9} \doteqdot -0.1209$ **k** $x = \log_6 1.4 \doteqdot 0.1878$
l $x = \log_{30} 2 \doteqdot 0.2038$ **m** $x = \log_{0.7} 0.1 \doteqdot 6.456$
n $x = \log_{0.98} 0.03 \doteqdot 173.6$
o $x = \log_{0.99} 0.01 \doteqdot 458.2$
4a $x > 5$ **b** $x \le 5$ **c** $x < 6$ **d** $x \ge 4$ **e** $x > 1$
f $x \le 0$ **g** $x < -1$ **h** $x \le -3$
5a $0 < x < 8$ **b** $x \ge 8$ **c** $x > 1000$ **d** $x \ge 10$
e $x > 1$ **f** $0 < x < 6$ **g** $0 < x \le 125$ **h** $x > 36$
6a $x > \log_2 12 \doteqdot 3.58$
b $x < \log_2 100 \doteqdot 6.64$
c $x < \log_2 0.02 \doteqdot -5.64$
d $x > \log_2 0.1 \doteqdot -3.32$
e $x < \log_5 100 \doteqdot 2.86$
f $x < \log_3 0.007 \doteqdot -4.52$
g $x > \log_{1.2} 10 \doteqdot 12.6$
h $x > \log_{1.001} 100 \doteqdot 4610$
7a After 1 year, the price is 1.05 times greater, after 2 years, it is $(1.05)^2$ times greater, and so on.
b $\log_{1.05} 1.5 \doteqdot 8.3$ years
8a $\log_8 x = \frac{\log_2 x}{\log_2 8} = \frac{1}{3}\log_2 x$
b $\log_{a^n} x = \frac{\log_a x}{\log_a a^n} = \frac{1}{n}\log_a x$
9a–c Answers will vary
10a $x = 3$ **b** $x = 2$ **c** $x < 1$ **d** $x \le 9$ **e** $x = 0$ **f** $x = \frac{1}{5}$
g $x < 4.81$ **h** $x > -2.90$
11a $x < 33.2$, 33 powers **b** $x < 104.8$, 104 powers
12a $10^2 < 300 < 10^3$ **b** $1 \le \log_{10} x < 2$ **c** 5 digits
d 27.96, 28 digits **e** $1000\log_{10} 2 = 301.03$, 302 digits

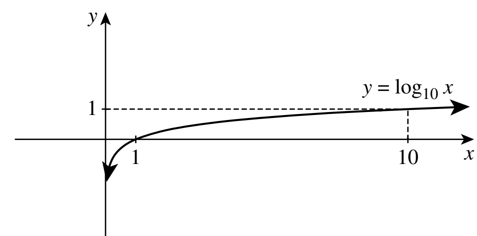
Exercise 7F

1a

x	0.1	0.25	0.5	0.75	1	2
$\log_{10} x$	−1	−0.60	−0.30	−0.12	0	0.30

x	3	4	5	6	7	8	9	10
$\log_{10} x$	0.48	0.60	0.70	0.78	0.85	0.90	0.95	1

b



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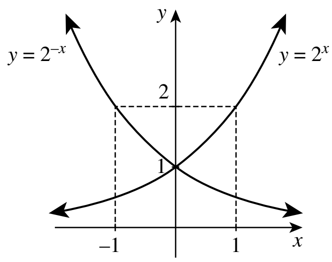
2a i

x	-3	-2	-1	0	1	2	3
2^x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

ii

x	-3	-2	-1	0	1	2	3
2^x	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

b



c The values of $y = 2^{-x}$ are the values of $y = 2^x$ in reverse order.

d The two graphs are reflections of each other in the y-axis, because x has been replaced with $-x$.

e **i** and **ii** For both, domain: all real x, range: $y > 0$

f **i** and **ii** For both, the asymptote is $y = 0$ (the x-axis).

g i 'As $x \to -\infty$, $2^x \to 0$.'

ii 'As $x \to \infty$, $2^x \to \infty$.'

h i 'As $x \to -\infty$, $2^{-x} \to \infty$.'

ii 'As $x \to \infty$, $2^{-x} \to 0$.'

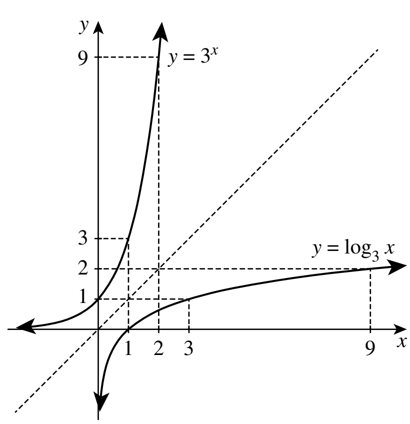
3a i

x	-2	-1	0	1	2
3^x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

ii

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$\log_3 x$	-2	-1	0	1	2

b



c The two rows have been exchanged.

d The two graphs are reflections of each other in the diagonal line $y = x$, because the two functions are inverses of each other.

e i domain: all real x, range: $y > 0$

ii domain $x > 0$, range: all real y

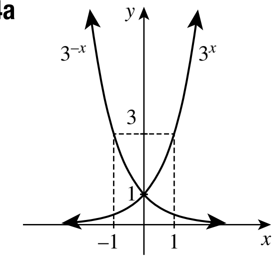
f i $y = 0$ (the x-axis)

ii $x = 0$ (the y-axis)

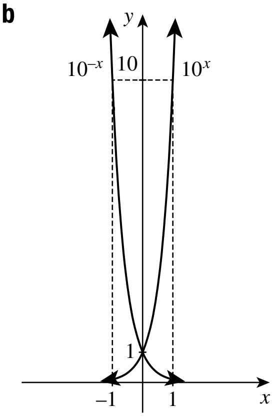
g i 'As $x \to -\infty$, $3^x \to 0$.'

ii 'As $x \to 0^+$, $\log_3 x \to -\infty$.'

4a



b



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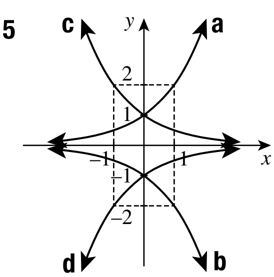
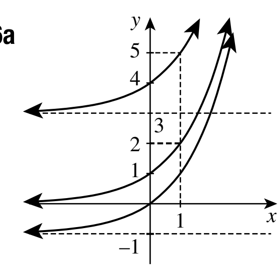


6a



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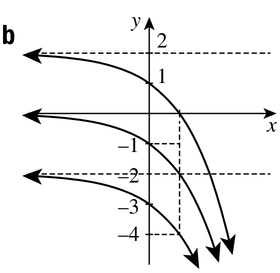

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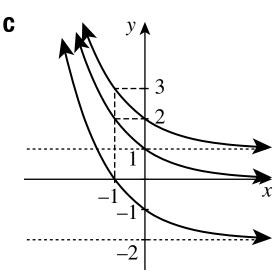

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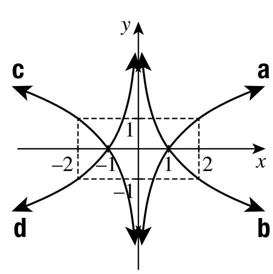

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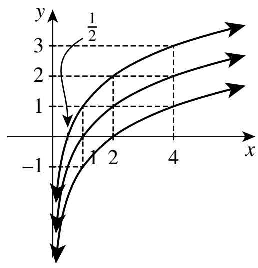
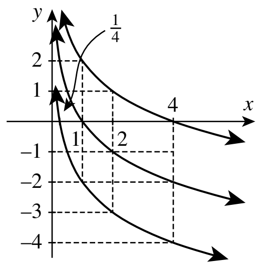
8a



b



9a i 4 **ii** $\frac{1}{4}$ **iii** 2.83 **iv** 1.32 **v** 0.66

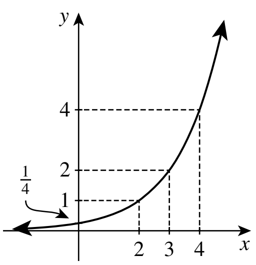
b i 1 **ii** 1.58 **iii** 0.26 **iv** −1.32

c i $0 \le x \le 2$ **ii** $0 \le x \le 1$ **iii** $0.58 \le x \le 1.58$

iv $-1 \le x \le 1$

d i 2 **ii** 1.58 **iii** 0.49 **iv** −0.32

10a



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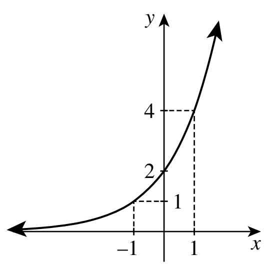

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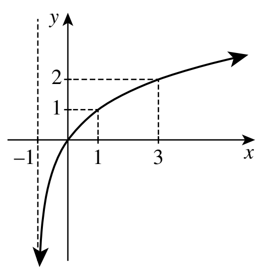
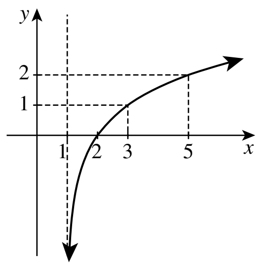
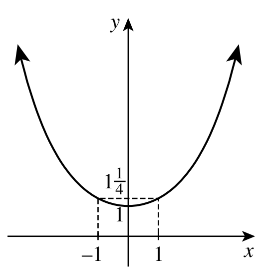


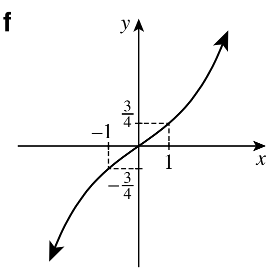
d



e



f



Exercise 7G

1a 5000, 2594 **b** $\frac{t}{2} = \log_{10}\frac{Q}{5}$, so $t = 2\log_{10}\frac{Q}{5}$

c 4, 3.419

2a 60, $20\log_2 12 = \frac{20\log_{10}12}{\log_{10}2} \doteqdot 71.70$

b $\frac{t}{20} = \log_2 2Q$, so $2Q = 2^{\frac{t}{20}}$, so $Q = \frac{1}{2} \times 2^{\frac{t}{20}}$

c 2, 2.378

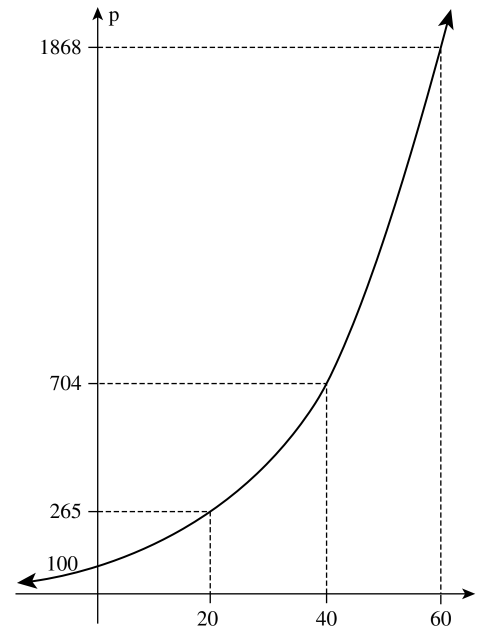
3a There are $\frac{n}{30}$ thirty–year intervals in n years.

b i 24 000 000 **ii** 30 000 000

c i 120 years **ii** $30\log_2 20 \doteqdot 130$ years

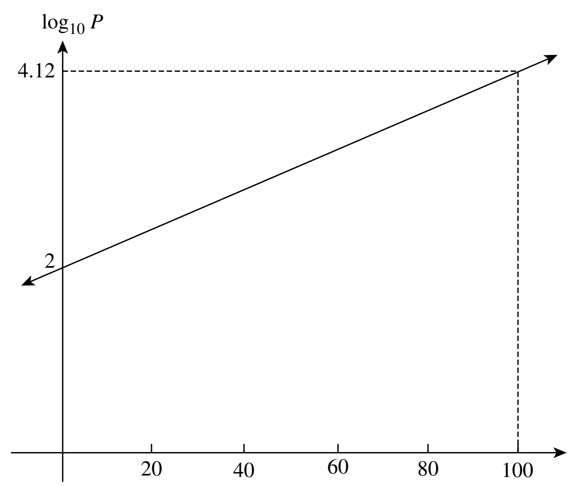
4a P: 100, 265, 704, 1868, 4956, 13 150

b $P = 100 \times (1.05)^n$



c The values are about 2, 2.42, 2.85, 3.27, 3.70, 4.12.

d $\log_{10} P$



e The new graph is a straight line, and $\log_{10} P$ is a linear function of n.

5a $\frac{n}{2}$ is the number of 2–year periods

b $D = 2^{20}D_0 \doteqdot 1\,050\,000\,D_0$

c $2^{\frac{n}{2}} = 10^7$, so $\frac{n}{2} = \log_2 10^7$, so $n = 2\log_2 10^7 \doteqdot 47$ years, that is, in 2022.

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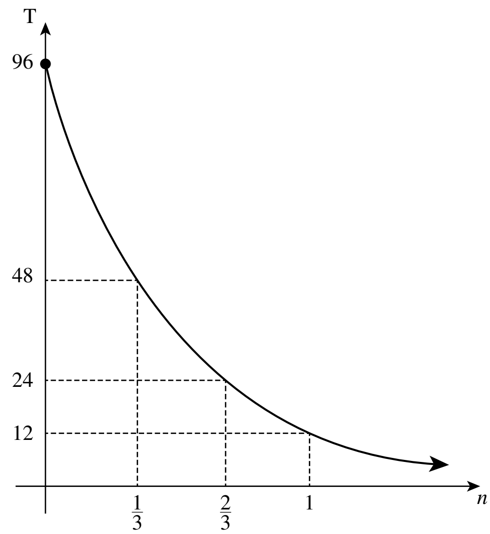
6a $3n$ is the number of 20-minutes periods in n hours.

b $T = 96 \times \left(\frac{1}{2}\right)^{3n}$



c $96 \times \left(\frac{1}{2}\right)^6 = 1\frac{1}{2}°C$

d $3n = \log_{\frac{1}{2}} \frac{T}{96}$ $\left(\text{Alternatively } n = -\frac{1}{3}\log_2 \frac{T}{96}.\right)$

e $n = \frac{1}{3}\log_{\frac{1}{2}} \frac{1}{96} = 2.1949\ldots \text{h} \doteqdot 2\text{h}\,12\,\text{min}$

7a The mass halves every 700 000 000 years.

b When $n = 4$ billion, $\frac{n}{700\,000\,000} = \frac{40}{7}$,

so $M = M_0 \times \left(\frac{1}{2}\right)^{\frac{40}{7}} \doteqdot 1.9\%$ of M_0

c When $n = -4.5$ billion, $\frac{n}{700\,000\,000} = -\frac{45}{7}$,

so $M = M_0 \times \left(\frac{1}{2}\right)^{\frac{-45}{7}} \doteqdot 86M_0$

8a 1000 **b** $1000^{\frac{3}{2}} \doteqdot 32\,000$

c Ratio of shaking amplitudes is 10^5, ratio of energies released is about 3.2×10^7.

9a $[H^+] = 10^{-pH}$ **b** About 10^{-7}mol/L

c About 10^{-2} mol/L, about 100 000 times more acidic than water

d About 7.94×10^{-9} mol/L, about 12.6 times more alkaline than water.

Chapter 7 review exercise

1a 125 **b** 256 **c** 1 000 000 000 **d** $\frac{1}{17}$ **e** $\frac{1}{81}$ **f** $\frac{1}{8}$ **g** $\frac{1}{81}$
h 1 **i** $\frac{8}{27}$ **j** $\frac{12}{7}$ **k** $\frac{36}{25}$ **l** 6 **m** 3 **n** 4 **o** 243 **p** $\frac{2}{7}$ **q** 1
r $\frac{5}{3}$ **s** $\frac{4}{9}$ **t** $\frac{1000}{27}$

2a x^{-1} **b** $7x^{-2}$ **c** $-\frac{1}{2}x^{-1}$ **d** $x^{\frac{1}{2}}$
e $30x^{\frac{1}{2}}$ **f** $4x^{-\frac{1}{2}}$ **g** yx^{-1} **h** $2yx^{\frac{1}{2}}$

3a x^{20} **b** $\frac{81}{a^{12}}$ **c** $5x^3$ **d** $\frac{2r}{t^2}$

4a x^3y^3 **b** $60xy^3z^5$ **c** $18x^{-1}y^{-2}$ **d** $4a^3b^3c^{-1}$ **e** x^2y^{-2}
f $2x^{-3}y$ **g** m^2n^{-1} **h** $72s^9t^3$ **i** $8x^3y^{-3}$

5a 4 **b** 2 **c** -1 **d** -5 **e** 2 **f** 3 **g** $\frac{1}{2}$ **h** $\frac{1}{3}$

6a $2^x = 8, x = 3$ **b** $3^x = 9, x = 2$ **c** $10^x = 10\,000, x = 4$
d $5^x = \frac{1}{5}, x = -1$ **e** $7^x = \frac{1}{49}, x = -2$ **f** $13^x = 1, x = 0$
g $9^x = 3, x = \frac{1}{2}$ **h** $2^x = \sqrt{2}, x = \frac{1}{2}$ **i** $7^2 = x, x = 49$
j $11^{-1} = x, x = \frac{1}{11}$ **k** $16^{\frac{1}{2}} = x, x = 4$ **l** $27^{\frac{1}{3}} = x, x = 3$
m $x^2 = 36, x = 6$ **n** $x^3 = 1000, x = 10$
o $x^{-1} = \frac{1}{7}, x = 7$ **p** $x^{\frac{1}{2}} = 4, x = 16$

7a 1 **b** 2 **c** 2 **d** -2 **e** 2 **f** 0

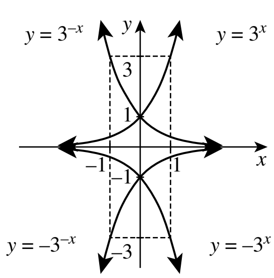
8a $\log_a x + \log_a y + \log_a z$ **b** $\log_a x - \log_a y$
c $3\log_a x$ **d** $-2\log_a z$ **e** $2\log_a x + 5\log_a y$
f $2\log_a y - \log_a x - 2\log_a z$ **g** $\frac{1}{2}\log_a x$
h $\frac{1}{2}\log_a x + \frac{1}{2}\log_a y + \frac{1}{2}\log_a z$

9a 1 and 2 **b** 2 and 3 **c** 4 and 5 **d** 5 and 6 **e** -1 and 0
f -3 and -2 **g** -4 and -3 **h** -2 and -1

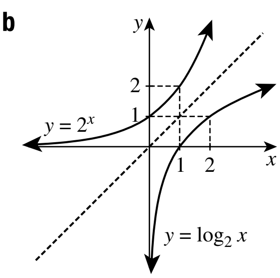
10a 2.332 **b** -2.347 **c** 2.010 **d** 9.966 **e** -0.9551
f 69.66 **g** -3 **h** 687.3

11a 3.459 **b** -4.644 **c** 3.010 **d** -0.3645 **e** 161.7
f -161.7 **g** 10.32 **h** 458.2

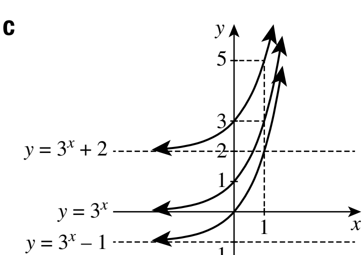
12a



b



c



13a There are $\frac{n}{4}$ four-hour periods in n hours.

b i 800 **ii** $100 \times 2^{3.25} \doteqdot 950$

c $\frac{n}{4} = \log_2 \frac{P}{100}$, so $n = 4\log_2 \frac{P}{100}$

d $4\log_2 100\,000 \doteqdot 66$ hours

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Chapter 8

Exercise 8A

1 The values of $f'(x)$ should be about -4, -3, -2, -1, 0, 1, 2, 3, 4. The graph of $y = f'(x)$ should approximate a line of gradient 2 through the origin; its exact equation is $f'(x) = 2x$.
2 Answers are the same as for question 1.
3 The values of $f'(x)$ should be about $1\frac{1}{2}$, 0, -0.9, -1.2, -0.9, 0, $1\frac{1}{2}$. The graph of $f'(x)$ is a parabola crossing the x-axis at $x = -2$ and $x = 2$.
4 The eventual graph of $f'(x)$ is a parabola with its vertex at the origin. Depending on the software, you may be able to see that it is $y = 3x^2$.

