

# ELEC 278 - Lecture Notes

## FUNDAMENTALS OF INFORMATION STRUCTURES

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# 1 What is a Data Structure

A data structure is a way to store and organize data in order to facilitate access and modification

It concerns the **representation, manipulation, and efficient management of data**. Use cases involve large databases and internet indexing services.

Efficient data structures are key to designing efficient algorithms.

Each data structure supports one or more algorithms for the **operations**, that are insert, delete, search, and modify

Conversely, an algorithm is a step by step procedure in performing a task

## 2 Overview of C - Review

Incorrect swap function

```
void swap(int a, int b){
    int temp = a;
    a = b;
    b = temp;
}

int x = 1;
int y = 2;
swap(x,y);
```

This function is swapping its own copy of the data, and these changes are not reflected outside the function. Pointers are necessary to affect the original variables.

### 2.1 Pointers

```
int *nptr = &total;
```

This pointer is pointing to the memory address of the total variable.

The & operator returns or takes the address of a variable.

Usage:

```
int total = 5;
float speed = 2.3;
int numbers[4];
int *nptr = &total;
*nptr = 12; # this changes 5 to 12
```

The \* operator uses or dereferences the value of the pointer.

### 2.2 Correct swap function

```
void swap(int *a, int *b){
    int tmp = *a;
    *a = *b;
    *b = *a;
}

int x = 1;
int y = 2;
swap(&x, &y); # x and y are changed
```

Pointer values are copied, and pointers are used to access original values.

## 2.3 Indexing Pointers

Neglecting `[]` of an array calls its first value (at index 0)

```
int numbers[4];  
int *nptr = numbers;
```

Notice, no `&` for pointing to arrays

You can add or subtract integers from pointers. When you do this, the pointers value is scaled by the size of its referenced type (e.g. `nptr + 2` is equivalent to adding 8 bytes if pointing to `int` since each `int` is 4 bytes)

You can access array elements using pointer arithmetic:

```
*(nptr+2)=10; # sets the third element of the array to 10
```

## 2.4 Global and Local Variables

Global variables are always initialized to zero

## 3 Overview of C - New Types

A data structure is a way to store and organize data in order to facilitate access and modification

### 3.1 C Structures

This groups items of possibly different types into a single type

```
struct structure_name{  
    data_type member_name1;  
    data_type member_name2;  
};
```

The items in the structure are called **members** or **fields**

We can define structure variables as

```
struct structure_name variable_name1;
```

or at the same time:

```
struct structure_name{  
    data_type member_name1;  
    data_type member_name2;  
} variable_name1;
```

Where *structure\_name* is the defined structure of data, and *variable\_name* is an instance of the struct

Use the dot (.) operator to access members

```
var_name.marital_status = 4;
```

and initialize individual fields with the dot notation:

```
struct Student student1={.name = "Alice", .section = 'A'};
```

## A More Detailed Example



```
#include <stdio.h>
#include <string.h>
struct student {
    int id;
    char name[20];
    float percentage;
};

int main() {
    struct student record;
    record.id=1;
    strcpy(record.name, "Raju");
    record.percentage = 86.5;
    printf(" Id is: %d \n", record.id);
    printf(" Name is: %s \n", record.name);
    printf(" Percentage is: %f \n", record.percentage);
    return 0;
}
```

Define the data type

Declare the structure variable

Assign values to structure members

Access structure members

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### 3.2 Typedef

A typedef looks like a variable definition, but defines a new name for an existing type. This can improve code readability.

```
typedef int studentNumberType;
studentNumberType studentNumber1;
```

Here, studentNumberType becomes an alias for int, so studentNumber1 is actually an int but is made clearer.

```
typedef struct {
    char name[50];
    int class;
    char section;
} Student;
Student s1, s2;
```

### 3.3 Nested Structures

If we typedef a struct, and use it in another structure, one of the members of a structure is itself another structure

```
typedef struct {  
    int imag;  
    float real;  
} complex;  
  
struct number {  
    int flags;  
    complex phase;  
} num1, num2;
```

Where *phase* is a *struct*

### 3.4 C Enumerated Types

This is a way to create a user-defined type consisting of a set of named integer constants

```
enum colors {RED, GREEN, BLUE}
```

You can also change the starting value by:

```
enum week {Monday = 1; ...} # Sunday is 0
```

We can also combine enums with typedef



## 4 Overview of C - More Structs and Dynamic Memory

### 4.1 More Structs

Structs can be used as the base type of an array

```
typedef struct {  
    int x;  
    int y;  
} point;  
  
point vertexes[100];  
  
vertexes[4].x=23;  
vertexes[4].y=18;
```

We can also point to members within a struct

```
typedef struct {  
    int x;  
    int y;  
} point;  
  
point a = {23,18};  
point *b = &a;  
(*b).x=34;
```

We can also return structs as a result of a function

### 4.2 Pointers to pointers

Pointers to pointers are used to reference the address of another pointer

```
void allocateInt(int **p) {  
    *p = malloc(sizeof(int))  
}
```

or

```
void swap(point **a, point **b){  
    point *tmp;  
    tmp = *a;  
    *a = *b;  
    *b = tmp;  
}
```

### 4.3 Dynamic Memory

So far, memory was either global or local. Now we add **Heap memory**, to request memory at runtime

```
tax_info *bob = (tax_info*) malloc(sizeof(tax_info));
bob->marital_status = single;
```

The arrow operator is shorthand for `(*ptr).field`

### 4.4 Arrays of pointers

Instead of storing 100 structs (3200 bytes), you can store 100 pointers (800 bytes) and allocate each struct individually:

```
tax_info *employees[100];
employees[0] = malloc(sizeof(tax_info));
```

### 4.5 Dynamic strings

To duplicate a string dynamically:

```
char *s1 = "Hello";
char *s2 = strdup(s1); # strdup is like copying into another string
```

### 4.6 Function pointers

Since functions also live in memory, we can point to them too:

```
int sum(int a, int b) { return a+b; }
int (*func_ptr)(int,int) = sum;
printf("%d\n", func_ptr(2,3));
```

## 5 Foundational Data Structures 1

Use structure to group array and count of number elements in the array \* Arrays are declared with a maximum length

Linked lists provide an ordered collection that grows with the number of data items \* each element is a node with data and a pointer to the next element \* Head points to the first element

### 5.1 Linked Lists

Linked lists are a means of dynamically allocating an ordered collection of elements

**Why?:** Arrays are rigid. If you dont know how many items youll need, arrays can overflow. Linked lists solve this with dynamic allocation.

