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Physical Modeling of Wind Instruments

A major theme of musical acoustics has been to understand, through physical models of self-sustained oscillation, how wind instruments function. The first significant advance in scientific understanding was associated with Helmholtz, who identified how wind instruments produce tones as a relevant scientific problem. Yet, by this time, the orchestral string and wind instruments had, with few exceptions, already congealed into their modern forms. Adaptation ceased. The revolution in design of Western musical instruments between the late 18th and 19th centuries occurred essentially without the benefit of a mature science of acoustics.

If the 19th century was the golden age of wind instrument design, then the 20th century might be described as a silver age of electronic musical instrument design. There is a fundamental distinction to be made, however. The designer of electronic musical instruments, both analog and digital, has access to explicitly scientific methods, spanning the range from the physical to the computational, perceptual, and cognitive. The 19th-century designers were keenly aware of what their technology could do, but when they attempted to apply acoustics, the results were inconclusive at best.

Current theories of wind instruments are formulated in a manner suitable to computation. The idea of using a physical model of a wind instrument as a music synthesis technique has arrived. This is due to the demonstrated musical flexibility of traditional wind instruments, the relatively sophisticated knowledge concerning how these instruments produce sound, and advances in computer technology and digital audio. The creative interaction between the points of view of musical acoustics and computer music on the subject of wind instruments is the focus of this tutorial.

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Early Research

With the publication of the first edition of *On the Sensations of Tone* in 1862, Helmholtz set forth the principles by which each class of instrument produced musical tones. He classified wind instruments as belonging to the reed pipe or the flue pipe class. The reed pipe instruments include three subclasses with quite different pressure-controlled valving actions: (1) the organ reed pipe, harmonium, and harmonica; (2) the cane-reed instruments including single-reed woodwinds (clarinet and saxophone) and double-reed woodwinds (oboe, English horn, bassoon, and shawm); and (3) the so-called lip-reed instruments including the trumpet, trombone, and other brass wind instruments, where lip reed is the term used to describe the valving action of the player's lips. The flue pipe class includes the organ flue pipes, recorders, and flutes. In the 1877 edition of his book, Helmholtz (1954) had formulated quantitative theories of the mechanism by which oscillations are sustained in reed pipes, which have formed the basis for all later research. These theories are not based on detailed dynamical models; rather, they embody constraints that must be satisfied by any self-sustained, steady oscillation.

Rayleigh pointed out the significance of nonlinear dynamics for theories of musical instruments, but he never considered musical applications in depth. The first quantitative description of self-sustained oscillations (Rayleigh 1883) might be considered the touchstone for nonlinear mechanics and a model for the path that musical acoustics would tread after the 1960s. In modern terminology, he analyzes a system with strong connections to the Van der Pol oscillator, and shows both the negative resistance condition for self-sustained oscillations and the existence of the Hopf bifurcation that sets the threshold of oscillation. Rayleigh also developed the theory of transverse instabilities on air jets, which would later be important for understanding flutes and organ flue pipes.

Helmholtz and Rayleigh understood that an essential feature in sustaining a tone on a wind instrument is the existence of dissipation—some of the energy is transmitted in the form of acoustical radiation, but most is lost in the form of frictional and thermal dissipation. The wind instrument must be supplied with energy from an external source, so that just as much energy is entering the system as is being dissipated. If more energy is supplied than is dissipated, then the amplitude of oscillation grows; if less is supplied, then the amplitude of oscillation diminishes. One of the simplest oscillatory systems to consider is that of an adult pushing a child on a swing. If the adult stops pushing, the swing will ultimately come to a stop and the child will protest. The adult pushes on the swing once per oscillatory cycle and gains an intuitive knowledge of the physics of oscillation by pushing just after the swing has risen to its maximum amplitude. Thus, there is an important phase relationship between the displacement of the swing and the phase at which the external force is supplied. The adult also learns that if the swing is pushed too vigorously, then the displacement becomes too large, and the child again protests. This is a saturation mechanism to limit the amplitude of oscillation. A steady-state oscillation results that is modifiable—"I want to go higher..."

Poincaré, at the end of the 19th century, followed Rayleigh as the foremost developer of nonlinear dynamics. After Poincaré's death, nonlinear dynamics entered a dormant period that extended to the invention of experimental nonlinear dynamics, associated most prominently with Van der Pol and his radio oscillator. This was an inactive period in the science of wind instruments with the important exception of Henri Bouasse, whose work marks a transition between Helmholtz and the modern era.

Technical, but nonquantitative reviews of wind instruments are given by Benade (1990) and Keefe (1990c), and a quantitative review of reed-driven wind instruments is given by Keefe (1990b).

Current Issues

To the researcher in musical acoustics, the primary appeal of time-domain models of wind instruments is the production of waveforms that include transient information. Such data are exceedingly difficult to extract from frequency-domain models. Transients from time-domain models have been compared, usually qualitatively, to those measured on actual instruments, but such comparisons have not included the reports of listeners on perceptions of similarities and differences. To the computer music researcher, some of the key features of wind instruments have been implemented as an experimental synthesis technique, but many of the details known to be important to the behavior of actual instruments are not included, and it is only recently that systems can respond in real time. To the researcher in music perception, a computational model of wind instrument sound production has the potential to generate expressive musical sounds such as those produced by traditional instruments, while also exercising precise control over the synthesis parameters.

A wind instrument simulation constructs a mapping from each physical parameter to its corresponding control parameter, including correspondence with the air column and the physiological control and response of the player. The gestural controllers by which the player communicates with the instrument simulation capture the control data important in the actual player-instrument system.

To the potential objection that there is no need to reinvent instruments that already exist, there are several responses. Carrying out a computationally based simulation of a known instrument-player system is a powerful method for testing the accuracy of the implicit assumptions and structure of the underlying physical model. Second, the best test of a simulated musical instrument is to listen to its sound output, since the skilled musical listener has a much more extensive knowledge base than any artificial measuring instrument. It may be that the simulation sounds dissimilar to the actual instrument, when compared within a well-defined musical context. Alternatively, it may be that a simplified model of a wind instrument produces sound exceedingly similar to that of the actual system. Either experiment provides useful information to the music researcher. One can have confidence in manipulating the parameters of a dynamical wind instrument oscillator model after these parameters have been validated against actual instruments. There is also interest in exploring computational models that do not have corresponding wood, metal, leather, felt, and plastic exemplars. Such models might lead to musically relevant classifications of timbre space and related instrument families, and even to the construction of new instruments, be they electronic or acoustical in design.

Organ Flue Pipe and Flute

The flute, recorder, and organ flue pipe produce sound through the interaction of an air-jet motion about the edge of the pipe mouth, with the oscillatory displacement of air through the mouth (or embouchure hole in the case of the flute). It is the air jet emerging from the player's lips into the flute, or from the flue slit into the organ pipe, that adds energy to the steady-state tone. A qualitatively correct discussion of the excitation mechanism was given as early as 1830 by Sir John Herschel, but it was Helmholtz and Rayleigh who developed more quantitative approaches.

In 1967, Cremer and Ising photographed the air-jet motion in an organ pipe–like system at various phases over a period of oscillation and were the first to model the organ pipe as a resonant system coupled by feedback to the air jet. The theory accounts for such properties of pipe organs as the association of the playing frequency with the organ pipe resonant modes, the gradual increase in playing frequency with gradual increases in blowing pressure, and the excitation of higher pipe modes with larger increases in blowing pressure.

In 1968 Coltman measured the source impedance of an artificially blown flute head joint as a function of jet velocity and frequency. In the steady state, this source impedance must balance the energy lost in viscous, thermal, and radiative losses. In impedance language, oscillation is possible only when the real part of the source impedance is negative, which occurs over a finite range of air jet velocities. These results account for the flutist's ability to overblow to create regimes of oscillation based on the octave and higher intervals, by increasing air-jet velocity. In general agreement with the Rayleigh's theory, transverse disturbances on the air jet grow in amplitude and move with a phase velocity that is slightly less than half the air-jet velocity.

A plausible model of air-jet dynamics is needed to construct a model of sound generation, and the underlying fluid mechanical description is at present controversial. Fletcher constructed a theoretical model of the nonlinear coupling of the air column to the air jet that accounts for some of the properties of organ, flute, and recorder tones. For example, the model shows the transition of overblowing to be approximately an octave, as the blowing pressure is increased. Such predictions are in qualitative agreement with the experience of organ pipe voicers and flutists. The full model developed by Fletcher and colleagues and a review of research through the mid-1980s, are found in Fletcher and Rossing (1991).

