McCombs Math 381 Exam Practice Problems

1. Prove that the conditional statement $p \rightarrow q$ and its inverse are not logically equivalent.

2. Use the Euclidean Algorithm to find integers x and y so that $156x + 65y = 104$.

3. Find the number of English words of length eight with no repetitions so that the words do not have both A and B in them.

4. A market sells 40 kinds of candy bars. You want to buy 15 candy bars.
   (i) How many possibilities are there?
   (ii) How many possibilities are there if you want at least three peanut butter bars and exactly five almond bars?

5. Suppose nine people, A, B, C, D, E, F, G, H, and J are in a room. Five of them line up for a picture.
   (i) In how many ways can this be done if B is to be in the picture?
   (ii) In how many ways can this be done if E and G must be in the picture, standing next to each other? This question is not related to part (i).

6. Find the number of subsets of S = $\{1,2,3,\ldots,10\}$ that contain exactly five elements, including 3 or 4 but not both.

7. Determine the inverse of 2641 in $\mathbb{Z}_{3793}$, if it exists.

8. Prove or disprove: If p and q are primes, with $p > 2$ and $q > 2$, then $pq + 1$ is never prime.

9. Solve the linear congruence, $30k + 14 \equiv 5 \pmod{11}$.

10. Let $A = \{a,b,c,d,e\}$ with the partition P given by $P_1 = \{a,d\}, P_2 = \{b,c\}, P_3 = \{e\}$.
    (i) Use a digraph to represent the equivalence relation R induced by partition P on set A.

    (ii) Use a matrix to represent the equivalence relation R induced by partition P on set A.

11. You have 50 pennies and three jars, labeled A, B, and C. In how many ways can you put the pennies in the jars, assuming that the pennies are identical and each jar must have at least two pennies put into it?

12. Given $S = \{\text{anagrams of } ABCD\}$. Let equivalence relation R on S be defined as follows.
    $\forall x, y \in S$, xRy means that x and y begin with the same letter.
    (i) Find the total number of distinct equivalence classes of R on S.
    (ii) Find the size of each equivalence class from part (i).

13. Find the number of subsets of $S = \{1,2,3,\ldots,10\}$ that contain exactly five elements, the sum of which is even.

14. Find the smallest positive integer solution for the system.

$$x \equiv 14 \pmod{30}$$
$$x \equiv 5 \pmod{11}$$

Math 381 Final Exam Practice Problems Key

15. Let $A=\{w,x,y,z\}$. Draw the digraph for relation $R:A\to A$ given by

$$R=\{(w,w),(w,x),(x,w),(x,x),(x,z),(y,y),(z,y),(z,z)\}$$

16. Let R be the relation on $A=\{1,2,3,4,5\}$ given by

$$R=\{(1,1),(1,3),(1,4),(2,2),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4),(5,5)\}$$

17. Prove or disprove: If $a\equiv b \pmod{2m}$, then $a\equiv b \pmod{m}$.

18. How many bit strings of length eight have more 0s than 1s?

19. Use Mathematical Induction to prove that 4 divides (9^n-5^n) for all $n\geq 0$.

20. Use linear congruences to prove the given statement.
The equation $x^2-5y^2=2$ has no integer solutions.

21. Determine whether the given statement is true or false.

$$p\to(\neg q\wedge r)\equiv\neg p\vee(r\to q)$$

22. Suppose p is prime and $a,b\in\mathbb{Z}$. Prove the given statement.
If p divides $a\cdot b$, then p divides a or p divides b.

23. Prove the given statement.
If x is rational and y is irrational, then $x+y$ is irrational.

24. Let $a,b\in\mathbb{Z}$. Prove the given statement.
If $a\cdot b$ is even and $a+b$ is even, then a and b are even.

25. Prove the given statement.
If k is and odd integer, then $3^k+2\equiv 1 \pmod{4}$.

26. Given the predicates

$P(x)$: "student x is a math major "

$Q(y,x)$: "class y contains student x"

Use quantifiers to express each statement.
(i) Every class contains a math major.
(ii) One particular class contains every math major.