CSC352 HW8

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Question 2

I created a file "my_chol.m" using recursive function.

Question 3

Since $\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, then $\mathbf{A} - \lambda \cdot \mathbf{I} = \begin{bmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$ and the corresponding characteristis polynomial will be,

$$\begin{aligned} \det(\mathbf{A} - \lambda \cdot \mathbf{I}) &= (2-\lambda)(2-\lambda)(2-\lambda) - (2-\lambda)(-1)(-1) - (-1)(-1)(2-\lambda) + 0 + 0 - 0 \\ &= (2-\lambda)^3 - (2-\lambda) - (2-\lambda) \\ &= (2-\lambda)^3 - 2 \cdot (2-\lambda) \\ &= -\lambda^3 + 6\lambda^2 - 10\lambda + 4 \end{aligned}$$

Let it be 0,

$$\begin{aligned} -\lambda^3 + 6\lambda^2 - 10\lambda + 4 &= 0 \\ -(\lambda - 2)(\lambda^2 - 4\lambda + 2) &= 0 \end{aligned}$$

We can get that $\lambda_1 = 2 + \sqrt{2}$, $\lambda_2 = 2$, and $\lambda_3 = 2 - \sqrt{2}$. Given that all eigenvalues are positive and $\mathbf{A}$ is symmetric, $\mathbf{A}$ is a symmetric positive definite matrix.

Question 4

The pseudocode will be:

```
function x = tri_mat(M,y)
    n = size(M,1)
    x = zeros(n,1)
    for i = 2 to n
        l = a_i/b_{i-1}
        b_i = b_i - l · c_{i-1}
        y_i = y_i - l · y_{i-1}
    end for
    x_n = y_n/b_n
    for j = n-1 to 1
```

$x_j = (y_j - c_j \cdot x_{j+1})/b_j$
```
    end for
end fucntion
```

The algorithm will first perform gaussian elimination, thus creating a new upper triangular matrix. After that uses back substituion to solve the linear system. In first loop, each iteration does 5 flops and there are total $n-1$ iterations, so the total flops will be $5n-5$. The second loop contains 3 flops in each iteration and the total flops will be $3n-3$. The time complexity for this algorithm will be $O(8n-8) = O(n)$.