CSC352 HW6

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Question 1

Given that $\tilde{x} = x(1+\varepsilon_x)$, and $\tilde{y} = y(1+\varepsilon_y)$, we can simplify the inequality,

$$\begin{aligned}
\left|\frac{xy - \tilde{x}\tilde{y}}{xy}\right| &\leq (2+\varepsilon)\varepsilon \\
\left|\frac{xy - (xy + xy\varepsilon_x + xy\varepsilon_y + xy\varepsilon_x\varepsilon_y)}{xy}\right| &\leq (2+\varepsilon)\varepsilon \\
\left|\frac{-xy\varepsilon_x - xy\varepsilon_y - xy\varepsilon_x\varepsilon_y}{xy}\right| &\leq (2+\varepsilon)\varepsilon \\
|-\varepsilon_x - \varepsilon_y - \varepsilon_x\varepsilon_y| &\leq (2+\varepsilon)\varepsilon \\
|\varepsilon_x + \varepsilon_y + \varepsilon_x\varepsilon_y| &\leq \varepsilon + \varepsilon + \varepsilon^2
\end{aligned}$$

Csae 1: $\varepsilon = \left|\frac{x-\tilde{x}}{x}\right|$
Since $\tilde{x} = x(1+\varepsilon_x)$, $\varepsilon_x = \left|\frac{\tilde{x}-x}{x}\right| = \left|\frac{x-\tilde{x}}{x}\right|$. Indicate $\varepsilon = \varepsilon_x$. Because $\varepsilon_x \geq \varepsilon_y$, $\varepsilon_x^2 \geq \varepsilon_x\varepsilon_y$, we can get that,

$$\varepsilon_x + \varepsilon_x + \varepsilon_x^2 \geq |\varepsilon_x + \varepsilon_y + \varepsilon_x\varepsilon_y|$$

which is the same as the simplified inequality.

Case 2: $\varepsilon = \left|\frac{y-\tilde{y}}{y}\right|$
Without loss of generality, we can apply the same proof on ε_y using ε_x's and it will have the same result.
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Question 2

Question 3