Exercise 8B

1a 3 **b** -7 **c** 5 **d** -3 **e** $\frac{1}{2}$ **f** 0
2a Answers will vary **b** $10h + 5h^2$
c Answers will vary **d** Answers will vary
3a Answers will vary **b** $10xh + 5h^2$
c, d and e Answers will vary
4a, b and c Answers will vary **d** At A, $f'(1) = -2$
e At B, $f'(3) = 2$; at C, $f'(2) = 0$. **f** Answers will vary
5a Answers will vary **b** $5h$ **c** 5
d The value of $\dfrac{f(x+h) - f(x)}{h}$ is a constant 5, so trivially its limit is 5 as $h \to 0$.
6a 10 **b** $\frac{2}{3}$ **c** -1
7a $2xh + h^2$ **b** $f'(x) = 2x$ **c** $f'(0) = 0$ **d** $f'(3) = 6$
8a $2x + h + 4, 2x + 4$ **b** $f'(0) = 4$, $f'(-2) = 0$
9a $2x + h - 2, f'(x) = 2x - 2$ **b** $f'(0) = -2, f'(2) = 2$
10a $2x + h + 6, f'(x) = 2x + 6$ **b** $f'(0) = 6, f'(-3) = 0$
11a $4 + h, f'(2) = 4$ **b** $2h + 3, f'(0) = 3$
c $-6 + h$, $f'(-1) = -6$
12a $3x^2$ **b** Answers will vary
13a $4x^3$ **b** Answers will vary
14a–d Answers will vary

Exercise 8C

1a $7x^6$ **b** $5x^4$ **c** $-24x^{23}$ **d** $45x^4$ **e** $6x$ **f** $-60x^{11}$
g $2x^5$ **h** $4x^7$ **i** $-6x^8$
2a Answers will vary **b** Answers will vary
3a 5 **b** -1 **c** $2x + 5$ **d** $6x - 5$ **e** $4x^3 - 10x$
f $-3 - 15x^2$ **g** $4x^3 + 3x^2 + 2x + 1$ **h** $x^3 + x^2 + x$
i $2x^5 - 2x^3 + 2x$
4a $2x + 7$ **b** $f'(0) = 7$
5a $f'(x) = -2x, f'(2) = -4$
b $f'(x) = 3x^2 + 6$, $f'(2) = 18$
c $f'(x) = 20x - 4x^3, f'(2) = 8$
6a 12 **b** 3 **c** 0 **d** 3 **e** 12
7a $f'(x) = 2x$ **b** $(0, -4)$ **c** $(3, 5)$ **d** $(-3, 5)$
8a $4 - 2x$ **b** $3x^2 + 1$ **c** $6x - 16x^3$ **d** $2x + 2$ **e** $8x$
f $4x^3 + 12x$ **g** $2x - 14$ **h** $3x^2 - 10x + 3$ **i** $18x - 30$
9a $2x + 1$ **b** $f'(0) = 1$ **c** 45°
10a $-1 + 2x$ **b** Answers will vary **c** 71°34′
11 $f'(x) = 2x - 3$ **a** 3, 71°34′ **b** 1, 45° **c** 0, 0°
d -1, 135° **e** -3, 108°26′
12a $f'(x) = 8 - 2x$
b It is a concave-down parabola with x-intercepts $x = 0$ and $x = 8$. **c** $f'(0) = 8, f'(8) = -8$ **d** $f'(4) = 0$
13a $2x + 8$ **b** $x = -4, (-4, -9)$ **c** $x = 2, (2, 27)$
14a $-4x$ **b** $x = 0, (0, 3)$ **c** $x = 5, (5, -47)$
15a $2x - 2, (1, 6)$ **b** $2x + 4, (-2, -14)$
c $2x - 10, (5, -10)$
16a $f'(x) = 2x - 6$ **b** It is a concave-up parabola with x-intercepts $x = 0$ and $x = 6$.
c $f'(0) = -6, f'(6) = 6$ **d** $(3, -9)$
17 $f'(x) = 2x - 5$ **a** $(4, -3)$ **b** $(0, 1)$ **c** $(3, -5)$
d $(2, -5)$
18a $f'(x) = 3x^2 - 3, (1, 0), (-1, 4)$
b $f'(x) = 4x^3 - 36x, (0, 0), (3, -81), (-3, -81)$
c $f'(x) = 3x^2, (5, 131), (-5, -119)$
19 The tangent has gradient $2a - 6$.
a i 3 **ii** 4 **iii** $3\frac{1}{4}$ **b** $2\frac{1}{2}$ **c i** $3\frac{1}{3}$ **ii** $2\frac{1}{4}$

Exercise 8D

1a $2x$ **b** $2x + 7$ **c** $3x^2 + 6x + 6$ **d** $4x^3 + 2x + 8$
e 4 **f** 0
2a $\dfrac{dy}{dx} = 6x^5 + 2, \dfrac{d^2y}{dx^2} = 30x^4, \dfrac{d^3y}{dx^3} = 120x^3$
b $\dfrac{dy}{dx} = 10x - 5x^4, \dfrac{d^2y}{dx^2} = 10 - 20x^3, \dfrac{d^3y}{dx^3} = -60x^2$
c $\dfrac{dy}{dx} = 4, \dfrac{d^2y}{dx^2} = 0, \dfrac{d^3y}{dx^3} = 0$
3a $f'(x) = 30x^2 + 1, f''(x) = 60x, f'''(x) = 60,$ $f^{(4)}(x) = 0$
b $f'(x) = 8x^3, f''(x) = 24x^2, f'''(x) = 48x, f^{(4)}(x) = 48$
c $f'(x) = 0, f''(x) = 0, f'''(x) = 0, f^{(4)}(x) = 0$
4 $\dfrac{dy}{dx} = 2x + 1,\ x = 3,\ (3, 12)$
5 $\dfrac{dy}{dx} = 3x^2,\ x = 2$ or $-2,\ (2, 7),\ (-2, -9)$
6a $2x - 3$ **b** 1 **c** $(2, -2)$ **d** $y = x - 4$ **e** -1 **f** $y = -x$
7a $5x^4 + 3x^2 + 2$ **b** $y = 0, \dfrac{dy}{dx} = 2$ **c** $y = 2x$ **d** $-\frac{1}{2}$
e $y = -\frac{1}{2}x$
8a They all have derivative $3x^2 + 7$. First to second, shift down 10. First to third, shift down $7\frac{1}{2}$. First to fourth, shift up 96.
b The third has derivative $-2x^3 + 6x$. The other three have derivative $2x^3 + 6x$.
9a $2x$ **b** $6x - 5x^4$ **c** $2x - 3$

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10a $3x^2 - 2,\ 1,\ 10$ **b** $45°,\ 84°17'$
11a $-2 + 2x,\ (1,\ 2)$ **b** $4x^3 + 36x,\ (0,\ 0)$
12 $\dfrac{dy}{dx} = 2x - 8$ **a** $y = -6x + 14,\ x - 6y + 47 = 0$
b $y = 4x - 21,\ x + 4y - 18 = 0$
c $y = -8x + 15,\ x - 8y + 120 = 0$
d $y = -1, x = 4$
13a $2x - 6, y = -6x, y = \frac{1}{6}x$
b $3x^2 - 4,\ y = 8x - 16,\ x + 8y - 2 = 0$
c $2x - 4x^3, y = 2x + 2, x + 2y + 1 = 0$
d $3x^2 - 3, y = 0, x = 1$
14a $y = 4x - 4, y = -\frac{1}{4}x + 4\frac{1}{2}$
b $A = (0, -4),\ B = \left(0, 4\frac{1}{2}\right)$
c $AB = 8\frac{1}{2}$, area $= 8\frac{1}{2}$ square units
15a $y = -2x + 10, x - 2y + 15 = 0$
b $A = (5, 0), B = (-15, 0)$
c $AB = 20$, area $= 80$ square units
16a Answers will vary
b At D, $y = 2x - 5$, at E, $y = -2x - 5$ **c** $(0, -5)$
17a $\dfrac{dy}{dx} = -2x, A = (-1, 3)$ and $B = (1, 3)$
b tangent at A: $y = 2x + 5$, tangent at B: $y = -2x + 5$. They meet at $(0, 5)$.
c normal at A: $y = -\frac{1}{2}x + 2\frac{1}{2}$, normal at B: $y = \frac{1}{2}x + 2\frac{1}{2}$. They meet at $\left(0, 2\frac{1}{2}\right)$.
18 $y = 3x - 2,\ x + 3y = 4,\ P = (0, -2)$, $Q = \left(0, 1\frac{1}{3}\right)$, $|\Delta QUP| = 1\frac{2}{3}$ square units
19a $5x^4, 20x^3, 60x^2, 120x, 120$. Five successive derivatives are non-zero. **b** 6 **c** n
20a $y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$ **b** $y = 2x^3 - 7x + C$
c $y = \frac{5}{4}x^4 + x^3 - 4x + C$ **d** $y = 2x^5 - 4x^3 - 24x + C$
21a $\dfrac{dy}{dx} = 2ax + b$ **b** $x = -\frac{b}{2a}$
c The vertex is the only point on the parabola where the tangent is horizontal.
22a $2a$ **b** $2a$
c Here is one way to express the result: 'Let M be the midpoint of a chord AB of a parabola, and let the line through M parallel to the axis of symmetry meet the parabola at P. Then the chord AB is parallel to the tangent at P.'
23a $b = 7, c = 0$ **b** $b = -2,\ c = -3$ **c** $b = -10,\ c = 25$
d $b = -1,\ c = -2$ **e** $b = -9,\ c = 17$ **f** $b = -5\frac{2}{3},\ c = 7$

Exercise 8E

1a $1x^{-3}$ **b** $-3x^{-4}$ **c** $-\frac{3}{x^4}$
2a $-x^{-2}$ **b** $-5x^{-6}$ **c** $-3x^{-2}$ **d** $-10x^{-3}$ **e** $4x^{-4}$
f $-4x^{-3} - 4x^{-9}$
3a $f(x) = x^{-1}, f'(x) = -x^{-2} = -\frac{1}{x^2}$
b $f(x) = x^{-2}, f'(x) = -2x^{-3} = -\frac{2}{x^3}$
c $f(x) = x^{-4}, f'(x) = -4x^{-5} = -\frac{4}{x^5}$
d $f(x) = 3x^{-1}, f'(x) = -3x^{-2} = -\frac{3}{x^2}$
4a $\dfrac{dy}{dx} = -6x^{-3}, -6$ **b** $\dfrac{dy}{dx} = -60x^{-5}, -60$
c $\dfrac{dy}{dx} = 2 + 2x^{-2}, 4$ **d** $\dfrac{dy}{dx} = 1 - 30x^{-7}, -29$
e $\dfrac{dy}{dx} = -x^{-3} - x^{-4}, -2$ **f** $\dfrac{dy}{dx} = 6x^5 - 6x^{-7}, 0$
5a $y = x^2 + x, \dfrac{dy}{dx} = 2x + 1$
b $y = x^{-2} + x^{-3}, \dfrac{dy}{dx} = -2x^{-3} - 3x^{-4}$
c $y = 4x^{-1} - 5x^2, \dfrac{dy}{dx} = -4x^{-2} - 10x$
d $y = 3x^{-4} + 3, \dfrac{dy}{dx} = -12x^{-5}$
6a $f'(x) = -\frac{1}{x^2}, f'(3) = -\frac{1}{9}, f'\left(\frac{1}{3}\right) = -9$
b $(1, 1),\ (-1, -1)$ **c** $\left(\frac{1}{2}, 2\right),\ \left(-\frac{1}{2}, -2\right)$
d No; the derivative $-\frac{1}{x^2}$ can never be zero.
e Yes, all of them; the derivative $-\frac{1}{x^2}$ is negative for all points on the curve.
7a $f'(x) = \frac{3}{x^2}, f'(2) = \frac{3}{4}, f'(6) = \frac{1}{12}$
b $(1, -3)$ and $(-1, 3)$
8a $f'(x) = -\frac{12}{x^2}, f'(2) = -3, f'(6) = -\frac{1}{3}$
b At $M(2, 6)$, tangent: $y = -3x + 12$, normal: $x - 3y + 16 = 0$. At $N(6, 2)$, tangent: $y = -\frac{1}{3}x + 4$, normal: $y = 3x - 16$.
c $(1,\ 12)$ and $(-1,\ -12)$
9a $-\frac{6}{x^7} + \frac{8}{x^9}$ **b** $-\frac{1}{3x^2}$ **c** $-\frac{15}{x^4}$ **d** $-\frac{4}{5x^5}$ **e** $\frac{7}{x^2}$ **f** $-\frac{7}{2x^2}$
g $\frac{7}{3x^2}$ **h** $\frac{3}{x^6}$
10a $4x^3 - 2x,\ 2$ **b** $2x^5 - 2x^3 + 2x,\ 2$
c $\frac{1}{2}x^4 - \frac{1}{2}x^2 + \frac{1}{2},\ \frac{1}{2}$ **d** $2x,\ 2$ **e** $6x - 5x^4,\ 1$
f $4x - 5,\ -1$ **g** $-4x^{-2},\ -4$ **h** $-2x^{-4},\ -2$
i $-x^{-2} + 2x^{-3},\ 1$
11a $y = 3x^3 - 5x, \dfrac{dy}{dx} = 9x^2 - 5$ **b** $y = x^2 - 4, \dfrac{dy}{dx} = 2x$
c $y = \frac{5}{3}x^3 + \frac{4}{3}x^2, \dfrac{dy}{dx} = 5x^2 + \frac{8}{3}x$
d $y = 3x - x^{-1}, \dfrac{dy}{dx} = 3 + x^{-2}$
e $y = x^{-3} + 7x^{-2}, \dfrac{dy}{dx} = -3x^{-4} - 14x^{-3}$
f $y = 3x^3 - 5x + x^{-1}, \dfrac{dy}{dx} = 9x^2 - 5 - x^{-2}$
12 $-\dfrac{a}{x^2} - \dfrac{2b}{cx^3}$

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13a $f'(x) = -x^{-2}, f'''(x) = 2x^{-3}, f'''(x) = -6x^{-4}$,
$f^{(4)}(x) = 24x^{-5}, f^{(5)}(x) = -120x^{-6}$

b $f'(1) = -1, f''(1) = 2, f'''(1) = -6, f^{(4)}(1) = 24$,
$f^{(5)}(1) = -120$

c Start with -1, then multiply by $-n$ to get each next term.

d Same as before, except that all the terms are negative.

15a The tangent has gradient $2a + 15$, and it passes through $(a, a^2 + 15a + 36)$. Now use point–gradient form.

b Solving $0 = 0 - a^2 + 36$ gives $a = 6$ or $a = -6$. Substituting these values into the equation of the tangent gives $y = 27x$ or $y = 3x$.

Exercise 8F

1a $y = 20x^{\frac{1}{2}}$ **b** $\dfrac{dy}{dx} = 10x^{-\frac{1}{2}}$ **c** $\dfrac{dy}{dx} = \dfrac{10}{x^{\frac{1}{2}}} = \dfrac{10}{\sqrt{x}}$

2a $\frac{1}{2}x^{-\frac{1}{2}}$ **b** $-\frac{1}{2}x^{-1\frac{1}{2}}$ **c** $\frac{3}{2}x^{\frac{1}{2}}$ **d** $4x^{-\frac{1}{3}}$ **e** $-4x^{-1\frac{1}{3}}$

f $x^{-\frac{3}{4}} - 2x^{-\frac{5}{4}}$ **g** $\frac{49}{3}x^{1\frac{1}{3}}$ **h** $-\frac{10}{3}x^{-1\frac{2}{3}}$ **i** $6x^{-1.6}$

3a $\frac{1}{2}x^{-\frac{1}{2}}$ **b** $\frac{1}{3}x^{-\frac{2}{3}}$ **c** $\frac{1}{4}x^{-\frac{3}{4}}$ **d** $5x^{-\frac{1}{2}}$

4 $\dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}, \dfrac{d^2y}{dx} = -\frac{1}{4}x^{-1\frac{1}{2}}$

5a $y = x\sqrt{x} = x^1 \times x^{\frac{1}{2}} = x^{1\frac{1}{2}} = x^{\frac{3}{2}}, \dfrac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

b $y = x^2\sqrt{x} = x^2 \times x^{\frac{1}{2}} = x^{2\frac{1}{2}} = x^{\frac{5}{2}}, \dfrac{dy}{dx} = \frac{5}{2}x^{1\frac{1}{2}}$

c $y = x^{-\frac{1}{2}}, \dfrac{dy}{dx} = -\frac{1}{2}x^{-1\frac{1}{2}}$

d $y = \dfrac{1}{x^1 \times x^{\frac{1}{2}}} = x^{-1\frac{1}{2}} = x^{-\frac{3}{2}}, \dfrac{dy}{dx} = -\frac{3}{2}x^{-2\frac{1}{2}}$

6a $y = x^{\frac{1}{2}}, \dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ **b** $\frac{1}{2}$ and $\frac{1}{4}$

c $y = \frac{1}{2}x + \frac{1}{2}, y = \frac{1}{4}x + 1$ **d** $-2, -4$

e $y = -2x + 3, y = -4x + 18$

7a $y = 4x^{-\frac{1}{2}}, \dfrac{dy}{dx} = -2x^{-1\frac{1}{2}} = -\frac{2}{x\sqrt{x}}$

b $y = -\dfrac{1}{4}x + 3, y = 4x - 14$

c $\dfrac{1}{\sqrt{x}}$ is undefined when $x \le 0$; $-\dfrac{2}{x\sqrt{x}} < 0$, for all $x > 0$

8a $\dfrac{1}{\sqrt{x}}, \dfrac{1}{2}, -2$ **b** $y = \dfrac{1}{2}x + 2, y = -2x + 12$

c $A(-4, 0),\ B(6, 0)$ **d** $AB = 10$, area $= 20$ square units

9a $(1, 1)$ and $(-1, -1)$ **b** $\left(1, \frac{1}{2}\right)$ **c** $\left(\frac{1}{4}, -\frac{1}{2}\right)$

d $(0, 0),\ \left(1, -1\frac{1}{4}\right), \left(-1, \frac{3}{4}\right)$

10a $y = x + 6x^{\frac{1}{2}} + 1, \dfrac{dy}{dx} = 1 + 3x^{-\frac{1}{2}}$

b $y = 3 - 3x^{\frac{1}{2}} - 8x, \dfrac{dy}{dx} = -\dfrac{3}{2}x^{-\frac{1}{2}} - 8$

c $y = 3x^{\frac{1}{2}} - 2, \dfrac{dy}{dx} = \dfrac{3}{2}x^{-\frac{1}{2}}$

11a $f(x) = 24x^{\frac{1}{2}}, f'(x) = 12x^{-\frac{1}{2}}$

b $f(x) = 8x^{\frac{1}{2}}, f'(x) = 4x^{-\frac{1}{2}}$

c $f(x) = 5x^{\frac{1}{2}}, f'(x) = \dfrac{5}{2}x^{-\frac{1}{2}}$ **d** $f(x) = 2x^{1\frac{1}{2}}, f'(x) = 3\sqrt{x}$

e $f(x) = 12x^{1\frac{1}{2}}, f'(x) = 18\sqrt{x}$

f $f(x) = 4x^{2\frac{1}{2}}, f'(x) = 10x^{\frac{3}{2}}$

g $f(x) = 24x^{\frac{1}{3}}, f'(x) = 8x^{-\frac{2}{3}}$ **h** $f(x) = x^{\frac{2}{3}}, f'(x) = \dfrac{2}{3}x^{-\frac{1}{3}}$

i $f(x) = 30x^{\frac{2}{3}}, f'(x) = 20x^{-\frac{1}{3}}$

j $f(x) = x^{-\frac{1}{2}}, f'(x) = -\dfrac{1}{2}x^{-1\frac{1}{2}}$

k $f(x) = 6x^{-\frac{1}{2}}, f'(x) = -3x^{-\frac{3}{2}}$

l $f(x) = 5x^{-1\frac{1}{2}}, f'(x) = -\dfrac{15}{2}x^{-2\frac{1}{2}}$

12a–c Answers will vary

13a At P, $\dfrac{dy}{dx} = 2a - 10$. **b** At P, $y = a^2 - 10a + 9$.