A consensus has not yet emerged, but there is a great deal of active work under way to improve our knowledge concerning the basic sound production mechanisms in the organ pipe and flute. Recently, a number of investigators have carried out time-domain simulations of organ and flute tones, but discussion is outside the scope of this tutorial, which focuses primarily on the reed-driven wind instruments.

Time-Domain Models

It is worthwhile to review some of the steps involved in creating, analyzing, and using time-domain models, as outlined in Table 1. The first step is in formulating the dynamical system, taken to be the specific set of coupled, nonlinear differential equations representing the wind instrument model. This first step is in some sense the most crucial step in the physics, and various simplifications of the underlying fluid mechanics are introduced to lead to a computationally tractable system. The class of models to be discussed are expressed as a set of three first-order differential equations with time delay, where the three variables are the reed displacement, reed velocity, and air flow through the reed aperture. Other physical variables may be expressed in terms of these three variables. Leaving aside the time delay properties associated with the assumed linear response of the air column,

Table 1. The Development of Time-Domain Models

1. Formulate physics of model
2. Choose parameter values
3. Calculate time-domain simulation
4. Parameter sensitivity and playing behavior
5. Technology
6. Music perception and cognition
7. Applications to musical instruments

the phase space of such a dynamical system is three-dimensional. The presence of time delay can lead to a phase space of much higher dimensionality, but musical tones tend to lie in a low-dimensional subspace.

In addition to the physical variables, there are a number of parameters in the dynamical system. These include the mass, stiffness, and damping of the reed, and the geometry of the reed and air column. A key control parameter is blowing pressure, which represents the source of external power needed to balance the dissipative losses in the dynamical system. From the standpoint of scientific understanding of sound production processes, one needs to know the physically plausible range of values for these parameters. For example, reed damping is dependent on the effect of the player's lips on a moist cane reed, or on the effect of the brass player's muscular control of the lips. Furthermore, these parameters may be slowly varying over the duration of a single tone, or may have different values as the performer produces tones at different pitches. Such slowly varying parameters are intrinsically linked with the expressive quality of wind instrument tones.

In the relatively simple cases to be explored, a set of physical parameter values is chosen, the linear response of the air column is calculated, and the dynamical system is integrated forward in time from some physically plausible set of initial conditions. This constitutes a time-domain simulation. If there are N variables characterizing the state of the system, one can construct the corresponding N-dimensional phase space of the system. At any particular time, the dynamical system is at a point in the phase space, and the evolution of the system in time can be represented by trajectories in the phase space. After the initial transient, the trajectories in the phase space approach a limit set that depends on the particular values of the parameters and the initial conditions. A limit set that is likely to be experimentally observed is called an attractor, and a periodic attractor is simply an attractor whose trajectory in phase space is a closed curve. Benade and Kouzoupis (1988) state that a regime of oscillation is a "stable, nonlinear, multicomponent oscillation in which several resonance peaks collaborate with a flow controller to maintain an oscillation whose components are members of an exact harmonic series." This is precisely the concept of a periodic attractor when applied to wind instruments. There remains a difference in the language used to express these concepts; it is sometimes useful to think in the frequency domain in terms of resonant modes and sometimes useful to think in the time domain in terms of trajectories.

Time-domain simulations can be used to assess the sensitivity of the system dynamics to changes in the parameters. Performers are able to elicit subtle changes in the dynamical system by physiological control of the parameters. It may be that a dynamical system can be simplified without sacrificing the musically important behavior of the actual instrument, or it may be that the existing dynamical system needs to be enhanced to better describe the underlying physical processes. Even the simple dynamical systems considered here are capable of modeling qualitative aspects of playing behavior.

Computer technology is developing rapidly, and the ability to listen to the sound output from a physical model already exists. More efficient sound production algorithms are needed to achieve greater throughput. While many questions relating to the correctness of the model can be answered by simulating short transients until the attractor is reached, other questions require real-time performance. The player continually alters blowing pressure, embouchure, and other physiological gestures that affect sound production in the actual wind instrument. Once the parameters of a dynamical system become slowly varying in time with respect to the period of oscillation, the complex response of the system cannot be assessed by calculating short transients. It becomes desirable to examine the behavior of the simulation in real time, to compare it with empirical measurements of the actual wind instrument performance situation, about which relatively few data are available.

A number of perceptual questions can be addressed by real-time systems. Just how physically realistic does an implementation of a dynamical system need to be in order to "sound" like the actual instrument? It is possible that one needs to model the physics correctly to produce similar perceptual responses from listeners, but other scenarios are also possible. McIntyre, Schumacher, and Woodhouse (1983) proposed a simplified clarinet model that possessed some, but not all, of the physical realism of the Schumacher (1981) time-domain model of the clarinet. They proposed that such time-domain models should be capable of real-time performance.

Real-Time Implementation

Park and Keefe (1988) presented a real-time model of the simplified clarinet implemented on first-generation digital signal processing (DSP) chip technology. The implementation on mid-1980s technology, described in Keefe and Park (1990), is summarized here. The computer was an 8 MHz IBM PC-AT computer with a Dalanco Spry Model 10 coprocessor. This coprocessor used a 6.25 MIPS Texas Instruments TMS32010 DSP, with 12-bit linear digital-to-analog converters. A Roland MPU-401 MIDI processing unit resided in the host computer to allow the input of MIDI gestural control information. The modulation wheel of a MIDI keyboard controller was used to generate a control signal that was interpreted to be linearly proportional to the player's blowing pressure. A Pascal program on the host read in the MIDI data from the modulation wheel and updated the blowing pressure input parameter value on the DSP processor. The host stored a number of different reflection functions with time delays such that a chromatic scale could be played. Whenever a MIDI note-on command was received from a MIDI keyboard or wind instrument controller, the corresponding reflection function for that note was loaded from the host computer memory into the DSP memory. The result was a real-time monophonic musical instrument with a two-octave range.

Given the superiority of current technology to the first-generation DSP technology, it is evident that fairly realistic time-domain models, based on a detailed model of the physics of the instrument, can be implemented in real time.There is no need to oversimplify the dynamical system to obtain a useful synthesis technique. The sound output from the simulation did not really begin to "sound" like a clarinet until the blowing pressure became time-varying, and the behavior of the model was in some ways different from that of a clarinet. This latter result could have been obtained without real-time performance, but the former result is essentially dependent on the "player" being able to "play" the model. The future availability of more sophisticated real-time computational models of musical instruments should be a tool for perceptual and cognitive investigations of how musicians and listeners organize the experience they gain from playing and hearing traditional musical instruments.

Research on the acoustics of wind instruments can be utilized in the design of new wind instruments, or in the modification of existing instruments. Another application lies in the use of nonlinear dynamical systems models as a music synthesis technique. Here, the intention is probably not to simulate an existing musical instrument but, rather, to use the framework of dynamical systems to construct alternative electroacoustic instruments that may or may not be physically realizable as traditional wind instruments.

Sound Production in Reed-Driven Wind Instruments

A fairly standard model of sound production in reed-driven woodwind and brass instruments has been developed by musical acousticians, and detailed discussion and references are found in Benade (1990) and Fletcher and Rossing (1991). For woodwinds, Backus measured in 1963 the nonlinear flow-control relation of a clarinet reed and developed the theory of the threshold of oscillation, Benade and Gans constructed in 1968 a nonlinear theory of sound regeneration, Nederveen measured in 1968 the flow-control relation of a double reed and contributed to the threshold theory, Worman developed in 1971 the frequency domain theory to make quantitative predictions of clarinet spectra, and Fletcher showed in 1978 how mode-locking into a regime of oscillation takes place. For brass instruments, Elliott and Bowsher constructed in 1982 a nonlinear theory of sound regeneration and estimated the lip-reed parameters of the trumpet and trombone, and Fletcher developed in 1979 a unified model applicable to brass and woodwind instruments.

Alternative time-domain formulations of the clarinet model of Worman were proposed by Thompson (1978) and were proposed and implemented by Schumacher (1981). Thompson derived a single nonlinear integral equation for the mouthpiece pressure, which could be integrated forward in time based on a set of initial conditions, but he did not obtain numerical results. Schumacher's method of solution used the air-column reflection function and handled the nonlinearity in an efficient computational manner. He obtained detailed numerical results that have served as a basis for more recent developments. A time-domain model for the simplified clarinet was adapted to the case of a simplified brass instrument by Hoekje (1986). Time-domain simulations of a double-reed instrument were first obtained by Barjau and Agulló (1989) for the Catalan tenora, a conical-bore woodwind, using the impulse response of the air column.