Each element (**a node**) stores: \* Data \* Pointer to the next node

The list is ended with a *NULL* ptr

```
struct node {
    int value;
    struct node *next;
};
struct node *head; // points to first node
```



### 5.2 Building a Linked List

To begin building a linked list, memory is first allocated for the head node using `malloc`, and its value field is set to 14

```
head = (struct node *) malloc (sizeof(struct node));
head-> value = 14;
```

Next, another node is added by allocating memory for `head->next`, and its value is set to 92

```
head -> next = (struct node *) malloc (sizeof(struct node));
head -> next -> value = 92;
```

After allocating each node, it is important to check if memory allocation succeeded; if head is NULL, the program prints failed and exits

```
if (head == NULL) {
    printf("failed");
    exit(1);
};
```

We can use linked lists with more complex datatypes:

**Example:**

```
typedef struct {
    float x;
    float y;
    int alt;
} aircraftPos;
```

We can build a node struct, which is the building block of the linked list, where each node contains a value of type `aircraftPos` and a pointer to the next node on the list

```
typedef struct node {
    aircraftPos value;
    struct node *next;
} node;
```

### 5.3 Traversing a linked list

Start function

```
void printList(struct node *head) {
```

To traverse a linked list, you begin by pointing a temporary node `tmp` to the head, then print an opening bracket to start the list display

```
struct node *tmp = head;
printf("[ ");
```

The traversal continues in a loop while `tmp` is not `NULL`; this ensures you stop once you reach the end of the list

```
while (tmp != NULL) {
```

Inside the loop, you first access the current nodes value by printing it, and then move forward by updating `tmp` to its `next` pointer

```
    printf("%d, ", tmp->value);
    tmp = tmp->next;
}
```

Finally, you print a closing bracket and a newline to finish displaying the entire list

```
print("]\n");
```

## 5.4 Adding a new node to an empty list

To simplify creating nodes, we can define a helper function `newNode` that allocates memory with `malloc`, assigns the given value, initializes the `next` pointer to `NULL`, and returns the new node

```
node *newNode(int value){
    node *tmp = (node *)malloc(sizeof(node))
    tmp -> value = value;
    tmp -> next = NULL;
    return tmp;
}
```

To add a new node to an empty list, we call `newNode` to create a node with value 14, and if the head is `NULL` (meaning the list is empty), we set `head` to point to this new node

```
new = newNode(14);
if (head != NULL) {
    head = new;
}
```

We can also insert at the start of an existing list by creating a new node, setting its `next` pointer to the current head, and then updating head to point to this new node

```
node *new = newNode(12);
new->next = *h;
*h = new;
```

This insertion-at-front logic can also be written as a reusable function, `insertFirst`, which takes a double pointer to the head, creates a new node, links it before the current head, and updates the head pointer.

```
void insertFirst(node **h, int data){
    node *new = newNode(data);
    new -> next = *h;
    *h = new;
}

insertFirst(&head, 14);
```

## 6 Foundational Data Structures 2

### 6.1 Appending to an existing list

Start function

```
void appendList(node **h, int data){  
    node *new = newNode(data);
```

Cover empty list case:

```
if (*h == NULL) { *h = new; return; }
```

To append to an existing linked list, we first set a pointer `tail` to the head of the list, and then create a new node with the desired value using `newNode`

```
node *tail = *h; // start at the head  
struct node *new = newNode(42);
```