c $a = 3$ and $y = -4x$, or $a = -3$ and $y = -16x$

Exercise 8G

1 $\dfrac{du}{dx} = 2x, \dfrac{dy}{du} = 5u^4$,

$\dfrac{dy}{dx} = 5(x^2 + 9)^4 \times 2x = 10x(x^2 + 9)^4$

2a $12(3x + 7)^3$ **b** $30(5x - 9)^5$ **c** $-28(5 - 4x)^6$

d $-4(1 - x)^3$ **e** $24x(x^2 + 1)^{11}$ **f** $14x(x^2 - 2)^6$

3a $-7(7x + 2)^{-2}$ **b** $-6(x - 1)^{-3}$ **c** $-12x^2(x^3 - 12)^{-5}$

d $-30x(5x^2 - 2)^{-4}$ **e** $-64x(7 - x^2)^3$

f $-18(3x^2 + 1)(x^3 + x + 1)^5$

4a $25(5x - 7)^4$ **b** $49(7x + 3)^6$ **c** $180(5x + 3)^3$

d $-21(4 - 3x)^6$ **e** $-22(3 - x)$ **f** $-28(4x - 5)^{-8}$

g $-30(3x + 7)^{-6}$ **h** $12(10 - 3x)^{-5}$ **i** $84(5 - 7x)^{-5}$

5a and **b** $2x - 6$

6a and **b** $24x - 12$

7 $f'(x) = 10(2x + 3)^4, f''(x) = 80(2x + 3)^3$

8a $-6x(5 - x^2)^2$ **b** $42x(3x^2 + 7)^6$ **c** $16x^3(x^4 + 1)^3$

d $45x^2(3x^3 - 7)^4$ **e** $-5(3x^2 - 2x)(x^3 - x^2)^{-6}$

f $-9(2x + 3)(x^2 + 3x + 1)^{-10}$

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9a $y = (2x+7)^{-1}, \dfrac{dy}{dx} = \dfrac{-2}{(2x+7)^2}$

b $y = (2-x)^{-1}, \dfrac{dy}{dx} = \dfrac{1}{(2-x)^2}$

c $y = (3+5x)^{-1}, \dfrac{dy}{dx} = \dfrac{-5}{(3+5x)^2}$

d $y = 7(4-3x)^{-1}, \dfrac{dy}{dx} = \dfrac{21}{(4-3x)^2}$

e $y = 4(3x-1)^{-5}, \dfrac{dy}{dx} = \dfrac{-60}{(3x-1)^6}$

f $y = -5(x+1)^{-3}, \dfrac{dy}{dx} = \dfrac{15}{(x+1)^4}$

10a $20(5x-4)^3$ **b** $y = 1, \dfrac{dy}{dx} = 20$

c $y = 20x - 19,\ x + 20y = 21$

11a $y = 24x - 16$ **b** $3x + y = 4$ **c** $x + 2y = 2$

12a $4(x-5)^3, (5, 0)$

b $6x(x^2-1)^2, (0, -1), (1, 0), (-1, 0)$

c $10(x+1)(2x+x^2)^4, (0, 0), (-2, 0), (-1, -1)$

d $\dfrac{-5}{(5x+2)^2}$, none **e** $6(x-5)^5, (5, 4)$

f $\dfrac{-2x}{(1+x^2)^2}, (0, 1)$

13a $2\frac{1}{2}$ and 1 **b** 2 and $1\frac{1}{2}$

14a $\dfrac{dy}{dx} = 3(6x+4)^{-\frac{1}{2}}, \dfrac{3}{4}$ **b** $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{2x+5}}, \dfrac{1}{3}$

c $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2-3}}, 2$

15a $-\dfrac{1}{\sqrt{3-2x}}$, none **b** $\dfrac{x-1}{\sqrt{x^2-2x+5}}, (1, 2)$

c $\dfrac{x-1}{\sqrt{x^2-2x}}$, none ($x = 1$ is outside the domain.)

16a $\dfrac{dy}{dx} = \dfrac{-x}{\sqrt{25-x^2}}$ **b** Answers will vary **c** The tangent at P has gradient $-\frac{3}{4}$, the radius OP has gradient $\frac{4}{3}$.

17a $\dfrac{11(\sqrt{x}-3)^{10}}{2\sqrt{x}}$ **b** $\dfrac{-3}{4\sqrt{4-\frac{1}{2}x}}$ **c** $\dfrac{3\sqrt{2}}{\left(1-x\sqrt{2}\right)^2}$

d $\frac{1}{2}(5-x)^{-1\frac{1}{2}}$ **e** $\frac{1}{2}a^2(1+ax)^{-1\frac{1}{2}}$ **f** $\frac{1}{2}b(c-x)^{-1\frac{1}{2}}$

g $-16\left(1-\dfrac{1}{x^2}\right)\left(x+\dfrac{1}{x}\right)^3$

h $3\left(\dfrac{1}{\sqrt{x}}-\dfrac{1}{x\sqrt{x}}\right)\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)^5$

18a $\dfrac{dy}{dx} = 3(x-a)^2, a = 4$, or $a = 8$

b $\dfrac{dy}{dx} = \dfrac{-1}{(x+a)^2}$, $a = -5$ or $a = -7$

19a $a = 8,\ b = 1$ **b** $a = \dfrac{1}{16},\ b = 12$

20a $x + y(b-4)^2 = 2b - 4$ **b i** $x + 4y = 0$ **ii** $x + y = 6$

Exercise 8H

1 Let $u = 5x$
and $v = (x-2)^4$
Then $\dfrac{du}{dx} = 5$
and $\dfrac{dv}{dx} = 4(x-2)^3$
Let $y = 5x(x-2)^4$
Then
$$\begin{aligned}\frac{dy}{dx} &= v\frac{du}{dx} + u\frac{dv}{dx}\\ &= (x-2)^4 \times 5 + 5x \times 4(x-2)^3\\ &= 5(x-2)^4 + 20x(x-2)^3\\ &= 5(x-2)^3((x-2)+4x)\\ &= 5(x-2)^3(5x-2)\end{aligned}$$

2a and **b** $2x^2(2x-3) = 4x^3 - 6x^2$

3a and **b** $4x - 9$

4a and **b** $4x^3$

5a $u' = 4x^3, v' = 10(2x-1)^4$ **b** Answers will vary

c Answers will vary **d** $x = 0,\ x = \frac{1}{2},\ x = \frac{2}{9}$

6a $(3x+5)^2(12x+5)$ **b** $x(x-1)^2(5x-2)$

c $2x^3(1-5x)^5(2-25x)$

7 $y = x, y = -x$

8a $(x-1)^3(5x-1), 1, \frac{1}{5}$ **b** $(x+5)^4(6x+5), -5, -\frac{5}{6}$

c $2(4-3x)^4(2-9x), \frac{4}{3}, \frac{2}{9}$

d $3(3-2x)^4(1-4x), 1\frac{1}{2}, \frac{1}{4}$

e $x^2(x+1)^3(7x+3), 0, -1, -\frac{3}{7}$

f $3x^2(3x-2)^3(7x-2), 0, \frac{2}{3}, \frac{2}{7}$

g $x^4(1-x)^6(5-12x), 0, 1, \frac{5}{12}$

h $(x-2)^2(4x-5), 2, \frac{5}{4}$

i $(x+5)^5(7x+17), -5, -\frac{17}{7}$

9a Answers will vary **b** $(0, 0), (1, 0)$, and $\left(\frac{3}{8}, \left(\frac{3}{8}\right)^3 \times \left(\frac{5}{8}\right)^5\right)$

10a Answers will vary **b** $y = 2x - 1,\ y = -\frac{1}{2}x + 1\frac{1}{2}$

11 $y = 8x + 8,\ x + 8y + 1 = 0$

12a $10x(x^2+1)^4, (x^2+1)^4(11x^2+1)$,

b $-8x(1-x^2)^3, 2x^2(1-x^2)^3(3-11x^2)$

c $3(2x+1)(x^2+x+1)^2$,
$-2(x^2+x+1)^2(7x^2+4x+1)$

d $(4-9x^4)^3(4-153x^4)$

13a $10x^3(x^2-10)^2(x^2-4)$

b $(0, 0), \left(\sqrt{10}, 0\right), \left(-\sqrt{10}, 0\right), (2, -3456), (-2, -3456)$

14a $\dfrac{3(3x+2)}{\sqrt{x+1}}, -\frac{2}{3}$ **b** $\dfrac{4(3x-1)}{\sqrt{1-2x}}, \frac{1}{3}$ **c** $\dfrac{10x(5x-2)}{\sqrt{2x-1}}$, 0 and $\frac{2}{5}$

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15a $(x+1)^2(x+2)^3(7x+10), -1, -2, -\frac{10}{7}$

b $6(2x-3)^3(2x+3)^4(6x-1),\ 1\frac{1}{2}, -1\frac{1}{2}, \frac{1}{6}$

c $\dfrac{1-2x^2}{\sqrt{1-x^2}}, \sqrt{\frac{1}{2}}$ and $-\sqrt{\frac{1}{2}}$

16a $y' = 2a(x-3)$ **b** $y'(1) = -4a, y'(5) = 4a$

c $y = -4ax + 4a, y = 4ax - 20a, M = (3, -8a)$

d $V = (3, -4a)$ **e** Answers will vary

Exercise 8I

1 Let $u = 2x + 3$

and $v = 3x + 2$

Then $u' = 2$

and $v' = 3$

Let $y = \dfrac{2x+3}{3x+2}$

$$\begin{aligned}\text{Then } \frac{dy}{dx} &= \frac{vu' - uv'}{v^2}\\ &= \frac{(3x+2)\times 2 - (2x+3)\times 3}{(3x+2)^2}\\ &= \frac{6x+4-6x-9}{(3x+2)^2}\\ &= \frac{-5}{(3x+2)^2}\end{aligned}$$

2a $\dfrac{1}{(x+1)^2}$ **b** $\dfrac{4}{(x+2)^2}$ **c** $\dfrac{1}{(1-3x)^2}$ **d** $\dfrac{-2}{(x-1)^2}$

e $\dfrac{-4}{(x-2)^2}$ **f** $\dfrac{4}{(x+2)^2}$ **g** $\dfrac{-5}{(2x-3)^2}$ **h** $\dfrac{-40}{(5+4x)^2}$

3a $\dfrac{x(x+2)}{(x+1)^2}, x = 0, x = -2$ **b** $\dfrac{3+x^2}{(3-x^2)^2}$, none

c $\dfrac{x(2-x)}{(1-x)^2}, x = 0, x = 2$ **d** $\dfrac{1+x^2}{(1-x^2)^2}$, none

e $\dfrac{4x}{(x^2+1)^2}, x = 0$ **f** $\dfrac{10x}{(x^2-4)^2}, x = 0$

4a and **b** $\dfrac{-3}{(3x-2)^2}$

5a Answers will vary **b** $5, 78°41'$

c $y = 5x - 12, x + 5y + 8 = 0$

6a Answers will vary **b** $\frac{4}{3}, 53°8'$

c $4x - 3y = 4,\ 3x + 4y = 28$

7a $y = x$ **b** Answers will vary **c** $A(-1, 0), B\left(0, \frac{1}{4}\right)$

d area $= \frac{1}{8}$ square units

e $\left(\frac{1}{3}, \frac{1}{3}\right)$

8a $y' = \dfrac{x^2+2x}{(x+1)^2}, c = 0$ or -2

b $y' = \dfrac{-4kx}{(x^2-k)^2}, \dfrac{12k}{(9-k)^2} = 1, k = 3$ or 27

9a $12(3x-7)^3$ **b** $\dfrac{x^2+2}{x^2}$ **c** $8x$ **d** $\dfrac{-2x}{(x^2-9)^2}$

e $4(1-x)(4-x)^2$ **f** $\dfrac{-6}{(3+x)^2}$ **g** $20x^3(x^4-1)^4$

h $\dfrac{1}{2(2-x)^{\frac{3}{2}}}$ **i** $6x^2(x^3+5)$ **j** $\dfrac{3x^2+x-1}{4x\sqrt{x}}$

k $\frac{2}{3}x(5x^3-2)$ **l** $\dfrac{5}{(x+5)^2}$ **m** $\frac{1}{2}\sqrt{x}(3+5x)$

n $\dfrac{2(x-1)(x+1)(x^2+1)}{x^3}$ **o** $x^2(x-1)^7(11x-3)$

p $\dfrac{(x+1)(x-1)}{x^2}$

10a Answers will vary

b The denominator is positive, being a square, so the sign of y' is the sign of $a - b$.

Exercise 8J

1a $\dfrac{dQ}{dt} = 3t^2 - 20t$ **b** When $t = 2, Q = -32, \dfrac{dQ}{dt} = -28$.

2a $\dfrac{dQ}{dt} = 2t + 6$ **b** When $t = 2, Q = 16, \dfrac{dQ}{dt} = 10$

c i $t = -3$ **ii** $t > -3$ **iii** $t < -3$

3a 7 and 15 **b** $\frac{15-7}{3-1} = 4$ **c** $\frac{7-15}{7-5} = -4$

4a 180 mL **b** When $t = 0,\ V = 0$. **c** 300 mL **d** 60 mL/s

e The derivative is a constant function.

5a 80 000 litres **b** 35 000 litres **c** 20 min

d 2 000 litres/min

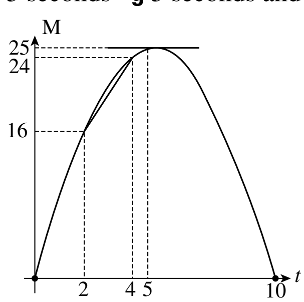
6a $\dfrac{dM}{dt} = 10 - 2t$ **b** $M = 24$ kg, $\dfrac{dM}{dt} = 2$ kg/s

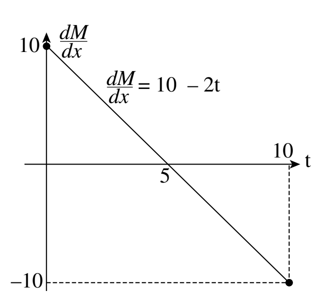
c $M = 16$ kg, average rate $= \dfrac{24-16}{4-2} = 4$ kg/s

d 0 seconds and 10 seconds **e** 10 seconds

f 5 seconds **g** 5 seconds and 5 seconds

h





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7a \$2 **b** \$5.60

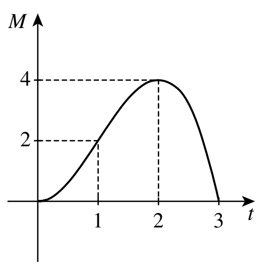
c $\dfrac{dP}{dt} = -0.8t + 4$, \$2.40 per annum

d $t = 5$, the start of 1975

e The price was increasing before then, and decreasing afterwards.

f $\dfrac{dP}{dt}$ is linear with negative gradient -0.8.

g At the start of 1980.

8a 

b $t = 2$

c $\dfrac{M}{t} = 6t - 3t^2, t = 1$

d $t = 1$

9 The scheme appears to have worked initially and the level of pollution decreased, but the rate at which the pollution decreased gradually slowed down and was almost zero in 2000. A new scheme would have been required to remove the remaining pollution.

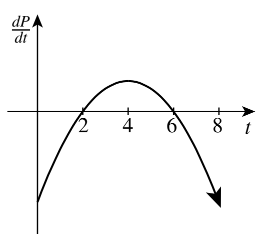
10a The graph is stationary on 1 July and 1 January.

b It is maximum on 1 July and on 1 January. The price is locally minimum on 1 March, but globally the graph has no minimum.

c It is increasing from 1 March to 1 July. It is decreasing from 1 January to 1 March and after 1 July.

d on 1 May

e from 1 March to 1 May

f 

11a $A = \pi r^2 = \pi\left(\frac{t}{3}\right)^2 = \frac{\pi}{9}t^2$ **b** $\dfrac{dA}{dt} = \frac{2\pi}{9}t$

c When $A = 5, t = \sqrt{\frac{45}{\pi}} \doteqdot 3.785$ s and

$\dfrac{dA}{dt} = \frac{2\pi}{9}\sqrt{\frac{45}{\pi}} \doteqdot 2.642$ km^2/s

12a When $t = 0, h = 80$, so the building is 80 m tall.

b When $h = 0, t = 4$, so it takes 4 seconds.

c $v = -10t$

d When $t = 4, v = -40$, so the stone hits the ground at 40 m/s.

e 10 m/s^2 downwards

13a Yes. $\dfrac{dv}{dt} = -\frac{1}{2}$, meaning his velocity decreased at a constant rate of $\frac{1}{2}$ m/s every second, just as he said.

b Yes. $\dfrac{dx}{dt} = -\frac{1}{2}t + 50$, which is what the truck's speed monitor recorded.

c Yes. $\dfrac{dy}{dt} = -\frac{1}{2}t + 50$, which also agrees with the truck's speed monitor.

d When $t = 0, x = 0$ and $y = 450$, so the truck was 450 m ahead.

e Solving $-\frac{1}{2}t + 50 = 0$ gives 100 seconds. When $t = 100, x = 2500$ m or 2.5 km.

f When $t = 0$, $v = 50$ m/s, which is 180 km/h. He was in court for speeding.

14a i Area $= h^2$cm^2 **ii** Volume $= 3000h^2$cm^3

b i $h = 3\sqrt{t}, \dfrac{dh}{dt} = \frac{3}{2\sqrt{t}}$

ii $h = 15$ cm, $\dfrac{dh}{dt} = \frac{3}{10}$cm/min

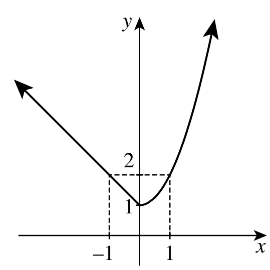
iii 100 min, $\frac{3}{20}$cm/min

Exercise 8K

1a $x = 6$ **b** $x = 1, x = 3, x = 5$. **c** $x = -2, x = -3$

2a $f(0) = 1$. First table: 3, 2, 1. Second table: 1, 2, 5

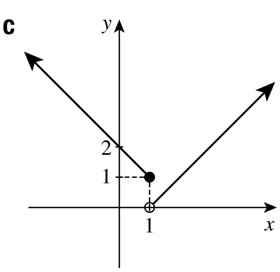
b Yes

c 

d domain: all real x, range: $y \geq 1$

3a $f(1) = 1$. When $x = 1, 2 - x = 1$ and $x - 1 = 0$.

b No

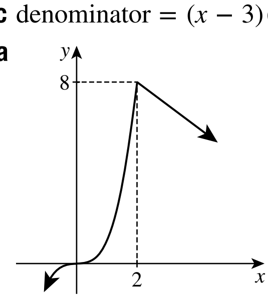
c 

d domain: all real x, range: $y > 0$

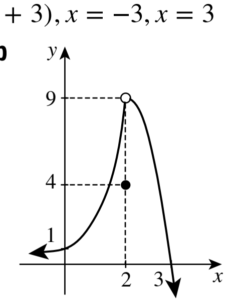
4a denominator $= x(x - 5), x = 0, x = 5$

b denominator $= (x - 2)(x - 3), x = 2, x = 3$

c denominator $= (x - 3)(x + 3), x = -3, x = 3$

5a 

$f(2) = 8$. When $x = 2, x^3 = 8$ and $10 - x = 8$. Thus $f(x)$ is continuous at $x = 2$. domain: all real x, range: $y \leq 8$

b 

$f(2) = 4$. When $x = 2, 3^x = 9$ and $13 - x^2 = 9$. Thus $f(x)$ is not continuous at $x = 2$. domain: all real x, range: $y < 9$

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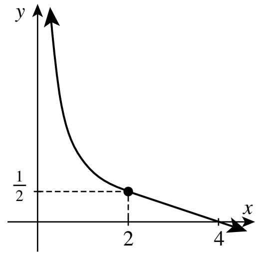
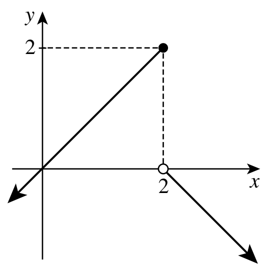
c



$f(2) = \frac{1}{2}$. When $x = 2, \frac{1}{x} = \frac{1}{2}$ and $1 - \frac{1}{4}x = \frac{1}{2}$.
Thus $f(x)$ is continuous at $x = 2$.
domain: $x > 0$, range: all real y

d



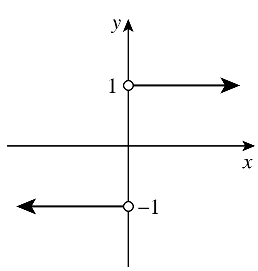
$f(2) = 2$. When $x = 2$, $x = 2$, but $2 - x = 0$.
Thus $f(x)$ is not continuous at $x = 2$.
domain: all real x, range $y \leq 2$

6a The table of values should make it clear that

$$y = \begin{cases} 1, & \text{for } x > 0, \\ -1, & \text{for } x < 0, \\ \text{undefined}, & \text{for } x = 0. \end{cases}$$

b The curve is not continuous at $x = 0$ — it is not even defined there.