A reed-driven, wind instrument model has three major components: the air column, the performer, and the reed. The air column of the wind instrument is typically characterized by its linear response, including radiative, viscous, and thermal losses. However, nonlinear air-column losses are significant (Keefe 1992). The performer acts as a source of blowing pressure, and the vocal tract is a second air column that can affect sound production (Hoekje 1986). It is the nonlinear valving action of the reed that allows a wind instrument to produce a sustained tone. The aperture between the cane reed tip and mouthpiece (or between the two cane reeds in double-reed woodwinds) admits flow from the performer into the air column, and the mechanical motion of the cane reed narrows or widens the aperture, thereby controlling the flow. This coupling of reed displacement to air flow is nonlinear, controlled by the pressure difference across the reed. In the brass instrument family, the mechanical motion of the player's lips forms an aperture through which a flow of air is admitted into the instrument, and the relationship between the pressure difference and the flow is again nonlinear.

A model used in the simulations is one in which the reed is assumed to be a simple harmonic oscillator. There are many refinements to this model that are beyond the scope of the present discussion. Let $x(t)$ represent the displacement of the reed, and $ur(t)$ represent the volume of air per unit time swept out by the reed. The reed is closed at $x = 0$, and the volume velocity is positive when the reed tip is opening. Let $u(t)$ represent the volume flow through the reed tip aperture. The dynamical system for these three variables is

$$
\begin{aligned}
\dot{x}(t) &= S_r^{-1} u_r(t), \\
\dot{u}_r(t) &= S_r \left\{ \begin{array}{l} -\left(g_r S_r^{-1} + \dfrac{\sigma}{u_r} Z_c \right) u_r(t) - \omega_r^2 [x(t) \\ - H] - \dfrac{\sigma}{\mu_r} [P_0 - p_h(t) - Z_c u(t) \end{array} \right\}, \\
\dot{u}(t) &= I_e^{-1}(x) \left\{ \begin{array}{l} P_0 - p_h(t) - Z_c [u(t) - u_r(t)] \\ - C|u|^{\alpha} x^{-\beta} Sgn(u) \end{array} \right\},
\end{aligned}
\tag{1}
$$

where the thermodynamic constants and model parameters are defined in Table 2, and $Sgn(u)$ is the sign function returning 1 if u is nonnegative and -1 otherwise. The cane reed in woodwinds is assumed to close with increasing blowing pressure, and this class of instruments is represented by choosing s = 1. The lip reed in brasses is assumed to open with increasing blowing pressure, and this is represented by choosing $\sigma = -1$. The volume flow u_d into the (downstream) air column is $u_d(t) = u(t) - u_r(t)$. The mouthpiece pressure $p(t)$ is calculated from

$$p(t) = p_h(t) + Z_c u_d(t). \tag{2}$$

Table 2. Parameters in Dynamical System of Wind Instruments

Parameter Symbol	*Parameter Name*	*Cane-Reed Value (SI units)*	*Lip-Reed Value (SI units)*
c	Phase velocity of sound	343	343
ρ	Equilibrium density of air	1.17	1.17
S	Area of air-column entryway	$1.77\ 10^{-4}$	$7.07\ 10^{-4}$
Z_c	$\rho c/S$, Characteristic impedance at entryway	$2.27\ 10^{6}$	$5.68\ 10^{5}$
S_r	Dynamic area of reed	$1.46\ 10^{-4}$	$2\ 10^{-4}$
ω_r	Reed resonance frequency (radians/sec)	Variable	Variable
f_r	ω_r, Reed resonance frequency (Hz)	Variable	Variable
μ_r	Dynamic mass per unit area of reed	0.0231	$0.001(232/f_r)/S_r$
g_r	ω_r/Q_r, Half-power bandwidth of reed resonance	$Q_r = 3$	Variable
H	Equilibrium opening of reed tip	$0.4\ 10^{-3}$	Variable
l	Length of reed tip aperture	0.002	0.01
w	Width of reed tip aperture	0.008	0.008
I_e	$\rho l/(wH)$, Inertance of reed tip	731	Variable
C	Flow-control constant	44.4	$0.5\rho/\omega^2$
α	1.5 (single reed), 2 (double reed and lip reed)	1.5	2
β	2 (single and double reeds, lip reed)	2	2
P_0	Blowing pressure	Variable	Variable
$r(t)$	Reflection function of air column at its entryway	—	—
$p_h(t)$	Mouthpiece pressure history function	—	—
δ	Cane reed (1) or lip reed (–1)	1	–1

The pressure history function $p_h(t)$ is the convolution of the air column reflection function $r(t)$ and the mouthpiece pressure and flow into the air column, given by

$$p_h(t) = r(t) * [p(t) + Z_c\, u_d(t)]. \qquad (3)$$

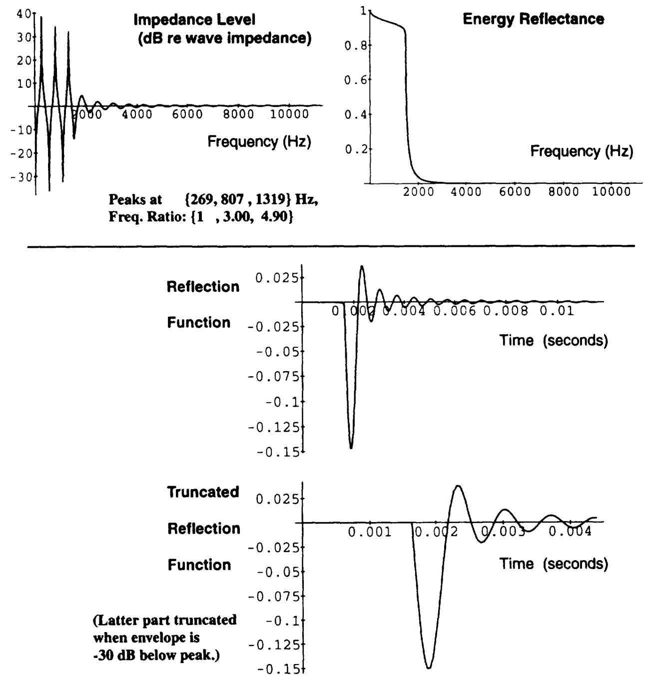
The theory underlying the solution of Eqs. (2 and 3) is presented (McIntyre, Schumacher, and Woodhouse (1983).

The method of numerical integration is a second-order implicit method applied to the three first-order, ordinary differential equations comprising the dynamical system, and results in a single nonlinear equation that can be solved using Newton's rule. The second-order method has better convergence properties than first-order forward or backward Euler difference methods, and better stability properties than fourth-order, explicit Runge-Kutta methods. The numerical method for dealing with the closing of the reed differs from Schumacher (1981); the reed displacement is not permitted to become negative, since zero displacement corresponds to a closed reed.

These simulations were implemented using the Mathematica programming language on a NeXT computer generating samples at 22.05 kHz. A simple representation of the air column is used. The soprano clarinet and trumpet are treble instruments in their respective families, and a generic input impedance function is used that has similarities to the input impedance of such treble instruments (Benade 1990). The air-column model is a cylindrical tube of variable length terminated by a semi-infinite open tone-hole lattice with viscothermal and radiative losses whose cutoff frequency is 1,500 Hz. The input impedance is calculated at one-half the sample rate. The cylindrical tube for the clarinet simulation has a 15 mm diameter, and that for the lip-reed-driven simulation has a 30 mm diameter, larger than the internal diameter of trumpet tubing but chosen equal to that used by Yoshikawa (1988) in an experimental study of self-oscillation in a simplified brass instrument.

