CSC355 PS3

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Problem 1

(a)

The gradient of $f(x, y)$ will be

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$\frac{\partial f}{\partial x} = -c(4x(y - x^2)) - 2(1 - x)$, $\frac{\partial f}{\partial y} = 2c(y - x^2) + 0$

$$\nabla f = \begin{bmatrix} -4c(x(y - x^2)) - 2(1 - x) \\ 2c(y - x^2) \end{bmatrix}$$

The Hessian of f will be

$$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x \partial x} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y \partial y} \end{bmatrix}$$

where $\frac{\partial^2 f}{\partial x \partial x} = -c(4y - 12x^2) + 2$, $\frac{\partial^2 f}{\partial y \partial x} = -4cx$, and $\frac{\partial^2 f}{\partial y \partial y} = 2c$

$$\nabla^2 f = \begin{bmatrix} -c(4y - 12x^2) + 2 & -4cx \\ -4cx & 2c \end{bmatrix}$$

(b)

Problem 2

(b)

When $c = 1$, the output is,

```
>> [X,FVAL,EXITFLAG,OUTPUT,GRAD] = fminunc(@(x) myfun(x,1),[.1;.1],options);
                                                        First-order
 Iteration  Func-count       f(x)        Step-size       optimality
     0           1           0.8181                          1.84
     1           3         0.255493       0.272171          0.617
     2           4         0.172388              1          0.491
     3           5        0.0215888              1          0.208
     4           6        0.0111538              1          0.449
     5           7      0.000208418              1         0.0618
```

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     6           8      1.69733e-06              1        0.00284
     7           9      5.79977e-09              1       0.000145
     8          10      6.11888e-12              1       9.22e-06
     9          11      1.81422e-16              1       5.51e-08

Local minimum found.

Optimization completed because the size of the gradient is less than
the value of the optimality tolerance.
```

When $c = 1000$, the output is,

```
   >> [X,FVAL,EXITFLAG,OUTPUT,GRAD] = fminunc(@(x) myfun(x,1000),[.1;.1],options);
                                                        First-order
 Iteration  Func-count       f(x)        Step-size       optimality
     0           1             8.91                          180
     1           3            0.987    0.000555556           29.3
     2           4         0.777219              1           1.75
     3           6         0.764588             10           2.68
     4           7         0.735046              1           15.4
     5           8         0.702228              1           7.45
     6           9         0.650868              1           2.95
     7          11         0.578572       0.618962           10.8
     8          12          0.53324              1           10.8
     9          13         0.448368              1            5.3
    10          15         0.385863       0.367553             10
    11          16         0.364507              1           4.81
    12          17         0.305381              1           10.3
    13          18         0.263112              1           3.55
    14          20         0.235968       0.480082           8.98
    15          21         0.199458              1           6.02
    16          22         0.162378              1           2.85
    17          24         0.152655        0.32329           8.84
    18          25         0.113748              1           8.99
    19          26         0.104463              1           9.91
                                                        First-order
 Iteration  Func-count       f(x)        Step-size       optimality
    20          27        0.0723461              1           8.18
    21          28        0.0666479              1            1.4
    22          29        0.0526213              1           2.72
    23          31        0.0447515       0.435248           7.91
    24          32        0.0343863              1           4.77
    25          33        0.0245107              1           5.56
    26          34        0.0180499              1           6.58
    27          36       0.00815378       0.416144           3.51
    28          37       0.00702843              1            2.8
    29          39       0.00484113       0.645221           4.64
    30          40       0.00213087              1           3.66
    31          42      0.000347732       0.486442          0.821
    32          43      0.000253146              1           1.12
    33          44      4.19498e-05              1          0.431
    34          45      5.90551e-06              1          0.103
```

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    35          46      4.22069e-07              1         0.0797
    36          47      1.24377e-07              1         0.0436
    37          48      1.60385e-12              1       3.36e-05

Local minimum found.

Optimization completed because the size of the gradient is less than
the value of the optimality tolerance.
```

(c)

i. fminunc uses 'quasi-newton' method to find local minimum

ii. When switching from 1 to 1000, fminunc seems to do one extra run because it fails to find a local minimum at first.

iii. It stops when the maximum absolute value in gradient is smaller than the tolerance($1e-06$).