CSC352 HW5

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Question 1

(a)

Since $\mathbf{x} \in \mathbb{R}^m$, let $\mathbf{q_1} = \frac{\mathbf{x_1}}{\|\mathbf{x_1}\|_2}$. Because $\mathbf{x}$ is a vector, it only has one column, the matrix $\mathbf{Q}$ is just $\mathbf{q_1}$. For $\mathbf{R}$, since there is only one columne, $\mathbf{R} = \mathbf{r}_{11} = \|\mathbf{x_1}\|_2$. The QR decomposition will be

$$\mathbf{x} = \frac{\mathbf{x_1}}{\|\mathbf{x_1}\|_2} \cdot \|\mathbf{x_1}\|_2$$

(b)

Given and orthogonal matrix $\mathbf{J} \in \mathbb{R}^{m \times n}$. Let $\mathbf{q}_1 = \frac{\mathbf{j}_1}{\|\mathbf{j}_1\|_2}$, and $\mathbf{q}_2$ is,

$$\mathbf{q}_2 = \frac{\mathbf{j}_2 - (\mathbf{q}_1^\top \mathbf{j}_2)\mathbf{q}_1}{\|\mathbf{j}_2 - (\mathbf{q}_1^\top \mathbf{j}_2)\mathbf{q}_1\|_2}$$

Since $\mathbf{J}$ is an orthogonal matrix, $\mathbf{q}_1 \cdot \mathbf{j}_2 = 0$, indicating

$$\mathbf{q}_2 = \frac{\mathbf{j}_2}{\|\mathbf{j}_2\|_2}$$

This case can be generalized into any column $\mathbf{q}_i$ for matrix $\mathbf{Q}$. For matrix $\mathbf{R}$, $\mathbf{r}_{ij} = \mathbf{q}_i^\top \mathbf{j}_j$. In this case, since $\mathbf{Q}$ is orthonormal matrix to $\mathbf{J}$, $\mathbf{r}_{ij} = 0 \; \forall i,\, j \leq m,\, n$. And for diagonal entries

$$\mathbf{r}_{jj} = \|\mathbf{j}_j - \sum_{i=1}^{i-1} \mathbf{r}_{ij}\mathbf{q}_i\|_2$$

Where $\sum_{i=1}^{i-1} \mathbf{r}_{ij}\mathbf{q}_i\|_2 = 0$ because $\mathbf{r}_{ij} = 0$. Therefore, $\mathbf{r}_{jj} = \|\mathbf{j}_j\|_2$.

Overall, after doing QR decomposition on an orthogonal matrix, we get $\mathbf{Q}$ is an orthonormal matrix and each column is the normal vector from $\mathbf{J}$. $\mathbf{R}$ is a diagonal matrix with each entry represents the 2-norm of the corresponding column vector in $\mathbf{J}$.

orthgonal. If it is orthgonal, Q is normalizing each (general proof needed). and the R is having the norm on main diagonal.

(c)

upper traingular. This will make Q to be a identity matrix and R is just the original matrix (proof needed).

Question 2