The air-column response functions for the lip-reed-driven instrument are illustrated in Fig. 1; the corre-

Fig. 1. Input impedance, energy reflectance, and reflection function for air column terminated by open tone-hole lattice.



the reflection coefficient. More than 90 percent of the energy is reflected below 1,400 Hz, and more than 90 percent of the energy is transmitted above 1,900 Hz. The time-domain reflection function is calculated using a 4,096-point inverse fast Fourier transform of the reflection coefficient. A convolution of the pressure and flow data with the reflection function is carried out for each sample using Eq. (3), and the computation time increases as the duration of the nonnegligible part of the reflection function increases. There is a fairly long tail to the reflection coefficient, but the main contribution to the convolution extends over only a few milliseconds of the reflection function. To reduce the computation time, a rule has been adopted that the reflection function coefficients are set to zero if they are less than some threshold value. In Fig. 1, the reflection function is set to zero if the absolute value is less than 30 dB (a factor of 0.03), compared to the maximum absolute value of the reflection function, that is, relative to the strong dip.

sponding functions for the clarinet are qualitatively similar. The cutoff frequency of an actual brass instrument air column is governed by the transition in the radiation impedance of the bell such that low frequencies are predominantly reflected and high frequencies are transmitted out of the bell. The input impedance in Fig. 1 is similar to an actual brass instrument in having several strong impedance peaks below the cutoff frequency whose frequencies are close to a harmonic alignment. However, the peak amplitudes below cutoff are quite different in a trumpet than in Fig. 1, and the lowest mode of a trumpet is significantly lower in frequency than the fundamental of an approximately aligned harmonic series (Benade 1990). This air-column impedance is used so as to compare more easily the results of simulations with Yoshikawa's observations concerning the lip reed, and the inclusion of the characteristic cutoff frequency enhances the similarity to brass instrument air columns.

The input impedance is transformed into a complex reflection coefficient by standard means (McIntyre, Schumacher, and Woodhouse 1983), and the energy reflectance is the squared magnitude of

Influence of the Clarinet-Reed Resonance Frequency

The player of any cane-reed woodwind exerts control by changes in embouchure. In terms of the standard model, such changes modify the reed resonance frequency and, possibly, the reed quality factor Q_r. Measurements by Worman (1971) show that the quality factor of a moist clarinet reed, including the influence of contact with the player's lip, is approximately $Q_r = 3$, and there are as yet no data on how this damping factor alters with changes in embouchure. Parameter values used in the simulations are typical of those used by previous investigators; they are shown in Table 2.

The woodwind player "shapes" the musical tone by alterations in embouchure, and this indicates an important role for the reed resonance frequency in sound production (Benade 1990). The resonance frequency for a clarinet reed is in the range of 2–3 kHz, well above the sounding frequencies of clarinet tone and well above the cutoff frequency of the clarinet air column. While multiple impedance peaks in near harmonic alignment (as in Fig. 1) tend to stabilize the

Fig. 2. Dependence of clarinet mouthpiece pressure on blowing pressure.

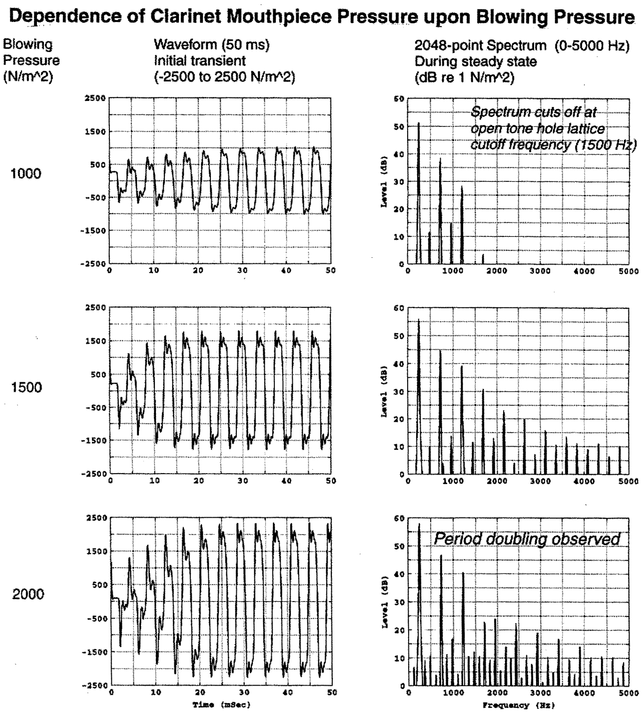
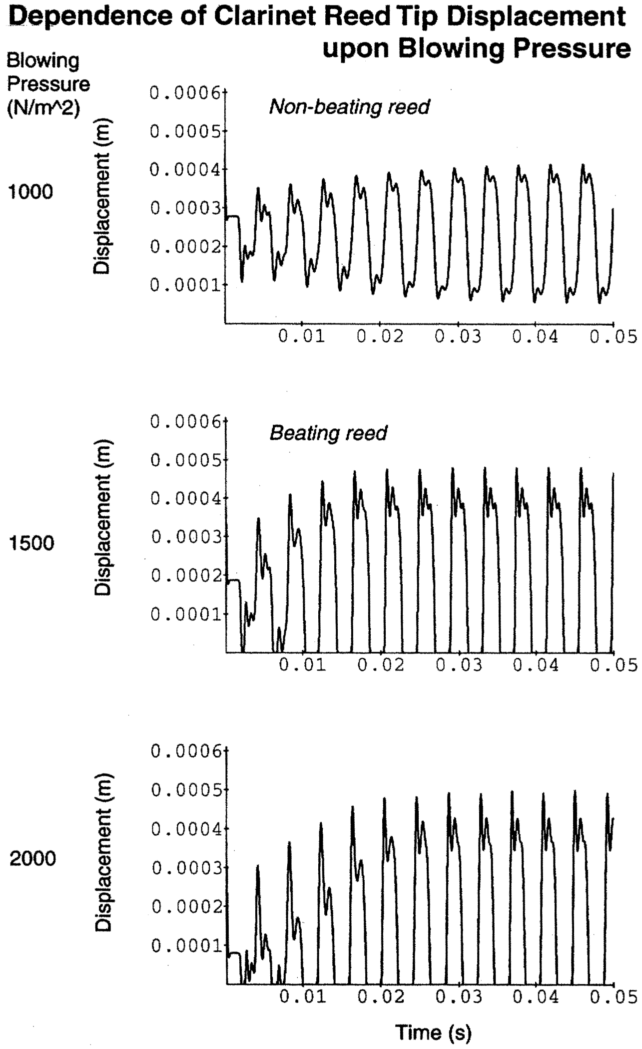


Fig. 3. Dependence of clarinet reed tip displacement on blowing pressure.

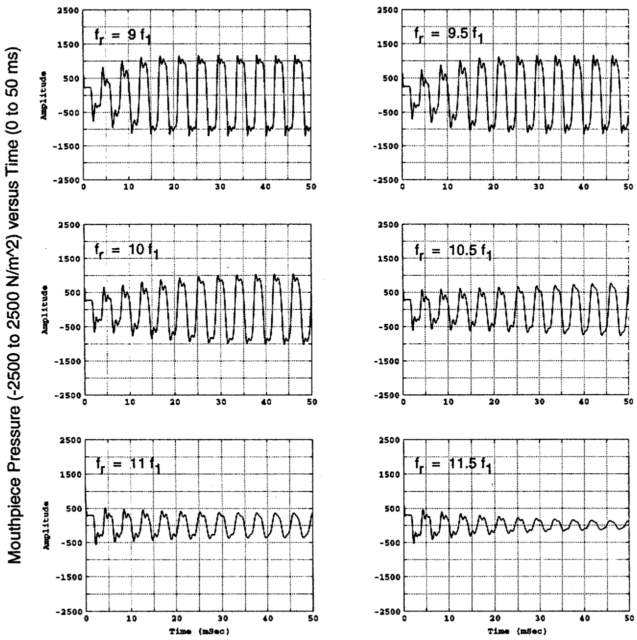


oscillation, the clarion (second) register of the clarinet has only a single impedance peak below cutoff. Aside from the reed resonance (and the upstream respiratory tract impedance resonances neglected in this discussion), there is no other resonant mode to stabilize the playing frequency in the vicinity of the single air-column input impedance peak. An analogous situation holds for an air column having several input impedance peaks that are far from harmonic alignment. Here also the reed resonance can be anticipated to play a major role on the sounding frequency as the player changes the embouchure. This is how the performer plays in tune on a mistuned instrument.

Thompson (1979) carried out experiments on a clarinet using a machine and artificial lip. He found that the sounding frequency of the clarion tone was such that the reed resonance frequency was within 50 cents of an upper harmonic of the sounding frequency. He concluded that harmonic alignment of the reed resonance frequency performs a similar function in stabilizing the oscillation that harmonic alignment of additional input impedance peaks perform, even if the dynamics are somewhat different. However, the quality factor of the reed on the artificial lip was much higher than in the actual system, so it is unknown whether these results generalize to actual clarinets.

