



# Ray-Tracing Soft Shadows Global Illumination

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## Course News

### Homework 8

- Ray-tracing, global illumination
- Discussed today, tomorrow in labs

### Assignment 3 (project)

- Due Friday!!
- Demos in labs starting this Friday
- Demos are MANDATORY(!)

### Reading

- Chapter 10 (ray tracing), except 10.8-10.10
- Chapter 14 (global illumination)

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## Ray-Tracing

### Basic Algorithm (Whithead):

for every pixel  $p_i$  {

    Generate ray  $r$  from camera position through pixel  $p_i$

$p_i = \text{background color}$

    for every object  $o$  in scene {

        if(  $r$  intersects  $o$  && intersection is closer than previously found intersections )

            Compute lighting at intersection point, using local normal and material properties; store result in  $p_i$

    }

}

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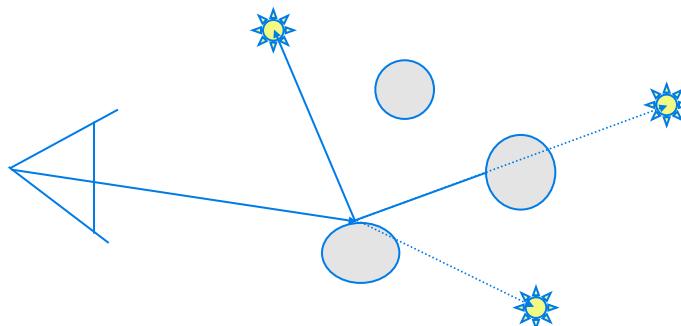
## Ray-Tracing Shadows



### Approach:

- To test whether point is in shadow, send out shadow rays to all light sources

— If ray hits another object, the point lies in shadow



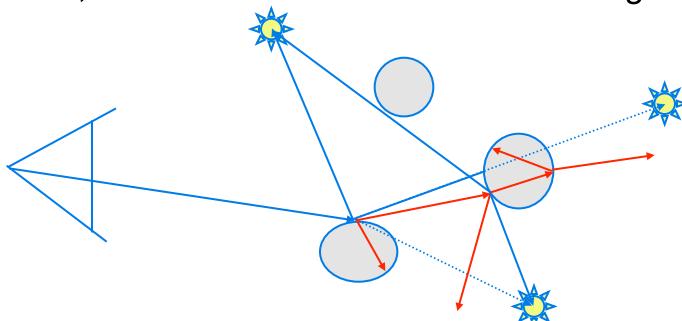
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## Ray-Tracing Reflections/Refractions

### Approach:

- Send rays out in reflected and refracted direction to gather incoming light
- That light is multiplied by local surface color and Fresnel term, and added to result of local shading



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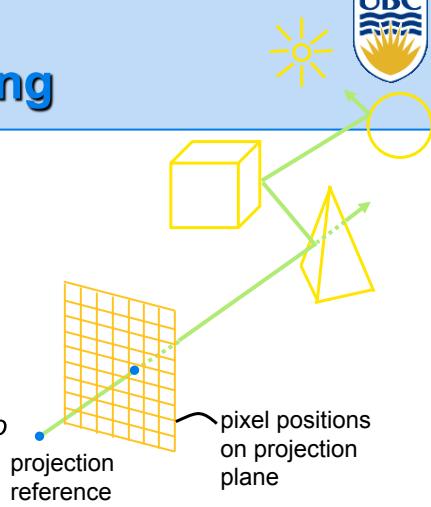
## Recursive Ray Tracing

