Statistical Analysis of the Black-Scholes Model vs. Real Call Option Prices for Target Corporation

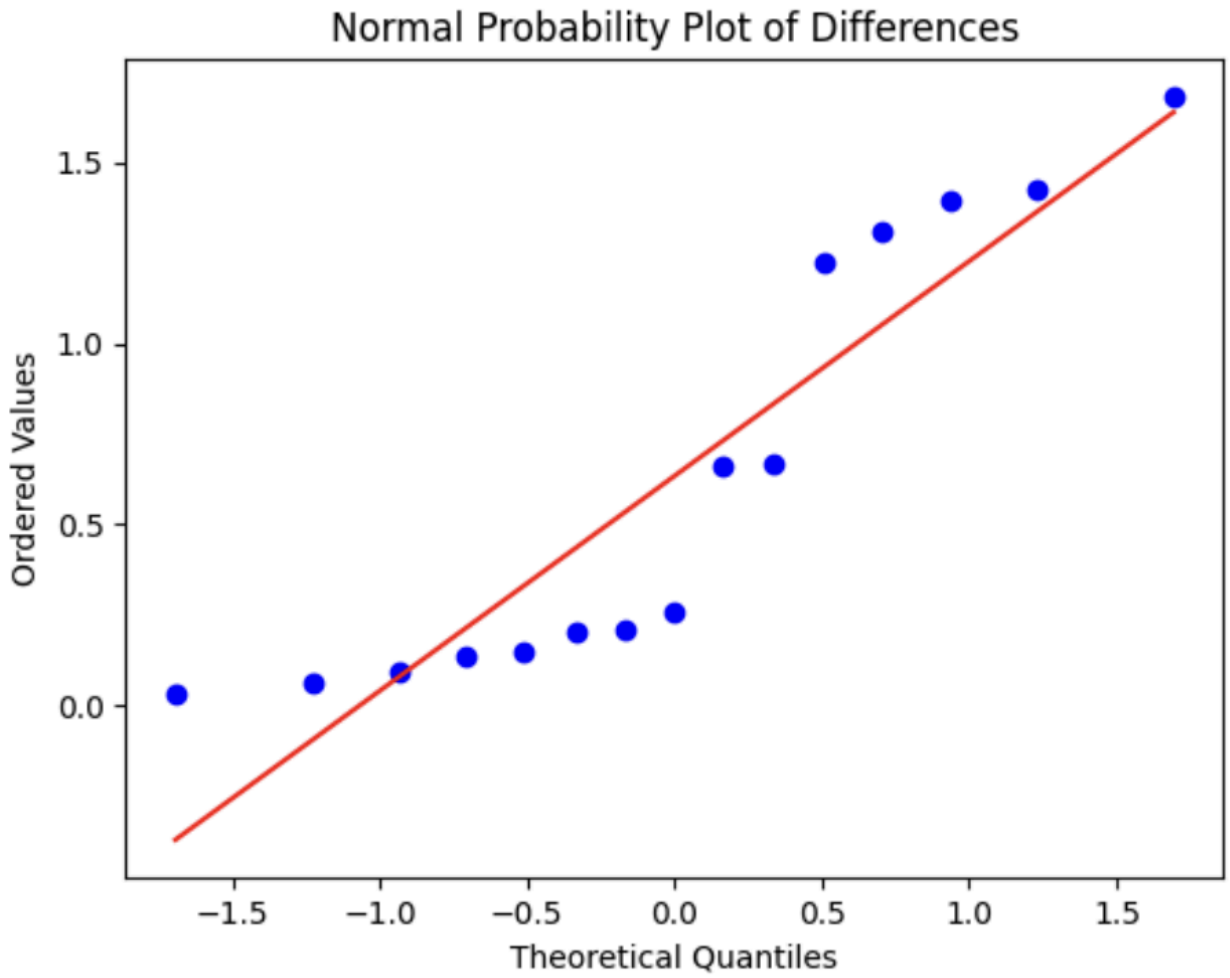
Introduction

The purpose of this project is to perform a comparative statistical analysis of the Black-Scholes model against real market call option prices for Target Corporation. The objective is to determine if the Black-Scholes model's pricing deviates significantly from actual market values, focusing on understanding the equality of means and variances between the two datasets.

Data Description

The data consists of 15 observations each for the Black-Scholes prices and real market prices of Target Corporation call options. The real data was collected on 12/02/24 from Robinhood, and the Black- Scholes prices were computed using a Python script. The Black-Scholes model inputs include the spot price ($130.49), risk-free rate (0.0441), time to expiration (32/365), and implied volatilities for different strike prices (K). The results are detailed below.

K (Strike Price)	Sigma (Implied Volatility)	Real Price ($)	Black-Scholes Price ($)	Difference ($)
100	0.4321	32.65	30.97	1.68
105	0.4056	27.50	26.08	1.42
110	0.3897	22.70	21.31	1.39
115	0.3213	17.70	16.39	1.31
120	0.2741	12.90	11.68	1.22
125	0.2550	7.80	7.54	0.26
130	0.2495	4.55	4.35	0.20
135	0.2503	2.45	2.24	0.21
140	0.2577	1.18	1.09	0.09
145	0.2698	0.59	0.53	0.06
150	0.2914	0.33	0.30	0.03
155	0.3334	0.40	0.25	0.15
160	0.3619	0.32	0.19	0.13
165	0.5258	1.36	0.69	0.67
170	0.5726	1.34	0.68	0.66



The normal probability plot demonstrates that the data points deviate notably from the theoretical quantiles, indicating that the differences between the Black-Scholes model and the real market prices are not normally distributed.

Sample Means and Variances

The mean and standard deviation for both the Black-Scholes and real market prices were calculated as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \, , s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)$$

Black-Scholes Mean: 8.28
Black-Scholes Standard Deviation: 10.17
Real Market Mean: 8.92
Real Market Standard Deviation: 11.08

Hypothesis Testing for Population Variances

To determine if the variances of the Black-Scholes and real market prices were equal, we conducted an F-test. The null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$ states that the variances are equal, while the alternative hypothesis $H_1: \sigma_1^2 \neq \sigma_2^2$ states that they are not equal. The F-statistic: $F = \frac{s_1^2}{s_2^2}$ was calculated as 0.90 with a corresponding p-value of 0.85. Since the p-value is greater than the significance level $\alpha < 0.05$, we fail to reject the null hypothesis and conclude that the variances are not significantly different.

Hypothesis Testing for Population Means

Given that the variances are assumed equal, we proceeded with a two-sample t-test for the equality of means. The null hypothesis $H_0: \mu_1 = \mu_2$ states that the means are equal, while the alternative hypothesis $H_1: \mu_1 \neq \mu_2$ states that they are different. The t-statistic: $t = \frac{\overline{X_1} - \overline{X_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ equals -0.16, and the p-value was 0.87. Since the p-value is greater than the significance level $\alpha < 0.05$, we fail to reject the null hypothesis and conclude that there is no significant difference between the means of the Black-Scholes prices and the real market prices.

Conclusion

In conclusion, the results of our analysis suggest that there is no statistically significant difference between the Black-Scholes model's pricing and the real market call option prices for Target Corporation, both in terms of variances and means. This implies that, within the scope of our sample, the Black-Scholes model provides an acceptable estimate of market prices. All calculations and findings were completed using a Python script shared below.

References

"TGT Options." *Robinhood*, robinhood.com/options/chains/TGT. Accessed 2 Dec. 2024.