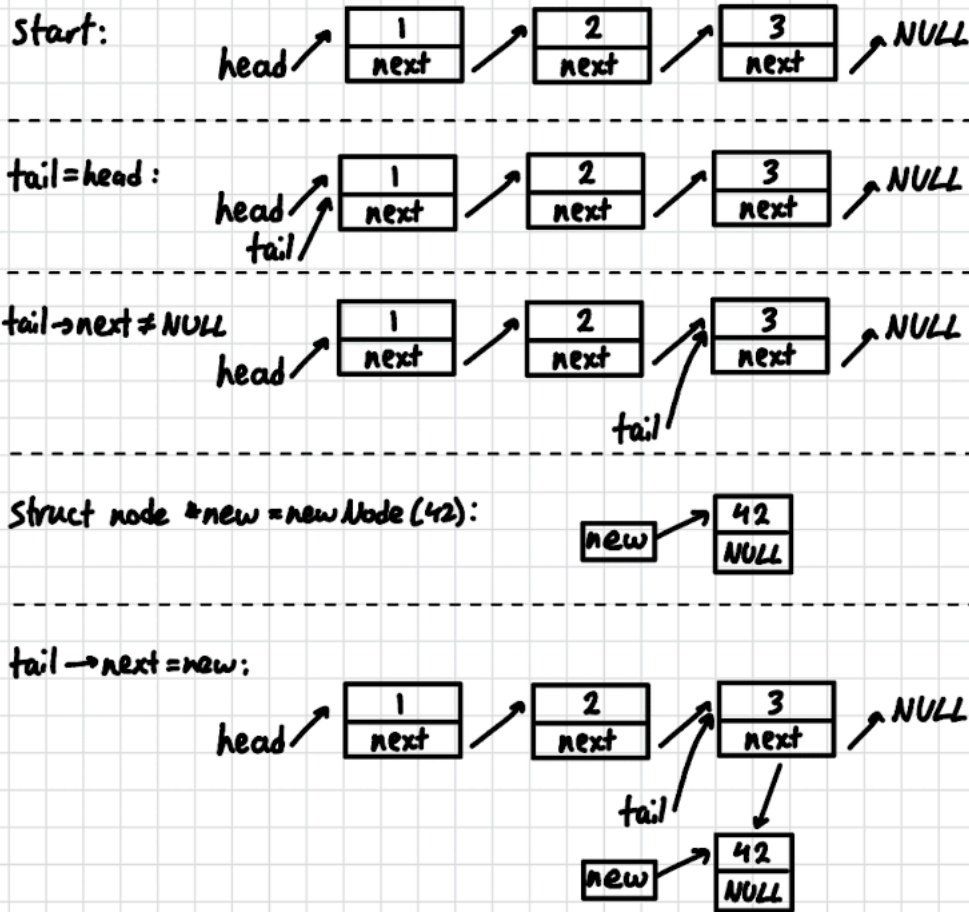
Next, we traverse the list by moving `tail` forward until we reach the last node, which is identified when `tail->next` becomes `NULL`

```
while (tail -> next != NULL) {  
    tail = tail -> next; // traverse to the last node  
}
```

Finally, once at the last node, we attach the new node by setting `tail->next` to point to it, completing the append operation

```
tail -> next = new; // link the new node at the end
```

### Appending to an existing list



## 6.2 Insert a Value in the Middle of a List (after a specific value)

To insert a new node after a specific value, we first create a temporary pointer `tmp` starting at the head, and then traverse forward while `tmp` is not NULL and the current nodes value is not equal to 92

```
struct node *tmp = head; // start at head
while (tmp != NULL && tmp->value != 92){
    tmp = tmp->next; // move to the next node
}
```

If the loop finds a node with value 92, then `tmp` points to the node. At this point, we create a new node with value 42, set its `next` pointer to the node after `tmp`, and then adjust `tmp->next` so that it links to the new node

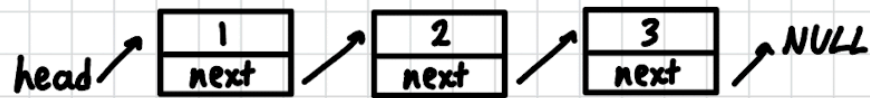
```

if (tmp != NULL){
    new = newNode(42); // create node with value 42
    new -> next = tmp -> next; // link new node to the node after tmp
    tmp -> next = new; // insert new node right after tmp
}

```

### Insert at a specific value

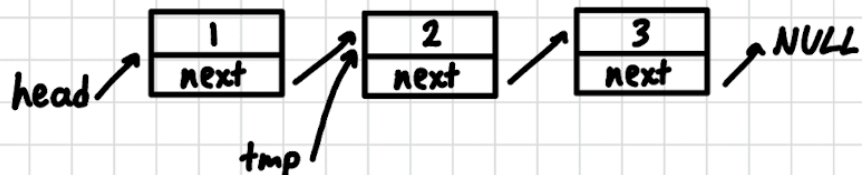
Start:



tail = head:



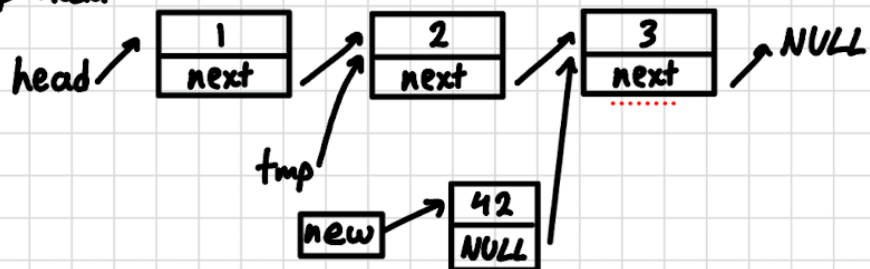
tail -> next ≠ 2



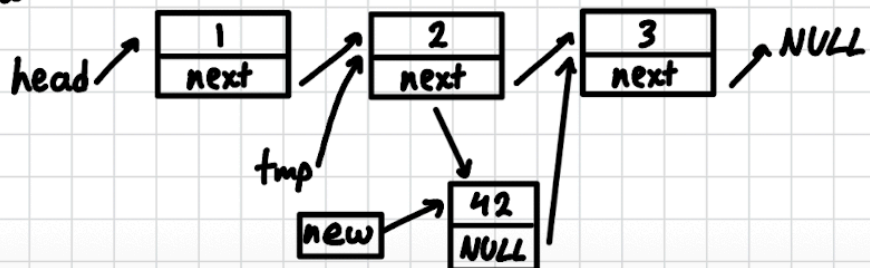
new = newNode(42):



new -> next = tmp -> next



tmp -> next = new





### 6.3 Insert Sorted Function

The function starts by creating a new node with the given value using `newNode`

```
node *new = newNode(value);
```

If the list is empty, the new node becomes the head, and the function immediately returns

```
if (*head == NULL){           // Case 1: Empty list
    *head = new;
    return;
}
```

If the new value is smaller than the current heads value, the new node is inserted at the front, and head is updated

```
if ((*head)->value > value){  // Case 2: Insert before the first
    ↪ node
    new->next = *head;
    *head = new;
    return;
}
```