The role of the reed resonance has been investigated for both chalumeau and clarion register tones. Simulations in the first set investigate the behavior of the model at blowing pressure of 1,000, 1,500, and 2,000 Pa, with the reed resonance frequency set to 2,500 Hz. The mouthpiece pressure is illustrated in Fig. 2, and the displacement of the tip of the reed is

Fig. 4. Clarinet mouthpiece pressure waveforms as reed resonance frequency f_r is varied by increasing stiffness.



illustrated in Fig. 3. At a blowing pressure of 1,000 Pa, there is a medium-amplitude regime of oscillation in which the reed does not beat; that is, the reed does not close against the mouthpiece tip. The corresponding spectrum has strong peaks in a frequency-locked 1:3:5 sequence, and there are no strong peaks above the 1,500 Hz cutoff frequency. There are even harmonics generated below cutoff whose amplitudes are low but increasing with increasing frequency. This behavior is as predicted from the frequency-domain theory, giving confidence that the simulations are correctly implemented.

At a blowing pressure of 1,500 Pa, the high-amplitude regime of oscillation is reached in which the reed remains closed over roughly 40 percent of the period of oscillation. The attack transients for the high-amplitude regimes have shorter durations than the attack transient for the medium-amplitude regime. The resulting steady-state spectrum is qualitatively different from the medium-amplitude spectrum, due to the wide-bandwidth nature of the closing of the reed.

At a physiologically plausible blowing pressure of 2,000 Pa, the steady-state spectrum shows the presence of period-doubling. That is, spectral peaks occur at half the frequency of the previous fundamental (just below 250 Hz), and halfway between the previous even and odd harmonics.

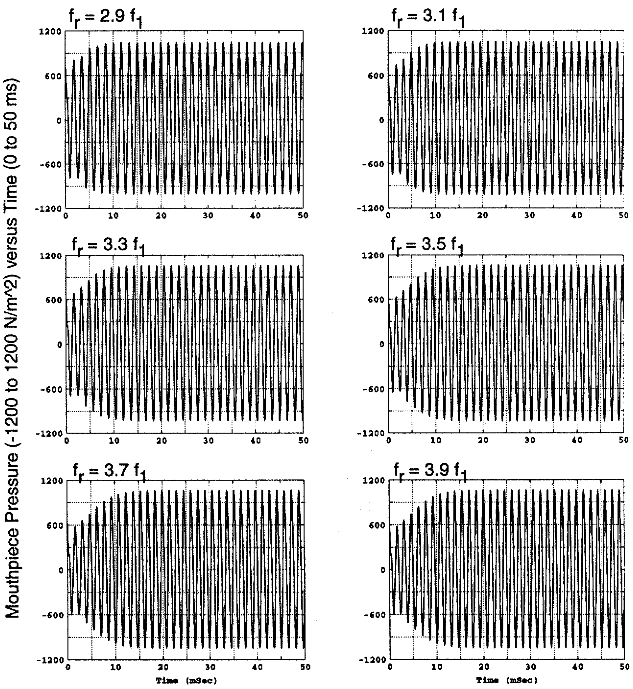
The reed resonance frequency is given in terms of the reed mass density and stiffness κ_r by $f_r = 1/2\pi\kappa_r/\mu_r$. The reed resonance frequency can be varied in the model by varying stiffness, or mass, or their combination. Simulations in the next set change f_r by varying reed stiffness for tones in the chalumeau register. The reed resonance frequency is expressed in Fig. 4 as a harmonic multiple of the lowest frequency, input impedance peak (f_1 = 255 Hz). The blowing pressure is 1,000 Pa. If there is a pronounced effect of alignment of f_r as a harmonic multiple of f_1, then the right column (maximal misalignment) of Fig. 4 should differ from the left column (maximal alignment). This is clearly not the case. Thedominant tendency is that oscillations are sustained at higher amplitude when the reed resonance frequency is low. Oscillations are not sustained at high reed resonance frequencies.

This is one case where the physical mechanisms are clarified by a frequency-domain analysis. The threshold blowing pressure P_t, below which no oscillation can be sustained, is given by $P_t = \kappa_r H/3$, where $\kappa_r H$ is the blowing pressure needed to close the reed tip. As reed stiffness κ_r is increased, the threshold blowing pressure is correspondingly increased, so the oscillation is not sustained.

A more physically appealing yet simple model of the controlling reed resonance frequency goes back to the geometry of the reed under the control of the performer. As the performer tightens the lip muscles to increase f_r on a single-reed mouthpiece, the effective vibrating length of the reed shortens and the reed tip aperture is decreased. This decrease in length both stiffens the reed and decreases its dynamic mass. In research on the lip reed, Elliott and Bowsher (1982) have assumed that an increase in f_r is produced by an equal fractional increase in stiffness and decrease in mass; this assumption is adopted here. The threshold blowing pressure should not vary much since the increase in κ_r and the decrease in H tend to cancel. In order to study the dynamics of reed resonance control apart from the threshold behavior, these shifts in κ_r and H are assumed to be equal and

Fig. 5. Clarinet mouthpiece pressure waveforms as reed resonance frequency f_r is varied by increasing reed stiffness and decreasing reed mass.

Fig. 6. Sounding frequency of clarinet tone in clarion register as the reed resonance frequency is varied (top). Spectral levels of first three harmonics of mouthpiece pressure as f_r is varied (bottom).



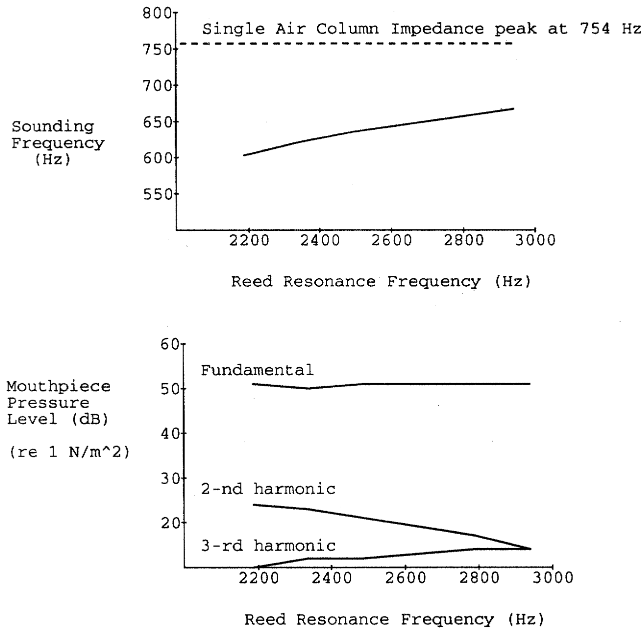


Table 3. Ratio of Reed Resonance Frequency and Sounding Frequency

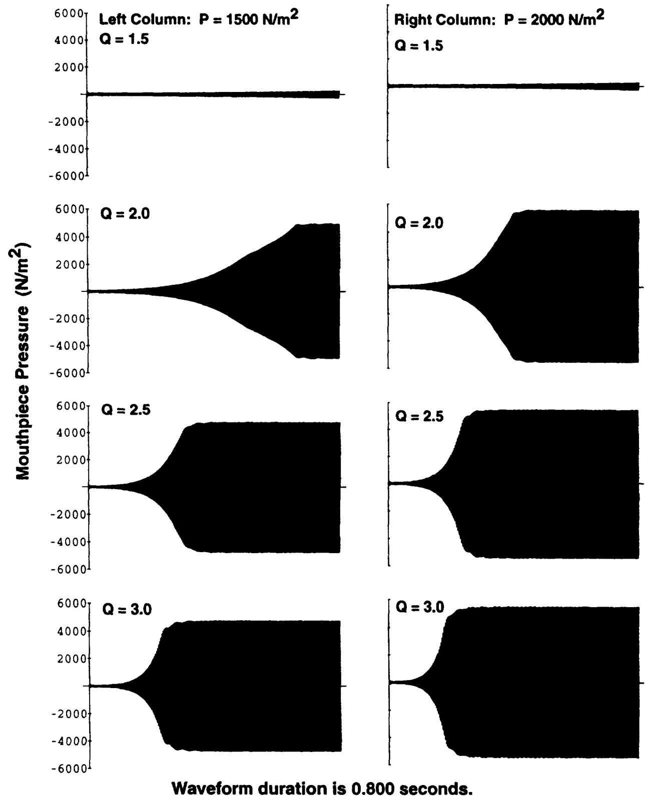
f_r/f_1	f_r/f_s
2.9	3.63
3.1	3.75
3.3	3.92
3.5	4.09
3.7	4.25
3.9	4.41

opposite, so that P_t is invariant. A more detailed model of reed dynamics interacting with the geometry of the mouthpiece is certainly desirable, but the qualitative shifts in all these parameters are in the appropriate direction.