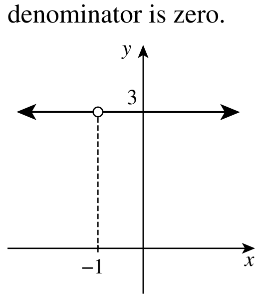
c



domain: $x \neq 0$, range: $y = 1$ or -1

7a The graph is not continuous at $x = -1$ because the denominator is zero.

b



domain: $x \neq -1$, range: $y = 3$

c $y = \dfrac{3(x+1)}{x+1} = \begin{cases} 3, & \text{when } x \neq -1 \\ \text{undefined}, & \text{when } x = -1 \end{cases}$

Hence the graph is the line $y = 3$, except that the point $(-1,\ 3)$ is removed.

Exercise 8L

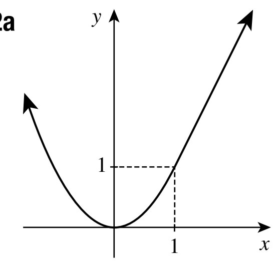
1a continuous and differentiable at $x = 0$, neither at $x = 2$

b continuous and differentiable at $x = 0$, continuous but not differentiable at $x = 2$

c neither at $x = 0$, continuous and differentiable at $x = 2$

d neither at $x = 0$, continuous but not differentiable at $x = 2$

2a

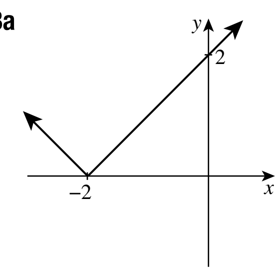


b $f(1) = 1$,
$x^2 = 1$ when $x = 1$,
$2x - 1 = 1$ when $x = 1$

c Answers will vary

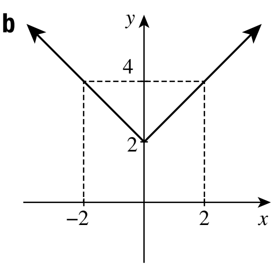
d $2x = 2$ when $x = 1$,
and $2 = 2$ when $x = 1$.
The tangent at $x = 1$ has gradient 2,
so $f'(1) = 2$.

3a



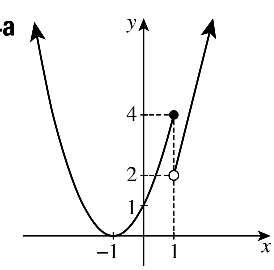
continuous but not differentiable at $x = -2$

b



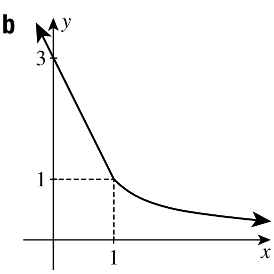
continuous but not differentiable at $x = 0$

4a



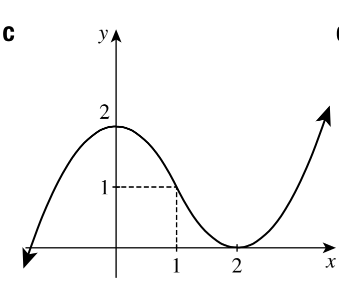
not even continuous at $x = 1$

b



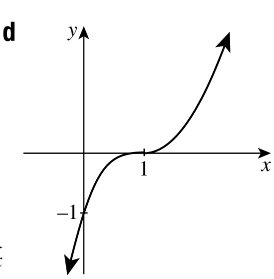
continuous but not differentiable at $x = 1$

c



differentiable at $x = 1$, $f'(1) = -2$

d



differentiable at $x = 1$, $f'(1) = 0$

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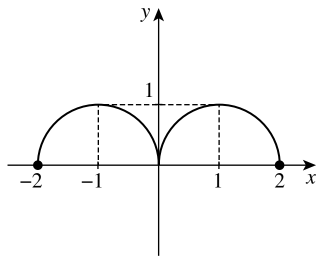
5



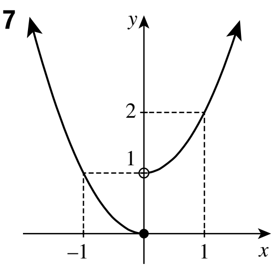
6a Differentiable at $x = 0$. x^2 is never negative, so $|x^2| = x^2$ for all x.
b Differentiable at $x = 0$. x^3 is flat at $x = 0$, so $|x^3|$ is also flat at $x = 0$.
c Continuous, but not differentiable, at $x = 0$. The graph of $y = \sqrt{x}$ becomes vertical near $x = 0$.
d Continuous, but not differentiable, at $x = 0$. The graph has a vertical tangent at the origin.

Chapter 8 review exercise

1a $2x + 5$ **b** $-2x$ **c** $6x - 2$
2a $3x^2 - 4x + 3$ **b** $6x^5 - 16x^3$ **c** $9x^2 - 30x^4$ **d** $2x + 1$
e $-12x + 7$ **f** $-6x^{-3} + 2x^{-2}$ **g** $12x^2 + 12x^{-4}$
h $\frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-1\frac{1}{2}}$ **i** $x^{-2} - 2x^{-3}$
3a $f'(x) = 4x^3 + 3x^2 + 2x + 1, f''(x) = 12x^2 + 6x + 2$
b $f'(x) = -10x^{-3}, f''(x) = 30x^{-4}$
c $f'(x) = -4x^{-\frac{3}{2}}, f''(x) = 6x^{-\frac{5}{2}}$
4a $y = x^3 + 4x + C$
b $y = 7x - 6x^2 - 4x^3 + C$
c $y = 4x^5 - 4x^3 + 4x + C$
5a $-\dfrac{3}{x^2}$ **b** $-\dfrac{1}{3x^3}$ **c** $\dfrac{7}{2\sqrt{x}}$ **d** $\dfrac{6}{\sqrt{x}}$ **e** $-\dfrac{9}{2}\sqrt{x}$ **f** $-\dfrac{3}{x\sqrt{x}}$
6a $6x - 2$ **b** $x - \frac{1}{2}$ **c** $10x + \dfrac{7}{x^2}$ **d** $-\dfrac{2}{x^2} - \dfrac{2}{x^3}$ **e** $\dfrac{2}{\sqrt{x}}$
f $3\sqrt{x} + \dfrac{3}{2\sqrt{x}}$
7a $9(3x + 7)^2$ **b** $-4(5 - 2x)$ **c** $-\dfrac{5}{(5x - 1)^2}$
d $\dfrac{14}{(2 - 7x)^3}$ **e** $\dfrac{5}{2\sqrt{5x + 1}}$ **f** $\dfrac{1}{2(1 - x)^{\frac{3}{2}}}$
8a $42x(7x^2 - 1)^2$ **b** $-15x^2(1 + x^3)^{-6}$
c $8(1 - 2x)(1 + x - x^2)^7$ **d** $-6x(x^2 - 1)^{-4}$
e $-\dfrac{x}{\sqrt{9 - x^2}}$ **f** $\dfrac{x}{(9 - x^2)^{\frac{3}{2}}}$

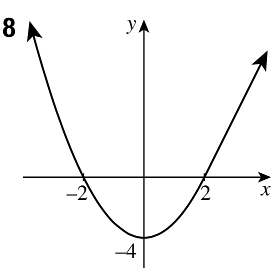
9a $x^8(x + 1)^6(16x + 9)$ **b** $\dfrac{x(2 - x)}{(1 - x)^2}$
c $2x(4x^2 + 1)^3(20x^2 + 1)$ **d** $\dfrac{12}{(2x + 3)^2}$
e $(9x - 1)(x + 1)^4(x - 1)^3$ **f** $\dfrac{(x - 5)(x + 1)}{(x - 2)^2}$
10 $\dfrac{dy}{dx} = 2x + 3$ **a** $3, 71°34'$ **b** $1, 45°$ **c** $-1, 135°$
11a tangent: $y = -3x$, normal: $3y = x$
b tangent: $y = -2$, normal: $x = 1$
c $(1, -2)$ and $(-1, 2)$
d $(2, 2)$ and $(-2, -2)$
12a $y = -x - 4, y = x - 8$ **b** $A(-4, 0), B(8, 0)$
c $AB = 12$, area $= 36$ square units
13 The tangent is $y = x$.
14a $\left(1, -6\frac{2}{3}\right), \left(-1, -7\frac{1}{3}\right)$ **b** $\left(-1, \frac{2}{3}\right)$
15 At $(1, -3)$ the tangent is $l: x + y + 2 = 0$, at $(-1, 3)$ the tangent is $x + y - 2 = 0$.
16a $V = \frac{4}{3}\pi \times \left(\frac{t}{3}\right)^3 = \frac{4\pi}{81}t^3$ **b** $\dfrac{dV}{dt} = \frac{4\pi}{27}t^2$
c $V \fallingdotseq \frac{4\pi}{81} \times 0.001 \fallingdotseq 0.00016\,\text{km}^3$,
$\dfrac{dV}{dt} \fallingdotseq \frac{4\pi}{27} \times 0.01 \fallingdotseq 0.0047\,\text{km}^3/\text{s}$ **d** $t^2 = \frac{27}{4\pi}, t \fallingdotseq 1.5\,\text{s}$

17



a $f(0) = 0$, $x^2 = 0$ when $x = 0$, $x^2 + 1 = 1$ when $x = 0$, so it is not continuous at $x = 0$.
b domain: all real x, range: $y \geq 0$

18



a $f(0) = 2$, $x^2 - 4 = 0$ when $x = 2$, $4x - 8 = 0$ when $x = 2$, so it is continuous at $x = 2$.
b $f'(2) = 4$ when $x < 2$ (substitute into $2x$), $f'(2) = 4$ when $x > 2$ (substitute into 4), so it is differentiable at $x = 2$, with $f'(2) = 4$
c domain: all real x, range: $y \geq 4$

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Chapter 9

Exercise 9A

1d

x	-2	-1	0	1	2
height y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
gradient $\frac{dy}{dx}$	0.17	0.35	0.69	1.39	2.77
$\frac{\text{gradient}}{\text{height}}$	0.69	0.69	0.69	0.69	0.69

e All the ratios are about 0.7. **f** $\frac{dy}{dx} \doteqdot 0.7y$

2b Both are equal to 1.

c

height y	$\frac{1}{2}$	1	2	3
gradient $\frac{y}{x}$	$\frac{1}{2}$	1	2	3
$\frac{\text{gradient}}{\text{height}}$	1	1	1	1

d They are all equal to 1. **e** $\frac{dy}{dx} = y$

3c The values are: 0.14, 0.37, 1, 2.72.

d The x-intercept is always 1 unit to the left of the point of contact.

4a i AB has gradient 1.

ii The curve is concave up, so the chord is steeper than the tangent.

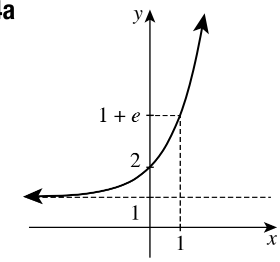
b i CA has gradient 1.

ii The curve is concave up, so the chord is not as steep as the tangent.

c As the base increases, the gradient at the y-intercept increases. With $y = 2^x$, the gradient at the y-intercept is less than 1, and with $y = 4^x$, the gradient at the y-intercept is greater than 1. Hence the base e for which the y-intercept is exactly 1 is between 2 and 4.

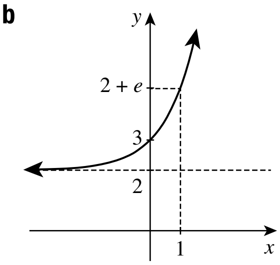
6 a–f The values get closer and closer to the limit: $\log_e 2 \doteqdot 0.69315$

4a



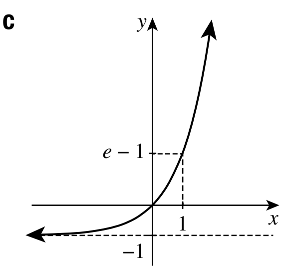
Shift up 1 unit, asymptote: $y = 1$, range: $y > 1$

b



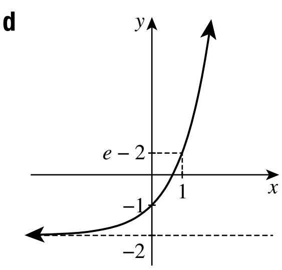
Shift up 2 units, asymptote: $y = 2$, range: $y > 2$

c



Shift down 1 unit, asymptote: $y = -1$, range: $y > -1$

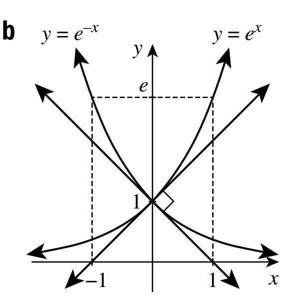
d



Shift down 2 units, asymptote: $y = -2$, range: $y > -2$

5a e^x: 0.14, 0.37, 1.00, 2.72, 7.39.
e^{-x}: 7.39, 2.72, 1.00, 0.37, 0.14

b



c Reflection in the y-axis.

Exercise 9B

1a 7.3891	**b** 20.0855	**c** 22026.4658
d 1.0000	**e** 2.7183	**f** 0.3679
g 0.1353	**h** 1.6487	**i** 0.6065
2a $e^1 \doteqdot 2.718$	**b** $e^{-1} \doteqdot 0.3679$	**c** $e^{-4} \doteqdot 0.01832$
d $e^{\frac{1}{2}} \doteqdot 1.649$	**e** $e^{\frac{1}{3}} \doteqdot 1.396$	**f** $e^{-\frac{1}{2}} \doteqdot 0.6065$
3a $5e^2 \doteqdot 36.95$	**b** $\frac{1}{64}e^6 \doteqdot 6.304$	**c** $7e^{\frac{1}{2}} \doteqdot 11.54$
d $\frac{3}{5}e^{\frac{1}{2}} \doteqdot 0.9892$	**e** $4e^{-1} \doteqdot 1.472$	**f** $\frac{5}{7}e^{-4} \doteqdot 0.01308$

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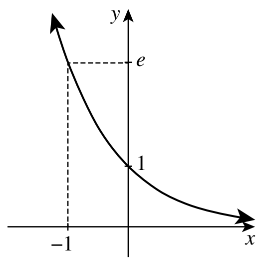
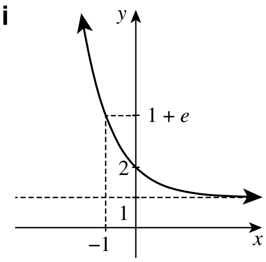
d The graph of $y = e^{-x}$ is the reflection of $y = e^x$ in the y-axis, so its gradient at the y-intercept is -1. Hence the two tangents are perpendicular because the product of their gradients is -1 (or because $45° + 45° = 90°$).

6a



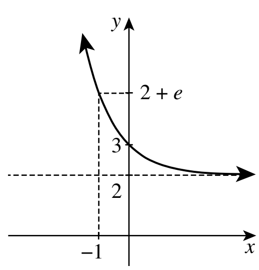
Asymptote: $y = 0$, range: $y > 0$

b i



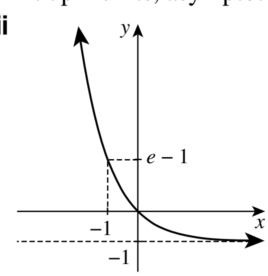
Shift up 1 unit, asymptote: $y = 1$, range: $y > 1$

ii



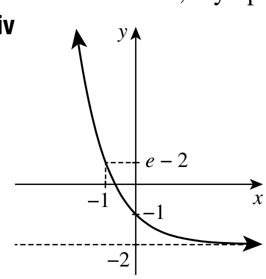
Shift up 2 units, asymptote: $y = 2$, range: $y > 2$

iii



Shift down 1 unit, asymptote: $y = -1$, range: $y > -1$

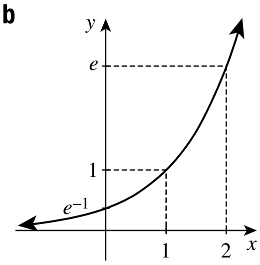
iv



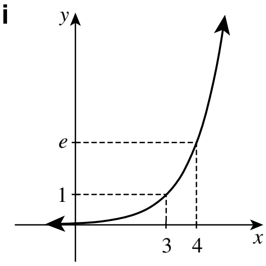
Shift down 2 units, asymptote: $y = -2$, range: $y > -2$

7a Shift right 1 unit

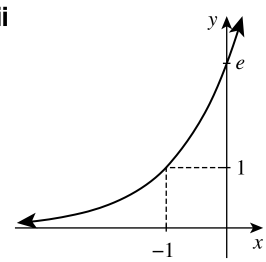
b



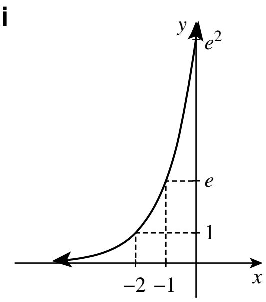
c i



ii



iii



8a $1, e, e^2, e^3$

b i gradient of $AB = e - 1$

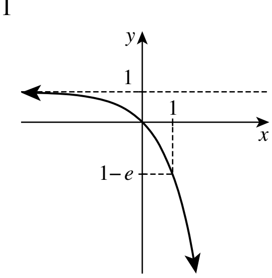
ii AB: $y - 1 = (e - 1)x$

iii Answers will vary

c gradient of $BC = e^2 - e$, BC: $y - e = (e^2 - e)(x - 1)$

d gradient of $CD = e^3 - e^2$, BC: $y - e^2 = (e^3 - e^2)(x - 2)$

9a $y < 1$



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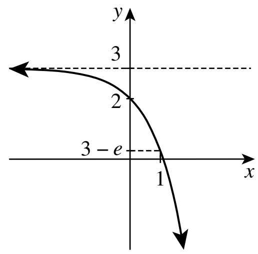
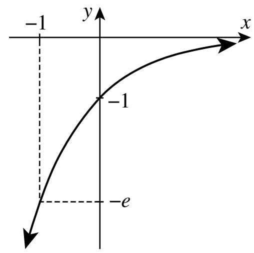
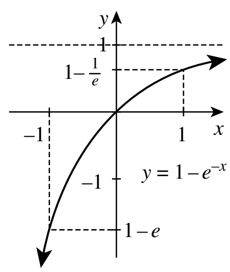
b $y < 3$



c $y < 0$



d $y < 1$



Exercise 9C

1a $2e^{2x}$ **b** $7e^{7x}$ **c** $-e^{-x}$ **d** $-5e^{5x}$
e $\frac{1}{2}e^{\frac{1}{2}x}$ **f** $2e^{\frac{1}{3}x}$ **g** $-\frac{1}{3}e^{-\frac{1}{3}x}$ **h** $e^{\frac{1}{5}x}$

2a $f'(x) = e^{x+2}$ **b** $f'(x) = e^{x-3}$
c $f'(x) = 5e^{5x+1}$ **d** $f'(x) = 2e^{2x-1}$
e $f'(x) = -4e^{-4x+1}$ **f** $f'(x) = -3e^{-3x+4}$
g $f'(x) = -3e^{-3x-6}$ **h** $f'(x) = e^{\frac{1}{2}x+4}$

3a $e^x - e^{-x}$ **b** $2e^{2x} + 3e^{-3x}$
c $e^{2x} + e^{3x}$ **d** $e^{4x} + e^{5x}$
e $\frac{e^x + e^{-x}}{2}$ **f** $\frac{e^x - e^{-x}}{3}$

4a $y' = 2e^{2x}$
b When $x = 0$, $f'(y') = 2$. When $x = 4$, $f'(y') = 2e^8$.