### Ray tracing can handle

- Reflection (chrome)
- Refraction (glass)
- Shadows

### Spawn secondary rays

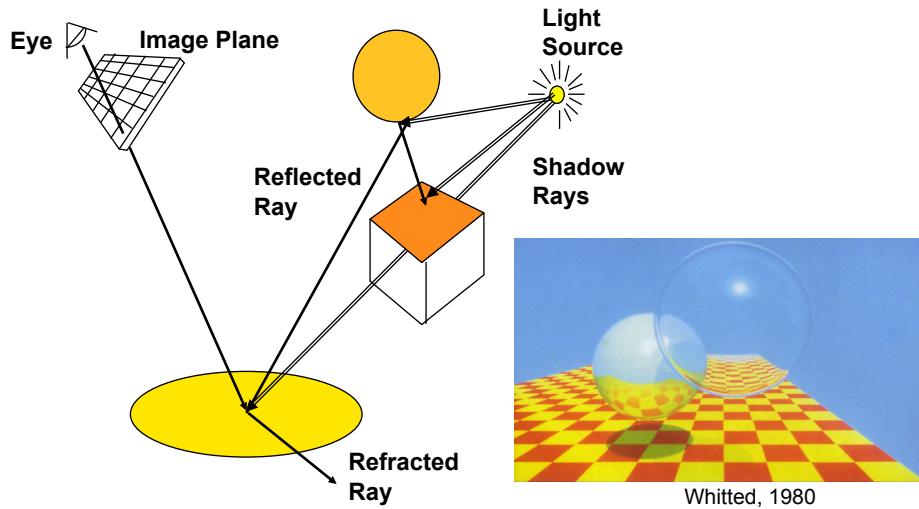
- Reflection, refraction
  - If another object is hit, recurse to find its color
- Shadow
  - Cast ray from intersection point to light source, check if intersects another object



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## Recursive Ray-Tracing



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## Recursive Ray-Tracing Algorithm

```
RayTrace(r,scene)
obj := FirstIntersection(r,scene)
if (no obj) return BackgroundColor;
else begin
    if ( Reflect(obj) ) then
        reflect_color := RayTrace(ReflectRay(r,obj));
    else
        reflect_color := Black;
    if ( Transparent(obj) ) then
        refract_color := RayTrace(RefractRay(r,obj));
    else
        refract_color := Black;
    return Shade(reflect_color,refract_color,obj);
end;
```

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## Algorithm Termination Criteria

### Termination criteria

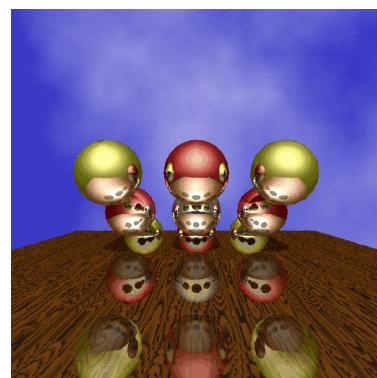
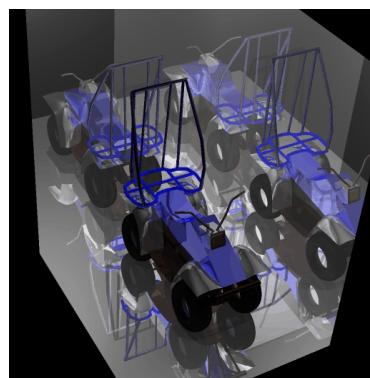
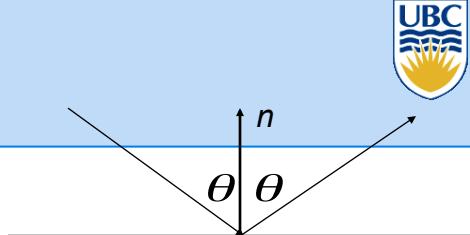
- No intersection
- Reach maximal depth
  - Number of bounces
- Contribution of secondary ray attenuated below threshold
  - Each reflection/refraction attenuates ray

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## Reflection

### Mirror effects

- Perfect specular reflection



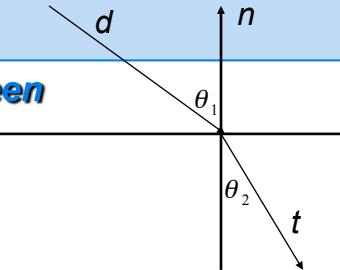
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## Refraction

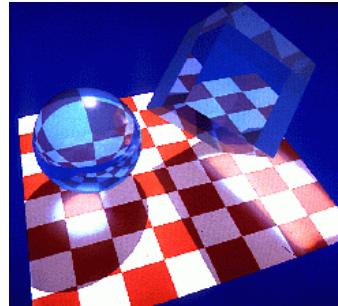
**Happens at interface between transparent object and surrounding medium**