Otherwise, the function traverses the list using a pointer `tmp`, always looking one node ahead. If it finds that the next nodes value is larger than the new value, it inserts the new node between `tmp` and `tmp->next`

```
node * tmp = *head;
while (tmp->next != NULL){    // Traverse the list
    if (tmp->next->value > value){ // Case 3: Insert in middle
        new->next = tmp->next;    // Point new node to next node
        tmp->next = new;         // Link previous node to new
    ↪ node
        return;
    }
    tmp = tmp->next;           // Advance to next node
}
```

If no larger value is found, the function appends the new node at the end of the list

```
tmp->next = new; // Case 4: Insert at the end
```

### 6.4 Removing Elements

Often, we want to remove the first node, a specific node, or the last node

#### Delete First Node

The function `removeFirst` begins by saving a pointer `tmp` to the current head of the list and initializing an integer variable `value` to 0, which acts as a default return value

```

struct node *tmp = *head;    // Save pointer to current head
int value = 0;               // Default return value if list is empty

```

It then checks if the list is non-empty, if so, the head pointer is advanced to the second node, the value of the original head is saved into `value`, and the memory allocated to the old head node is freed

```

if (*head != NULL){
    *head = (*head)->next;    // Move head to next node
    value = tmp->value;        // Save the value of the old head
    free(tmp);                // Free memory of the old head
}

```

Finally, the function returns the integer `value`, which is either the removed nodes value or 0 if the list was empty

```

return value;

```

### Delete Specific Value

The function `deleteValue` starts by checking if the head of the list is `NULL`, and if so, it immediately returns because there is nothing to delete

```

if (*head == NULL) return;

```

Next, it handles the special case where the first node contains the target value: it saves a pointer to the head in `tmp`, updates `head` to the second node, frees the old head, and then returns.

```

if ((*head)->value == val) {
    node * tmp = *head;
    *head = (*head)->next;
    free(tmp);
    return;
}

```

If the value is not in the head, the function creates a pointer `prev` starting at the head, and then iterates while `prev->next` is not `NULL`

```

node * prev = *head;
while (prev -> next != NULL) {

```

Inside the loop, it checks if the next node holds the target value; if so, it saves that node in `tmp`, bypasses it by updating `prev->next`, frees the node, and returns.

```
if (prev -> next -> value == val){  
    node * tmp = prev -> next;  
    prev -> next = tmp -> next;  
    free(tmp);  
    return;  
}
```

If the value was not found, the loop moves `prev` to the next node and continues searching until the end of the list.

```
prev = prev->next;
```

## 7 Double Linked Lists and Dynamic Arrays

### 7.1 Double Linked List

A single linked list has a single entry point (head)

**Idea:** Each node has `next` and `prev`, plus you often keep `head` and `tail` pointers so you can traverse both ways.

Hence a Double Linked List, where one linked list goes forward and one goes backwards

```
typedef struct node {
    int value;           // payload
    struct node * next;  // forward link
    struct node * prev;  // backward link
} node;

node * head;           // first node pointer
node * tail;           // last node pointer
```

### 7.2 Dynamic Arrays

**Idea:** Wrap an array with a count field so you know how many elements are live

```
typedef struct {
    int *array;
    size_t capacity; // available space
    size_t length;   // number of elements stored in the array
} Array;
```

#### Initialization

Allocate an initial buffer, starting at length 0 and capacity at initial size

```
void initArray(Array *a, int initialSize){
    a->array = (int *)malloc(initialSize * sizeof(int)); // allocate
    ↪ buffer
    a->length = 0;           // nothing used yet
    a->capacity = initialSize; // remember capacity
}
```

#### Append an Element to the Array

```
void addToArray(Array *a, int element){
```

The function begins by checking if the array is already full. It compares the number of used entries (`length`) to the total slots available (`capacity`). If they are equal, the array must grow.

```
if (a->length == a->capacity){
```

In that case, it doubles the capacity and resizes the buffer with `realloc`, ensuring that all existing elements are preserved while making space for new ones

```
a->capacity *= 2;
a->array = (int*)realloc(a->array, a->capacity * sizeof(int));
}
```

After ensuring enough capacity, the function places the new element at the current end of the array (`a->array[a->length]`), then increments `length` so the array correctly reflects its new size

```
a->array[a->length++] = element;
```

And free the dynamic array

```
void freeArray(Array *a){
    free(a->array);           // release heap buffer
    a->array = NULL;          // null out dangling pointer
    a->capacity = a->length = 0; // reset counters
}
```

**Note:** Arrays are contiguous memory. Inserting an element into the memory of the list means moving the remaining elements up one position

We use `memmove` to copy `n` bytes from the memory pointed to by `src`, to the memory pointed to by `dst`. This function understands memory blocks overlapping

```
void * memmove(void *dst, const void *src, size_t n)
```

## 8 Double Linked Lists and Dynamic Arrays (cont.)

### 8.1 Inserting an Element in the Middle

```
bool insertAt(Array *a, size_t index, int value){
```

The function begins by validating the requested index. If the index is greater than `length`, the function returns `false`

```
if (index > a->length)    // inserting past the end is invalid
    return false;
```

Next, it checks if the array is already full. If `length == capacity`, the capacity is doubled, and the buffer is resized using `realloc`. This ensures there is enough space for the new element

```
if (a->length == a->capacity){
    a->capacity *= 2;
    a->array = realloc(a->array, a->capacity * sizeof(int));
}
```

To make room for the new element, all elements from `index` up to the last used position are shifted one slot to the right. This is efficiently handled with `memmove`, which safely copies overlapping memory regions

```
memmove(&a->array[index + 1],    // shift destination
        &a->array[index],        // shift source
        (a->length - index) * sizeof(int)); // number of bytes to
        ↪ shift
```

