CSC352 HW8

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April 2023

Question

Question 2

Since $\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, then $\mathbf{A} - \lambda \cdot \mathbf{I} = \begin{bmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$ and the corresponding characteristis polynomial will be,

$$\begin{aligned} \det(\mathbf{A} - \lambda \cdot \mathbf{I}) &= (2-\lambda)(2-\lambda)(2-\lambda) - (2-\lambda)(-1)(-1) - (-1)(-1)(2-\lambda) + 0 + 0 - 0 \\ &= (2-\lambda)^3 - (2-\lambda) - (2-\lambda) \\ &= (2-\lambda)^3 - 2 \cdot (2-\lambda) \\ &= -\lambda^3 + 6\lambda^2 - 10\lambda + 4 \end{aligned}$$

Let it be 0,

$$\begin{aligned} -\lambda^3 + 6\lambda^2 - 10\lambda + 4 &= 0 \\ -(\lambda - 2)(\lambda^2 - 4\lambda + 2) &= 0 \end{aligned}$$

We can get that $\lambda_1 = 2 + \sqrt{2}$, $\lambda_2 = 2$, and $\lambda_3 = 2 - \sqrt{2}$. Given that all eigenvalues are positive and $\mathbf{A}$ is symmetric, $\mathbf{A}$ is a symmetric positive definite matrix.

Question