Definitions. *Repliers* of a node are the addressees of its tweets. *Mentioners* of a node are those who tweet to it. If a tweet from *@Alice* mentions *@Bob*, then *@Bob* is a replier of *@Alice* and *@Alice* is a mentioner of *@Bob*. Replier of a node is someone that that node repliers *to*. In other words, from a node's perspective, repliers are out-degree, mentioners are in-degree.

symbol	definition
S_v^t	Social Capital of node v at time t. Superscript t generally denotes "by time t." Specifically *during* time step t is denoted as $@t$.
$G^t(V,E)$	graph G with nodes V and edges E
$w_{uv}^{@t}$	total weight of directed edges $u \to v$, i.e. the number of tweets from u to v during time step $@t$
W_{uv}	total number of undirected edges between u and v: $W_{uv} = w_{uv} + w_{vu}$
B_{uv}	Balance of back and forth tweets from u to v: $B_{uv} = w_{uv} - w_{vu}$
M_u	$\{v \vert w_{vu} > 0\}$, i.e. the mentioners of u
$R_u^{@t}$	$\{v \vert w_{uv}^{@t} > 0\}$, i.e. repliers of u specifically during the timestep $@t$
$O_{uv}^{@t}$	outgoing activity of a node rewarded by social capital at timestep t
$A_{uv}^{@t}$	incoming activity in this cycle rewarded just for mentions (all)
$B_{uv}^{@t}$	incoming mentions in this cycle repaying previous replies (balance)
α, β, γ	model parameters

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\begin{aligned}
O_u^{@t} &= \frac{1}{\sum_{V^{t-1}} O^{@t-1}} \sum_{v \in M_u^{t-1} \cap R_u^{@t-1} \vert B_{uv}^{t-1} < 0} \vert B_{uv}^{t-1} \vert w_{uv}^{@t-1} W_{uv}^{t-1} S_v^{t-1} \\
B_u^{@t} &= \frac{1}{\sum_{V^{t-1}} B^{@t}} \sum_{v \in M_u^{@t-1} \vert B_{uv}^{t-1} > 0} B_{uv}^{t-1} w_{vu}^{@t-1} W_{uv}^{t-1} S_v^{t-1} \\
A_u^{@t} &= \frac{1}{\sum_{V^{t-1}} B^{@t}} \sum_{v \in M_u^{@t-1}} w_{vu}^{@t-1} W_{uv}^{t-1} S_v^{t-1} \\
I_u^{@t} &= \gamma B_u^{t-1} + (1-\gamma) A_u^{t-1} \\
S_u^t &= \alpha S_u^{t-1} + (1-\alpha)(\beta O_u^{t-1} + (1-\beta)(\gamma B_u^{t-1} + (1-\gamma) A_u^{t-1})
\end{aligned}
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Some notes on the definitions. O_u^t is the node u's output gaining social capital, thus we want to reward those who redress an imbalance of input and answer those who addressed you more than you had answered them. The summation is defined exactly over those with whom you have a deficit in replying: $v \in M_u^{t-1} \cap R_u^{@t-1} \vert B_{uv}^{t-1} < 0$. It means the target node mentioned you at some point prior, you replied to it in this cycle, and before that, you owed it a reply since it tweeted more to you than you did to it. For each such deficit node you replied to, finally, we multiply the number of replies in that cycle, $w_{uv}^{@t-1}$, by the balance you owed, $\vert B_{uv}^{t-1} \vert$, the value of the relationship, W_{uv}^{t-1}, and the importance of the replier S_v^{t-1}. We normalize the O's so that they all sum to 1, and reward each node proportionally to the value of the redress in the reply imbalance it actively contributed in this cycle.

Similarly, I_u^t is the node u's input worth of social capital. We generally consider all input as good – we can't distinguish bad publicity, or consider it all good anyways – but we distinguish mentions redressing the mentioners' own deficit with us as worthing more than just any mention. Those repaying mentions we reward with a multiplier for the balance owed additionally to the usual cycle contribution, relationship value, and the mentioner's social capital.

Note that in the output, we don't have a general activity term for all replies, even those not redressing an imbalance, as we do in the second term of the input. Thus we don't reward random replying, and you won't get social capital by just addressing everybody in volume.

It's easy to see that such a definition of social capital allows for an iterative economy by launching the update rule defining S_u^t in terms of S_u^{t-1}as shown in the last formula above.