CSC352 HW9

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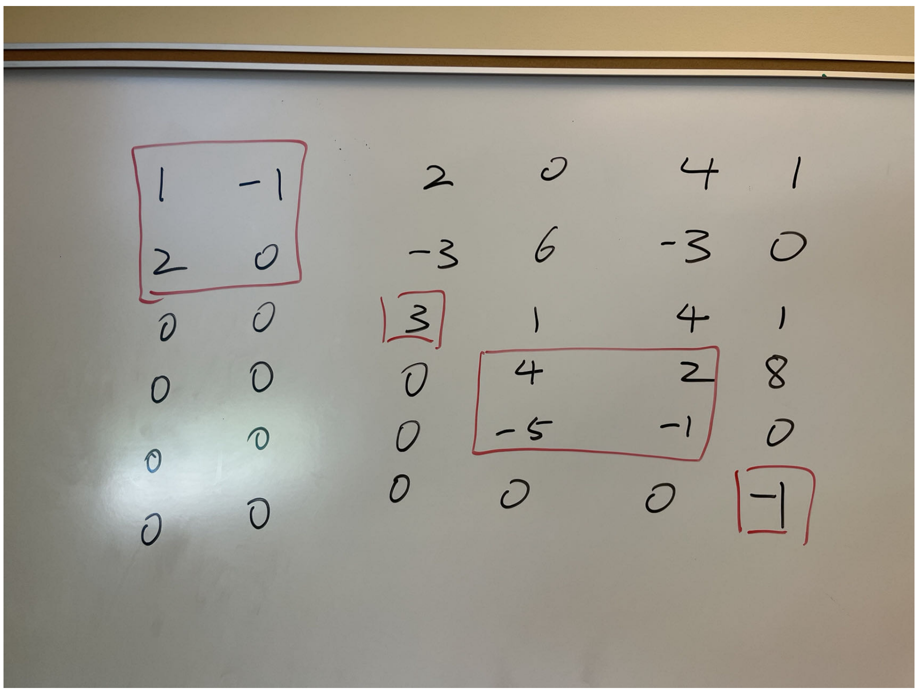
Question 1

Based on obvervation, I find out matrix $\mathbf{A}$ is a quasi-upper triangular matrix.



which I can do real schur decomposition on $\mathbf{A}$ which $\mathbf{A} = \mathbf{IAI}^{\top}$. $\mathbf{I}$ is identity matrix with same dimension of $\mathbf{A}$, and they are orthorgonal matrix. Therefore we can first get two eigenvalues 3 and -1. Then I will calculate the eigenvalues for rest two 2 by 2 matrices. For matrix $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, eigenvalues will be

$$1/2 \pm \sqrt{0.25 - 2}$$

$$1/2 \pm \sqrt{7}/2i$$

The eigenvalues for matrix $\begin{bmatrix} 4 & 2 \\ -5 & -1 \end{bmatrix}$ will be:

$$3/2 \pm \sqrt{2.25 - 6}$$

$$3/2 \pm \sqrt{15}/2i$$

Overll, the eigenvalues of this matrix will be 3, -1, $\frac{1}{2} \pm \frac{\sqrt{7}i}{2}$, and $\frac{3}{2} \pm \frac{\sqrt{15}i}{2}$.

Question 2

The pseudocode for Golub Kahan bidiagonalization will is below:

```
function B = GK_bidiagonalization(A)
    [m,n] = size(A)
    for j = 1:n
        x = A(j:m, j)
        u = x + norm(x) * e1
        u = u / norm(u)
        A(j:m,j:n) = A(j:m,j:n) - 2 * u * (u' * A(j:m,j:n))
        if j < n-1
            x = A(j, j+1:m)
            v = x + norm(x) * e1
            v = v / norm(v)
            A(j:m, j+1:n) = A(j:m, j+1:n) - 2 * (A(j:m, j+1:n) * v) * v'
        end if
    B = A
end function
```

Question 4

(a)

After doing SVD on **A**, the first singular value is 156.4358 and the second one is 8.7658. I think becuase the largest singular value is very large compared to the second. There will be one principal component that relates to the first singular value.

(b)

The rank-one approximation for matrix **A** will be,

$$A_1 = \begin{bmatrix} 46.7021 & 15.8762 \\ 94.0315 & 31.9657 \\ 52.0806 & 17.7046 \\ 43.3857 & 14.7488 \\ 68.2871 & 23.2139 \\ 40.6964 & 13.8346 \end{bmatrix}$$

The relative error is 0.056. Based on this number, I think the approximation is good or at least not bad.