Finally, the new value is written into the open slot at `index`, and `length` is incremented to account for the insertion. The function returns `true` to indicate success

```
a->array[index] = value;
a->length++;
return true;
```

### 8.2 Deleting from the middle

The function begins by validating the index.

```
if (index >= a->length)    // invalid index (out of range)
    return false;
```

If the index is valid, the function reduces the logical size of the array by decrementing `length`. This means the element at the given index will effectively be removed

```
a->length--;
```

To fill the gap, all elements after the removed one are shifted left by one slot. `memmove` is used here because it safely handles overlapping regions of memory

```
memmove(&a->array[index],      // overwrite deleted slot
        &a->array[index + 1],   // start from next element
        (a->length - index) * sizeof(int)); // shift the suffix left
```

After the shift, the function optionally applies a shrink policy. If the number of elements is at or below one-quarter of the current capacity, the capacity is halved and the buffer resized with `realloc`

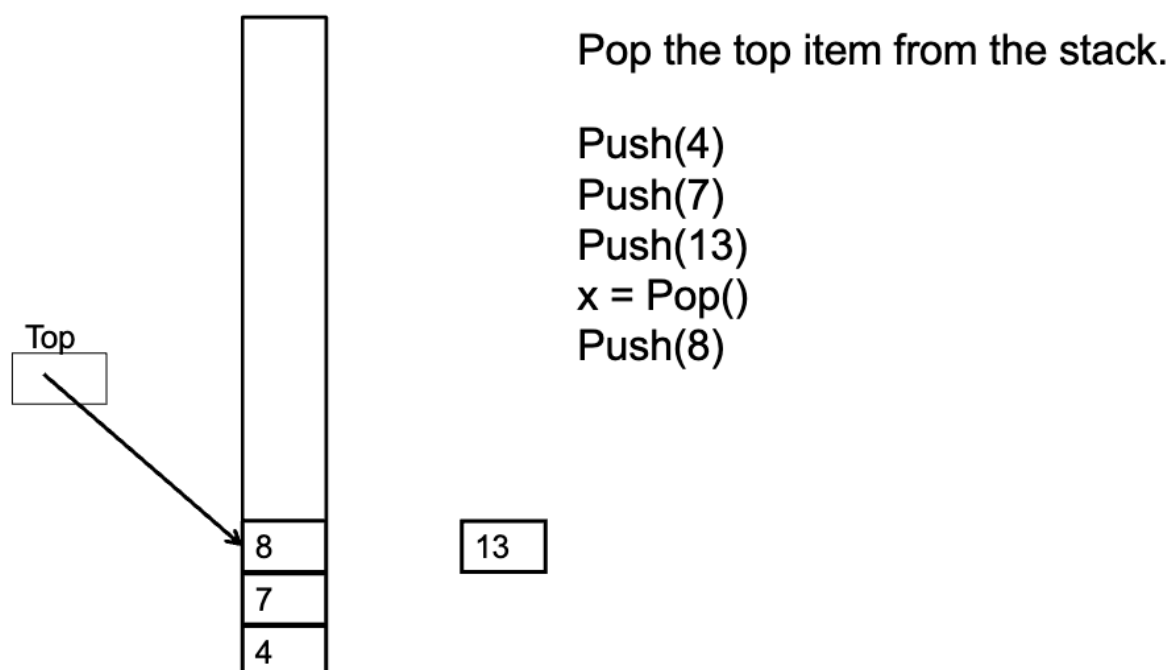
```
if (a->length * 4 <= a->capacity){
    a->capacity /= 2;
    a->array = realloc(a->array, a->capacity * sizeof(int));
}
return true;
```

## 9 Stack

**Definition:** A data structure for storing a collection of data items, where items can be added to and removed from the collection, but *only the last item added to the stack can be accessed or removed*. This is a last-in, first-out (LIFO) data structure

**Common Functions:** `push(item)` puts an item on the top of the stack `item=pop()` retrieves the top item from the stack `peek()` retrieves the top item of the stack without removing it

### Stack



LIFO is important for use cases such as Reversing a word, web browsers to store addresses of recently visited sites, and undo functions in applications

#### Implementation requirements

Top variable always indicates to the top element of the stack - expect when the stack is empty - special value to indicate the stack is empty

Some mechanism to indicate errors: - attempt to pop an empty stack - out of memory when pushing an element

#### Example:

`typedef struct { ... } ArrayStk` defines a new struct type for a stack with an alias, holds an index of the current top element in the stack `top`, and sets an array `data` of size 100 to store the stacks elements