5a $f'(x) = -e^{-x+3}$
b When $x = 0$, $f'(x) = -e^3$. When $x = 4$, $f'(x) = -e^{-1}$.

6a $y' = 3e^{3x}$, $y'(2) = 3e^6 \doteqdot 1210.29$
b $y' = -2e^{-2x}$, $y'(2) = -2e^{-4} \doteqdot -0.04$
c $y' = \frac{3}{2}e^{\frac{3}{2}x}$, $y'(2) = \frac{3}{2}e^3 \doteqdot 30.13$

7a $-e^{-x}, e^{-x}, -e^{-x}, e^{-x}$
b Successive derivatives alternate in sign.
More precisely, $f^{(n)}(x) = \begin{cases} e^{-x} & \text{if } n \text{ is even,} \\ -e^{-x} & \text{if } n \text{ is odd.} \end{cases}$

8a $2e^{2x}, 4e^{2x}, 8e^{2x}, 16e^{2x}$
b Each derivative is twice the previous one.
More precisely, $f^{(n)}(x) = 2^n e^{2x}$.

9a e^x, e^x, e^x, e^x
b All derivatives are the same, and are equal to the original function.

10 $y' = e^x + 2x + 1$, $y'' = e^x + 2$, y''' and all subsequent derivatives are e^x.

11a $5e^{5x} + 7e^{7x}$ **b** $4e^{4x+2} + 8e^{5+8x}$
c $-4e^{-x} - 12e^{-3x}$ **d** $-12e^{-2x-3} + 42e^{5-6x}$
e $10x - 4 + 3e^{-x}$ **f** $\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}x^{-\frac{1}{2}}$

12a $y' = ae^{ax}$ **b** $y' = -ke^{-kx}$
c $y' = Ake^{kx}$ **d** $y' = -B\ell e^{-\ell x}$

13a $y' = \frac{1}{2}\sqrt{e^x}$ **b** $y' = \frac{1}{3}\sqrt[3]{x}$
c $y' = -\dfrac{1}{2\sqrt{e^x}}$ **d** $y' = -\dfrac{1}{3\sqrt[3]{e^x}}$

14a $y' = pCe^{px+q}$ **b** $e^{ax} - e^{-px}$

15a a–c Answer is in question

16a a–c Answer is in question

Exercise 9D

1a 1 **b** $y = x + 1$

2a e **b** $y = ex$

3a $\frac{1}{e}$ **b** $y = \frac{1}{e}(x + 2)$

4a $A = \left(\frac{1}{2}, 1\right)$ **b** $y' = 2e^{2x-1}$ **c** $y = 2x$

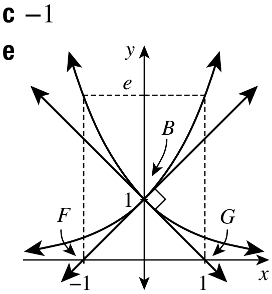
5a $y' = e^x$, which is always positive.
b $y' = -e^{-x}$, which is always negative.

6a $e - 1$ **b** $\dfrac{dy}{dx} = e^x$. When $x = 1$, $\dfrac{dy}{dx} = e$.
c $y = ex - 1$
d i never **ii** all real x **iii** never

7a $R = \left(-\frac{1}{3}, 1\right)$ **b** $y' = 3e^{3x+1}$
c $-\frac{1}{3}$ **d** $3x + 9y - 8 = 0$.

8a $-e$ **b** $\frac{1}{e}$
c $x - ey + e^2 + 1 = 0$
d x-intercept $-e^2 - 1$, y-intercept $e + e^{-1}$
e $\frac{1}{2}(e^3 + 2e + e^{-1})$

9a 1 **b** $y = x + 1$
c -1 **d** $y = -x + 1$
e



f isosceles right triangle, 1 square unit

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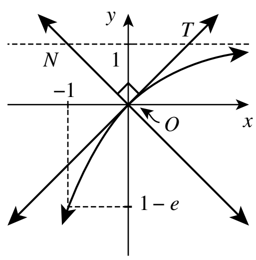
10a 1, 45° **b** e, 69°48′
c e^{-2}, 7°42′ **d** e^5, 89°37′

11 $A = (1, e^{-2})$, $B = (2, 1)$, $y' = 2e^{2x-4}$
a $y' = 2e^{-2}$ **b** $y' = 2$ **c** $1 - e^{-2}$

12a $y = e^t(x - t + 1)$ **b and c** Answers will vary

13a Answers will vary **b** $y = -x$ **c** $y = 1$

d



e 1 square unit

14a Answers will vary
b $B(1, 0)$, $C(1 + e^2, 0)$, $E(0, e)$, $F\left(0, e + \frac{1}{e}\right)$
c i–iv Answers will vary
d i $A = \frac{e}{2}(e^2 + 1)$ **ii** $A = \frac{1}{2e}(e^2 + 1)$

Exercise 9E

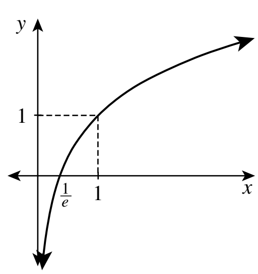
1a 0 **b** 0.6931 **c** 1.0986 **d** 2.0794 **e** −0.6931
f −1.0986 **g** −2.0794 **h** −2.3026

2, 3 a–f Answer is in question

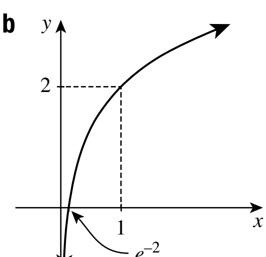
4a $e^x = 1, x = 0$ **b** $e^x = e, x = 1$ **c** $e^x = e^2, x = 2$
d $e^x = \frac{1}{e}, x = -1$ **e** $e^x = \frac{1}{e^2}, x = -2$ **f** $e^x = \sqrt{e}, x = \frac{1}{2}$

5a $2\log_e e = 2$ **b** $5\log_e e = 5$ **c** $200\log_e e = 200$
d $-6\log_e e = -6$ **e** $\log_e e^{-6} = -6\log_e e = -6$
f $-\log_e e = -1$ **g** $\log_e e^{-1} = -\log_e e = -1$
h $\frac{1}{2}\log_e e = \frac{1}{2}$ **i** $\frac{1}{2}\log_e e = \frac{1}{2}$ **j** $\log_e e^{-\frac{1}{2}} = -\frac{1}{2}\log_e e = -\frac{1}{2}$

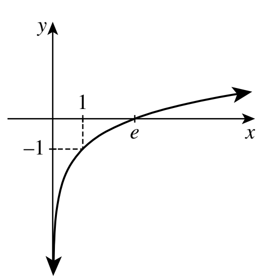
6a



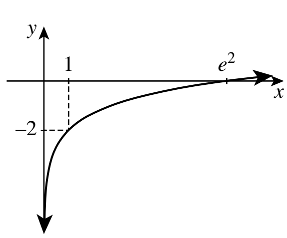
b



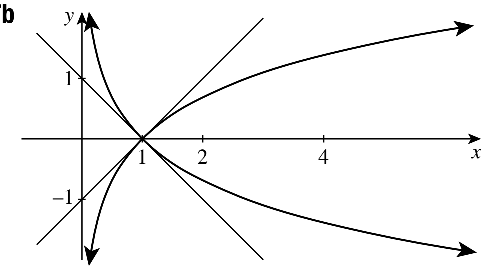
c



d



7b

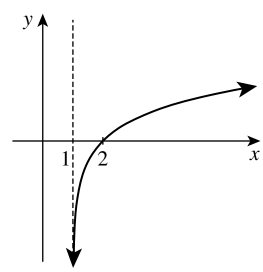


c The graph of $y = -\log_e x$ is obtained by reflecting the first in the x-axis. Hence its tangent has gradient −1, and the two are perpendicular.

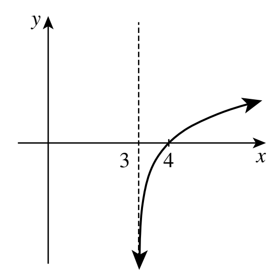
8a e **b** $-\frac{1}{e}$ **c** 6 **d** $\frac{1}{2}$ **e** $2e$ **f** 0

9a $\log_e 6$ **b** $\log_e 4$ **c** $\log_e 4$ **d** $\log_e 27$

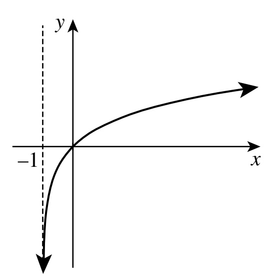
10a $x > 1$



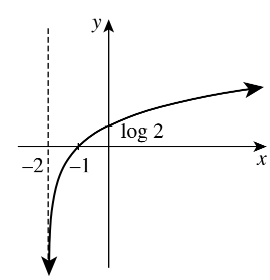
b $x > 3$



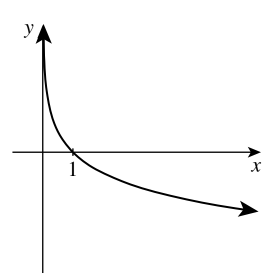
c $x > -1$



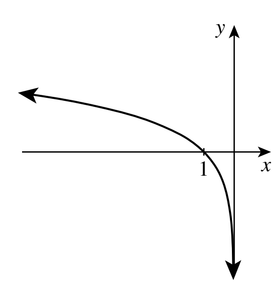
d $x > -2$



e $x > 0$



f $x < 0$



11a $\log_e \frac{a}{b} = \log_e a - \log b$ and $-\log_e \frac{b}{a} = -\log_e b + \log_e a$

b $\log_{\frac{1}{e}} x = \dfrac{\log_e x}{\log_e \frac{1}{e}} = \dfrac{\log_e x}{-1} = -\log_e x$

c Using part **b**, $\log_{\frac{1}{e}} x^{-1} = -\log_e x^{-1} = +\log_e x$

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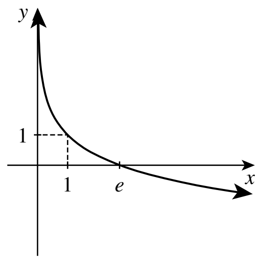
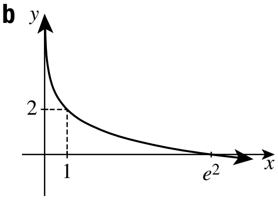
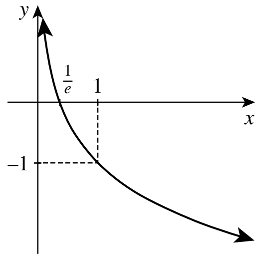
12a



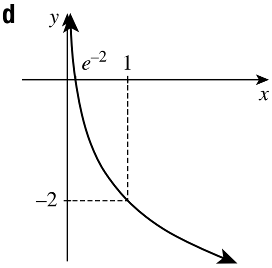
b



c



d



Exercise 9F

1a $\dfrac{dQ}{dt} = 900e^{3t}$

b $Q = 300e^6 \doteqdot 121\,000$, $\dfrac{dQ}{dt} = 900e^6 \doteqdot 363\,100$

c 60 360

2a $\dfrac{dQ}{dt} = -20\,000e^{-2t}$

b $Q = 10\,000e^{-8} \doteqdot 3.355$, $\dfrac{dQ}{dt} = -20\,000e^{-8} \doteqdot -6.709$

c −2499

3a $\dfrac{dQ}{dt} = 10e^{2t}$ **b** Put $1000 = 5e^{2t}$, $t = \frac{1}{2}\log 200 \doteqdot 2.649$

c Put $1000 = 10e^{2t}$, $t = \frac{1}{2}\log 100 \doteqdot 2.303$

4a $P = 2000e^{1.5} \doteqdot 8963$ individuals **b** $\dfrac{dP}{dt} = 600e^{0.3t}$

c $\dfrac{dP}{dt} = 600e^{1.5} \doteqdot 2689$ individuals/hour

d 1393 individuals per hour

5a $C = 2000e^{-4} \doteqdot 36.63$ **b** $\dfrac{dC}{dt} = -4000e^{-2t}$

c $\dfrac{dC}{dt} = -4000e^{-4} \doteqdot -73.26$ per year

d −981.7 per year

6a $t = 25\log_e 2 \doteqdot 17.33$ years **b** $\dfrac{dP}{dt} = 6e^{0.04t}$

c $t = 25\log_e 50 \doteqdot 97.80$ years

7a $\dfrac{dP}{dt} = 400e^{0.4t}$

b $P = 1000e^2 \doteqdot 7400$ cats, $\dfrac{dP}{dt} = 400e^2 \doteqdot 3000$ cats/year

c $t = \frac{5}{2}\log_e 20 \doteqdot 7.5$ years **d** $t = \frac{5}{2}\log_e 50 \doteqdot 9.8$ years

8a $\dfrac{dQ}{dt} = e^t$, which is always positive, so Q is increasing. Also $\dfrac{dQ}{dt}$ is increasing, so Q is increasing at an increasing rate.

b $\dfrac{dQ}{dt} = -e^{-t}$, which is always negative, so Q is decreasing. Also $\dfrac{dQ}{dt}$ is increasing, so the rate of change of Q is increasing, thus Q is decreasing at a decreasing rate. (The language here is not entirely satisfactory — more on this in year 12.)

9a $t = -10\log_e\left(\frac{1}{2}\right) = 10\log_e 2 \doteqdot 6.931$ years

b $\dfrac{dM}{dt} = -\frac{1}{10}M_0e^{-0.1t}$

c $(1 - e^{-0.1}) \times 100\% \doteqdot 9.516\%$

d When $\dfrac{dM}{dt} = -\frac{1}{100}M_0$,

$t = -10\log_e\left(\frac{1}{10}\right) = 10\log_e 10 \doteqdot 23.03$ years

Exercise 9G

1a $\frac{\pi}{2}$ **b** $\frac{\pi}{4}$ **c** $\frac{\pi}{6}$ **d** $\frac{\pi}{3}$ **e** $\frac{2\pi}{3}$ **f** $\frac{5\pi}{6}$ **g** $\frac{3\pi}{4}$ **h** $\frac{5\pi}{4}$ **i** 2π

j $\frac{5\pi}{3}$ **k** $\frac{3\pi}{2}$ **l** $\frac{7\pi}{6}$

2a 180° **b** 360° **c** 720° **d** 90° **e** 60° **f** 45° **g** 120°

h 150° **i** 135° **j** 270° **k** 240° **l** 315° **m** 330°

3a 0.84 **b** −0.42 **c** −0.14 **d** 0.64 **e** 0.33 **f** −0.69

4a 1.274 **b** 0.244 **c** 2.932 **d** 0.377

e 1.663 **f** 3.686

5a 114°35′ **b** 17°11′ **c** 82°30′ **d** 7°3′

e 183°16′ **f** 323°36′

6a $\frac{1}{2}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$ **d** $\sqrt{3}$ **e** 1 **f** $\frac{1}{2}$ **g** $\sqrt{2}$ **h** $\frac{1}{\sqrt{3}}$

7a $\frac{\pi}{9}$ **b** $\frac{\pi}{8}$ **c** $\frac{\pi}{5}$ **d** $\frac{5\pi}{9}$ **e** $\frac{5\pi}{8}$ **f** $\frac{7\pi}{5}$

8a 15° **b** 72° **c** 400° **d** 247.5° **e** 306° **f** 276°

9a $\frac{\pi}{3}$ **b** $\frac{5\pi}{6}$

10 $\frac{4\pi}{9}$

11a $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{2}$ **c** $-\frac{\sqrt{3}}{2}$ **d** $\sqrt{3}$ **e** −1 **f** $\frac{1}{2}$ **g** $-\frac{1}{\sqrt{2}}$ **h** $\frac{1}{\sqrt{3}}$

12a Hour hand: 30° or $\frac{\pi}{6}$ radians, minute hand: 360° or 2π radians.

b i 60° or $\frac{\pi}{3}$ radians **ii** $22\frac{1}{2}$° or $\frac{\pi}{8}$ radians

iii 105° or $\frac{7\pi}{12}$ radians **iv** $172\frac{1}{2}$° or $\frac{23\pi}{24}$ radians

13a 0.283 **b** 0.819

14a 0.733 **b** 0.349

Exercise 9H

1a $\frac{\pi}{4}$ **b** $\frac{\pi}{6}$ **c** $\frac{\pi}{4}$ **d** $\frac{\pi}{6}$ **e** $\frac{\pi}{3}$ **f** $\frac{\pi}{3}$

2a $x \doteqdot 1.249$ **b** $x \doteqdot 0.927$ **c** $x \doteqdot 1.159$ **d** $x \doteqdot 0.236$

e $x \doteqdot 0.161$ **f** $x \doteqdot 1.561$

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3a $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ **b** $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ **c** $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ **d** $x = \frac{\pi}{2}$
e $x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ **f** $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$ **g** $x = \pi$ **h** $x = \frac{5\pi}{4}$ or $\frac{7\pi}{4}$
4a $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ **b** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ or $\frac{7\pi}{4}$
c $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$ **d** $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$
5a $u^2 - u = 0$ **b** $u = 0$ or $u = 1$ **c** $\theta = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ or 2π
6a $u^2 - u - 2 = 0$ **b** $u = -1$ or $u = 2$
c $\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ or $\theta \doteqdot 1.11$ or 4.25
7a $\theta = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ or 2π **b** $\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$ or 2π
c $\theta = \frac{\pi}{2}$ **d** $\theta \doteqdot 1.11, 1.89, 4.25$ or 5.03
e $\theta = \frac{\pi}{3}, \pi$ or $\frac{5\pi}{3}$ **f** $\theta = \frac{\pi}{2}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$
g $\theta \doteqdot 0.34$ or 2.80 **h** $\theta \doteqdot 1.91$ or 4.37
8 Compare to the answer to question **2**
9a $x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}$ or $\frac{5\pi}{3}$ **b** $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$, or $x \doteqdot 1.25$ or 4.39
c $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, or $x \doteqdot 0.25$ or 2.89 **d** $x \doteqdot 0.84$ or 5.44
10a $\alpha = \frac{\pi}{2}$, or $\alpha \doteqdot 3.48$ or 5.94
b $\alpha \doteqdot 1.11, 2.82, 4.25$ or 5.96
11a $x = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ or 2π
b $x \doteqdot 1.11, 1.25, 4.25$ or 4.39

Exercise 9I

1a 3π **b** $\frac{5\pi}{2}$ **c** 7.5 **d** 24 **e** $\frac{\pi}{4}$ **f** 2π
2a 2π **b** $\frac{4\pi}{3}$ **c** 2 **d** $\frac{2\pi}{3}$ **e** 6 **f** 20
3a 12 cm **b** 3 cm **c** 2π cm **d** $\frac{3\pi}{2}$ cm
4a 32 cm^2 **b** 96 cm^2 **c** $8\pi\text{ cm}^2$ **d** $12\pi\text{ cm}^2$
5 4 cm
6 1.5 radians
7a 2.4 cm **b** 4.4 cm
8 8727 m^2
9a 8π cm **b** $16\pi\text{ cm}^2$
10 84°
11 11.6 cm
12a $6\pi\text{ cm}^2$ **b** $9\sqrt{3}\text{ cm}^2$ **c** $3\left(2\pi - 3\sqrt{3}\right)\text{ cm}^2$
14 15 cm^2
15a $4(\pi + 2)$ cm **b** $8\pi\text{ cm}^2$
16a 720 metres
b 2.4 radians (about 137°31′)
c 559.22 metres
d 317°31′T
17a $\frac{2\pi}{3}$ cm **b** $\frac{2\pi}{3}\text{ cm}^2$ **c** 2π cm **d** $\sqrt{3}\text{ cm}^2, 2\left(\pi - \sqrt{3}\right)\text{ cm}^2$
18 $\frac{4}{3}\left(4\pi - 3\sqrt{3}\right)\text{ cm}^2$
19a, b Answers will vary **c** $3\sqrt{55}\pi\text{ cm}^3$ **d** $24\pi\text{ cm}^2$

