CSC352 HW7

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Question 1

1. The relative error for $\mathbf{Q}$ is 2.
2. The relative error for $\mathbf{R}$ is 1.036.
3. The relative error for $\mathbf{Q} * \mathbf{R}$ is $1.1194e-15$.

I'm surprised with the first two relative errors. For $\mathbf{Q}$'s relative error, it should be 0 ideally, but I got 2, which shows there is a difference between true $\mathbf{Q}$, and calculated $\mathbf{Q}$. For $\mathbf{R}$, I think it still should be 0 for $\|0\|_p = 0$. Base on the two relative errors, I think for HouseHolder QR, $\mathbf{Q}$ and $\mathbf{R}$ are not accurate. However, the relative error for $\mathbf{Q} * \mathbf{R}$ is really small so their product is accurate. Based on this small relative error, we can also conclude that QR factorization using HouseHolder is stable.

Question 2

1. For QR factorization with HouseHolder, the distance is $8.6905e-16$.
2. For QR factorization with modified Gram-Schmidt, the distance is 1.

The distance using HouseHolder is very small and therefore reasonable. However, the result for using mgs is quiet big. One reason is when doing Gram-Schmidt process, calculating matrix $\mathbf{Q}$ involves multiplication and normalization. This process will make $\mathbf{Q}$ not be strictly orthonormal matrix, and the result will be affected then.

Changing the value η, orthongonality value of $\mathbf{Q}$ computed by mgs will still be 1. The distance of $\mathbf{Q}$ through HouseHolder will sometimes only vary 10^{-16}.

Question 3

Given that $\mathbf{L}$ and $\mathbf{B}$ are $\in \mathbb{R}^{n\times n}$, we write a pseudocode function called LB:

```
function X = LB(L,B)
    for i = 1: length(B)
        for j = 1: length(B)
            x(i,j) = (B(i,j)-∑_{k=1}^{j-1}L(j,k)*x(i,k))/L(j,j)
        end
    end
end fucntion
```

It eccentially adds a outer loop that helps store calculated $\mathbf{x}$ vector in each column.