CSC301 HW9

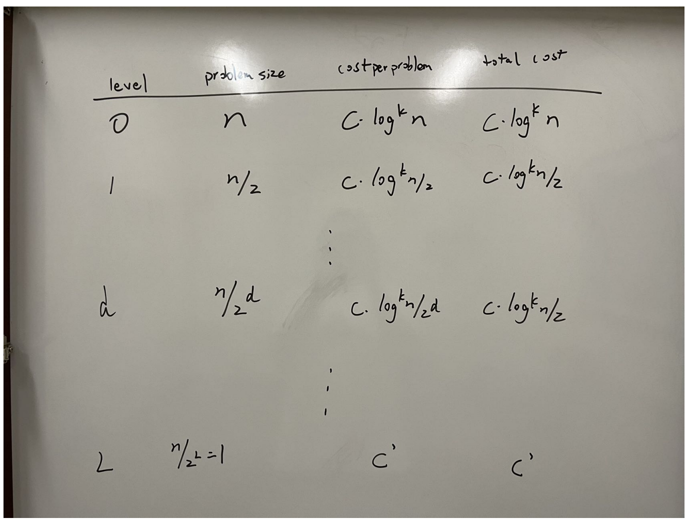
Alex Zhang

April 2023

Question 1

Question 2

Based on the recurrence, I can draw the following recursive table:



From this table, since there are total $L + 1$ levels and $L = \log_2 n$, the total work will be,

$$c \cdot \sum_{d=0}^{\log_2 n - 1} \log^k(n/2^d) + c'$$

doing transformation for each addition part,

$$c \cdot \sum_{d=0}^{\log_2 n - 1} (\log_2 n - \log_2 2^d)^k + c'$$

Case 1: Big-Oh
Since $\log_2 2^d = d$ and $d \geq 0$, $(\log_2 n - \log_2 2^d)^k$ will always smaller than $\log_2^k n$. This shows that

$$\begin{aligned} c \cdot \sum_{d=0}^{\log_2 n-1} (\log_2 n - \log_2 2^d)^k + c' &\leq c \cdot \sum_{d=0}^{\log_2 n-1} (\log_2 n)^k + c' \\ c \cdot \sum_{d=0}^{\log_2 n-1} \log^k(n/2^d) + c' &\leq c \cdot \sum_{d=0}^{\log_2 n-1} (\log_2 n)^k + c' \\ &= c \cdot \log_2 n \cdot (\log_2 n)^k + c' \\ &= c \cdot \log_2^{k+1} n + c' \end{aligned} \tag{1}$$

Because c and c' are both constant, let $g(n) = \log_2^{k+1} n$, $f(n) = \sum_{d=0}^{\log_2 n-1} \log^k(n/2^d) + c'$, if c,$N > 0$

$$f(n) \leq c \cdot g(n)$$

for all $n \geq N$, then

$$f(n) = O(g(n)) = O(\log_2^{k+1} n)$$

Case 2: Big-Omega
Generally speaking, the following inequality,

$$\frac{\sum_{d=0}^{\log_2 n-1} \log_2^k(n/2^d)}{10} \leq \sum_{d=0}^{\log_2 n-1} \log_2^k(n/2^d)$$

holds true.

Question 3