- E.g. glass/air boundary



### Snell's Law

- $c_1 \sin \theta_1 = c_2 \sin \theta_2$
- Light ray bends based on refractive indices  $c_1, c_2$



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## Area Light Sources

### So far:

- All lights were either point-shaped or directional
  - Both for ray-tracing and the rendering pipeline
- Thus, at every point, we only need to compute lighting formula and shadowing for **ONE** light direction

### In reality:

- All lights have a finite area
- Instead of just dealing with one direction, we now have to **integrate** over all directions that go to the light source

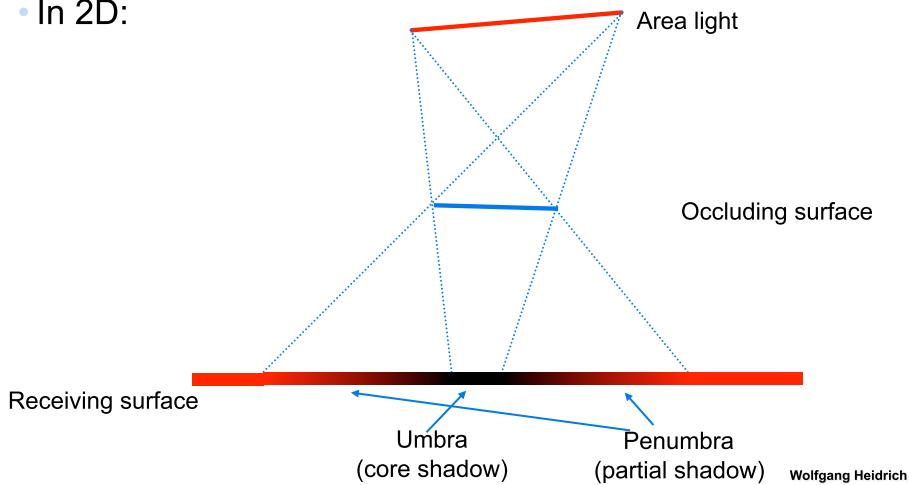
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## Area Light Sources

**Area lights produce soft shadows:**

- In 2D:



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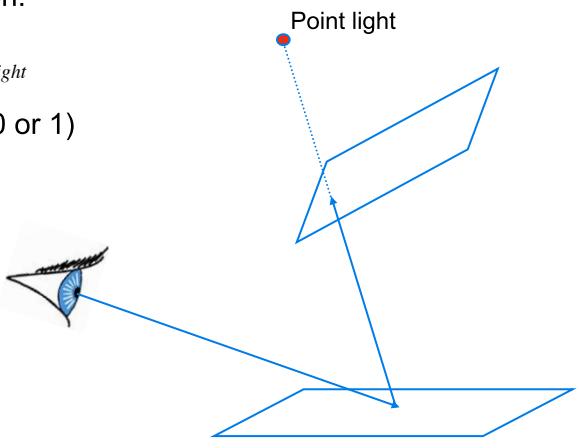
## Area Light Sources

**Point lights:**

- Only one light direction:

$$I_{reflected} = \rho \cdot V \cdot I_{light}$$

- $V$  is visibility of light (0 or 1)
- $\rho$  is lighting model (e.g. diffuse or Phong)



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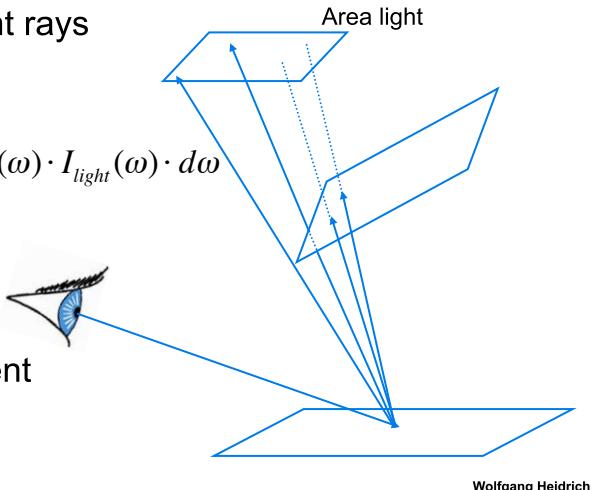
## Are Light Sources

### Area Lights:

- Infinitely many light rays
- Need to integrate over all of them:

$$I_{reflected} = \int_{\text{light directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

- Lighting model visibility and light intensity can now be different for every ray!