A clarion register tone has been simulated on the basis of an input impedance function with a single peak at f_1 = 754 Hz. The mouthpiece pressure waveform transients at a blowing pressure of 1,000 Pa show little variation as f_r is increased from 2.9 to 3.9 f_1 (see Fig. 5). At reed frequencies close to harmonic alignment with the input impedance peak (2.9, 3.1, and 3.9 f_1), there is no apparent growth in the amplitude of oscillation or reduction in the attack time relative to when the reed frequency is not harmonically aligned. In contrast, there is an overall weak trend for shorter attack times at lower values of f_r irrespective of the degree of harmonic alignment.

There is, however, strong control of the sounding frequency f_s by the reed resonance frequency, as shown in the top plot of Fig. 6. As the performer increases the reed resonance frequency, the sounding frequency increases, bounded above by the frequency f_1 = 754 of the single-input impedance peak. The frequency-locking effect reported by Thompson (1979) is not observed; that is, there is no particular tendency for the ratio f_r/f_s to be close to harmonic alignment (see Table 3). The reason for the discrepancy is that the reed quality factor used here is typical of clarinet reeds, whereas Thompson (1979) pointed out that his Q_r was higher than this. With a Q_r of 3, the reed resonance is so broad that there is no selective enhancement at harmonic alignment. The reed resonance frequency increased in the six simulations

Fig. 7. Mouthpiece pressure of brass instrument transient at low blowing pressure (left) and high blowing pressure (right), and as the quality factor of the lip reed is increased from 1.5 (top) to 3.0 (bottom).



from 2,187 to 2,941 Hz, an interval of 513 cents; the sounding frequency increased from 603 to 668 Hz, an interval of 177 cents. More recent work has shown that the spectra of saxophone tones are sensitive to the reed resonance when Q_r is approximately 10, which is consistent with Thompson's observations.

The bottom plot of Fig. 6 shows the variability in the mouthpiece pressure level of the first three harmonics due to changes in f_r. Since the reed does not beat, the spectrum cuts off at 1,500 Hz, and only the first three harmonics have significant levels. The fundamental is 25 dB larger than any of the other harmonics and is essentially constant over all conditions. The second harmonic decreases, and the third harmonic increases, with increasing f_r.

These results help explain the clarinet vibrato, rarely used in orchestral situations, but often used in other musical styles. The performer produces a vibrato by a slowing varying fluctuation (approximately 4–6 Hz) in the embouchure. This can be modeled as a modulation in the reed resonance frequency, and thus Fig. 6 represents the (frequency) modulation of the vibrato and the resulting amplitude modulation of each of the harmonics. The inclusion of slowly varying control functions for the reed parameters allows a physical model to produce musically realistic vibratos. A typical vibrato width might be 25 cent. If 177 cents variation in f_s is produced by 513 cents variation in f_r, then an approximate 25 cents variation in fs is produced by a 72 cent variation in f_r. This corresponds to a 4 percent variation in the reed resonance frequency. Similar methods are applicable to other instruments.

Influence of the Lip Reed Resonance Frequency

Whereas the cane reed closes with increasing pressure difference across it, the lip reed is assumed to open with increasing pressure difference across it. This behavior is included in the model by the choice of σ. The frequency-domain threshold behavior is quite different. Analysis of the linearized dynamical system at threshold in the case where there is a single, strong impedance peak shows that the cane-reed model cannot sustain an oscillation unless the reed resonance frequency is greater than the frequency f_1 of the input impedance peak, and that the lip-reed model cannot sustain an oscillation unless f_r is less than f_1.

In a frequency-domain approach, the complex ratio of lip displacement to mouthpiece pressure is the dynamic compliance D, and it is defined at each harmonic of the steady-state periodic tone. Yoshikawa (1988) has measured the phase angle of dynamic compliance. This angle is predicted to be positive according to the model of the lip reed as an outwardly beating reed with increasing mouthpiece pressure (Fletcher 1979), but Yoshikawa observed both negative and positive values. He concluded that a two-dimensional dynamical model of the lip was needed, with displacement both along the axis of flow and transverse to it. A two-dimensional model has been proposed by Keefe (1990a), where, following the arguments of Ingard and Ising (1967), the transverse force is assumed to be the mean of the upstream blowing

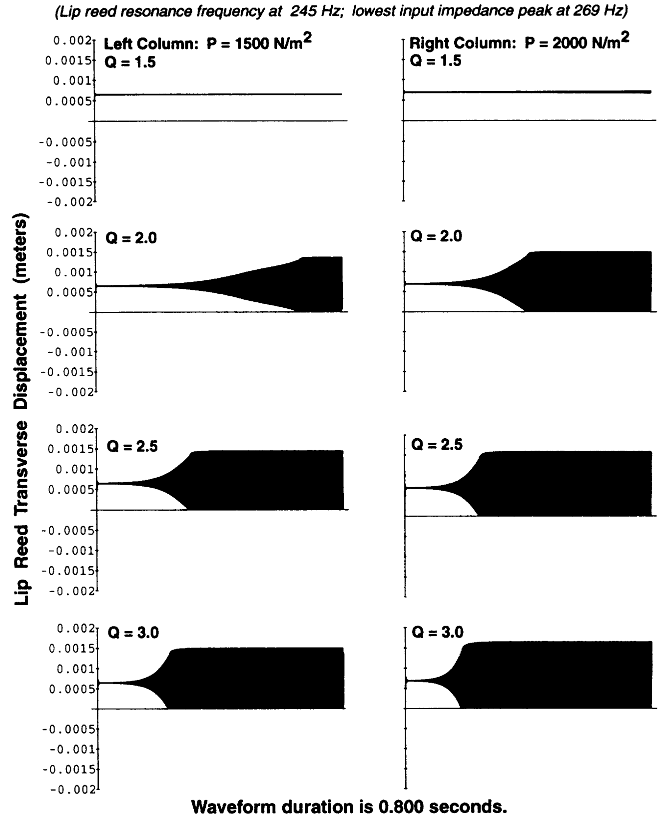


Fig. 8. Lip displacement of brass instrument transient at low blowing pressure (left) and high blowing pressure (right), and as the quality factor of the lip reed is increased from 1.5 (top) to 3.0 (bottom).

pressure and the downstream mouthpiece pressure acting on the transverse surface area of the lip. A two-dimensional model has also been proposed by Strong (1990), where the transverse force is assumed to be due to the Bernoulli force, as was also suggested by Yoshikawa. Strong has obtained detailed numerical results with this model, concluding that the transverse force term is significant. These two models are similar in positing a rolling or "swinging-door" model of lip motion.

Time-domain simulations using the model in Eq.(1) have been compared with these predictions and measurements. The parameters used in the simulations are as listed in Table 2 with the following additions, reflecting data on the lip reed of the trumpet (Elliott and Bowsher 1982). The dynamic mass of the lip is 0.001 kg at 232 Hz and decreases inversely with frequency. The equilibrium opening H (in meters) of the reed tip aperture is taken to be the following function of f_r:

$$H = 0.0001 + \mathrm{Max}\,[0.0002,\ 0.00042 + 6.0 * 10^{-7}\,(f_r - 232)],$$

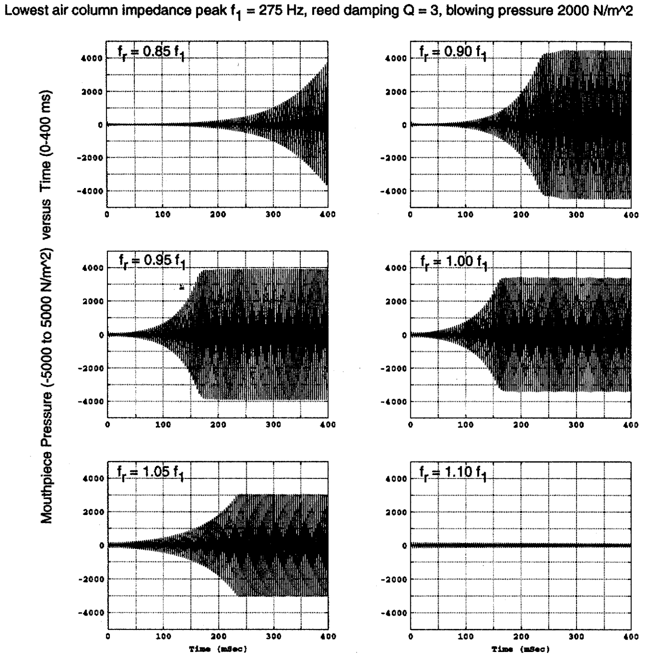
where Max$[a,b]$ returns the larger of the quantities a or b.

