

CSC301 HW4

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Question 1

Proof: $\mathbf{B} = \mathbf{A}\mathbf{A} = \mathbf{A}^2$

If there exists a path of length two between vertex u and w , then $\exists i$ which in adjacency matrix \mathbf{A} , both \mathbf{a}_{ui} and \mathbf{a}_{iw} exists. In order to ensure both \mathbf{a}_{ui} and \mathbf{a}_{iw} exists, their product $\mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$ has to be 1.

Since \mathbf{B} is counting the number of two-paths in the given graph. With given vertex u , and vertex w , \mathbf{B}_{uw} is just the sum of all exist two-paths. In mathematical expression

$$\mathbf{B}_{uw} = \sum_{i=0}^n \mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$$

Also by the definition of matrix multiplication,

$$\mathbf{A}\mathbf{A}_{uw} = \sum_{i=0}^n \mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$$

for all vertex u and w . This implies that, $\mathbf{B} = \mathbf{A}\mathbf{A} = \mathbf{A}^2$. ■

Question 2

Question 3

(a)

1. For a, the best recommend should be