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## Integrating over Light Source

### Rewrite the integration

- Instead of integrating over directions

$$I_{reflected} = \int_{\text{light directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

we can integrate over points on the light source

$$I_{reflected}(q) = \int_{s,t} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{light}(p) \cdot ds \cdot dt$$

where q: point on reflecting surface, p= F(s,t) is a point on the area light

- We are integrating over p
- Denominator: quadratic falloff!

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## Integration

### Problem:

- Except for the simplest of scenes, either integral is **not solvable analytically!**
- This is mostly due to the visibility term, which could be arbitrarily complex depending on the scene

### So:

- Use numerical integration
- Effectively: approximate the light with a whole number of point lights

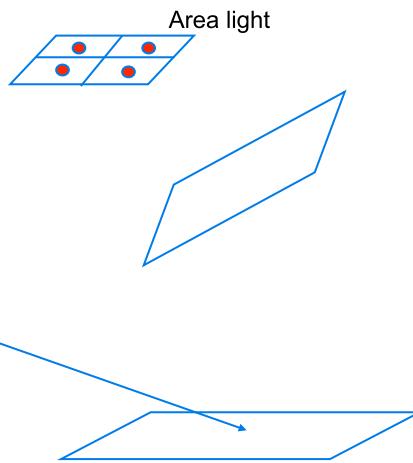
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## Numerical Integration

### Regular grid of point lights

- Problem:  
will see 4 hard  
shadows rather than  
as soft shadow
- Need LOTS of points  
to avoid this problem



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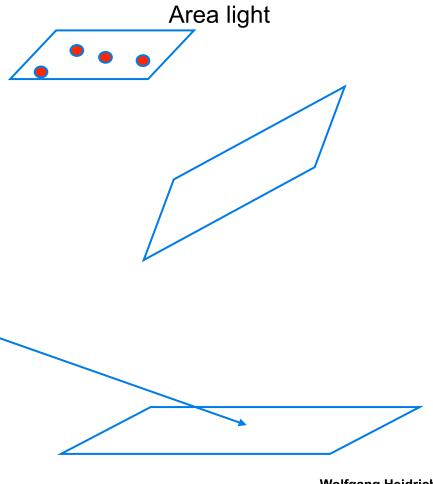


## Monte Carlo Integration

### Better:

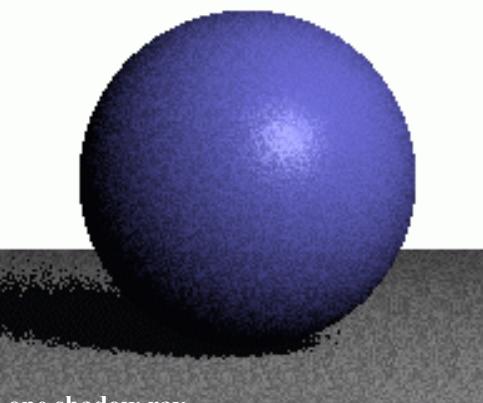
- **Randomly** choose the points
- Use different points on light for computing the lighting in different points on reflecting surface

