L3 APSC221 - Time Value of Money

How to make comparisons and decisions on tradeoff that occur at different times fairly?

Interest

Interest is compensation to the lender of money for their inconvenience.
The inconvenience is the loss of productivity when lending money in the present.

Interest is expressed as a percentage rate for a given period of time.

Notation	
Present Value / Present Worth / Principal	P
Future Value	F
Interest	I
Interest rate	i
Total Number of Interest Periods	N
Nominal Interest Rate	r
Effective Interest Rate per Period	i_e
Annuity Amount	A
Discrete Payment at the end of an Interest Period	A_n

The base unit of time for interest calculation is known as the interest period.
Periods include annually, semiannually, quarterly, monthly, weekly, daily, etc.
Continuous interest period is theoretical, but not practical. This rate limit approaches zero, and the function is exponential.

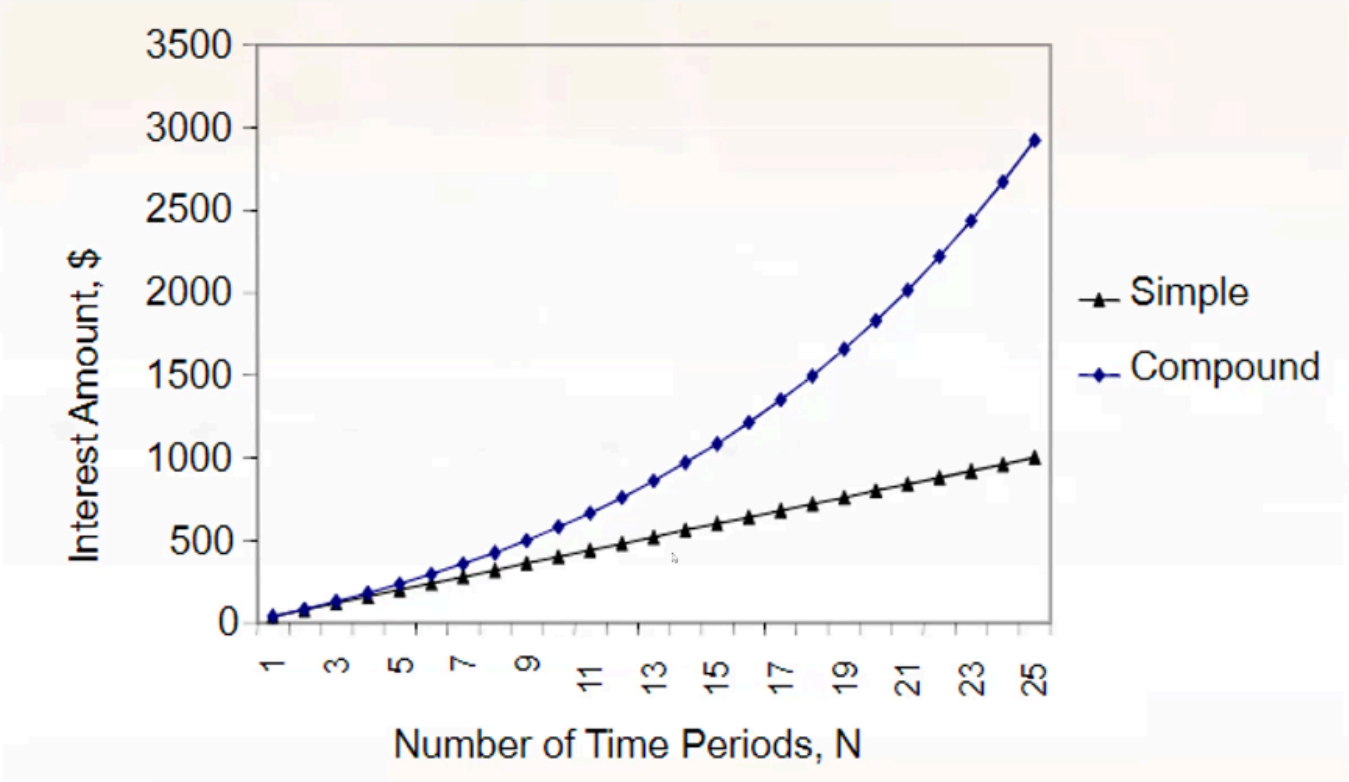
$$F = P + I$$
$$F = P + PI$$
$$F = P(1 + i)$$

Interest is payed for every period.
Simple interest is constant for every period and is payed on the principal. This is basically inapplicable.

$$I = PiN$$

Compound interest is paying interest on the principal and accrued interest value. This interest compounds on itself, over each period.

$$F = P(1 + i)^N$$



Nominal and Effective Interest Rates

Interest rates are stated for a given period of time.
The "stated" rate is the nominal interest rate, and is assumed to be **annual**.
The effective interest rate can be found through conversion.