Exercise 9J

1a and **b** Refer to teacher.
2a All six graphs are many-to-one
b i $\pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi$ **ii** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$
iii $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}, \frac{21\pi}{2}$ **iv** There are no solutions.
3a $x = \frac{\pi}{2}$, w$x = -\frac{\pi}{2}, x = \frac{3\pi}{2}, x = -\frac{3\pi}{2}, x = \frac{5\pi}{2}, x = -\frac{5\pi}{2}, \ldots$
b $y = \operatorname{cosec} x$, the reciprocal of $y = \sin x$.
c Neither graph has any line symmetries.
4a $x = 0, x = \pi, x = -\pi, x = 2\pi, x = -2\pi, \ldots$
b Line symmetry in the y-axis $x = 0$
c $y = \sec x$, the reciprocal of $y = \cos x$.
5a $(0, 0), (\pi, 0), (-\pi, 0), (2\pi, 0), (-2\pi, 0), \ldots$
b Point symmetry in the origin $(0, 0)$
c $y = \operatorname{cosec} x$, the reciprocal of $y = \sin x$.
6a $\left(\frac{\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), \left(-\frac{3\pi}{2}, 0\right), \ldots$
b $y = \sec x$, the reciprocal of $y = \cos x$.
7a $(0, 0), \left(\frac{\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), (\pi, 0), (-\pi, 0), \left(\frac{3\pi}{2}, 0\right), \left(-\frac{3\pi}{2}, 0\right), \ldots$
b Both functions are odd, because both have point symmetry in the origin. Neither is even, because neither have line symmetry in the y-axis.
8a Translations left or right by multiples of 2π.
b $y = \cos x$, $y = \operatorname{cosec} x$ and $y = \sec x$.
c $y = \tan x$ and $y = \cot x$ can each be mapped onto themselves by translations left or right by multiples of π.
d $y = \sin x$, $y = \cos x$,
$y = \operatorname{cosec} x$, $y = \sec x$ each has period 2π.
$y = \tan x$, $y = \cot x$ each has period π.
9a $x = \frac{\pi}{4}, x = -\frac{3\pi}{4}, x = \frac{5\pi}{4}, x = -\frac{7\pi}{4}, \ldots$
b $y = \operatorname{cosec} x$ and $y = \sec x$
c $x = \frac{\pi}{4}, x = -\frac{\pi}{4}, x = \frac{3\pi}{4}, x = -\frac{3\pi}{4}, x = \frac{5\pi}{4}, x = -\frac{5\pi}{4}, \ldots$
10a Translations left $\frac{\pi}{2}, \frac{5\pi}{2}, \ldots$, and right $\frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$
b $y = \sin(x - \theta)$ is $y = \sin x$ shifted right by θ, so $\sin(x - \theta) = \cos x$ for $\theta = \frac{3\pi}{2}, -\frac{\pi}{2}, \frac{7\pi}{2}, -\frac{5\pi}{2}, \frac{11\pi}{2}, -\frac{9\pi}{2}, \ldots$
c There are none.
11 There are none.
12a $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right), \left(-\frac{3\pi}{4}, -\frac{1}{\sqrt{2}}\right), \left(\frac{5\pi}{4}, \frac{1}{\sqrt{2}}\right), \left(-\frac{7\pi}{4}, -\frac{1}{\sqrt{2}}\right), \ldots$
b $\sin x = \cos x$, so $\tan x = 1$.
13a $(0, 0), (\pi, 0), (-\pi, 0), (2\pi, 0), (-2\pi, 0), \ldots$
b $\sin x = \frac{\sin x}{\cos x}$, so $\sin x \cos x = \sin x$, so $\sin x(\cos x - 1) = 0$, so $\sin x = 0$ or $\cos x = 1$.
14 Roughly 0.7 (radians).
15a They touch each other at their maxima and minima.
b $y = \cos x$ and $y = \sec x$.
c $y = \sin x$ & $y = \sec x$, $y = \cos x$ & $y = \operatorname{cosec} x$,
$y = \tan x$ & $y = \sec x$, $y = \cot x$ & $y = \operatorname{cosec} x$

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16a $$\cos x = \frac{\sin x}{\cos x}$$
$$\boxed{\times \cos x}\ \cos^2 x = \sin x \text{ and } \cos x \neq 0$$
$$1 - \sin^2 x = \sin x$$
$$\sin^2 x + \sin x - 1 = 0$$
$$\Delta = 1 + 4 = 5$$
$$\sin x = \frac{-1+\sqrt{5}}{2}$$
giving solutions in the first and second quadrants.
$\left(\dfrac{-1-\sqrt{5}}{2} < -1\text{, so } \sin x = \dfrac{-1-\sqrt{5}}{2}\right.$ has no solutions.)

b $$\frac{\sin x}{\cos x} = \frac{1}{\cos x}$$
$$\times \cos x \quad \sin x = 1 \text{ and } \cos x \neq 0$$
There are no solutions,
because if $\sin x = 1$, then $\cos x = 0$.

Chapter 9 review exercise

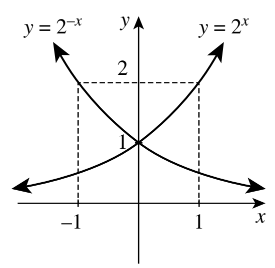
1a 3^9 **b** 3^{12} **c** 3^5 **d** 6^5

2a $\frac{1}{5}$ **b** $\frac{1}{100}$ **c** $\frac{1}{x^3}$ **d** $\frac{1}{3^x}$

3a 3 **b** 3 **c** 4 **d** $\frac{1}{4}$ **e** $\frac{1}{9}$ **f** $\frac{1}{1000}$

4a 2^{3x} **b** 2^{4x} **c** 2^{6x} **d** 10^x **e** 2^{2x+3} **f** 2^{2x-1}

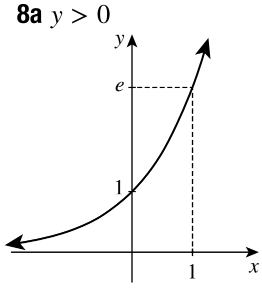
5 Each graph is reflected onto the other graph in the line $x = 0$.



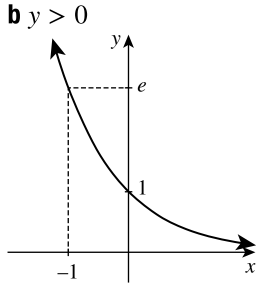
6a 2.718 **b** 54.60 **c** 0.1353 **d** 4.482

7a e^{5x} **b** e^{6x} **c** e^{-4x} **d** e^{9x}

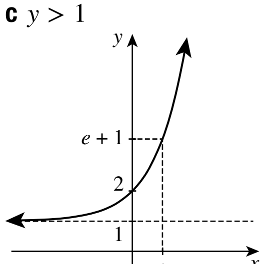
8a $y > 0$



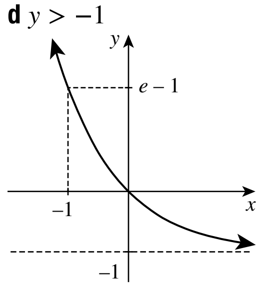
b $y > 0$



c $y > 1$



d $y > -1$



9a e^x **b** $3e^{3x}$ **c** e^{x+3} **d** $2e^{2x+3}$ **e** $-e^{-x}$ **f** $-3e^{-3x}$
g $-2e^{3-2x}$ **h** $6e^{2x+5}$ **i** $2e^{\frac{1}{2}x}$ **j** $4e^{6x-5}$

10a $5e^{5x}$ **b** $4e^{4x}$ **c** $-3e^{-3x}$ **d** $-6e^{-6x}$

11 2

12 $y = e^2x - e^2$, x-intercept 1, y-intercept $-e^2$.

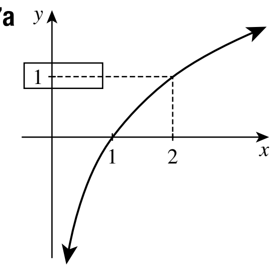
13a 1.4314 **b** −0.3010 **c** 0.6931 **d** 2.6391

14a 1.1761 **b** 0.4771 **c** 1.9459 **d** −1.0986

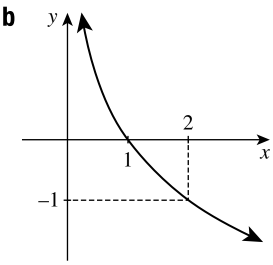
15a 5 **b** $-\frac{1}{4}$ **c** 3 **d** $\frac{1}{5}$

16a e **b** 3 **c** −1 **d** e

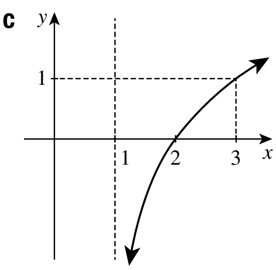
17a



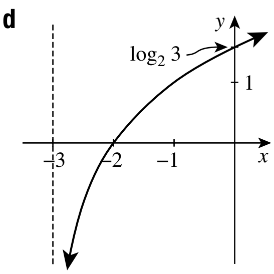
b



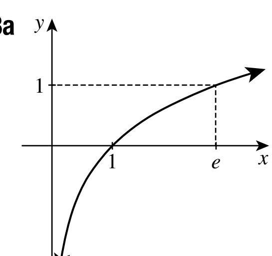
c



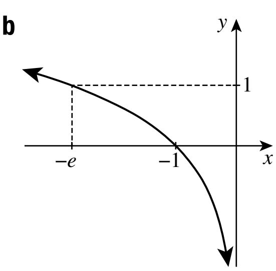
d



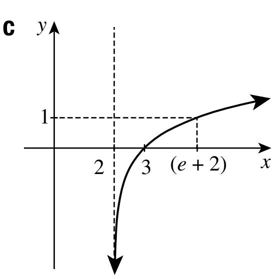
18a



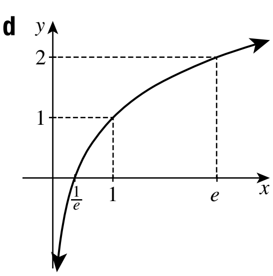
b



c



d



19a $\dfrac{dP}{dt} = -\frac{1}{100}P_0e^{-0.01t}$

b $\dfrac{dP}{dt} = -\frac{1}{100}P_0e^{-0.45} = -0.0064P_0$ lizards per year.

c $P = P_0e^{-0.45} \doteqdot 64\%$ of the original population.

d $e^{-0.01t} = \frac{1}{10}$, so $t = 100\log_e 10 \doteqdot 230$ years

20a π **b** $\frac{\pi}{9}$ **c** $\frac{4\pi}{3}$ **d** $\frac{7\pi}{4}$

21a 30° **b** 108° **c** 540° **d** 300°

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22a $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{\sqrt{3}}$

23a $x = \frac{\pi}{4}$ or $\frac{7\pi}{4}$ **b** $x = \frac{2\pi}{3}$ or $\frac{5\pi}{3}$

24a $\sin\theta = 0$ or $-\frac{1}{2}$, $\theta = 0,\ \pi,\ \frac{7\pi}{6},\ \frac{11\pi}{6}$ or 2π

b $\cos\theta = -1$ or 2, $\theta = \pi$ ($\cos\theta = 2$ has no solutions.)

c $\tan\theta = \frac{1}{2}$ and $\theta \doteqdot 0.46$ or 3.61, or $\tan\theta = -3$ and $\theta \doteqdot 1.89$ or 5.03

25a 3π cm **b** $12\pi\,\text{cm}^2$

26 $16(\pi - 2) \doteqdot 18.3\,\text{cm}^2$

27 148°58′

28a $y = \sin x$ and $y = \cos x$ both have amplitude 1.

b $y = \sin x$, $y = \cos x$, $y = \operatorname{cosec} x$ and $y = \sec x$ all have period 2π, $y = \tan x$ and $y = \cot x$ both have period π.

c $y = \sin x$, $y = \tan x$, $y = \operatorname{cosec} x$ and $y = \cot x$ are all odd, $y = \cos x$, and $y = \sec x$ are both even.

29 a $\theta = \frac{3\pi}{2}$ **b** $\theta = \frac{\pi}{2}$ **c** $x = \frac{\pi}{4}$

Chapter 10

Exercise 10A

1a $\frac{1}{20}$ **b** $\frac{19}{20}$

2a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** 1 **d** 0

3a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{1}{3}$

4a $\frac{5}{12}$ **b** $\frac{7}{12}$ **c** 0

5a $\frac{4}{9}$ **b** $\frac{5}{9}$ **c** $\frac{11}{18}$

6a $\frac{4}{9}$ **b** $\frac{5}{9}$ **c** $\frac{11}{18}$ **d** $\frac{7}{18}$ **e** $\frac{1}{3}$ **f** $\frac{1}{6}$

7a $\frac{3}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$

8a $\frac{1}{26}$ **b** $\frac{5}{26}$ **c** $\frac{21}{26}$ **d** 0 **e** $\frac{3}{26}$ **f** $\frac{5}{26}$

9 78%

10a $\frac{4}{7}$ **b** 32

11a 8 **b** $\frac{14}{15}$

12a 10 sixes

b i $\frac{18}{60} = 30\%$

ii The experiment suggest a probability of about 30%

iii The theoretical probability suggests that for an unbiased die, we would expect to get a six on one-sixth of the throws, that is, 10 times. The large number of sixes turning up suggests that this die is biased.

13a $\frac{100}{400} = \frac{1}{4} = 25\%$ **b** $\frac{8}{20} = \frac{2}{5} = 40\%$

c We would expect him to get chicken one-quarter of the time, that is on 5 occasions. He may have got more chicken sandwiches because of the way the canteen makes or sells the sandwiches, for example making the chicken sandwiches early and placing them at the front of the display, or making more Vegemite sandwiches as they sell out. Possibly also the sample is too small and the result would approach $\frac{1}{4}$ if the experiment were continued over a longer time. The experimental probability is only an estimate, and in fact it is possible he may have got no chicken sandwiches over the 20 days.

14a $\frac{1}{20}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{2}{5}$ **f** $\frac{1}{5}$ **g** $\frac{1}{4}$ **h** 0 **i** 1

15a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{13}$ **d** $\frac{1}{52}$ **e** $\frac{1}{4}$ **f** $\frac{3}{13}$ **g** $\frac{1}{2}$

h $\frac{1}{13}$ **i** $\frac{3}{13}$ (counting an ace as a one)

16a $\frac{1}{15}$ **b** $\frac{7}{150}$ **c** $\frac{1}{2}$ **d** $\frac{4}{25}$ **e** $\frac{1}{75}$ **f** $\frac{17}{50}$

17a $\frac{1}{5}$ **b** $\frac{3}{40}$ **c** $\frac{9}{20}$ **d** $\frac{7}{100}$ **e** $\frac{7}{50}$ **f** $\frac{1}{200}$

18a $\frac{3}{4}$ **b** $\frac{1}{4}$

19 187 or 188

20a The argument is invalid, because on any one day the two outcomes are not equally likely. The argument really can't be corrected.

b The argument is invalid. One team may be significantly better than the other, the game may be played in conditions that suit one particular team, and so on. Even when the teams are evenly matched, the high-scoring nature of the game makes a draw an unlikely event. The three outcomes are not equally likely. The argument really can't be corrected.

c The argument is invalid, because we would presume that Peter has some knowledge of the subject, and is therefore more likely to choose one answer than another. The argument would be valid if the questions were answered at random.

d The argument is only valid if there are equal numbers of red, white and black beads, otherwise the three outcomes are not equally likely.

e The argument is missing, but the conclusion is correct. Exactly one of the four players will win his semi-final and then lose the final. Our man is as likely to pick this player as he is to pick any of the other three players.

21a $\frac{2}{9}$ **b** $\frac{\pi}{18}$

Exercise 10B

1a HH, HT, TH, TT **b i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$

2a H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

b i $\frac{1}{4}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{4}$

3a TO, OT, OE, EO, ET, TE

b i $\frac{1}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{2}{3}$

4a AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, DC

b i $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{6}$ **v** $\frac{1}{4}$ **vi** $\frac{3}{4}$

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5a 23, 32, 28, 82, 29, 92, 38, 83, 39, 93, 89, 98

b i $\frac{1}{12}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{6}$ **v** $\frac{1}{4}$ **vi** 0

6a The captain is listed first and the vice-captain second: AB, AC, AD, AE, BC, BD, BE, CD, CE, DE, BA, CA, DA, EA, CB, DB, EB, DC, EC, ED

b i $\frac{1}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{5}$ **iv** $\frac{1}{5}$

7 HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** $\frac{1}{2}$

8 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66

a $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{36}$ **d** $\frac{1}{6}$ **e** $\frac{1}{6}$ **f** $\frac{1}{4}$ **g** $\frac{11}{36}$ **h** $\frac{4}{9}$ **i** $\frac{5}{36}$ **j** $\frac{1}{6}$

9a i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **b i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{2}$

10a $\frac{1}{16}$ **b** $\frac{1}{4}$ **c** $\frac{11}{16}$ **d** $\frac{5}{16}$ **e** $\frac{3}{8}$ **f** $\frac{5}{16}$

11a $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{1}{5}$

12a 24 **b i** $\frac{2}{3}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{12}$ **iv** $\frac{1}{6}$

13a $\dfrac{1}{2^n}$ **b** $1 - 2^{1-n}$

Exercise 10C

1a $\{1, 3, 5, 7, 9\}$ **b** $\{6, 12, 18, 24, 30, 36\}$

c $\{1, 2, 3, 4, 5, 6\}$ **d** $\{1, 2, 4, 5, 10, 20\}$

2a $A \cup B = \{1, 3, 5, 7\}$, $A \cap B = \{3, 5\}$

b $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A \cap B = \{4, 9\}$

c $A \cup B = \{\text{h, o, b, a, r, t, i, c, e, n}\}$, $A \cap B = \{\text{h, o, b}\}$

d $A \cup B = \{\text{j, a, c, k, e, m}\}$, $A \cap B = \{\text{a}\}$

e $A \cup B = \{1, 2, 3, 5, 7, 9\}$, $A \cap B = \{3, 5, 7\}$

3a false **b** true **c** false **d** false **e** true **f** true

4a 3 **b** 2 **c** $\{1, 3, 4, 5\}$ **d** 4 **e** $\{3\}$ **f** 1

g $\{2, 4\}$ **h** $\{1, 2, 5\}$

5a students who study both Japanese and History

b students who study either Japanese or History or both

6a students at Clarence High School who do not have blue eyes

b students at Clarence High School who do not have blond hair

c students at Clarence High School who have blue eyes or blond hair or both

d students at Clarence High School who have blue eyes and blond hair

7a $\varnothing$, $\{a\}$ **b** $\varnothing$, $\{a\}$, $\{b\}$, $\{a, b\}$

c $\varnothing$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$

d $\varnothing$

8a true **b** false **c** true **d** false **e** true

9a $\{2, 4, 5, 6, 8, 9\}$ **b** $\{1, 2, 3, 5, 8, 10\}$

c $\{7\}$ **d** $\{1, 2, 3, 4, 5, 6, 8, 9, 10\}$

e $\{1, 3, 4, 6, 7, 9, 10\}$ **f** $\{2, 5, 8\}$

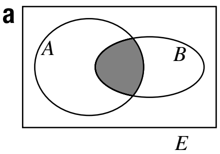
10a $\{2, 4, 5, 7, 9, 10\}$ **b** $\{1, 2, 5, 8, 9\}$

c $\{1, 2, 4, 5, 7, 8, 9, 10\}$ **d** $\{2, 5, 9\}$

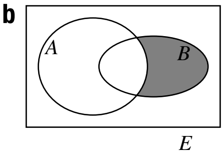
e $\{1, 3, 4, 6, 7, 8, 10\}$ **f** $\{2, 5, 9\}$

g $\{3, 6\}$ **h** $\{1, 2, 4, 5, 7, 8, 9, 10\}$

11a



b



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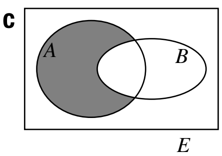

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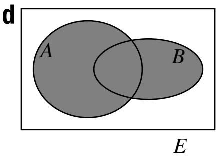

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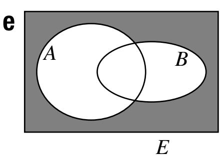

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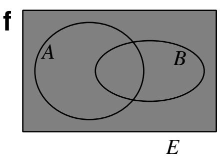


12a true **b** true

13a Q **b** P

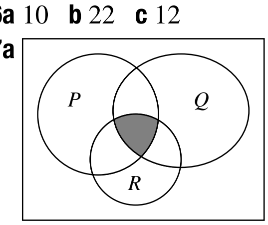
14a III **b** I **c** II **d** IV

15a $|A \cap B|$ is subtracted so that it is not counted twice.