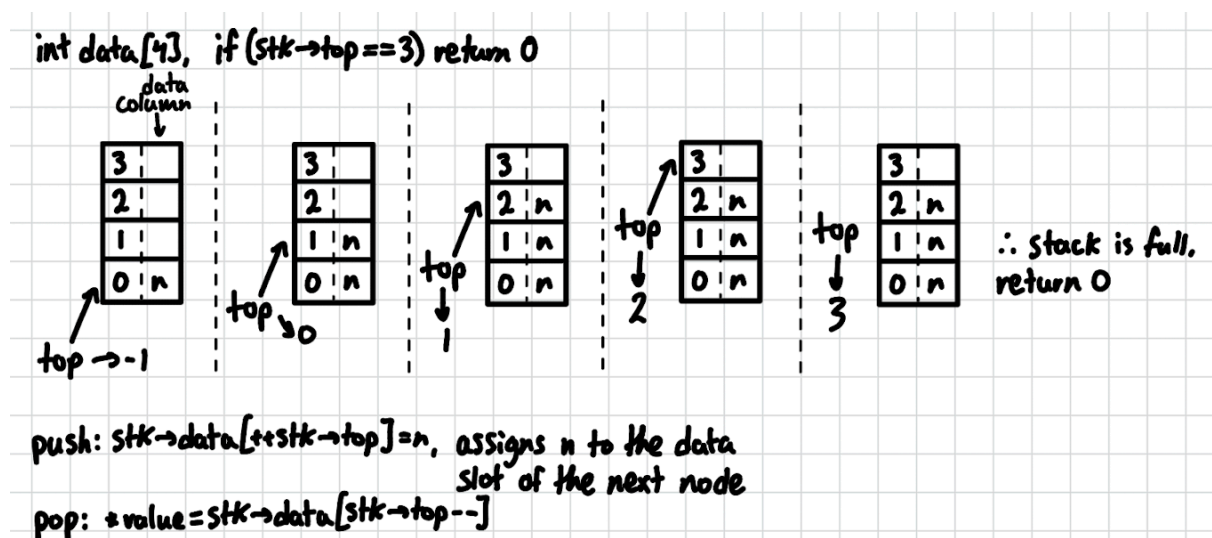
```
typedef struct {
    int top;           // index of current top, -1 when empty
    int data[4];       // storage
} ArrayStk;
```

This function `push` tries to insert a value `n` onto the stack, and returns 1 if successful and 0 if not. The function also pre-increments `stk->top` (moves it up one position) and stores the new value `n` in that position in the `data` array.

```
int push(ArrayStk *stk, int n){
    if (stk->top == 3) return 0;           // full
    stk->data[++stk->top] = n;             // push
    return 1;
}
```

The `pop` function checks if full, stores it in that variable pointed to by value, then post-decrements `stk->top` (moves it down one position), effectively removing it from the stack

```
int pop(ArrayStk *stk, int *value){
    if (stk->top == -1) return 0;          // empty
    *value = stk->data[stk->top--];        // pop
    return 1;
}
```



## 9.1 Modifying the top element

Setup:

```
ArrayStk stk = { .top = -1 };           // empty stack
int i, value, *ptr_top;                 // local vars
```

Pushes 20, 21, 22, 23, 24 onto the stack. Then pops them all off, printing in *reverse order* because of LIFO

```
for (i=0; i<5; i++) { push(&stk, 20+i); }
while (pop(&stk, &value)) {
    printf("%d\n", value);
}
```

This pushes 100, 101, 102, 103, 104

```
for (i=0; i<5; i++) { push(&stk, 100+i); }
```

Pops the top into `value`, calls `tos` to return a pointer to the new top element after the pop, then adds 50 to that element in place

pops 104 153 152 151 150

```
while (pop(&stk, &value)) {
    printf(" %d", value);
    if (tos(stk, &ptr_top))
        *ptr_top = *ptr_top + 50;
}
```

where `tos` is:

```
int tos(ArrayStk stk)(int **ptop) {
    if (stk.top == -1) return 0;
    *ptop = &stk.data[stk.top];
    return 1;
}
```

## 9.2 Implementing Stack using Linked List

Building the stack data structure on top of a linked list instead of using an array:

This defines a struct with an alias, and a pointer to the next node in the stack (meaning the node below the top). This allows us to chain nodes like a linked list. We also have `nodval`, which is the actual data being stored in the stack

```
typedef struct _stk_node {
    struct _stk_node *next;    // link to previous top
    int nodval;                // payload
} StkNode;
```

- Function takes a pointer to the stack head pointer to modify the pointer outside the function
- Allocate memory for a new node on the heap, and stores the new value inside the node

- Links the new node to the current stack and updates the head pointer so the new node is now the stacks top.

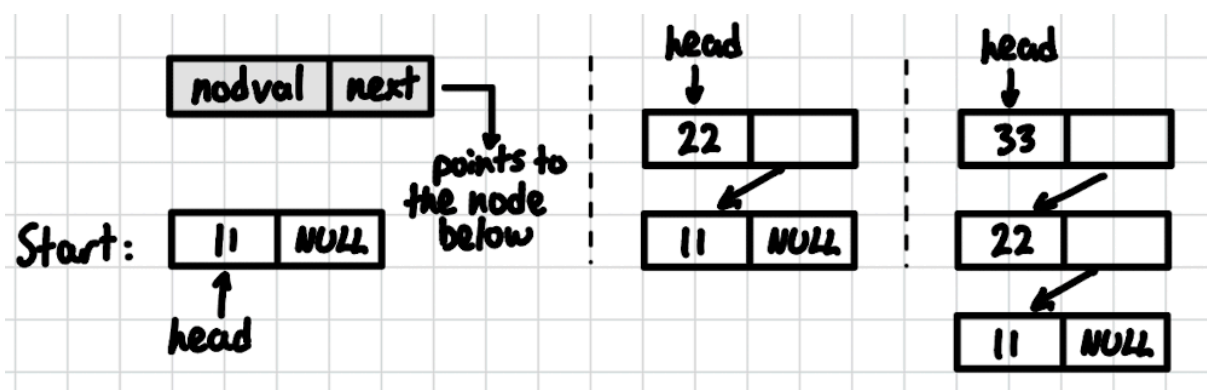
```
void push(StkNode **stkHead, int n){
    StkNode *p = malloc(sizeof *p); // allocate new node
    p->nodval = n; // set payload
    p->next = *stkHead; // chain to old top
    *stkHead = p; // new node becomes top
}
```

This pop function removes the top node, and returns 0 if the stack is empty (checks for underflow). It firstly saves the current top node pointer, moves the stack head down to the next node, and releases memory for the removed node.

```
int pop(StkNode **stkHead, int *out){ // (slide shows int return;
    ↪ safer to output)
    if (*stkHead == NULL) return 0; // underflow guard
    StkNode *p = *stkHead; // old top
    *out = p->nodval; // capture value
    *stkHead = p->next; // drop node from stack
    free(p); // free storage
    return 1; // success
}
```