Suppose that:
r is the nominal state rate for the period (annual)
m is the equal compounding sub-periods (target period)
the compound interest rate per sub period is i_s = r/m

$$F = P(1 + i_s)^m$$

The effective annual interest rate i_e :

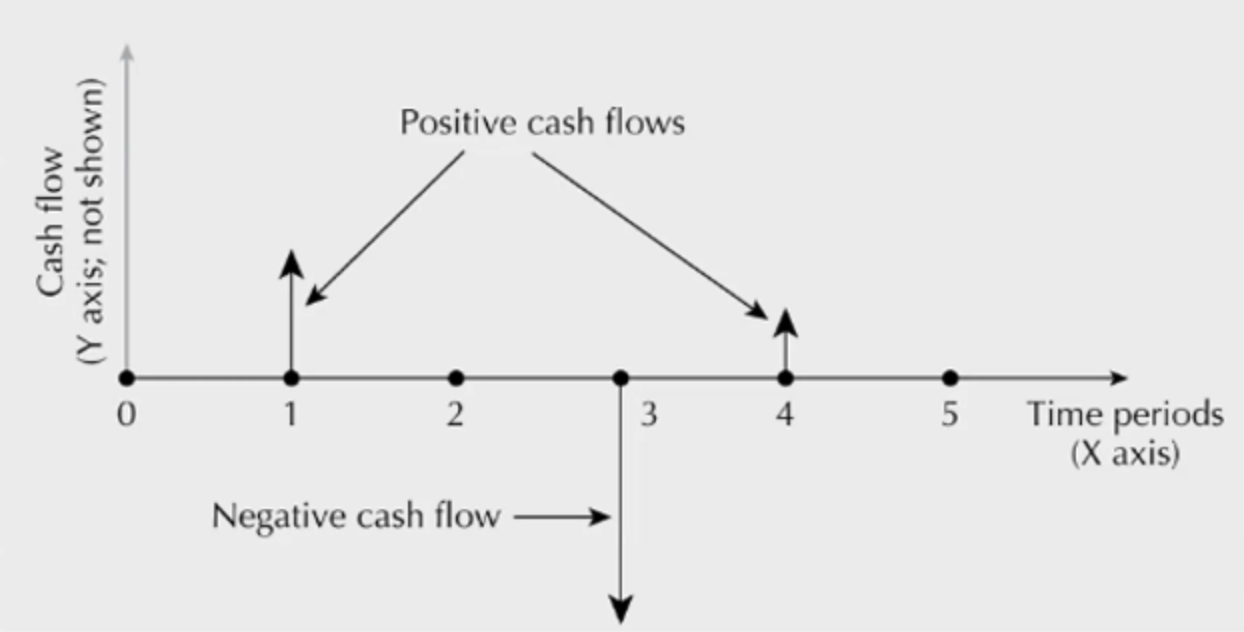
$$F = P(1 + i_s)^m = P(1 + i_e)$$

$$i_e = (1 + i_s)^m - 1$$

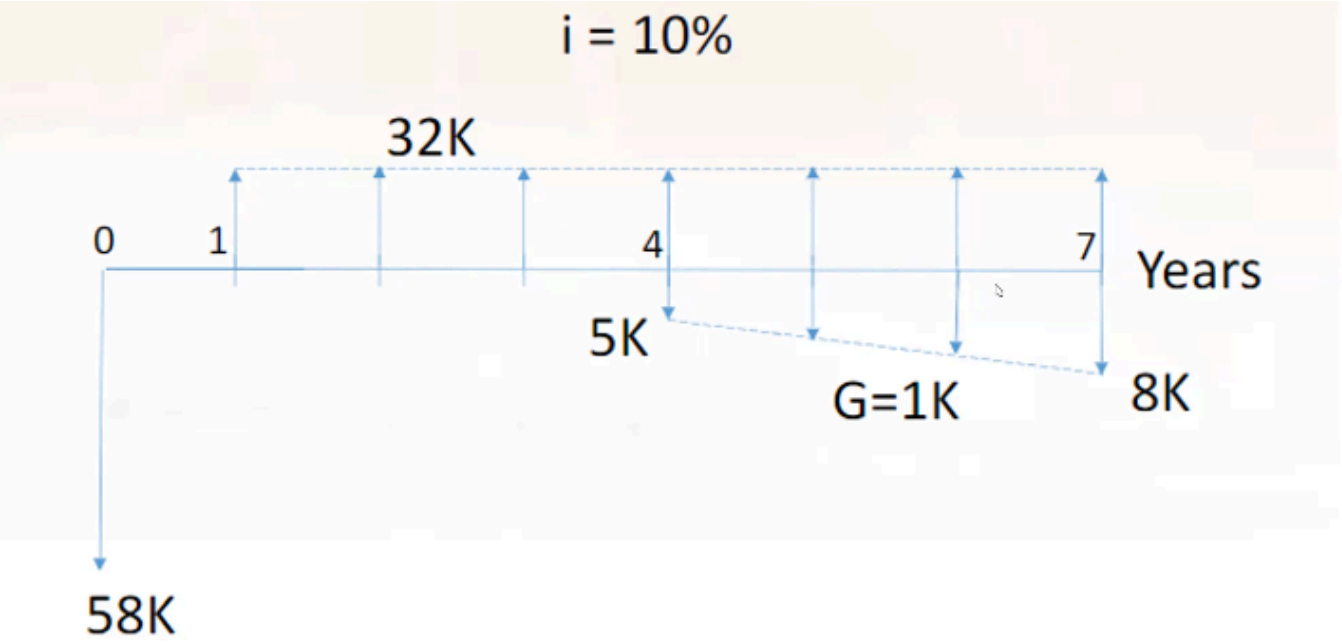
$$\text{or } i_e = (1 + r/m)^m - 1$$

Cash Flow Diagrams

Graphical representation to summarize the timing and magnitude of cash flows



Cash transactions occur at the end of a period
The magnitude of the arrow is also important and should be measurable.



Equivalence

This is the idea that different sums of money at different points in time can be considered equal in value if adjusted properly using interest rates.

Mathematical Equivalency

Calculations we are conducting that equates present and future values.

Decisional and Market Equivalency

These assumptions must hold true for the mathematical equivalence to be meaningful.

1. Decision maker is indifferent between money now or in the future
2. Decision maker can exchange cash flows at zero cost (ignore admin costs)