3. Tossing 10 coins

1. Assumed prior distribution

Program Flow

(1) define variables
- e.g. coin toss outcome = [1,1,0,0,0,0,0,0,0,0,0,0] where 1=head, 0=tail
- assume distribution of prior is [1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11]

(2) calculate likelihood according to coin toss outcome

(3) calculate marginal probability p(x) according to prior and likelihood

(4) calculate posterior by using Bayes Theorem

(5) Estimate p using mle & map

(6) repeat from (1) but assume distribution of prior is [0.01, 0.01, 0.05, 0.08, 0.15, 0.4, 0.15, 0.08, 0.05, 0.01, 0.01]

Theory

$$\frac{p(\theta)p(x|\theta)}{p(x)} = p(\theta|x)\,, where\ p(x) = \sum_{\theta} p(x,\theta) = \sum_{\theta} p(\theta)p(x|\theta)$$

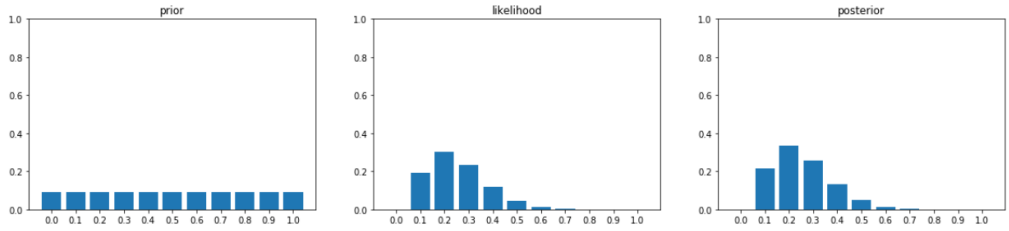
$p(\theta)$: prior ， $p(x|\theta)$: likelihood , $p(\theta|x)$: posterior

Maximum Likelihood Estimation: p with the highest likelihood

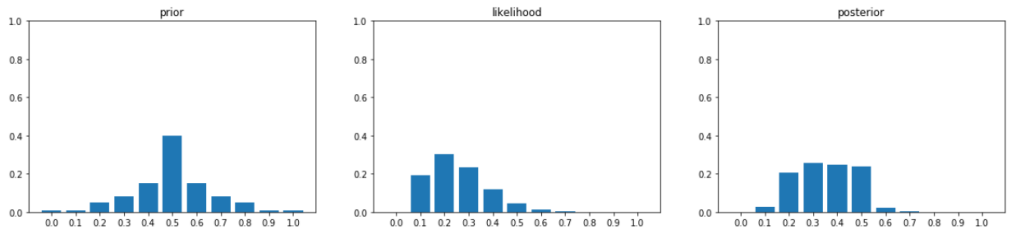
Maximum a Posteriori Estimation: p with the highest posterior

Outcome

```
(a) Assuming the distribution of the prior is [ 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11 ]
Estimated p using mle: 0.2; using map: 0.2
```




```
(b) Assuming the distribution of the prior is [ 0.01, 0.01, 0.05, 0.08, 0.15, 0.4, 0.15, 0.08, 0.05, 0.01, 0.01 ]
Estimated p using mle: 0.2; using map: 0.3
```



2. Toss 10 coins 50 times

Program Flow

(1) define variables
- assume distribution of prior is [1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11]