The dependence of mouthpiece pressure transients on blowing pressure and the quality factor of the lip reed is shown in Fig. 7. The lowest frequency input impedance peak is f_1 = 269 Hz, and the reed resonance frequency is f_r = 245 Hz. For fixed Q_r, the amplitude of oscillation is slightly higher and the attack transient is slightly shorter for a blowing pressure of 2,000 Pa, compared to the waveforms at a blowing pressure of 1,500 Pa.

For fixed blowing pressure, the attack time is shorter as the quality factor of the resonance increases. This may seem counterintuitive, because the amplitude of a passive resonator achieves maximum response after a delay on the order of $Q_r T$, where T is the period, and the amplitude of the resonator is proportional to Q_r. The reason is that the sharper resonant tuning of the lip reed is able to transfer more energy into the mouthpiece pressure during the early transient. The corresponding lip displacement for the same set of simulations is shown in Fig. 8. Independent of Q_r, the oscillation saturates when the lip displacement closes only once per cycle, and this saturation occurs earlier in the transient for the higher Q_r tones. The strong dependence of the transients on Q_r suggests that the brass instrument performer's adjustment of lip damping plays a significant role during attack transients.

The mouthpiece pressure spectra show strongly excited odd harmonics below the 1,500 Hz cutoff frequency, and both odd and even harmonics above cutoff. This results from the particular air column chosen and is not a feature of actual brass instrument spectra. One other interesting feature is that there is another component in the spectrum just below the fifth harmonic. Referring to Fig. 1, the third impedance peak is tuned at a frequency that is 4.90 times the lowest impedance peak frequency, owing to the frequency-dependent changes in the impedance of the open tone-hole lattice. According to the frequency-domain theory, energy regeneration is favored at frequencies where the input impedance is large, and thus independent regeneration can take

Fig. 9. Mouthpiece pressure of brass instrument transient as the lip-reed resonance frequency is varied.



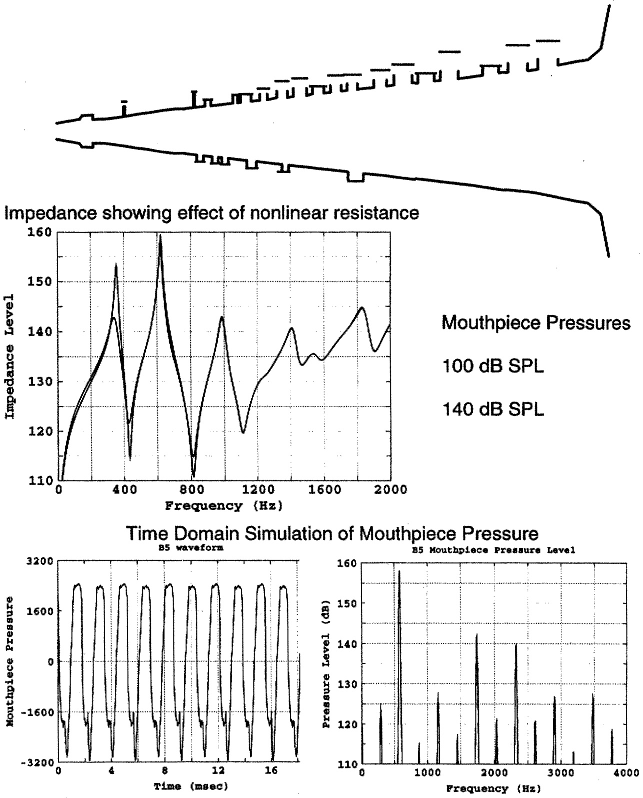
place in the vicinity of each impedance peak. The waveforms produced by the simulations show this phenomenon. The dynamical system is not fully phase-locked for this set of parameters, and the inharmonicity is clearly audible when one listens to the sound output. The "burbling" noise created in brass instruments is related to this effect, where sound production occurs at two frequencies that are not quite harmonically related, and complex modulation effects result.

The effect of manipulating the reed resonance frequency on the mouthpiece pressure waveform is addressed in the last set of simulations, for which f_1 = 275 Hz, Q_r = 3, and the blowing pressure is 2,000 Pa (see Fig. 9). The reed resonance frequency f_r is varied from 0.85 to 1.05 times the frequency f_1=275 Hz of the lowest input impedance peak. The oscillation reaches the steady state for $f_r = 0.85\,f_1$, but at longer times than the 400 msec transient displayed in Fig. 9. The oscillation is sustained for all values of f_r in the range from 0.85 f_1 through 1.05 f_1. That an oscillation can be sustained for f_r larger than f_1 contradicts the frequency-domain analysis of the linearized system near threshold. However, the blowing pressure is fairly high in these simulations, and thus the problem is that it is not permissible to generalize the threshold relationships to the largeamplitude regime. An oscillation can be sustained when the reed resonance frequency is greater than the input impedance peak frequency, but only over a restricted range in f_r; that is, an oscillation is not sustained for $f_r = 1.10\,f_1$. Furthermore, the peak amplitudes are higher when $f_r < f_1$, and the largest peak amplitude occurs when $f_r = 0.90\,f_1$. Although not illustrated, an oscillation is also sustained when $f_r = 2.90\,f_1$, corresponding to energy regeneration associated with the second input impedance peak at $f_2 = 3.0\,f_1$ (see Fig. 1).

A second prediction of the frequency-domain model is that the phase angle of the dynamic compliance is positive. The dynamic compliance has been calculated from the steady-state displacement and pressure waveforms output from the simulation. A 4,096-point fast Fourier transform of each time series is calculated, and the *D* volume is calculated as the ratio of the Fourier components of lip displacement and mouthpiece pressure at the fundamental frequency. Results are reported for the case corresponding to Fig. 9, and the phase angle of *D* is always 90°. Analysis shows that the dynamic compliance *D* at the fundamental frequency f_s is approximately equal to $+j\ Q_r/\kappa_r$, when $1-(f_s/f_r)^2 < (f_s/f_r)\,Q_r^{-1}$. The unit imaginary number is j, and the $+j$ factor explains the 90° phase angle. This inequality is well- satisfied because f_s/f_r is only several percent larger than unity and Q_r is not too large. The magnitude of Q_r/κ_r for the case $f_r = 0.95\,f_1$ is $4.6 * 10^6$ m/Pa. The corresponding magnitude of the dynamic compliance calculated from the ratio of Fourier components is 5.1 * 106 m/Pa, which is acceptably close in value.

While all the predictions of the frequency-domain threshold analysis do not generalize to the large-amplitude regime, the experimental measurements of Yoshikawa (1988) concerning the phase of dynamical compliance are inconsistent with the dynamical system of Eq. (1). A two-dimensional lip model (Keefe 1990a) can account for the phase relationships observed by Yoshikawa, but more research is clearly needed.

Fig. 10. Detailed geometry of alto saxophone (not including air-column curvature) for written note B5, with 4× magnification in transverse direction (top); input impedance at mouthpiece sound pressures of 100 and 140 dB SPL showing nonlinear reduction in amplitude (middle); mouthpiece pressure waveform (Pa) and spectrum (dB SPL) from time domain simulation (bottom).



Saxophone Simulation

One simplification in the above models lies in the representation of the air column. It is straightforward to generalize the air column model to include the realistic cross-sectional area of the main bore including taper and flare, and the presence of tone holes (Keefe 1990d). Nothing changes in the dynamical system except for the substitution of another reflection function, and real-time performance depends mainly on the time needed to calculate the convolution of the pressure response with the reflection function of the air column.