Top-of-stack returns the address of nodval to the head node, else return NULL

```
int *tos(StkNode *stkHead){
    return stkHead ? &stkHead->nodval : NULL; // pointer to top value
    ↪ or NULL
}
```



## 10 Queue

- A linear data structure that is open at both ends and the operations are performed in First In First Out (FIFO)
- Operations: `enqueue` adds an element to the end, `dequeue` removes an element from the front, `isEmpty`
- We need to track the Front (head) and End (tail), thus we need a pointer to the end and front

### 10.1 Implement Queue with Linked List

**Enqueue** inserts a node at the rear (end) of the queue

```
void enqueue (int n) {
```

Allocate memory for a new node, assign value and next pointer to NULL

```
struct item *pnew = malloc(sizeof(struct item));
pnew->value = n;
pnew->next = NULL;
```

Nudge  $n$  to  $pnew$

```
if (end != NULL){
end->next = pnew;
}
if (front == NULL){ // if front is NULL, the list is empty and set
↪ front to new
front = pnew;
}
count++;
```

**Dequeue** removes an element from the front (head) of the queue

```
bool dequeue (int *n){
```

Set new node `temp` to front Give the pointer value `*n` to front

```
struct item *temp = front;
*n = (front -> value);
```

Move `front` to the next node

```
front = front-> next;
```

Free memory of old front (`temp`)

```
free(temp);
count--;
return true;
```

## 10.2 Implement Queue with Array

Common way to implement queue with array is with a circular model. This fixes the idea of a fixed-size array

```
# initialize
// define max size
// define queue array
// define count, front, end = 0
// define isEmpty and isFull functions
```

```
bool enqueue(int n){
// if full, return false
// add to end of queue
end = (end+1) % MAX_SIZE;
count++;
return true;
}
```

```
bool dequeue(int *n){
// if empty, return false
// remove from front of queue
front = (front + 1) % MAX_SIZE;
count--;
return true;
}
```

## 10.3 Dequeues

- a double-ended queue, can be done with either an array or double linked list
- Operations: enqueueHead, enqueueTail, dequeueHead, dequeueTail, getHead, and getTail

**EnqueueHead** inserts a new node before the head

```

void enqueueHead (int n) {
    struct item *pnew = malloc(sizeof(struct item)); // allocate memory
    ↪ for node

    pnew -> value = n;
    pnew -> next = NULL;
    pnew -> prev = NULL; // assign value and pointers

    // update head node
    if (head != NULL){
        head -> prev = pnew;
        pnew -> next = head;
    }
    head = pnew;

    // if queue is empty, set tail
    if (tail == NULL){
        tail = pnew;
    }
    count++;
}

```

**EnqueueTail** inserts a new node after the tail

```

void enqueueTail (int n){
    struct item *pnew = malloc(sizeof(struct item)); // allocate memory
    ↪ for node

    pnew -> value = n;
    pnew -> next = NULL;
    pnew -> prev = NULL; // assign value and pointers

    // update tail node
    if (tail != NULL){
        pnew -> prev = tail;
        tail -> next = pnew;
    }
    tail = pnew;

    // if queue is empty, set front node
    if (head == NULL){
        head = pnew;
    }
    count++;
}

```

**DequeueHead** removes a node from the head

```

bool dequeueHead(int *n){
    struct item *temp = head; // create temp pointer to head

    if (count == 0) return false; // queue is empty, can't remove
    ↪ anything

    *n = head -> value; // store head value to n

    if (head -> next){
        head -> next -> prev = NULL; // detach old head
        head = head->next; // move head to next node
    }

    if (head==NULL) tail = NULL; // if head is NULL, queue is empty

    free(temp); // deallocate old head node
    count--;
    return true;
}

```

DequeueTail removes a node from the tail

```

bool dequeueTail (int *n){
    struct item *temp = tail; // create temp pointer to head

    if (count == 0) return false; // queue is empty, can't remove
    ↪ anything

    *n = tail -> value; // store head value to n

    if (tail -> next){
        tail -> next -> prev = NULL; // detach old head
        tail = tail->next; // move head to next node
    }

    if (tail==NULL) head = NULL; // if head is NULL, queue is empty

    free(temp); // deallocate old head node
    count--;
    return true;
}

```

## 11 Recursion

Sometimes a problem can be solved by first solving a smaller version of the same problem. Recursion means to define something in terms of itself

When the problem is small enough, then it can be solved directly, called the *base case*

### Example:

Base Case:  $1! = 1$

```
if (n==1){
    temp=1;
}
```

Recursive Case  $n! = n * (n - 1)!$

### Iterative vs. Recursive:

Iterative

```
def fact(n){
    result = 1;
    for i in range(1,n+1):
        result *= i;
    return result;
}
print(fact(i))
```

Recursive

```
def fact(n){
    if n==0 or n==1: // Base Case
        return 1;
    else:
        return n * fact(n-1); // Recursive Call
}
print(fact(n))
```



## 12 Recursion 2

### 12.1 Linked List - Recursive Insert Sorted

newNode function

```
node *newNode(int value, node *next){
    node *tmp = malloc(sizeof(node));
    tmp -> value = value;
    tmp -> next = next;
    return tmp;
}
```

