CSC301 HW7

Alex Zhang

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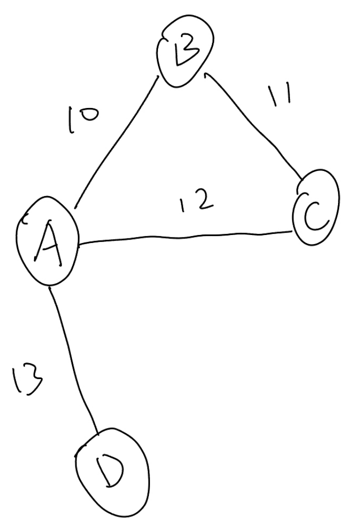
Question 1

(a)

The statement is incorrect by the following counterexample:



The unique heaviest edge 13 is in part of an MST (AB, BC, AD).

(b)

The statement is true. For finding the minimum spanning tree, we have to go through all vertices of the graph. Assume there are two endpoints that have path contains e.

Case 1: No cycle between two endpoints

If there is not cycle among these two endpoints, when we have to go through e because there is only one path that contains e connects these two endpoints. Therefore e is in the MST.

Case 2: A cycle connects two vertices of e

If there exist a path other than a path contains e for two vertices, by property of MST, it will remove the heaviest edge in the cycle and add e instead for creating MST. Because this will create a lighter tree and break cycle given that e is the lightest edge and unique.

This holds true for every MST because they have to go through two vertices have path containing e.

(c)

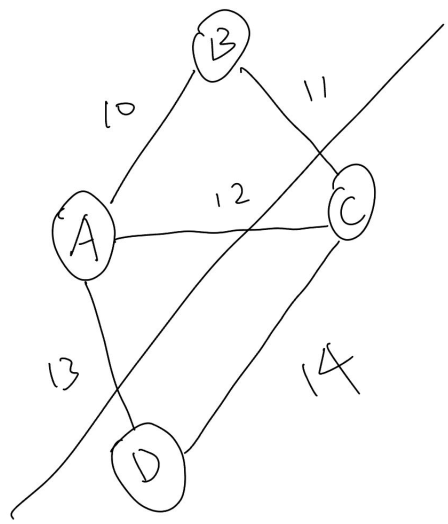
The statement is correct.

Assume a subset of edge X which is part of an MST and not include e. We can use the cut property to partition G. In this case, we will going to find the lightest edge e' crosses the partition. We also know that $X \cup e'$ will also be part of that MST.

For each cut, e' is the lightest edge among all possible edges and after the cut, e' is part of an MST. We can say that if e' is part of an MST, it needs to be the lightest edge for some cut of G. It is also true for edge e as long as it's in some MST.

(d)

The statement is incorrect by the following counterexample:



At first $|V_1|$ is A and B, $|V_2|$ is C and D. E_1 is 10, and E_2 is 14. In this case we find edge e to be 11. With recursion, we will eventually go into base case which for A and B, e will be 10, and for C and D, e will be 14.

Combining all edges in E_c, we find a tree has sum of weights $10 + 11 + 14 = 35$. However, in this graph, the MST has the sum of weights with $10 + 11 + 13 = 34$. This divide-and-conquer algorithm will not produce the correct MST.

Question 2

The pseudocode of my algorithm:

```
Function No111(A)
    n = size(A)
    init array dp of size A
    dp[1] = 2
    dp[2] = 4
    dp[3] = 7
    for i = 4 to n do
```

```
        dp[i] = dp[i-1] + dp[i-2] + dp[i-3]
    end for
    return dp[n];
end function
```

Proof of Correctness

Base cases:

1. 1-bits string: both "0" and "1" satisfy, therefore dp[1] = 2.
2. 2-bits string: "00", "01", "10", and "11" satisfy, so dp[2] = 4.
3. 3-bits string: "000", "001", "010", "011", "100", "101", and "110" satisfy, dp[3] = 7.

Inductive hypothesis:

Assume n-bits string has m1 strings that not include 111, n-1-bits string has m2 strings that not include 111, and n-2-bits string has m3 strings does not include. We need to show for n+1-bits string, it has $(m1 + m2 + m3)$ number of string that does not include 111.

For creating any n+1-bits string, it is eccentially adding 0 or 1 in front of a n-bits string.

Case 1: Adding 0

If we add a 0 in front of any n-bits string that doesn't include 111, the new n+1-bits string will also not have 111. Since the maximum consecutive 1's in that n-bits string is 2. Adding a extra 0 will not create three consecutive 1's.

Because we assume that there are m1 number of n-bits string that satisfy this condition, the number of n+1-bits string will be at least m1.

Case 2: Adding 1

In this case the first bit of n+1-bits string will be 1. And there are two cases.

Case 1: Second bit is 0

In this case, this means the first two bits of n+1-bits string is "10". In order to make n+1-bits strings not have three consecutive 1's, we can add all n-1-bits string that does not include 111 after "10". For this case, all n+1-bits string satisfy this condition will not have three consecutive 1's. There are total m2 number of strings in this case as we asuumed. So currently the overall number for n+1-bits is $m1 + m2$.

Case 2: Second bit is 1

In this case, the first two bits of n+1-bits string will be "11". Since we aim to count the number of n+1-bit strings that does not include 111. The thrid bit has to be 0 in this situation. Thererfore, the first three bits of n+1-bits strings have to be "110". If we could add all n-2-bits string behind "110", it will create n+1-bits string that does not include 111.

We assume that there are total m3 number of n-2-bits strings satisfy this condition. There are also m3 number of n+1-bits strings in this condition.

Overall, sum all numbers of possible n+1-bits strings, we conclude that there are $m1 + m2 + m3$ number of n+1-bits strings that doesn't have substring 111.

In representation of the algorithm, for any bit string of length n where $n \geq 4$, the total number of n-bits string that doesn't have substring 111 will be represented as the sum for bit string of length $n-1$, $n-2$, and $n-3$ that also not include 111.

Cost Analysis

Based on my pseudocode, the initialize of array has linear complexity, and there is a for loop up to length n. In the for loop, the addition also costs linear time. So overall the time complexity will be $O(n + n^2)$, generalized into dominating term, the time complexity will be

$$O(n^2)$$

Question 3

(a)

The file "knapsack.java" used a 2-D array to store the values and weights. The space complexity will be $O(nW)$, and it runs in $O(nW)$.

(b)

The file "knapsackMem.java" used an 1-D array to store the maximumx value. The space complexity will be $O(W)$, and it runs in $O(nW)$.

(c)