One specifies the geometry of the air column and calculates the response function. For example, a representation of a saxophone whose dimensions were measured by Nederveen (1968) is illustrated in Fig. 10 for the fingering corresponding to written B5. Note that the first register hole near the mouthpiece is opened to produce a second register tone, and the effect on the input impedance is to lower the amplitude of the lowest frequency input impedance peak and displace it upward in frequency (Benade 1990). The influence of nonlinear losses is shown for assumed frequency-independent mouthpiece pressure levels of 100 and 140 dB SPL (Keefe 1992). The lowest frequency resonance peak is significantly reduced by nonlinear losses particularly at the open register hole, whereas the other peaks are hardly affected. The corresponding reflection functions are calculated as before, but it is the impedance representation that best illustrates how these nonlinear losses enhance the tendency for the second mode to control the playing frequency.

Interestingly, the tendency for subharmonic generation (i.e., period doubling) from a wind instrument model is usually reduced when a realistic reflection function is used; in particular, low-register saxophone tones do not possess subharmonics. Thus, subharmonic generation in sounds generated by physical models is due in part to the oversimplified representation of the air column. Nevertheless, it is possible to use certain embouchure settings to produce subharmonics, a phenomenon well known to the player, and this sort of subharmonic generation is illustrated in Fig. 10. For example, the saxophonist or flutist can play a written B4, and produce what sounds like two tones, one at B4 and the other at B5. This is a physical manifestation of period doubling and a subjective manifestation of the analytical mode of pitch perception. Subharmonics can be produced whenever an input impedance peak of sufficient amplitude is placed near a subharmonic of the sounding frequency, and examples are found in essentially all families of wind instruments.

The bottom of Fig. 10 shows a time-domain simulation of a saxophone tone for a B5 fingering (corresponding to concert D5) at a reed resonance frequency near 1,500 Hz and blowing pressure of 2,000 Pa, calculated at a sample rate of 44.1 kHz. The mouthpiece pressure steady-state waveform and spectrum are shown for a fundamental frequency near 600 Hz, and a subharmonic is evident near 300 Hz. Note also that the mouthpiece pressure at the fundamental does exceed the 140 dB SPL at which nonlinear losses become significant. Including such nonlinear losses may help explain the playing behavior of wind instruments in their second and third registers, and suggests that it is not generally valid that the air column can be considered to have a linear response, particularly when mean flow effects are considered. The transformation from a mouthpiece pressure spectrum, as illustrated here, to the perceptually important room-averaged spectrum is reviewed by Keefe (1990c).

One can use desirable properties of the input impedance to help constrain the design of the instrument. In this inverse method, one begins with the principle that the lower resonant modes, including the frequency-perturbing effect of the reed, should have harmonically related resonance frequencies. This principle is known to be highly desirable for a wind instrument (Benade 1990). Then, the geometrical dimensions of the air are modified to approximate this optimal tuning, an idea originally formulated by Benade. These approaches have been applied by Barjau, Martínez, and Agulló (1989) to the tenora, and by Keefe (1989) to the flute. Although originally presented for the redesign of traditional wind instruments, these inverse methods are well suited for the design of synthetic air columns with desirable tuning properties.

Conclusion

An interesting physical model of a wind instrument has been specified by a relatively simple dynamical system. Sounds generated from such physical models are successful in accounting for some of the distinctive acoustic features of wind instruments. Although our knowledge of the physics of wind instruments is incomplete, physical models have considerable appeal as a new paradigm for music synthesis using computer-based instruments. Conversely, computer technology might be applied in the 21st century to design improved, or even new, families of acoustic wind instruments.

The emphasis on the role of the player's control of the reed to adjust intonation and tone color demonstrates how physical models differ from other synthesis techniques. The user gives up direct control over concepts such as oscillator frequency, but gains control over performance variables such as embouchure adjustment. A software synthesis system based on physical models must include feedback of the sound output by the "performer" to the "listener." Each "performer" must "listen" to his or her own instrument as well as all the other instruments, so that the ensemble plays in tune and is well blended. This is distinguished from a real-time system in which the human performer can make necessary adjustments. In this sense, the analogy with physical models must be extended to include perceptual modeling of performer responses.

Acknowledgments

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References

Barjau, A., J. Martínez, and J. Agulló. 1989. "Reed Woodwinds Redesign. A CAD Approach." In *Proceedings of the 13th International Congress on Acoustics,* vol. 3. Sabac, Yugoslavia: Dragan Srnic Press, pp. 95–98. .

Barjau, A., and J. Agulló. 1989. "Calculation of the Starting Transients of a Double-Reed Conical Woodwind." *Acustica* 69:204–210.

Benade, A. H. 1990. *Fundamentals of Musical Acoustics.* New York: Dover Publishers (updated reprinting of 1976 edition with corrections).

Benade, A. H., and S. N. Kouzoupis. 1988. "The Clarinet Spectrum: Theory and Experiment." *Journal of the Acoustical Society of America* 83:292–304.

Elliott, S. J., and J. M. Bowsher. 1982. "Regeneration in Brass Wind Instruments." *Journal of Sound and Vibration* 83:181–217.

Fletcher, N. H. 1979. "Excitation Mechanisms in Woodwind and Brass Instruments." *Acustica* 43:63–72.

Fletcher, N. H., and T. D. Rossing. 1991. *The Physics of Musical Instruments.* New York: Springer-Verlag.

Helmholtz, H. L. F. 1954. *On the Sensations of Tone.* Translation of 1877 German edition by A. J. Ellis. New York: Dover Publishers.

Hoekje, P. L. 1986. "Intercomponent Energy Exchange and Upstream/Downstream Symmetry in Nonlinear Self-sustained Oscillations of Reed Instruments." Ph.D. diss., Case Western Reserve University. Ann Arbor, Michigan: University Microfilms.

Ingard, U,. and H. Ising. 1967. "Acoustic Nonlinearity of an Orifice." *Journal of the Acoustical Society of America* 42:6–17.

Keefe, D. H. 1989. "Woodwind Design Algorithms to Achieve Desired Tuning." *Journal of the Catgut Acoustical Society* 1(3):14–22.

Keefe, D. H. 1990a. "On Sound Production in Brass Instruments." *Journal of the Acoustical Society of America* 87:S138.

Keefe, D. H. 1990b. "On the Theory of Sound Production in Reed-Driven Wind Instruments." Systematic Musicology Technical Report 9003. University of Washington School of Music.

Keefe, D. H. 1990c. "Tutorial on Physical Models of Wind Instruments: I, Introduction." Systematic Musicology Technical Report 9001. University of Washington School of Music.

Keefe, D. H. 1990d. "Woodwind Air Column Models." *Journal of the Acoustical Society of America* 88: 35–51.

Keefe, D. H. 1992. "Woodwind Models Incorporating Nonlinear Air-Column Losses." *Journal of the Acoustical Society of America* 91:S2449.

Keefe, D. H., and M. Park, 1990. "Tutorial on Physical Models of Wind Instruments: II, Time Domain Simulations." Systematic Musicology Technical Report 9002. University of Washington School of Music

McIntyre, M. E., R. T. Schumacher, and J. Woodhouse. 1983. "On the Oscillations of Musical Instruments." *Journal of the Acoustical Society of America* 74:1325–1345.

Nederveen, C. J. 1968. *Acoustical Aspects of Woodwind Instruments.* Amsterdam: Frits Knuf Publishers.

Park, M., and D. H. Keefe. 1988. "Woodwind Instrument Simulation in Real-Time." *Journal of the Acoustical Society of America* 83:S120.

Rayleigh, J. W. S. 1883. "On Maintained Vibrations." *Philosophical Magazine* XV:229–235. Reprinted in *Scientific Papers,* in six volumes bound as three, vol. II, pp. 188–193, including 1899 corrections. New York: Dover Publishers.

Schumacher, R. T. 1981. "Ab Initio Calculations of the Oscillations of a Clarinet." *Acustica* 48:72–85.

Strong, W. J. 1990. "Computer Simulation of a Trumpet." *Journal of the Acoustical Society of America* 87:138.

Thompson, S. C. 1978. "Reed Resonance Effects on Woodwind Nonlinear Feedback Oscillations." Ph.D. diss, Case Western Reserve University. Ann Arbor, Michigan: University Microfilms.

Thompson, S. C. (1979). "The Effect of the Reed Resonance on the Woodwind Tone Production." *Journal of the Acoustical Society of America* 66:1299–1307.

Worman, W. E. 1971. "Self-sustained Nonlinear Oscillations of Medium Amplitude in Clarinet-like Systems." Ph.D. diss., Case Western Reserve University. Ann Arbor, Michigan: University Microfilms.

Yoshikawa, S. 1988. "On the Modeling of Self-Oscillation in Brass Instruments." *Journal of the Acoustical Society of America* 84:161.