- This produces random noise
- Visually preferable to structured artifacts

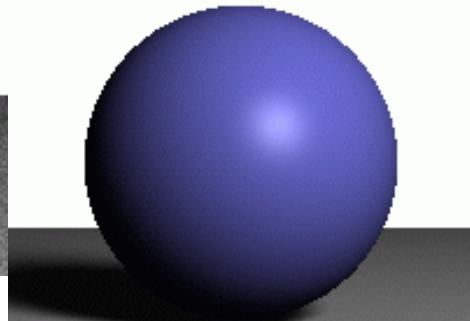


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## Monte Carlo Integration



one shadow ray



lots of shadow rays



## Monte Carlo Integration

### Formally:

- Approximate integral with finite sum

$$\begin{aligned} I_{reflected}(q) &= \int_{s,t} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{light}(p) \cdot ds \cdot dt \\ &\approx \frac{A}{N} \sum_{i=1}^N \frac{\rho(p_i-q) \cdot V(p_i-q)}{|p_i-q|^2} \cdot I_{light}(p_i) \end{aligned}$$

where

- *The  $p_i$  are randomly chosen on the light source*
  - With equal probability!
- *A is the total area of the light*
- *N is the number of samples (rays)*

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## Sampling

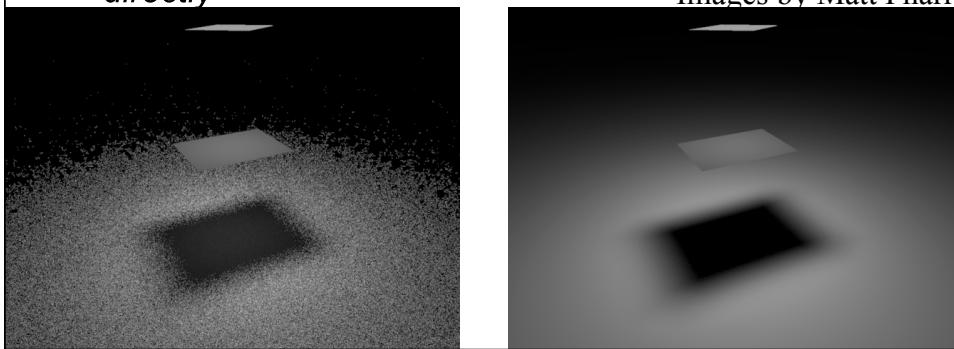


### Sample directions vs. sample light source

- Most directions do not correspond to points on the light source

- *Thus, variance will be higher than sampling light directly*

Images by Matt Pharr





## Monte Carlo Integration

### Note:

- This approach of approximating lighting integrals with sums over randomly chosen points is much more flexible than this!
- In particular, it can be used for global illumination
  - *Light bouncing off multiple surfaces before hitting the eye*

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## Global Illumination

### So far:

- Have considered only light directly coming from the light sources
  - *As well as mirror reflections, refraction*

### In reality:

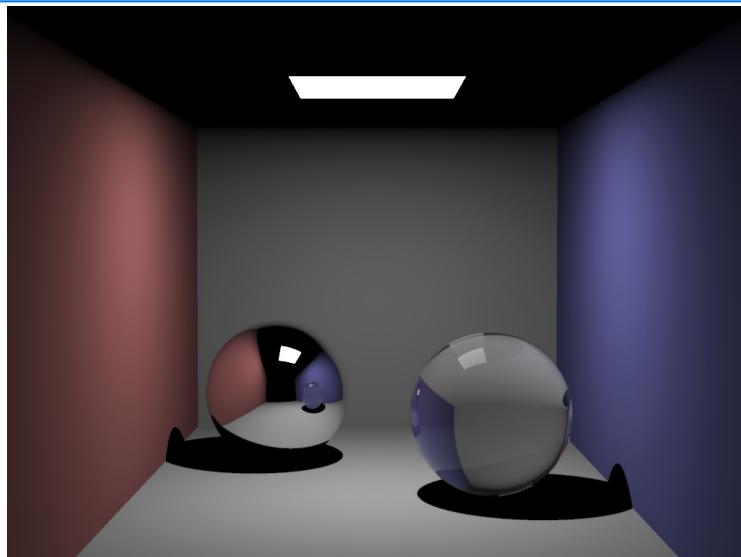
- Light bouncing off diffuse and/or glossy surfaces also illuminates other surfaces
  - *This is called global illumination*