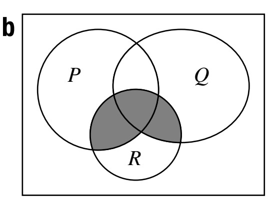
b 5 **c** LHS = 7, RHS = 5 + 6 – 4 = 7

16a 10 **b** 22 **c** 12

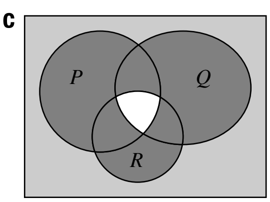
17a



b



c



18 4

Exercise 10D

1a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{3}$ **d** 0 **e** 1 **f** 0 **g** $\frac{1}{6}$ **h** $\frac{2}{3}$

2a $\frac{1}{13}$ **b** $\frac{1}{13}$ **c** $\frac{2}{13}$ **d** 0 **e** $\frac{11}{13}$ **f** $\frac{1}{2}$

g $\frac{3}{13}$ **h** $\frac{3}{26}$ **i** $\frac{8}{13}$ **j** $\frac{5}{13}$

3a $A = \{HH\}$, $B = \{HT, TH\}$, $P(A \text{ or } B) = \frac{3}{4}$, $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{4}$

b $A = \{RS\}$, $B = \{RS, ST\}$, $P(A \text{ or } B) = \frac{3}{3}$, $P(A) = \frac{1}{3}$, $P(B) = \frac{2}{3}$

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4a no **b i** $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{5}{6}$

5a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{3}{4}$ **e** $\frac{1}{4}$ **f** $\frac{1}{6}$
g $\frac{1}{6}$ **h** $\frac{1}{36}$ **i** $\frac{11}{36}$ **j** $\frac{25}{36}$

6a i $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{2}$ **v** $\frac{1}{2}$
b i $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{5}$ **iv** 0 **v** 1
c i $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$ **v** $\frac{5}{6}$

7a $\frac{7}{15}$ **b** 0 **c** $\frac{3}{5}$ **d** $\frac{5}{7}$

8a i no **ii** $\frac{1}{2}, \frac{1}{4}, \frac{3}{20}, \frac{3}{5}$
b i no **ii** $\frac{1}{2}, \frac{3}{10}, \frac{3}{20}, \frac{13}{20}$
c i yes **ii** $\frac{1}{4}, \frac{9}{20}, 0, \frac{7}{10}$

9a $\frac{9}{25}$ **b** $\frac{7}{50}$ **c** $\frac{17}{50}$

10a 10 **b i** $\frac{4}{21}$ **ii** $\frac{1}{3}$

11 $\frac{1}{4}$

12 2, 3, 5, 7, 11,13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
a $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{11}{100}$ **d** $\frac{39}{100}$

13a $\frac{7}{12}$ **b** $\frac{13}{60}$ **c** $\frac{3}{10}$ **d** $\frac{7}{60}$

Exercise 10E

1a $\frac{1}{24}$ **b** $\frac{1}{28}$ **c** $\frac{1}{12}$ **d** $\frac{1}{96}$ **e** $\frac{1}{42}$ **f** $\frac{1}{336}$

2a $\frac{1}{12}$ **b** $\frac{1}{12}$ **c** $\frac{1}{4}$ **d** $\frac{1}{3}$

3a $\frac{1}{25}$ **b** $\frac{2}{25}$ **c** $\frac{3}{25}$ **d** $\frac{3}{25}$ **e** $\frac{4}{25}$ **f** $\frac{2}{25}$ **g** $\frac{1}{25}$

4a $\frac{15}{49}$ **b** $\frac{8}{49}$ **c** $\frac{6}{49}$

5a $\frac{1}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{10}$ **d** $\frac{3}{10}$

6a $\frac{1}{36}$ **b** $\frac{1}{12}$ **c** $\frac{1}{36}$ **d** $\frac{1}{9}$ **e** $\frac{1}{6}$

7a $\frac{1}{7}$ **b** $\frac{180}{1331}$

8a i $\frac{13}{204}$ **ii** $\frac{1}{17}$ **iii** $\frac{4}{663}$ **iv** $\frac{1}{2652}$
b $\frac{1}{16}, \frac{1}{16}, \frac{1}{169}, \frac{1}{2704}$

9a i $\frac{2}{3}$ **ii** $\frac{1}{3}$ **b i** $\frac{8}{27}$ **ii** $\frac{1}{27}$ **iii** $\frac{4}{27}$

10a $\frac{3}{4}$ **b** $\frac{31}{32}$ **c** $\frac{1023}{1024}$

11a The argument is invalid, because the events 'liking classical music' and 'playing a classical instrument' are not independent. One would expect that most of those playing a classical instrument would like classical music, whereas a smaller proportion of those not playing a classical instrument would like classical music. The probability that a student does both cannot be discovered from the given data — one would have to go back and do another survey.

b The argument is invalid, because the events 'being prime' and 'being odd' are not independent — two out of the three odd numbers less than 7 are prime, but only one out of the three such even numbers is prime. The correct argument is that the odd prime numbers amongst the numbers 1, 2, 3, 4, 5 and 6 are 3 and 5, hence the probability that the die shows an odd prime number is $\frac{2}{6} = \frac{1}{3}$.

c The teams in the competition may not be of equal ability, and factors such as home-ground advantage may also affect the outcome of a game, hence assigning a probability of $\frac{1}{2}$ to winning each of the seven games is unjustified. Also, the outcomes of successive games are not independent — the confidence gained after winning a game may improve a team's chances in the next one, a loss may adversely affect their chances, or a team may receive injuries in one game leading to a depleted team in the next. The argument really can't be corrected.

d This argument is valid. The coin is normal, not biased, and tossed coins do not remember their previous history, so the next toss is completely unaffected by the previous string of heads.

12a $\frac{1}{36}$ **b** $\frac{1}{6}$ **c** $\frac{1}{4}$ **d** $\frac{1}{36}$ **e** $\frac{1}{36}$ **f** $\frac{1}{18}$ **g** $\frac{1}{12}$ **h** $\frac{1}{12}$ **i** $\frac{1}{6}$

13 HHH, HHM, HMH, MHH, HMM, MHM, MMH, MMM
a $P(\text{HHH}) = 0.9^3 = 0.729$ **b** $P(\text{MMM}) = 0.1^3 = 0.001$
c $P(\text{HMM}) = 0.9 \times 0.1^2 = 0.009$
d $P(\text{HMM}) + P(\text{MHM}) + P(\text{MMH} =$
$3 \times 0.009 = 0.027$
e 0.081 **f** 0.243

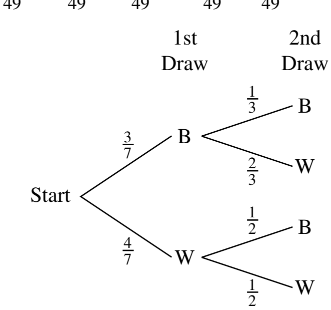
14a $\frac{9}{25}$ **b** 11
c Compare it with question 13 above, replacing 90% there with 80% here.

15a $\frac{1}{12960000}$ **b** 233

16a $\frac{1}{9}$ **b** $\frac{1}{9}$. Retell as 'Nick begins by picking out two socks for the last morning and setting them aside'.
c $\frac{1}{9}$. Retell as 'Nick begins by picking out two socks for the third morning and setting them aside'.
d $\frac{1}{63}$ **e** $\frac{1}{9 \times 7 \times 5 \times 3}$ **f** zero

Exercise 10F

1a i $\frac{9}{49}$ **ii** $\frac{12}{49}$ **iii** $\frac{12}{49}$ **iv** $\frac{16}{49}$ **b i** $\frac{25}{49}$ **ii** $\frac{24}{49}$ **c i** $\frac{3}{7}$ **ii** $\frac{4}{7}$



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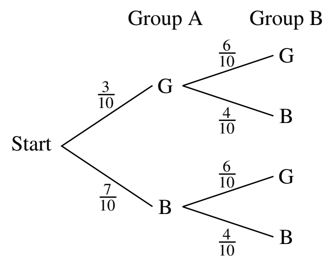
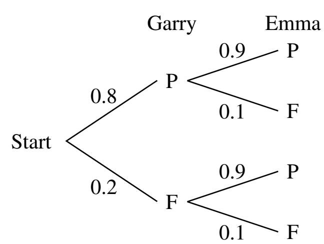
2a **i** 90.25% **ii** 4.75% **iii** 4.75% **iv** 0.25%
b 99.75%

3a **i** $\frac{6}{25}$ **ii** $\frac{9}{25}$ **iii** $\frac{4}{25}$ **iv** $\frac{6}{25}$ **b** **i** $\frac{12}{25}$ **ii** $\frac{13}{25}$

4a **i** $\frac{9}{50}$ **ii** $\frac{3}{25}$ **iii** $\frac{21}{50}$ **iv** $\frac{7}{25}$ **b** **i** $\frac{23}{50}$ **ii** $\frac{27}{50}$



5a 8% **b** 18% **c** 26% **d** 28%



6a $\frac{9}{25}$ **b** $\frac{21}{25}$

7 4.96%

8a 0.01 **b** 0.23

9 0.35

10 $\frac{4}{7}$

11a $\frac{21}{3980}$ **b** $\frac{144}{995}$

12a $\frac{3}{10}$ **b** $\frac{7}{24}$ **c** $\frac{21}{40}$

13a $\frac{1}{11}$ **b** $\frac{14}{33}$ **c** $\frac{10}{33}$ **d** $\frac{19}{33}$

14a $\frac{5}{6}$ **b** $\frac{5}{12}$ **c** $\frac{1}{6}$

15 The term 'large school' is code for saying that the probabilities do not change for the second choice because the sample space hardly changes.
a 0.28 **b** 0.50

16a $\frac{1}{25}$ **b** $\frac{3}{5}$

17a $\frac{1}{20}$ **b** $\frac{57}{8000}$

18a 31.52% **b** 80.48%

19a **i** $\frac{5}{33}$ **ii** $\frac{5}{22}$ **iii** $\frac{19}{33}$ **iv** $\frac{1}{4}$ **v** $\frac{19}{66}$ **vi** $\frac{47}{66}$
b **i** $\frac{25}{144}$ **ii** $\frac{5}{24}$ **iii** $\frac{5}{9}$ **iv** $\frac{1}{4}$ **v** $\frac{25}{72}$ **vi** $\frac{47}{72}$

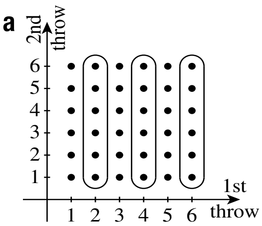
20a $\frac{1}{36}$ **b** $\frac{1}{46\,656}$ **c** $\frac{11}{36}$

21a $\frac{1}{216}$ **b** $\frac{5}{72}$ **c** $\frac{5}{12}$ **d** $\frac{5}{9}$

22 $\frac{1}{3}$

Exercise 10G

1a



b $\frac{1}{18}$ **c** $\frac{4}{9}$ **d** $\frac{1}{9}$

2a $\frac{340}{1000} = \frac{17}{50} = 0.34$ **b** $\frac{190}{420} = \frac{19}{42} \doteqdot 0.45$
c $\frac{130}{340} = \frac{13}{34} \doteqdot 0.38$ **d** $\frac{20}{130} = \frac{2}{13} \doteqdot 0.15$

3a Totals in last column: 56, 137, 193
Totals in last row: 124, 69, 193
b **i** $\frac{42}{193} \doteqdot 0.22$ **ii** $\frac{29}{124} \doteqdot 0.23$
iii $\frac{29}{56} \doteqdot 0.52$ **iv** $\frac{95}{137} \doteqdot 0.69$

4a $\frac{1}{16}$ **b** HH, HD, HC, HS; $\frac{1}{4}$
c HH, HD, HC, HS, DH, CH, SH; $\frac{1}{7}$
d HH, HD, HC, HS, DH, DD, DC, DS; $\frac{1}{8}$

5a

	1	2	3	4	5	6
HH	3	4	5	6	7	8
HT	2	3	4	5	6	7
TH	2	3	4	5	6	7
TT	1	2	3	4	5	6

b $\frac{1}{24}$ **c** $\frac{1}{6}$ **d** $\frac{1}{2}$

6a $\frac{5}{7}$ **b** $\frac{3}{8}$ **c** $\frac{16}{19}$

7a $P(A \cap B) = 0.24$ **b** $P(A \cap B) = 0.15$
c $P(A|B) = 0.4$ **d** $P(A|B) = 0.7$

8a dependent **b** independent **c** dependent
d independent
e impossible; $P(A \cap B)$ cannot be bigger than $P(A)$ or $P(B)$
f independent

9a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b The cases $1 + 4$, $2 + 3$, $3 + 2$ and $4 + 1$ make up the reduced sample space.
i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** 1

10a **i** 0.1 **ii** $\frac{1}{3}$ **iii** $\frac{1}{4}$ **b** $\frac{3}{7}$ **c** $\frac{1}{2}$ **d** $\frac{5}{9}$

11 $\frac{4}{11}$

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12 $\frac{5}{8}$ or 62.5%

13a $\frac{1}{2}$ **b** $\frac{1}{3}$

14a BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG
b $\frac{4}{7}$ **c** $\frac{2}{3}$

15a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{11}{153}$

16a and **b** $P(A|B) = P(A \cap B)/P(B) = \frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$, while $P(A) = \frac{1}{2}$. Hence the events are independent.

17a $P(A \cup B) = 0.76$ **b** $P(A \cup B) = 0.72$

18a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{5}$

19 $\frac{7}{15}$

20 $\frac{9}{23}$

21 $\frac{3}{7}$

22a 5.75% **b** 4.95% **c** 86% **d** 0.21%
e It is most important that the number of false negatives is low — that almost all patients with the disease are picked up. False positives are scary for the patient, but further tests should determine that they do not have the disease.

23a and **b**
$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(B \cap A)}{P(B)} \\ &= \frac{P(B|A)P(A)}{P(B)} \\ &= \frac{P(B|A)}{P(B)} \times P(A) \end{aligned}$$

24 If B is independent of A then,
$$\begin{aligned} P(A|B) &= \frac{P(B|A)}{P(B)} \times P(A) \\ &= \frac{P(B)}{P(B)} \times P(A) \\ &= P(A) \end{aligned}$$
which states that A is independent of B.

25 Suppose first that the contestant changes her choice. If her original choice was correct, she loses, otherwise she wins, so her chance of winning is $\frac{2}{3}$. Suppose now that the contestant does not change her choice. If her original choice was correct, she wins, otherwise she loses, so her chance of winning is $\frac{1}{3}$. Thus the strategy of changing will double her chance of winning.

26 Let $G1$ be, 'A girl is born on a Sunday', let $B1$ be, 'A boy is born on a Sunday', let $G2$ be, 'A girl is born on a Monday', …, giving 14 equally likely events at the birth of every child. In this particular family, there are two children, giving $14^2 = 196$ equally likely possible outcomes for the two successive births in this family. Draw up the 2×2 sample space, showing at least all the entries in the row indexed by $G2$ and the column indexed by $G2$.
Let A be, 'At least one child is a girl born on a Monday.' There are 27 favourable outcomes for A.
Let B be, 'Both children are girls.' There are 13 favourable outcomes for the event $A \cap B$.
Hence $P(B|A) = |A \cap B| / |A| = \frac{13}{27}$

Chapter 10 review exercise

1a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{6}$ **d** $\frac{1}{2}$

2a $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{3}{10}$ **d** 0 **e** 1 **f** $\frac{3}{10}$

3a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{13}$ **d** $\frac{1}{52}$ **e** $\frac{1}{2}$ **f** $\frac{12}{13}$

4 37%

5a $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

6a $\frac{1}{36}$ **b** $\frac{1}{9}$ **c** $\frac{1}{6}$ **d** $\frac{11}{36}$ **e** $\frac{4}{9}$ **f** $\frac{1}{9}$ **g** $\frac{1}{6}$ **h** $\frac{11}{36}$

7a $\frac{17}{60}$ **b** $\frac{19}{60}$ **c** $\frac{1}{6}$

8a No **b i** $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{5}{6}$

9a $\frac{1}{12}$ **b** $\frac{1}{5}$ **c** $\frac{3}{20}$ **d** $\frac{1}{20}$

10a i $\frac{13}{204}$ **ii** $\frac{1}{17}$ **iii** $\frac{4}{663}$ **iv** $\frac{1}{2652}$
b i $\frac{1}{16}$ **ii** $\frac{1}{16}$ **iii** $\frac{1}{169}$ **iv** $\frac{1}{2704}$

11a 14% **b** 24% **c** 38% **d** 6%

12a $\frac{2}{21}$ **b** $\frac{11}{21}$ **c** $\frac{10}{21}$ **d** $\frac{2}{7}$

13a $\frac{19}{12475}$ **b** $\frac{979}{12475}$

14a independent **b** dependent
c independent, with $P(A \cap B) = 0.18$

15 $\frac{3}{11}$

Chapter 11

Exercise 11A

1a numeric, discrete **b** numeric, continuous
c categorical **d** numeric, continuous
e categorical **f** categorical
g On a standard scale of shoes sizes, this is numeric and discrete. The length of a person's foot would be a continuous distribution.
h Numeric, discrete. Reported ATAR scores are between 30 and 99.95 in steps of 0.05. There are about 1400 different scores awarded.

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2a

Outcome	HH	HT	TH	TT
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Uniform distribution (and categorical).

b

Outcome	2 heads	1 head and 1 tail	2 tails
Probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

3a

Outcome	red	green
Probability	$\frac{4}{7}$	$\frac{3}{7}$

b

Outcome	J	K	L	O
Probability	0.06	0.08	0.04	0.82

c

Outcome	P	A	R	M	T
Probability	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{5}$

d

Outcome	1	2	3	4
Probability	$\frac{9}{1000}$	$\frac{90}{1000}$	$\frac{900}{1000}$	$\frac{1}{1000}$

e

Outcome	even	prime	neither
Probability	$\frac{5}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

4a Let X be the number of letters in a randomly chosen word.

Outcome x	3	4	6
Probability $P(X = x)$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

b Let X be the number of heads recorded when two coins are thrown.

x	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c Let X be the digits recorded from the first 12 digits of $\sqrt{2}$.

x	1	2	3	4	5	6	7
$P(X = x)$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

d Let X be the number selected.

x	1	2	3	4	5
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(Note that the answer is the same if the sets are amalgamated. Why?)