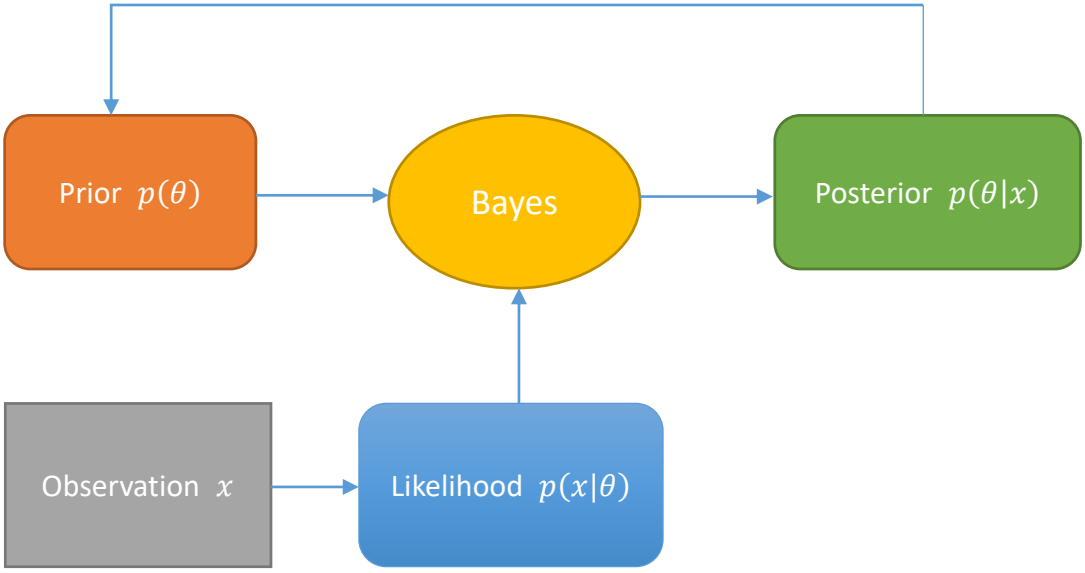
(2) toss 10 coins using coin_sim()
1. pick p (assume distribution of p is equally 1/11, not affected by prior)
2. toss coins using p as probability of tossing a head

(3) calculate posterior using Bayes Theorem as mentioned above

(4) draw graph if iteration is 10 times
- graph is saved to local in png format

(5) repeat from (2) using current posterior as new prior for 50 times

Theory



For every iteration (coin toss), the posterior will become the prior for the next iteration.

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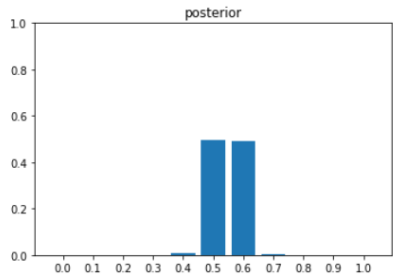
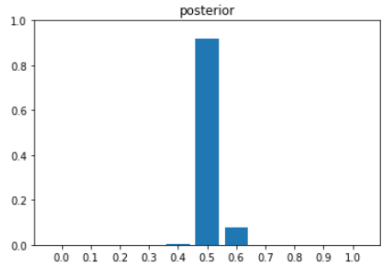
Outcome

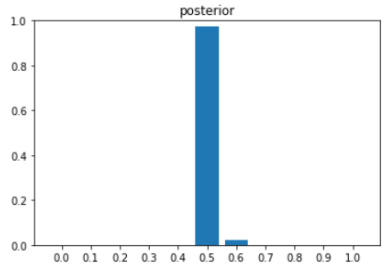
```
Assuming the initial distribution of the prior is [ 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11 ]

~~~~~~~~~~~~~~~~~  10 iteration  ~~~~~~~~~~~~~~~~~
```



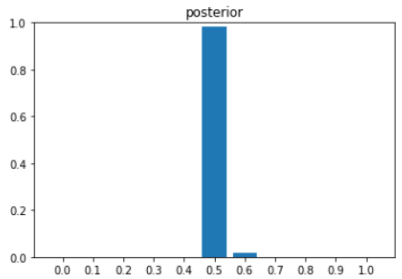

```
Figure saved to '10 toss.png'.
~~~~~~~~~~~~~~~~~  20 iteration  ~~~~~~~~~~~~~~~~~
```



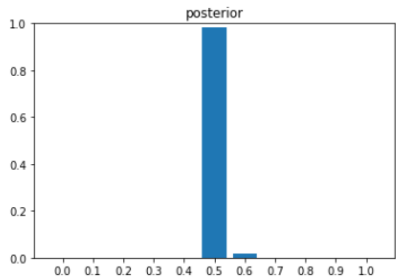

```
Figure saved to '20 toss.png'.
~~~~~~~~~~~~~~~~~  30 iteration  ~~~~~~~~~~~~~~~~~
```




```
Figure saved to '30 toss.png'.

~~~~~~~~~~~~~~~~~  40 iteration  ~~~~~~~~~~~~~~~~~
```




```
Figure saved to '40 toss.png'.
~~~~~~~~~~~~~~~~~  50 iteration  ~~~~~~~~~~~~~~~~~
```




```
Figure saved to '50 toss.png'.
```