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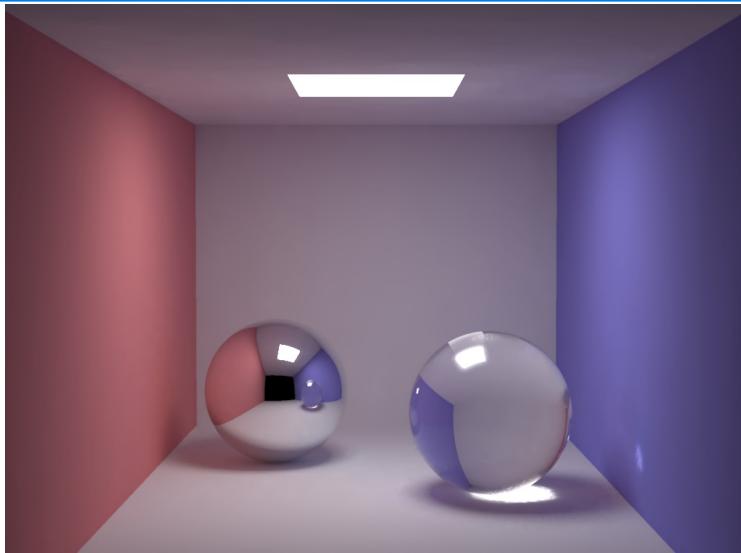
## Direct Illumination

Image by  
Henrik Wann Jensen



## Global Illumination

Image by  
Henrik Wann Jensen





## Rendering Equation

### Equation guiding global illumination:

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_0) L_i(\omega_i) d\omega_i$$

$$= L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_0) L_o(R(x, \omega_i), -\omega_i) d\omega_i$$

#### Where

- $\rho$  is the reflectance from  $\omega_i$  to  $\omega_o$  at point x
- $L_o$  is the outgoing (i.e. reflected) **radiance** at point x in direction  $\omega_i$ 
  - Radiance is a specific physical quantity describing the amount of light along a ray
  - Radiance is constant along a ray
- $L_e$  is the emitted radiance (=0 unless point x is on a light source)
- R is the “ray-tracing function”. It describes what point is visible from x in direction  $\omega_i$

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## Rendering Equation

### Equation guiding global illumination:

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_0) L_i(\omega_i) d\omega_i$$

$$= L_e(x, \omega_o) + \int_{\Omega} \rho(x, \omega_i, \omega_0) L_o(R(x, \omega_i), -\omega_i) d\omega_i$$

#### Note:

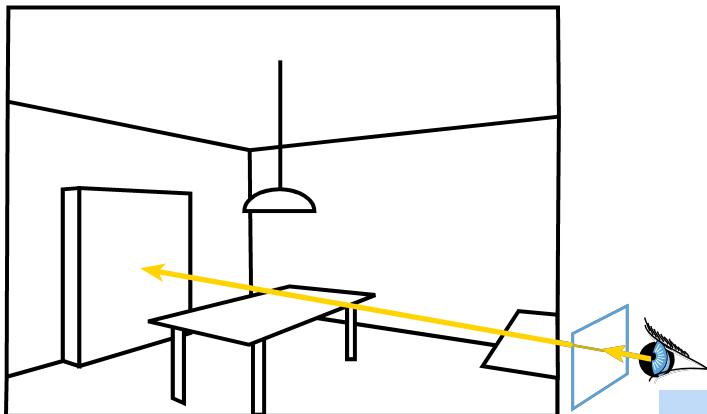
- The rendering equation is an **integral equation**
- This equation cannot be solved directly
  - Ray-tracing function is complicated!
  - Similar to the problem we had computing illumination from area light sources!

