HOMEWORK 9

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Given: n chemical substances $(1, \ldots, n)$,
2 bottles B1, B2 used for storage,
2 disjoint sets L1, L2, signifying which chemicals (if any) must be stored in either bottle,
and a list of energies $e_{ij} = e_{ji}$ for $i, j \in [n]$

GOAL: maximize the energy produced from combining the given substances in B1, B2;

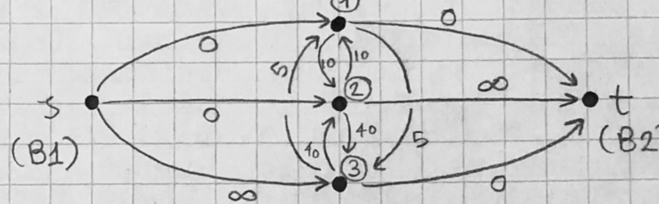
maximize $\sum_{K=1}^{2} \sum_{i,j \in B_K} e_{ij}$

MODEL
- Vertex for each chemical substance
- SOURCE side represents bottle 1
- SINK side represents bottle 2
- Forward and backward edges between every pair of chemical substances. Each edge going between vertices i and j has capacity e_{ij}. Similarly, an edge going from j to i has capacity of $e_{ji} = e_{ij}$.
- The SOURCE has outgoing edges to ALL chemical substances. If the given substance is included in L_1, the edge capacity is ∞. However, if not included in L_1, the edge capacity is 0.
- The SINK has incoming edges from all chemical substances. If a given chemical substance is included on list L_2, the edge capacity is ∞, otherwise – 0.

We are approaching this problem as a network flow problem, and are looking to find the minimum cut. This will allow us to maximize the total energy, because we are finding an optimal partition of the chemical substances (into 2 bottles). To reiterate, we are creating a division which minimizes the potential energy produced by the SEPARATED elements.

To illustrate our approach, the model below represents our Solution to the example provided in the prompt.

$n=3$
$L_1 = \{3\}$
$L_2 = \{2\}$
$e_{12} = 10$
$e_{13} = 5$
$e_{23} = 40$



Our approach determines the min. cut using the network flow algorithm discussed in class.

The following are 4 possible partitions and their corresponding cut-capacities:

① The partition includes vertices s, ③, and ① on the S side, while ② and t are on the T side: The cut capacity of the given S-T cut is $10 + 40 = 50$.

② This partition includes vertices s, ③ on the S side, and ①, ②, t on the T side. The cut capacity is $40 + 5 = 45$.

③ This partition includes no vertices (besides s) on the S side, and all remaining vertices are on the T side. This results in a cut capacity of $0 + 0 + \infty = \infty$. We have a result of ∞, because of the given prerequisite for substance ③ to be in B1.

④ This partition include all vertices on the S side, and results in a cut capacity of ∞, for the same reasoning as partition #3.

It is important to note that these are NOT all possible partitions, but they cover the important cases. Finally, since partition 2 has the smalles cut capacity, we conclude that it maximizes energy. This process results in the following division:

$B1 = S \setminus \{s\}$
$B2 = T \setminus \{t\}$

TIME COMPLEXITY:

The time in which the algorithm draws the vertices is $O(n)$, where n is the number of given chemical substances. The time to create edges between all the vertices is $O(n^2)$, because we consider every possible pair of vertices. Thus, the total time to construct the model is $O(n+n^2) = O(n^2)$. As was mentioned during lecture, Fulkerson's path augmentation algorithm finds the max flow and min cut in a given network. Using this approach, our method results in $O(f(n^2))$ runtime, which satisfies the requirement for polynomial time complexity.