

CSC301 HW4

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Question 1

Proof: $\mathbf{B} = \mathbf{A}\mathbf{A} = \mathbf{A}^2$

If there exists a path of length two between vertex u and w , then $\exists i$ which in adjacency matrix \mathbf{A} , both \mathbf{a}_{ui} and \mathbf{a}_{iw} exists. In order to ensure both \mathbf{a}_{ui} and \mathbf{a}_{iw} exists, their product $\mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$ has to be 1.

Since \mathbf{B} is counting the number of two-paths in the given graph. With given vertex u , and vertex w , \mathbf{B}_{uw} is just the sum of all exist two-paths. In mathematical expression

$$\mathbf{B}_{uw} = \sum_{i=0}^n \mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$$

Also by the definition of matrix multiplication,

$$\mathbf{A}\mathbf{A}_{uw} = \sum_{i=0}^n \mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$$

for all vertex u and w . This implies that, $\mathbf{B} = \mathbf{A}\mathbf{A} = \mathbf{A}^2$. ■

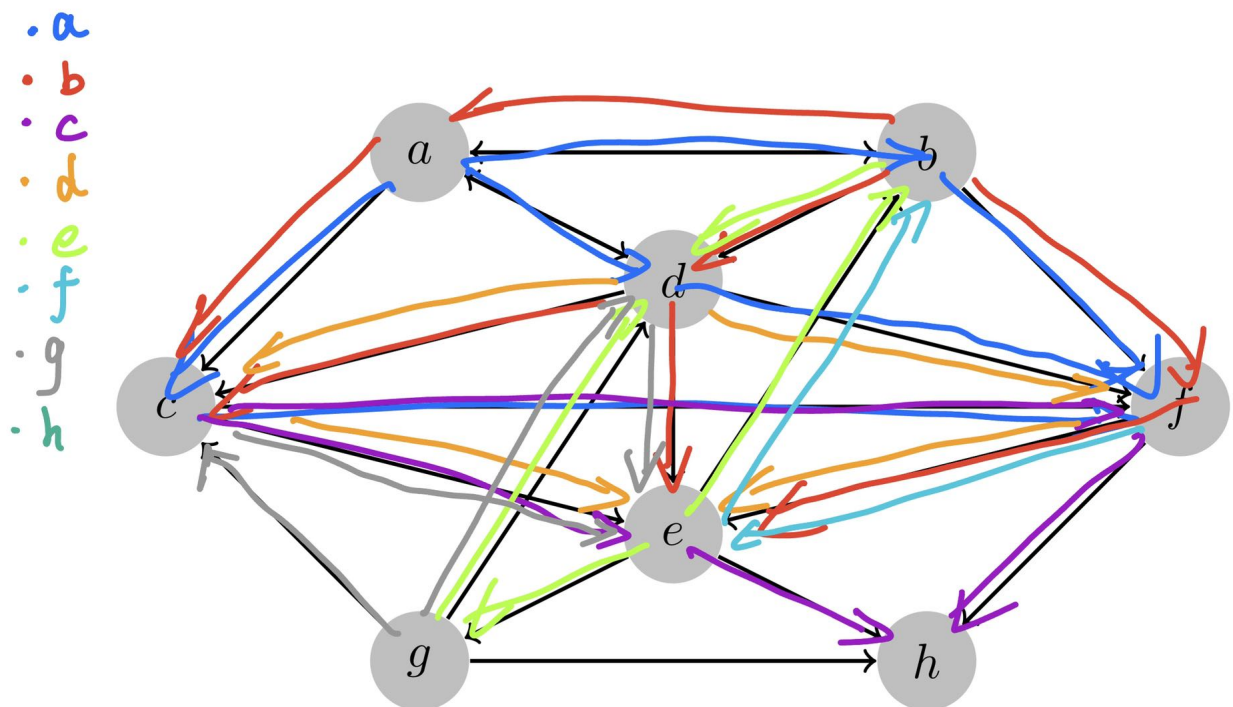
Question 2

In matrixMult.java file, different methods are created. The randMat method is used to random generate a n by n dimension matrix and put it into text file. The matTransform method is used to transform text file matrix into array representation. The multiplication method is matrix multiplication using strassen's method. The print method is just printing matrix in 2-D order. The user can change the value of n in order to perform matrix multiplication on different dimension. There are also two unit test included.

I have added more descriptions of my code through comments in JAVA file.

Question 3

(a)



1. For a , the best recommender should be f since the number of two-paths is 3.
2. For b , the best recommender should be c since it has the most two-paths 2.
3. For c , the best recommender should be h since it has 2 two-paths which is the most
4. For d , the best recommender should be h . b and e also have the same number of two-paths, but they have already connected to d which we should recommend h .
5. For e , the best recommender should be d since it has the most number of two-paths of 2.
6. For f , the best recommender should be b . Because b , e , and f all have same two-paths, by alphabetical order, we should choose b .
7. For g , the best recommender should be e since e and f have the same two-paths and by alphabetical order we choose e .
8. For h , the best recommender should be a . Because for any letter the number of two-paths is 0 which lead us to choose a .

(b)

Based on the graph, we can create an adjacency matrix

$$\mathbf{A} = \begin{bmatrix} & a & b & c & d & e & f & g & h \\ a & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ c & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ e & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ f & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ g & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(c)

After doing the computation in Task(2), I got the following matrix \mathbf{B} ,

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 2 & 1 & 1 & 2 \\ 1 & 0 & 1 & 2 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(d)

Based on the \mathbf{B} we have calculated, we can just look at the entries to determine best recommender.

1. For a , recommender should be f .
2. For b , recommender should be c .
3. For c , recommender should be h .
4. For d , recommender should be h .
5. For e , recommender should be d .
6. For f , recommender should be b .
7. For g , recommender should be e .
8. For h , recommender should be a .

I think based on the matrix \mathbf{B} and the ties rule, the recommendation from \mathbf{B} matches the recommendation from Task(3a).