5a { T }, { F1 }, { F2 }, { T, F1 }, { T, F2 }, { F1, F2 }, { T, F1, F2 }

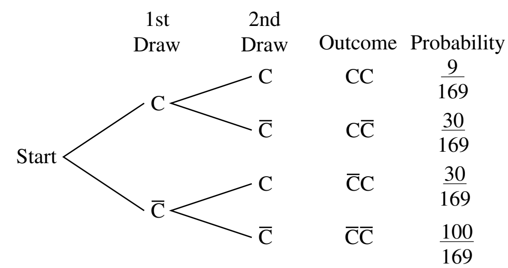
b

x	5	10	15	20
$P(X = x)$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

6a yes **b** no **c** yes **d** yes **e** no **f** yes

7a 0.2 **b** 0.6 **c** 0.75 **d** 0 **e** 0.6 **f** 0.85 **g** 0.9 **h** 0.7 **i** 0.45

8a i Let C be the event, 'A court card is drawn.'



ii

x	0	1	2
$P(X = x)$	$\frac{100}{169}$	$\frac{60}{169}$	$\frac{9}{169}$

b i The eight outcomes EEE, EEO, EOE, EOO, OEE, OEO, OOE, OOO each have probability $\frac{1}{8}$.

ii

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

c GGG has probability $\frac{8}{125}$, GGB, GBG, BGG each have probability $\frac{12}{125}$, GBB, BGB, BBG each have probability $\frac{18}{125}$, BBB has probability $\frac{27}{125}$.

x	0	1	2	3
$P(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$	$\frac{54}{125}$	$\frac{27}{125}$

d Let S be the event, 'A wallaby from Snake Ridge was selected'. SSS has probability 0.027,
$\overline{S}SS$, $S\overline{S}S$, $SS\overline{S}$ each have probability 0.063,
$\overline{S}\,\overline{S}S$, $\overline{S}S\overline{S}$, $S\overline{S}\,\overline{S}$ each have probability 0.147,
$\overline{S}\,\overline{S}\,\overline{S}$ has probability 0.343.

x	0	1	2	3
$P(X = x)$	0.343	0.441	0.189	0.027

9a $a = \frac{1}{25}$ **b** $a = \frac{1}{14}$ **c** $a = \frac{1}{27}$ **d** $a = \frac{1}{10}$ **e** $a = 1$

10a i EE and OO each have probability $\frac{1}{5}$,
EO and OE each have probability $\frac{3}{10}$.

ii

x	0	1	2
$P(X = x)$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$

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b i BB has probability $\frac{2}{5}$, BG and GB each have probability $\frac{4}{15}$, GG has probability $\frac{1}{15}$.

ii

x	0	1	2
$P(X = x)$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$

c i EE has probability $\frac{3}{10}$, ER, RE, ET, TE each have probability $\frac{3}{20}$, RT and TR each have probability $\frac{1}{20}$.

ii

x	0	1	2
$P(X = x)$	$\frac{1}{10}$	$\frac{3}{5}$	$\frac{3}{10}$

11

x	22	44	55	24 or 42	25 or 52	45 or 54
$P(X = x)$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{36}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{6}$

12a

Outcome	RR	RG	GR	GG
Probability	$\frac{16}{49}$	$\frac{12}{49}$	$\frac{12}{49}$	$\frac{9}{49}$

b

Outcome	RR	RG	GR	GG
Probability	$\frac{12}{42}$	$\frac{12}{42}$	$\frac{12}{42}$	$\frac{6}{42}$

c

Outcome	HH	DD	SS	CC
Probability	$\frac{1}{17}$	$\frac{1}{17}$	$\frac{1}{17}$	$\frac{1}{17}$

HS or SH	HC or CH	HD or DH
$\frac{13}{102}$	$\frac{13}{102}$	$\frac{13}{102}$
SC or CS	SD or DS	CD or DC
$\frac{13}{102}$	$\frac{13}{102}$	$\frac{13}{102}$

13 a–c Answers will vary

d There is no guarantee that the results will be identical, though you would expect more *trials* (repeats of the experiment) would bring the results closer to each other and to the theoretical probabilities.

e Theoretical results: $P(X = 0) = 0.3$, $P(X = 1) = 0.6$, $P(X = 2) = 0.1$

f It might be easier to perform the experiment with coloured balls or tokens. Running the experiment in pairs with a nominated recorder also helps. The paper pieces need to be indistinguishable and well mixed in the bag. You could increase the number of trials or combine the class results.

14 EEE and OOO each have probability $\frac{1}{20}$, the other six possible outcomes each have probability $\frac{3}{20}$,

x	0	1	2	3
$P(X = x)$	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{20}$

15a The condition that the sum of the probabilities is 1 gives $a = \frac{1}{4}$ or $a = 1$. But $a = 1$ gives probabilities outside the interval $0 \le p \le 1$, so the only valid answer is $a = \frac{1}{4}$.

b $a = 1$ or $\frac{7}{6}$ (both are valid)

16a Let X be the sum of the numbers on the three cards. This question is best done by asking what card is *discarded.*

x	20	21	22
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

b

x	20	21	22
$P(X = x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{3}{10}$

Exercise 11B

1a

x	0	1	2	3	Sum
$p(x)$	0.4	0.1	0.2	0.3	1
$xp(x)$	0	0.1	0.4	0.9	1.4

Hence $E(X) = 1.4$.

b

x	2	4	6	8	Sum
$p(x)$	0.1	0.4	0.4	0.1	1
$xp(x)$	0.2	1.6	2.4	0.8	5

Hence $E(X) = 5$.

c

x	−50	−20	0	30	100	Sum
$p(x)$	0.1	0.35	0.4	0.1	0.05	1
$xp(x)$	−5	−7	0	3	5	−4

Hence $E(X) = -4$.

2a

x	−40	0	30	60	Sum
$p(x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1
$xp(x)$	−20	0	5	10	−5

b Expected value $= -5$

c The average cost to the player per game is 5 cents.

d $100 \times (-5) = -500$ cents. Thus the player expects to lose 500 cents and the casino expects to make 500 cents profit. This is an expected average value, not guaranteed.

3a–d Answers will vary

4a

x_i	2	4	6	8	10	Sum
p_i	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	1
$x_i p_i$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	$\frac{8}{5}$	$\frac{10}{5}$	6

So $E(X) = 6$.

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b

x_i	-3	1	2	5	6	Sum
p_i	0.1	0.3	0.2	0.3	0.1	1
$x_i p_i$	-0.3	0.3	0.4	1.5	0.6	2.5

So $\text{E}(X) = 2.5$.

5a

x	1.50	2.10	2.40	Sum
$p(x)$	$\frac{5}{12}$	$\frac{4}{12}$	$\frac{3}{12}$	1
$xp(x)$	0.625	0.7	0.60	1.925

The expected value is \$1.925.

b If 100 purchases are made at random, the expected cost is \$192.50.

6a $\text{E}(X) = 3$

b **i** $\text{E}(Y) = 6$ **ii** Yes

c **i** $\text{E}(Z) = 4$ **ii** Yes

7a 15 **b** 10 **c** $\frac{5}{2}$ **d** 3 **e** 0 **f** 18

8

x	0	1	2	3	Sum
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1
$xp(x)$	0	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{3}{8}$	$\frac{12}{8}$

The expected value is $1\frac{1}{2}$, as might be expected from the symmetry of the table of probabilities.

9

x	0	1	2	Sum
$p(x)$	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{1}{17}$	1
$xp(x)$	0	$\frac{13}{34}$	$\frac{2}{17}$	$\frac{17}{34}$

The expected value is $\frac{1}{2}$.

10a–c Answers will vary

d

x	0	1	2	3	4	5	Sum
$p(x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	1
$xp(x)$	0	$\frac{10}{36}$	$\frac{16}{36}$	$\frac{18}{36}$	$\frac{16}{36}$	$\frac{10}{36}$	$\frac{70}{36}$

Hence $\text{E}(X) = \frac{35}{18}$.

e Answers will vary

f In any dice experiment, it is important to check the randomness of your dice rolls. This can depend on your rolling technique. Try throwing a die 12 times and see if every outcome is equally likely. Does each outcome seem independent of the last?

g Answers will vary

11a $\frac{3}{15}, \frac{3}{15}, \frac{3}{15}, \frac{2}{15}, \frac{2}{15}, \frac{2}{15}$

b -12, so the casino expects to make 12 cents each game, on average.

12a $P(\text{Orange}) = \frac{1}{6}$, $P(\text{Strawberry}) = \frac{2}{6}$, $P(\text{Apple}) = \frac{3}{6}$

b

Outcome	OOO	SSS	AAA	Other	Sum
x	$11k$	$2k$	k	0	—
$p(x)$	$\frac{1}{216}$	$\frac{8}{216}$	$\frac{27}{216}$	$\frac{180}{216}$	1

c The payout will be \$44 and their profit would be \$43, accounting for the \$1 entry fee.

13a $\mu = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{16} + 5 \times \frac{1}{32} + 6 \times \frac{1}{64} + \ldots \quad (1)$

Doubling:

$2\mu = 1 \times 1 + 2 \times \frac{1}{2} + 3 \times \frac{1}{4} + 4 \times \frac{1}{8} + 5 \times \frac{1}{16} + 6 \times \frac{1}{32} + \cdots \quad (2)$

Subtracting (1) from (2):

$\mu = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \quad (3)$

b Doubling:

$2\mu = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots \quad (4)$

Subtracting (3) from (4): $\mu = 2$.

c On average, we would expect to get a head on the second throw. You could test this by recording how many throws it takes over say 50 trials and averaging the results.

14 $\text{E}(X) = 2 \times \frac{1}{2} + 4 \times \frac{1}{4} + 8 \times \frac{1}{8} + 16 \times \frac{1}{16} + \cdots$

$= 1 + 1 + 1 + 1 + \cdots$

The expected value 'increases without bound', that is, $\text{E}(X) \to \infty$ as the game continues.

This suggests that there is no reasonable price the casino could put on this game and expect to break even. There are various issues with this scenario in real life. Casinos would not provide a game which had no upper limit to the payout. Patrons would also be unwilling to pay a large price for a game with such low apparent probabilities for the later stages of the game. The calculation of a simple expected value may not be the best way to analyse this game.

Exercise 11C

1a

x	1	2	3	4	Sum
$p(x)$	0.3	0.5	0.1	0.1	1
$xp(x)$	0.3	1	0.3	0.4	2
$(x-\mu)^2$	1	0	1	4	—
$(x-\mu)^2\,p(x)$	0.3	0	0.1	0.4	0.8

$\mu = 2$, $\text{Var}(X) = 0.8$

b $\sigma = \sqrt{0.8} \doteqdot 0.89$

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2a

x	1	2	3	4	Sum
$p(x)$	0.3	0.5	0.1	0.1	1
$xp(x)$	0.3	0.1	0.3	0.4	2
x^2	1	4	9	16	—
$x^2 p(x)$	0.3	2	0.9	1.6	4.8

b $\text{Var}(X) = 4.8 - 2^2 = 0.8$

3a $\text{E}(X) = 2, \text{Var}(X) = 2$ **b** $\text{E}(X) = 3, \text{Var}(X) = 1$

c $\text{E}(X) = 0, \text{Var}(X) = 2.6$

d $\text{E}(X) = 2.8, \text{Var}(X) = 1.36$

4a i $\text{E}(Y) = 2, \text{Var}(Y) = 1, \sigma = 1$

ii $\text{E}(Z) = 2, \text{Var}(Z) = 4, \sigma = 2$

iii $\text{E}(V) = 1, \text{Var}(V) = 0.8, \sigma \doteqdot 0.89$

iv $\text{E}(W) = 3, \text{Var}(W) = 0.8, \sigma \doteqdot 0.89$

b i Both sets of data are centred around 2 and the expected value of each is, unsurprisingly, 2. The second data set is more spread out — in fact in moving from Y to Z the distances from the mean to each data point have been doubled and the standard deviation is doubled.

ii The data has been 'flipped over', but is no more spread out than before; the variance is unchanged, the expected value has changed. You may notice that $W = 4 - V$.

5 $\text{E}(X) = 2, \text{Var}(X) = 0$

6a $\text{E}(J) = 1.55, \text{Var}(J) = 2.05, \text{E}(L) = 1.4, \text{Var}(L) = 0.84$

b Over the season John might be expected to score more baskets, because his expected value is higher.

c Liam is the more consistent player, with the lower variance. Coaches may prefer a more consistent player, particularly if it is more important to score *some* goals, rather than the maximum number. This may also be a sign that John needs to work on the consistency of his game.

7a Each outcome has probability $\frac{1}{3}$. This is a uniform distribution. **b** $\text{E}(X) = 2$ **c** $\text{Var}(X) = \frac{2}{3}$

8a Two standard deviations

b It is one and a half standard deviations below the mean.

c The English score was more standard deviations below the mean than the Mathematics result, so it may be considered less impressive.

9a Visual Arts is 1 standard deviation below the mean, Music is 1.75 standard deviations below the mean, hence the Visual Arts score is better.

b Earth Science is 2 standard deviations above the mean, Biology is 1.5 standard deviations above the mean, hence the Earth Science score is more impressive.

c Chinese is 2 standard deviations above the mean, Sanskrit is also 2 standard deviations above the mean, hence the scores are equally impressive.

10a $E(X) = 3.3, \sigma = 1.45$

b 8 appears to be a long way from 3.3 and well removed from the rest of the data.

c 8 is 3.2 standard deviations above the mean and thus would be an outlier by this definition.

d $E(X) = 3.15, \sigma = 1.06$

e The mean and standard deviation have changed significantly, especially the standard deviation.

f Outliers are interesting values in any distribution and should be a flag to investigate more closely. Were results recorded correctly? Was there an error in the experiment; e.g. Jasmine used a more powerful bow with greater range, or perhaps she used a new set of arrows with better fletching? It may, however, be that Jasmine is inconsistent, occasionally getting much better results, but often getting fairly poor results — in which case the large standard deviation is warranted as a measure of this distribution. Over 20 trials, a probability of 0.05 only represents one set of 10 shots, so a larger set of results may give a better picture of her long term accuracy and reduce the impact of one strong result amongst many other weaker scores.

11 $k = \frac{1}{10}, E(X) = 3, \sigma = 1$

12a $\frac{1}{n}$ **b** $\frac{n+1}{2}$ **c** $\frac{1}{12}(n^2 - 1)$ **d** Answers will vary

13a Because $Z = X + a$:

$$\begin{aligned}
\text{E}(Z) &= \sum zP(Z = z) \\
&= \sum (x + a)P(X + a = x + a) \\
&= \sum (x + a)P(X = x) \\
&= \sum xP(X = x) + \sum aP(X = x) \\
&= \sum xP(X = x) + a\sum P(X = x) \\
&= \mu + a
\end{aligned}$$

because $\sum P(X = x) = 1$.

b Because $Z = kX$:

$$\begin{aligned}
\text{E}(Z) &= \sum zP(Z = z) \\
&= \sum (kx)P(kX = kx) \\
&= \sum (kx)P(X = x), \\
&= k \times \sum xP(X = x) \\
&= k\mu
\end{aligned}$$

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14a The mean of Z is $\mu + a$, by the previous question.

Hence $\mathrm{Var}(Z) = \mathrm{E}((Z - (\mu + a))^2)$
$= \mathrm{E}((Z - a - \mu)^2)$
$= \mathrm{E}((X - \mu)^2)$
$= \mathrm{Var}(X)$

Hence the standard deviation of the new distribution remains σ. This is to be expected, because the distribution is no more spread out than previously.

b The mean of Z is $k\mu$, by the previous question. Hence

$\mathrm{Var}(Z) = \mathrm{E}((Z - k\mu)^2)$
$= \mathrm{E}((kX - k\mu)^2)$
$= k^2 \times \mathrm{E}((X - \mu)^2)$
$= k^2\mathrm{Var}(X)$

The standard deviation of the new distribution is $\sqrt{k^2\sigma^2} = k\sigma$.

Exercise 11D

1a

x	0	1	2	3	Sum
$p(x)$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$	1
$xp(x)$	0	$\frac{12}{27}$	$\frac{12}{27}$	$\frac{3}{27}$	1
$x^2p(x)$	0	$\frac{12}{27}$	$\frac{24}{27}$	$\frac{9}{27}$	$1\frac{2}{3}$

$\mu = 1, \sigma^2 = 1\frac{2}{3} - 1^2 = \frac{2}{3}, \sigma \doteqdot 0.82$

b

x	0	1	2	3	Sum
f	33	47	16	4	100
f_r	0.33	0.47	0.16	0.04	1
xf_r	0	0.47	0.32	0.12	0.91
x^2f_r	0	0.47	0.64	0.36	1.47

$\bar{x} = 0.91, s^2 = 1.47 - (0.91)^2 = 0.6419, s \doteqdot 0.80$

c The sample results are a little below what is predicted by the theoretical probabilities.

2a $\mu = 7, \sigma^2 = \frac{35}{6}, \sigma \doteqdot 2.42$ **b–f** Answers will vary

3a–f Answers will vary

4a Answers will vary **b** Answers will vary

c

x	0	1	2	3	4	5	Sum
$p(x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	1
$xp(x)$	0	$\frac{10}{36}$	$\frac{16}{36}$	$\frac{18}{36}$	$\frac{16}{36}$	$\frac{10}{36}$	$\frac{70}{36}$
$x^2p(x)$	0	$\frac{10}{36}$	$\frac{32}{36}$	$\frac{54}{36}$	$\frac{64}{36}$	$\frac{50}{36}$	$\frac{210}{36}$

$\mu \doteqdot 1.94, \sigma^2 = \frac{210}{36} - \left(\frac{70}{36}\right)^2, \sigma \doteqdot 1.43$

i–iv Answers may vary should be suitable

5a Answers will vary **b** Answers will vary

6a–d Answers will vary

7a $\mu = 3.441, \sigma \doteqdot 2.46$ **b** Answers will vary

8 Answers will vary

9a–c Answers will vary

10a–c Answers will vary

11 Answers will vary

12a Later people taking part in the experiment will be influenced by earlier guesses, particularly if the previous guesses have been measured for accuracy. Perhaps students could record their estimate, or draw their estimated shape, at the same time and before any measuring occurs. Perhaps students go into a separate room for the experiment.

b i–iii Answers will vary

13a–c Answers will vary

14a $m - k$ is the number of serial numbers not yet discovered in the range from 1 to m. If these serial numbers are spread between the k gaps, the average size of the gap (number of undiscovered serials) is $\frac{m-k}{k}$.

b The gap of $\frac{m-k}{k}$ integers should extend past m to $m + \frac{m-k}{k}$. Using this estimate the last serial will be:

$N = m + \frac{m-k}{k}$
$= m + \frac{m}{k} - 1$

c i–iii Answers will vary

Chapter 11 review exercise

1a numeric, continuous **b** numeric, discrete

c numeric, discrete (and infinite)

d categorical

2a yes **b** no **c** no

3 The probabilities are not all positive, do not sum to 1, and are not all less than 1.

4a $\mathrm{E}(X) = 1.4$ **b** $\mathrm{E}(X) = -0.8$

5a $\mathrm{E}(X) = 27.22$

b His expected cost is \$27.22 × 52 = \$1415.56.

6a $E(X) = 2, \mathrm{Var}(X) = 1, \sigma = 1$

b $E(X) = 5.1, \mathrm{Var}(X) = 0.69, \sigma \doteqdot 0.83$

7a $E(X) = 2, E(X^2) = 5, \mathrm{Var}(X) = 1$

b $E(X) = 5.1, E(X^2) = 26.70, \mathrm{Var}(X) = 0.69$

8a $E(X) = 1.9, \mathrm{Var}(X) = 0.49, \sigma = 0.7$

b $E(X) = 2, \mathrm{Var}(X) = 2.6, \sigma \doteqdot 1.61$

9 Expected value is a measure of central tendency — it measures the centre of the data set. It may also be thought of as a weighted mean (weighted by the probabilities of the distribution). If the experiment is carried out experimentally a large number of times we would expect that the average of the outcomes would approach the expected value.

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10 The standard deviation is the square root of the variance. Both measure the spread of the data, so that a distribution with a larger standard deviation is more spread out than a distribution with a smaller standard deviation. Both are zero if the distribution only takes one value — that is. if it is not spread out at all. If the distribution is stretched (multiplied) by a constant k the standard deviation also increases by a factor k.

11a 12, 8, $2\sqrt{2}$ **b** 11, 2, $\sqrt{2}$ **c** 17, 18, $3\sqrt{2}$

12a

x	5	6	7	8	9
$p(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$E(X) = 7$, $\mathrm{Var}(X) = 2$, $\sigma = \sqrt{2}$

b Answers will vary

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