(c)

I created two bar charts and find out that for height column, the approximated data is closer to true data with greatest difference by 1.7. For weight column, the approximated data is not so accurate compared to height's data. It has maximum difference by 4.75.

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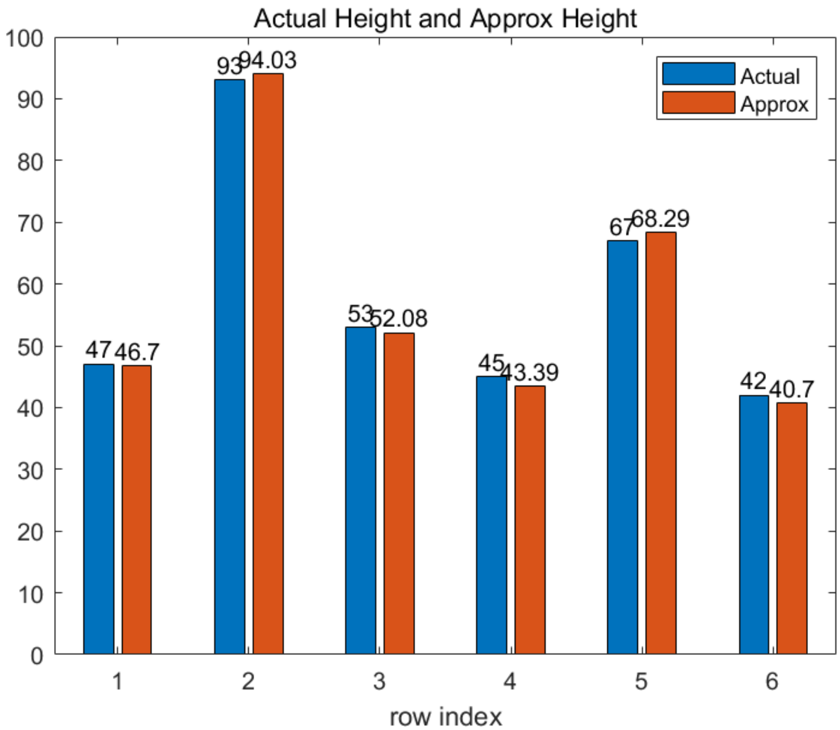

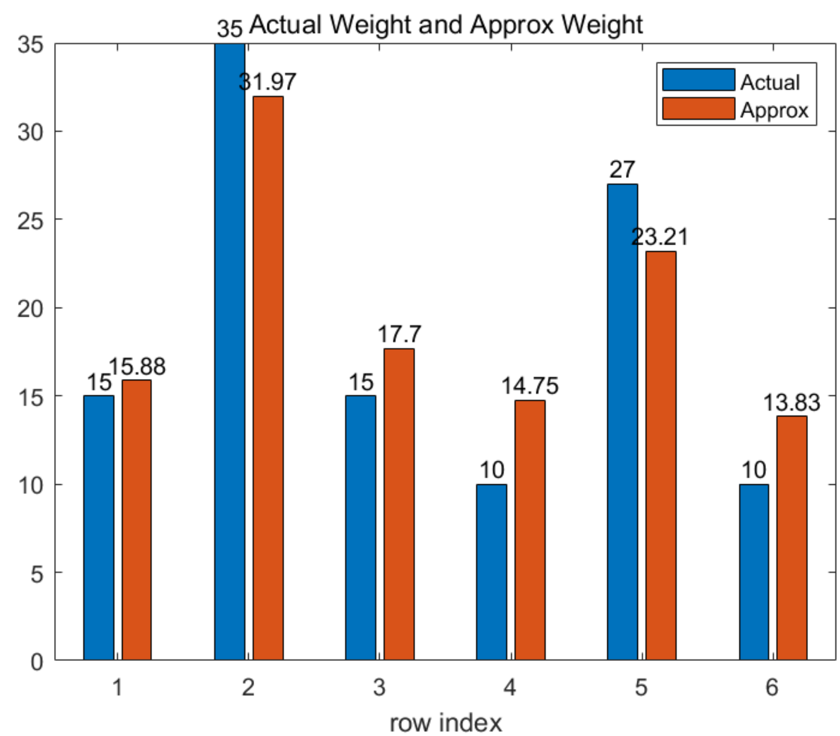

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$$z = v' * A(:, J)$$

$$z =$$

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