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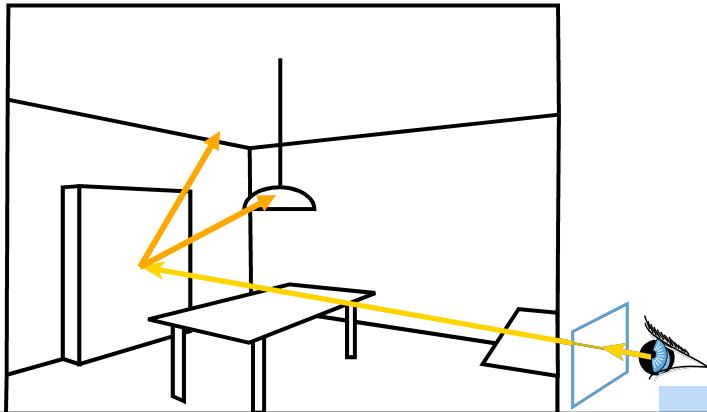
## Ray Casting

- Cast a ray from the eye through each pixel
- The following few slides are from Fred Durand (MIT)



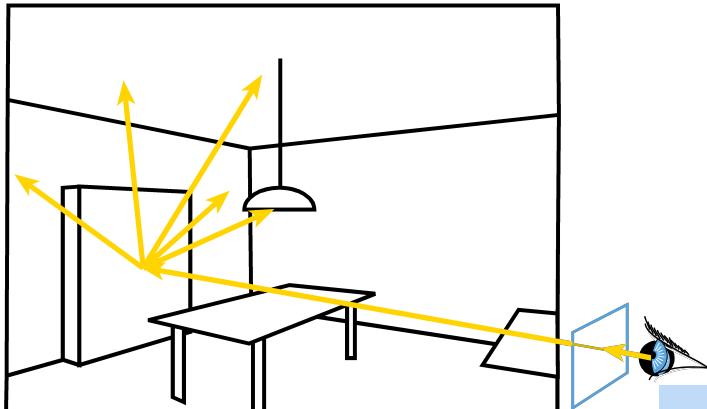
## Ray Tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)



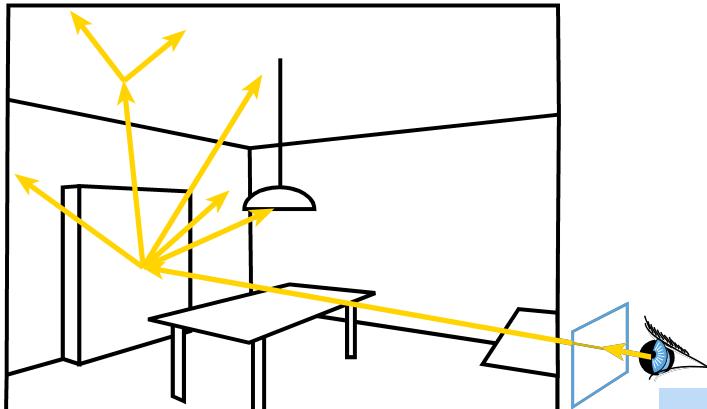
## Monte Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
  - Accumulate radiance contribution



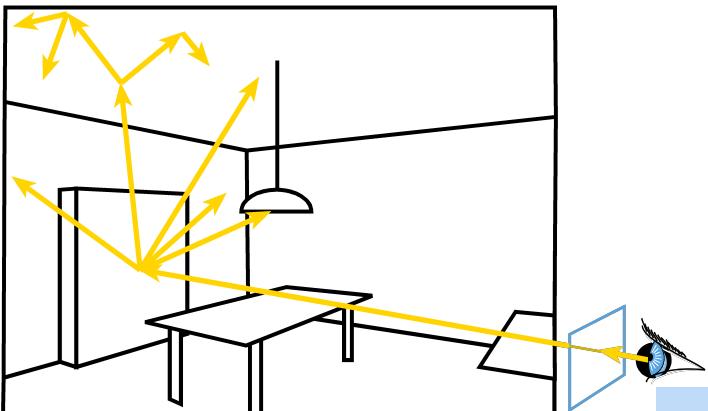
## Monte Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