Insert function

```
int insert(struct node **list, int value){

    // if list empty, or the current node's value is greater than the
    // value to insert, create new node
    if (*list == NULL || (*list)->value > value){
        *list = newNode(value, *list);
        return 1;
    }

    // if new value is smaller than current, insert it before the
    // current node
    if (*list->value == value) {
        return 0;
    }

    // otherwise, move one step deeper and try to insert (recursive
    // step)
    else {
        return insert(&((*list)->next), value);
    }
    return 1;
}

int main(){
    struct node *head = NULL;
    insert(&head, 27); // head: 27
    insert(&head, 92); // head: 27 -> 92
    insert(&head, 12); // head: 12 -> 27 -> 92
    insert(&head, 14); // head: 12 -> 14 -> 27 -> 92
}
```

## 13 Algorithm Analysis

Algorithm analysis is about measuring how much computing resources (like time or memory) an operation or algorithm uses

### 13.1 Axioms

1. Fetching or storing an integer from memory takes a constant time.  $y = x + 1$  takes  $T_{fetch} + T_{store}$
2. Basic operations (add, subtract, multiply, divide, compare) on integers all take constant time.  $y = y + 1$  takes  $2T_{fetch} + T_{op} + T_{store}$
3. Function call and return times are constant ( $T_{call}, T_{return}$ ), passing an integer argument is like fetching it.  $y = f(x)$  takes  $T_{fetch} + T_{call} + T_{f(x)} + T_{store}$
4. Array subscripting address calculation is constant  $T$ , not including time to compute  $i$  or fetch/store the element.  $y = a[i]$  takes  $3T_{fetch} + T_{store} + T$
5. Allocating memory is constant time  $T_{new}$ , not including initialization

### 13.2 Examples

- Finding the largest element in an array  $\max_{0 \leq i < n} a_i$

Statement	Time
1 int FindMaximum(int a [], int n)	
2 {	3 $3T_{fetch} + T_{[]} + T_{store}$
3     int result = a[0];	4a $T_{fetch} + T_{store}$
4     for(int i = 1; i < n; ++i)	4b $(2T_{fetch} + T_{<}) \times n$
5         if(a[i] > result)	4c $(2T_{fetch} + T_{+} + T_{store}) \times (n - 1)$
6             result = a[i];	5 $(4T_{fetch} + T_{[]} + T_{<}) \times (n - 1)$
7     return result;	6 $(3T_{fetch} + T_{[]} + T_{store}) \times ?$
8 }	7 $T_{fetch} + T_{store}$

Line 6 executed only if

$$a_i > (\max_{0 \leq j < i} a_j)$$

? - depends on the actual elements of the array,  $a_0, a_1, \dots, a_{n-1}$   
 $T(n, a_0, a_1, \dots, a_{n-1}) = t_1 + t_2 n + \sum_{i=1}^{n-1} t_3$   
 $a_i > (\max_{0 \leq j < i} a_j)$

$$\begin{aligned} t_1 &= 2T_{store} - T_{fetch} - T_{+} - T_{<} \\ t_2 &= 8T_{fetch} + 2T_{<} + T_{[]} + T_{+} + T_{store} \\ t_3 &= 3T_{fetch} + T_{[]} + T_{store} \end{aligned}$$

$$T_{average}(n) = t_1 + t_2 n + t_3 \sum_{i=1}^{n-1} \frac{1}{i+1}$$

The probability that  $a_i$  is the largest of the  $i + 1$  values, which is  $\frac{1}{i+1}$  from  $p_i = P[a_i > (\max_{0 \leq j < i} a_j)]$

$$T_{worsecase}(n) = (t_1 - t_3) + (t_2 + t_3) \times n \quad T_{bestcase}(n) = t_1 + t_2 n + \sum_{i=1}^{n-1} p_i t_3 \quad \text{when } p_i = 0, \text{ thus } = t_1 + t_2 n$$

Instead of detailing each parameter, we let  $T$  be the *clock cycle* of a machine, where  $T_{fetch} = kT$

Assume that all timing parameters expressed in units of clock cycles. Thus  $T = 1$ .  $k$  is assumed to be the same for all parameters, thus  $k = 1$

### 13.3 Time Functions

The time function  $T(n)$  describes how the running time of an algorithm grows as a function of the input size  $n$

$f(n) = O(g(n))$  means for large enough  $n$ ,  $f(n)$  is at most a constant times  $g(n)$

**Example:** Given  $8n + 128$ , show that  $f(n) = O(n^2)$ , find constants  $n_0 > 0$  and  $c > 0$  such that  $\forall n \geq n_0, f(n) \leq cn^2$

If  $c = 1$ , then  $f(n) \leq cn^2 \Rightarrow 8n + 128 \leq n^2 \Rightarrow 0 \leq (n - 16)(n + 8)$

Since  $(n + 8) > 0$  for all values  $n \geq 0$ , then  $(n_0 - 16) \geq 0, i.e. n_0 = 16$

Possible solution: for  $c = 1, n_0 = 16, f(n) \leq cn^2$  for all integers  $n \geq n_0$ , hence  $f(n) = O(n^2)$

### 13.4 Properties of $O()$

If two functions have the same  $O(g(n))$ , the functions are not necessarily equal, they just share the same upper bound on their growth

When adding functions, we take  $f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$

If we multiply two functions, we get  $O(n^2) + O(n^3) = O(n^5)$

If we have a polynomial time or space functions, we take the term with the highest order without the constant, e.g.  $f(n) = O(n^m)$

Big  $O$ : upper bound (worst-case growth) Big  $\Omega$ : lower bound (best-case growth) Big  $\Theta$ : tight bound (exact growth)

### 13.5 Sum Example

This algorithm sums elements in an array, let  $n = 5$

```
int sum(a,n){
    s=0;
    for(int i = 0; i<n; i++){
        s=s+a[i];
    }
    return s;
}
```

frequency count method:

```
// 1
// for (1, n+1, n), use the greatest count (n+1)
// n
// 1

// time function f(n) = 2n+3
// O(n)
```