CSC301 HW4

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Question 1

Proof: $\mathbf{B} = \mathbf{A}\mathbf{A} = \mathbf{A}^2$

If there exists a path of length two between vertex u and w, then $\exists\ i$ which in adjacency matrix $\mathbf{A}$, both $\mathbf{a}_{ui}$ and $\mathbf{a}_{iw}$ exists. In order to ensure both $\mathbf{a}_{ui}$ and $\mathbf{a}_{iw}$ exists, their product $\mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$ has to be 1.

Since $\mathbf{B}$ is counting the number of two-paths in the given graph. With given vertex u, and vertex w, $\mathbf{B}_{uw}$ is just the sum of all exist two-paths. In mathematical expression

$$\mathbf{B}_{uw} = \sum_{i=0}^{n} \mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$$

Also by the definition of matrix multiplication,

$$\mathbf{A}\mathbf{A}_{uw} = \sum_{i=0}^{n} \mathbf{a}_{ui} \cdot \mathbf{a}_{iw}$$

for all vertex u and w. This implies that, $\mathbf{B} = \mathbf{A}\mathbf{A} = \mathbf{A}^2$.$\blacksquare$

Question 2