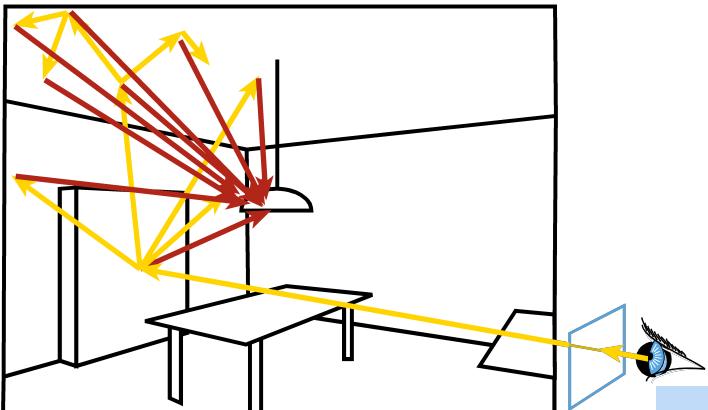
## Monte Carlo

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



## Monte Carlo

- Systematically sample primary light





## Monte Carlo Path Tracing

### In practice:

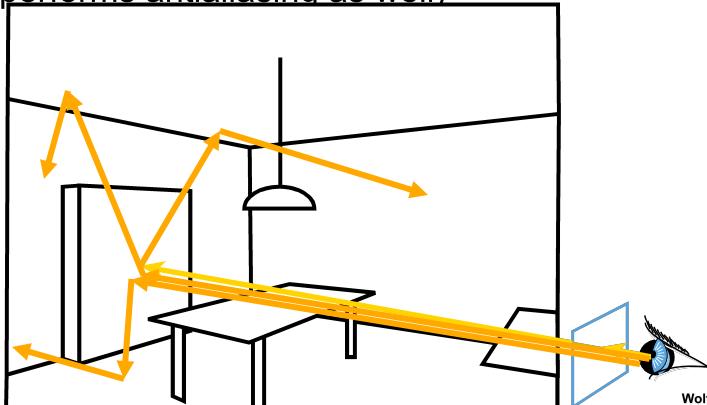
- Do not branch at every intersection point
  - This would have exponential complexity in the ray depth!
- Instead:
  - Shoot some number of primary rays through the pixel (10s-1000s, depending on scene!)
  - For each pixel and each intersection point, make a **single, random** decision in which direction to go next

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## Monte Carlo Path Tracing

- Trace only one secondary ray per recursion
- But send many primary rays per pixel
- (performs antialiasing as well)



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## How to Sample?

### **Simple sampling strategy:**

- At every point, choose between all possible reflection directions with equal probability
- This will produce very high variance/noise if the materials are specular or glossy
- Lots of rays are required to reduce noise!

### **Better strategy: importance sampling**

- Focus rays in areas where most of the reflected light contribution will be found
- For example: if the surface is a mirror, then only light from the mirror direction will contribute!
- Glossy materials: prefer rays near the mirror direction

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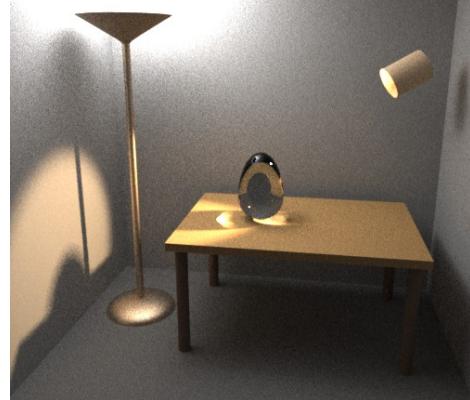
## How to Sample?

- Images by Veach & Guibas

Naive sampling strategy



Multiple importance sampling





## How to Sample?

***Sampling strategies are still an active research area!***

- Recent years have seen drastic advances in performance
- Lots of excellent sampling strategies have been developed in statistics and machine learning
  - *Many are useful for graphics*

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## How to Sample?

### ***Objective:***

- Compute light transport in scenes using stochastic ray tracing
  - *Monte Carlo, Sequential Monte Carlo*
  - *Metropolis*

[Burke, Ghosh, Heidrich '05]  
[Ghosh, Heidrich '06]  
[Ghosh, Doucet, Heidrich '06]





## How to Sample?

- E.g: importance sampling (left) vs. Sequential Monte Carlo (right)



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## More on Global Illumination

***This was a (very) quick overview***

- More details in CPSC 514 (Computer Graphics: Rendering)
- Not offered this year, but in 2008/9

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## Coming Up...

### **Next Week:**

- Global illumination

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