2. This is Geometric distribution. So $\mu = \frac{1}{p}$ where p is probability of getting six on each toss. So $\mu = 6$.

In the normal way, $P(x=n) = (\frac{5}{6})^{n-1} \cdot \frac{1}{6}$ $E(x) = \sum_{n=1}^{\infty} n \cdot (\frac{5}{6})^{n-1} \cdot \frac{1}{6} = \underline{6}$

4. a. Let X_i represent the event ~~one~~ exactly one person gets off on ith floor. $P(x_i) = \underbrace{n}_{\text{1 out of n person}} \cdot \underbrace{(\frac{1}{10})}_{\text{ith floor is choosen by that person}} \cdot \underbrace{(\frac{9}{10})^{n-1}}_{\text{other 9 floors chosen by 9 person}}$

b. $E(X) = \sum_{i=1}^{10} E(X_i) = n \cdot (\frac{9}{10})^{n-1}$

6. Let $X_i = \begin{cases} 1 & i\text{th student gets his/her own bed.} \\ 0 & \text{otherwise.} \end{cases}$

$P(X_i = 1) = \frac{1}{n}$ $E(X_i) = 1 \cdot \frac{1}{n} + 0 \cdot (\frac{n-1}{n}) = \frac{1}{n}$

$E(x) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n} \cdot \frac{1}{n} = 1$

8. (a) $E(Z) = \sum_{i=1}^{6} i \cdot P(i) = (1+2+3+4) \cdot \frac{1}{8} + (5+6) \cdot \frac{1}{4} = 4$

~~(b)~~ $var(Z) = E(Z^2) - [E(Z)]^2 = (1+4+9+16) \cdot \frac{1}{8} + (25+36) \cdot \frac{1}{4} - 16 = 3$

(b). $E(X) = \sum_{i=1}^{10} E(Z_i) = 10 \times 4 = 40$

$var(X) = var(\sum_{i=1}^{10} Z_i) = \sum_{i=1}^{10} Var(Z_i) = 30$

because Z_i are independent

(c). $A = \frac{1}{n} \sum_{i=1}^{n} Z_i$

$E(A) = E(\frac{1}{n} \sum_{i=1}^{n} Z_i) = \frac{1}{n} \sum_{i=1}^{n} E(Z_i) = 4$

$Var(A) = var(\frac{1}{n} \sum_{i=1}^{n} Z_i) = \frac{1}{n^2} Var(\sum_{i=1}^{n} Z_i) = \frac{1}{n^2} \sum_{i=1}^{n} Var(Z_i) = \frac{3}{n}$

10. (a). $\Omega = n^m$ $|X_i \overset{=0}{}| = (n-1)^m$ $P(X_i=0) = \left(\frac{n-1}{n}\right)^m$

(b). $\Omega = n^m$ $|X_i=1| = C_m^1 \cdot (n-1)^{m-1}$ $P(X_i=1) = C_m^1 \cdot \frac{(n-1)^{m-1}}{n^m}$

$= C_m^1 \frac{1}{n}\left(1-\frac{1}{n}\right)^{m-1}$

(c). Let $Y_j = \begin{cases} 1 & \text{jth ball fall in bin } i \\ 0 & \text{jth ball not fall in bin } i \end{cases}$

$X_i = Y_1 + Y_2 + \cdots Y_m$

$E(X_i) = E\left(\sum_{j=1}^{m} Y_j\right) = m \cdot E(Y_j) = \frac{m}{n}$

(d) $var(X_i) = var\left(\sum_{j=1}^{m} Y_j\right) = m \cdot var(Y_j) = m \cdot \left(E(Y_j^2) - [E(Y_j)]^2\right)$

$= m \cdot \left(\frac{1}{n} - \frac{1}{n^2}\right)$

12. ~~Let $Y_i = \{1$~~

$P(X=1)=0$ $P(X=n) = \frac{2}{2^n}$ $n>1$

$E(X) = 1\times 0 + \sum_{2}^{\infty} n \cdot \frac{2}{2^n} = \sum_{2}^{\infty} n \cdot \frac{2}{2^n}$

$\underline{E(X)+1} = \sum_{1}^{\infty} n \cdot \frac{2}{2^n} = \sum_{1}^{\infty}\left((n+1) \cdot \frac{2}{2^{n+1}} \cdot 2 - \frac{2}{2^n}\right) = 2\sum_{2}^{\infty} n \cdot \frac{2}{2^n} - \sum_{0}^{\infty} \frac{1}{2^n}$

$\underline{= 2 \cdot E(X) - 2}$

$E(X) = 3$